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To the Reader

The reader's attention is called to the Glossary, printed at the end of Volume II. Schillinger sometimes uses conventional terms in special senses. It will facilitate the study of certain passages if the student bears in mind that explanations are available there. It is felt that no table of abbreviations is needed since the significance of each symbol (which sometimes recurs in varying senses in different parts of the book) is always made clear in its context.

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK I

THEORY OF RHYTHM

PRELIMINARY REMARKS ON THE THEORY OF RHYTHM

The Theory of Rhythm is the foundation of Schillinger's system. But for him, rhythm is not simply a matter of time-rhythm, which is what is ordinarily meant by the term. Schillinger begins by applying rhythm to time durations, and then extends it to all other phases of composition—the way in which block-harmonies change, intervals in scales and melody, entrances of counterthemes in counterpoint, distribution of parts through a score, and other processes of composition. Schillinger's statements are clear provided the reader takes the trouble to work them out, rather than merely read them. It must be borne in mind at this stage that the individual processes worked out in this book are *all to be used* in the actual composition of music.

The Schillinger System of Musical Composition has the integrated construction of a closely reasoned work of science or mathematics. Beginning with Book I, *Theory of Rhythm*, Schillinger successively presents techniques relating to the various phases of composition. Book II develops the *Theory of Pitch Scales*; Book IV, *Melody*; Book V, *Harmony*; Book VI, *Correlation of Melody and Harmony*; Book VII, *Counterpoint*; etc.

Mastery of the materials of any one of these books will provide the student with undreamed-of new resources. However, the Schillinger System places its emphasis on *composition*, that is, on the procedure for integrating elements and structures, and not on the detached and uncoordinated techniques. The method for integrating the individual techniques is presented in Book XI, *Theory of Composition*, which is the crowning summit of this work, as the Theory of Rhythm is its foundation.

It should be emphasized that study of the Theory of Rhythm is the prerequisite to any real understanding of the entire work. Each of the succeeding books employs devices initially presented in the Theory of Rhythm, so that the student who skips ahead in an effort to cover ground quickly will find it necessary to retrace his steps. Thereafter, each book in turn requires a thorough understanding of preceding books.

Readers who are interested in knowing how Schillinger came to devise the system of notation he employs are referred to Chapters 1 and 2 of Book IV *Theory of Melody*. In the first chapter Schillinger presents an engrossing analysis of the physical components of music. In the second chapter he traces the history of musical notation and demonstrates the inadequacy which caused him to search for a new and more exact system of notation. Both these chapters contain insights which will assist the reader in understanding details of the Schillinger system. (Ed.)

BOOK I

THEORY OF RHYTHM

CHAPTER I

NOTATION SYSTEM

THE CUSTOMARY method of musical notation, which is a product of the "trial and error" method, is inadequate for the analysis and study of rhythmic patterns. It offers no common basis for computations. The history of creative experience in music shows that even the greatest composers have been unnecessarily limited in their rhythmic patterns because they thought in terms of ordinary musical notation.*

The arrangement of time-durations, which constitutes the theory of rhythm, may be studied through three parallel systems of notation: (1) numbers, (2) graphs, (3) musical notes.

Understanding the nature of these group formations helps us to compose, to arrange any given musical material, and to play the most involved rhythmic patterns.

Number values will be used in this system in their normal mathematical operations (such as the four actions—addition, subtraction, multiplication, and division—, raising to powers, extracting roots, permutations, etc.)**

A. Graphing Music

The graph method used in this system is the general method of graphs, *i.e.*, a record of variation of special components, such as pitch or intensity in music, stocks in finance , diseases in medicine, etc., during a given time-period. In our theory of rhythm we shall deal with time only. The horizontal coordinate (known as *abscissa*) reads always from left to right. Here it will express *time*. The vertical coordinate (known as *ordinate*) will express the recurrence of a phase, *i.e.*, the moment of attack. This graph method is a general method and therefore objective.

Any wave motion records itself automatically. Let the pendulum of a clock swing uniformly over a strip of paper while the latter is being moved uniformly—and in a direction perpendicular to the movements of the pendulum itself.

Such record will have approximately this appearance:

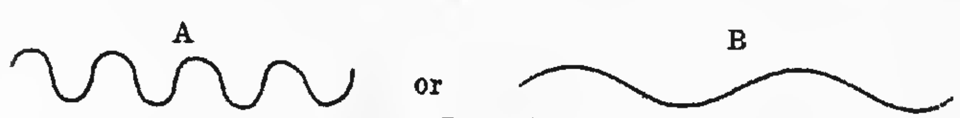


Figure 1.

*If, from experience outside the musical field, you already know how graphs are used, it will be sufficient to say at this point that (a) music can be graphed by allowing the lengths of a number of horizontal lines to stand for the *durations* of tones, and causiog the distance up or down (ordinate) to stand for the *pitch* levels of the tones; and (b) when graphing duration only, as in these studies, the end of one duration and beginning of the next may be indicated by a 'turn' (phase change) in the line, as shown in Figure 47. (Ed.)

**Although Schillinger makes much use of mathematics in this work, the reader is not presumed to be a student of mathematics. Each mathematical operation is carefully explained so that those who possess the most elementary knowledge of mathematics will not encounter difficulty either in understanding the text or in performing the necessary operations. (Ed.)

depending on the speed with which the strip of paper is moving. In case A (see Figure 1) the speed is less than in case B.

Similar configurations of curves of different degrees of complexity may be observed in the projected oscillograms of sound waves. The complexity of a wave depends upon the number of components in such a wave. The simplest wave has the form which is shown in Figure 1. All clock mechanisms produce such waves (pendulum, sewing machine, etc.). In frequencies which produce musical pitch, the simplest wave may be found in the sound of tuning forks and of the flute-stops of a pipe organ.

The general form of the analysis of wave-motion is the Fourier method which Fourier developed in 1822 for the purpose of analyzing *heat*-waves. This method is very precise. It is used in all fields dealing with oscillatory phenomena. Yet it is a very complicated method to use for the purpose of analyzing the music of human performers. It takes about twelve hours to analyze a wave of thirty components. Machines known as harmonic analyzers have been devised. These machines perform the work of an expert mathematician in about ten minutes without any possibility of error. They are used in various fields of physics and in meteorological departments, mainly to predict tidal variations.

The simplest (i.e., one-component) wave of *one period* (recurrence group) has this appearance:

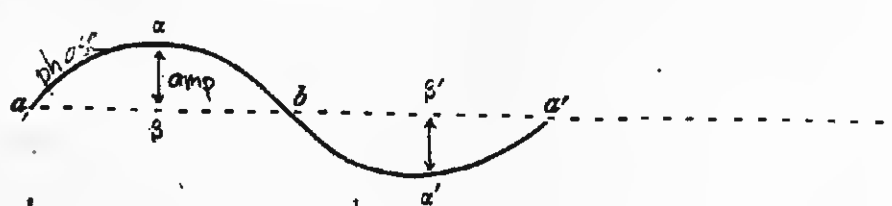


Figure 2.

The distances, a α b and b α' a', are equal. These curves are *phases* of the wave. Two phases constitute a period. For the purpose of studying periodic groups and their recurrences, we shall use *phases* as units of measurement. In continuous sequence they constitute the *periodicity of phases*.

The distances, αβ and α' β', are equal, and constitute *amplitudes*. The latter are physical expressions of *intensity*.

We shall consider intensity in the study of durations in reference to accents only. The coincidence of phases of two different periodicities intensifies the attack. The recurrence of intensified attacks ("accents") will constitute musical measures ("bars"). The reality of "bars" depends actually on the placement of attacks, not on the placement of bar lines on music paper.

By assuming that the arrangement of durations does not necessitate the expression of amplitudes, we shall use rhythm graphs in the following form:

Figure 3.

Here the horizontal lines are a simplification of the general curve; they express time. The vertical segments express the moment of attack. In the following graphs the forms of attack will be constant, and the time durations will assume various values.

B. Forms of Periodicity

Continuous recurrence of a group constitutes *periodicity*. Periodicity in which all groups are identical constitutes *uniform periodicity*. The difference between various forms of uniform periodicity may be distinguished by the number of *terms* (places) in a recurring group.

Groups with one term (a *monom*) constitute *monomial periodicity*.

The algebraic expression for monomial periodicity is:

$$at_1 + at_2 + at_3 + \ . \ . \ . \ + at_n$$

where a is the recurring monom and where t_1, t_2 . . . are the consecutive time moments; a may assume different values. In the field of musical time durations these values are integers; a may equal 1, 2, 3, . . . n.

When the forms of such periodicities are expressed in number-values, they have this appearance:

$$\begin{array}{l} 1 + 1 + 1 + 1 + \ . \ . \ . \ . \\ 2 + 2 + 2 + 2 + \ . \ . \ . \ . \\ 3 + 3 + 3 + \ . \ . \ . \ . \ . \ . \\ n + n + n + \ . \ . \ . \ . \ . \ . \end{array}$$

Their graph expression is –

Figure 4.

—where each rectilinear segment represents a time-unit expressed in some space unit (inches, centimeters, etc.).

When a unit is defined, the respective values of units in different monomial periodicities will be:

1 + 1 + 1 + . . .
2 + 2 + 2 + . . .
3 + 3 + 3 + . . .

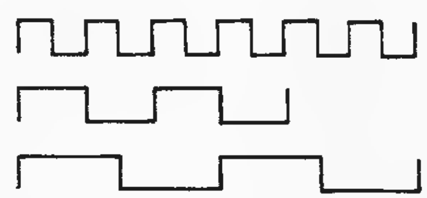


Figure 5.

Musical notation will serve as a *final* form into which number and graph expressions will be translated.

Thus, if 1 represents ♩ (or 1 = ♩), 2 = 𝅗𝅥 , 3 = 𝅗𝅥. , 4 = 𝅝 , etc.

CHAPTER 2

INTERFERENCES OF PERIODICITIES

WE ARE now concerned with what may technically be called the "generation of resultant rhythmic groups as produced by the interference of two synchronized monomial periodicities"—that is to say, the way in which one monomial periodicity (say, 3, 3, 3, 3) may be combined with another (say, 4, 4, 4, 4) so as to produce still another rhythm.

A periodicity consisting of *greater* number values will be denoted by the term, "major generator"; the smaller of the two will be called, "minor generator." The way in which we will express two synchronized generators producing one interference-group is a ÷ b.* The expression for the *resultant* of interference is $r_{a \div b}$.

A. Binary Synchronization

To synchronize two monomial periodicities it is necessary:

(1) to find the *common product* or common *denominator* (c.p. or c.d.)

(2) to find *complementary factors* of both generators; the complementary factor of *a* is $\frac{ab}{a} = b$, and the complementary factor of *b* is $\frac{ab}{b} = a$.

After this is completed, it is necessary to draw a graph of both generators in their synchronization. To find the resultant (r), drop perpendiculars from all points of attack on both generators. The resultant is discovered by drawing lines through these points. The common product is then added to the diagram, and the number-values of the resultant are indicated. The entire diagram is then translated into musical notation.

When *a* equals *any number-value,* and *b* equals one, the resultant expresses a musical "bar," whether or not this bar-line would actually be drawn on music paper. Thus, a formula for a musical bar (or measure) is:

$$T = r_{a \div 1}$$

(*read:* musical bar (T) is the resultant of *a* to *one.*)

First Case

2 ÷ 1

Find the resultant, $r_{2 \div 1}$

Common product (c.p.) $2 \times 1 = 2$

Complementary factor of *a* $\frac{2}{2} = 1$ 1(2)

Complementary factor of *b* $\frac{2}{1} = 2$ 2(1)

a consists of two's

b consists of one's

*Although Schillinger here and elsewhere uses the division sign to indicate the a and b relationship (a ÷ b), it should be noted that Schillinger on other occasions uses a *colon* in place of the division sign (a:b).

Neither the colon nor the division sign is employed as in ordinary arithmetic. 4:3 means, in ordinary arithmetic, 4 divided by 3 or 4/3—and 4 ÷ by 3 means the same. In Schillinger's use of these signs, neither a ratio nor division is meant. He meant *interference.* In arithmetic, 4:3 = 1⅓. In Schillinger, 4:3 = $\frac{3+1+2+2+1+3}{12} = \frac{12}{12} = 1$. (Ed.)

Here is the operation expressed in numbers, in graph, and in musical notation:

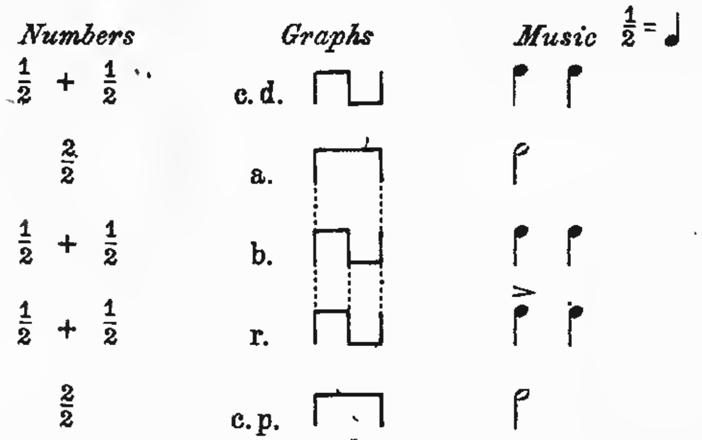


Figure 6.

The resultant differs from *b* with respect to accent (which results from the coincidence of attacks of both generators).

Musically, the *first case* establishes a bar in which the musical numerator is 2, i.e., $\frac{2}{2}$ (¢), $\frac{2}{4}$, $\frac{2}{8}$. When the bar is $\frac{2}{2}$, $\frac{1}{2}$ = 𝅗𝅥 ; when the bar is $\frac{2}{4}$, $\frac{1}{2}$ = ♩ when the bar is $\frac{2}{8}$, $\frac{1}{2}$ = ♪

Second Case

3 ÷ 1

Find the resultant, $r_{3 \div 1}$.

$3 \times 1 = 3$

1 (3)

3 (1)

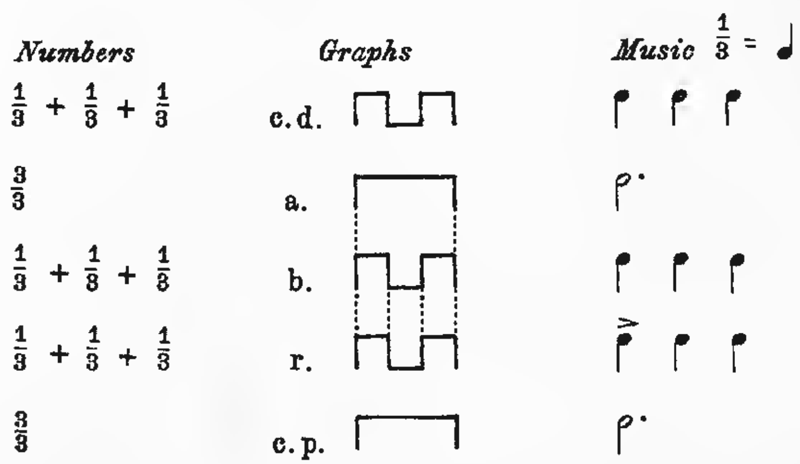


Figure 7

In this case, using *a* and *b* we hear the resultant, i.e., three uniform durations, with the accent on the first. This produces all bars with musical numerator three, i. e., $\frac{3}{2}$, $\frac{3}{4}$, $\frac{3}{8}$.

Figure 8.

Third Case

$4 \div 1$

Find the resultant, $r_{4 \div 1}$

4×1

1 (4)

4 (1)

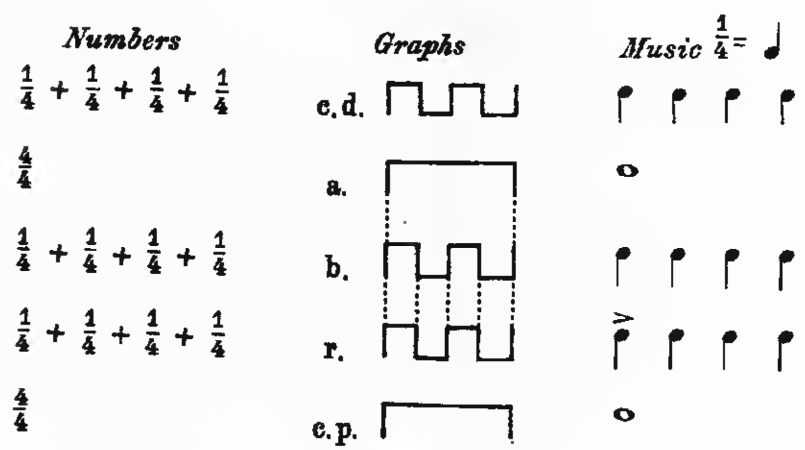


Figure 9.

The importance of this procedure lies in the fact that even the most noted composers of today do not seem to know that to express a bar before a non-uniform group is offered *is to represent the scheme of uniformity with respect to the periodicity of accents.* This means that an accent should not be forced but should result from superimposition of *a* on *b*.

When it comes to the application of higher numerators—such as 5, 7, or 11—the entire music becomes incomprehensible to the average listener, and the composer is the one to blame. When it comes to the shifting of accents which are not correctly expressed (i.e., through the use of *a* and *b*), the performance is never adequate; the performer suffers (for example, hear Stokowski in Stravinsky's *Rites of Spring*), and the listener wonders what it is all about.

Non-uniform rhythmic resultants occur when b ≠ 1. Through the procedure described above, one may obtain *all* the rhythmic patterns of the past, present and future, including all the possible rhythms of the Orient or of the primitives.

$3 \div 2$

Find the resultant, $r_{3 \div 2}$

$3 \times 2 = 6$

2 (3)

3 (2)

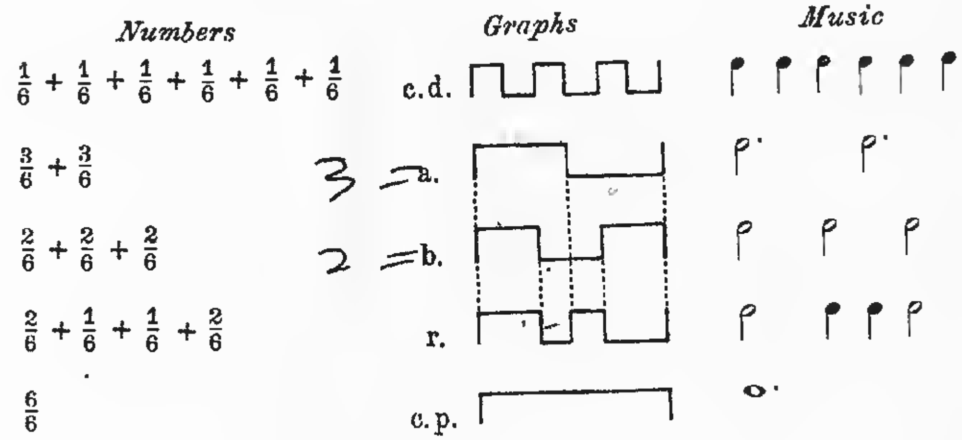


Figure 10.

B. Grouping

Three forms of grouping are available.*

(1) **Grouping by c.p.** In this case, c.p. = 6, which may express musical quarters or eighths.

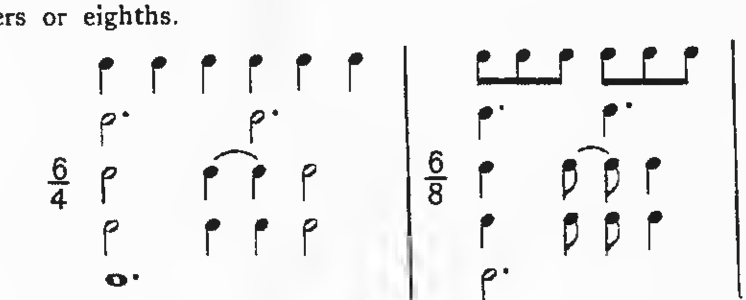


Figure 11.

Six may also express six units in $\frac{3}{2}$ or $\frac{3}{4}$ time, then:

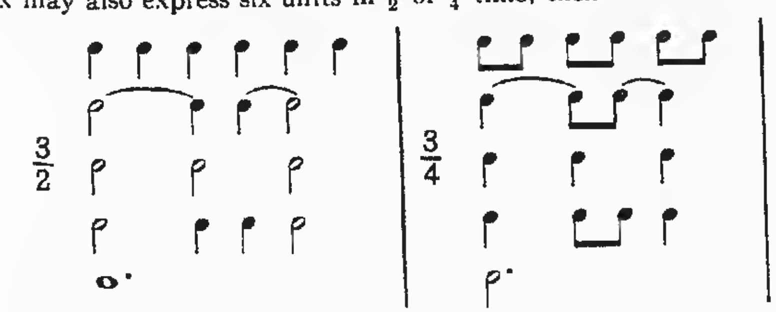


Figure 12.

*The technique and theory of grouping are described in detail in Chapter 3. (Ed.)

(2) **Superimposition of a.** a = 3. In order to get the reality of such superimposition, c.p. must be excluded and b becomes merely an optional component.

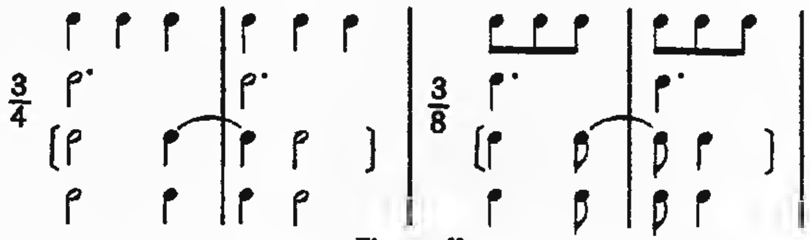


Figure 13.

(3) **Superimposition of b.** b = 2; c.p. is excluded; a becomes optional.

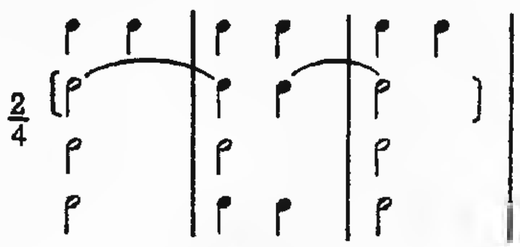


Figure 14.

$4 \div 3$

Find the resultant, $r_{4 \div 3}$

3 (4)

4 (3)

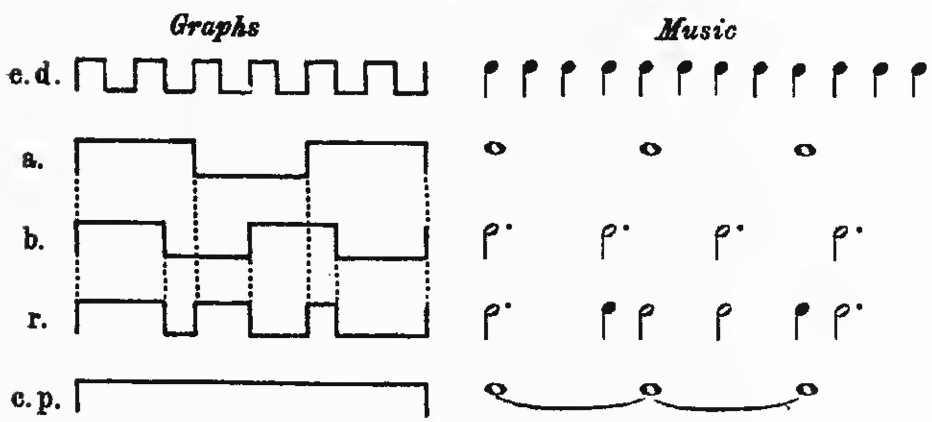


Figure 15.

$\frac{1}{12}$ = ♪ superimposition of c.p.

Figure 16.

$\frac{1}{12}$ = ♩ superimposition of a.

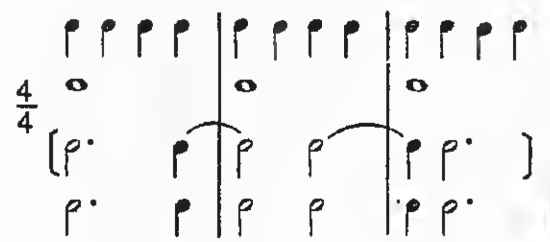


Figure 17.

superimposition of b.

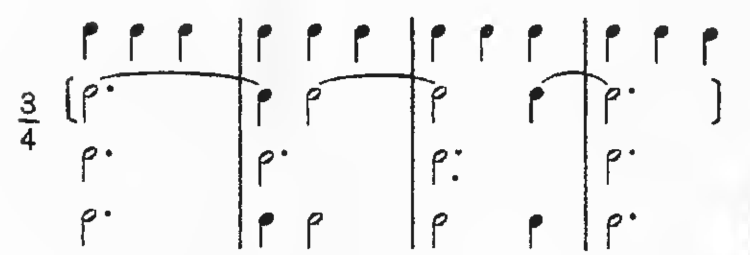


Figure 18.

All these diagrams represent the *natural nucleus* of a musical score,* in which c.d. units are arpeggio or obligato figures, *a* and *b* are chords, *r* rhythms of the theme, and c.p. sustained tones ("pedal point"). The resultants have the following characteristics:

*It may be useful to stress the fact that Schillinger means just what he says in this sentence (and other sentences); that is, he means not only that these patterns *could* be the prototype of a musical score, but—as he will show much later—they actually are the bases of scoring. (Ed.)

(1) recurrence
(2) balance (in $r_{4\div3}$: $2+2$)
(3) contrasts (in $r_{4\div3}$: $3+1$)
(4) inversion, through the axis of symmetry (center):

$$\overleftarrow{3+1+2}+\overrightarrow{2+1+3}$$

Thus, esthetic efficiency (harmony of form) is a product of physical efficiency.

All rhythmic patterns in music are either complete or incomplete resultants. Take, for example, a figure $\frac{3}{4}$; it is $r_{3\div2}$. Take ; it is two-thirds of the $r_{4\div3}$.

When all the resultants up to $a = 9$ have been found, one can obtain all the patterns of the past and present, and, to some extent, of the future.

In making your own diagrams, make them on graph paper, eliminating the c.d. units; they are the units of the cross sections.*

All the necessary generators for practical purposes are:**

$3\div2$				
$4\div3$				
$5\div2$	$5\div3$	$5\div4$		
$6\div5$				
$7\div2$	$7\div3$	$7\div4$	$7\div5$	$7\div6$
$8\div3$	$8\div5$	$8\div7$		
$9\div2$	$9\div4$	$9\div5$	$9\div7$	$9\div8$

Figure 19

When c.p. is greater than 15, use *a* and *b* superimposition only.

When the numbers get large, a musical eighth (♪) becomes the most practical musical denominator. All the reducible fractions are excluded from the above chart, for they give recurrences of the previous cases. For example, $6\div4$ would simply give $3\div2$ twice.

The *a* and *b* components present a clear idea of how "cross-rhythms" should be performed. Beating *a* and *b* with both hands, listen to the resultant, i.e., playing 3 against 2, play one-two, one, one, one-two ($2+1+1+2=$ ♩♫♩), alternating hands.

*In the foregoing, Schillinger has given an extremely *rigorous* (as the logicians say) statement of the case so as to satisfy the most exacting demands of mathematicians and other scientists. It may be helpful to state the process in another, and less rigorous, way—for the benefit of those who are not directly interested in the scientific aspects of the matter. The process is this: (1) take a piece of graph paper, and regard each square from left to right as some unit in time, whether it be a sixteenth note, an eighth note, or what not; (2) mark off the larger of the two generators in a fashion similar to that seen in Figure 10 or Figure 47, that is, breaking the line—if the major generator be "4"—every four units; (3) then mark off the smaller generator until the two "come out even"; (4) then mark off the *resultant*, by making a line which breaks wherever either (or both) of the other two lines breaks. This can be done very rapidly, and the process is best learned by actually carrying it out. Schillinger generally used graph paper with twelve squares to the inch. (Ed.)

**When Schillinger presents a list of this kind, it is his intention that the student should work out all of these instances for himself. He planned the present manuscript so as to have it serve the double function of a workbook and theoretical study. (Ed.)

Let me add a few words on primitive rhythm: the true "primitive" rhythm (such as the rhythm used by some African cannibalistic tribes) is a *combination of various monomial periodicities in time-continuity.*

For example:

$2+2+\ldots$
$3+3+\ldots$
$4+4+\ldots$ etc.

These, when combined in sequence, produce such rhythmic patterns as:

$$(2+2)+1+(1+1+1)+(1+1+1+1)+\ldots=\tfrac{4}{4}$$

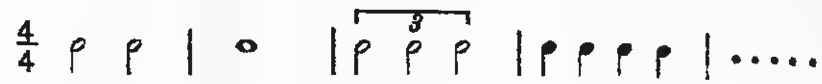


Figure 20.

CHAPTER 3

THE TECHNIQUES OF GROUPING

HAVING SEEN how two monomial periodicities produce a resultant, we have now to consider the manner in which these patterns may be grouped. There are three fundamental forms of grouping of $a \div b$.

(1) Grouping by the product (by ab);
(2) Grouping by the major generator (by a);
(3) Grouping by the minor generator (by b).

In order to group m elements by n, it is necessary to divide m by n. Thus grouping by ab is the *quotient* of $\frac{m}{ab}$.

As in the case of binary synchronization the duration of the entire score equals ab. The formula for grouping by ab is:

$$\frac{ab}{ab} = T \qquad (1)$$

i.e., grouping by ab produces one T with abt.

Example:

$3 \div 2 \qquad \frac{ab}{ab} = \frac{6}{6} = T$, one measure with 6t.

The 6t can be represented in musical notation as any measure with 6 single units. For instance, $\frac{3}{4}$ time, where t = ♪ , or $\frac{6}{4}$ time, where t = ♩ , or $\frac{6}{8}$ time, where t = ♪ .

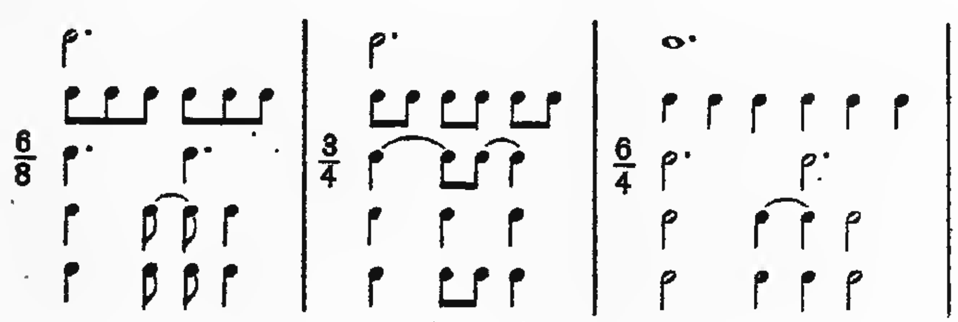


Figure 21.

$$\text{Grouping by a:} \qquad \frac{ab}{a} = bT \qquad (2)$$

In grouping by a, ab must be excluded from the score, as the presence of the latter neutralizes one of the accents, which as a result makes it sound like one T.

$3 \div 2 \qquad \frac{ab}{a} = \frac{6}{3} = 2T$, i.e., two measures with 3t.

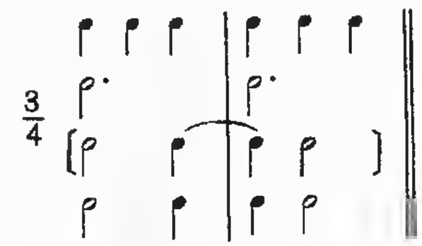


Figure 22.

When grouped by a, b becomes an optional component, causing an effect known as syncopation.

$$\text{Grouping by b:} \qquad \frac{ab}{b} = aT \qquad (3)$$

Exclude ab from the score and assign a as an optional component.

$3 \div 2 \qquad \frac{ab}{b} = \frac{6}{2} = 3T$, i.e., three measures with 2t.

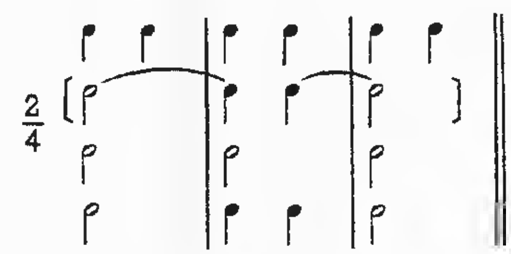


Figure 23.

It is practical to score all the 19 cases of binary synchronization by ab, by a, and by b, with the exception of cases in which ab is too great to be used as one T. The latter consideration is merely a concession to musical habits.

The following table includes all the necessary scores. The reason that some of the forms of T, like $\frac{10}{8}$ or $\frac{14}{8}$, are not in common use is merely due to the lack of adequate rhythmic patterns for their representation.

$a \div b$	Grouping by ab	Grouping by a	Grouping by b
$3 \div 2$	$\frac{6}{8}$; $\frac{3}{4}$ t = ♪	$\frac{3}{4}$	$\frac{2}{4}$
$4 \div 3$	$\frac{12}{8}$; $\frac{3}{4}$ t = ♪	$\frac{4}{4}$	$\frac{3}{4}$
$5 \div 2$	$\frac{10}{8}$	$\frac{5}{4}$	$\frac{2}{4}$
$5 \div 3$	$\frac{15}{8}$	$\frac{5}{4}$	$\frac{3}{4}$
$5 \div 4$	——	$\frac{5}{4}$	$\frac{4}{4}$
$6 \div 5$	——	$\frac{6}{8}$; $\frac{3}{4}$ t = ♪	$\frac{5}{4}$
$7 \div 2$	$\frac{14}{8}$	$\frac{7}{8}$	$\frac{2}{4}$
$7 \div 3$	——	$\frac{7}{8}$	$\frac{3}{4}$
$7 \div 4$	——	$\frac{7}{8}$	$\frac{4}{4}$
$7 \div 5$	——	$\frac{7}{8}$	$\frac{5}{4}$
$7 \div 6$	——	$\frac{7}{8}$	$\frac{6}{8}$; $\frac{3}{4}$ t = ♪
$8 \div 3$	——	$\frac{8}{8}$	$\frac{3}{4}$
$8 \div 5$	——	$\frac{8}{8}$	$\frac{5}{4}$
$8 \div 7$	——	$\frac{8}{8}$	$\frac{7}{8}$
$9 \div 2$	$\frac{18}{8}$	$\frac{9}{8}$	$\frac{2}{4}$
$9 \div 4$	——	$\frac{9}{8}$	$\frac{4}{4}$
$9 \div 5$	——	$\frac{9}{8}$	$\frac{5}{4}$
$9 \div 7$	——	$\frac{9}{8}$	$\frac{7}{8}$
$9 \div 8$	——	$\frac{9}{8}$	$\frac{8}{8}$

Figure 24.

CHAPTER 4

THE TECHNIQUES OF FRACTIONING

THE FIRST process by which rhythmic resultants are generated—the process just explained in the foregoing—is not entirely satisfactory for all musical purposes; it is too "rich" in its variety for all uses, and one may feel the need for a higher degree of uniformity which would complement this variety. Thus the second process by which rhythmic resultants may be generated is now offered with this purpose in mind.

Groups arrived at by means of this second process will be known as rhythmic resultants with fractioning around the axis of symmetry.

Symbols: $\underline{a \div b}$ (underlined) and $r_{\underline{a \div b}}$

The process of synchronization is:

(1) Take the product of a by a, i.e., a^2 (read: "a square"). a becomes its own complementary factor.

(2) Use a as a complementary factor of b, i.e., b appears a times.

(3) The minor generator completes itself before the major generator. Call the *first group* of the minor generator b_1 (the first b). Start the second b (b_2) at the beginning of the *second phase* of a. Start the third b (b_3) at the beginning of the *third phase* of a, when present. This procedure is continued until both generators complete at the same time. $b_1, b_2, b_3, \ldots$ always appear a times.

To find the total number of b groups this formula is used:

$\boxed{N_b = a - b + 1}$ i.e., the number

of b groups equals a minus b plus *1*.

Figure 25.

Example:

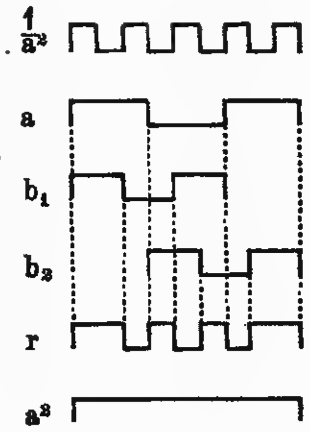
$\underline{3 \div 2}$ find $r_{\underline{3 \div 2}}$

$3 \times 3 = 3^2 = 9$

3 (3)

3 (2)

$N_2 = 3 - 2 + 1 = 2$, i.e., b_1 and b_2



$$a^2\left(\frac{1}{a^2}\right) = 9\left(\frac{1}{9}\right)$$

$$= 3\times 3 \quad a = \frac{3}{9} + \frac{3}{9} + \frac{3}{9}$$

$$= 3\times 2 \quad b_1 = \frac{2}{9} + \frac{2}{9} + \frac{2}{9}$$

$$= 3\times 2 \quad b_2 = \frac{2}{9} + \frac{2}{9} + \frac{2}{9}$$

$$r = \frac{2}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{2}{9}$$

$$a^2 = \frac{9}{9}$$

Figure 26.

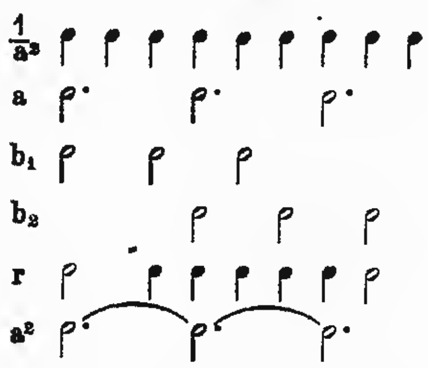


Figure 27.

Fundamental Grouping by a^2 or a only

Grouping by a^2

$$\boxed{\frac{a^2}{a^2} = T}$$

$a^2 = 9$ $\quad \frac{1}{a^2} = \frac{1}{9} = ♪;\ \frac{9}{9} = T$

Figure 28.

Grouping by a

$$\boxed{\frac{a^2}{a} = aT}$$

Exclude a^2 from the score.

$a = \frac{3}{9}$ $\quad \frac{1}{a} = \frac{1}{9} = ♩;\ \frac{9}{3} = 3T$

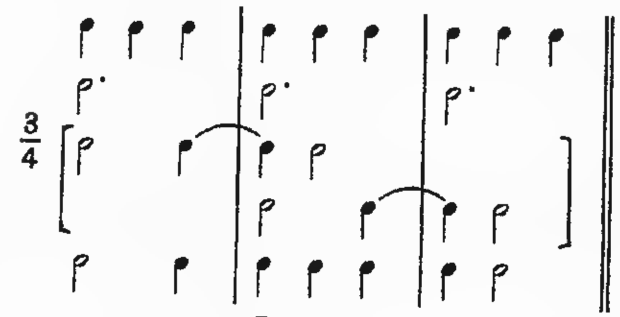


Figure 29.

Grouping by *b of the resultants with fractioning* serves the purpose of producing syncopated rhythms. In such cases the resultant and the bar do not close simultaneously in the first run of the resultant. Therefore, the resultant should be repeated from the point where it stops.

Just when the resultant and the bar come out even may be found in the following manner:

$$\boxed{\frac{a^2}{b} = Q}$$

Figure 30.

The "Q" stands for the quotient which indicates the number of bars. It always has a remainder. The denominator of the remainder indicates how many times the resultant will have to run. For the *b* grouping, the resultant is used alone.

$$\boxed{b \times Q = bQ}$$

Figure 31.

Example: (1)

$\frac{a^2}{b} = \frac{9}{2} = 4\frac{1}{2}$. $4\frac{1}{2}$ indicates the number of bars. 2 indicates the number of groups of r. $2\left(4\frac{1}{2}\right) = 9$.

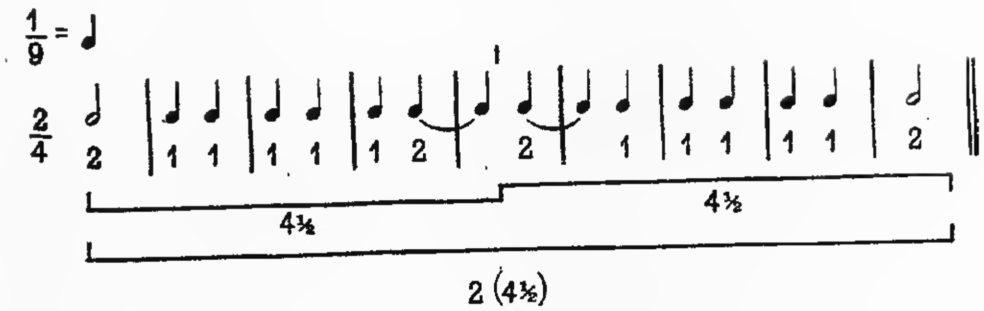


Figure 32.

Example: (2)

$\underline{4 \div 3}$ Find $r_{\overline{4 \div 3}}$

$4^2 = 16$

4 (4)
4 (3)

$N_3 = 4 - 3 + 1 = 2$

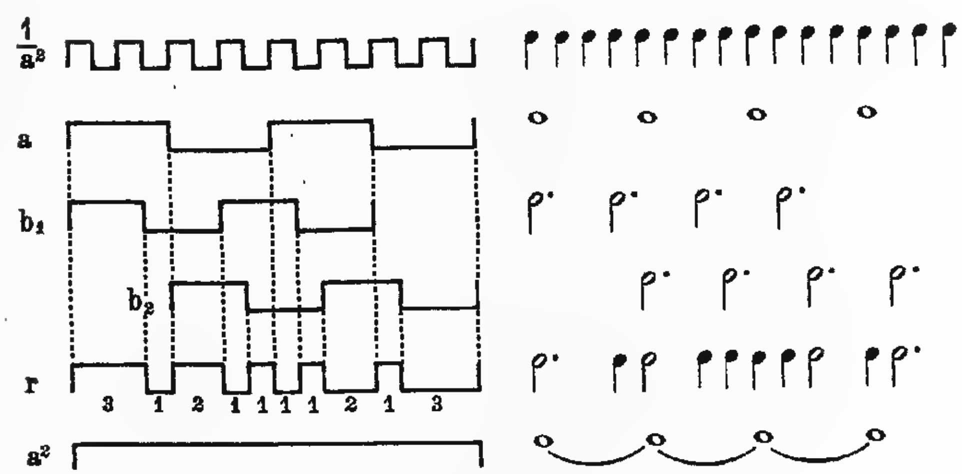


Figure 33.

Grouping by a^2

Figure 34.

Grouping by a

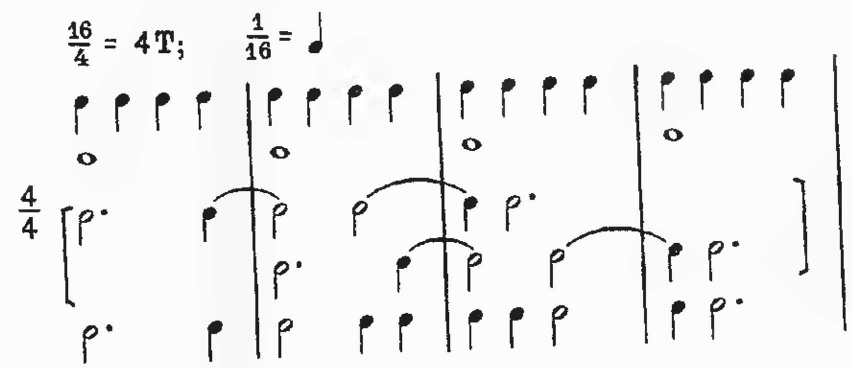


Figure 35.

Grouping by b

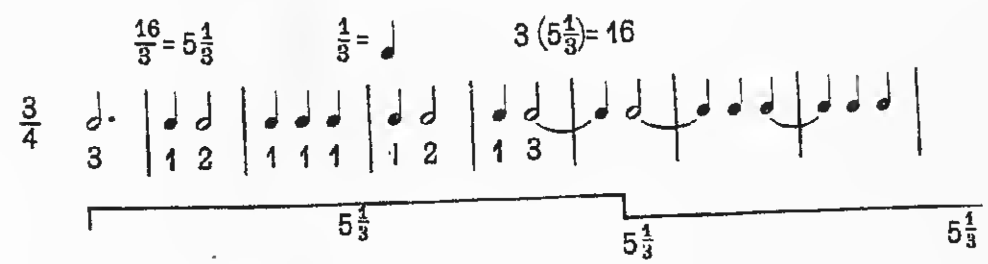


Figure 36 (continued).

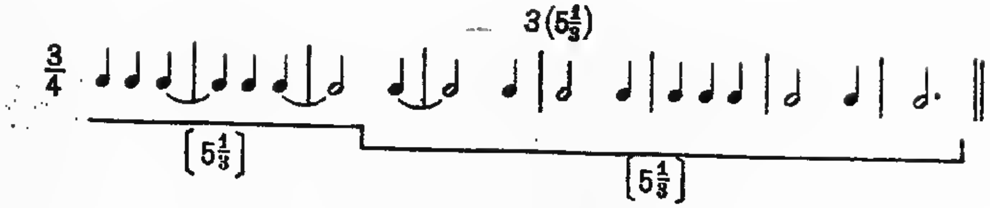


Figure 36 (concluded)

CHAPTER 5

COMPOSITION OF GROUPS BY PAIRS

THESE TECHNIQUES of obtaining resultants may be extended further so as to evolve processes by which we may compose rhythmic resultants in pairs.

In the ordinary exposition of a musical theme, it is customary to state the theme twice in such a way that for the first time the theme does not sound entirely completed, while for the second time it is brought to a completion. As composers of the past (as well as composers of the present) do not know how to do it, they usually resort to variations of the cadence *harmonically*. But it remains a pure problem of rhythm nevertheless.

Composers have also been confronted with the problems of expansion and contraction in the two adjacent groups. Moving from a long to a short group is what we mean by *contraction;* the opposite is *expansion.*

These procedures were performed crudely even by well-reputed composers. For instance, L. van Beethoven in his piano sonata, No. 1, in the first movement at the end of exposition, states a two-bar group three times. On the third statement, he makes an expansion by merely holding the chord through the whole bar (a whole note), thus adding one more bar. In his piano sonata, No. 7, (in the beginning of the finale) he has a four-bar group. There are many rests in this group, and the rests are injected *a priori* with the idea of taking them out afterwards. Thus he makes a three-bar group out of a four-bar group. Even this crude form of contraction was rarely attempted by Beethoven in his long career.

Here, we shall present a general method of *balancing*, *expanding*, and *contracting* a pair of adjacent groups, no matter what the rhythmic constitution of such groups may be.

As the resultants which have identical generators have a great deal in common, such performance gives the utmost esthetic satisfaction.

(A) **Balance**

B = balance

$$\boxed{B = r_{\underline{a \div b}} + r_{a \div b} + a(a-b)}$$

Figure 37.

The above means that in order to balance two resultants with identical generators, take the resultant of a to b, with fractioning, add the resultant of a to b and add a times a minus b. Grouping for such pairs is through a only.

Example:

$B = r_{\underline{3 \div 2}} + r_{3 \div 2} + 3(3-2) = [(2+1)+(1+1+1)+(1+2)] + [(2+1)+(1+2)+3]$

Figure 38.

$B = r_{\underline{4\div3}} + r_{4\div3} + 4(4-3) = [(3+1)+(2+1+1)+$
$(1+1+2)+(1+3)] + [(3+1)+(2+2)+(1+3)+4].$

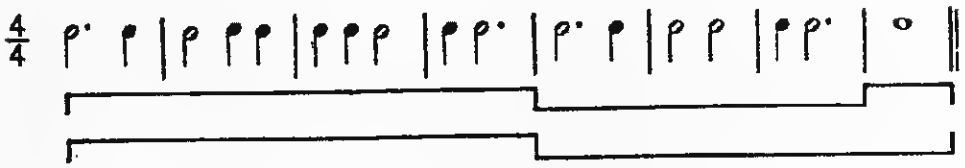


Figure 39.

Balance does not seem natural when $a > 2b$, $a > 3b$, i.e., when a is greater than 2b or greater than 3b. Yet it may be accomplished through a general procedure.

(1) Take $r_{\underline{a\div b}}$
(2) Take $r_{a\div b}$ as many times as it enters (as divisor) into a^2.
(3) Add one total duration which equals the difference between a^2 and 2ab, a^2 and 3ab, etc.

$$\boxed{B_{a>mb} = r_{\underline{a\div b}} + m r_{a\div b} + (a^2 - mab)}$$

Figure 40.

Example:

$5 \div 2$ $\quad 5>2(2)$

(1) $r_{\underline{5\div2}} = (2+2+1)+(1+1+1+1+1)+(1+1+1+1+1)$
$+(1+1+1+1+1)+(1+2+2) = 25$

(2) $\frac{25}{10} = 2\frac{1}{2}$

(3) $25 - 20 = 5$

$B = r_{\underline{5\div2}} + r_{5\div2} + r_{5\div2} + 5 = [(2+2+1)$
$+(1+1+1+1+1)+(1+1+1+1+1)+(1+1+1+1+1)$
$+(1+2+2)] + [(2+2+1)+(1+2+2)] + [(2+2+1)$
$+(1+2+2)] + 5$

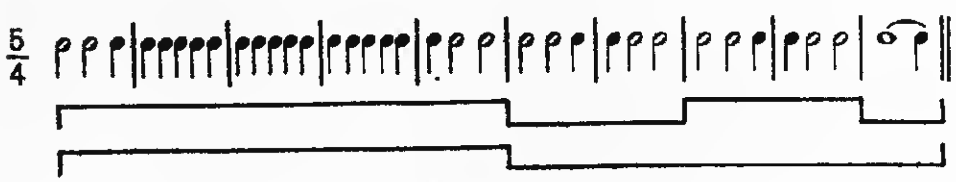


Figure 41.

(B) Expansion E = expansion

$$\boxed{E = r_{a\div b} + r_{\underline{a\div b}}}$$

Grouping by *a* only.

Example:

$E = r_{3\div2} + r_{\underline{3\div2}} = [(2+1)+(1+2)] + [(2+1)+(1+1+1)+(1+2)]$

Figure 42.

$E = r_{4\div3} + r_{\underline{4\div3}} = [(3+1)+(2+2)+(1+3)] + [(3+1)$
$+(2+1+1)+(1+1+2)+(1+3)]$

Figure 43.

(C) Contraction C = contraction

$$\boxed{C = r_{\underline{a\div b}} + r_{a\div b}}$$

Grouping by *a* only.

Figure 44.

Example:

$C = r_{\underline{3\div2}} + r_{3\div2} = [(2+1)+(1+1+1)+(1+2)]$
$+[(2+1)+(1+2)]$

Figure 45.

$C = r_{\underline{4\div3}} + r_{4\div3} = [(3+1)+(2+1+1)+(1+1+2)$
$+(1+3)] + [(3+1)+(2+2)+(1+3)]$

Figure 46.

CHAPTER 6

UTILIZATION OF THREE OR MORE GENERATORS

IT IS CLEAR that just as rhythmic groups may be developed by the use of two generators, so, too, may they be based on the use of three—or more than three—generators. In such a case, the selection of the third generator becomes important.

It happens that *all generators pertaining to one family of rhythm belong to the same series of number-values.** Such series are the series of growth; they control not only music and the arts in general, but also the proportions of the human body, as well as various forms of growth in nature. Horns, antlers, cockleshells, maple leaves, sunflower seeds and many other natural developments are controlled by the series of growth. Mathematically, one can produce an infinite number of *types* of the series of growth, and an infinite number of series of each type.

The series referring to the developments mentioned above constitute one specific type of *summation series*. In this type of summation series, every third number-value is the sum of the two preceding number-values. For instance, if in some series, numbers 2 and 3 occur, then the next number is 5, i.e., $2 + 3$. The best known of all series of this type is:

1, 2, 3, 5, 8, 13, 21, 34, 55, 89 . . .

For example, the spiral tangent to a maple leaf grows through 90-degree arcs and each consecutive radius of each arc follows this very series. Formation of the seeds in a sunflower follows the same series. Professor Church of Oxford University devoted his life to this problem. He found that only slight deviations may be found and then in only two cases out of a thousand, the deviations being caused by exceptionally unfavorable climatic conditions.

An important portrait painter of New York City, Wilford S. Conrow, devoted many years of research in order to find out how this series works in relation to the human body. He found an overwhelming amount of material in the ancient Greek theories of proportions. Conrow's deductions are that it is this particular series that makes the human body beautiful to us.

I have found in the field of music that each style (or family) of rhythm evolves through the series of such types. Here are all the series that are useful for musical purposes:

I. 1, 2, 3, 5, 8, 13, ...
II. 1, 3, 4, 7, 11, 18, ...
III. 1, 4, 5, 9, 14, 23, ...

As previously mentioned, all rhythmic groups (or patterns) of one style are the resultants of the generators of the same series. For example, if a certain rhythmic group is identified with $r_{3 \div 2}$, then groups of the same style will be produced by $r_{5 \div 3}$ or $r_{5 \div 3 \div 2}$.

*For obvious reasons, Schillinger does not here present fully the entire case surrounding this statement, which is a statement of crucial significance in esthetics. His fuller statement is contained in his 'Mathematical Basis of the Arts,' which is to be published shortly. (Ed.)

The following are the important and practical combinations of generators to be worked out:

SERIES I. $2 \div 3 \div 5$ $3 \div 5 \div 8$
SERIES II. $3 \div 4 \div 7$
SERIES III $4 \div 5 \div 9$

A. The Technique of Synchronization

In order to synchronize three or more generators, it is necessary first to find their common product and their complementary factors.

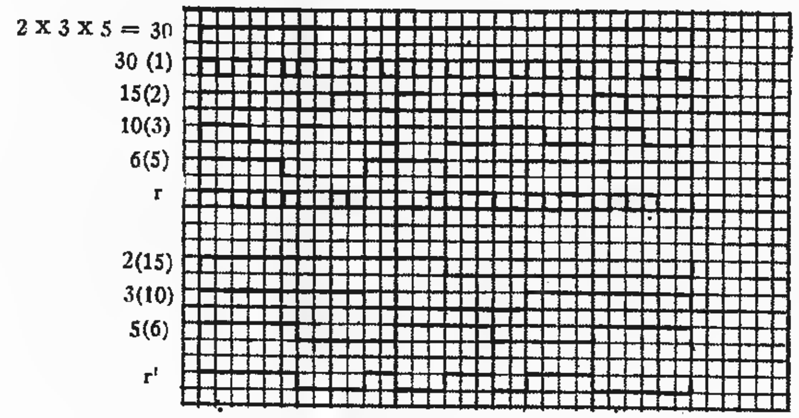
Let us take $2 \div 3 \div 5$
The product is $2 \times 3 \times 5 = 30$

The complementary factors are the quotients of the product by a generator. Thus, $\frac{30}{2} = 15$ means that 15 is a complementary factor of 2.

Therefore: 15 (2)
10 (3)
6 (5)

This method offers two resultants (r and r′) at a time, one serving as a theme, the other as a countertheme. Generators produce r, and the complementary factors produce r′.

$2 \div 3 \div 5$



$$r = 2+1+1+1+1+2+1+1+2+2+1+1+ \\ 2+2+1+ . +2+1+1+1+1+2$$
$$r^1 = 6+4+2+3+3+2+4+6$$

Figure 47.

The rule of grouping is: *group by any generator or any of the complementary factors.* In the case of $2 \div 3 \div 5$, grouping is available through 2, 3, 5, 6, 10, 15, i.e., $\frac{2}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, $\frac{6}{8}$, $\frac{10}{8}$, $\frac{15}{8}$.

Grouped through $\frac{6}{8}$, r′ appears as follows:

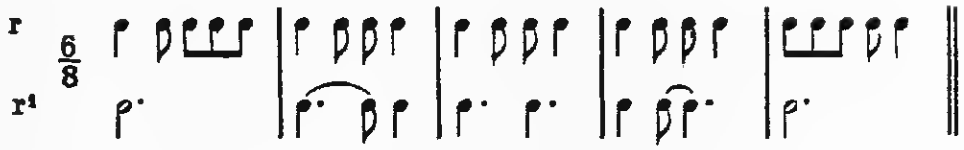


Figure 48.

It can be seen from this example that no more rhythmically suitable countertheme can be devised. The theme makes three recurrences while the countertheme makes continuous changes in much longer values. The listener has the opportunity to hear *both themes* together.

All resultants from three or more generators have these recurrences and variations as their chief characteristics.

CHAPTER 7

RESULTANTS APPLIED TO INSTRUMENTAL FORMS

WHEN WE speak of *time rhythm* we are referring to the periodicity of attacks, that is, the intervals of time at which the attacks occur.

A. Instrumental Rhythm

Instrumental rhythm is made up of the number of *places* of attack; for example, in beating two differently pitched kettle drums in sequence, we are dealing with two places of attack.

Synchronizations of these two types of rhythm—i.e., time rhythm and instrumental rhythm—are subject to the same laws of synchronization and interference as the time periodicity previously discussed.*

When the number of places in an instrumental group does not coincide with the number of terms in a time group, then a common denominator will define the number of time groups—and the number of instrumental groups—until their recurrence. For example, if we use two differently tuned kettle drums on $r_{3\div2}$, the entire figure will close after the first group is over because the number of *places* in the instrumental group is two (kettle drums) and the number of terms in the time group is four ($4 \div 2 = 2$). This means that while the instrumental group appears twice, the rhythmic resultant will appear once.

$r_{3\div2}$

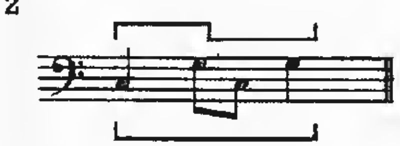


Figure 49.

Taking the same case of the two kettle drums for $r_{\underline{3\div2}}$, we get a totally different resultant. The number of attacks in the instrumental group remains the same (2). The number of terms in the rhythmic resultant is 7 ($2+1+1+1+1+1+2$). $7 \times 2 = 14$. Seven has a complementary factor 2, and 2 has a complementary factor 7. The kettle drum 2-attack figure will appear 7 times, while the rhythmic resultant appears twice.

*Here we see the first of what will come to be a great many examples of the way in which Schillinger's theory of rhythm goes much further than the simple question of *time* rhythm. Note that Schillinger here states that instrumental rhythm is one thing (that is, the pattern according to which instruments enter or drop out of the ensemble) and time rhythm is another (that is, the pattern according to which the sounds are produced, regardless of which instruments are producing these sounds). Schillinger here discusses the application of his theory not only to the rhythm of the sounds produced, but also to the allocation of parts among various instruments. (Ed.)

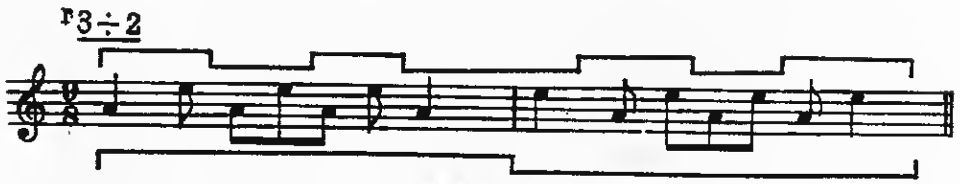


Figure 50.

This principle may be carried out to any desired degree of complexity, depending on the common denominator between the number of terms in a rhythmic group and the number of attacks in an instrumental group. The difference between two kettle drums and any melody or any instrumental form of harmony (accompaniment) with respect to this calculation is merely a quantitative difference.

Let us take a motif consisting of four different pitches, (for example: c, d, e, f); such sequence of pitches is merely one of the possible forms of melody. But superimposing $r_{3\div2}$ we obtain one group without recurrence because the number of pitches (intonation attacks), and the number of terms in the rhythmic resultant (time attacks), are equal ($4 \div 4 = 1$). Taking the same four notes of the melody and superimposing $r_{3\div2}$, we get $7 \times 4 = 28$. The rhythmic group having 7 attacks acquires the complementary factor 4, i.e., it will run 4 times until its own recurrence, while the melody having 4 attacks will acquire the complementary factor 7, i.e., it will run 7 times until its own recurrence will coincide with the recurrence of the rhythmic resultant.

Figure 51.

This technique makes it possible to run a very simple motif practically to infinity, as the duration of continuous variability depends solely on a common denominator. A simple example of rhythmic continuity through instrumental interference may be found in many arrangements of fox-trots. The figure of 6 uniform attacks (two false triplets) placed in a common time measure ($=\frac{8}{8}$) produces an interference of $8 \div 6$. $8 \div 6$ reduces to $4 \div 3$. Six acquires the complementary factor 4, and 8 acquires the complementary factor 3, i.e., the instrumental figure with 6 attacks runs 4 times in the course of 3 $\frac{8}{8}$ measures.

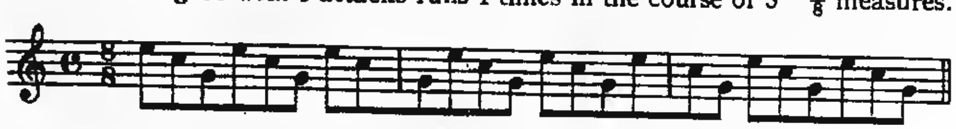


Figure 52.

The principles of rhythmic (time) and instrumental interference have been known since time immemorial. They constitute one of the most striking characteristics in the composition of rhythmic continuity as it exists in the music of the Orient as well as through the entire African continent. This tendency is almost as fundamental as the superimposition of a major generator on any uniform group—as for example, the imaginary grouping of the attacks of a ticking clock by 2, 3, 4, or any other simple number we can think of.

B. Applying the Principles of Interference to Harmony

The principles of interference of rhythmic and instrumental groups, when applied to harmony, produce the most effective forms of accompaniment. They make it possible, as well, to correlate a number of accompaniments simultaneously.

At this point, the illustrations of harmony are restricted to three of the simplest instrumental forms. However, in the later part of the course, details of the instrumental forms of harmony will be discussed. Here we will cite:

(1) The two-attack instrumental figure (as in the polka). The first attack is the detached bass of harmony. The second attack includes all the remaining upper parts of a chord.

(2) The four-attack instrumental figure (as in the fox-trot). The first attack is the lower bass. The second attack, the upper part of the chord. The third attack is another detached bass. The fourth attack, the upper part of a chord.

(3) The six-attack instrumental figure (as in the rhumba). The first attack is the lower detached bass. The second attack is the upper part of the chord. The third attack is the middle detached bass. The fourth attack is the upper part of the chord. The fifth attack is the upper detached bass. The sixth attack is the upper part of the chord.

Figure 53.

It is easy to see that the waltz accompaniment figure is merely (1) above, with the upper chord having two attacks instead of one. The old tango and habanera figures are like (2), except that the last attack is made on the lower bass.

The following diagrams illustrate the continuous run of these instrumental forms of harmony with various simpler rhythmic resultants, all used on one chord:

3 ÷ 2

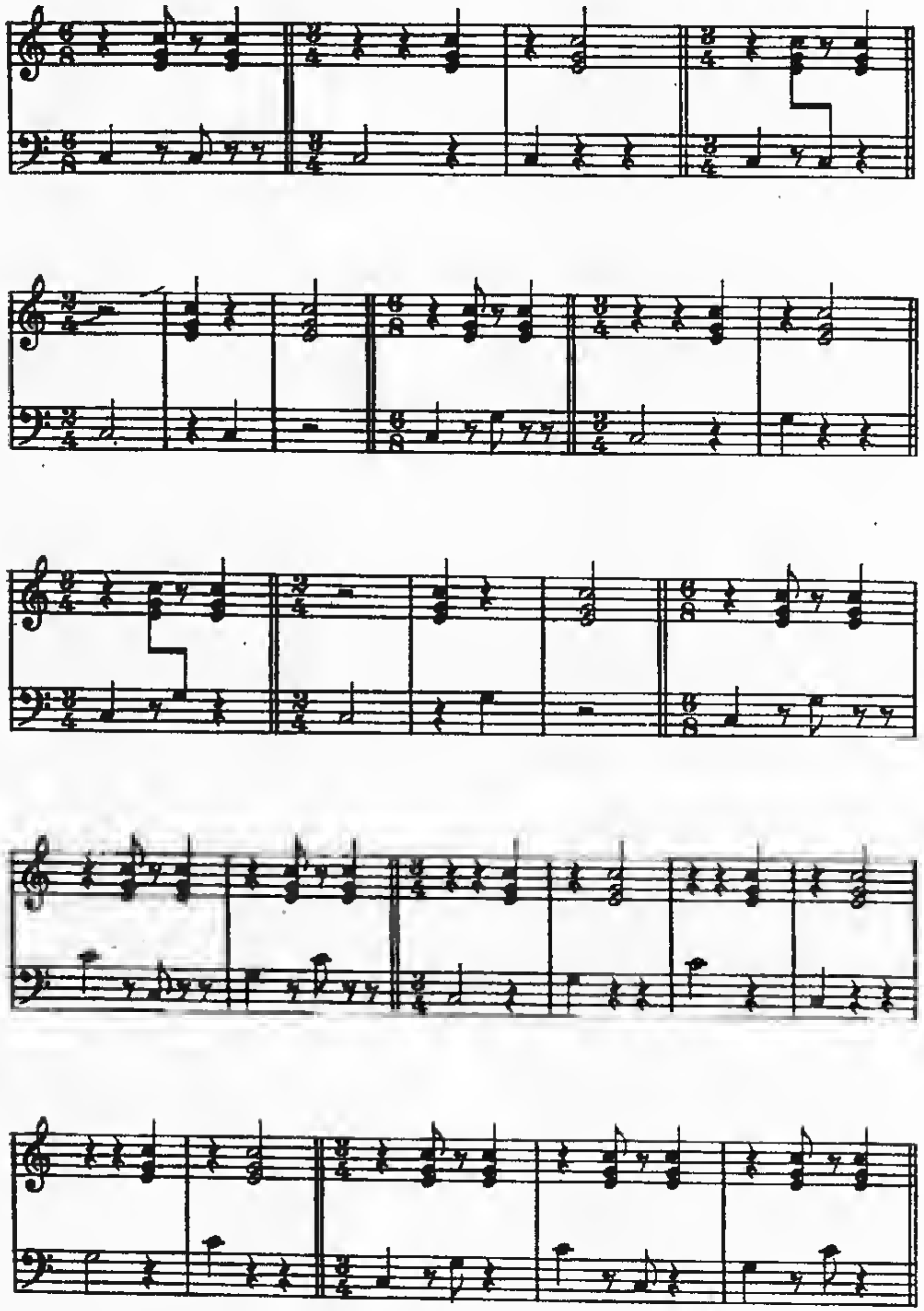


Figure 54 (continued).

4 ÷ 3

Figure 54 (continued).

Figure 54 (concluded).

One may also compose other instrumental forms of harmony with as many as 16 attacks—such as an alternation of the four different notes in the bass, with the upper part of the chord doubled in two octaves:

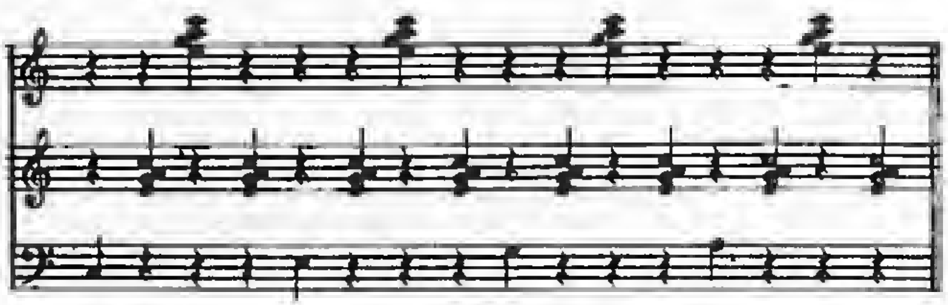


Figure 55.

A still greater number of attacks in an instrumental figure may be produced by the common technique of arpeggio. Technically, any longer motif presents the same problem, except that its pitch commonly has a more limited range.

When one time-group is distributed through the different places of attack, different individual parts become *the resultants of interference between the time and the instrumental groups*. For example, if we have a figure of 4 places, as referred to in Item (2), page 29, and superimpose a time group, $2 + 1 + 1$ (3 attacks), we obtain through the common denominator 12, 2 different instrumental resultants. One is the sequence of attacks on chords; the other, the sequence of attacks in the bass, when all the bass attacks are tied over. The upper part produces the resultant $2\ (2 + 3 + 3)$ and the bass, $2\ (3 + 3 + 2)$. This is a striking example of transformation of one type of rhythm into another —a result of the phenomenon of instrumental and time interference.

The $2 + 1 + 1$ is a traditional classical figure, and, as expressed in the following musical example, consists of a quarter and two eighths. Yet the result

sounds like a rhumba. This is due to the *new* resultant which appears as a sequence of the attacks of the bass notes.

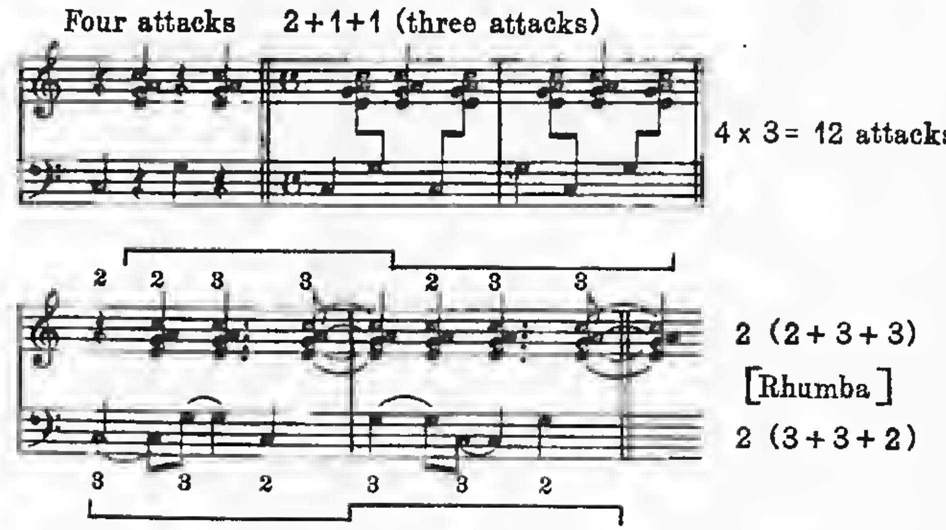


Figure 56.

The preceding technical items may also be treated in combination. The following example represents the application of two generators and their resultants, combined with the instrumental interference. The accompaniment represents the minor generator (2). The sustained chords represent the major generator (5). The melody represents the resultant $(5 \div 2)$. In addition, the whole score is carried out through an *alien measure grouping*, $\frac{4}{4}$. While the entire rhythmic score would occupy 4 bars in $\frac{5}{4}$ time, it takes 5 bars in $\frac{4}{4}$ time. This example illustrates* the possibility of introducing various rhythmic resultants into music which is supposed to be written in common time.

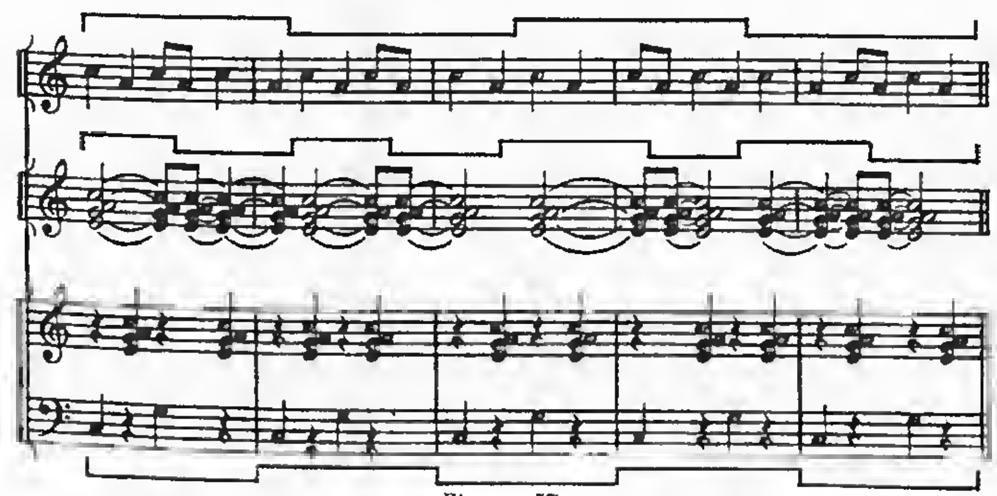


Figure 57.

*The example in Figure 57 is of more than passing interest because it foreshadows the way in which full orchestral scores of unprecedented richness and complexity are developed logically and organically from the rhythmic raw materials now being discussed in this section. (Ed.)

CHAPTER 8

COORDINATION OF TIME STRUCTURES

MOTION—that is, changeability in time—is the most important intrinsic property of music. Different cultures of different geographical and historical localities have developed many types and forms of intonation. The latter varies greatly in tuning, in quantity of pitches employed, in quantity of simultaneous parts, and in the ways of treating them.

The types are as diversified as drum-beats, instrumental and vocal monody (one part music), organum, discantus, counterpoint, harmony, combinations of melody and harmony, combinations of counterpoint and harmony, different forms of coupled voices, simultaneous combinations of several harmonies, and many others. Any of these types—as well as any combinations of them—constitute the different musical cultures. In each case, musical culture crystallizes itself into a definite combination of types and forms of intonation. The latter crystallize into habits and traditions.

For example, people belonging to a *harmonic* musical culture want every melody harmonized. But people belonging to a monodic musical culture are disturbed by the very presence of harmony. Music of one culture may be *music* (meaningful sound) to the members of that culture; but the very same music may be *noise* (meaningless sound) to the members of another. The functionality of music is comparable to a great extent to that of a language.

Nevertheless, *all* forms of music have one fundamental property in common: *organized time.* The plasticity of the temporal structure of music, as expressed through its attacks and durations, defines the quality of music. Different types and forms of intonation—as well as different types of musical instruments—come and go like the fashions, while the everlasting strife for *temporal plasticity* remains a *symbol of the "eternal" in music.*

The temporal structure of music, usually known as *rhythm*, pertains to two directions: *simultaneity* and *continuity*. The rhythm of *simultaneity* is a form of coordination among the different components (parts). The rhythm of *continuity* is a form of coordination of the successive moments of one component (part).

People of our civilization have developed the power of reasoning at the price of losing many of the instincts of primitive man. Europeans have never possessed the "instinct of rhythm" with which the Africans are endowed. So-called European "classical music" has never attained the ideal it strived for, that ideal being: the utmost plasticity of the temporal organization. When J. S. Bach, for example, tried to develop a coordinated independence of simultaneous parts, he succeeded in producing only a resultant which is uniformity.* We find evidence of the same failures in Mozart and Beethoven. But a score in which the several coordinated parts produce, together, a resultant which

*That is to say, when the separate rhythms of the separate parts of a Bach score are "added up," the result tends to be simple uniformity. Schillinger suggests the desirability of scores, and develops a method of scoring, so that the separate parts, while satisfactory rhythmically by themselves, all "add up" to a new rhythm which is *not* uniformity. (Ed.)

has a distinct pattern—has been a "lost art" of the aboriginal African drummers. The age of this art can probably be counted in tens of thousands of years!

Today in the United States, owing to the transplantation of Africans to this continent, there is a *renaissance* of rhythm. Habits form quickly—and the instinct of rhythm in the present American generation surpasses anything known throughout European history. Yet our professional "coordinators of rhythm," specifically in the field of dance music, are slaves to, rather than masters of, rhythm. There is plenty of evidence that the urge for coordination of the whole through individualized parts is growing. The so-called "pyramids" (sustained arpeggio produced by successive entrances of several instruments) is but an incompetent attempt to solve the same problem.

Fortunately, we do not have to feel discouraged or moan over this "lost art." The *power of reasoning* offers us a complete *scientific solution.*

This problem can be formulated as the *distribution of a duration-group through instrumental and attack-groups.*

The entire technique consists of five successive operations with respect to the following:

(1) The number of individual parts in a score;
(2) The quantity of attacks appearing with each individual part in succession;
(3) The rhythmic patterns for each individual part;
(4) The coordination of all parts (which become the resultants of instrumental interference) into a form which, in turn, results in a specified rhythmic pattern (the resultant of interference of all parts); and
(5) The application of such scores to any type of musical measures (bars).

Any part of such a score can be treated as melody, coupled melody, block-harmony, harmony, instrumental figuration—or as a purely percussive (drum) part. Aside from the temporal structure of the score, *the practical uses of this technique in intonation* depend on the composer's skill in the respective fields concerned, i.e., melody, harmony, counterpoint and orchestration.

Distribution of a Duration-Group (T) through Instrumental (i) and Attack (a) groups.

Notation

pl_i number of places in the instrumental group.
pl_a number of places in the attack-group.
a_a number of attacks in the attack-group.
a_T number of attacks in the duration-group.
PL the final number of places.
A the synchronized attack-group (the number of attacks synchronized with the number of places).
A′ the final attack group (number of attacks synchronized with the duration-group).
T the original duration-group.
T′ the synchronized duration-group.
T″ the final duration-group.
$N_{T''}$ the number of final duration-groups.

Procedures:

(1) **Interference between the number of places in the instrumental group (pli) and the number of places in the attack-group (pla).**

$$PL = \frac{pli}{pla}; \qquad \frac{pla\ (pli)}{pli\ (pla)}$$

(2) **The product of the number of attacks in the attack group (a_a) by the complementary factor to the number of places in the attack-group (pli after reduction).**

$$A = a_a \cdot pli$$

(3) **Interference between the synchronized attack-group (A) and the number of attacks in the original duration-group (a_T).**

$$A' = \frac{A}{a_T} = \frac{a_a \cdot pli}{a_T}$$

(4) **The product of the original duration-group (T) by the complementary factor to its number of attacks (A').**

$$T' = T \cdot A' = \frac{T \cdot a_a \cdot pli}{a_T}$$

(5) **Interference between the synchronized duration-group (T') and the final duration-group (T'').**

$$N_{T''} = \frac{T'}{T''}$$

A. Synchronization of an Attack-Group (a) with a Duration-Group (T).

Distribution of attacks of an attack-group (a_a) through the number of attacks of a duration-group (a_T).

First Case: $\frac{a_a}{a_T} = 1$

$A = a_T$

$T' = T$

Example:

$a_a = 4a$; $T = r_{3 \div 2} = 6t$; $a_T = 4a$

$A = 4a$

$T' = 6t$

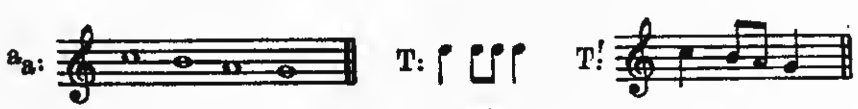


Figure 58.

Second Case: $\frac{a_a}{a_T} \neq 1$

$A = a_T \cdot a_a$

$T' = T \cdot a_a$

Example:

$a_a = 5a$; $T = r_{3 \div 2} = 6t$; $a_T = 4a$

$\frac{5}{4}$

$A = 5a \cdot 4 = 20a$

$T' = 6t \cdot 5 = 30t$

Figure 59.

Third Case: $\frac{a_a}{a_T} = \frac{a_{a'}}{a_{T'}}$ i.e., a reducible fraction

$A = a_{T'} \cdot a_{a'}$

$T' = T \cdot a_{a'}$

Example:

$a_a = 6a$; $T = r_{3 \div 2} = 6t$; $a_T = 4a$

$\frac{6}{4} = \frac{3}{2}$

$A = 4a \cdot 3 = 12a$

$T' = 6t \cdot 3 = 18t$

Figure 60.

B. Distribution of a Synchronized Duration-Group (T') through the Final Duration-Group (T'').

First Case: $\frac{T'}{T''} = 1$

$T'' = T'$

Example:

$T' = 6t$; $T'' = 6t$

Figure 61.

Second Case: $\frac{T'}{T''} \neq 1$

$N_{T''} = T'$

Example:

$T' = 6t;\ T'' = 5t$

$N5t = 6$

T':


T'':


Figure 62.

Third Case: $\frac{T'}{T''} = \frac{T_{\prime}}{T_{\prime\prime}}$ i.e., a reducible fraction

$N_{T''} = T_{\prime}$

Example:

$T' = 6t;\ T'' = 4t$

$\frac{6}{4} = \frac{3}{2}$

$N_{4t} = 3$

T':

T'':


Figure 63.

Example:

$a_a = 5a$ $T = r_{5\div2} = 10t;$ $a_T = 6a$

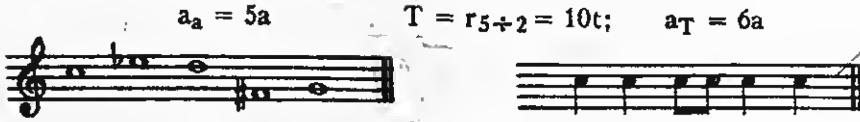


Figure 64.

(1) $\frac{6}{5}$ $\frac{5}{6}\frac{(6)}{(5)}$
(2) 6 attacks are equivalent to 10t; 10t × 5 = 50t
(3) When $T'' = \frac{8}{8}$, $\frac{50t}{8} = \frac{25\cdot4}{4} = 25T''$

Figure 65 (continued).

Figure 65 (concluded).

C. Synchronization of an Instrumental Group (pli) with an Attack-Group (pla).

Example:

pli = 4; pla = 3; $a_a = 3+2+3 = 8$; $T = r_{5\div2} = 10t$; 6a

(1) $\frac{4}{3}$; $\frac{3}{4}\frac{(4)}{(3)}$ (3) $\frac{32}{6} = \frac{16}{3}$
(2) $8 \times 4 = 32$ (4) $\frac{16\cdot10}{3} = \frac{160}{3}$
(5) $T'' = 8t$; $\frac{160}{3\cdot8} = \frac{20}{3}$; $\frac{20\cdot3}{3} = 20T''$

Figure 66.

Example:

pli = 3; pla = ~~3~~ 4; $a_a = 3+2+2+3 = 10$; $T = r_{\underline{4\div3}} = 16t$; 10a

(1) $\frac{3}{3} = 1$

(2) $10 \cdot 1 = 10$

(3) $\frac{10}{10} = 1$

(4) $16 \cdot 1 = 16$

(5) $T'' = 8t$; $\frac{16}{8} = 2T''$

Figure 67.

Example:

pli = 6; pla = 8; $a_a = r_{5\div4} = 20$; $T = r_{\underline{4\div3}} = 16t$; 10a $T'' = 8t$

(1) $PL = \frac{6}{8} = \frac{3}{4}$; $\frac{4\,(8)}{3\,(6)}$

(2) $A = 20 \cdot 3 = 60$

(3) $A' = \frac{60}{10} = 6$

(4) $T' = 16t \cdot 6 = 96t$

(5) $\frac{96}{8} = 12T''$

(See Fig. 73, p. 43 for an example based on this formula)

Example of composition of the resultant of instrumental interference.

pli = pla = 2

Form of distribution: 5+3

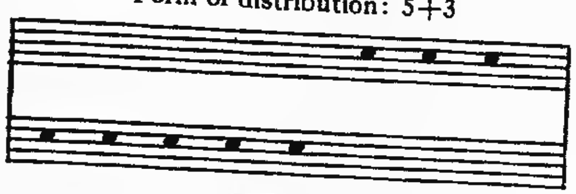


Figure 68.

(1) $\frac{2}{2} = 1$

(2) 2 is an equivalent of $5+3 = 8$

(3) Duration-group: $T = r_{5\div2} = 10t$

$a_T = 6$

$\frac{8}{6} = \frac{4}{3}$ $\frac{3\,(8)}{4\,(6)}$

(4) $10t \times 4 = 40t$

(5) When $T'' = \frac{8}{8}$, $\frac{40t}{8} = 5T''$

Preliminary Scoring

Final Scoring

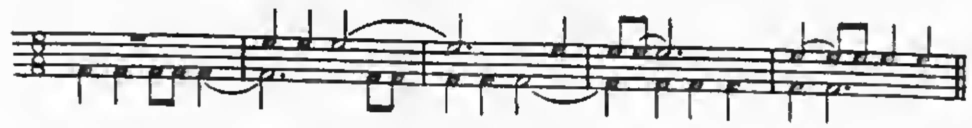


Figure 69.

Example of composition of the resultant of instrumental interference.

pli = 3 pla = ~~X~~ 4

Form of distribution: 8+3+5+2

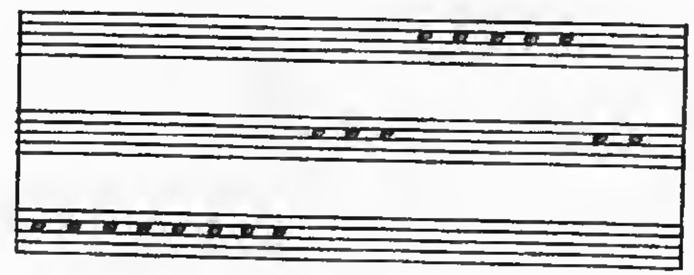


Figure 70.

(1) $\frac{3}{3} = 1$

(2) 4 is an equivalent of $8+3+5+2 = 18$

$18 \times 1 = 18$

(3) Duration group: $r_{5\div2} = 10t$ $\frac{18}{6} = 3$ 3(6)

$a_T = 6$

(4) $10t \times 3 = 30t$

(5) When $T'' = \frac{8}{8}$, $\frac{30t}{8} = \frac{15}{4}$; $\frac{15 \cdot 4}{4} = 15T''$

Preliminary Scoring

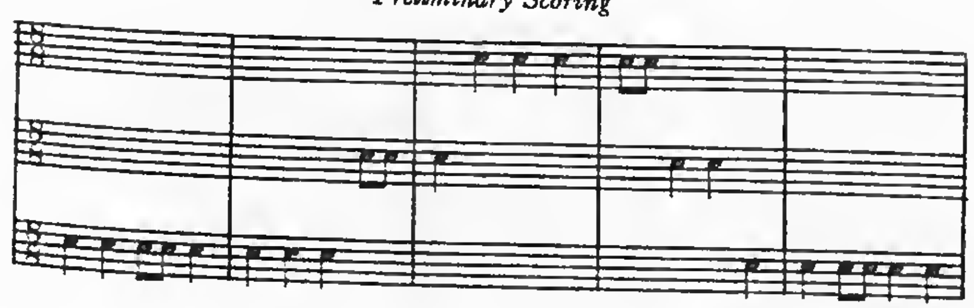


Figure 71 (continued).

Figure 71 (concluded).

Final Scoring

Figure 72 (continued).

Figure 72 (concluded).

Example of composition of the resultant of instrumental interference.

pli = 6; pla = 8;

Form of distribution: $r_{5 \div 4}$

Figure 73. See p. 40 for additional comment.

(1) $\frac{6}{8} = \frac{3}{4} \quad \frac{4\,(6)}{3\,(8)}$

(2) 8 is equivalent to 20 in $r_{5 \div 4}$ $\quad 20 \times 3 = 60$

(3) Duration-group $= r_{\underline{4 \div 3}}$ $\quad a_T = 10; \quad \frac{60}{10} = 6 \quad 6(10)$

$r_{\underline{4 \div 3}} = 16t$

(4) $16t \times 6 = 96t$; a given $T'' = \frac{8}{8}$

(5) $\frac{96t}{8t} = 12T''$

Preliminary Scoring

Figure 74.

Final Scoring

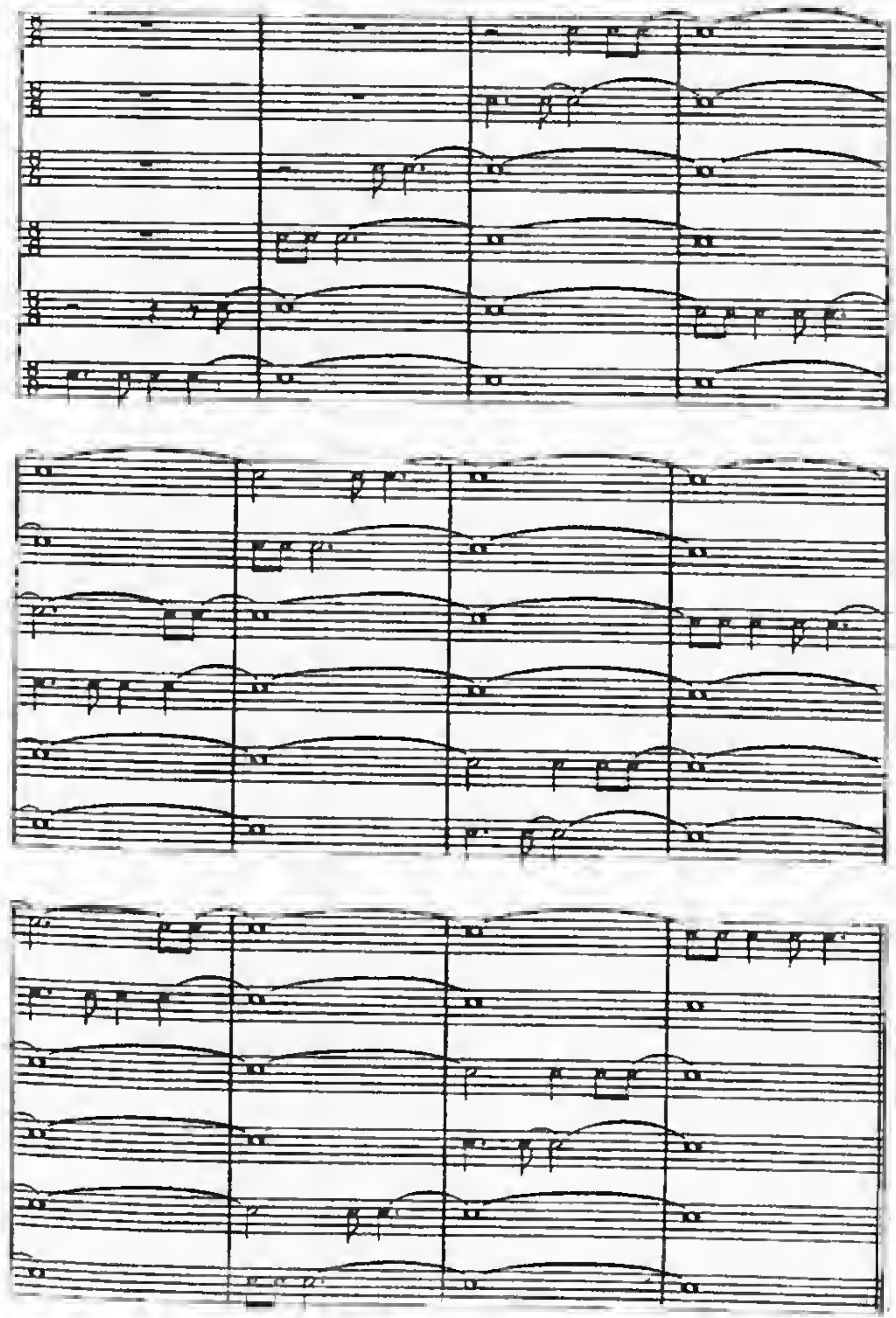


Figure 75.

CHAPTER 9

HOMOGENEOUS SIMULTANEITY AND CONTINUITY (VARIATIONS)

THE PRECEDING discussions show us that all rhythmic groups or rhythmic patterns are necessarily either the resultants of interferences or portions of such resultants.

A figure such as $2+1+1$ may be conceived as one of the elementary rhythmic patterns in $\frac{4}{4}$ time. Yet it is possible, with this method of analysis, to assign it directly to a definite place in a definite resultant—the second bar of $r_{\underline{4 \div 3}}$. The longer patterns, such as the resultants produced by higher number-values or by more than two generators, possess enough variation in themselves.

Musical memory does not emphasize a group of 20 or more bars as one indivisible pattern. Therefore, the recurrence of such pattern seems to be less monotonous than the recurrence of a short pattern. Short patterns obviously call for variations. There are many outstanding compositions in which direct recurrence of a short pattern is used throughout the entire composition—for example, the first movement of Beethoven's symphony No. 5; Chopin's waltz No. 7, the second theme. In such compositions, rhythmic monotony is usually compensated for by the variety of devices used on some other components—it may be the dynamic, the harmonic, or the melodic composition of a piece that makes this music sound interesting. The best method by which to detect the effect of the purely rhythmic patterns is to isolate them from all other components, i.e., to take a fragment of a composition, or the entire composition, and to perform the rhythm of it in a percussive manner.

The musical components of rhythm include durations, rests, accents, split-unit groups and groups in general. The inherent variability of any of these components of the time rhythm depends solely on their quantitative form, i.e., whether there are two or three, or more, elements involved in the pattern subjected to variations—for example, two elements, two durations, two forms of accent, as well as binary combinations of rests with durations, or durations with accents. The variability of groups follows the general principles of *permutations.*

A. General and Circular Permutations

There are two fundamental forms of permutations: first, general permutations; second, circular permutations (displacement). The quantity of general permutations is the product of all integers from unity up to the number expressing the quantity of the elements in a group. For example, the general number of permutations produced by 5 elements equals the product of $1 \times 2 \times 3 \times 4 \times 5$, i.e., 120. The number of circular permutations equals the number of elements in a group. Thus, five elements produce five circular permutations.

When an extremely large amount of material is used, general permutations become very practical. But in cases where limitations are imposed by a certain

type of esthetic necessity, circular permutations may solve the problem better than a vague selection from the entire number of general permutations.

In the following exposition, a bi-coordinate method will be applied to the composition of continuity. A linear sequence of the modified versions of one pattern produces the time coordinate (continuity). A correlation of the modified patterns produces the coordinate of simultaneity (or pitch). In other words, all modified forms of the original pattern may grow through the bi-coordinate system, i.e., they appear one after another in different parts, thus producing compensatory balance.

In terms of music the above simply means that a score may be evolved with a continuous variation of the original pattern following through the different parts.

Variations
2 Elements
Table of Permutations:

ab	ba

2 permutations

Examples of application:

(1) **Durations:** Binomial $2+1$ $\quad a = 2; b = 1$

$$\frac{(2+1)+(1+2)}{(1+2)+(2+1)} =$$

Figure 76.

Binomial $5+3$ $\quad a = 5; b = 3$

$$\frac{(5+3)+(3+5)}{(3+5)+(5+3)} =$$

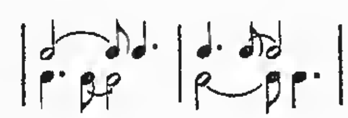


Figure 77.

(2) **Rests:** [indicated with a circle around the number]:
Binomial $1+1$ $\quad a = ①; b = 1$

$$\frac{(①+1)+(1+①)}{(1+①)+(①+1)} =$$

Figure 78.

Binomial $2+1$

$$\frac{(②+1)+(2+①)}{(2+①)+(②+1)} =$$

Figure 79.

Combined variations of durations and rests:

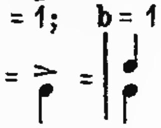
$$\frac{\frac{\frac{(2+1)+(1+2)}{(1+2)+(2+1)}}{(2+1)+(1+2)}}{(1+2)+(2+1)} =$$

Figure 80.

(3) **Accents** [through superimposition of an additional component]

Binomial 1 + 1 $a = \overset{>}{1}$; $b = 1$

a =



b =

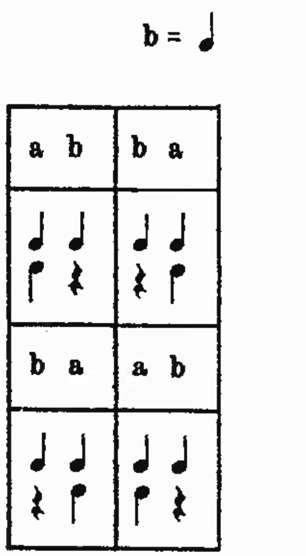
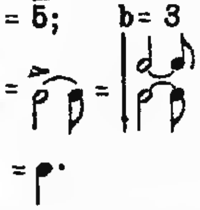


Figure 81.

Binomial 5 + 3

$a = \overset{>}{5}$; $b = 3$

a =



b =

Figure 82 (continued)

$$\frac{(\overset{>}{5}+3)+(5+\overset{>}{3})}{(5+\overset{>}{3})+(\overset{>}{5}+3)} =$$

Figure 82 (concluded).

The additional component may emphasize the entire duration of the accented attack, as in the previous example, or be considerably shorter (just to single out the moment of attack).

Example:

Binomial 2 + 1

or

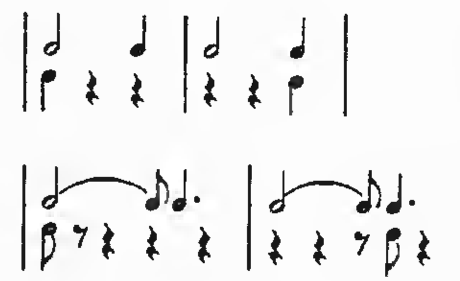


Figure 83.

Variations of rests may be combined with variations of the previous components.

(4) **Split-unit groups**

Binomial 2 + 2 a = 1 + 1
b = 2

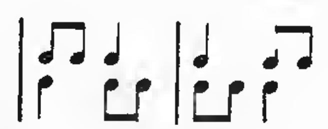


Figure 84.

When durations are non-uniform, either value may be split in a binomial.

5 + 3 a = 3 + 2 b = 3

$$\frac{[(3+2)+3]+[3+(3+2)]}{[3+(3+2)]+[(3+2)+3]} =$$

Figure 85.

$5+3 \qquad a=5 \qquad b=2+1$

$$\frac{[5+(2+1)]+[(2+1)+5]}{[(2+1)+5]+[5+(2+1)]} =$$

Figure 86.

(5) **Groups in General** .

Any rhythmic group may become an element and be permuted with its converse.

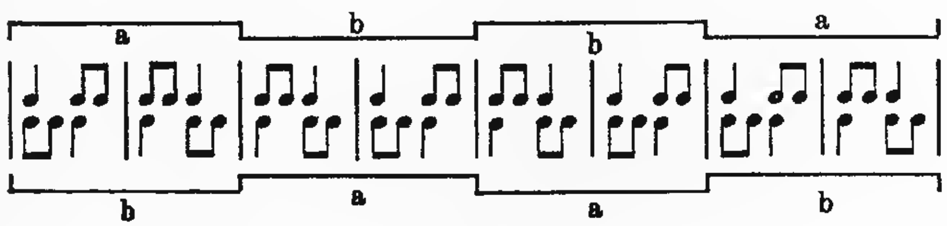


Figure 87.

* Song: "Pennies from Heaven"**

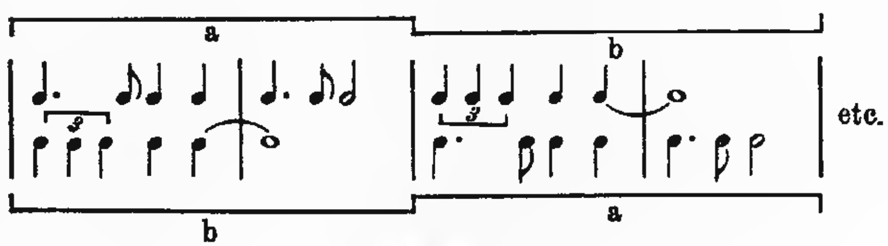


Figure 88.

r4+3

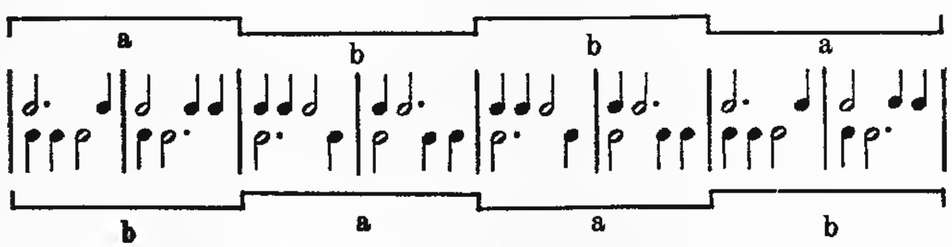


Figure 89.

*Schillinger's study of musical styles and the development of music took him from the earliest forms of recorded sound to contemporary popular American song. With an unusual catholicity of interest, Schillinger chooses illustrative materials frequently from popular songs. (Ed.)

As can be easily observed from these examples, the converse variation-group produces a rhythmic counterpart.

3 Elements

Table of General Permutations:

a b c	b a c
a c b	b c a
c a b	c b a

6 permutations

Figure 90.

Table of Circular Permutations:

(a) Clockwise circular permutations:

a b c	b c a	c a b

3 permutations

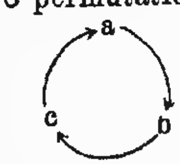


Figure 91.

(b) Counter-clockwise circular permutations:

a c b	c b a	b a c

3 permutations

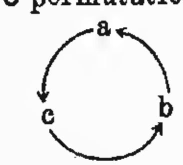


Figure 92.

When two elements in a group of three are identical, circular permutations *either* in clockwise or counter-clockwise direction are the only possible ones.

a a b	a b a	b a a

3 permutations

a b b	b a b	b b a

3 permutations

Figure 93.

Examples of Application:

(1) **Durations:**

Trinomial 2 + 1 + 1; a = 2; b = 1

(2+1+1) + (1+2+1) + (1+1+2)

(1+2+1) + (1+1+2) + (2+1+1)

(1+1+2) + (2+1+1) + (1+2+1)

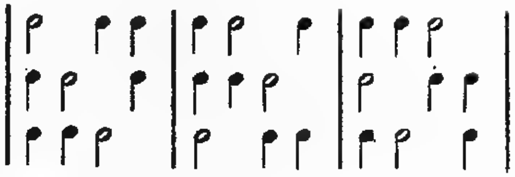


Figure 94.

Trinomial from $r_{8 \div 3}$ 3 + 3 + 2
a = 3; b = 2

(3+3+2) + (3+2+3) + (2+3+3)

(3+2+3) + (2+3+3) + (3+3+2)

(2+3+3) + (3+3+2) + (3+2+3)

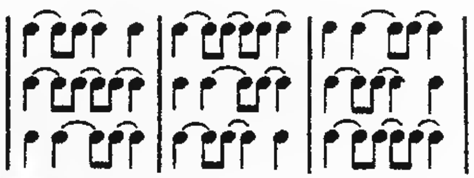


Figure 95.

Trinomial from $r_{4 \div 3}$ 3 + 1 + 2
a = 3; b = 1; c = 2

(a+b+c) + (a+c+b) + (c+a+b) + (b+a+c) + (b+c+a) + (c+b+a)

Figure 96.

Using circular permutations of this continuity, we obtain the following simultaneity:

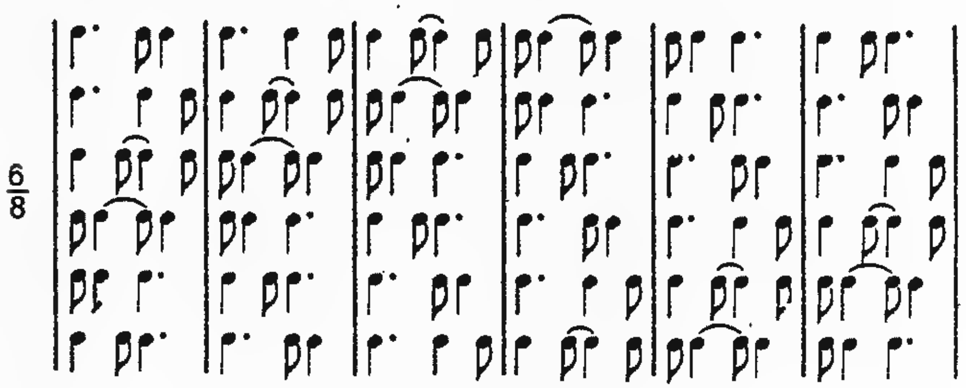


Figure 97.

(2) **Rests:**

Trinomial 1 + 1 + 1 a = (1); b = 1

((1)+1+1) + (1+(1)+1) + (1+1+(1))

(1+(1)+1) + (1+1+(1)) + ((1)+1+1)

(1+1+(1)) + ((1)+1+1) + (1+(1)+1)

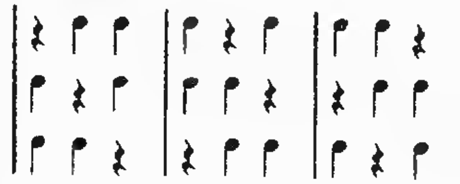


Figure 98.

Trinomial 2 + 1 + 1

((2)+1+1) + (2+(1)+1) + (2+1+(1))

(2+(1)+1) + (2+1+(1)) + ((2)+1+1)

(2+1+(1)) + ((2)+1+1) + (2+(1)+1)

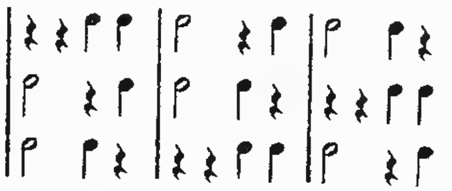


Figure 99.

(3) **Accents:**

Trinomial 1 + 1 + 1 a = $\overset{>}{1}$; b = 1

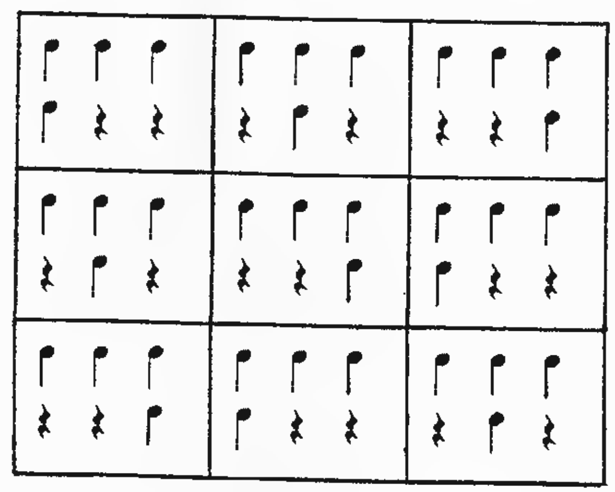


Figure 100.

Trinomial $2 + 1 + 1$

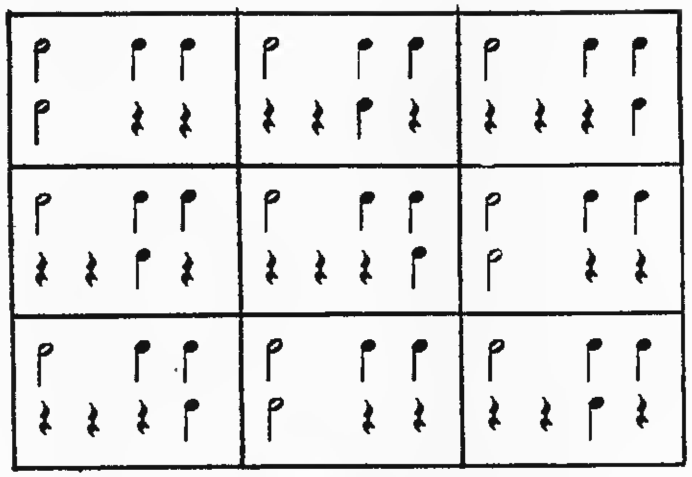


Figure 101.

Each group (with its additional component) of shifting accents may be used individually. Simultaneous application of all groups requires instruments of a different tone-quality for each group.

(4) **Split-Unit Groups:**

Trinomial $2 + 2 + 2$ $a = 1 + 1$; $b = 2$

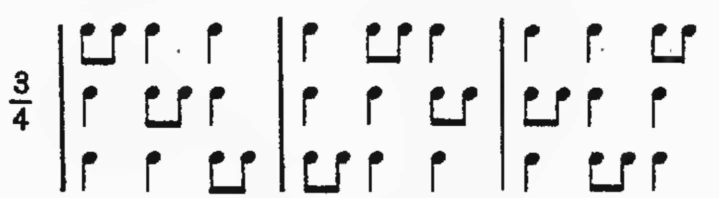


Figure 102.

Trinomial $4 + 1 + 3$ $a = 4$; $b = 1$; $c = 2 + 1$

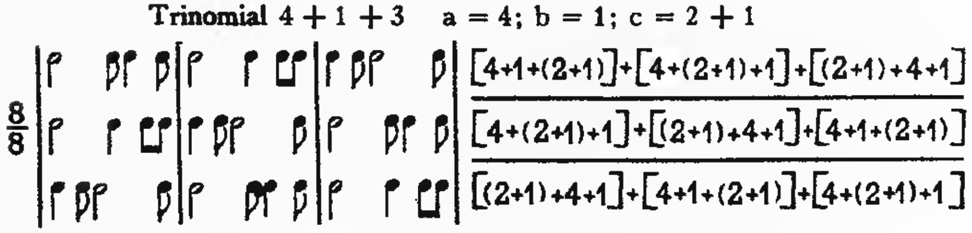


Figure 103.

(5) **Groups in general:**

$a = 2 + 1 + 1$; $b = 1 + 2 + 1$;. $c = 1 + 1 + 2$

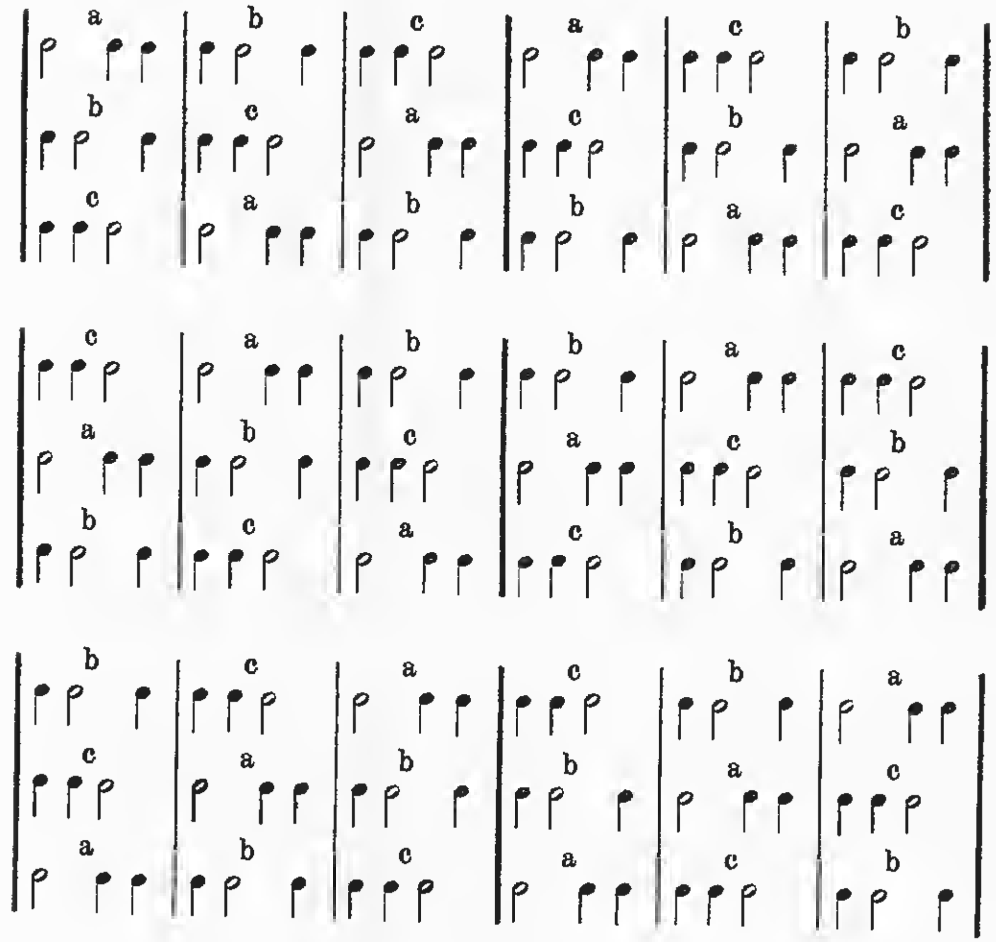


Figure 104.

Song: "Pennies from Heaven"*

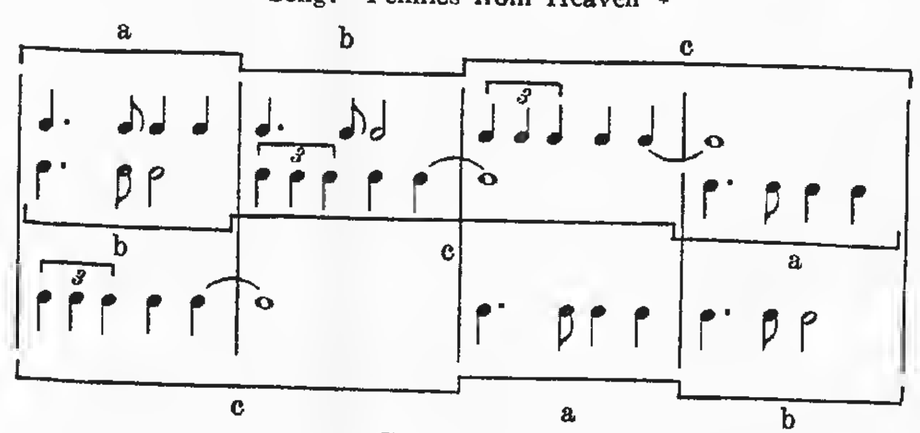


Figure 105.

4 Elements
Table of Permutations:

(1) **All elements different:**

abcd	acbd	cabd	bacd	bcad	cbad
abdc	acdb	cadb	badc	bcda	cbda
adbc	adcb	cdab	bdac	bdca	cdba
dabc	dacb	dcab	dbac	dbca	dcba

24 permutations

Figure 106.

(2) **Two elements identical:**

aabc	aacb	abac
abca	acba	baca
bcaa	cbaa	acab
caab	baac	caba

12 permutations

Figure 107.

(3) **Two pairs identical:**

aabb	abab
abba	baba
bbaa	
baab	

6 permutations

Figure 108.

(4) **Three elements identical:**

aaab	aaba	abaa	baaa

4 permutations

Figure 109.

Assuming that any of the permutations is an original group, each of the above groups may be limited to four circular permutations.

Example:

abcd	bcda	cdab	dabc

Figure 110.

Examples of application:

(1) **Durations:**

(a) All four elements different.

Quadrinomial from $r_{5\div4}$; $4+1+3+2$

$a=4;\ b=1;\ c=3;\ d=2$

$$(4+1+3+2)+(4+1+2+3)+(4+2+1+3)+(2+4+1+3)+$$
$$+(4+3+1+2)+(4+3+2+1)+(4+2+3+1)+(2+4+3+1)+$$
$$+(3+4+1+2)+(3+4+2+1)+(3+2+4+1)+(2+3+4+1)+$$
$$+(1+4+3+2)+(1+4+2+3)+(1+2+4+3)+(2+1+4+3)+$$
$$+(1+3+4+2)+(1+3+2+4)+(1+2+3+4)+(2+1+3+4)+$$
$$+(3+1+4+2)+(3+1+2+4)+(3+2+1+4)+(2+3+1+4)$$

This 24-group continuity produces a 24-part simultaneity in 24 bars of $\frac{10}{8}$ time.

By limiting the original group $(4+1+3+2)$ to circular clockwise permutations, we obtain 4 parts in 4 bars of $\frac{10}{8}$ time.

(b) Two elements identical.

Quadrinomial from $r_{4\div3}$; $3+1+2+2$

$a=2;\ b=3;\ c=1$

Form: $b+c+a+a$

Starting with the third permutation of the corresponding table, we obtain:

$$(3+1+2+2)+(1+2+2+3)+(2+2+1+3)+(2+1+3+2)+$$
$$+(1+3+2+2)+(3+2+2+1)+(2+3+2+1)+(3+2+1+2)+$$
$$+(2+1+2+3)+(1+2+3+2)+(2+2+3+1)+(2+3+1+2)$$

This 12-group continuity produces a 12-part simultaneity in 12 bars of $\frac{8}{8}$ time or in 24 bars of $\frac{4}{4}$ time.

(c) Two pairs identical.

Quadrinomial $r_{3\div2}$; $2+1+1+2$

$a=2;\ b=1$

Form: $a+b+b+a$

Starting with the second permutation of the corresponding table, we obtain:

$$(2+1+1+2)+(1+1+2+2)+(1+2+2+1)+(2+1+2+1)+$$
$$+(1+2+1+2)+(2+2+1+1)$$

This 6-group continuity produces a 6-part simultaneity in 6 bars of $\frac{6}{8}$ time or in 12 bars of $\frac{3}{4}$ time (1 = ♩). Clockwise circular permutations give 4 parts in 4 bars of $\frac{6}{8}$ time or 4 parts in 8 bars of $\frac{3}{4}$ time (1 = ♩).

(d) Three elements identical.

Quadrinomial: $3 + 1 + 1 + 1$
$a = 1; b = 3$
Form: $b + a + a + a$

Starting with the fourth permutation of the corresponding table we obtain:

$(3+1+1+1) + (1+1+1+3) + (1+1+3+1) + (1+3+1+1)$

This 4-group continuity produces a 4-part simultaneity in 4 bars of $\frac{6}{8}$ time or in 8 bars of $\frac{3}{4}$ time.

Assigning different symbols to the same group we obtain the form $a + b + b + b$.

Then: $(a+b+b+b) + (b+a+b+b) + (b+b+a+b) + (b+b+b+a)$

This produces a continuity of perfect musical quality:

Figure 111.

Similar modification of the symbols assigned is possible with any group containing identical terms.

(2) **Rests:**

Quadrinomial: $1 + 1 + 1 + 1$
$a = ①; b = 1$

$(①+1+1+1) + (1+①+1+1) + (1+1+①+1) + (1+1+1+①)$
$(1+①+1+1) + (1+1+①+1) + (1+1+1+①) + (①+1+1+1)$
$(1+1+①+1) + (1+1+1+①) + (①+1+1+1) + (1+①+1+1)$
$(1+1+1+①) + (①+1+1+1) + (1+①+1+1) + (1+1+①+1)$

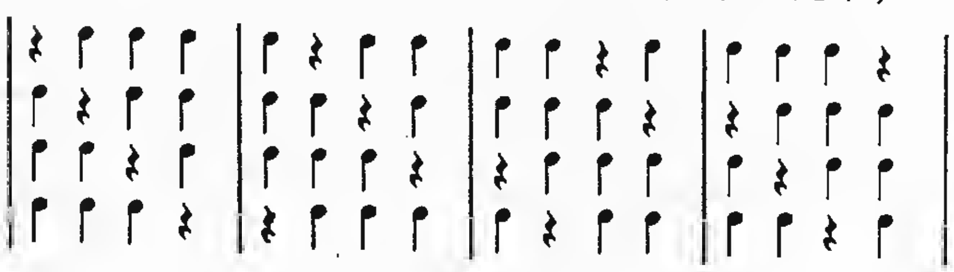


Figure 112.

Analogous permutations of rests may be devised in non-uniform groups.

(3) **Accents:**

Quadrinomial: $1 + \overset{>}{1} + 1 + 1$
$a = \overset{>}{1}; b = 1$

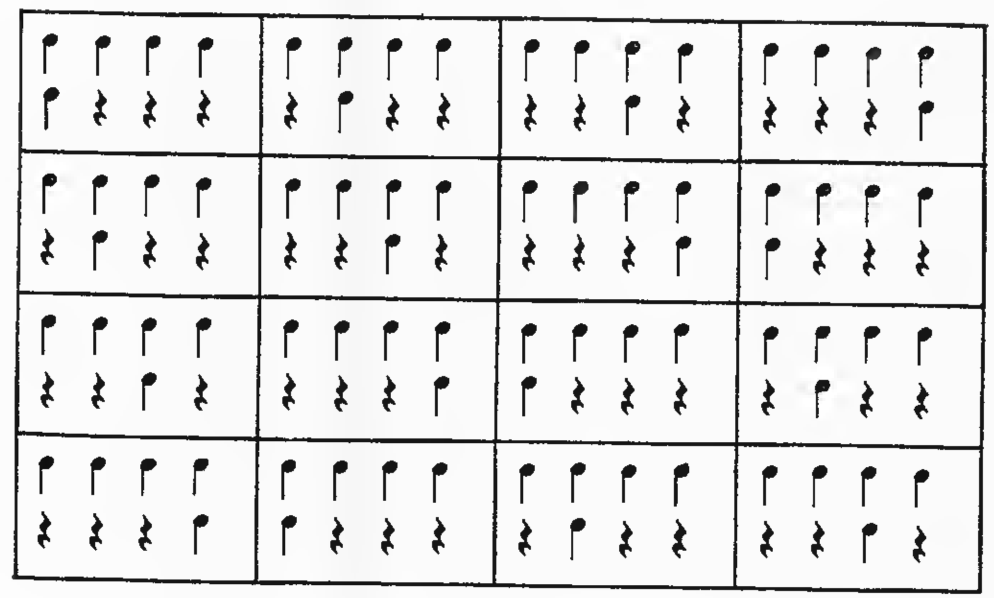


Figure 113.

Analogous permutations of accents may be devised in non-uniform groups.

(4) **Split-unit groups:**

Quadrinomial: $2 + 2 + 2 + 2$
$a = 1 + 1; b = 2$

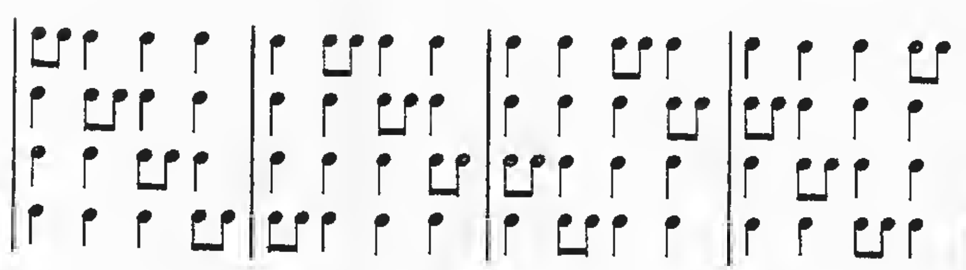


Figure 114.

Analogous permutations may be devised in non-uniform groups originally consisting of four places.

Example: $r_{\frac{5\div4}{2}} = 4 + 1 + 3 + 2$

Either of the numbers may be split into a group:

(a) $4 = 2+2$	$(2+2)$	$+1+3+2$
$4 = 2+1+1$	$(2+1+1)$	$+1+3+2$
$4 = 1+2+1$	$(1+2+1)$	$+1+3+2$
$4 = 1+1+2$	$(1+1+2)$	$+1+3+2$
$4 = 1+1+1+1$	$(1+1+1+1)$	$+1+3+2$
(b) $1 = \frac{1}{2}+\frac{1}{2}$	$4+(\frac{1}{2}+\frac{1}{2})+3+2$	
(c) $3 = 2+1$	$4+1+(2+1)+2$	
$3 = 1+2$	$4+1+(1+2)+2$	
$3 = 1+1+1$	$4+1+(1+1+1)+2$	
(d) $2 = 1+1$	$4+1+3+(1+1)$	

Any of these versions may be used. Each version contains 4 circular and 24 general permutations.

(5) **Groups in general:**

Figure 115.

This group produces the following simultaneity and continuity:

Simultaneity—4 parts.
Continuity through circular permutations—16 bars.
Continuity through general permutations—96 bars.

The original 4 bars take the appearance of the example in (4) [split-unit groups].

Any rhythmic resultant placed in 4 bars may constitute such a group. For example:

$r_{4\div3}$ grouped by a in $\frac{4}{4}$ time:

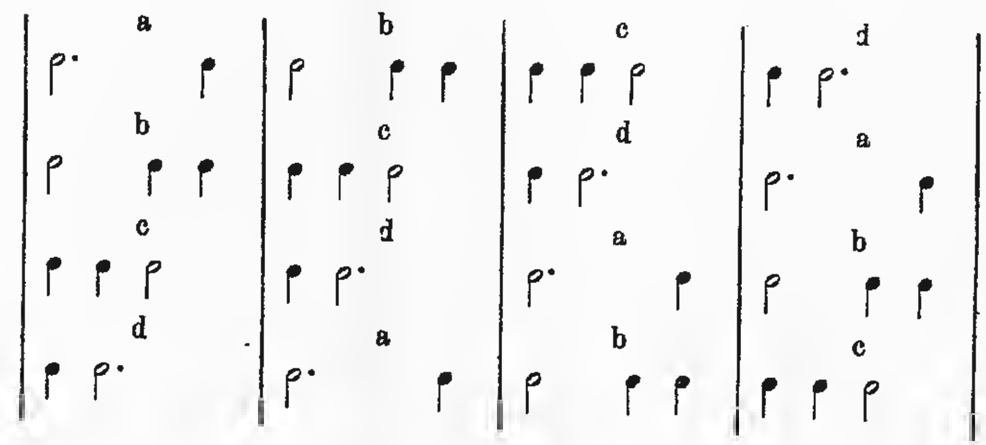


Figure 116.

The number of variations is the same as in the preceding group.

A group consisting of 4 elements may be produced from any rhythmic resultant, providing a non-uniform distribution is applied:

For example: $r_{5\div3}$ grouped by b in $\frac{3}{4}$ time:

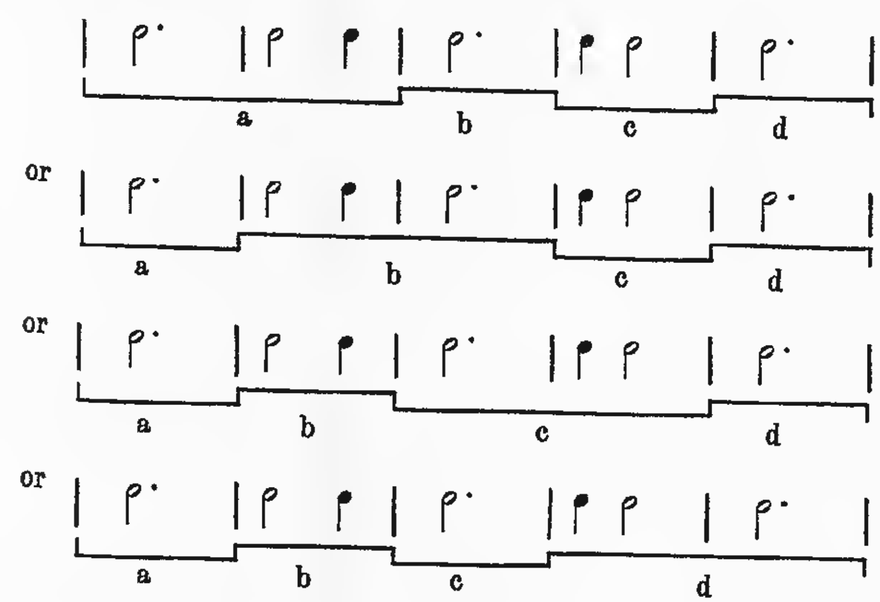


Figure 117.

Example from the popular song, *Pennies From Heaven.** When necessary a tie between the notes may be omitted, though it is not necessary if the same group repeats itself.

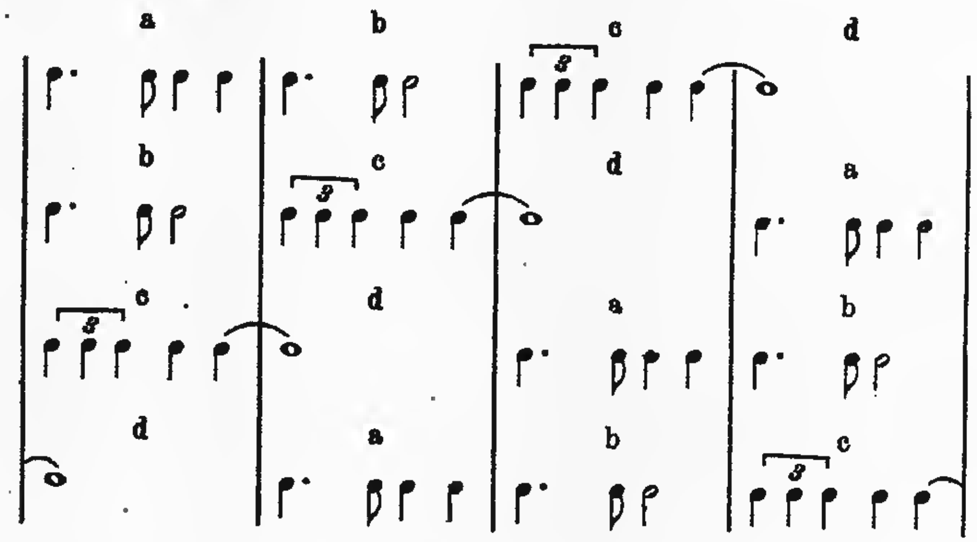


Figure 118.

You can see what extraordinary variety may be secured by a group as simple as this through this variation method.**

As the general *velocity* of musical time (tempo) is most essential in establishing one or another characteristic, many of the preceding examples, although similar in numbers, produce musical continuities as remote from each other in character as Händel is remote from the Cuban rhumba.

For example the group: (①+1+1+1) + (1+①+1+1) + (1+1+①+1) + (1+1+1+①) being written and performed as *largo* in $\frac{4}{4}$ time.


as compared with:

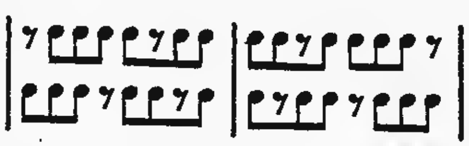

in the tempo of a fast rhumba.

Figure 119.

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**The tables are worked out in detail on these and other pages not only for the sake of clarity; it is also a way of furnishing the practical composer with ready-made calculations so that each pattern need not be re-calculated whenever it is needed in actual composition. The time-saving way is to refer to the tables in this book, although a composer should also know how to calculate them afresh for himself, if necessary. (Ed.)

CHAPTER 10

GENERALIZATION OF VARIATION TECHNIQUES

A. Permutations of the Higher Order.

In order to increase the quantity of material evolving through the variation method from the original group, the method of *permutations of a higher order* may be used. The original element or group produces variations which in turn become the elements of the next order. The quantity of elements in the next successive order equals the square of the number of the elements of the preceding order. If the original number of elements in a group is 3, there will be 9 elements on the second order, 27 on the third, etc., through circular permutations. If the original number of elements in a group is 3, and general permutations are used, this will give 6 elements in the second order, 720 in the third order, etc.

Indicating the original elements as *a* of the first order (a_1), *b* of the first order (b_1) . . . and permuting them, the elements of the following order, which represent a group of the elements of the preceding order, are acquired. The technique of evolving the elements of the following order acquires this appearance:

$$a_1 + b_1 = a_2$$
$$\underline{b_1 + a_1 = b_2}$$
$$a_2 + b_2 = a_3$$
$$\underline{b_2 + a_2 = b_3}$$
$$\dots\dots\dots\dots$$
$$\boxed{\begin{array}{c} a_{n-1} + b_{n-1} = a_n \\ b_{n-1} + a_{n-1} = b_n \end{array}}$$

Figure 120.

This device is particularly important when one wishes to evolve a large quantity of material from the original group, or when the number of elements in the original group is exceedingly small. If the procedure of the permutations carried out through the sixth order concerned only 2 elements in a group, we would obtain ultimately only $2^6 = 64$ elements.

Music of animated motion often contains a much greater quantity of rhythmic elements (durations, rests, etc.). For example, take an average waltz. In ordinary printing we get at least 4 bars to a line, and 5 lines to a page. In music moving in eighth notes for 3 pages, we would get 360 durations

Example:

Application of the Permutations of the Higher Orders to the Original Group.

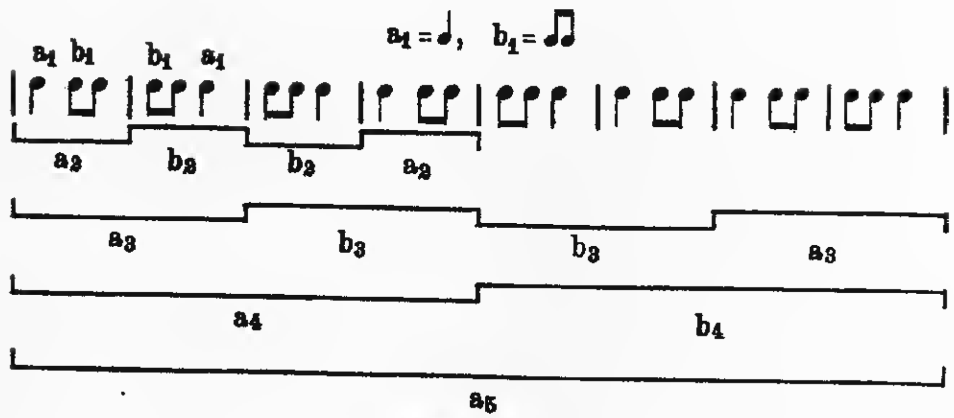

Figure 121.

Moving for 8 more bars, i.e., carrying out the permutations of the 4th order to their completion, we obtain 16 bars containing great variety as compared to the usual continuous recurrence. This device is particularly useful when the character of music must be retained for considerably longer time than the original rhythmic group permits. Instead of making continuous repetitions of the original group, or recurrences of the larger groups, it is possible with this device to go on continuously for an indefinite period of time.

In musical backgrounds for motion picture photoplays, when the scene develops in a definite locality—associated with definite rhythmic forms of expression, it may be desirable to extend this homogeneous rhythmic character to 10 or 15 minutes. In the case of a "Cuban" scene, rhumba rhythms are considered characteristic of the locality. The audience is distracted from action on the screen by the musical background when a definite dance composition is played repeatedly. This annoys the audience and never helps to bring out the dramatic plot. On the contrary, it produces conflicts with the plot. A neutral background, being homogeneous and yet continuously varied, will serve the purpose much better.

Permutations of the higher orders based on the original group with 3 elements (a_1, b_1, c_1) offer the following combinations by 2: $a_1 + b_1$, $a_1 + c_1$, $b_1 + c_1$. These are the three possible alternatives when 2 elements out of 3 are used. The 2 elements form a group of 3, following the regulations described in the preceding paragraph concerning the higher orders of the 2 elements.

The original group containing 3 elements has only one combination by 3: $a_1 + b_1 + c_1$. The second order permutations on the 3 elements appear as follows:

$$a_1 + b_1 + c_1 = a_2$$
$$a_1 + c_1 + b_1 = b_2$$
$$c_1 + a_1 + b_1 = c_2$$
$$b_1 + a_1 + c_1 = d_2$$
$$b_1 + c_1 + a_1 = e_2$$
$$c_1 + b_1 + a_1 = f_2$$

These 6 elements of the second order produce, in turn, combinations by 2, by 3, by 4, by 5 and by 6.

Combinations by 2:

$a_2 + b_2$	$b_2 + c_2$	$c_2 + d_2$	$d_2 + e_2$	$e_2 + f_2$
$a_2 + c_2$	$b_2 + d_2$	$c_2 + e_2$	$d_2 + f_2$	
$a_2 + d_2$	$b_2 + e_2$	$c_2 + f_2$		
$a_2 + e_2$	$b_2 + f_2$			
$a_2 + f_2$				

The total number of cases: $15 \times 2 = 30$

Combinations by 3:

$a_2 + b_2 + c_2$	$a_2 + c_2 + d_2$	$a_2 + d_2 + e_2$	$a_2 + e_2 + f_2$
$a_2 + b_2 + d_2$	$a_2 + c_2 + e_2$	$a_2 + d_2 + f_2$	
$a_2 + b_2 + e_2$	$a_2 + c_2 + f_2$		
$a_2 + b_2 + f_2$			
$b_2 + c_2 + d_2$	$b_2 + d_2 + e_2$	$b_2 + e_2 + f_2$	
$b_2 + c_2 + e_2$	$b_2 + d_2 + f_2$		
$b_2 + c_2 + f_2$			
$c_2 + d_2 + e_2$	$c_2 + e_2 + f_2$		
$c_2 + d_2 + f_2$			
$d_2 + e_2 + f_2$			

The total number of cases: $20 \times 6 = 120$

Combinations by 4:

$a_2 + b_2 + c_2 + d_2$	$a_2 + c_2 + d_2 + e_2$	$a_2 + d_2 + e_2 + f_2$
$a_2 + b_2 + c_2 + e_2$	$a_2 + c_2 + d_2 + f_2$	
$a_2 + b_2 + c_2 + f_2$		
$a_2 + b_2 + d_2 + e_2$	$a_2 + c_2 + e_2 + f_2$	
$a_2 + b_2 + d_2 + f_2$		
$a_2 + b_2 + e_2 + f_2$		
$b_2 + c_2 + d_2 + e_2$	$b_2 + d_2 + e_2 + f_2$	
$b_2 + c_2 + d_2 + f_2$		
$b_2 + c_2 + e_2 + f_2$		
$c_2 + d_2 + e_2 + f_2$		

Total number of cases: $15 \times 24 = 360$

Combinations by 5:

$a_2 + b_2 + c_2 + d_2 + e_2$ $\quad a_2 + b_2 + d_2 + e_2 + f_2$ $\quad a_2 + c_2 + d_2 + e_2 + f_2$

$a_2 + b_2 + c_2 + d_2 + f_2$

$a_2 + b_2 + c_2 + e_2 + f_2$

$b_2 + c_2 + d_2 + e_2 + f_2$

Total number of cases: $6 \times 120 = 720$

Combinations by 6:

$a_2 + b_2 + c_2 + d_2 + e_2 + f_2$

Total number of cases: $1 \times 720 = 720$

All the recurring elements are eliminated from these charts, which may be consulted for coefficients of recurrence. For example, a trinomial combination from 2 elements, a_1 and b_1, with a coefficient 2 for the first element becomes $2a_1 + b_1$. This is a trinomial with 2 identical elements, and is subjected to circular permutations only. Similar cases occur with 4 elements having 2 identical terms, 2 identical pairs or 3 identical terms. Similar cases occurring with 5 and 6 elements may contain 2, 3 and more identical elements. They will be treated as coefficients of recurrence.

Example:

Trinomial of the Third Order

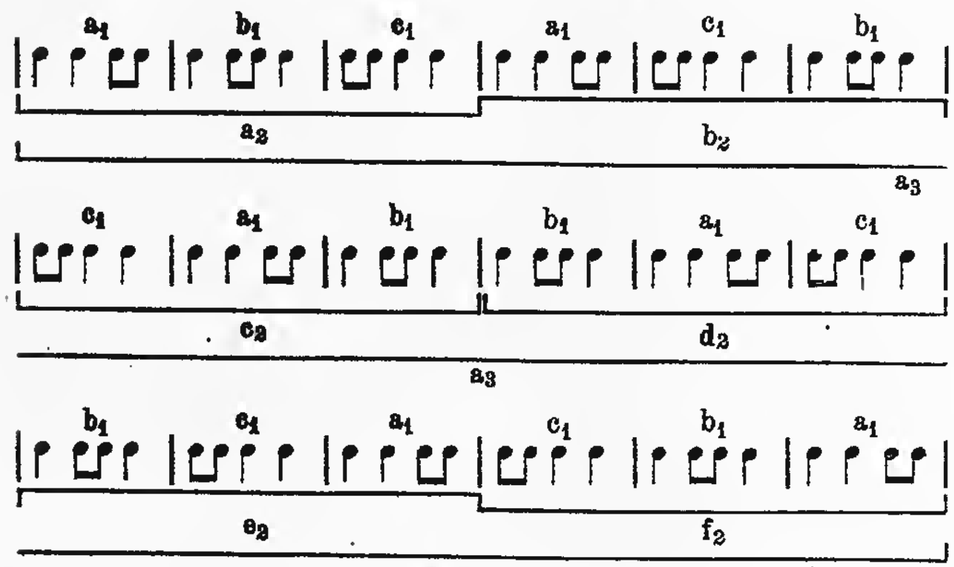

Figure 122.

When the quantities exceed the necessary amount, one can limit the number of variations by reducing them to circular permutations only. The illustrations above are applicable to rests, accents and other group formations.

CHAPTER 11

COMPOSITION OF HOMOGENEOUS RHYTHMIC CONTINUITY

ANY rhythmic group may be adapted to the processes of growth in simultaneity and continuity. There are three fundamental procedures, varying with regard to the quantity of material to be evolved. The first process gives the minimum quantity; the second, the intermediate; and the third, the maximum quantity. Select them in accordance with the requirements of each specific case.

(1) We may produce elements from a given rhythmic group by means of splitting the group through the *simplest divisor*. For example, the group $r_{\underline{4 \div 3}}$ (grouped by 4) represents a 4-bar continuity in $\frac{4}{4}$ time. 4 may be divided by 2 and thus we obtain two groups: a_1 comprising the first two bars, and b_1 comprising the second two bars. This gives us an 8-bar, 2-part continuity, i.e., the quantity of the original material is doubled both in simultaneity and continuity.

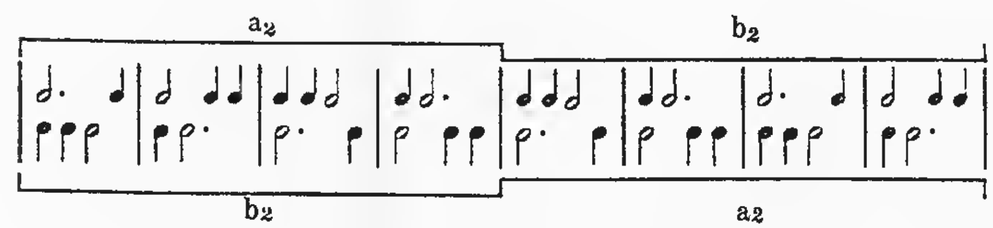

Figure 123.

When a group is not divisible by 2, like $r_{5 \div 3}$ (grouped by 5), it may become divisible by 3. In this case it produces 3 bars in $\frac{5}{4}$, $\frac{5}{8}$ or any other quintuple time—the first bar being a_1, the second b_1 and the third c_1.

(2) We may produce elements from a given rhythmic group by means of splitting it through individual bars. For example, in $r_{\underline{4 \div 3}}$ grouped in 4 bars, each individual bar becomes an element. The first is a_1, the second b_1, the third c_1, and the fourth d_1. This splitting process produces a 16-bar continuity in 4 parts—i.e., both simultaneity and continuity of the original group become quadrupled.

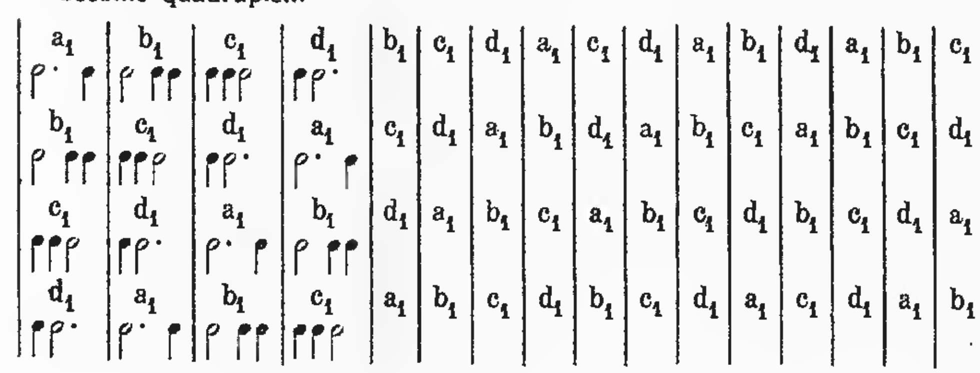

Figure 124.

This continuity is the result of circular permutations. Using general permutations for this group, and splitting it in this particular fashion, we obtain 4 bars in 4 parts with 24 different variations, i.e., 96 bars in 4 parts. In a case in which the simplest divisor corresponds to the splitting by individual bars, as in the above-mentioned case of r_{5+3}, this becomes the only possible procedure.

Any bar splitting will ultimately give a score in which the number of parts equals the number of bars, and the number of bars equals the number of circular or general permutations available for such number. For example, taking $\underline{r_{8+5}}$ and having it grouped in 8 bars, we obtain 8-part simultaneity in 64-bar continuity through circular permutations, and 322,560 bar continuity through general permutations as the total number of permutations of 8 elements equals 40,320.

(3) We may produce elements from a given rhythmic group by means of splitting the group through the *individual attacks* (terms). For example, if we take the group $\underline{r_{4+3}}$, we obtain 10 individual terms. These 10 terms are subjected to growth in simultaneity and continuity. The original group arranged in 4 bars of the $\frac{4}{4}$ time produces a 10-part simultaneity. These 4 bars evolve into a 40-bar continuity (4 × 10). Thus the total original score has 40 bars in 10 parts.

$\underline{r_{4+3}}$	3	1	2	1	1	1	1	2	1	3
	a_1	b_1	c_1	d_1	e_1	f_1	g_1	h_1	i_1	j_1

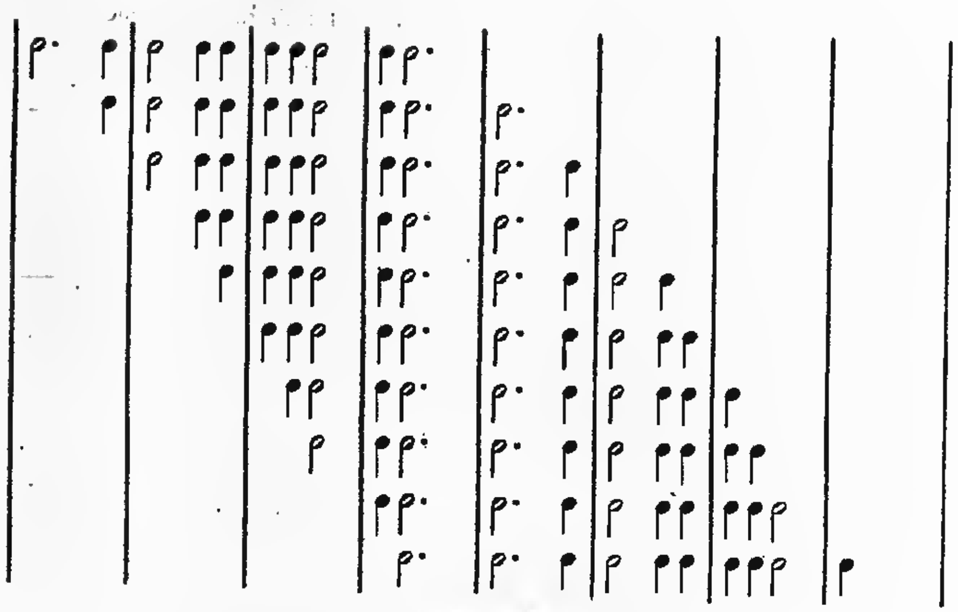

Figure 125 (continued).

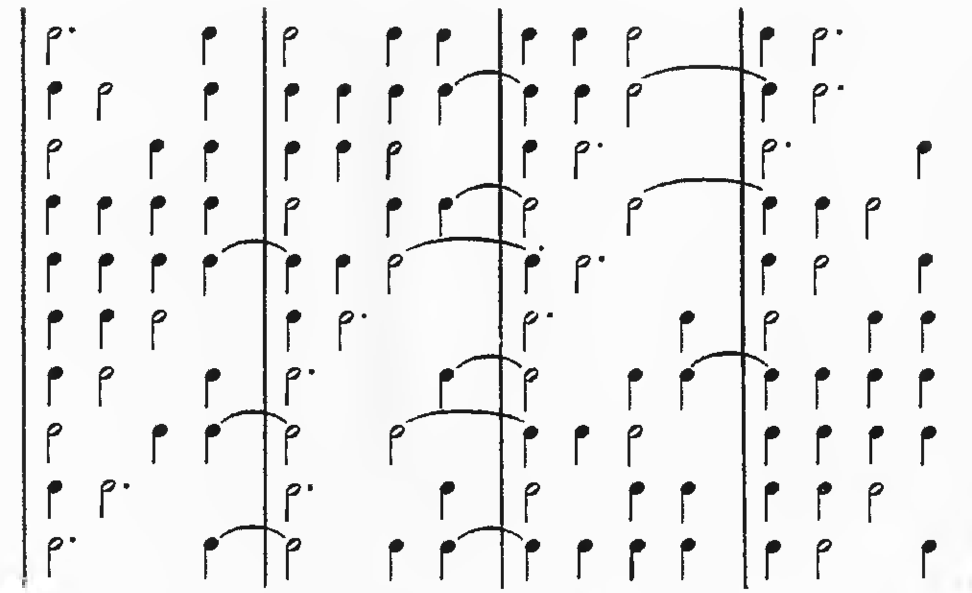

Figure 125(concluded).

While in this case there is a coincidence of the figure 1 + 2 + 1, the number of parts moving simultaneously obscures it entirely to the human ear. This 40-bar, 10-part score produces 10 elements of 4 bars each. 10 elements give 3,628,800 permutations, which give a total of 145,152,000 bars in 10 parts.

CHAPTER 12

DISTRIBUTIVE POWERS

A. Continuity of Harmonic Contrasts

THE PROBLEM of producing contrasts in a rhythmic continuity concerns the two fundamental methods of evolving rhythm: *one*, the patterns generated through the common denominator and constituting rhythmic continuity within musical measures ($\frac{t}{t}$); and *two*, the patterns of the measures themselves growing into a complete form expressing the rhythm of measures and of groups of measures. The first form of continuity is called *fractional continuity;* the second, *factorial continuity.**

While rhythm evolves within musical measures, musical measures themselves also evolve their own rhythm. The correlation of the two in time sequence will be incorporated into the *series of factorial-fractional continuity*. Homogeneous series of factorial-fractional continuity are *power-series*. The original value ($\frac{t}{t}$) represents the *determinant of a series*. Powers express the evolution of a number through its own continuous factoring. Algebraic treatment of the power processes is quantitative and—being applied—does not bring the solution of esthetic problems. Esthetic problems are essentially the problems of *distribution* and *coordination*, and not problems of mere quantity. The process of evolving any initial ratio through its own factoring lies within the field of *distributive powers*.** The distributive powers organize not only the value but also the *quantity of the values* harmonically. Any binomial under distributive powers becomes a quadrinomial on the square ($2^2 = 4$). It becomes a group of 8 terms on the cube; a trinomial becomes a polynomial with 9 terms on the square ($3^2 = 9$). . . . It happens to be the fact that the art of music, with regard to its rhythm, has not yet exceeded the series with the $\frac{2}{2}$ determinant. In the later exposition of the evolution of rhythmic families, this subject will be treated in detail.

*This is an idea fundamental to Schillinger's system. He does not regard what is ordinarily called "musical form"—i.e., the organization of the entire composition by "phrases," etc.—as something separate from the rhythms of the measures themselves. Rather, he regards the two — fractional (rhythms *within* the measure) and factorial (rhythms *of* the measures)—as two aspects of the same central situation. (Ed.)

**For example, the algebraic square of a + b is: $a^2 + 2ab + b^2$. But the *distributive* square would be $a^2 + ab + ab + b^2$. In other words, the magnitudes are *distributed* rather than being grouped after coefficients. The use by Schillinger of *distributive powers* is one of the most extraordinary aspects of his system; these are used as well in other arts, especially in the spatial arts. (Ed.)

The following is a survey of the series which have been employed to date from the beginning of music by the inhabitants of this planet.

$\frac{1}{t^n}$ . . .	$\frac{1}{t^3}$ . . .	$\frac{1}{t^2}$ . . .	$\frac{1}{t}$ . . .	$\frac{t}{t}$ . . .	t . . .	t^2 . . .	t^3 . . .	t^n
. . . $\frac{1}{16}$ . . .	$\frac{1}{8}$ . . .	$\frac{1}{4}$ . . .	$\frac{1}{2}$ . . .	$\frac{2}{2}$ . . .	2 . . .	4 . . .	8 . . .	16 . . .
	. . . $\frac{1}{27}$ . . .	$\frac{1}{9}$ . . .	$\frac{1}{3}$ . . .	$\frac{3}{3}$ . . .	3 . . .	9 . . .	27 . . .	
	. . . $\frac{1}{64}$ . . .	$\frac{1}{16}$ . . .	$\frac{1}{4}$ . . .	$\frac{4}{4}$ . . .	4 . . .	16 . . .	64 . . .	
		. . . $\frac{1}{25}$ . . .	$\frac{1}{5}$ . . .	$\frac{5}{5}$ . . .	5 . . .	25 . . .		
		. . . $\frac{1}{36}$ . . .	$\frac{1}{6}$ . . .	$\frac{6}{6}$ . . .	6 . . .	36 . . .		
		. . . $\frac{1}{49}$ . . .	$\frac{1}{7}$ . . .	$\frac{7}{7}$ . . .	7 . . .	49 . . .		
		. . . $\frac{1}{64}$ . . .	$\frac{1}{8}$ . . .	$\frac{8}{8}$ . . .	8 . . .	64 . . .		
		. . . $\frac{1}{81}$ . . .	$\frac{1}{9}$ . . .	$\frac{9}{9}$ . . .	9 . . .	81 . . .		

. . . .

Each of the series whose determinants are indicated above represents centuries, sometimes millennia, of musical evolution. The most familiar of all is the series with the determinant 2. The $\frac{2}{2}$ series represents our own musical civilization, known to us as an important and glorious period of musical history, but it certainly does not appear very inventive from the viewpoint of objective analysis. The $\frac{2}{2}$ series has, in fact, caused more damage to the evolution of our musical culture than it has helped the development of our culture—with respect to rhythm. The number-values found on the right side of the determinant represent the constant growth of *factorial* groups (measures and their multiples). The left side represents the formation of rhythmic patterns within each consecutive measure.

The real reason for our musical civilization's being so elementary is the system of notation evolved in Europe during recent centuries. Just a few hundred years ago, the very idea of recording rhythm in relative durations seemed to be quite revolutionary (Mensural system). Even in our own day our schools teach that a whole note consists of two half-notes, and the two half-notes consist of four quarter notes, etc. But why mention notes, when this is an ordinary process of arithmetical division by two? The habit of thinking in two's and their multiples has retarded the development of our musical civilization to such an extent that the rhythms used on the African continent, perhaps thirty thousand years ago, seem to us to be quite exciting even today. The general field of classical music deals with division by two and multiplication by two. All classical rhythmic patterns are based on halfs, quarters, eights, sixteenths, etc. Measures accumulate through the same multiples.

Measure-groups (known as "phrases") appear in 2's, 4's, 8's, etc. The inefficiency of the accepted system of musical notation was sufficiently discussed at the beginning of this theory.* When a symbol called a quarter-note appears in musical writing, such a quarter-note does not necessarily represent a quarter of anything. It may be a half, a third, a fifth, or any other fraction.

*See Chapter 1 of *Theory of Rhythm*. See also Chapter 2 of Book IV, *Theory of Melody* which presents a history of musical notation and fully describes the inadequacy of the accepted system of notation that caused Schillinger to search for a new system. (Ed.)

Classical music developed very little efficiency in the $\frac{3}{3}$ series. The right side is entirely untouched, because when we find 3-bar phrases in such music, it is usually a 4-bar or 2-bar phrase—modified by means of expansion or contraction. The left side of the $\frac{3}{3}$ series is somewhat better developed. There are bars with three beats ($\frac{3}{4}$ time), there are bars with nine beats ($\frac{9}{8}$ time), and there are even a few rare cases when $\frac{27}{16}$ appears as a musical measure, as in some works of J. S. Bach. If music has been developed so consistently up to the seventh power on the determinant $\frac{2}{2}$, why should it not develop with the same consistency on any other determinant in use? Why has the $\frac{3}{3}$ series reached only its cube, and that only on very rare occasions? Why has it not developed beyond the first power on the factorial side? The answer is obvious: it is the system of musical notation attached to the $\frac{2}{2}$ determinant that has caused this conservatism.

Racial and national instincts in music, in contrast to acquired musical theories, work with much greater consistency although evolution by this means often requires centuries. Some of the American Indians, for example, exhibited such a degree of consistency with regard to the $\frac{3}{3}$ series. Their evolution did not reach high powers, yet these Indians are uniformly consistent as to both the factorial and fractional side. They use the first, the second, and the third powers of the above-mentioned series—see the musical example in Helen Roberts' book, *Form in Primitive Music*, page 39.*

The $\frac{4}{4}$ series, being a multiple of the $\frac{2}{2}$ determinant, does not exhibit strikingly new characteristics. We find such music frequently in many compositions. Groups like 4 + 1 + 1 + 1 + 1 may be found in any music entitled "March" (♩ ♬♬). The accumulation of bars in groups of 4 and 16 is also quite common.

Classical music of the past evolving from the series with the $\frac{3}{3}$ determinant, deviating from the natural consistency of powers, resorts to simplification. The common case of music written in $\frac{3}{4}$ time is not a quarter-note split into a triplet of eighths but into two eighths. This means that $\frac{1}{3}$, instead of being multiplied by $\frac{1}{3}$ and becoming $\frac{1}{9}$, is multiplied by $\frac{1}{2}$ and becomes $\frac{1}{6}$. This is typical of a hybrid resulting from ***unintentional*** simplification. The eighth in $\frac{3}{4}$ time more frequently becomes ♫ , and not ♬ as it should.

This tendency toward simplification is philosophically puzzling. We must ask: is this number 2 an ***unavoidable*** condition in evolving any series, as in the multiplication of spermatozoa, microbes and lower organisms? Or is the determinant 2 not as vital as it may seem at first, merely serving as an outlet for simplification?

If the former were true, one could not find any pure folk music with any other form of fractional development than that achieved through the determinant 2. Yet the music of Hindustan, very old and very traditional, uses a great many triplets representing a split-unit group of one beat in $\frac{5}{4}$ time (♩ = ♫) This shows that the $\frac{5}{5}$ determinant, which is characteristic of many old Asiatic civilizations, acquires its simplified fractioning through the $\frac{3}{3}$ series, i.e., $\frac{1}{5}\times\frac{1}{3}=15$. The $\frac{5}{5}$ series, in addition to being characteristic of Hindustan, is also characteristic of Java, Bali and Siam, whence it moved westward influencing Afghanistan, Persia, Arabia and Russia.

*American Library of Musicology, 1932.

The present state of development of the $\frac{5}{5}$ series is still very elementary. Five-bar groups are as rare as the quintuplets in $\frac{5}{5}$ time (♩ = ♬♬ (5)). It is difficult now to make a definite statement on the origin of the $\frac{5}{5}$ series. It may have been influenced by the forms of poetical rhythm known as pentameter; and it may have been influenced by the very same factors that influenced the formation of a pentacle in starfish.

The $\frac{6}{6}$ series, being a multiple of $\frac{2}{2}$ by $\frac{3}{3}$, is a typical European hybrid. It may be found throughout the southern coast of Europe, and especially in Portugal, Spain and Italy. Most of the barcarolles of the last-mentioned are written in $\frac{6}{8}$ time.

The $\frac{7}{7}$ series is also of eastern origin. In its trans-Asiatic travel it has crossed the Ural mountains and reached central Russia (Borodinc, Rimsky-Korsakov).

The $\frac{8}{8}$ series is of African origin and is the most popular in dance music in the United States today. Thcse patterns undoubtedly penetrated through the imported Negro slaves, as the patterns are common in South America, Puerto Rico, Cuba and the United States. In ancient times, these rhythms traveled northward and reached Arabia. During the late Middle Ages they got as far as North Russia and slowed down their pace, in the literal sense of the word. Folk music in the region of the White Sea and the Arctic Ocean on the north coast of European Russia has patterns identical with the Cuban rhumba of today; but the absolute *velocity* of the rhumba is doubled as compared with Russian music. This means that by taking a rhumba and slowing it down exactly twice, you will get the rhythms of North Russia, constructed in $\frac{8}{8}$ time and even with the same duration values ($\frac{8}{8} = \frac{3}{8} + \frac{2}{8} + \frac{3}{8}$). To make such music sound like a real rhumba, it would simply be necessary to transcribe it into a different pitch-scale.

The application of this method of series leads me to the conclusion that a consistent form of what is known as "jazz" is music which must be written in $\frac{8}{8}$ time, having $\frac{1}{8}$ as a common denominator and 8-bar phrases accumulating by eighths. The standard form of popular song usually includes a 32-bar chorus. The perfect structure is achieved when the chorus comprises a unit of factorial continuity and consists of 64 bars (8^2)—see *Cheek to Cheek*, and other 64-bar choruses.

The $\frac{9}{9}$ series is now in the making. There are some symptoms of it disclosing itself through different channels of musical time, one being the Viennese waltz and the other, the fox-trot. Today we have a hybrid of the old $\frac{8}{8}$ series and the coming $\frac{9}{9}$ series, which bears the trade-name of "swing."

A complete analysis of the phenomenon known as "swing," so prominent today, will be given at the end of the rhythm theory. It is a hybrid trying to crystallize itself through the intuitive efforts of musicians into the pure style of the $\frac{9}{9}$ series.

There are no difficulties in the way of producing any type of pure or hybrid series, because any of the series-determinants may become either major or minor generators of the rhythmic resultants, and may be incorporated in many ways Any doubts as to the construction of a perfect 5-bar phrase may be dissolved by the utilization of the devices previously offered, such as $r_{5\div3\div2}$, grouped by 6; $r_{6\div5}$ grouped by 6, etc.

The above survey of the series of *factorial* and *fractional* continuity shows that these series belong to the category of power series. Since each number in any of these series represents a monomial, further evolution of the monomial into a polynomial will express the more developed patterns of factorial and fractional continuity. The latter, like the original, are subjected to powers.

The method of *distributive powers* offers a solution for producing harmonic contrasts developed from the original polynomial ratio. This solves the rhythmic problem of composing counterthemes to any theme, whether the contrast appears in simultaneity (counterpart) or continuity (sequence). The law of distributive powers is a common esthetic law of proportionate distribution of harmonic contrasts.

B. Composition of Rhythmic Counterthemes by Means of Distributive Powers

1. *Square of a Binomial*

Formula: (a) Factorial: $(a+b)^2 = a^2 + ab + ab + b^2$

(b) Fractional: $\left(\frac{a}{a+b} + \frac{b}{a+b}\right)^2 =$

$$\frac{a^2}{(a+b)^2} + \frac{ab}{(a+b)^2} + \frac{ab}{(a+b)^2} + \frac{b^2}{(a+b)^2}$$

To obtain the distributive second power of a binomial, it is necessary to multiply the first term of a binomial by itself, then by the second term; then the second term by the first, then the second by the second.

Formula for Synchronization:

(a) Factorial: $S = a(a+b) + b(a+b)$

(b) Fractional: $S = \frac{a}{a+b} \cdot \left(\frac{a+b}{a+b}\right) + \frac{b}{a+b} \cdot \left(\frac{a+b}{a+b}\right)$

To synchronize the initial binomial with its distributive square, it is necessary to multiply the first term by the sum of the binomial, then the second term by the sum of the binomial.

Example:

Series $\frac{3}{3}$ Factorial binomials: $2 + 1$ and $1 + 2$

Fractional binomials: $\frac{2}{3} + \frac{1}{3}$ and $\frac{1}{3} + \frac{2}{3}$

$\left(\frac{2}{3} + \frac{1}{3}\right)^2 = \frac{4}{9} + \frac{2}{9} + \frac{2}{9} + \frac{1}{9}$ (squaring)

$\frac{3}{3}\left(\frac{2}{3} + \frac{1}{3}\right) = \frac{6}{9} + \frac{3}{9}$ (synchronization)

$\left(\frac{1}{3} + \frac{2}{3}\right)^2 = \frac{1}{9} + \frac{2}{9} + \frac{2}{9} + \frac{4}{9}$ (squaring)

$\frac{3}{3}\left(\frac{1}{3} + \frac{2}{3}\right) = \frac{3}{9} + \frac{6}{9}$ (synchronization)

The initial binomials synchronized with their distributive squares represent the themes. The distributive squares represent the counterthemes.

The proportion $\frac{a^2}{ab} = \frac{ab}{b^2}$ produces harmonic contrast, and gives esthetic satisfaction as to both simultaneity and continuity.

Here is a graph and the musical notation of the entire score:

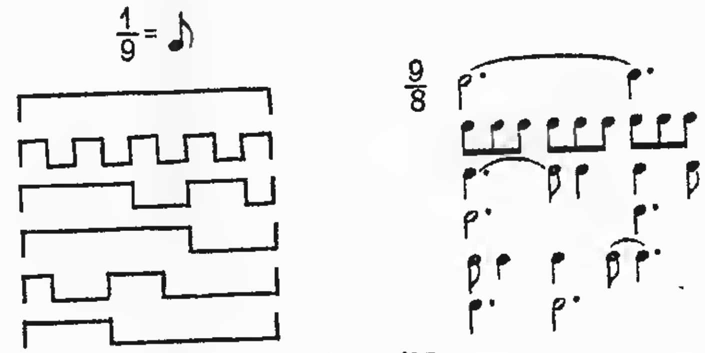

Figure 126.

The same score may be expressed in $\frac{3}{4}$ time.

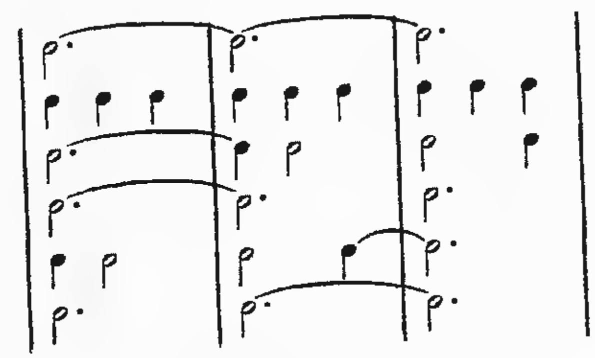

Figure 127.

$\frac{6}{9} + \frac{3}{9}$ and $\frac{4}{9} + \frac{2}{9} + \frac{2}{9} + \frac{1}{9}$ in continuity:

Figure 128.

Musical intuition in some cases approximates these harmonic groups. Here is a musical pattern which is the nearest approximation to the case above:

i.e., $7 + 1 + 3 + 1$

instead of $4 + 2 + 2 + 1$.

Figure 129.

As the numbers grow, it becomes practical to find the resultants of interference between the initial binomials (synchronized with their squares) and the resultants of the distributive squares. Having these resultants available, such power groups may later be utilized in scoring (when more than one orchestral part is desirable), and the resultants of such groups may be used when one part must express the same rhythm.

In addition to this, it is important to supplement the score by $r_{a \div b}$ where a is the determinant of a series. For example, in the foregoing case the determinant is 3; therefore $r_{3 \div 2}$ may be added to the score.

Here is a complete graph and musical notation of the power groups, their resultants and $r_{a \div b}$.

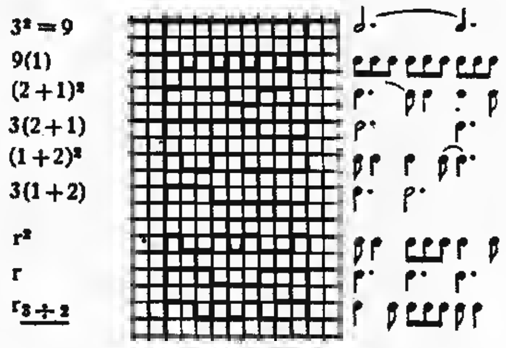

Figure 130.

Chart of the binomials for squaring and synchronization:

$\frac{3}{8}$ 2+1 1+2	$\frac{4}{4}$ 3+1 1+3	$\frac{5}{8}$ 3+2 4+1 2+3 1+4	$\frac{6}{8}$ 5+1 1+5
$\frac{7}{4}$ 4+3 5+2 6+1 3+4 2+5 1+6	$\frac{8}{8}$ 5+3 7+1 3+5 1+7	$\frac{9}{8}$ 5+4 7+2 8+1 4+5 2+7 1+8	

Factorial groups of rhythm build the entire continuity in terms of bars, while fractional groups build the bars in terms of duration-units (attacks).

II. *Square of a Trinomial*

Formula: $(a + b + c)^2 = (a^2 + ab + ac) +$
$+ (ab + b^2 + bc) + (ac + bc + c^2)$.

The distributive square of a trinomial is the sum of the products of a by itself, of a by b, of a by c, of b by a, of b by itself, of b by c, of c by a, of c by b and of c by itself.

The number of terms in a distributive square of any polynomial equals the square of this number. Thus, a binomial gives 4 terms ($2^2 = 4$), a trinomial gives 9 terms ($3^2 = 9$), etc. The denominator of all terms in the distributive power-groups equals the quantitative square of the sum. In a trinomial it equals $(a + b + c)^2$, like $(3 + 2 + 1)^2 = 36$.

In order to synchronize any initial polynomial with its distributive square, it is necessary to find the products of each term by the sum of the polynomial. For example, to synchronize a trinomial with its distributive square:

$$\frac{a}{a+b+c} \cdot \frac{(a+b+c)}{(a+b+c)} + \frac{b}{a+b+c} \cdot \frac{(a+b+c)}{(a+b+c)} + \frac{c}{a+b+c} \cdot \frac{(a+b+c)}{(a+b+c)}$$

Series: $\frac{4}{4}$

$\frac{2}{4} + \frac{1}{4} + \frac{1}{4} \quad \frac{1}{4} + \frac{2}{4} + \frac{1}{4} \quad \frac{1}{4} + \frac{1}{4} + \frac{2}{4}$

$$\left(\tfrac{2}{4} + \tfrac{1}{4} + \tfrac{1}{4}\right)^2 = \left(\tfrac{4}{16} + \tfrac{2}{16} + \tfrac{2}{16}\right) + \left(\tfrac{2}{16} + \tfrac{1}{16} + \tfrac{1}{16}\right) + \left(\tfrac{2}{16} + \tfrac{1}{16} + \tfrac{1}{16}\right)$$
$$\tfrac{4}{4}\left(\tfrac{2}{4} + \tfrac{1}{4} + \tfrac{1}{4}\right) = \tfrac{8}{16} + \tfrac{4}{16} + \tfrac{4}{16}$$
$$\left(\tfrac{1}{4} + \tfrac{2}{4} + \tfrac{1}{4}\right)^2 = \left(\tfrac{1}{16} + \tfrac{2}{16} + \tfrac{1}{16}\right) + \left(\tfrac{2}{16} + \tfrac{4}{16} + \tfrac{2}{16}\right) + \left(\tfrac{1}{16} + \tfrac{2}{16} + \tfrac{1}{16}\right)$$
$$\tfrac{4}{4}\left(\tfrac{1}{4} + \tfrac{2}{4} + \tfrac{1}{4}\right) = \tfrac{4}{16} + \tfrac{8}{16} + \tfrac{4}{16}$$
$$\left(\tfrac{1}{4} + \tfrac{1}{4} + \tfrac{2}{4}\right)^2 = \left(\tfrac{1}{16} + \tfrac{1}{16} + \tfrac{2}{16}\right) + \left(\tfrac{1}{16} + \tfrac{1}{16} + \tfrac{2}{16}\right) + \left(\tfrac{2}{16} + \tfrac{2}{16} + \tfrac{4}{16}\right)$$
$$\tfrac{4}{4}\left(\tfrac{1}{4} + \tfrac{1}{4} + \tfrac{2}{4}\right) = \tfrac{4}{16} + \tfrac{4}{16} + \tfrac{8}{16}$$

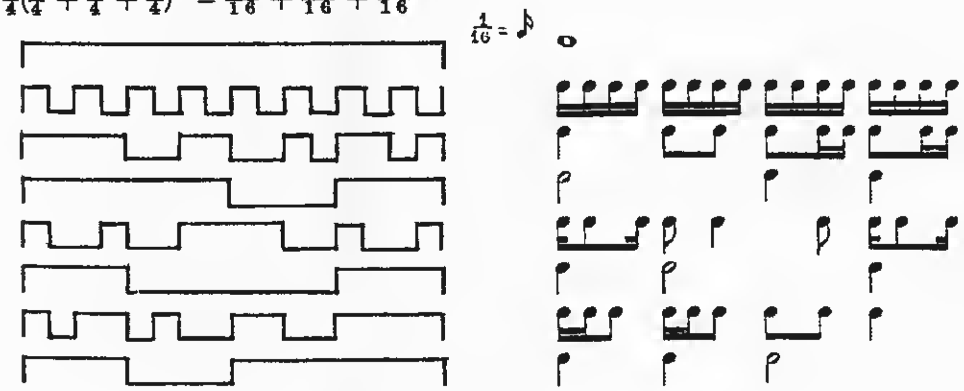

Figure 131.

The same score may be expressed in four bars in $\frac{4}{4}$, assuming $\frac{1}{16} = $ ♩ The above computation can be made in integers, i.e., using the numerators only.

As in the case of binomials, it is desirable to supplement this score by the first and second power resultants and the $r_{a \div b}$.

Here is the entire score:

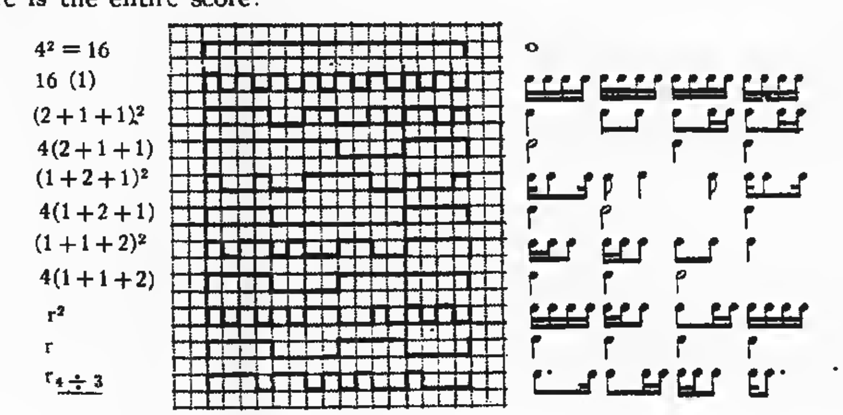

Figure 132.

It is interesting to note that in this particular case, i.e., $2 + 1 + 1$ and $1 + 1 + 2$, classical composers found *intuitively* the exact distributive squares. As you can see from this score, they could not find the square of $1 + 2 + 1$. This figure, i.e., $(1 + 2 + 1) + (2 + 4 + 2) + (1 + 2 + 1)$, or assuming $\frac{1}{16} =$ ♪), $\frac{4}{4}$  is very practical for the tango.

Chart of Trinomials

	2+1+1		2+2+1	3+1+1		4+1+1
$\frac{4}{4}$	1+2+1	$\frac{5}{8}$	2+1+2	1+3+1	$\frac{6}{8}$	1+4+1
	1+1+2		1+2+2	1+1+3		1+1+4

	3+3+1	2+2+3	5+1+1		3+3+2	6+1+1
$\frac{7}{4}$	3+1+3	2+3+2	1+5+1	$\frac{8}{8}$	3+2+3	1+6+1
	1+3+3	3+2+2	1+1+5		2+3+3	1+1+6

	4+4+1	2+2+5	7+1+1
$\frac{9}{8}$	4+1+4	2+5+2	1+7+1
	1+4+4	5+2+2	1+1+7

The reason for selecting these particular trinomials will be given later when we discuss the evolution of style in rhythm.

III. *Generalization of the Square*
(Any Polynomial)

Formula:

$$(a + b + c + \ldots + m)^2 = (a^2 + ab + ac + \ldots + am) + \\ + (ab + b^2 + bc + \ldots + bm) + (ac + bc + c^2 + \ldots + cm) + \\ + \ldots +) \, am + bm + cm + \ldots + m^2)$$

The following graphs and scores on quintinomials of the $\frac{8}{8}$ series should be worked out.

$2 + 1 + 2 + 1 + 2$
$2 + 1 + 2 + 2 + 1$
$2 + 2 + 1 + 2 + 1$
$1 + 2 + 1 + 2 + 2$
$1 + 2 + 2 + 1 + 2$

The following is an illustration of the first one:

$$(2+1+2+1+2)^2 = (4+2+4+2+4) + (2+1+2+1+2) + (4+2+4+2+4) + \\ + (2+1+2+1+2) + (4+2+4+2+4)$$

Synchronization:

$$8(2+1+2+1+2) = 16 + 8 + 16 + 8 + 16$$

Assuming $\frac{1}{64} =$ ♪

Figure 133.

This is the square of the real "hot" rhythms and it has the utmost plasticity. Nobody realizes, listening to this, that the eight bars are over.

Any bar of $\frac{4}{4}$ treated as $\frac{8}{8}$ will give a perfect countertheme for 8 bars. Take, for example, the song used earlier, *Pennies from Heaven*.* The first bar (it may be any bar) is ♩. ♪♩ ♩ , i.e., $3 + 1 + 2 + 2$. It is now squared in order to obtain a countertheme for the first eight bars.

$$(3 + 1 + 2 + 2)^2 = (9 + 3 + 6 + 6) + (3 + 1 + 2 + 2) + \\ + (6 + 2 + 4 + 4) + (6 + 2 + 4 + 4)$$

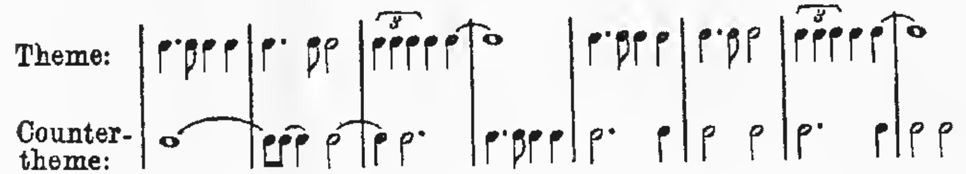

Figure 134.

IV. *Cube of a Binomial*

Cubes produce a new degree of harmonic contrasts. Distributive cubes serve as a new countertheme to the groups of the first and the second power with which they will be synchronized. Cubes are related to squares as the squares are related to the first powers. The number of terms in a distributive binomial of the third power equals 2^3. The recurrence of the central binomial is an invariant of a distributive binomial of the third power.

To obtain the distributive third power of binomials, multiply the distributive second power binomial by the first term of the first power binomial, then by the second term of the first power binomial, and add the products in the same sequence.

Formula:

$$(a+b)^3 = a(a^2 + ab + ab + b^2) + b(a^2 + ab + ab + b^2) = \\ = a^3 + a^2b + \underline{a^2b + ab^2} + \underline{a^2b + ab^2} + ab^2 + b^3$$

The denominator is the quantitative cube of the sum.

To synchronize the distributive square with the distributive cube, it is necessary to multiply each term of the square by the sum of the first power binomial.

To synchronize the first power binomial with its distributive cube, it is necessary to multiply each term of the first power binomial by the square of the sum of the binomial.

$$(2 + 1)^3 = 2(4 + 2 + 2 + 1) + (4 + 2 + 2 + 1) = \\ = (8 + 4 + 4 + 2) + (4 + 2 + 2 + 1) = 27$$

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Synchronization of the square with the cube:

$3(4 + 2 + 2 + 1) = 12 + 6 + 6 + 3 = 27$

Synchronization of the first power with the cube:

$9(2 + 1) = 18 + 9 = 27$

$1 + 2$ gives the converse of these groups. Assuming $\frac{1}{27} =$ ♪ , we obtain 3 bars in $\frac{9}{8}$ time.

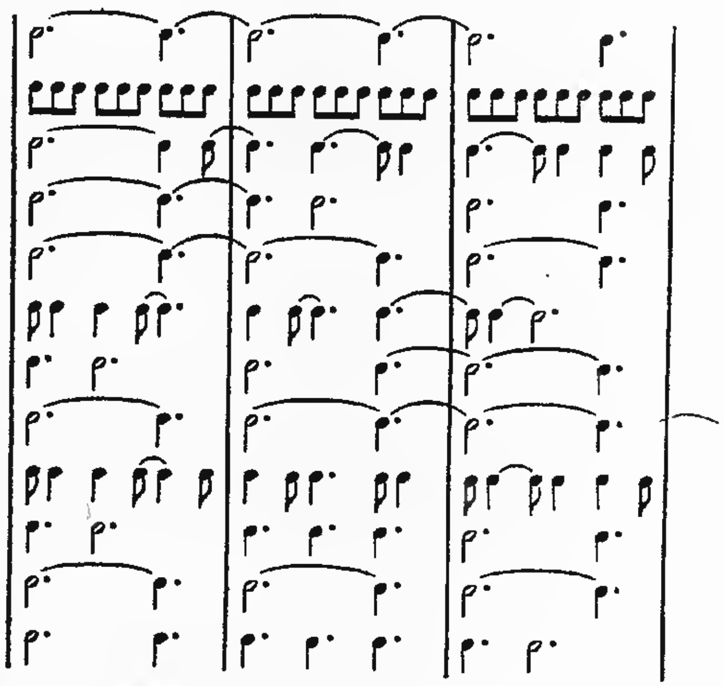


Figure 135.

This produces three harmonically contrasting pairs. Using the first, the second and the third power groups in sequence, we obtain a harmonically growing animation.

As cubes become relatively great number-values, it is practical to limit them for musical purposes by the value 3. Thus, the only practical binomials are:

in $\frac{3}{3}$	in $\frac{4}{4}$	in $\frac{5}{5}$
$2 + 1$	$3 + 1$	$3 + 2$
$1 + 2$	$1 + 3$	$2 + 3$

The previous second power resultants can be easily synchronized, being multiplied by the corresponding determinants.

V. *Cube of a Trinomial*

The procedure remains the same, i.e., each term of second power groups must be multiplied consecutively by each term of the first power groups, and the products added in sequence.

Formula:

$$(a + b + c)^3 = a[(a^2 + ab + ac) + (ab + b^2 + bc) + (ac + bc + c^2)] +$$
$$+ b[(a^2 + ab + ac) + (ab + b^2 + bc) + (ac + bc + c^2)] +$$
$$+ c[(a^2 + ab + ac) + (ab + b^2 + bc) + (ac + bc + c^2)] =$$
$$= (a^3 + a^2b + a^2c + a^2b + ab^2 + abc + a^2c + abc + ac^2) +$$
$$+ (a^2b + ab^2 + abc + ab^2 + b^3 + b^2c + abc + b^2c + bc^2) +$$
$$+ (a^2c + abc + ac^2 + abc + b^2c + bc^2 + ac^2 + bc^2 + c^3).$$

The denominator equals the quantitative sum of the trinomial cubed.

Synchronization of the first and the second power trinomials with the distributive third power trinomial must be performed by consecutive multiplication of each term of the first power trinomial by the square of the sum of its terms —and for synchronization of the square—by the sum of its terms.

Example:

$\frac{4}{4}\ 2 + 1 + 1$

$$(2 + 1 + 1)^3 = 2[(4 + 2 + 2) + (2 + 1 + 1) + (2 + 1 + 1)] +$$
$$+ [(4 + 2 + 2) + (2 + 1 + 1) + (2 + 1 + 1)] +$$
$$+ [(4 + 2 + 2) + (2 + 1 + 1) + (2 + 1 + 1)] =$$
$$= [(8 + 4 + 4) + (4 + 2 + 2) + (4 + 2 + 2)] +$$
$$+ [(4 + 2 + 2) + (2 + 1 + 1) + (2 + 1 + 1)] +$$
$$+ [(4 + 2 + 2) + (2 + 1 + 1) + (2 + 1 + 1)].$$

Synchronization of the square:

$4(4 + 2 + 2) + (2 + 1 + 1) + (2 + 1 + 1) = (16 + 8 + 8) + (8 + 4 + 4) + (8 + 4 + 4).$

Synchronization of the first power:

$16(2 + 1 + 1) = 32 + 16 + 16$

Assuming $\frac{1}{64} =$ 𝅘𝅥𝅯

Figure 136.

Trinomials to be cubed and synchronized with their second and first power groups:

$\frac{4}{4}$	$\frac{5}{5}$	
2 + 1 + 1	2 + 2 + 1	3 + 1 + 1
1 + 2 + 1	2 + 1 + 2	1 + 3 + 1
1 + 1 + 2	1 + 2 + 2	1 + 1 + 3
$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$
3 + 2 + 1	2 + 2 + 3	3 + 3 + 2
3 + 1 + 2	2 + 3 + 2	3 + 2 + 3
1 + 3 + 2	3 + 2 + 2	2 + 3 + 3
2 + 3 + 1		
2 + 1 + 3		
1 + 2 + 3		

VI. *Generalization of the Cube*

(Any Polynomial)

To obtain the distributive cube of any group (polynomial) it is necessary to obtain the distributive square first, and multiply all its terms by the terms of the first power polynomial consecutively; then add the products in sequence.

Formula:

$$
\begin{aligned}
(a + b + c + \ldots + m)^3 = a[\,&(a^2 + ab + ac + \ldots + am) + \\
&+ (ab + b^2 + bc + \ldots + bm) + \\
&+ (ac + bc + c^2 + \ldots + cm) + \ldots \\
\ldots + b[\,&(a^2 + ab + ac + \ldots + am) + \\
&+ (ab + b^2 + bc + \ldots + bm) + \\
&+ (ac + bc + c^2 + \ldots + cm)\,] + \ldots \\
\ldots + c[\,&(a^2 + ab + ac + \ldots + am) + \\
&+ (ab + b^2 + bc + \ldots + bm) + \\
&+ (ac + bc + c^2 + \ldots + cm)\,] + \ldots \\
\ldots + m[\,&(a^2 + ab + ac + \ldots + am) + \\
&+ (ab + b^2 + bc + \ldots + bm) + \\
&+ (ac + bc + c^2 + \ldots + cm)\,] \ldots
\end{aligned}
$$

Synchronization must be obtained in the manner previously described, i.e., through consecutive multiplication by the square of the sum, or by the sum respectively.

One bar in $\frac{5}{8}$ will give an entire countertheme of 64 bars. ***Charts and scores should be made on the following quintinomials:***

2 + 1 + 2 + 1 + 2
2 + 1 + 2 + 2 + 1
2 + 2 + 1 + 2 + 1
1 + 2 + 1 + 2 + 2
1 + 2 + 2 + 1 + 2

VII. *Generalization of All Powers*

(Any polynomial to any power)

When further development of contrasting parts is desirable, powers higher than the cube may be used. In practical application they will concern mostly small groups and small number-values.

The procedure remains the same. To obtain the distributive n^{th} power of any group, it is necessary to obtain the distributive $n - 1$ power of the same group, multiply each term of such group by the terms of the first power group consecutively, and then add the products in sequence.

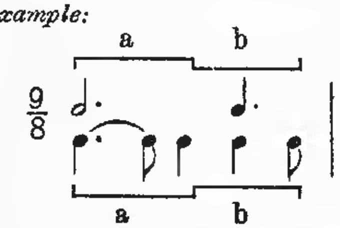
If G stands for a group, this may be expressed through the formula:

$$\boxed{G^n = G(G^{n-1}) \text{ with distribution.}}$$

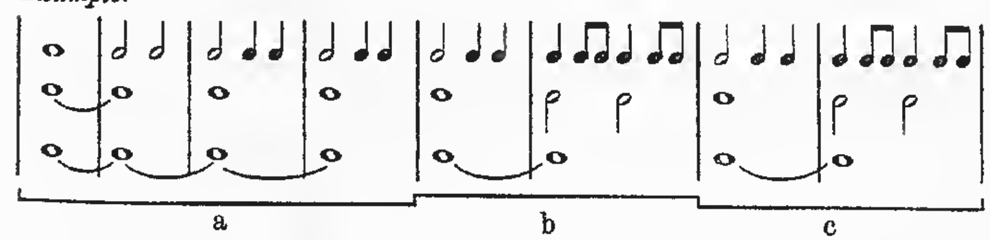
To synchronize the first power group with the n^{th} power group, it is necessary to multiply each term of the first power group by the quantitative $n - 1$ power of the same group. To synchronize the second power group with the n^{th} power group, it is necessary to multiply each term of the second power group by the quantitative $n - 2$ power of the same group, etc.

All permutations in the power groups must be performed through the terms of the preceding power.

Example:



Example:



Example:

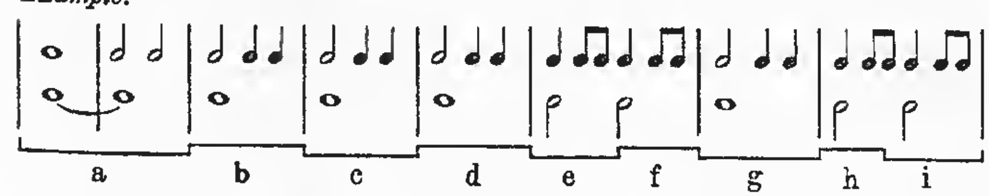


Figure 137

CHAPTER 13

EVOLUTION OF RHYTHM STYLES (FAMILIES)

WE MAY note that uniform groups, as well as non-uniform groups, generate various resultants. Whereas synchronized monomial periodicities generate symmetric polynomial resultants, distribution within any T (the determinant of a series) produces binomials or trinomials characteristic of all resultants where such T is a major generator.

Taking all or some of the possible *binomials* of a certain T and synchronizing them with their converses, trinomial resultants may be obtained. Through synchronizing all permutations of such trinomial resultants (of one series), quintinomials are obtained. The resultants of quintinomials and their permutation-groups produce groups with *nine terms*.

This is a normal serial development as observed in various phenomena (for instance, in crystal formation).

Formula:

$$\boxed{i_n = 2nt_{n-1} - 1}$$

The number of terms in the n^{th} interference-group equals the product of the number of terms in the $n-1^{st}$ interference-group by 2, minus 1.

Example:

The first interference-group	$i_1 = 2$
The second " "	$i_2 = (2\times2) - 1 = 3$
The third " "	$i_3 = (2\times3) - 1 = 5$
The fourth " "	$i_4 = (2\times5) - 1 = 9$
The fifth " "	$i_5 = (2\times9) - 1 = 17$

With the limit 9 as a determinant of a series, the maximum non-uniform resultants are quintinomials. Uniform resultants follow the maximum non-uniform resultants. The greater the number-value of a determinant, the more interference-groups it produces. While the determinant 3 produces only one non-uniform interference-group, 9 produces three non-uniform interference-groups.

All the consecutive interference-groups generated by one determinant constitute the evolution of all rhythmic patterns in the corresponding family (style).

This makes it possible to predict all future rhythmic patterns of one family as well as to trace the origin of more involved rhythms.

As previously mentioned, the original (binomial) interference-groups may be obtained directly from a determinant. For example, the distribution of a determinant 5 gives 3 + 2 and 4 + 1, and their permutations. These binomials are the first and the last binomials of the resultants obtained from two uniform monomial generators in which the determinant of a series is a *major* generator (a).

Therefore, 3 + 2 are the first two terms of a resultant where a = 5; 4 + 1 are the last two terms of a resultant where a = 5.

In order to trace the origin of a binomial with respect to two uniform generators, it is necessary to take the greater number-value of the binomial and to assign it as a minor generator (b). The sum of the binomial is the major generator.

Example:

 is a given binomial.

Find the determinant of the series.
$5 + 3 = 8$ The determinant is $\frac{8}{8}$
Find the *a* and *b* generators.
$b = 5$ $a = 5 + 3 = 8$

The binomial represents the first two terms of $r_{8\div5}$.

Existing music often works on more than one determinant, thus producing various hybrids. It is very easy to trace the origin of any rhythmic hybrid, as such groups which are alien to the family are indicated in musical notation by the numbers. For instance, the triplets in $\frac{4}{4}$ time; the duplets in $\frac{3}{4}$ time, etc.

Leaving theories aside for the moment, I believe that the actual cause of any new interference-binomial appearing in the world is the urge toward *unbalancing*, that is, the *centrifugal* tendency.

In the light of such a hypothesis, the origin of the "Charleston" 5 + 3 binomial may be explained as a tendency to disturb the balance of $\frac{2}{4} + \frac{2}{4}$ in $\frac{4}{4}$ or $\frac{4}{8} + \frac{4}{8}$ in $\frac{8}{8}$.

Chronologically, the more unbalanced binomials (such as $\frac{7}{8} + \frac{1}{8}$) appear later than the balanced ones (such as $\frac{5}{8} + \frac{3}{8}$), regardless of their structural complexity. While 5 + 3 has been known in the American dance-music for some time, 1 + 7 appeared as a prominent pattern only with the song, "Organ Grinder's Swing."

The prediction of *new rhythmic families* to come is based on the principle of the growth-through-power series.

So far we have had, during the entire range of recorded history, the evolution of $\frac{2}{2}$ into its second power $\frac{4}{4}$, and into its third power $\frac{8}{8}$. Most probably $\frac{9}{9}$ will take its place in the near future as the second power of $\frac{3}{3}$. The series, $\frac{6}{6}$ is an exhausted European hybrid, being the product of 2×3. The $\frac{5}{5}$ and $\frac{7}{7}$ series are Oriental series of old origin. They may become fashionable for a while in the Western musical world.

Thus, the series of factorial-fractional continuity express the evolutionary forms in the two-coordinate system.

A. "Swing" Music

The following is an analysis of the phenomenon known as "swing music"—it is an analysis of "swing" as it is *performed*, not as it is written out on paper.

In view of the fact that triplets of eighths in common time are very prominent in this type of playing, particular attention must be given to the value 3, its multiples and its powers. Knowing from the previous analysis that $\frac{9}{8}$ is the most probable candidate for the new style, I have studied all the "waltz-like" phenomena which have appeared during the last few decades. The utmost plasticity of the Viennese type of waltz (such as the waltz from *Rosenkavalier* by Richard Strauss) is due to this figure:

Figure 138.

The above is $3 + 1 + 5 + 1 + 5 + 1 + 2$ etc., where the characteristic grouping is one appearing between the two fives.

The trinomial $4 + 1 + 4$ can be found in the second interference group in the $\frac{9}{9}$ series. Here again (as in the case of powers) is the *intuitive approximation* of the correct patterns. The *waltz* pattern is "trying to evolve" into its second-power. The idea of unity between the two greater number-values is right, but the number-values are only approximately correct.

There are other hybrids which are characteristic of Viennese waltzes. For example, a hybrid between $\frac{3}{3}$ and $\frac{4}{4}$ series: $3 + 1$ coming from $\frac{4}{4}$ series and placed into $\frac{3}{3}$ series ($\frac{3}{4}$ time).

Figure 139.

Most jazz ("Charleston Rhythm") is a hybrid between $\frac{8}{8}$ and $\frac{6}{6}$ series. Examine the following:

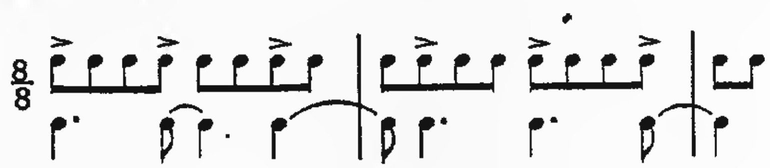
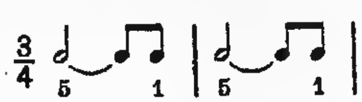


Figure 140.

And so, too, with all patterns of $\frac{6}{6}$ series put into $\frac{8}{8}$ time.

The original binomial of this style is most characteristic of Viennese waltzes:



Also the trinomial of $\frac{6}{6}$ series: $1 + 1 + 4$

See "Rosenkavalier"

Figure 141.

$5 + 1$, being placed into $\frac{8}{8}$ time, produces:

etc.

Figure 142.

An approach to the $\frac{9}{9}$ family from another angle is "swing." The foundation of the latter is the fox-trot in triplets. Rhythms of $\frac{4}{4}$ and $\frac{8}{8}$ are modified on a basis of $\frac{12}{12}$ or $\frac{12}{8}$ and $\frac{4}{4}$ (in triplets) musically.

The common denominator units are the eighths.

Figure 143.

Through syncopation tendencies, plus the $\frac{3}{3}$ series binomial, we obtain all the possible patterns of "swing."

The original patterns:

and

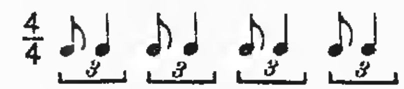


Figure 144.

The syncopated patterns:

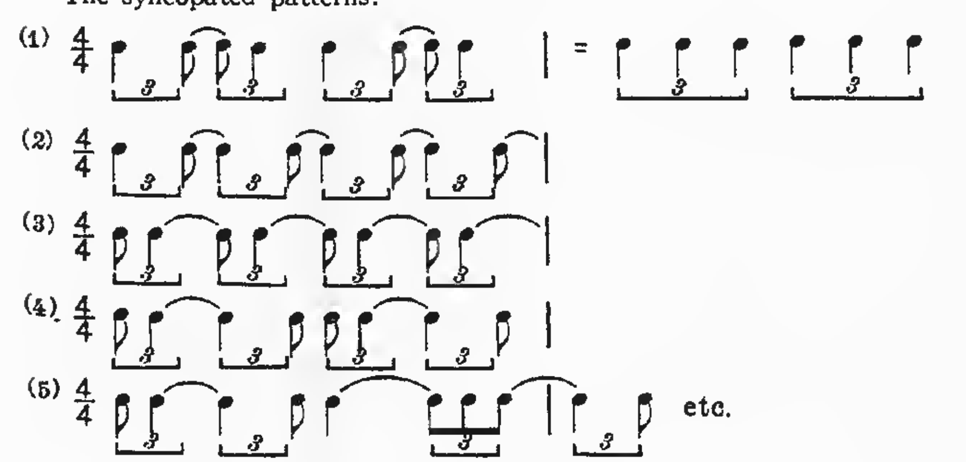


Figure 145.

The characteristic values are:

2, i.e., ♪♪ or ♩ (an eighth tied to an eighth, or a quarter).

3, i.e., ♩♪ or ♪♩ (a quarter tied to an eighth, or an eighth tied to a quarter).

4, i.e., ♩♩ (a quarter tied to a quarter).

Often some of these number-values appear as rests.

It is interesting to note that, even in bands such as Benny Goodman's, all orchestral parts are *written* either in the $\frac{4}{4}$ series patterns or $\frac{9}{8}$ series patterns, but are then translated into "swing" while being played.

Figure 145, line 4, is the first true pattern of a $\frac{9}{8}$ series trinomial $(4 + 1 + 4)$. This pattern, with greater consistency, would appear as in line 4a. The number-values are correct but the group unit is wrong. It is applied in the wrong type of measure, $\frac{12}{8}$ instead of $\frac{9}{8}$.

Thus, we can see that both the Viennese waltz and the fox-trot are engaged in a struggle for crystallization of the $\frac{9}{8}$ family.

All rhythmic interference groups have as the only alternatives in their evolution: ***either to evolve the higher powers of the same patterns, or to evolve into the higher powers of the same determinant.***

The entire process of the evolution of rhythmic families may be expressed as follows:

$$
\begin{array}{llll}
r_1 \ldots & Pr_1 \ldots & SPr_1 \ldots & i_1 \\
r_2 \ldots & Pr_2 \ldots & SPr_2 \ldots & i_2 \\
\ldots & \ldots & \ldots & \ldots \\
r_1^2 \ldots & Pr_1^2 \ldots & SPr_1^2 \ldots & i_1 \\
r_2^2 \ldots & Pr_2^2 \ldots & SPr_2^2 \ldots & i_2 \\
\ldots & \ldots & \ldots & \ldots \\
r_1^n \ldots & Pr_1^n \ldots & SPr_1^n \ldots & i_1^n \\
\ldots & \ldots & \ldots & \ldots \\
r_n^n \ldots & Pr_n^n \ldots & SPr_n^n \ldots & i_n^n \\
\ldots & \ldots & \ldots & \ldots
\end{array}
$$

r—is the resultant
P—Permutations
S—Synchronization
i—Interference

Continuous dotted line represents uniformity.

The first resultant (r_1) produces its permutations (Pr_1) which form the first interference-group; these being synchronized (SPr_1) produce the first interference. The resultant of this interference is the second resultant (r_2), etc.

The following graphs should be converted into musical notation:

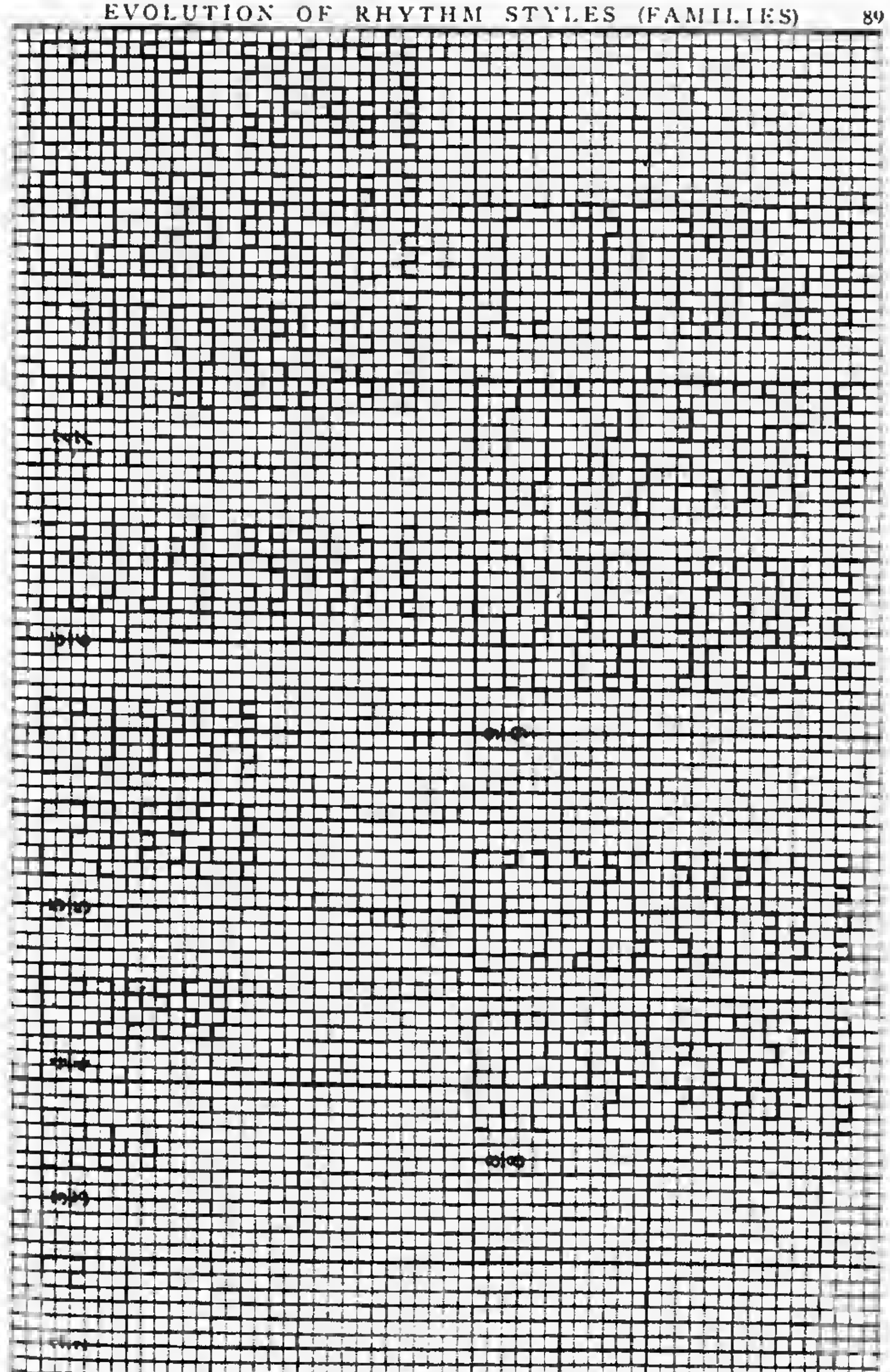


Figure 146.

CHAPTER 14

RHYTHMS OF VARIABLE VELOCITIES

THE only constant velocity known in the physical world is that of light. By introducing *constant* velocities into the art forms we try to simplify the scheme of surrounding phenomena. But different forms of progressive, *variable* speed are also well known to us—through biological growth, through gravity, through different series of acceleration and different ratios of acceleration.

The ratios of acceleration through gravity grow rapidly. Series like the natural harmonic series reveal a much greater graduality in their rates of acceleration. The urge for freedom in a musical performance often reveals itself by the speeding up or slowing down of a certain passage written out in musical notation as a uniform group. The increase and the decrease of speed may be, however, merely two reciprocals of the same series.

Rhythms based on constant velocities are either continuous repetitions of the terms of a monomial periodicity, or of several monomial or binomial periodicities synchronized. But rhythms based on *variable velocities* or progressions are single terms belonging to different periodicities.

In the following list of different mathematical series, the series of gravity has been eliminated as it is too "intense" for esthetic purposes. The simpler the ratios of acceleration, the more obvious the forms of acceleration will appear to the listener of music. Such is the case of doubling or quadrupling the original speed. In all three forms of representation—number, graph and music—there is a constant speed which reveals various forms of acceleration through the actual number-values. Music expressed in such a way may be performed or conducted in a constant tempo—counting out all the durations exactly as they are represented by the number-values.

Here is a list of various series of acceleration:

(1) *Natural Harmonic Series.*
1, 2, 3, 4, 5, 6, 7, 8, 9 . . . .

(2) *Arithmetical Progressions.*
+ n constant
+ 2 — 1, 3, 5, 7, 9 . . . .
+ 3 — 1, 4, 7, 10, 13 . . .

(3) *Geometrical Progressions.*
× n
× 2 — 1, 2, 4, 8, 16 . . . .
× 3 — 1, 3, 9, 27 . . . .
× 2 — 3, 6, 12, 24, 48 . . . .
× 3 — 2, 6, 18, 54. . . .

(4) *Power Series.*
n^{th} power
2, 4, 8, 16, 32 . . . .
3, 9, 27, 81 . . . .
5, 25, 125. . . .

(5) *Summation Series.*
1, 2, 3, 5, 8, 13, 21 . . . .
1, 3, 4, 7, 11, 18 . . . .
1, 4, 5, 9, 14, 23 . . . .

(6) *Arithmetical Progressions with Variable Differences.*
$1^{+1}, 2^{+2}, 4^{+3}, 7^{+4}, 11^{+5}, 16^{+6}, 22^{+7}$, ...

(7) *Prime Number Series.*
1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37 . . . .

It is important to note that the rate of acceleration varies within each given series. For instance, in the first series, the ratio between the second and the first value is $2 \div 1$, while the ratio between the sixth and the seventh number-values is $7 \div 6$. Therefore, when a greater speed of acceleration is desired, it is advisable to take a number-value considerably higher than unity. This concerns mostly the type of series that grow with greater speed, such as arithmetical progressions.

Although any series may be used for carrying out different types of accelerando and rallentando, the most suitable are those belonging to a given rhythmic family. For example, to obtain a proper type of "accel. rall." in the $\frac{8}{8}$ series ("Charleston" family), we should use the most suitable series, which is the first summation series—for all the number-values of this series represent generators of the $\frac{8}{8}$ series. Likewise, for any music written in the $\frac{4}{4}$ series, such as marches, polonaises, mazurkas, etc., we shall have the most appropriate "accel. rall." expressed through the second summation series. Similarly, music in the $\frac{9}{9}$ series requires the third summation series for its "accel. rall."

In many cases the freedom of the performer leads to the use of numbers *alien* to the series in which a given piece of music is composed; this causes an obvious dissatisfaction and many listeners—with a natural sense of rhythm—feel that something is wrong but cannot explain the cause of such rhythmic irregularity. Speeding up and slowing down is a natural tendency in much folk music. Some of the most striking examples may be found in Hungarian music (see Liszt's *Second Hungarian Rhapsody*) and music of gypsies in various countries.

The technique of variable speeds becomes extremely important in dealing with stage productions, compositions for the dance, and especially film music.

In film music, the animation technique in particular requires absolute precision of timing. In illustrating a "chase," one has to time the corresponding music in such a fashion that the whole period of acceleration will be limited to a definite portion of time, with the precision of $\frac{1}{24}$ of a second (the duration of a single shot in a film synchronized with sound). In some instances of descriptive music, especially those dealing with the speeding up and slowing down of mechanisms, similar precision adds a great deal to the esthetic effects. This method permits the use of the "accel. and rall." of the same rate and series as counter-rhythms, as well as the resultants of their interference (we shall call them the *resultants of acceleration*).

Thus, we acquire four fundamental forms to be used as material for acceleration groups:

(1) Increasing velocity (accelerando).
(2) Decreasing velocity (rallentando).
(3) The two [(1) and (2) velocities] combined.
(4) The resultant of acceleration.

Forms (1) and (2) may be used for the introductions and conclusions (codas); forms (3) and (4)—for the climaxes.

In either case it is more practical to find the decreasing velocity (increasing number-values) first, as it is more practical to have a definite *initial* number-value.

For example, if we use the natural harmonic series and start with unity, we find a practical stopping point at 8, because $1 + 2 + 3 + 4 + 5 + 6 + 7 + + 8 = 36$. The sum must be in a simple relation to the multiples of the group-unit (measure). The value 36 offers a number of possibilities. Four-and-a-half of $\frac{4}{4}$ time (in eighths); nine bars in $\frac{4}{4}$ time (in quarters); four bars of $\frac{9}{8}$ time (in eighths); six bars in $\frac{6}{8}$ time (in eighths); twelve bars in $\frac{3}{4}$ time (in quarters).

Using the first summation series for accelerando in the $\frac{8}{8}$ family, we obtain a practical value of 32 by summing up $1 + 2 + 3 + 5 + 8 + 13$. This offers exactly four bars of $\frac{8}{8}$ time, and in music of eight-bar groups, the ratio becomes very simple ($\frac{8}{4} = \frac{2}{1}$).

A. Acceleration in Uniform Groups.

Examples:

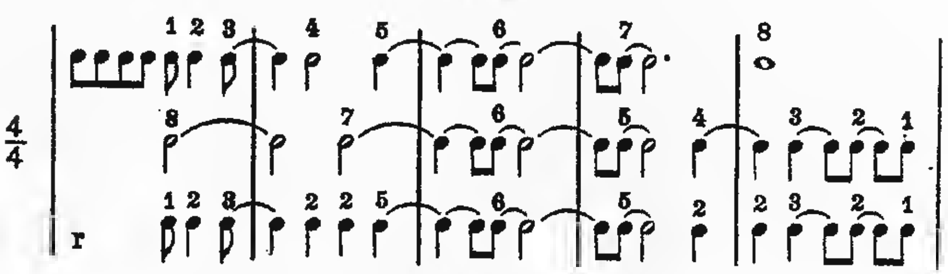


Figure 147.

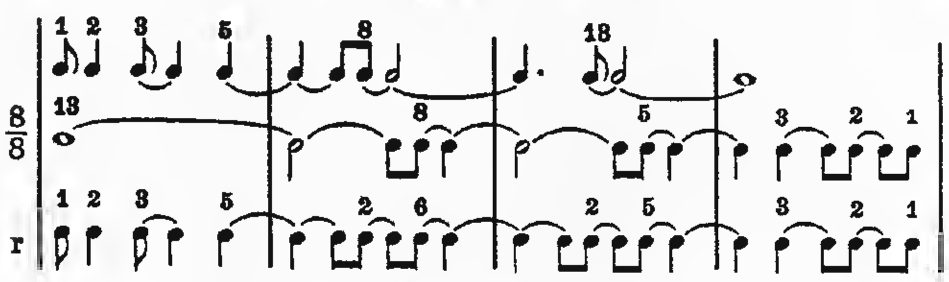


Figure 148.

B. Acceleration in Non-Uniform Groups.

The technique of progressive addition to the original number-value now becomes an addition of respective values to the terms of the original group.

Here is an example of progressive addition through natural harmonic series:

The original group: $3 + 1 + 2$
$(3 + 1 + 2) + (6 + 2 + 4) + (9 + 3 + 6) + \ldots$

Figure 149.

Thus, the $3 + 1 + 2$ group appears with the coefficients which are the terms of a certain series (natural series in this case).

$$(3 + 1 + 2) = 6$$
$$2\,(3 + 1 + 2) = 12$$
$$3\,(3 + 1 + 2) = 18$$

C. Rubato.

"Rubato" is the process of unbalancing a balanced binomial, or the process of balancing an unbalanced binomial. In terms of quantities, the first process *increases the complexity* of an original ratio; the second—*decreases the complexity* of an original ratio.

The process of unbalancing a balanced binomial must be carried out by means of a *unit of deviation*. This unit of deviation, supposedly an infinitesimal (in the calculus, dx) becomes a rational fraction in the field of musical rhythm. The most satisfactory results are produced by means of a *standard unit of deviation*, which is defined in this theory as $\frac{1}{t}$, i.e., the unit of a series of fractional continuity. We shall call it τ (the Greek letter, tau).

Formula for a standard unit of deviation:

Example I: $\tau = \frac{1}{t}$

Take the second theme of Chopin's *Valse in c♯ minor*. All bars of this theme have the following construction: $\frac{3}{4}$. Many performers play the first bar of this theme like this: . Let us see what causes the transformation of , i.e., $2 + 2$ into , i.e., $3 + 1$. The way this binomial is written makes it $\frac{2}{4} + \frac{2}{4}$. In this case we have $\frac{4}{4}$ series, where $\tau = \frac{1}{4}$. Therefore, the process of unbalancing the original binomial $(\frac{2}{4} + \frac{2}{4})$ may be expressed as follows:

$(\frac{2}{4} + \tau) + (\frac{2}{4} - \tau) = (\frac{2}{4} + \frac{1}{4}) + (\frac{2}{4} - \frac{1}{4}) = \frac{3}{4} + \frac{1}{4}$, which means .

Example II.

Take any fox-trot where you find $\frac{4}{4}$ in the printed copies. In a performance you hear it as or . Let us follow the previous procedure. The two half-notes in a fox-trot time belong in reality to the $\frac{8}{8}$ series. They must be expressed as $\frac{8}{8} = \frac{4}{8} + \frac{4}{8}$. In this case $\tau = \frac{1}{8}$. By adding $\frac{1}{8}$ to the first term and subtracting from the second we obtain $(\frac{4}{8} + \frac{1}{8}) + (\frac{4}{8} - \frac{1}{8}) = \frac{5}{8} + \frac{3}{8}$.

In both examples the process of unbalancing may be reversed, i.e., $\frac{1}{4} + \frac{3}{4}$ and $\frac{3}{8} + \frac{5}{8}$.

The process of balancing an unbalanced binomial is a typical case of the ratio simplification as we find it in "swing" performance. Write in $\frac{4}{4}$ time, and the "swingsters" will play it in the same $\frac{4}{4}$ time. The same thing happens in "boogie-woogie," where the written accompaniment of broken octaves $\frac{8}{8}$ is played $\frac{4}{4}$, and the upper parts are handled as "Charleston." The ratio $3 + 1$ becomes $2 + 1$, which is closer to balance.

Some of the other forms of "rubato" playing are small groups of accelerando and rallentando.

D. Fermata (Hold).

There are two types of fermata; the two may seem to have an entirely different character, but in reality this difference is purely quantitative.

The first type produces the effect of a full stop. It is commonly used at the very beginning, at the very end, at the moment of a climax, or before a new theme enters.

In writing out such a fermata, it is best to make it a simple multiple of the preceding or the following values, or the sum of the preceding group of uniform values.

An example of transcription of a fermata of the first type:

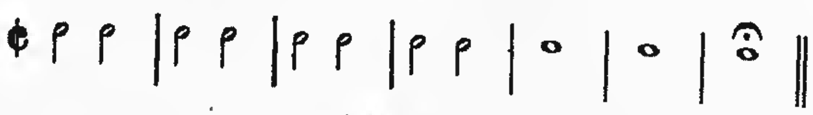


Figure 150.

The first four bars move in halves, the following two—in wholes. By assigning a double value to the last note, we satisfy two requirements. *One:* we produce a simple ratio of $1 + 1 + 2$ in the last three bars, i.e., our last note is the simplest multiple for each of the two preceding notes; *two:* such a multiple compensates the first four bars, thus creating a balance $4 + 4$. Transcription:

Figure 151.

Obviously, this gives the utmost satisfaction.

The second type of fermata is a *temporary delay.* The method of creating simple ratios is most effective in transcribing this type of fermata. Here is a transcription of the following example:

Figure 152.

By isolating the group preceding the fermata we obtain $\frac{3}{4}$; by isolating the group with a fermata we obtain $\frac{2}{8}$

Now the entire group appears as follows:

Figure 153.

This being transcribed into number-values gives:

$$(\tfrac{11}{16} + \tfrac{1}{16}) + (\tfrac{3}{16} + \tfrac{1}{16}) + \tfrac{16}{16}$$

The first bar is related to the second bar as $3 \div 1$ because $\frac{3}{4} = \frac{6}{8}$; $\frac{6}{8} \div \frac{2}{8} = \frac{3}{8} \div \frac{1}{8}$.

Simplifying the ratio $3 \div 1$ into $2 \div 1$, we obtain the following musical measures: $\frac{3}{4} + \frac{3}{8}$. Making the first duration in the second bar (the fermata) longer, we obtain a binomial $\frac{2}{8} + \frac{1}{8}$, or in musical notation:

Figure 154.

This procedure makes the absolute value of the fermata note increase in a very subtle way, $\frac{1}{16}$ longer than the original duration. Here are the numbers from the musical transcription:

$$(\tfrac{11}{16} + \tfrac{1}{16}) + (\tfrac{4}{16} + \tfrac{2}{16}) + \tfrac{16}{16}$$

By comparing the original and the transcription, it is easy to see that the fermata note (which originally was $\frac{3}{16}$) became $\frac{4}{16}$, thus gaining $\frac{1}{16}$.

The rhythm of variable velocities presents a fascinating field for study and exploration. The very thought that various rhythmic groups may speed up and slow down at various rates, appearing and disappearing, is overwhelming.

This idea stimulates one's imagination towards the complex harmony of the universe, where different celestial bodies (comets, stars, planets, satellites) coexist in harmony of variable velocities.*

*So ends the exposition of Schillinger's theory of rhythm, to be followed next by the theory of pitch-scales. Although the casual reader may not be entirely aware of it, the specific techniques have now been set forth whereby all possible rhytnms, of any nature whatsoever, may be derived. And these "all possible" rhythms have been grouped into related families, sub-families and "styles," so that what is an infinity of rhythms may be rapidly and practically utilized in the actual composition of music. (Ed.)

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK II

THEORY OF PITCH-SCALES

BOOK TWO

THEORY OF PITCH-SCALES

PRELIMINARY REMARKS ON THE THEORY OF PITCH-SCALES

Just as the first book developed the theory and practice of rhythms, which concern durations in *time*, so does the present portion of the Schillinger system develop the theory and practice of that other basic factor in music, *pitch*. The theory of pitch-scales concerns pitch considered in continuity, i.e., one tone sounding after another. When pitch is considered in simultaneity, i.e., tones sounding at the same time, questions of harmony and counterpoint are involved. These are discussed in later sections of the entire work. Schillinger approaches the pitch question as, first, a problem in primary selective systems—or tuning; then, as a problem of abstracting from all the tones made available by the tuning system those particular tones which are to be used in any composition. These sets of tones, called pitch-scales or—more commonly—just "scales," furnish raw material for both melody and harmony. (Ed.)

CHAPTER 1

PITCH SCALES AND EQUAL TEMPERAMENT

THE INTONATION units and the intervals between them constitute the elements of the pitch-scales. The intonation units are named *pitch-units* (p) in the following exposition, and the intervals between the pitch-units are called *pitch-intervals* (i).

A pitch-scale is a sequence of pitch-units following in consecutive order (increasing or decreasing frequency). The number of pitch-units in scales, constructed within the equal temperament of twelve, ranges from 1 to 144. Families of pitch-scales, as well as families of time-scales (rhythm), serve as esthetic material for racial, national and local expressions.

The subject of this portion of my theory consists of the following items:

(1) Construction of melodic forms from pitch-scales;

(2) Modification of melodic forms;

(3) Composition of melodic continuity;

(4) Deduction of harmonic forms from pitch-scales.

All pitch-scales may be classified into the following four groups:

Group One:	One root-tone. Range limit = 11.
Group Two:	One root-tone. Range over 12.
Group Three:	More than one root-tone. Range = 12.
Group Four:	More than one root-tone. Ranges: 24, 36, 60, 132.

The number values here express the number of semitone units which will serve as standard units for measuring pitch within the equal temperament of 12.*

*A few additional words of explanation may be useful here. Pitch is, of course, a question of the frequencies of the sound waves, i.e., the number of vibrations per second. In order to produce music, it is first necessary to determine *which particular* frequencies will be used as points of reference. We take this for granted now, but working it out was a subject of much theoretical struggle over the centuries. From the set of *all possible* frequencies (in this case, all audible frequencies), it was thus necessary to *select* a smaller set which becomes a *primary selective system*.

In equal temperament tuning, the 12 "tones" comprising the system are c, c sharp, d, d sharp, e, f, f sharp, g, g sharp, a, a sharp, b—followed by another c, the latter c being one octave higher than the former (one octave higher means that the frequency is exactly *doubled*). The flatted tones are considered in equal temperament to be identical to (that is, enharmonics of) the sharped tones.

How are these twelve basic tones tuned, that is, what are their frequency ratios? They are related in the following manner: if we construct a series of the twelfth root of 2, $\sqrt[12]{2}$, in such a fashion that the root remains 12 while the power of 2 increases from zero to 12, we will have a series which corresponds to the actual frequencies of equal temperament tuning. Note that the first term is 1, for the zero power of 2 is 1 and the 12th root of 1 is also 1. Here

The mathematical expression for this system of tuning (developed by Andreas Werckmeister in Germany, in 1691) is $\sqrt[12]{2}$. Two expresses the octave ratio of frequencies, i.e., $2 \div 1$; the exponent 12 expresses the number of the uniform ratios within one octave. The semitones are integers when they express the logarithms to the base $\sqrt[12]{2}$.

we will express the root powers as fractions, in which the denominators represent roots and the numerators represent powers. The series is:

$2^{\frac{0}{12}}$	$2^{\frac{1}{12}}$	$2^{\frac{2}{12}}$	$2^{\frac{3}{12}}$	$2^{\frac{4}{12}}$	$2^{\frac{5}{12}}$	$2^{\frac{6}{12}}$
C	C#	D	D#	E	F	F#
$2^{\frac{7}{12}}$	$2^{\frac{8}{12}}$	$2^{\frac{9}{12}}$	$2^{\frac{10}{12}}$	$2^{\frac{11}{12}}$	$2^{\frac{12}{12}}$	
G	G#	A	A#	B	C	

The frequencies are logarithmically related as the terms of the above series are related. In reality, as Schillinger points out elsewhere, there are at least two additional matters to keep in mind: (1) music as actually performed exhibits a much higher variety of intonation than the equal temperament system would suggest, the tuning system supplying simply the points of reference; (2) music tends toward a greater fluidity of intonation more nearly approximating actual curves, as in the instance of what are popularly called "scoops" in pitch in violin performances, or in "hot" trumpet intonations. (Ed.)

CHAPTER 2

FIRST GROUP OF PITCH-SCALES

"Diatonic" and Related Scales

IN THE FOLLOWING discussion, all the scales are constructed from *c*; they are classified according to the number of pitch-units and the number-values for the intervals.

In each case the full number of technical possibilities is described.

A. One-unit Scales. Zero Intervals.

[The number of scales: one.]

Scales with one constant pitch-unit constitute the so-called "monotone" music and may actually be found among the primitives.

The natives of southern Patagonia (Tierra del Fuego) have one pitch-unit scale and are not familiar with any other form of musical intonation. This music has been recorded on dictaphone cylinders by Erich von Hornbostel, Berlin University, and the records are—or were—located in the phonogram archive of the Psychological Institute of Berlin University. One copy exists in the archives of the New School in New York City. This music compensates for its lack of variety in intonation by the variety of its rhythm.

Music of our civilization quite frequently deals with one pitch-unit scales. Instances are to be found in sustained tones (pedal points) and many rhythmic passages executed by individual instruments, such as rhythmic trumpet passages.

The only technical procedure possible with such scales is: *superimposition of the time-rhythm.*

Scale:0; Time-Rhythm = $r_{\underline{4 \div 3}}$

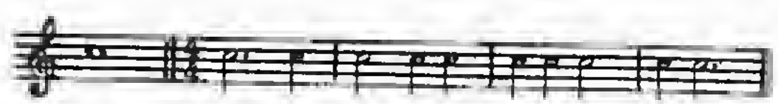


Figure 1.

B. Two-unit Scales. One Interval.

[The number of scales: eleven.]

Table of Intervals

Scale	1	2	3	4	5	6	7	8	9	10	11
Interval	c–d♭	c–d	c–e♭	c–e	c–f	c–f#	c–g	c–a♭	c–a	c–b♭	c–b

Technical procedures:
(1) Definition of the number of melodic forms.
(2) Combinations of melodic forms.
(3) Continuity of melodic forms through permutations.
(4) Coefficients of recurrence of the melodic forms.
(5) Superimposition of the time-rhythm.

(1) There are two melodic forms; one combination of the latter is possible with the two-unit scales.

(2) The melodic forms are: $a_1 + b_1$ and $b_1 + a_1$, where a_1 and b_1 are the pitch-units. Thus, the two forms become a_2 and b_2 respectively.

(3) A neutral continuity of melodic forms may be obtained by means of permutations of the higher order. In order to give individual expression to the continuity of melodic forms, it is necessary to introduce a specified recurrence into such continuity.

(4) Two elements (a_2 and b_2) permit the application of binomial and polynomial coefficients (with an even number of terms). For example:

$3a_2 + b_2$ or $5a_2 + 3b_2$ etc.;
or $3a_2 + b_2 + 2a_2 + 2b_2 + a_2 + 3b_2$
or $4a_2 + b_2 + 3a_2 + 2b_2$ etc.

By placing such a continuity of melodic forms into various types of bars and using uniform durations for each pitch-unit, one may achieve an extraordinary diversity of effects.

Scale: 5; Melodic Form: $2a_2 + b_2$

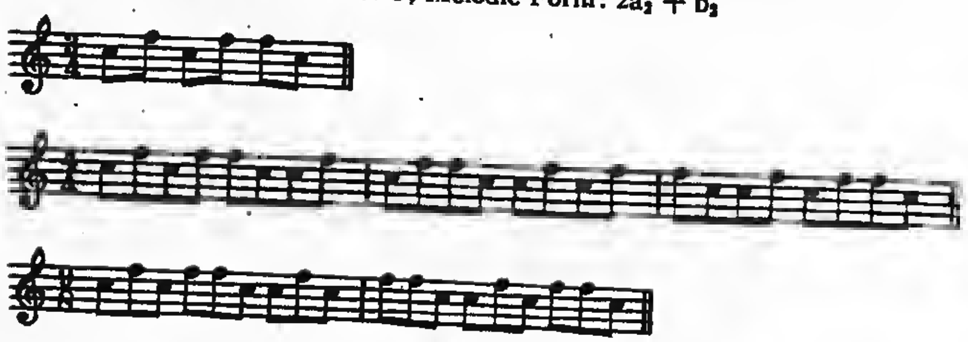


Figure 2.

(5) The final procedure is superimposition of time-rhythm on the preselected form of melodic continuity.

As it follows from the Theory of Rhythm [see Book I], such a continuity is subjected to synchronization and interference. The components of synchronization are the number of attacks in melodic continuity and the number of attacks in the rhythmic group.

Rhythm: $r_{4\div3}$; Measure: $\frac{3}{4}$

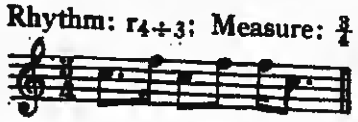


Figure 3.

Rhythm: $r_{3\div2}$; Measure: $\frac{2}{8}$

Figure 4.

C. Three-unit Scales. Two Intervals.
[The number of scales: 55.]

Table of Intervals

1+1	2+1	3+1	4+1	5+1	6+1	7+1	8+1	9+1	10+1
1+2	2+2	3+2	4+2	5+2	6+2	7+2	8+2	9+2	
1+3	2+3	3+3	4+3	5+3	6+3	7+3	8+3		
1+4	2+4	3+4	4+4	5+4	6+4	7+4			
1+5	2+5	3+5	4+5	5+5	6+5				
1+6	2+6	3+6	4+6	5+6					
1+7	2+7	3+7	4+7						
1+8	2+8	3+8							
1+9	2+9								
1+10									

Material:

(1) The number of melodic forms = 6.

$a_1 + b_1 + c_1 = a_2$
$a_1 + c_1 + b_1 = b_2$
$c_1 + a_1 + b_1 = c_2$
$b_1 + a_1 + c_1 = d_2$
$b_1 + c_1 + a_1 = e_2$
$c_1 + b_1 + a_1 = f$

(2) Combinations of melodic forms.

(a) Combinations by two:

$a_2 + b_2$	$b_2 + c_2$	$c_2 + d_2$	$d_2 + e_2$	$e_2 + f_2$
$a_2 + c_2$	$b_2 + d_2$	$c_2 + e_2$	$d_2 + f_2$	
$a_2 + d_2$	$b_2 + e_2$	$c_2 + f_2$		
$a_2 + e_2$	$b_2 + f_2$			
$a_2 + f_2$				

Each combination has two permutations. The number of cases: $15 \times 2 = 30$.

(b) Combinations by three:

(α) two places identical:

$a_2 + a_2 + b_2$

------ Total number: $15 + 15 = 30$

$a_2 + b_2 + b_2$ Three permutations each.

The number of cases: $30 \times 3 = 90$.

(β) all three places different:

$a_2 + b_2 + c_2$ $a_2 + c_2 + d_2$ $a_2 + d_2 + e_2$ $a_2 + e_2 + f_2$
$a_2 + b_2 + d_2$ $a_2 + c_2 + e_2$ $a_2 + d_2 + f_2$
$a_2 + b_2 + e_2$ $a_2 + c_2 + f_2$
$a_2 + b_2 + f_2$

$b_2 + c_2 + d_2$ $b_2 + d_2 + e_2$ $b_2 + e_2 + f_2$
$b_2 + c_2 + e_2$ $b_2 + d_2 + f_2$
$b_2 + c_2 + f_2$

$c_2 + d_2 + e_2$ $c_2 + e_2 + f_2$
$c_2 + d_2 + f_2$

$d_2 + e_2 + f_2$

Twenty combinations, six permutations each. The number of cases: $20 \times 6 = 120$.

(c) Combinations by four:

(α) Three identical places:

$a_2 + a_2 + a_2 + b_2$

--------- Total number: $15 + 15 = 30$.

$a_2 + b_2 + b_2 + b_2$ Four permutations each.

The number of cases: $30 \times 4 = 120$.

(β) Two identical pairs:

$a_2 + a_2 + b_2 + b_2$ Total number $= 15$.

Six permutations each.

The number of cases: $15 \times 6 = 90$.

(γ) Two identical places:

$a_2 + a_2 + b_2 + c_2$

--------- Total number: $20 \times 3 = 60$

$a_2 + b_2 + b_2 + c_2$

--------- Twelve permutations each.

$a_2 + b_2 + c_2 + c_2$

The number of cases: $60 \times 12 = 720$.

(δ) All four places different.

$a_2 + b_2 + c_2 + d_2$ $a_2 + b_2 + d_2 + e_2$ $a_2 + b_2 + e_2 + f_2$
$a_2 + b_2 + c_2 + e_2$ $a_2 + b_2 + d_2 + f_2$
$a_2 + b_2 + c_2 + f_2$

$a_2 + c_2 + d_2 + e_2$ $a_2 + c_2 + e_2 + f_2$
$a_2 + c_2 + d_2 + f_2$

$a_2 + d_2 + e_2 + f_2$

$b_2 + c_2 + d_2 + e_2$ $b_2 + c_2 + e_2 + f_2$
$b_2 + c_2 + d_2 + f_2$

$b_2 + d_2 + e_2 + f_2$

$c_2 + d_2 + e_2 + f_2$

Fifteen combinations, 24 permutations each. The number of cases: $15 \times 24 = 360$.

(d) Combinations by five may contain four, three or two identical places, or two identical pairs. As the material begins to grow to enormous quantities, this exposition will be limited by referring to the combinations with five different places.

$a_2 + b_2 + c_2 + d_2 + e_2$ $a_2 + b_2 + c_2 + e_2 + f_2$
$a_2 + b_2 + c_2 + d_2 + f_2$

$a_2 + b_2 + d_2 + e_2 + f_2$

$a_2 + c_2 + d_2 + e_2 + f_2$

$b_2 + c_2 + d_2 + e_2 + f_2$

Six combinations, 120 permutations each. The number of cases: $6 \times 120 = 720$.

(e) One combination by six:

$a_2 + b_2 + c_2 + d_2 + e_2 + f_2$

720 permutations.

The number of cases: $1 \times 720 = 720$.

(3) Continuity of melodic forms through permutations.

Circular permutations can be used as well. They give the best combinations by the number of elements.

$a_1 + b_1 + c_1 = a_2$
$b_1 + c_1 + a_1 = b_2$ Three melodic forms
$c_1 + a_1 + b_1 = c_2$

Combinations by 2:

$a_2 + b_2$ $b_2 + c_2$
$a_2 + c_2$

Three combinations, 2 permutations each. Total: $3 \times 2 = 6$.

Combinations by 3:

$a_2 + b_2 + c_2$

One combination, 6 permutations. Total: $1 \times 6 = 6$.

(4) **Coefficients of recurrence of the melodic forms.**

Figure 5.

(5) **Superimposition of time-rhythm.**

Melodic Form: $3a_2 + e_2 + 2c_2$; Rhythm: $r_{4 \div 3}$

Measure: $\frac{3}{4}$

Rhythm: $r_{5 \div 3}$ Measure: $\frac{3}{4}$

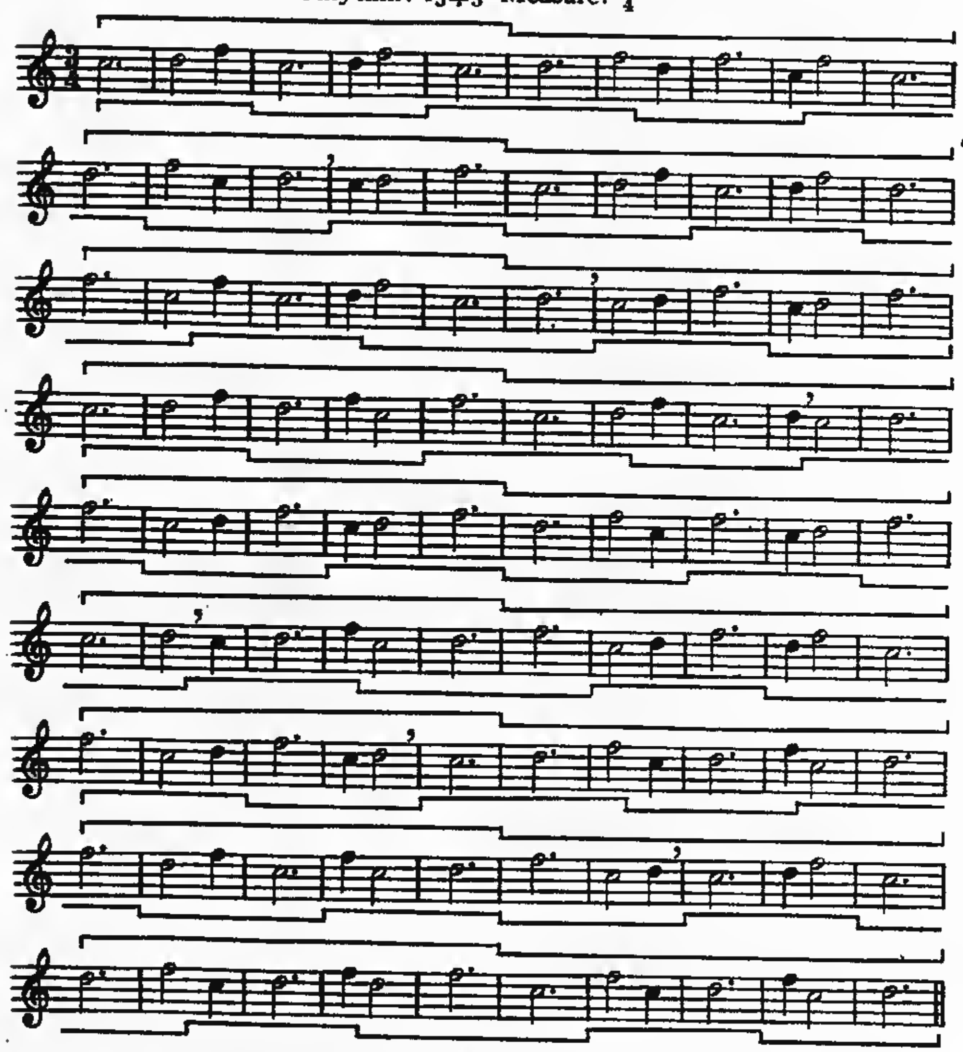


Figure 6.

D. Four-unit Scales. Three Intervals.

[The number of scales: 165.]

Table of Intervals

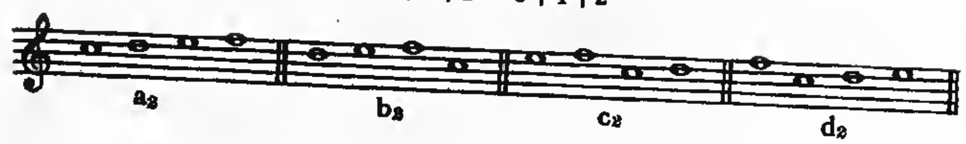
1+1+1	2+2+1	3+3+1	4+4+1	5+5+1
1+1+2	2+1+2	3+1+3	4+1+4	5+1+5
1+2+1	1+2+2	1+3+3	1+4+4	1+5+5
2+1+1	2+2+2	3+3+2	4+4+2	
1+1+3	2+2+3	3+2+3	4+2+4	
1+3+1	2+3+2	2+3+3	2+4+4	
3+1+1	3+2+2	3+3+3	4+4+3	
1+1+4	2+2+4	3+3+4	4+3+4	
1+4+1	2+4+2	3+4+3	3+4+4	
4+1+1	4+2+2	4+3+3		
1+1+5	2+2+5	3+3+5		
1+5+1	2+5+2	3+5+3		
5+1+1	5+2+2	5+3+3		
1+1+6	2+2+6			
1+6+1	2+6+2			
6+1+1	6+2+2			
1+1+7	2+2+7			
1+7+1	2+7+2			
7+1+1	7+2+2			
1+1+8				
1+8+1				
8+1+1				
1+1+9				
1+9+1				
9+1+1				

1+2+3	1+2+4	1+2+5	1+2+6	1+2+7	1+2+8
1+3+2	1+4+2	1+5+2	1+6+2	1+7+2	1+8+2
3+1+2	4+1+2	5+1+2	6+1+2	7+1+2	8+1+2
2+1+3	2+1+4	2+1+5	2+1+6	2+1+7	2+1+8
2+3+1	2+4+1	2+5+1	2+6+1	2+7+1	2+8+1
3+2+1	4+2+1	5+2+1	6+2+1	7+2+1	8+2+1

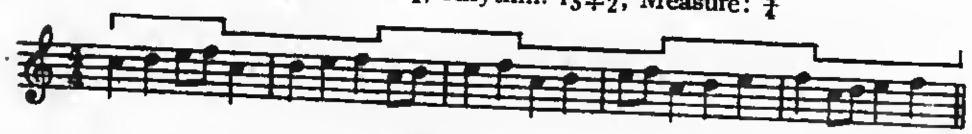
1+3+4	1+3+5	1+3+6	1+3+7
1+4+3	1+5+3	1+6+3	1+7+3
4+1+3	5+1+3	6+1+3	7+1+3
3+1+4	3+1+5	3+1+6	3+1+7
3+4+1	3+5+1	3+6+1	3+7+1
4+3+1	5+3+1	6+3+1	7+3+1

1+4+5	1+4+6
1+5+4	1+6+4
5+1+4	6+1+4
4+1+5	4+1+6
4+5+1	4+6+1
5+4+1	6+4+1

2+3+4	2+3+5	2+3+6	2+4+5
2+4+3	2+5+3	2+6+3	2+5+4
4+2+3	5+2+3	6+2+3	5+2+4
3+2+4	3+2+5	3+2+6	4+2+5
3+4+2	3+5+2	3+6+2	4+5+2
4+3+2	5+3+2	6+3+2	5+4+2



Melodic Form: a_2; Rhythm: r_{5+2}; Measure: $\frac{4}{4}$



Melodic Form: $a_2 + b_2 + c_2 + d_2$

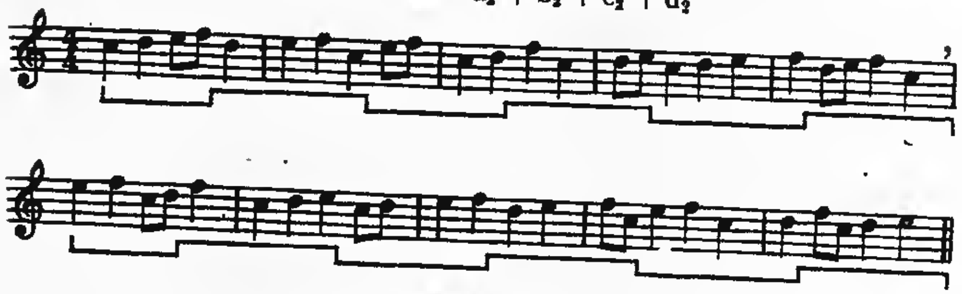


Figure 7.

Material:

(1) The number of melodic forms = 24 by all permutations, and 4 by circular permutations.

(2) Combinations by 2, 3, 4, . . . 24 through the first group, and by 2, 3, and 4 through the second group.

When it is desirable to *limit* the material, use circular permutations. Limited material is practical for thematic development of motifs. In writing studies for an instrument (in animated motion), often hundreds of attacks are necessary. Then a very large amount of material becomes desirable and all the permutations may be used.

(3) Continuity of melodic forms through permutations.

(4) Coefficients of recurrence of the melodic forms.

(5) Superimposition of time-rhythm.

The technique described above may be applied to variations, thematic development and composition of melodic continuity.

In arranging a song, the technique of varying melodic forms can be utilized for modification of the theme, or for the counter-motifs, (thus becoming modified thematic motifs), to fill in the time intervals during the long durations and the rests in a theme.

The following is an illustration of this procedure. [George Gershwin's *The Man I Love*, first four bars of the refrain.]*

Figure 8.

All four motifs have the form a_2 ($= a_1 + b_1 + c_1$), i.e., the sequence of appearance of the pitch-units is a_2 despite the recurrences. The scale is obviously a three-unit scale, and in the fourth bar the scale shifts its root-tone, following the harmony.

The next step is the modification of the second, the third and the fourth bars, using b_2, c_2 and d_2 respectively, and preserving the original form of recurrence.

The melody then acquires the following appearance:

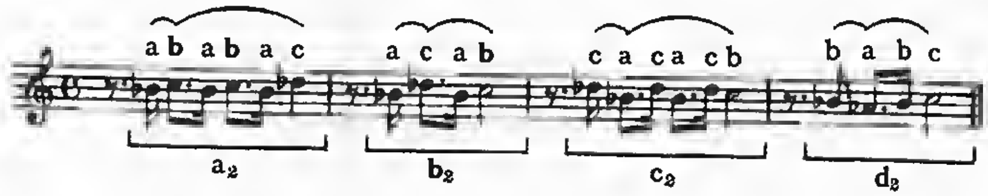


Figure 9.

Such modified motifs, being placed in any of the parts of harmony, produce a thematic "fill-in." It may be compared with the original neutral scalewise "fill-in" in Gershwin's own version.

E. Scales of Seven Units

Technical procedures similar to the foregoing are possible with the scales having more than four pitch-units. Any desired number of scales can be built with pitch-units exceeding 4.

There is no need to have complete charts of all 2,048 scales of this group, as all the necessary procedures will be generalized in the succeeding pages.

Seven-unit scales constitute the musical language of our civilization and serve as the material of harmony. There are 462 seven unit scales, but only a few will be offered in the following description.

Major and minor scales are constructed from four-unit scales known as "tetrachords."

By uniting two tetrachords separated by the interval 2, one can produce all major and minor scales with the repeated upper tonic. This form of presentation of the scale material helps to emphasize the different structures of so-called "major" and "minor" scales.

The three fundamental tetrachords are:

Major (M) = 2 + 2 + 1
Minor I (m_1) = 2 + 1 + 2
Minor II (m_2) = 1 + 2 + 2

In addition to these tetrachords, European music of the last few centuries also uses the tetrachord coming from the Mohammedan East (Arabia, Persia). This is a tetrachord which penetrated into Europe partly through the Crusaders and partly through the immediate influence of the Turks upon the Balkans. It still prevails in the southern part of Europe (Jugoslavia, Hungary, Rumania). It can be found in the music of Franz Liszt, Ludwig van Beethoven and many other composers. We shall call it the *harmonic* tetrachord (h). Its structure is: 1 + 3 + 1.

All major and minor scales are classified according to musical tradition into:

(1) Natural
(2) Harmonic
(3) Melodic

Though melody may be based on one unaltered scale, *hybrids* appear quite frequently. There is no law or reason for playing the melodic minor upward—and the natural minor downward, the way many instrumentalists do. As long as one intends to use hybrids, any hybrids may be used.

Major

Upward	*Downward*
natural	harmonic
natural	melodic
harmonic	natural
harmonic	melodic
melodic	natural
melodic	harmonic

Analogous hybrids exist in the minor group.

Table of Scales

Major		Minor
	Natural	
M + 2 + M		$m_1 + 2 + m_2$
	Harmonic	
M + 2 + h		$m_1 + 2 + h$
	Melodic	
M + 2 + m_2		$m_1 + 2 + M$

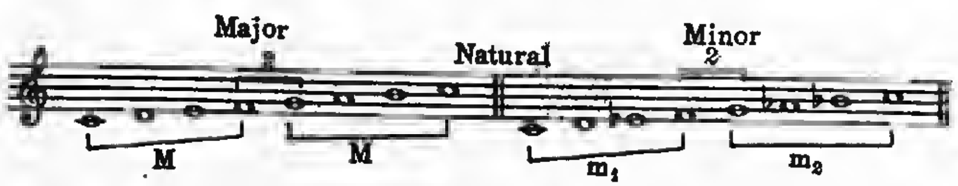


Figure 10 (continued).

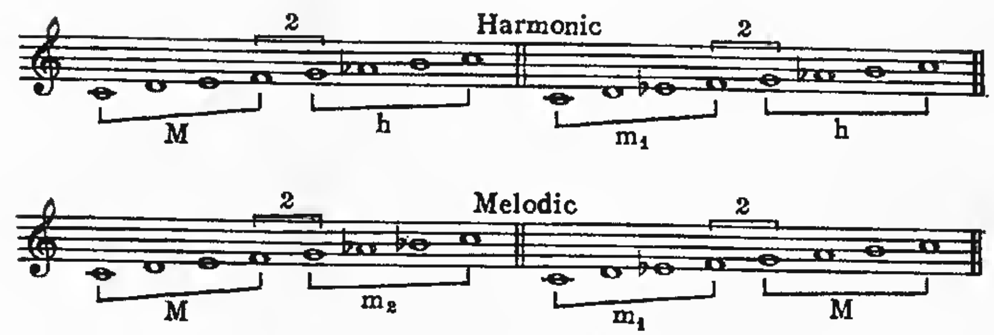


Figure 10 (concluded).

Comparing the two groups one finds that all lower parts of the major groups are M; all lower parts of the minor groups are m_1; all connections in all groups are 2; the natural scales in both groups have individual upper tetrachords; the upper tetrachords are in common for all harmonic scales; the melodic scales in both groups have individual upper tetrachords; the upper tetrachords in the natural and melodic scales exchange their structures, being, in the natural scales 2+2+1 for the major, and 1+2+2 for the minor; in the melodic scales, 1+2+2 for the major, and 2+2+1 for the minor.

Here are a few more scales in common use.

Neapolitan Minor: $m_2 + 2 + h$

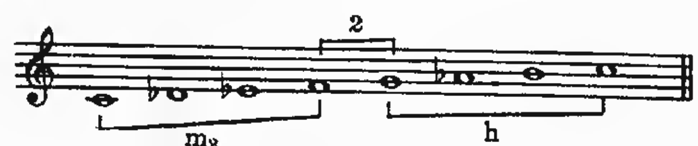


Figure 11.

Hungarian Minor: 2 + (1 + 3 + 1) + (1 + 3 + 1) = 2 + h + h

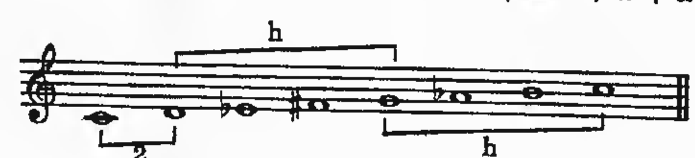


Figure 12.

Hungarian Major or "Blue": 3 + 1 + 2 + 1 + 2 + 1 + 2

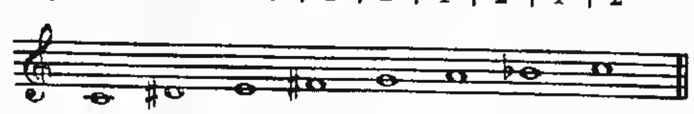


Figure 13.

Persian or Double Harmonic Scale: h + 2 + h

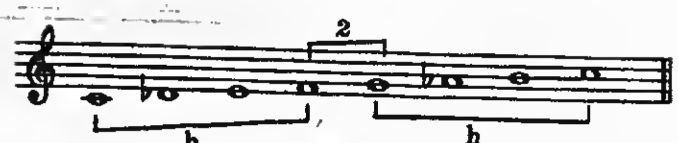


Figure 14.

The so-called "ecclesiastic modes" may be regarded as derived from natural major.

The entire technique of scale derivation, as well as the evolution of scales within the families, will be explained in the succeeding pages.

CHAPTER 3

EVOLUTION OF PITCH-SCALE STYLES

PITCH-SCALES, like time-scales (rhythms), are subject to serial development. The number-values express pitch intervals. Each scale with two pitch-units and one interval becomes a *generator* of its family. Splitting the number-value expressing the interval into a binomial, we acquire a three-unit scale with two intervals. The modified forms of the binomial interval fall into synchronization and produce a resultant scale with four units and three intervals. The modified forms of the trinomial interval fall into synchronization and produce a resultant scale with six units and five intervals. The modified forms of the quintinomial interval fall into synchronization and produce a resultant scale with ten units and nine intervals.

A. Relating Pitch-Scales through the Identity of Intervals

All scales identified by the original interval, or the consequent resultants, belong to one family. This is the process of relating pitch-scales through the *identity of intervals.*

Example:

Two-unit scale. Interval = 5 = c − f

(a) 5 = 3 + 2 = c − e♭ − f
5 = 2 + 3 = c − d − f

This interference group produces the resultant trinomial =
= 2 + 1 + 2 = c − d − e♭ − f
2 + 2 + 1 = c − d − e − f
1 + 2 + 2 = c − d♭ − e♭ − f

The following quintinomial (the resultant of the second interference-group) is uniformity, i.e., = 1 + 1 + 1 + 1 + 1 = c − d♭ − d♮ − e♭ − e♮ − f
Uniformity, being neutral, belongs to all families (as the last interference) and does not possess any distinctive characteristics.

(b) 5 = 4 + 1 = c − e − f
5 = 1 + 4 = c − d♭ − f

The resultant of this interference-group =
= 1 + 3 + 1 = c − d♭ − e − f
1 + 1 + 3 = c − d♭ − d♮ − f
3 + 1 + 1 = c − d♯ − e − f

The following quintinomial is neutral.

Example:

Trinomial:
4 + 4 + 3 = c − e − g♯ − b
4 + 3 + 4 = c − e − g − b
3 + 4 + 4 = c − e♭ − g − b

The resultant quintinomial of 4 + 4 + 3 with permutations, equals:

3 + 1 + 3 + 1 + 3 = c − e♭ − e♮ − g − g♯ − b
1 + 3 + 1 + 3 + 3 = c − d♭ − e − f − a♭ − b
3 − 1 + 3 + 3 + 1 = c − d♯ − e − g − a♯ − b
1 + 3 + 3 + 1 + 3 = c − d♭ − e − g − a♭ − b
3 + 3 + 1 + 3 + 1 = c − e♭ − f♯ − g − a♯ − b

The resultant nine-term polynomial equals:

1+2+1+1+1+1+1+2+1 = c − d♭ − e♭ − f♭ − f♮ − g♭ − g♮ − a♭ − b♭ − b♮
2+1+1+1+1+1+2+1+1 = c − d − d♯ − e − f − f♯ − g − a − a♯ − b
1+1+1+1+1+2+1+1+2 = c − c♯ − d − d♯ − e − f − g − g♯ − a − b
1+1+1+1+2+1+1+2+1 = c − c♯ − d − d♯ − e − f♯ − g − g♯ − a♯ − b
1+1+1+2+1+1+2+1+1 = c − d♭ − d♮ − e♭ − f − g♭ − g♮ − a − b♭ − b♮
1+1+2+1+1+2+1+1+1 = c − c♯ − d − e − f − f♯ − g♯ − a − a♯ − b
1+2+1+1+2+1+1+1+1 = c − d♭ − e♭ − e♮ − f − g − a♭ − a♮ − b♭ − b♮
2+1+1+2+1+1+1+1+1 = c − d − d♯ − e − f♯ − g − g♯ − a − a♯ − b
1+1+2+1+1+1+1+1+2 = c − c♯ − d − e − f − f♯ − g − g♯ − a − b

Thus, one may start with a monomial, trinomial or quintinomial, and evolve scales of corresponding complexity which *belong to one family*, i.e., which provide a homogeneous melodic continuity.

Taking a scale from folklore, one may compose music of different degrees of complexity—yet secure an authentic style.

This method also provides material from which one may evolve themes of different complexity, to be used in one musical continuity (as primary and secondary subjects or counter-subjects). For example, if we use the original scales of the Stony Indians (Alberta, Canada) [3 + 2 = c − e♭ − f and 2 + 3 = c − d − f] for one subject, it is desirable to adopt the resultant scale [2 + 1 + 2 = c − d − e♭ − f] for another subject.

B. Relating Pitch-Scales Through the Identity of Pitch-units

Another device through which scales of one family may be evolved is *the process of circular permutations* of the pitch-units of the original scale. This is the process of relating pitch-scales through the *identity of pitch-units*.

Take the scale c − d − e − g − a. Let the original scale be indicated as d_0 (zero displacement.) The derivative scales resulting from circular permutations of these pitch-units will be indicated as d_1 (the first displacement scale), d_2 (the second displacement scale), etc. The number of displacement scales resulting from the original scales equals the number of units in the original, minus one.

$$\boxed{N_d = N_u - 1}$$

There are 5 units in the c − d − e − g − a scale.

$N_d = 5 - 1 = 4$

Here is a chart of derivative scales:

d_0 c − d − e − g − a
d_1 d − e − g − a − c
d_2 e − g − a − c − d
d_3 g − a − c − d − e
d_4 a − c − d − e − g

As can be seen from the above chart, each of the derivative scales has a different group of intervals.

The following chart represents the transposition of these scales to the tonic *c*.

d_0 c − d − e − g − a
d_1 c − d − f − g − b♭
d_2 c − e♭ − f − a♭ − b♭
d_3 c − d − f − g − a
d_4 c − e♭ − f − g − b♭

Scales derived from permutations of the *intervals* are different from scales derived from permutations of the *pitch-units*, though there are *some* coincidences.

Original scale:

c − d − e − g − a = 2 + 2 + 3 + 2

Permutation of the Intervals:

c − d − e − g − a
2 2 3 2

c d e f♯ a
2 2 2 3

c e♭ f g a
3 2 2 2

c d f g a
2 3 2 2

Only the last scale in this group coincides with one in the preceding group (d_3).

Using either form of producing derivative scales of the same family, one can evolve a melodic continuity. Other devices previously presented, such as permutations of pitch-units in the scales following as one group, combinations of such melodic forms, coefficients of recurrence and superimposition of time-rhythm, can be used in the composition of continuity.

The following examples offer illustrative comparisons of the application of these different devices.

The original scale: Melodic forms derived from circular permutations:

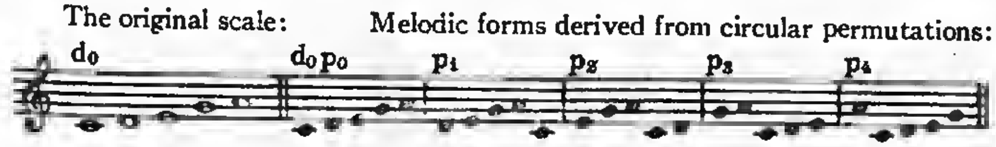
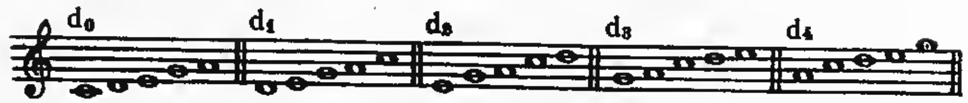
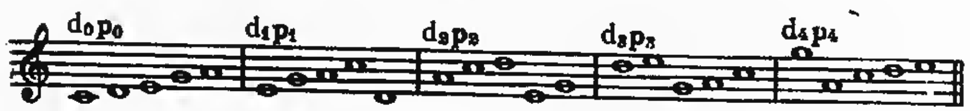


Figure 15. (continued).

The original scale: Derivative scale through pitch permutations:



Melodic forms derived from pitch permutations:



Transposition of the pitch permutation scales:

Melodic forms derived from the above scales:

The original scale: Derivative scales through interval permutations:

Melodic forms derived from the above scales:

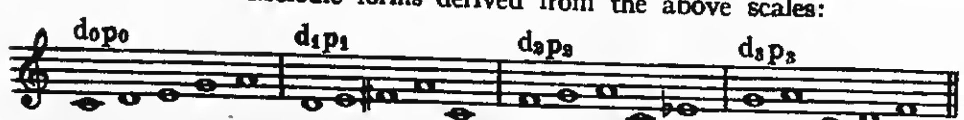


Figure 15 (concluded).

C. Evolving Pitch-Scales through the Method of Summation

There are two methods of evolving scales each with a different number of units but belonging to the same family.

The first method was described as the method of interference, as applied to the number-values expressing intervals. Through this method we can evolve scales with a greater number of units than in the original one. When a scale with many units is the original scale, the simpler derivative scales may be evolved through reversal of the first procedure, i.e., through *summing up* the number-values expressing intervals. For example, if the original scale is: 2 + 2 + 1 + + 2 + 2 + 1, i.e., c − d − e − f − g − a − b♭, simpler scales may be evolved in the following ways:

(2+2)+1+2+2+1 = 4+1+2+2+1 = c − e − f − g − a − b♭
2+(2+1)+2+2+1 = 2+3+2+2+1 = c − d − f − g − a − b♭
2+2+(1+2)+2+1 = 2+2+3+2+1 = c − d − e − g − a − b♭
2+2+1+(2+2)+1 = 2+2+1+4+1 = c − d − e − f − a − b♭
2+2+1+2+(2+1) = 2+2+1+2+3 = c − d − e − f − g − b♭

(2+2+1)+2+2+1 = 5+2+2+1 = c − f − g − a − b♭
2+(2+1+2)+2+1 = 2+5+2+1 = c − d − g − a − b♭
2+2+(1+2+2)+1 = 2+2+5+1 = c − d − e − a − b♭
2+2+1+(2+2+1) = 2+2+1+5 = c − d − e − f − b♭

(2+2+1+2)+2+1 = 7+2+1 = c − g − a − b♭
2+(2+1+2+2)+1 = 2+7+1 = c − d − a − b♭
2+2+(1+2+2+1) = 2+2+6 = c − d − e − b♭

(2+2+1+2+2)+1 = 9+1 = c − a − b♭
2+(2+1+2+2+1) = 2+8 = c − d − b♭

(2+2+1)+(2+2+1) = 5+5 = c − f − b♭
etc.

D. Evolving Pitch-Scales through the Selection of Intervals

The second method consists of taking a smaller group of intervals or units from the original scale in the sequence of their appearance.

(a) We may evolve partial scales through selecting pitch-units from the original scale.

The Original scale:
c − d − e − f − g − a − b♭
Partial six-unit scales:
c − d − e − f − g − a
d − e − f − g − a − b♭
Partial five-unit scales:
c − d − e − f − g
d − e − f − g − a
e − f − g − a − b♭

Partial four-unit scales:

c – d – e – f
d – e – f – g
e – f – g – a
f – g – a – b♭

Partial three-unit scales:

c – d – e
d – e – f
e – f – g
f – g – a
g – a – b♭

Partial two-unit scales:

c – d
d – e
e – f
f – g
g – a
a – b♭

(b) We may evolve partial scales through selecting intervals from the original scale, and in the sequence of their appearance.

The original scale:

2 + 2 + 1 + 2 + 2 + 1

Partial Scales:

2 + 2 + 1 + 2 + 2	= c – d – e – f – g – a
2 + 1 + 2 + 2 + 1	= c – d – e♭ – f – g – a♭
2 + 2 + 1 + 2	= c – d – e – f – g
2 + 1 + 2 + 2	= c – d – e♭ – f – g
1 + 2 + 2 + 1	= c – d♭ – e♭ – f – g♭
2 + 2 + 1	= c – d – e – f
2 + 1 + 2	= c – d – e♭ – f
1 + 2 + 2	= c – d♭ – e♭ – f
(2 + 2 + 1)	
2 + 2	= c – d – e
2 + 1	= c – d – e♭
1 + 2	= c – d♭ – e♭
(2 + 2)	
(2 + 1)	

Scales with identical structures are omitted (numbers in parentheses).

E. Historical Development of Scales

Analysis of historic material in the field of melody reveals that the laws of identity described above (pitch, interval) develop intuitively with different races and civilizations.

Primitive American Indian music, such as that of the Canadian Stony Indians in Alberta already cited, has two 3-unit scales, both belonging to the same family through identity of intervals (3 + 2 and 2 + 3). The ancient Greeks had their fundamental tetrachord (4-unit scale) 2 + 2 + 1. They called it a "Lydian" tetrachord. Through their own procedures, which were quite different from the procedures described in this theory, they found two other fundamental tetrachords: the "Phrygian" (2 + 1 + 2), and the "Dorian" (1 + 2 + 2). This is another case of evolving scales through interval identity. Ancient China used a scale which has still survived and which is used throughout Asia among the Mongols. It is usually known as a "pentatonic" scale. Naturally, this is only one of the large number of the "pentatonic"—i.e., 5-unit—scales. The construction of this scale is 2 + 2 + 3 + 2. Another scale used by the Chinese has the construction, 2 + 3 + 2 + 2.

It is interesting to note, also, that the last-mentioned two scales have frequently been employed in many American popular songs in the course of the last two decades.

What is still more important is that the Americans have developed intuitively—and perhaps even through the channels of harmony—two other scales used together with the two Chinese scales and incorporated into the same musical continuity. These scales have been described in the preceding text, and possess the following structures:

2 + 2 + 2 + 3 and	3 + 2 + 2 + 2
c – d – e – f♯ – a	c – e♭ – f – g – a

A similar analysis of the more developed scales, such as the 7-unit scales of our major and minor groups, and the Greek and the Ecclesiastic modes, reveals that musical intuition, with the investment of centuries of experience, has led to the evolution of scale families through a proper channel.

Through our method of analysis, we find that the so-called "Ecclesiastic modes," i.e., scales used during the Middle Ages in Europe, are *displacement* scales of the natural major. Natural major was known as the Ionian mode; d_1 was known as the Dorian mode; d_2 was known as the Phrygian mode; d_3 was known as the Lydian mode; d_4 was known as the Mixolydian mode; d_5 was known as the Aeolian mode; and d_6 was the Locrian or Hypo-Phrygian mode. These scales all conform to one family through the identity of their pitch-units.

There are two different systems of terminology which conflict with each other in relation to the above-mentioned scales (modes). The one offered here is the *medieval* terminology used by musicians. The other is the ancient Greek terminology used by historians only with reference to the ancient Greek music. When such discrepancies occur as the Ecclesiastic Dorian mode being called the Greek Phrygian, the explanation is quite apparent—that when Greek manuscripts were studied during the Middle Ages many things were misinterpreted, and this change of the names is merely due to misunderstanding of the Greek terms.

Taking advantage of the fact that the whole European culture of music is an outcome of circular pitch displacement in the natural major or Ionian mode, this evolution can be continued from any other forms of major and minor, thus yielding 21 more displacement-scales: 7 from harmonic major; 7 from harmonic minor; and 7 from melodic major. Upon comparison of the major and the minor natural scales, it may be observed that the natural minor is the d_5 of the natural major, and the melodic minor is the d_3 of the melodic major.

As it follows from the previous text, all the pitch-displacement scales may be transposed to the same pitch-axis (key note). When we apply this method to natural major scale and its derivative modes, this entire group appears in different normal key signatures. Starting the natural major (Ionian) scale on c, the key signature is zero. Starting the Dorian (d_1) on c places this music in the key of B♭ major, to which the two flats (b♭ and e♭) belong. The Phrygian mode (d_2) starting on c acquires the four normal flats pertaining to A♭ major. The Lydian mode (d_3) acquires one sharp pertaining to G major. The Mixolydian mode (d_4) acquires one flat pertaining to F major. The Aeolian mode (d_5) acquires three flats pertaining to E♭ major. The Locrian mode (d_6) acquires five flats pertaining to D♭ major.

All the displacement scales derivative from the natural major (through which my system of key signatures, not commonly in use, has been evolved) may be automatically transposed to one axis, in which the different displacement scales will have the same name for their pitch-units, but differ in their key signatures. If the great composers of the past had known anything about this procedure (i.e., that the same music can acquire different characteristics without loss of any of its ingredients and without distortion of any of its components), they would have overcome difficulties in finding the proper type of chords, their progressions and the forms of voice leading—all of which was one of the most difficult tasks they faced in their intuitive attempts at modal writing. Their difficulty was not only in finding the proper chord relations, but also in finding *all* the chords belonging to any one of the displacement scales.

Rimsky-Korsakov, who is considered one of the best composers in modal writing, is helpless enough when he tries to find the proper chord progressions for such modes as Dorian, or Mixolydian, but he becomes entirely helpless when he attempts to *modulate* through various modes. The first problem is merely a problem of automatic key signature adjustment; the second will be explained in the next chapter.

TABLE OF MODAL TRANSPOSITIONS

Original Key	Derivative Scale (Mode)	Derivative Key	Relative Signature
c	Dorian d_1	B♭	2♭
c	Phrygian d_2	A♭	4♭
c	Lydian d_3	G	1♯
c	Mixolydian d_4	F	1♭
c	Aeolian d_5	E♭	3♭
c	Locrian d_6	D♭	5♭

Figure 16.

The above signature variations are relative to their original keys. All the additional sharps mean the addition of sharps to the naturals, and the addition of naturals to the flats. All the additional flats mean the addition of flats to the naturals, and the addition of the naturals to the sharps.

For example, if one desires to play music written in the key of A major directly in Phrygian mode, and A major contains three sharps in its key signature (f♯, c♯, g♯), translation into the Phrygian mode will require the addition of four flats, i.e., the cancellation of the three sharps into naturals and the addition of one flat (b♭). Music originally written in the key of natural C minor (Aeolian), to be played in Mixolydian scale, requires cancellation of e♭ and a♭. C minor is d_5 in the key of E♭ major. E♭ major has three flats in its key signature (b♭, e♭, a♭). The Mixolydian mode starting on c belongs to F major, which has one flat in its key signature (b♭). The difference between the Aeolian of E♭ major, and the Mixolydian of F major excludes the two above-mentioned flats from the key signature. This explains how through a more complicated procedure one can perform modal transpositions automatically.

There is room in this description to present one illustration of the inadequate modal manipulations of the composers of the past—manipulations considered to be acceptable only by reason of the present level of musical competence.

For a classical example, take a record or the music of the *Song of the Viking* from Rimsky-Korsakov's opera, *Sadko*. Play it first as it is written by the composer; then cancel all the accidentals. The two versions should be compared, and the component scales analyzed. It will be sufficient to take the first refrain where modulation returns it to the original Dorian d (C major).

As musical key signatures in their customary form refer only to the natural scales, all other alterations of pitch appear as accidentals. Therefore, automatic modal transposition refers only to the Ionian scale and its derivatives. But if the musical world faced the fact squarely, it would agree that most key signatures are a pure myth; that there is scarcely a piece of music which really evolves in a natural scale throughout; that scales change and are modified, and so does the key-axis. Then all could agree that the application of *real* key signatures would solve the problem of universal automatic transposition which is possible now only for the natural scales. For example, if one would like to play music written in natural major, in the scale which is d_3 of G melodic minor, it would be necessary only to add both b♭ and f♯ to the key signature.

Existing musical theories offer such vague notions on this matter that they even explain such scales as being in the key of F major and confuse the f♯ alteration with g♭. This is true of one of the attempts to explain music written by Scriabine.

CHAPTER 4

MELODIC MODULATION AND VARIABLE PITCH AXES

OUR SENSORY orientation—with respect to static and kinetic forms—is based on our ***general associative orientation.*** The prerequisite of the latter is *memory.* Real or imaginary guiding lines help us to apprehend, to analyze, to study and to construct different forms. In geometry, we use the coordinates, the bisector, the directrix, the radius-vector; in astronomy, we use the geodetics (equator, ecliptic); in painting, design and sculpture, we apply geometrical lines and centers (the coordinates, the medians, the area boundaries, the center of gravity, the harmonic [rhythmic] center).

When it comes to music, we must all confess that previously accepted musical "theories" do not provide us with such luxuries. Musical notation does not suggest any quantitative or directional data. Fortunately for the art of music and for musicians, our general associative orientation is not obscured by our own acquired musical education.

A. Primary Axis

When we listen to a melody we hear and identify (owing to our memory) that pitch-unit which is more predominant. Our auditory centers register the quantity of attacks and durations on various sound-wave frequencies which constitute a certain melody. Then our memory sums them up, thus producing an imaginary line (which can be registered graphically)—*the primary axis of a melody.*

A *primary axis* (P.A.) may be defined as the *pitch-time* maximum of an entire melody or of any portion of it. This means that, when we hear only the first two measures of a certain melody, the axis may be one pitch-unit, but when we hear the first eight measures of the same melody, it may be *another.* We re-orientate ourselves as time flows. It is very noticeable that while we move away from certain nearby objects, the center of scenery modulates—as, for example, on the ferry-boat trip from Manhattan to Staten Island.

The P.A. of a melody is the root-tone (the tonic) of a *real* scale. If a melody is written in the standard signature of three flats (b♭, e♭, a♭), it may be in any of the displacement scales of the natural E♭ Major. If the P.A. of such melody is *g*, then it is a case of Phrygian *g* scale. Only through associations with harmony may we think of e♭ being a root-tone under such circumstances. But any of the derivative scales may be harmonized by the chords of any other derivative scale from the same d_0. Thus, ***the number of axis-relations between a melody and its harmony equals the square of the number of derivative scales*** (from one d_0 and including d_0). Therefore, any of the five-unit scales offers 25 axis-relations between melody and harmony. Any seven-unit scale offers 49. A melody in d_0 may be accompanied by harmony in d_0, d_1, d_2 . . . A melody in d_1 may be accompanied by harmony in d_0, d_1, d_2 . . . etc.

There are four forms of these axis-relations which will be considered in this discussion in time-continuity (as modulations from one axis to another). Later they will be considered in the theory of simultaneous correlation of melodier (counterpoint)* and correlation of melodies with harmonies (melodization of harmony and harmonization of melody).**

B. The Key-Axis

When harmony is absent, the P.A. of a melody is the real *key-axis*. The term "modal" in the following classification will pertain to *intervals;* the term "tonal" pertains to *pitch-units*. "Unimodal" means "in identical mode," i.e., the scale remains the same. "Polymodal" means "in different modes," i.e., the scale varies. "Unitonal" means "in identical tonality," i.e., the key remains the same. "Polytonal" means "in different tonalities," i.e., the key varies.

Modulations may be performed from key to key without a change in the scale-structure, and modulations from a scale of one structure into a scale with a different structure may be accomplished without a change in key.

C. Four Forms of Axis-Relations:

(1) Unitonal — Unimodal U — U
(2) Unitonal — Polymodal U — P
(3) Polytonal — Unimodal P — U
(4) Polytonal — Polymodal P — P

The key does not change in the first two forms. U — U (1) represents the affirmation of P.A. and zero modulation. The process of establishing a key and a scale includes: 1) introduction of the *pitch-units* of a selected scale in any desirable sequence; 2) movement of the *leading tones* (pitch-units adjacent to the tonic—P.A.) into P.A.; and 3) the quantitative predominance of P.A.

Time: ($r_{3 \div 2}$) Pitch: 2 + 2 + 3 + 2

1. *Melodic continuity (circular permutations of the scale):*

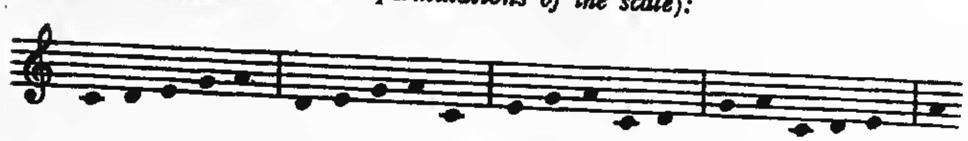


Figure 17.

2. *Melodic continuity with time-rhythm superimposed:*

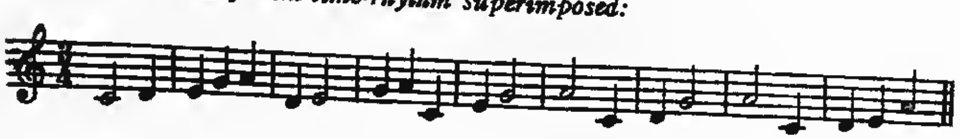


Figure 18.

*See Book VII.
**See Book VI.

Unit *a*, whose durations sum up to 7, forms the P.A. of this melody. This shows that any pitch-unit of a scale may become a P.A. U — P (2) represents modulations on scales derived from one common d_0. Such modulations may be achieved through one procedure: transposition of the melody into any derivative scale.

The key-axis of the scale in figure 18 is *c*, while the P.A. of the melody is *a*. The key-axis of d_1 scale is *d*, while the P.A. of the melody becomes *c*. The key-axis of d_2 scale is *e* while the P.A. of the melody becomes *d*, etc.

The following is the original melody together with its transposed versions to all the other axes:

Figure 19.

These five different axes become elements of continuity. Five elements produce 120 permutations. Any of these 120 forms may be used for esthetic purposes.

Here is a composition employing the following arrangement: $d_3 - d_2 - d_1 - d_4 - d_0$.

Figure 20.

The entire continuity of all 120 forms would extend to 5,400 measures (45 × 120).

By varying the key signatures (which remain constant every time for the entire continuity), we can multiply the number of possible compositions by 330, the number of all five-unit scales.

The relationship, P – U (3), represents a more general form of variation of the key-axis. In this case all or some of the pitch-units are not in common in the two adjacent key-axes (the preceding and the following keys). The structure of the scale remains the same.

P – P (4) represents a case in which both the *key-axis and the scale vary*, and *the pitch-units are not entirely in common*.

Cases (3) and (4) emphasize modulations as they are usually known, i.e., from one key to another, with or without modification of the scale structure.

When these axis-relations concern seven-unit scales, some of the pitch-units are in common because all the combinations by seven, taken from twelve elements and with more or less uniform distribution, have some elements in common. For example, natural C Major and natural E♭ Major have four units in common: c, d, f, g. Pitch-units which are written differently, but sound the same (enharmonics), must be considered identical.

Thus in modulations from natural C Major to harmonic f♯ minor, four units are in common: a, b, d, f (= e♯). Scales with fewer pitch-units, being constructed from different key-axes, may not have any tones in common. Such is the case in 2 + 3 + 2 = c – d – f – g and an identical scale from an *e* key-axis: *e* – f♯ – a – b.

In the older civilizations, where the number of units in a scale is restricted to a very few, modulations exist in the (1) and (2) type of the axis relations only. Types (3) and (4) are unknown. *To make such scales practical for type (3) and (4) modulations, the themes must already be modulating through type (2).* This increases the number of pitch-units in a theme and produces potential common tones.

The technique of transition from one key-axis to another for types (3) and (4) consists of three different devices, each having a different esthetic value:

(a) common units
(b) chromatic alterations
(c) identical motifs

D. Modulating through Common Units

In order to modulate through common units, it is necessary:

(1) to detect the pitch-units which are in common between the preceding and the following key.
(2) to produce motifs on common units long enough to eliminate the potential discrepancy between units of the preceding and the following key that are not in common (i.e., long enough to let the memory forget the possible discrepancy). The motifs are melodic forms with time rhythm superimposed. It is best to take rhythm material from the theme.

The theory of planning variable key-axes will be fully explained in the *Special Theory of Harmony.** For the present, it is best to modulate into *any key-axis which is identical with one of the pitch-units of the original scale.*

If c – d – e – g – a is the original scale, the best modulations are to the keys of d, e, g and a. The corresponding scales assume the following appearances:

Key of c = c – d – e – g – a
Key of d = d – e – f♯ – a – b
Key of e = e – f♯ – g♯ – b – c♯
Key of g = g – a – b – d – e
Key of a = a – b – c♯ – e – f♯

The sequence of different keys in one melodic continuity composes the possible permutations. For contrast, use as adjacent keys those which have fewer units in common; for similarity, do the contrary. In the case above, with the following planning—Key c – Key g – Key e – Key a—similarity is obtained by modulating between the first two keys, extreme contrast between the second and the third keys, and much less contrast between the last two keys.

The following is an example of modulatory continuity obtained through the application of *common units*. It is desirable *not* to show the axis of the following key in the course of modulation. The reasons for this will appear later in the *Theory of Melody*.**

* See Book V. **See Book IV.

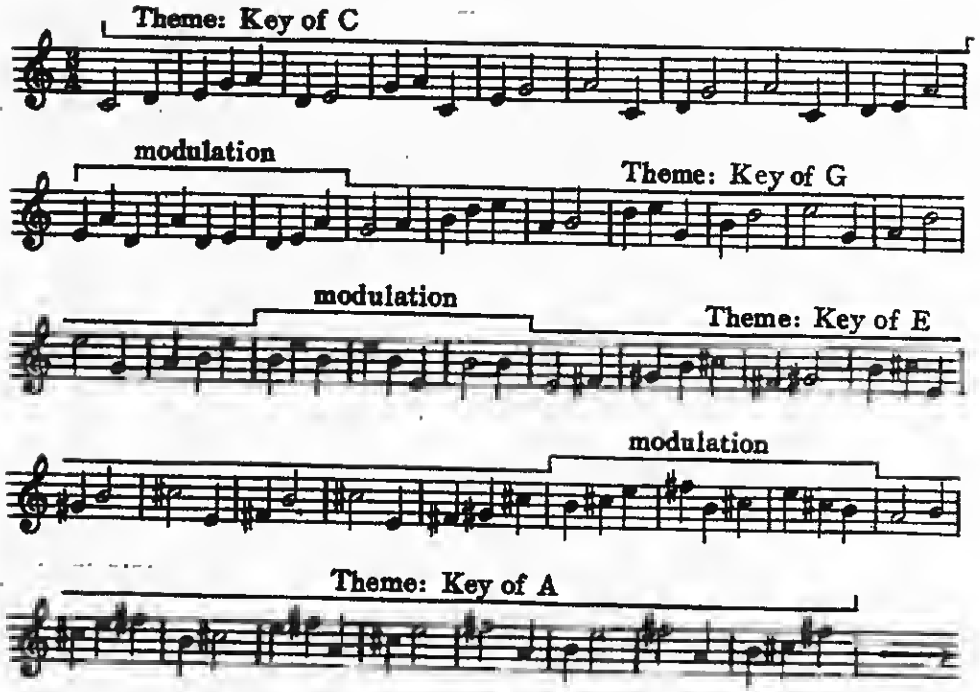


Figure 21.

Modulation through common units is the most subtle of all modulatory devices. A question may arise as to *how long* a modulation should be. The only answer to this question is: long enough to let the listener forget the potential pitch-discrepancy of the adjacent keys. The example given above illustrates a reasonable average. The length of modulation depends on the absolute velocity (tempo) of music, as well as on the audience for which the music is written. It must be longer for the conservative listener and shorter for the advanced one. At the present time there are many listeners who object to any modulatory transition from key to key; instead, they prefer to go there directly. Some modern composers object to the very phenomenon of the axis.

E. Modulating through Chromatic Alteration

If the common-unit method of transition is regarded as the process of *dodging* the conflicts (as in diplomacy) then the chromatic alteration method of transition is entirely bold; it takes advantage of the possible conflict and goes about it directly (as in war).

In order to modulate through chromatic alterations, it is necessary:

(1) to find the units that are not in common but have identical musical names (like c — c♯).

(2) to perform one or more chromatic operations with such units. A single chromatic operation consists of *demonstrating the preceding and the following units in reasonably long durations* with their following into the next unit bearing a different musical name (like c — c♯ — d or c — c♭ — b♭). In the case of *more than one* chromatic operation, it is necessary to *proceed immediately* with the other intended chromatic operations and to use the *third term* of a chromatic group *in the last group only.*

Example:

From natural C Major to natural E♭ Major. Units not in common: b — b♭; e — e♭; a — a♭.

One operation: b — b♭ — a♭; e — e♭ — d; a — a♭ — g.

More than one operation: b — b♭ — e — e♭ — a — a♭ — g.

Modulatory Continuity Obtained Through the Application of Chromatic Alterations:

(Theme: from the preceding example of modulation through common units; key-sequence: C — E).

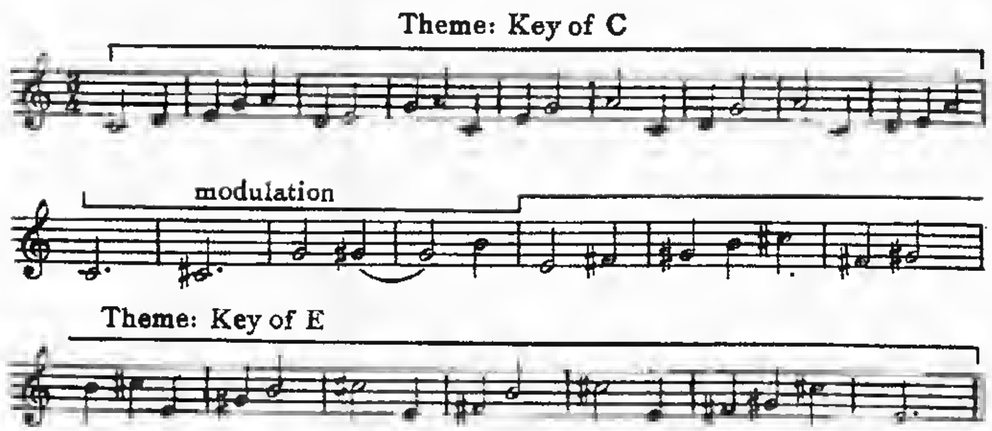


Figure 22.

F. Modulating through Identical Motifs

The *identical-motif* method of transition is the process of imitating appearances and is like adapting oneself to a surrounding medium which constantly varies (as in mimicry; compare with the behavior of a chameleon). It is the most obvious and the most commonly used of all three methods of transition.

In order to modulate through identical motifs, it is necessary:

(1) to select a motif from the theme which immediately precedes the modulation.

(2) to construct another motif identical or similar in appearance aod to adapt it to the signature of the succeeding key. The second motif may consist of the pitch-unit bearing the same musical names as the first motif, or it may be located in the adjacent lower or higher position.

This method of transition is very typical of popular songs, or of anything that must have an obvious character. It is also found in most of the well-known classical symphonies.

The following illustration is one of modulatory continuity obtained through the application of ***identical motifs.*** The theme is from the preceding example: Key sequence: C – E – G – A.

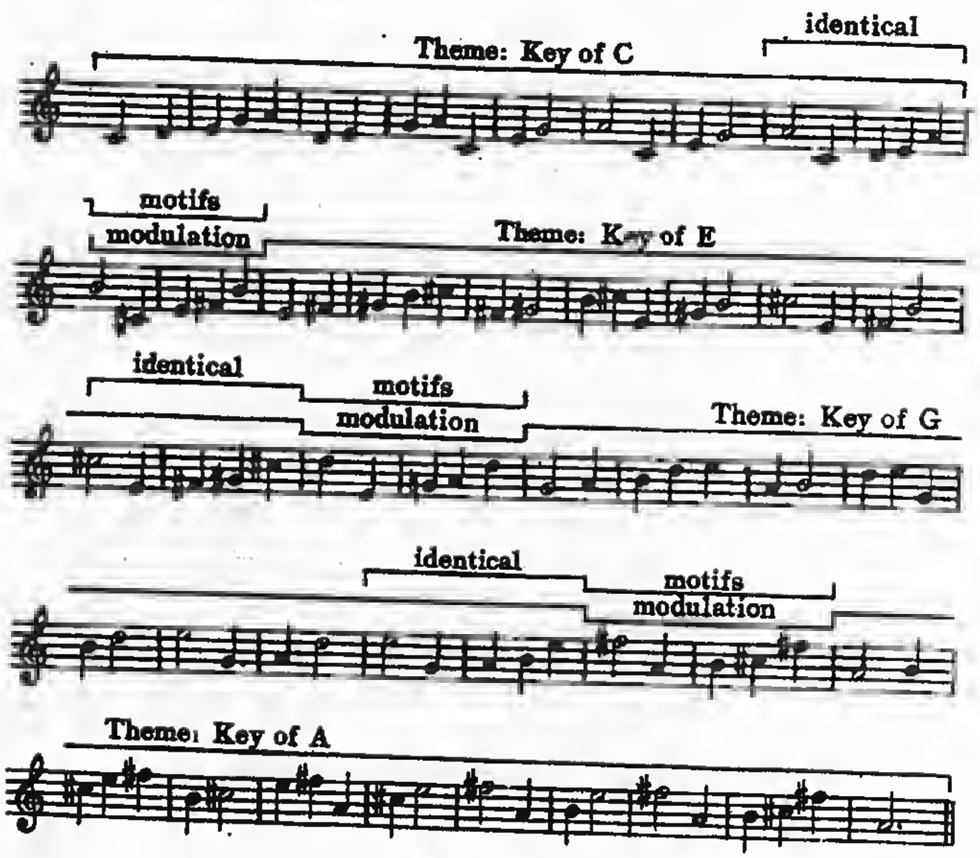


Figure 28.

CHAPTER 5

PITCH-SCALES: THE SECOND GROUP

Scales in Expansion

THE second group of pitch-scales emphasizes scales produced from one constant pitch-unit, and ***exceeding*** the range of one octave. These scales do not necessarily conform to two- or three-octave range. The range may be more than one and less than two octaves; more than two and less than three, etc.

A. Methods of Tonal Expansion

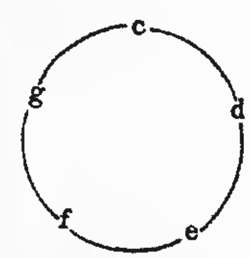
Scales constituting this group may be obtained by means of *tonal expansion* (expansion of invariant pitch-units through rearrangement of their mutual positions) of the scales of the first group.

The first expansion (E_1) of a scale may be obtained through circular permutation ***over*** one pitch-unit of the original scale.

There are two cases: first, when the number of units in a scale is an odd number.

Example:

Scale: c – d – e – f – g
Circular arrangement:

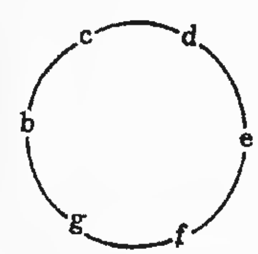


The first expansion: c – e – g – d – f

Second, when the number of units is even. Then, through the same form of permutation over one unit, ***the recurring unit is omitted*** in addition to the normal omission of the respective number of units.

Example:

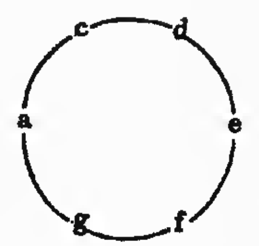
Scale: c – d – e – f – g – b
Circular arrangement:



The first expansion: c – e – g – d – f – b

Scale: c – d – e – f – g – a

Circular arrangement:



The first expansion: c – e – g – d – f – a

With the increase of the number of units omitted between the selected units, further expansions may be obtained. The total number of tonal expansions of one scale equals the number of units therein minus one.

$$\boxed{N_E = N_P - 1}$$

This includes the original scale.

A scale that cannot be contracted in a given system of tuning will be considered as being in the *zero* expansion (E_0).

All further expansions will be E_1, E_2, ... E_n, where the subnumeral represents the number of units omitted between the number of units selected in circular permutation.

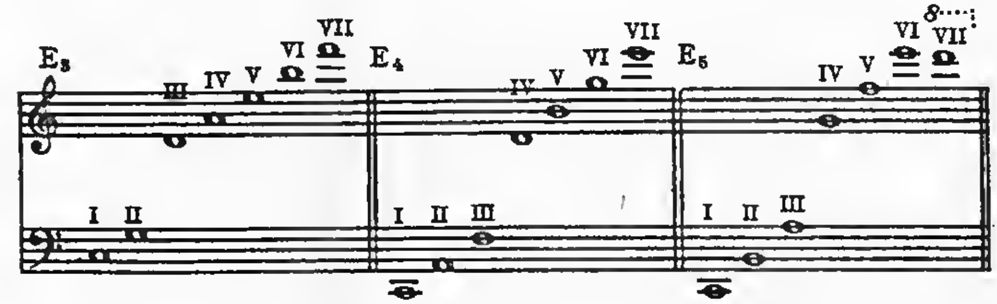
The process of tonal expansion is applicable to any melodic form—a scale being merely a special case of melodic form. Different expansions of a melody provide means for *variation* as well as for composition of *melodic continuity*.

The technique of *transcribing a melody from one expansion into another* consists in finding the scale in both expansions, in enumerating all the units in consecutive order from the root-tone (scale axis) in both scales, and in translating units of one melody into the units of another through the identical numbers.

The octave adjustment (range) for compounding continuity out of different expansions must be performed by placing the root-tone (the axis from which expansions have been obtained) of all the expansions on the same pitch level. With the adjustment, fragmentary melodies in different expansions become elements of one intonation-group, and as such, are permutable in time continuity.

Examples of Tonal Expansions

Natural Major Scale



Chinese Scale

Melodic Minor (g) d_3

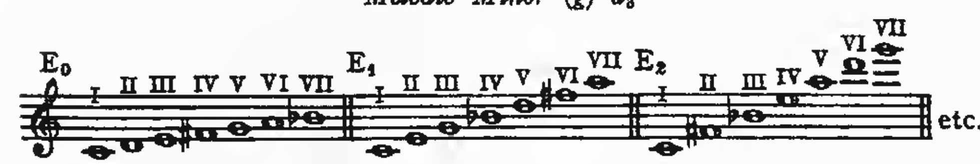


Figure 24.

The last case (melodic minor) is particularly interesting, as it illustrates how music written in the 17th or 18th century can be transformed directly into the style of Debussy or Ravel by means of E_1; how music written by Handel or Bach may be converted into the style of Scriabine's *Poem of Ecstasy* by means of E_2.

This device of tonal expansion is the device for modernization of the music of the past. If the music of the present were written consistently, following its own tendency in any of the expansions, it could be contracted back into E_0. Thus, two styles three centuries apart could be compared under the same coefficients of expansion. This device gives the critics of music something to think about. One cannot really draw any comparisons between music of the present and music written two or three centuries ago because they exist in different states of expansion.

B. Translation of Melody Into Various Expansions

Melody is a special type of scale. When a melody contains, among others, the adjacent musical names (musical seconds), it may be considered part of a complete 7-unit scale (containing all musical names). The following melody may be considered part of a natural major scale with the root-tone on c. In such a case the expansion must be performed from c as the axis of expansion. This melody is formed on both sides of the axis and the same pattern will remain in all expansions.

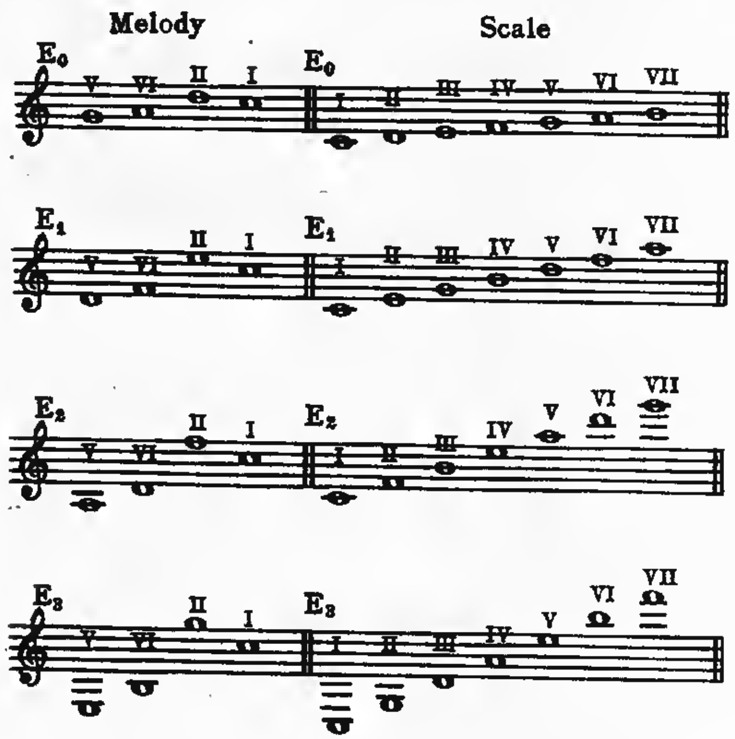


Figure 25.

Different expansions of the same melody produce melodic continuity in similar forms, evolving in different ranges. They are permutable in time continuity.

The following is an example of melodic continuity produced by different expansions.

The original setting:

Figure 26.

Continuity produced by circular permutations:

Figure 27.

As presented in the foregoing example, this device may be employed to produce studies for a solo instrument and is particularly suitable for instruments with wide ranges, such as the violin, clarinet and French horn. In order to obtain more expressive melodies, time-rhythm must be superimposed on the melodic continuity. The interference between the number of units in a melodic layout and a rhythmic group often results in a complete solo composition of considerable length. By playing this type of melody in different modal transpositions, one may obtain a number of compositions, each distinctly different in character, and each esthetically equivalent to the original.

Examples of the tendency toward tonal expansion resulting from purely intuitive processes may be found extensively in the works of modern composers. For example, Prokofiev in his *Song, Opus 27, No. 2* for voice and piano, has a melody evolving mostly in the E_2, while the accompaniment is a hybrid of E_2, E_3, and E_0. The last two bars on the first page reveal E_0 in the melody, E_1 in the right hand of the piano accompaniment. These forms are hybrid and naturally produce various deviations from the pure style. In No. 3 of the same *Opus*, the vocal part is a hybrid between E_0 and E_1 while the right hand of the piano accompaniment is consistently carried out in E_1, and the left hand in E_3.

C. Variable Pitch Axes (Modulation)

All techniques with regard to changes of scale structure or key signature are applicable to the second group of scales as well. Modal transpositions as well as modulations can be carried out in any form of tonal expansion, providing that the expansion remains constant in the two portions of melodic continuity connected by any form of modulatory transition.

It is unsatisfactory to vary expansion in the two portions of melodic continuity belonging to two different axes with modulatory transitions between them. Therefore, all the variations of E must be performed from one axis. The entire scheme of modulatory continuity, including expansions, may appear as follows:

Key I E_0 + Key I E_1 + Mod. +
+ Key II E_1 + Key II E_2 + Mod. +
+ Key III E_2 + Key III E_1 +
+ Key III E_0 + ...

In a melodic continuity evolving from one thematic melody, this device becomes invaluable as it introduces variety into unity. It eliminates the necessity of having several melodies as different themes in one composition. One or two subjects are enough to evolve a diversified melodic continuity when tonal expansions and modulations are used. This form of composition may be applied on a limited scale for the purpose of arranging music where the "fill-in" groups are to appear as imitations of a preceding motif in one or another tonal expansion.

There are many popular melodies which are intuitively written in the first expansion. For example, *Without a Song*, *You Hit the Spot*, and others. Note also Debussy's *La fille aux cheveux de lin*. Such themes present the possibility of reversing the whole procedure, i.e., tonal *contraction* of the original theme. Vincent Youmans' *Without a Song** starts on *c* in the key of F (F is the axis and being in the E_1 is the third degree of E_1). The same melody, being rewritten into E_0 and translated into the corresponding degrees, acquired a new musical appearance that can be utilized wherever thematic motifs are desired. It may serve as an introduction or provide the interludes between the portions of thematic continuity. The processes of expanding and contracting music often lead to startling discoveries. For example, in the case of *Without a Song*, this melody when translated into E_0 has a great deal in common with the theme by Rimsky-Korsakov from his opera *Coq d'Or* commonly known as *Hymn to the Sun.*

Contraction: E_0

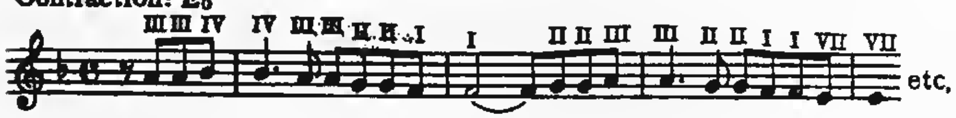


Figure 28.

D. Technique of Modulation in Scales of the Second Group

As transition from key to key—based on chromatic alteration—does not offer any definite procedure for tonal expansion and may lead to pitch-units alien to both the preceding and the succeeding key, it has to be eliminated. Thus, the two available devices are:

1. The common tones.
2. The identical motifs.

*The process of tonal contraction, as described by Schillinger, is most easily executed in the following manner. The first expansion (E_1) of the scale of F has the following notes: f-a-c-e-g-b♭-d-f. These are numbered from 1 to 8. The zero expansion (E_0) of the scale of F—f-g-a-b♭-c-d-e-f—is likewise numbered from 1 to 8. Now, we take the notes of Vincent Youman's song *Without A Song* and number them according to their position in the expanded scale. To discover the tonal contraction of this melody, we simply substitute the notes of the contracted scale corresponding to these numbers. When this has been done to *Without A Song*, the notes of the opening bars change as follows: c-c-e-e-c-c-a-a-f become a-a-b♭-b♭-a-a-g-g-f, etc. The latter will, of course, be readily identified with the theme of *Hymn to the Sun*. (Ed.)

As previously stated, both the preceding and the succeeding key have the same coefficient of expansion (whatever it is). The common tones can be easily found. For any given pair of keys, these common tones are invariant in any given scale, since tonal expansion does not alter the original pitch-units but merely arranges them in a new fashion. It is important, however, to realize that any usage of such common tones for a transition from one key axis to another must be carried out within the type of intervals inherent in the selected tonal expansion. For example, in the major or minor diatonic scales, the first expansion intervals are 3rds, 5ths, 7ths, 9ths, etc. There are no 2nds or 4ths in the same octave. One should refrain from using 2nds when they are really 9ths. This concerns all the intervals

Here is an example of melodic continuity modulating through common tones.

Figure 29 (continued).

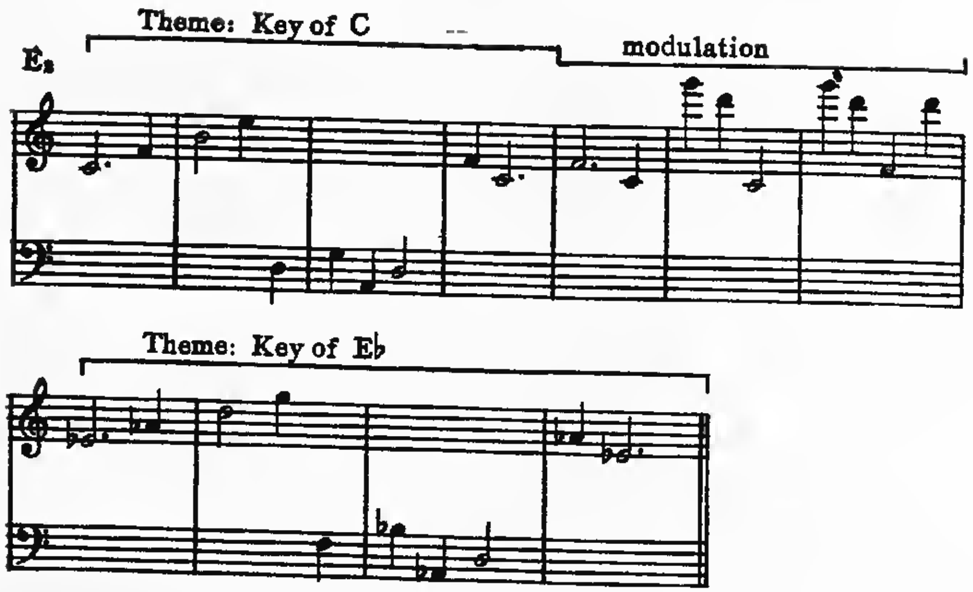


Figure 29 (concluded)

The use made of common tones in the above example was different in each case, i.e., E_1 and E_2 were not direct translations of the melodic pattern of the modulation in E_0 into the corresponding expansions. Modulation through identical motifs in any given expansion can be accomplished in a similar way, providing there are enough common tones to manipulate, and depending on the complexity of the structure of the motif itself.

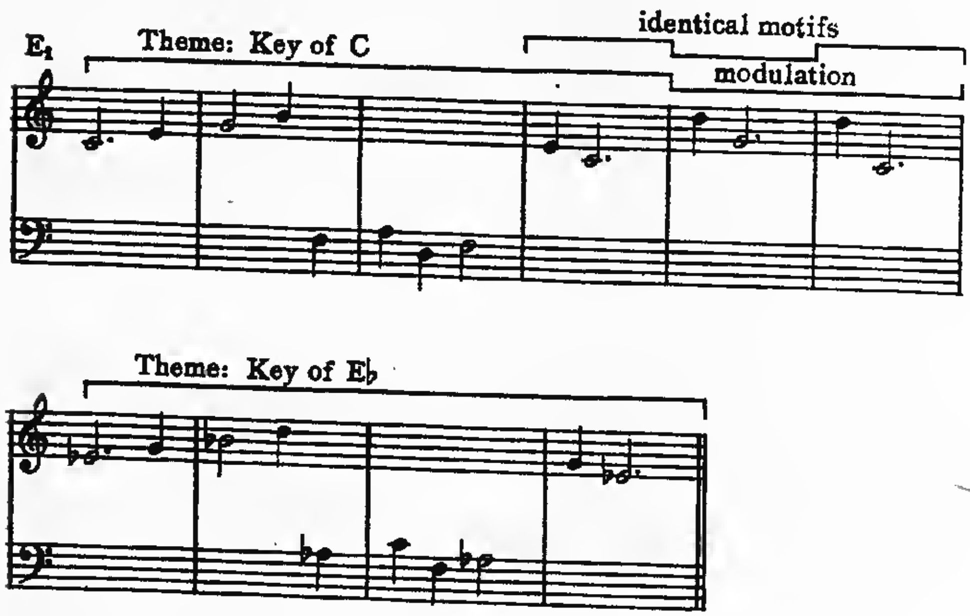


Figure 30.

The two forms of modulation may be combined in the same melodic continuity.

In order to translate melodic continuity which already contains modulations based on common tones or identical motifs, it is necessary to introduce the principle of *common degrees*, as the corresponding degrees of one expansion do not correspond to the respective degrees of another. For example, e in the key of c is the second degree of E_1, and the same e is the third degree of E_0, and the same e is the fourth degree of E_2. Naturally when a certain tone does not correspond to itself in one key, by reason of the different arrangements produced by different expansions, it will not correspond in the same relation to any other key. Take f, which is the fourth degree of the key of C in E_0, and the fifth degree of the key of B♭ in the same expansion; the note f in the key of C is the sixth degree of the E_1 and the same note is the third degree in the key of B♭ in the same expansion; the note f in the key of C on E_2 is the second degree while the same note in the key of B♭ in the corresponding expansion is the seventh degree.

Modulatory continuity must be translated into any other expansion by means of *common degrees*. A new pitch-unit representing the identical degrees of the original expansion must be used directly in place of the corresponding pitch-unit of the same expansion. For example, if the modulation in E_0 was carried out from the key of C to B♭, through the common tone f—f being the fourth degree of the first and the fifth degree of the second key—it would change its pitch-units in such a way that the identity of degrees, i.e., IV = V, would be preserved.

The fourth degree of the key of C in the first expansion is b while the fifth degree of the key of B♭ in E_1 is c. Therefore, the transition must take place through these two pitch-units placed in immediate sequence. The corresponding modulation in E_2 will take the following form: the fourth degree of the key of C in E_2 is e while the fifth degree in the key of B♭ in E_2 is g. The immediate sequence from e to g constitutes the transition. In this case, g following e must be placed one-tenth above e as this is the proper placement of a third in E_2.

Key of C	IV = V	Key of B♭
E_0	f – f	
E_1	b – c	
E_2	e – g	

Figured 31 (continued)

Figure 31 (concluded)

The identity of motifs in the process of modulating through the different forms of expansions, has dual significance. Firstly, it permits modulation through common tones, yet preserves the identity of the melodic material. This effect was illustrated above with reference to identical motif modulation. Secondly, through direct changes of key signatures in the adjacent identical motifs, one may achieve arpeggio-like modulatory progressions. To the listener's ear, the latter will appear as the customary modulations moving through arpeggio chords.

etc.

Figure 32.

In the above example, a group of three identical motifs gradually becomes modified through variation of the key signatures from zero signature (key of C)

through one-flat signature (key of F) to a two-flat signature calling for e♭ and d♭ (which would permit the motif's being interpreted as fitting into the key of harmonic b♭ minor, which in its full form has four flats).

The whole field of tonal expansion technique is suggestive of harmony, and therefore presents more elaborate forms of arpeggio-making than the usual harmonic arpeggio.

This device may be successfully utilized when the effect of forming, of growing or of decreasing has to be expressed through a thematically homogeneous melodic form. These effects—when combined with corresponding dynamic treatment—suggest any mechanical form associated with spiral development, i.e., forming, increasing in size, or becoming louder on the one hand, and moving away, decreasing in size, or fading out on the other. Motion picture backgrounds offer a very fertile field for the application of such devices.

CHAPTER 6

SYMMETRIC DISTRIBUTION OF PITCH-UNITS

THE problem of the symmetric distribution of sequences within a given acoustical range of a simple ratio is not new. Musical cultures of the Orient —such as the Javanese, Siamese, Balinese, and Arabian—attempted to produce such symmetries in their systems of tuning. They were not mathematically equipped to solve this problem in its general form, i.e., by means of logarithms, but they intentionally sought to distribute the pitch relations of an octave into five and seven uniform intervals, or to produce more complex forms of periodicity of pitch, such as in the Arabian scale introduced in the 7th century A.D. The latter differs from the Javanese and Siamese scales. The first two are symmetrical systems of tuning (primary selective systems), while the Arabian is a scale constructed within a given tuning system (secondary selective system).

Ancient civilizations were fascinated by the properties of prime numbers. This perhaps explains why they used a symmetric breaking-up of an octave into such numbers as 5 and 7. The actual motivation behind the use of these particular numbers may be an inclination which results from the primeval pentadic and heptadic forms of symmetry. The creation by nature of lower forms of animal life in forms of pentagonal symmetry and snow-flakes in hexagonal symmetry, is merely an outcome of electro-chemical processes which may also take place in our brain-functioning as well as in the general evolution of species.

There is no acoustical reason, or "natural inclination" in the human ear, for differentiating the octave into heptadic or pentadic symmetric relations. Intervals thus produced do not conform to a simple acoustical ratio. Habit and heredity are more important factors in the development of artistic taste than is the perfection of the structural constitution of the raw material. Listening to Javanese, Siamese or Balinese music—authentically recorded from the original sources, one can easily get accustomed to it in a very short time.

While the apparent reasons for the tuning symmetry in Oriental musical cultures were religious and symbolic considerations, the apparent reasons behind the system now in use in the civilized world are acoustical considerations. But these apparent reasons are misleading. They are not true in the light of unbiased scientific analysis. The real reason for the evolution from the system of the symmetry of 12 to an octave is the versatility of the number 12 as compared to 5 and 7. While 5 and 7 are prime numbers, i.e., they may be divided by themselves or by unity only, the number 12 has *four additional* divisors, (2, 3, 4, 6). The next number which would have one more divisor is 60, i.e., no other number between 12 and 59 exhibits greater versatility with respect to combinations of the sub-systems than does the number 12 itself. Being a limited value, it becomes very practicable for the solution of many problems of musical composition. The lack of versatility in the prime numbers, with respect to tuning, becomes apparent after a continuous experience of listening to Javanese or Balinese music. Music of our culture also becomes monotonous, regardless of its esthetic quality, and for the same reason.

When a composer like Debussy begins to use the symmetry of 6 (whole-tone scale) *consistently*, his music becomes monotonous—despite the abundant use of various devices. Using 6 instead of 12 makes the system lose one divisor; the loss of this one divisor makes such music considerably more monotonous to our ear.

All the above-mentioned systems of symmetry are evolved within the range of a ratio of $2 \div 1$, which, being the simplest ratio, produces the effect of greatest likeness to our ear. Musical experience considers this likeness so great that all the tones of such ratio bear identical musical names. With the further evolution of pitch discrimination, this likeness may become assigned to ratios of somewhat greater complexity, such as $3 \div 2$, $5 \div 4$, etc. Then it will be possible to evolve the primary selective systems on the basis of symmetry within such ratios.

Generalizing this idea (symmetric splitting into uniform ratios) we can express it in a formula:* $S = \sqrt[m]{\frac{a}{b}}$

Thus, the system of Javanese music is a special case of symmetry in which $S = \sqrt[5]{2}$; Siamese, in which $S = \sqrt[7]{2}$: and the European so-called "equal temperament," $S = \sqrt[12]{2}$. The latter was developed by Andreas Werckmeister in 1691.

The need for such a system in Europe in the 17th century was created by the desire to produce greater versatility of the pitch axes. The limited key relations satisfactory to the community at that time were compensated for by the acoustical perfection of the system then in use. This system, known as "mean temperament," was a bi-coordinate acoustical system of tuning. The two ratios were $3 \div 2$ and $5 \div 4$, one giving a so-called "perfect 5th," and the other, a so-called "major 3rd"—and between the two coordinate systems developed from these two ratios, compromises were reached.

While in full agreement with the requirements of the Church—as well as with the simpler natural phenomena, this system gave the utmost satisfaction with regard to the consonant quality of harmony. The ideal of early homophonic music was consonant quality of a few chords rather than versatility of harmony at the price of an acoustical compromise. The technical expediency of the new system won, and the entire cultural inheritance of the preceding century's vocal music was automatically transplanted to the new system to which the instruments were tuned, even at the time of J. S. Bach.

J. S. Bach was the first composer to take advantage of the key versatility offered by the new system. Variation of key relations was used by him with the boldness of a catalogue rather than in the form of harmonious continuity. Each prelude and fugue is in a different key in place of a greater variety of key modulations within a single composition.

Musical culture, the stronghold of which was consonance, had eventually to give up its way in favor of the harmonic versatility offered by the new system. Simple harmonic forms used in music of the period preceding equal temperament lost their acoustical perfection in the new system to such an extent that at later times treatises were written trying to explain the reason why certain simple chord structures and chord progressions were "false" in the equal temperament of 12.

*See footnotes on pages 101-2.

It took two hundred years to realize, at least intuitively, the nature and the possibilities of $\sqrt[12]{2}$ system. Even today we are dealing with hybrids produced by music—the sources of which go centuries back—and by forms derived from equal temperament. The intuitive start on the new track was due not to any discrimination in favor of symmetry, but rather to the consequences of a habit formed in the early 16th century. For the interval of an augmented 4th, which occurs between steps II and V of the "Neapolitan" minor scale, was the actual stimulus which historically influenced music to take this particular direction. This interval which exhibits the *symmetry of 2* within one octave ($\sqrt{2}$), is at the same time the simplest form of symmetry within the $\sqrt[12]{2}$ system.*

The first phases of this evolution of harmonic forms produced the $\sqrt[4]{2}$ (Wagner), and the $\sqrt[3]{2}$ (Liszt). While Wagner operated on the $\sqrt[4]{2}$ with $4 + 3$ structures (major triad), he attempted and failed in the application of the $\sqrt[3]{2}$, using the structure $3 + 4$ (minor triad). Liszt used the $\sqrt[3]{2}$ on $4 + 3$ structures exclusively; it took a few decades until the $3 + 4$ structures on the same roots came into existence with Rimsky-Korsakov.

An early application of the $\sqrt[6]{2}$ to melodic forms, as well as to harmonic forms of the $\sqrt[3]{2}$ ($4 + 4$ structure: equals augmented triad), was in the opera, *Stone Guest* written by Dargomishsky in the middle of the 19th century. Further application of this system appeared at the end of the 19th century in the music written by Debussy and Ravel. The $\sqrt[12]{2}$ in $4 + 3$ structures is characteristic of Wagner and of post-Wagnerian opera written by Russian composers.

All the group forms of symmetry within the $\sqrt[12]{2}$ are the derivatives (sub-systems) of this system. Various combinations of the various sub-roots of the $\sqrt[12]{2}$ produce various forms of group symmetry—such as binomials, trinomials and more complicated polynomials. At the beginning of the 20th century the binomial form of symmetry becomes quite apparent (as in Rimsky-Korsakov's *Coq D'Or*). The influence of symmetry on chord structures as well as on chord progressions begins to flourish with Debussy and Ravel.

Chord structures comprising five and more functions (such as 9th chords and 11th chords) take the place of the more archaic triads and 7th chords. Where Beethoven would move his melodies in the inversions of triads, Debussy prefers the 7th and 9th chords.

The most recent forms of symmetry of pitch belong to the type of writing known as "polytonality." Polytonality is a symmetric superimposition of chord structures related through the roots of the octave. The sub-roots of the $\sqrt[12]{2}$ become autonomous tonalities. Such simultaneous superimposition of sym-

*Referring once more to the footnote on pages 101-2, the symmetry of 2 is mathematically analogous to—and much simpler than—the symmetry of 12 characteristic of equal temperament. The ratio series for symmetry of 2 is (expressed in fractional powers): $2^{\frac{0}{2}}$, $2^{\frac{1}{2}}$, $2^{\frac{2}{2}}$. In the first term, the zero power of 2 is 1, and the square root of 1 is 1; in the third and final term, the square root of two squared is, of course, simply 2. The middle term is the point of symmetry. It fits the symmetry of 12 because the square root of 2 to the first power is the same as $2^{\frac{6}{12}}$, which is the middle term of the longer series described in the footnote cited earlier. The same procedure may be generalized to cover the symmetry of 3, of 4, of 6. (Ed.)

metrically related keys may be observed in homophonic as well as polyphonic writing.

The first intentional superimposition of chords belonging to two symmetrical roots of the octave occurred in Stravinsky's *Petrouchka*. The maximum saturation caused by symmetric superimposition derives from the simultaneous composition of all the roots of the octave. Equal temperament of 12 is the complete expression of the symmetry of 12 in one octave. The sub-systems of this general symmetry are: the sub-systems derived from $\sqrt[6]{2}$ as a ratio unit; the $\sqrt[4]{2}$; the $\sqrt[3]{2}$; and the $\sqrt{2}$, the latter being the simplest form of octave symmetry.

CHAPTER 7

PITCH-SCALES: THE THIRD GROUP

Symmetrical Scales

THE THIRD group of pitch-scales is made up of those scales derived from various roots of the number 2—the square (second) root, the cube (third) root, the fourth root, the sixth, and the twelfth roots.

The first table below shows the manner in which the interval between one tone (say, c) and its octave may be divided into *twelve symmetric* parts, thus producing a 12-tonic system; the second table shows the same octave symmetrically split into six tonics; the third, division into four symmetric tonics; the fourth, division into three tonics; the fifth, into two tonics.*

A. Table of Symmetric Systems Within $\sqrt[12]{2}$.

(1)

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_1
1	$\sqrt[12]{2}$	$\sqrt[6]{2}$	$\sqrt[4]{2}$	$\sqrt[3]{2}$	$\sqrt[12]{2^5}$	$\sqrt{2}$	$\sqrt[12]{2^7}$	$\sqrt[3]{2^2}$	$\sqrt[4]{2^3}$	$\sqrt[6]{2^5}$	$\sqrt[12]{2^{11}}$	2
C	C#	D	Eb	E	F	F#	G	Ab	A	Bb	B	C^1

(2)

T_1	T_2	T_3	T_4	T_5	T_6	T_1
1	$\sqrt[6]{2}$	$\sqrt[3]{2}$	$\sqrt{2}$	$\sqrt[3]{2^2}$	$\sqrt[6]{2^5}$	2
C	D	E	F#	Ab	Bb	C^1

(3)

T_1	T_2	T_3	T_4	T_1
1	$\sqrt[4]{2}$	$\sqrt{2}$	$\sqrt[4]{2^3}$	2
C	Eb	F#	A	C^1

(4)

T_1	T_2	T_3	T_1
1	$\sqrt[3]{2}$	$\sqrt[3]{2^2}$	2
C	E	Ab	C^1

(5)

T_1	T_2	T_1
1	$\sqrt{2}$	2
C	F#	C^1

The capital "T's" in the preceding table represent the corresponding tonics (axis-points of the corresponding symmetric systems). These tonics serve as root tones of the structures evolving in simultaneity and continuity.

The first such evolution (in simultaneity) produces *chord* structures. The second (in continuity) produces the individual *pitch-scales* of one compound symmetric scale and also the progression of roots for the chord sequence.

*These symmetric scales and the symmetric harmony derived therefrom are of the utmost importance in modern and future music; they constitute one of the most brilliant theoretical and practical discoveries of the Schillinger System. (Ed.)

All *sectional scales* of the third group, starting from their symmetrical points, have *identical construction*. The number of scales is limited by the intervals between the two adjacent symmetrical roots.

B. Table of Arithmetical Values Expressing Intervals in Semitones.

The Third Group

Number of roots.	Complete range.	Interval between roots.	Limit-range of sectional scales.
2	12	6	5
3	12	4	3
4	12	3	2
6	12	2	1
12	12	1	0

Scales of the Third Group

2 Tonics

Figure 33 (continued).

Four-Unit

Five-Unit

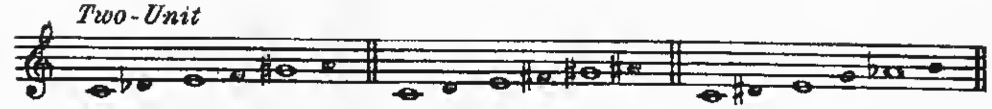
Six-Unit

3 Tonics

One-Unit

Figure 33 (continued)

Two-Unit



Three-Unit

Four-Unit

4 Tonics

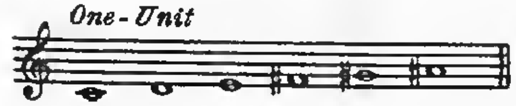
One-Unit

Two-Unit

Three-Unit

6 Tonics

One-Unit



Two-Unit

Figure 33 (continued).

12 Tonics

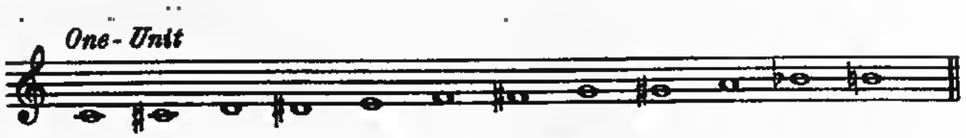


Figure 33 (concluded).

The scale on $\sqrt{2}$ with four-units in each sectional scale of the structure $2+1+2$ was known to the Arabs in the 7th century A.D. Their conception of the structural scheme was: a large step and a small step. Thus, they had obtained a binomial periodicity which, in its nearest approximation to our tuning system, produces $4(2+1)$, and the derivative of it, $4(1+2)$.

This scale came into existence in our music through the realization of $\sqrt{2}$ and $\sqrt[4]{2}$ as influenced by harmonic structures. It results automatically from a continuous chain of the simple chord-structures following the above-mentioned roots. We find portions of it as far back as the music of Bellini.

No composer until Rimsky-Korsakov was aware of this by-product of harmony. It is evidenced in his operas, *Kasche*ɪ and *Mlada*.

Wagner used the scale on $\sqrt[3]{2}$ with three-unit sectional scales $(2+1)$ in his prelude to *Parsifal*. Naturally, neither of these composers was conscious of the symmetric systems as such.

Arabians called their $4(2+1)$ scale a "string of pearls" (Zer ef Kend), drawing an analogy between the alternation of large and small beads in a string of pearls and the large and small steps between the pitch-units of the scale.

Further study of this and other symmetric scales as by-products of chord progressions will be found in the *Special Theory of Harmony*.*

C. Composition of Melodic Continuity in the Third Group

The Third Group of scales offers the following possibilities for composition of melodic continuity:

Scales with Two Tonics:
Total number equals 32.

1-Unit sectional scales on two tonics produce 1^2 equals 1 melodic form. Total number of scales 1.
2-Unit sectional scales on two tonics produce 2^2 equals 4 melodic forms. Total number of scales 5.
3-Unit sectional scales on two tonics produce 6^2 equals 36 melodic forms. Total number of scales 10.
4-Unit sectional scales on two tonics produce 24^2 equals 576 melodic forms. Total number of scales 10.
5-Unit sectional scales on two tonics produce 120^2 equals 14,400 melodic forms. Total number of scales 5.
6-Unit sectional scales on two tonics produce 720^2 equals 518,400 melodic forms. Total number of scales 1.

*See Book V.

Scales with Three Tonics:
Total number equals 8.

1-Unit sectional scales on three tonics produce 1^3 equals 1 melodic form. Total number of scales 1.
2-Unit sectional scales on three tonics produce 2^3 equals 8 melodic forms. Total number of scales 3.
3-Unit sectional scales on three tonics produce 6^3 equals 216 melodic forms. Total number of scales 3.
4-Unit sectional scales on three tonics produce 24^3 equals 13,824 melodic forms. Total number of scales 1.

Scales with Four Tonics:
Total number equals 4.

1-Unit sectional scales on four tonics produce 1^4 equals 1 melodic form. Total number of scales 1.
2-Unit sectional scales on four tonics produce 2^4 equals 16 melodic forms. Total number of scales 2.
3-Unit sectional scales on four tonics produce 6^4 equals 1,296 melodic forms. Total number of scales 1.

Scales with Six Tonics:
Total number equals 2.

1-Unit sectional scales on six tonics produce 1^6 equals 1 melodic form. Total number of scales 1.
2-Unit sectional scales on six tonics produce 2^6 equals 64 melodic forms. Total number of scales 1.

Scales with Twelve Tonics:
Total number equals 1.

1-Unit sectional scales on twelve tonics produce 1^{12} equals 1 melodic form. Total number of scales 1.

Scales of the roots, the exponents of which are multiples of the original roots, give coincidences in the corresponding symmetric points. Thus the scales built through the $\sqrt[4]{2}$ coincide with some of the scales built on the $\sqrt{2}$ where the sectional scales move through the points coinciding with the points of the $\sqrt[4]{2}$. If the two tonics are c and f♯, then all sectional scales which include e♭ and a coincide with the four tonics having identical c — e♭ — f♯ — a as their roots.

The technique of evolving a continuity in symmetric scales must be carried out through sectional scales used either in their complete form or in parts. The complete utilization of the sectional scales follows the methods of circular or general permutations of melodic forms, application of the coefficients of recurrence of melodic forms, superimposition of time rhythm on melodic form, etc. When some of the sectional scales, or all of the sectional scales, are used in parts, a definite rhythmic procedure must be established. The method of elimination of pitch-units must follow with a system of circular permutations or any other pre-arranged method of distribution.

Example of composition of melodic continuity from a scale of the third group:

c – d – f – f♯ – g♯ – b

Through circular permutations we obtain:

c–d–f – g♯–b–f♯ – f–c–d – f♯–g♯–b – d–f–c – b–f♯–g♯

Using 1-unit at a time on the second tonic and all three units on the first tonic, and applying the method of circular permutations, we obtain:

c–d–f – f♯ – d–f–c – g♯ – f–c–d – b

Example of Melodic Continuity:

Scale: Melodic form through circular permutations

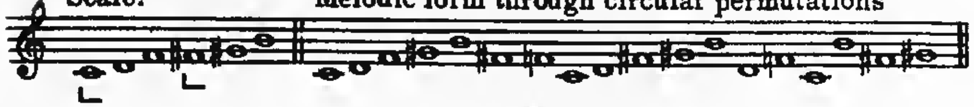


Figure 34.

Rhythm of durations: $3(2+1)+(2+1)^2$

Melody: 18 attacks
Durations: 6 attacks
Interference: $\frac{18}{6} = 3$

Continuity:

Melodic forms

Durations

Figure 35.

CHAPTER 8

PITCH SCALES: THE FOURTH GROUP

Symmetrical Scales of More Than One Octave in Range

THE FOURTH group of pitch-scales is based on the following roots: $\sqrt[3]{4}$, $\sqrt[4]{8}$, $\sqrt[6]{32}$, $\sqrt[12]{2048}$. The ranges of these systems are, respectively: 2 octaves (3 tonics); 3 octaves (4 tonics); 5 octaves (6 tonics); 11 octaves (12 tonics).

(1)

T_1	T_2	T_3	T_1
1	$\sqrt[3]{4}$	$\sqrt[3]{4^2}$	4
C	A♭	E	C^1

Read the tones upward; the C^1 is *two* octaves above the C.

(2)

T_1	T_2	T_3	T_4	T_1
1	$\sqrt[4]{8}$	$\sqrt{8}$	$\sqrt[4]{8^3}$	8
C	A	F♯	E♭	C^1

The C^1 is three octaves above C.

(3)

T_1	T_2	T_3	T_4	T_5	T_6	T_1
1	$\sqrt[6]{32}$	$\sqrt[3]{32}$	$\sqrt{32}$	$\sqrt[3]{32^2}$	$\sqrt[6]{32^5}$	32
C	B♭	A♭	F♯	E	D	C^1

The C^1 is five octaves above C.

(4)

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9
1	$\sqrt[12]{2048}$	$\sqrt[6]{2048}$	$\sqrt[4]{2048}$	$\sqrt[3]{2048}$	$\sqrt[12]{2048^5}$	$\sqrt{2048}$	$\sqrt[12]{2048^7}$	$\sqrt[3]{2048^2}$
C	B	B♭	A	A♭	G	F♯	F	E

T_{10}	T_{11}	T_{12}	T_1
$\sqrt[4]{2048^3}$	$\sqrt[6]{2048^5}$	$\sqrt[12]{2048^{11}}$	2048
E♭	D	D♭	C^1

The C^1 is eleven octaves above C.

All sectional scales of the fourth group starting from their symmetrical points have *identical construction.* The number of scales is limited by the interval between the two adjacent symmetrical roots.

The Fourth Group

The number of roots	The complete range	The interval between the roots	The limit-range of sectional scales
3	24	8	7
4	36	9	8
6	60	10	9
12	132	11	10

The tonality of all scales of the fourth group may be discovered by utilizing all combinations by 2, 3 . . . for each sectional scale until it fills out the sectional limit-range.

The total number of scales of the fourth group amounts to two thousand. Here are a few illustrations:

The Fourth Group of Pitch Scales

Excerpts from Complete Table

THREE TONICS

Figure 36 (*continued*).

FOUR TONICS

Figure 36 (*continued*).

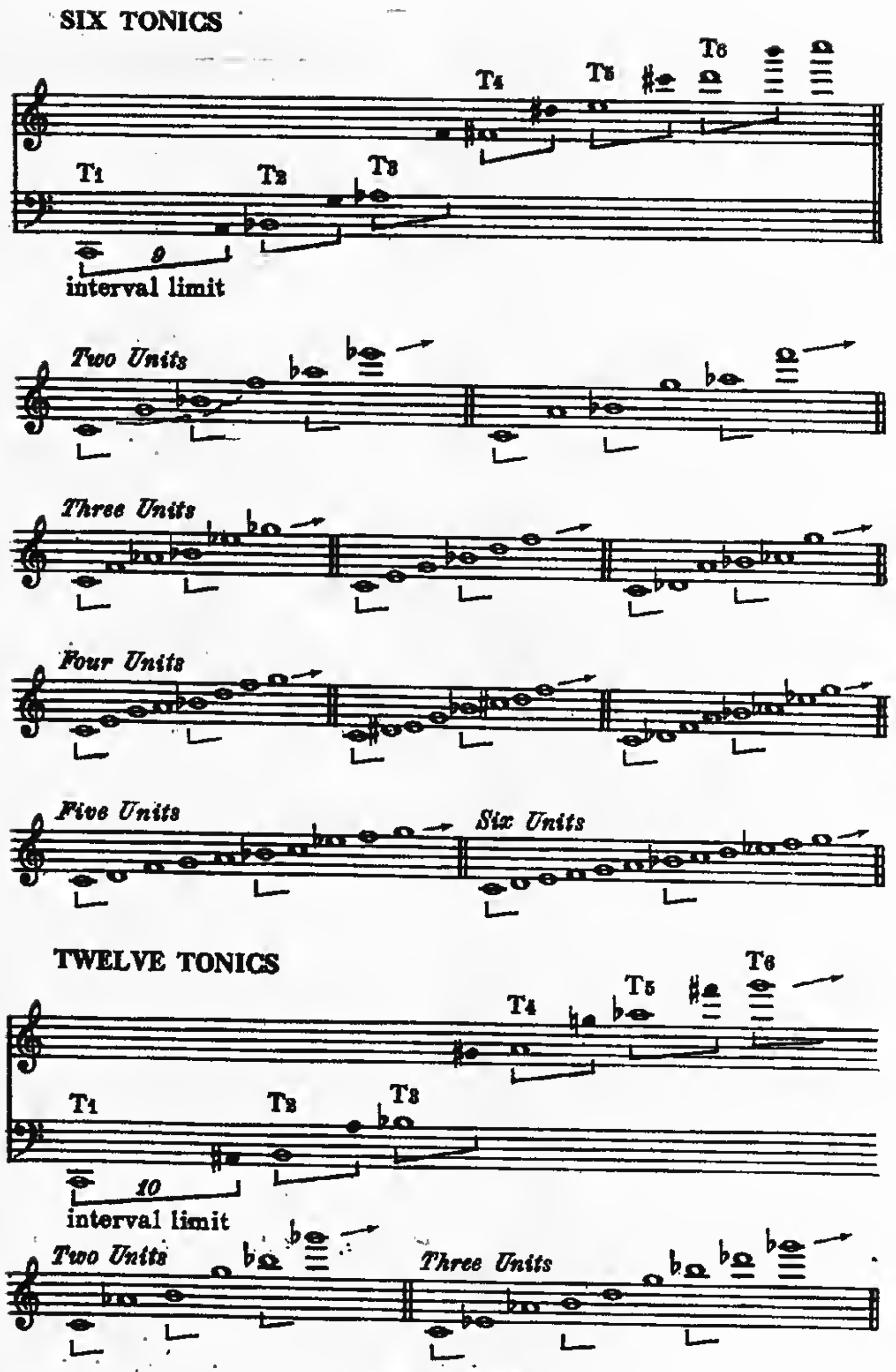


Figure 36 (continued).

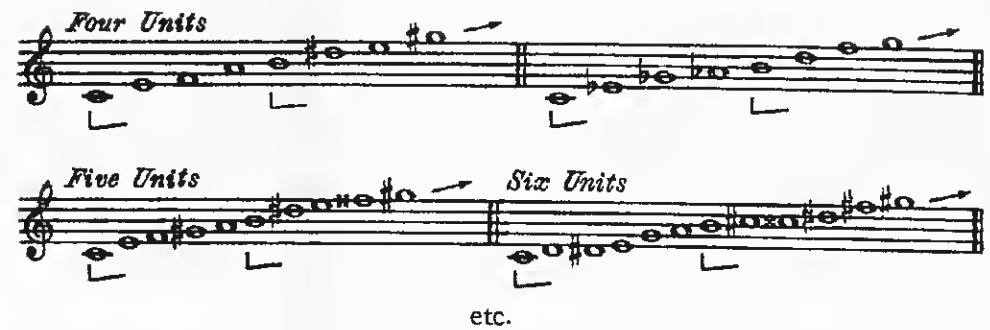


Figure 36 (concluded)

The definition of the quantity of melodic forms in the fourth group is based on the same method of computation as in the third group. The general number of melodic forms available from any symmetric scale equals the number of permutations of the units of the sectional scale to the power expressing the number of roots (tonics).

$$N_m = (u!)^{NT}$$

A numeral with an exclamation mark on its right represents the product of all integers from 1 to such numeral value, i.e., 5! equals $1 \times 2 \times 3 \times 4 \times 5 = 120$. For example, the number of melodic forms in a 5-unit sectional scale on four tonics equals 207,360,000 melodic forms [$(5!)^4 = 120^4 = 207{,}360{,}000$].

A. Melodic Continuity in Scales of the Fourth Group

Composition of melodic continuity from the scales of the fourth group may originate from the three forms of settings:

(1) The original scale
(2) The first contraction
(3) The final contraction

The procedure from the setting (1) is the usual procedure as described for the scales of the third group.

The second setting can be obtained in the following way; first, construct the sectional scale on the first tonic; then the sectional scale above it on the tonic nearest in pitch to the first tonic; then the sectional scale below it on the tonic nearest in pitch to the first tonic. When the number of tonics is even, the further addition of the remaining sectional scales may be either above or below the first tonic. This always offers two forms of distribution for the second setting. For example, take a 4-unit sectional scale on four tonics like c – d – f – g – a – – b – d – e – f# – g# – b – c# – d# – e# – g# – a#. Centering the first tonic and surrounding it by the nearest tonics, we obtain the following setting of type (2):

– d# – e# – g# – a#
– c – d – f – g –
a – b – d – e

Figure 37

Adding to this the remaining tonic (f#), we may place it either above or below:

(a)

− f# − g# − b − c#
− d# − e# − g# − a# −
− c − d − f − g −
a − b − d − e −

(b)

− d# − e# − g# − a#
− c − d − f − g −
− a − b − d − e −
f# − g# − b − c# −

Figure 38

The second setting is always an overlapping one. There are definite contractions corresponding to each system of tonics. Scales on three tonics in their first contraction emphasize the range of 15 semitones. Scales on four tonics in their first contraction emphasize the range of 17 semitones. Scales on 6 tonics in their first contraction emphasize the range of 21 semitones. Scales on 12 tonics in their first contraction emphasize the range of 22 semitones.

The final contraction (3) generally produces a complete chromatic scale with the same range as the first contraction when the distribution of tonics must be preserved. The pitch-units of the sectional scales become rest tones and all the intermediate tones become auxiliary. Each sectional scale consists of *directional units.* Rest tones may move into the auxiliary tones, but their return to the rest tones is required. Otherwise, the auxiliary tones become passing notes between the rest tones.

In the following example of the final contraction on the scale previously illustrated, the black notes indicate the tones to be used as passing or auxiliary, and the white notes indicate the rest tones, i.e., the original pitch-units of the sectional scale in their original sequence.

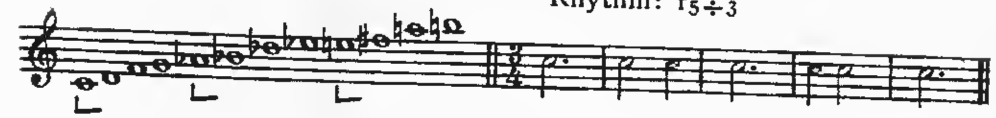
Figure 39.

The final contraction of symmetrical scales, though it produces a chromatic scale, is different from a chromatic scale used on a basis of atonality, where all tonal centers are completely eliminated. Composition of melodic continuity from melodic forms obtained through either of the melodic settings of the scales of the fourth group follows the same principle as the composition of melodic continuity in the third group of scales.

Hybrid forms of melodic continuity in the fourth group of scales may be derived by the application of mixed settings, i.e., different states of contraction the rest following the usual methods of variation of melodic forms.

Examples of Composition of Melodic Continuity from Scales of the Fourth Group

Scale: Rhythm: $r_{5 \div 3}$

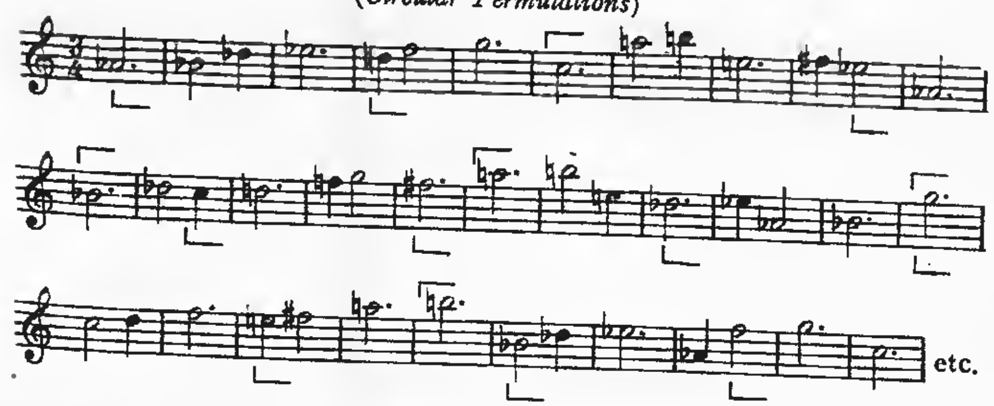


Continuity from the Original Setting (Circular Permutations)

etc.

The First Contraction

Continuity from the First Contraction (Circular Permutations)



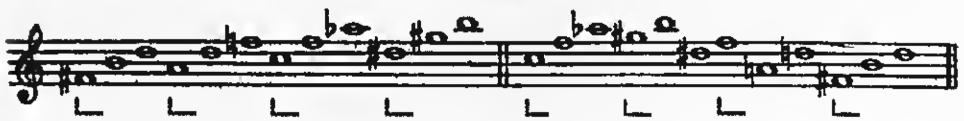
etc.

Figure 40 (continued)

Rhythm: $(2+1+1)^3$; $\frac{1}{64}$ = ♪

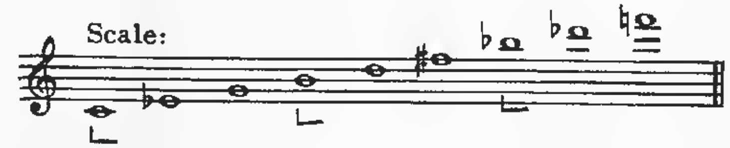
Continuity from the Original Setting (without Permutations)

The First Contraction *Melodic Form (Circular Permutations)*

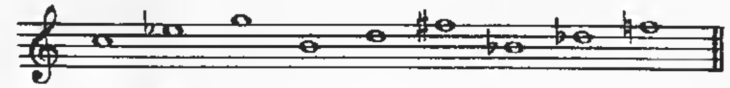


Continuity from the First Contraction

Figure 40 (continued).



The First Contraction



Melodic Form

Rhythm: $3(2+1) + (2+1)^2$

Continuity from the First Contraction (Circular Permutations)

Figure 40 (concluded).

The procedure of composing melodic continuity from sectional scales of the symmetric system may also be reversed for the purpose of evolving symmetric continuity from a given motif. A root-tone of such motif corresponds to a root-tone of the symmetric sectional scale. The selection of the system of symmetry is left to the composer's discretion. However, the choice of such system may well be suggested by the *harmonic* structure of the original composition.

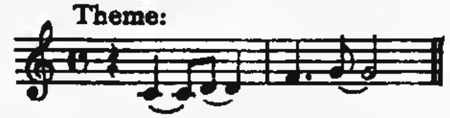
Almost any music contains some symmetric chord-progressions, at least in a fragmentary form. For example, in the repertory of popular songs, you find as a characteristic progression (often the beginning of the chorus) the $\sqrt[3]{2}$. See such songs as *Night and Day*, *Everything I Have Is Yours*, and *High on a Windy Hill* which uses $\sqrt{2}$. In such cases, the above form of symmetry would be the one most organically related to the whole harmonic constitution of the chorus.

By varying the rhythm of such motifs, it is possible to build different kinds of "cadenzas" which may be used in the introductory section or in transitions.

The following illustration exemplifies an application derived from a well-known motif:

GEORGE GERSHWIN'S *I Got Rhythm**

Theme:



Four Tonic Setting (without Permutations)

Four Tonic Setting (with Circular Permutations)

Figure 41.

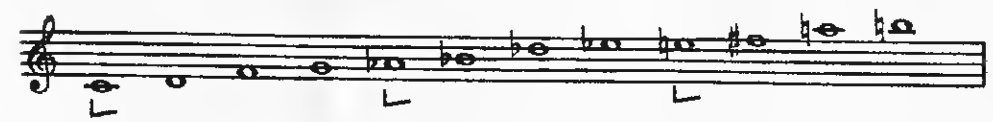
B. Directional Units

A scale which contains directional units may appear in all of its three settings. The directional units consist of the original tones of a sectional scale and of the tones of the allied sectional scales. The tones of the original sectional scale may be surrounded on both sides by the tones of the allied sectional scales. In such a case, a system of upper and lower auxiliary tones to the same tones of the original sectional scale is possible. In some cases, the units from the allied sectional scales appear once between the tones of the original sectional scale—and in some cases twice. In other cases, the lower one serves as the upper auxiliary tone to the preceding tone of the original sectional scale, and the upper one as the lower auxiliary tone to the following tone of the original scale.

A *directional unit* consists of a tone of the original sectional scale together with any of the auxiliary tones leading into it. When permutations are used, such a directional unit does not change its own form, i.e., if the original directional unit consists of a lower auxiliary tone leading into the tone of the original sectional scale, this particular form of sequence of the directional unit must be preserved. Permutations refer to different directional units only.

Figure 42, which follows, contains examples of this technique. The arrows indicate the direction taken by the auxiliary tone as it moves into the original tone. The letters indicate directional units.

The Original Scale

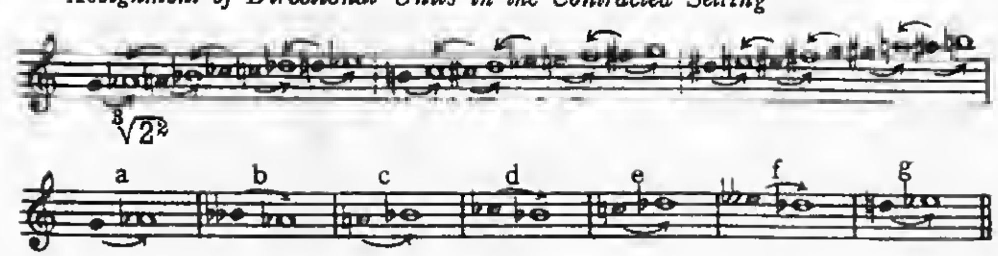


The First Contraction

The Original Scale with Auxiliary Tones

The First Contraction with Auxiliary Tones

Assignment of Directional Units in the Contracted Setting



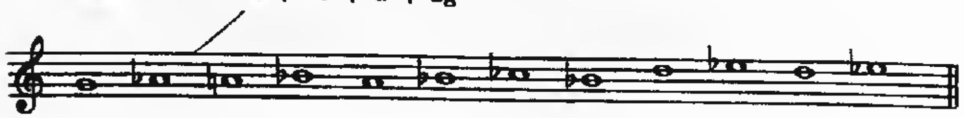
The Number of Melodic Forms: $2^7 = 128$

Figure 42.

By selecting coefficients of recurrence for certain directional groups (as shown in the *Theory of Rhythm*, Book I) and superimposing the rhythm of durations, we may obtain a different type of melodic continuity from that presented in previous examples evolved from the scales of the Fourth Group.

When such melodies are harmonized, the original tones of the sectional scale become rest tones of a chord (chordal functions), and the auxiliary tones become the elements of melodic *figuration*.

Melodic Form: $a + 2c + d + 2g$



Time rhythm: $r_{\underline{3\div2}}$—with circular permutations

Continuity:

Melodic Form: $T_1\,a + T_3\,2c + T_1\,d + T_2\,2g$

Time rhythm: $r_{4\div3}$

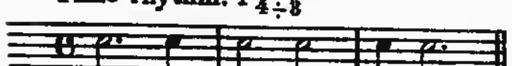


Figure 43 (continued).

Continuity:

$(T_1 + T_3 + T_1 + T_2) + (T_3 + T_1 + T_2 + T_1) + (T_1 + T_2 + T_1 + T_3) +$
$+ (T_2 + T_1 + T_3 + T_1)$

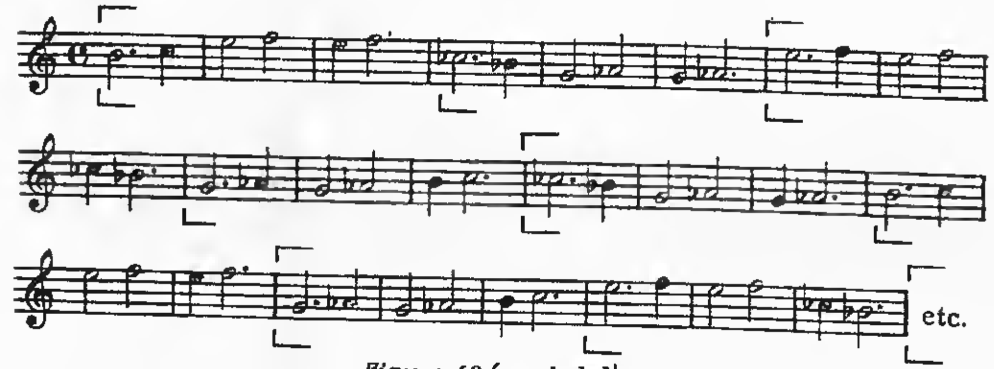


Figure 43 (concluded).

This process is *reversible* when applied to a given melody. In such a case it is necessary before harmonizing to determine which are the auxiliary and which are the rest tones, and then to assign the rest tones as definite chordal functions.

CHAPTER 9

MELODY-HARMONY RELATIONSHIPS IN SYMMETRIC SYSTEMS

CHORDS, or "harmony", and pitch-scales are interrelated. There is sufficient evidence that *simultaneous pitch aggregations* (groups which usually are called "chords") are *tonal expansions* of the corresponding pitch scales.* Such expansions produce an acoustically more suitable form of distribution of pitch-units. Wider intervals are characteristic of the lower groups of harmonics and our ear accepts narrow intervals more easily in the higher frequencies.

Certain groups produce an unsatisfactory effect merely because of their low pitch placement. Our hearing is not capable of discriminating simultaneous groups of pitch when their position is so low in frequency that the corresponding fundamental tone of the series cannot be heard in reality, i.e., when it would be below 30 cycles per second. It is easy to verify this phenomenon by simply placing such intervals as thirds, which are supposed to be "consonant," in an unusually low register. Yet when even the most "dissonant" intervals are located according to the series of harmonics, our hearing accepts them as consonant intervals. For example, the c of the large octave sounding simultaneously with the c♯ of the second octave on the scale of harmonics are the fundamental and the 17th harmonics. In this case the correspondence to the actual intonation of harmonics is so close that the effect is definitely consonant to our ear. It may be easily verified through placing any melody in such parallel couplings.

The process of building harmonic groups (chords) is a process of redistributing pitch-units so that the latter are spread through a greater tonal range. As many scales in symmetric systems of pitch appear in already wide interval-distribution, many of these scales sound like chords when played simultaneously. Any simultaneous combinations of pitch-units of such scales or of their sectional scales produce chords of varying complexity and therefore of varying *tension* (the degree of dissonant quality).

The following figure is an illustration of a scale belonging to the fourth group and based on the symmetry of 3 tonics ($\sqrt[3]{2}$, i = 4).

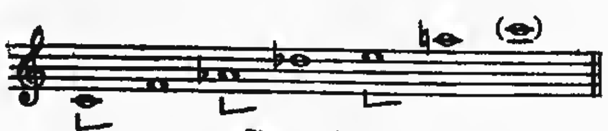


Figure 44.

*This chapter introduces matters having to do with harmony, and it should therefore be ooted that the complete development of the theory of harmony occurs at a later point in the text. When all the tones of any scale are sounded simultaneously, the resulting sound provides the raw material for harmonization of melodic forms in that scale. But such a sounding of all tones at once is acoustically dissonant, except in the case of scales of comparatively few tones. Therefore, the scale is subjected to the first expansion, E_1, in order to locate the tooes in a manner that will yield better acoustic results. The result of the expansion is the master-structure denoted by the Greek letter, sigma. From this total master-structure, sections are taken to provide the raw material for specific groups ("chords"). But this approach, although fuodamental to all harmony, does not constitute the entire Schillinger theory of harmony which is very much more extensive than this chapter would indicate. (Ed.)

Σ (sigma) indicates the compound chord which emphasizes *all* the pitch units of the scale, whether such chord appears as a scale in its original setting or in any of the tonal expansions.

The original setting of the above scale appears in the form of the following Σ:

Figure 45.

From this compound harmonic group, smaller groups may be devised of different degrees of complexity and classified on the basis of the number of component units. Starting each time with the succeeding pitch-unit of the original scale-setting, harmonic groups for all degrees of the scale are obtained. Being classified on the basis of the number of pitch-units, they become: diads, triads, tetrads, pentads, or hexads—for this particular scale consists of 6 units (3 tonics, 2-unit scales).

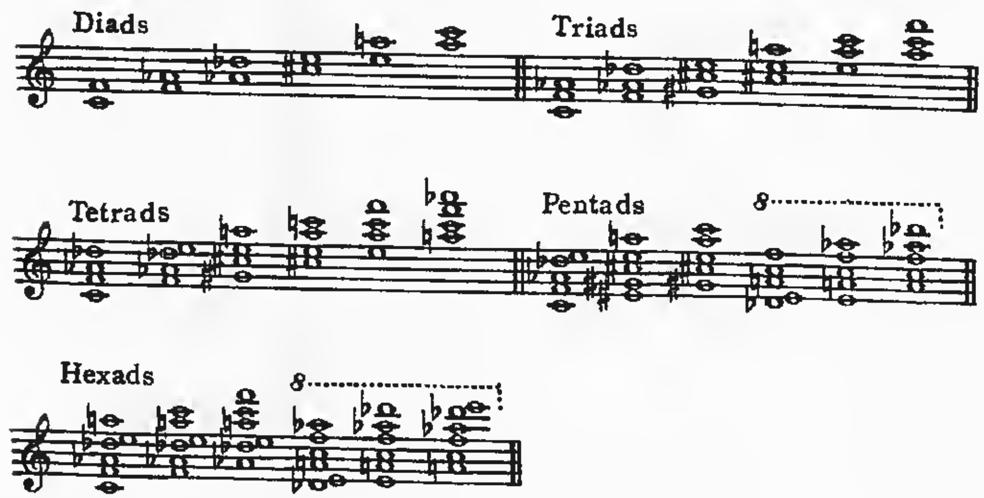


Figure 46.

Without studying harmony as such at all, one can produce perfect forms of harmonic continuity through the application of consecutive pitch-units in the form of harmonic groups of the different degrees of tension. In order to perform what is usually known as "voice-leading", it is necessary to find the *nearest* pitch-units of the succeeding group as they appear in relation to the preceding group. Groups of the different degrees of tension, as treated later in the *Special Theory of Harmony**, have many forms of transformations constituting their positions and voice-leading. In this case the principle of nearest tones is merely *one* of the special cases of such transformations.

*See Book V and IX.

Harmonic Continuity of Diads:

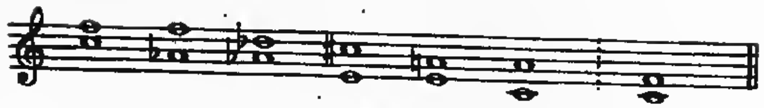


Figure 47.

By knowing which is the fundamental tone for each of such diads and through the addition of this same constant fundamental tone in the bass, a *hybrid form* of harmony is obtained (2p var. + 1 const.)

Hybrid Three-Part Harmony

Figure 48.

Likewise, a continuity of triads may be devised.

Harmonic Continuity of Triads:

Figure 49.

By adding a constant fundamental to 3-part harmony, a hybrid 4-part harmony is obtained. (3p var. + 1 const.)

Hybrid Four-Part Harmony:

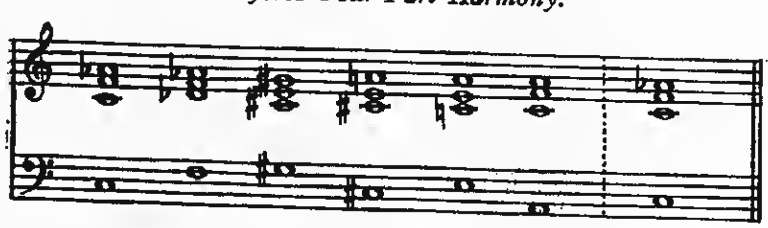


Figure 50.

The following example represents a harmony of tetrads evolved through the same principle:*

Harmonic Continuity of Tetrads:

Figure 51.

Through analogous addition of a constant fundamental, a hybrid 5-part harmony thereof is obtained (4p var. + 1 const.)

Hybrid Five-Part Harmony:

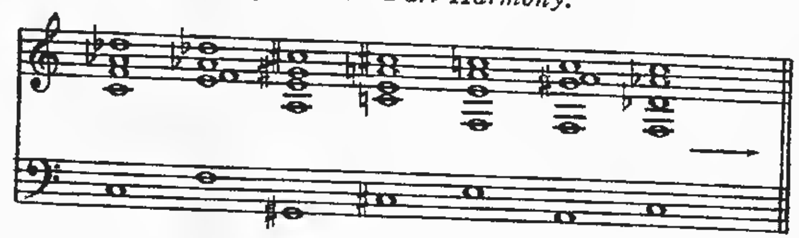


Figure 52.

The following examples represent the harmony of pentads and hexads respectively. Through the addition of the constant fundamental an additional part is obtained, thus producing a hybrid 6-part harmony (5p var. + 1 const.) and 7-part harmony (6p var. + 1 const.)

Harmonic Continuity of Pentads:

Figure 53.

*With the tetradic examples given here, structures of considerable tension are introduced—and their dissonant characteristics may be too "extreme" for some, but hardly for all tastes. Schillinger here is developing, however, *a general method*, which in this case happens to be applied to a master-structure (denoted by sigma) of high tension. Exactly the same methods may be applied to structures of much lower tension, if one desires to produce progressions, etc., which seem "less extreme." For example, the same processes set forth here may be applied to a master structure consisting of the first expansion, E_1, of G melodic minor—i.e., the minor scale starting on G with a one-flat signature and a sharped F. When expanded this produces (reading upward in thirds) c — e — g — b flat — d — f sharp — a. This master-structure happens to be a very "popular" one; examples which use this as raw material will sound "less extreme." (Ed.)

Harmonic Continuity of Hexads:

Figure 54.

In addition to harmonic possibilities as the by-products of symmetric systems of pitch, there is a method of harmonization of melody and melodization of harmony, based on the different relations of tension with respect to tonics used in melody and harmony. The word "tension" refers *both* to the harmonic group, with respect to its dissonant quality, and to the relationship which exists between melody and harmony.

In the following text, M will signify a sectional scale or a melodic form derived therefrom. H will express a harmonic group (chord structure) built on the pitch-units of one or more sectional scales (treated as total groups).

Different forms of relations between M and H produce different degrees of tension. The ***minimum*** tension occurs when M and H have ***identical*** groups (like $\frac{M}{H} = T_1$). The increase of tension depends upon the remoteness of the T's expressing M and H. The system of tension relations is symmetrical, i.e., it follows the arrangement of the tonics: T_1 is followed by T_2, T_2 is followed by T_3 . . . . T_n is followed by T_1.

The relationship between $\frac{M}{H}$ may be constant (with specified degrees of tension) or variable (with specified range of tension). The variable range of tension is subject to distributive processes assuming centrifugal or centripetal form. Forms are centrifugal when moving from the center to periphery, and centripetal when moving from periphery to the center. With regard to a scale of tension, a centrifugal form means movement from medium tension to low tension, and then to high tension, and then to low tension. Centripetal form means from high tension to low tension, and then to medium tension, or from low tension to high tension, and then to medium tension.

SYMMETRIC SYSTEMS OF PITCH $\frac{M}{H}$ RELATIONS

(1) $\frac{M}{H}$

$$\frac{M}{H}=T_1;\ \frac{M}{H}=T_2;\ \frac{M}{H}=T_3\ \ldots\ \frac{M}{H}=T_n$$

(2) $\frac{2M}{H}$

$$\frac{2M}{H}=\frac{T_1+T_2}{T_1};\ \frac{2M}{H}=\frac{T_2+T_3}{T_1};\ \ldots\ \frac{2M}{H}=\frac{T_{n-1}+T_n}{T_1};\ \frac{2M}{H}=\frac{T_n+T_1}{T_1}$$

(3) $\frac{M}{2H}$

$$\frac{M}{2H}=\frac{T_1}{T_1+T_2};\ \frac{M}{2H}=\frac{T_1}{T_2+T_3};\ \ldots\ \frac{M}{2H}=\frac{T}{T_{n-1}+T_n};\ \frac{M}{2H}=\frac{T_1}{T_n+T_1}$$

(4) $\frac{3M}{H}$

$$\frac{3M}{H}=\frac{T_1+T_2+T_3}{T_1};\ \ldots\ \frac{3M}{H}=\frac{T_{n-2}+T_{n-1}+T_n}{T_1};\ \frac{3M}{H}=\frac{T_{n-1}+T_n+T_1}{T_1};\ \frac{3M}{H}=\frac{T_n+T_1+T_2}{T_1}$$

(5) $\frac{M}{3H}$

$$\frac{M}{3H}=\frac{T_1}{T_1+T_2+T_3};\ \ldots\ \frac{M}{3H}=\frac{T_1}{T_{n-2}+T_{n-1}+T_n};\ \frac{M}{3H}=\frac{T_1}{T_{n-1}+T_n+T_1};\ \frac{M}{3H}=\frac{T_1}{T_n+T_1+T_2}$$

(6) $\frac{nM}{H}$

$$\frac{nM}{H}=\frac{T_1+T_2+T_3+\cdots+T_n}{T_1};\ \frac{nM}{H}=\frac{T_2+T_3+\cdots+T_n+T_1}{T_1};\ \frac{nM}{H}=\frac{T_3+\cdots+T_n+T_1+T_2}{T_1};$$
$$\frac{nM}{H}=\frac{T_n+T_1+T_2+T_3+\cdots}{T_1}$$

(7) $\frac{M}{nH}$

$$\frac{M}{nH}=\frac{T_1}{T_1+T_2+T_3+\cdots+T_n};\ \frac{M}{nH}=\frac{T_1}{T_2+T_3+\cdots+T_n+T_1};\ \frac{M}{nH}=\frac{T_1}{T_3+\cdots+T_n+T_1+T_2};$$
$$\frac{M}{nH}=\frac{T_1}{T_n+T_1+T_2+T_3+\cdots}$$

(8) $\frac{aM}{bH}$

$$\frac{aM}{bH}=\frac{T_1+T_2+\cdots+T_a}{T_1+T_2+\cdots+T_b}$$

Figure 55

The lower forms of tension pertain to music which corresponds chronologically to the earlier forms of $\frac{M}{H}$ relations. Such music is typical of Scarlatti, Haydn, Mozart, etc. The higher forms of tension lead to modernity of effect. The actual musical effect depends on the original structure of the sectional scales and their compound sonority from all symmetrical points simultaneously. Many of the scales in their $\frac{M}{H} = \frac{T_2}{T_1}$ relation produce the effect of moderately modern music in the way it sounds to the listener today. It belongs to the type of music in which the tension is analogous to that of Chausson, Debussy, Ravel, early Stravinsky, etc. Further forms of tension are characteristic of the later Stravinsky, of Casella, Malipiero, Auric, Poulenc, Milhaud, etc.

By using the multiple-tonic system, such as six or twelve tonics, still higher tensions than those mentioned above can be obtained.

As it follows from the table, the emphasis of more than one group of M against one group of H, and vice-versa, may include different degrees of tension as a constant characteristic of style. In the esthetic sense such a method offers a moderation of the extremities. Thus, any symmetrical scale offers a multiplicity of styles where each individual style is an outcome of the forms of setting of the original scale as well as the specifications of $\frac{M}{H}$ relations.

In the following examples, melodies from the 2-unit sectional scales are produced from two melodic forms ($a_1 + b_2$). Constant T appears in various degrees of tension, occurring between melody and harmony.

In the first example, the melody emphasizes the pitch-units of the corresponding harmonic group only.

In the second example the 2-unit melody group is displaced one phase upward, i.e., it emphasizes the second unit of the first sectional scale and the first unit of the second sectional scale.

$\frac{M}{H} =$ T const.

$\frac{M}{H} =$ T const.

Figure 56.

The constant degrees of tension acting within the restricted limits of sectional scales represent different degrees of tension between melody and harmony, according to the table of $\frac{M}{H}$ relations set forth on page 173.

Melodic form is realized through the same structure as in the preceding example.

$$\frac{M}{H} = \frac{T_2}{T_1} + \frac{T_3}{T_2} + \frac{T_1}{T_3}$$

$$\frac{M}{H} = \frac{T_3}{T_1} + \frac{T_1}{T_2} + \frac{T_2}{T_3}$$

$$\frac{M}{H} = \frac{T_1+T_2+T_3}{T_1} + \frac{T_1+T_2+T_3}{T_2} + \frac{T_1+T_2+T_3}{T_3} + \dots$$

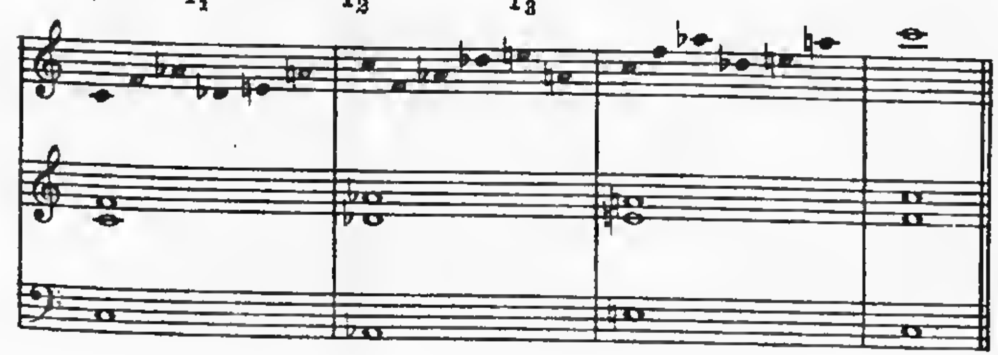


Figure 57.

As previously stated, when the original setting of a symmetrical scale is *not acoustically acceptable*, its sectional scales must undergo tonal expansions in order to acquire the acoustical appearance of harmonic groups. For example, take a scale of the fourth group with 4-unit sectional scales on 4 tonics ($\sqrt[4]{2}$, $i = 2 + 3 + 2$).

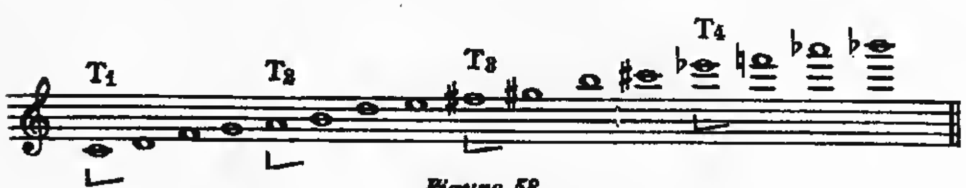


Figure 58.

Through E_1, harmonic groups may be obtained on all 4 tonics. Here are the chord structures (S) obtained through tonal expansion of the sectional scales.

Figure 59.

These tetrads produce the following form of harmonic continuity when voice-leading is obtained through the same principle (moving to the nearest tone).

Figure 60.

By establishing various relations of tension between melody and harmony, different forms of accompanied melody may be devised.

Figure 61.
(continued)

$$\frac{M}{M} = \frac{T_4}{T_1} + \frac{T_1}{T_2} + \frac{T_2}{T_3} + \frac{T_3}{T_4} + \dots$$

$$\frac{M}{H} = \frac{T_1+T_2}{T_1} + \frac{T_2+T_3}{T_2} + \frac{T_3+T_4}{T_3} + \frac{T_4+T_1}{T_4} + \dots$$

Figure 61 *(concluded).*

In this procedure, the removal of harmony from melody produces the same effect of increasing tension as does the removal of melody from harmony. The entire scale may thus be harmonized in two fundamental ways: when a chord is constant and the sectional scale varies, and when the sectional scale is constant and the chord varies.

The following example illustrates the combination of both procedures, i.e., the first sectional scale is accompanied by all chords; then the second sectional scale is accompanied by all chords, etc.

$$\frac{M}{H} = \frac{T_1}{T_1+T_2+T_3+T_4} + \frac{T_2}{T_1+T_2+T_3+T_4} + \frac{T_3}{T_1+T_2+T_3+T_4} + \frac{T_4}{T_1+T_2+T_3+T_4} + \dots$$

Figure 62.

The reversal of this procedure is applicable to various phases of arranging and composing music. An illustration might be taken from George Gershwin's song, *I Got Rhythm*. As a possible form of introduction, the first two bars of melody represent the original two-bar motif; the following three two-bar motifs represent circular permutations of the original motif on the $\sqrt[4]{2}$ setting. The original harmony is left for accompaniment which naturally undergoes, under such conditions, one of the procedures described in $\frac{M}{H}$ relations. In this particular example, the degree of tension between melody and harmony is constant.

This method is applicable in many ways and potentially includes an inconceivable amount of music, as the number of scales consists of two thousand, and practically every scale gives an infinite number of melodies and a great number of $\frac{M}{H}$ relations.

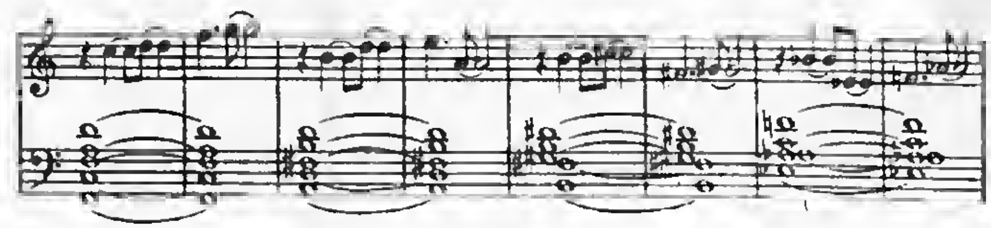


Figure 63.

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK III

VARIATIONS OF MUSIC
BY MEANS OF GEOMETRICAL PROJECTION

BOOK THREE

VARIATIONS OF MUSIC BY MEANS OF GEOMETRICAL PROJECTION

CHAPTER 1

GEOMETRICAL INVERSIONS

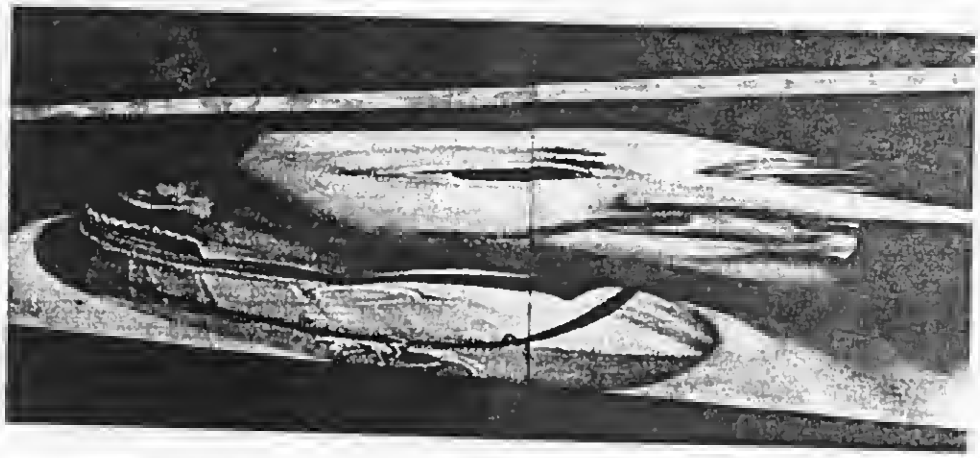
MUSIC in any equal temperament, when it is recorded graphically in rectangular projection, expresses the equivalent of musical *notation* in equal temperament. Such a geometrical projection of music is expressed on a plane, and as such is subject to quadrant rotation of the plane through three dimensional space. Rotation may be either *clockwise* or *counterclockwise*.*

The conception of time, which is based on the common denominator and not on the logarithmic series, implies two possible positions: (1) the original, under zero degrees to the field of vision (parallel to the eyes); (2) the 180° position derived from the first one through rotation around the ordinate axis. Such an ordinate axis is either the starting or the ending limit of the vertical cross-section of the graph (duration limits). If the original (zero degree position) is conceived as a forward motion of music in time continuity, then the respective variation of it (180° position) is the backward motion of the original, when the ordinate is the ending limit in time.**

The *logarithmic* contraction of time corresponds to the logarithmic contraction of space on the graph—and if our music were not bound to a common denominator system of measurement, it would be possible to apply such projection practically. This same form of variation has been known in *visual* art since about 1533 A.D., in skillful paintings made by German and Italian artists. They are based on the principle of angle-perspective and have to be looked at (that is, held at an angle) from right to left, instead of under the zero angle to the field of vision.

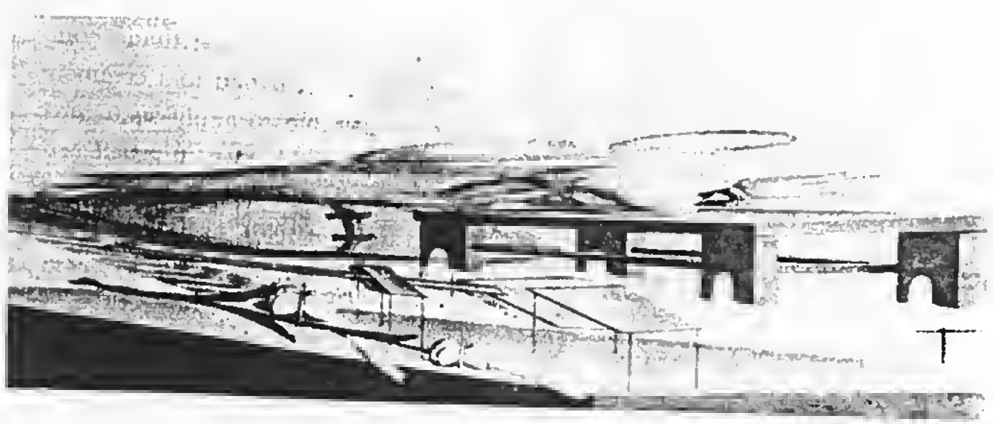
*It may be helpful to add at this point the following: geometric inversion of music consists of "a" of the original form of the music, to start with; then, as the "b" inversion, the same thing *backwards*; as the "c" inversion, the original but backwards and *upside down*; as the "d" version, forwards and upside down. (Ed.)

**When you turn a left-hand page of this book as you normally do, you are revolving it through 180° around—what may be conceived as—its "ordinate" axis. Now if the page were transparent and there was reading matter on only one side, you would find—after you turned the page—that the material at the right side would then be at the left side: that is, you would be reading it backwards. This is position ⓑ of geometrical inversion. See part A of figure 4. (Ed.)



*German School, 16th Century; Charles V, 1533**

Figure 1.



*Unknown Master, 16th Century: St. Anthony of Padua **

Figure 2.

*Courtesy of Museum of Modern Art, exhibited in *Fantastic Art, Dada and Surrealism*, 1937.

By revolving the second position of a musical graph through the abscissa (which becomes the axis of rotation) 180° in a clockwise direction, we obtain the third position of the original. The axis of rotation must represent a *pt* (pitch-time) maximum and the direction of the third position is backwards upside-down of the original, and forward upside-down of the second position.* Further 180° clockwise rotation of the third position about its ordinate produces the fourth position, which is the backwards of the third position, the backwards upside-down of the second position and the forward upside-down of the original.** The respective four positions will be expressed in the following exposition through ⓐ, ⓑ, ⓒ and ⓓ.

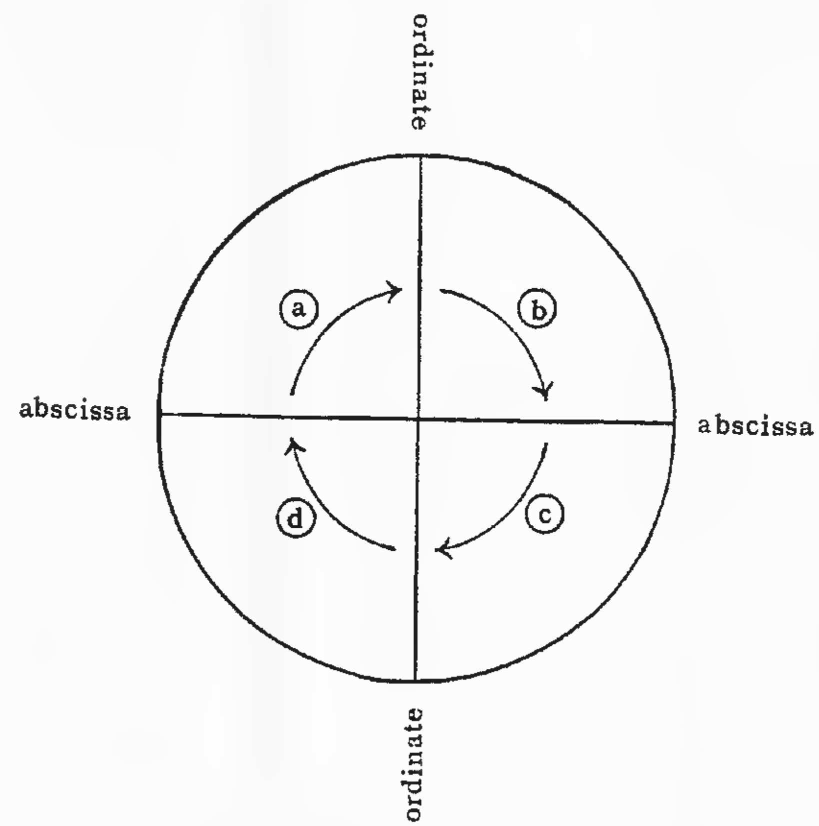


Figure 3.

*To continue from the point at which we stopped in the footnote on page 185, imagine that you could turn the transparent page downward. You would then be revolving it 180° around—what may be conceived as—its "abscissa" axis. The reading matter would appear upside down and backwards with reference to its original position on the left-hand page. This is the third position ⓒ of geometrical inversions. See part A of figure 4. (Ed.)

**Continuing with the illustration of the previous footnote, imagine that you could now turn our transparent page back toward the front cover. The reading matter would then appear upside down and forwards with reference to the original left-hand page, position ⓐ. It would appear backwards with regard to position ⓒ—and upside down and backwards with regard to position ⓑ. See part A of figure 4. (Ed.)

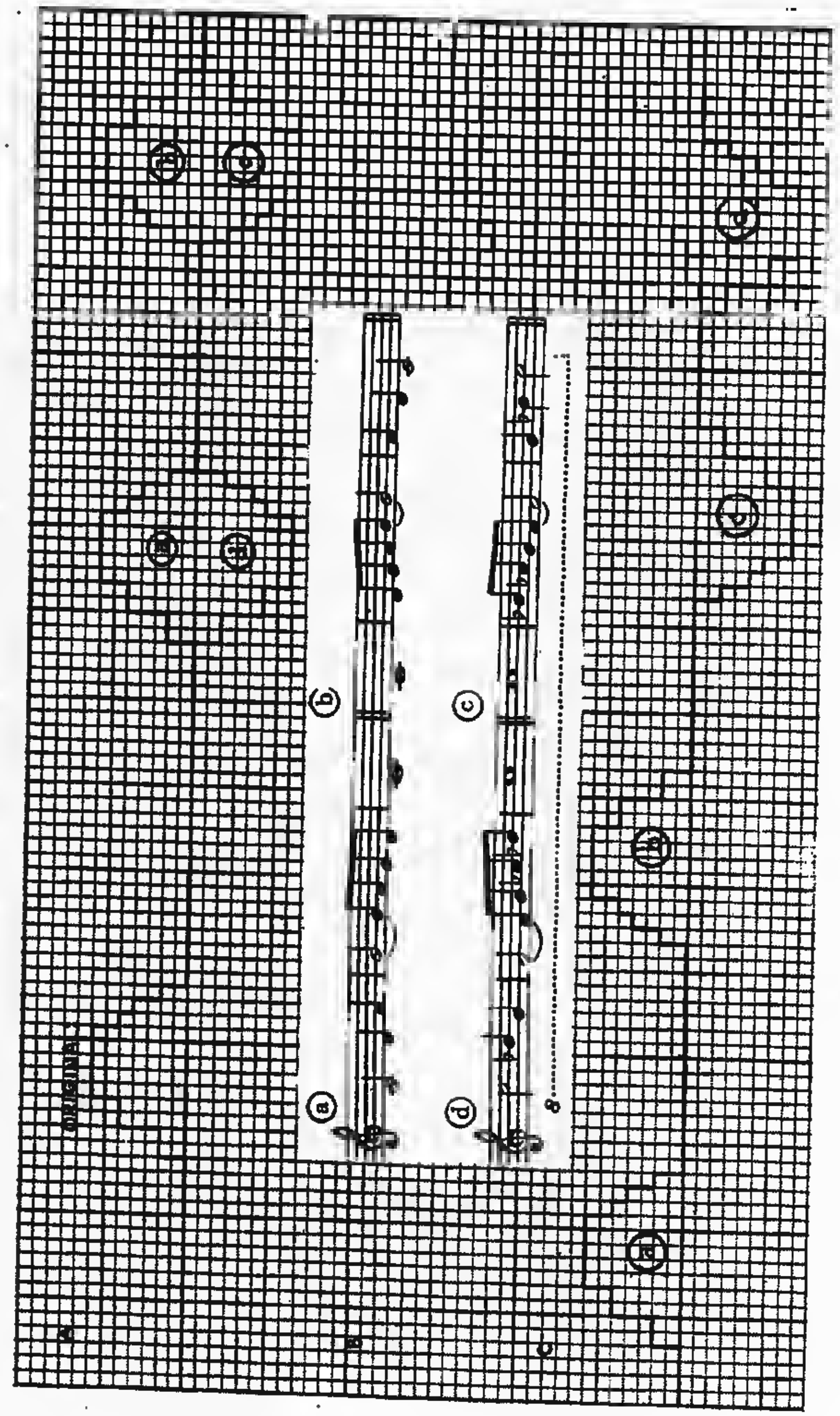


Figure 4. Evolving the Four Geometrical Inversions of a Given Melody.

*Graphs on this page are slightly reduced in scale.

These four geometrical inversions may be used individually as variations of a given melody. They may also be developed into a continuity in which the different positions are given different coefficients. Under such conditions the recurrence of the different positions is subject to rhythm.

Melodic continuity derived from geometrical inversions is exemplified in the following illustrations:

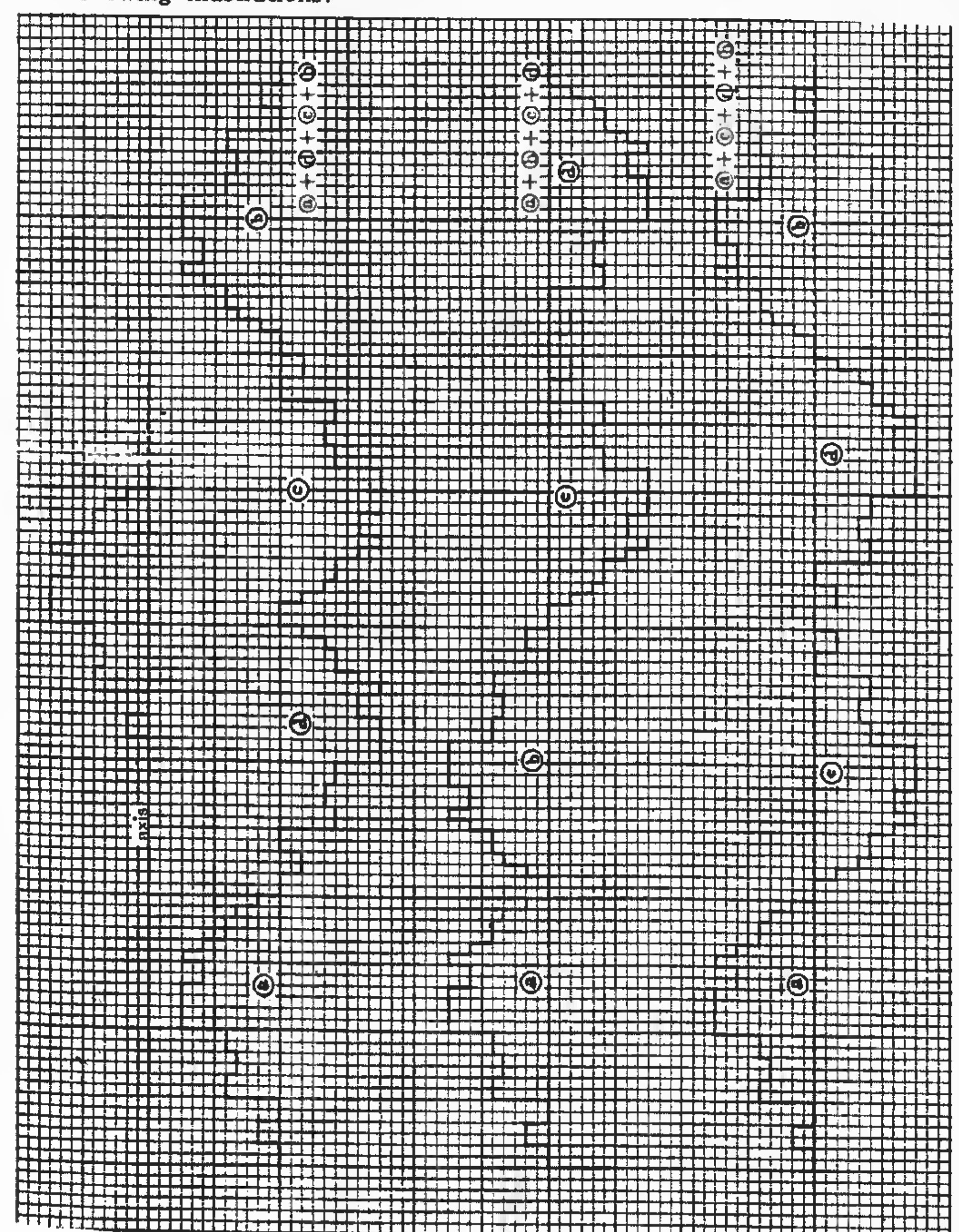


Figure 5. Some permutations of geometrical inversions. Graph representation.

Figure 6. Musical representation of graphs in Figure 5.

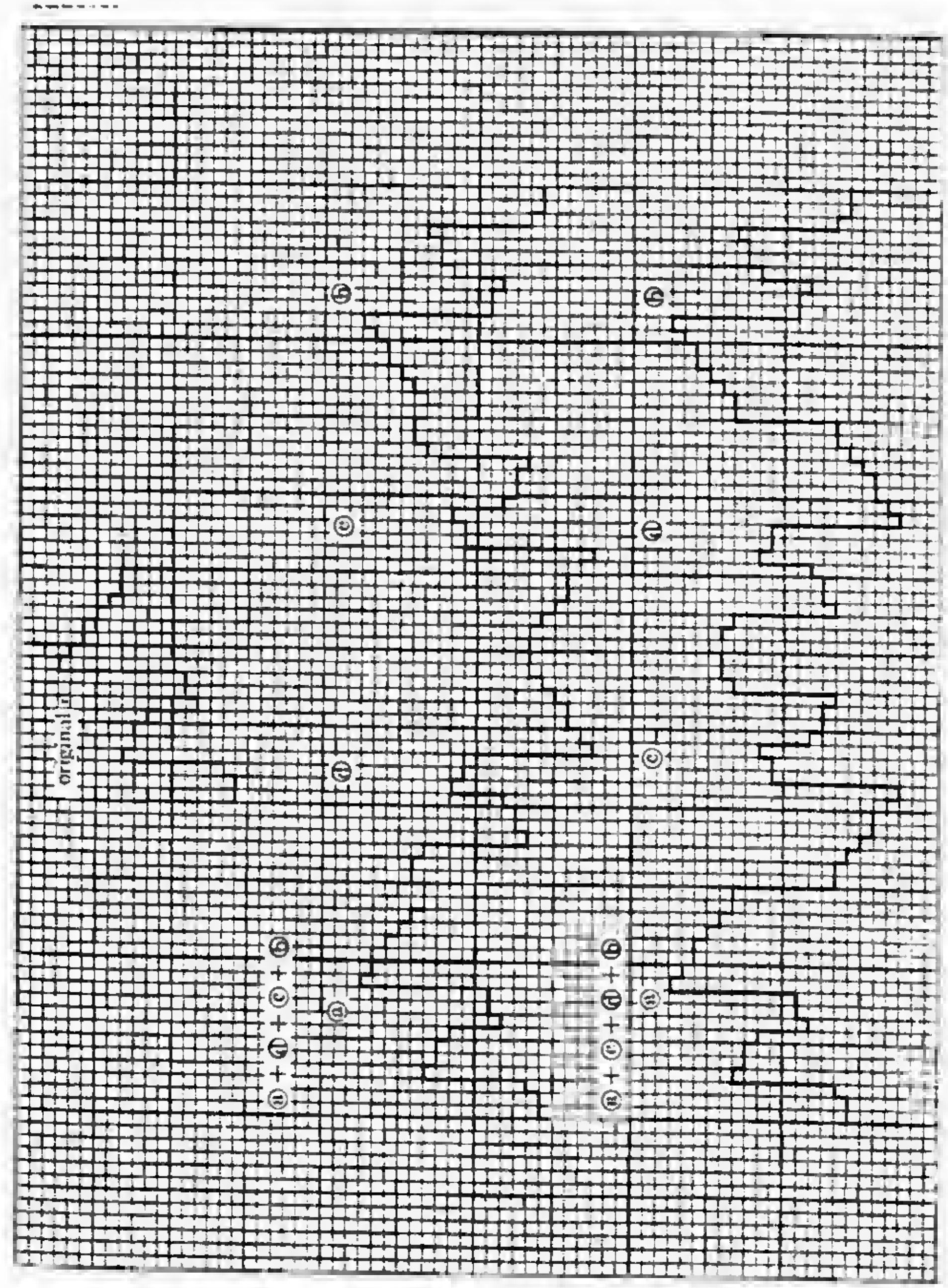


Figure 7. Geometrical inversions represented graphically.

Figure 8. Musical representation of Figure 7.

This method of geometrical inversion, when applied to the composition of melodic continuity, offers much greater versatility—yet preserves the unity more—than any composer in the past was able to achieve. For example, by comparing the music of J. S. Bach with the following illustrations, the full range of what he could have done by using the method of geometrical inversions becomes clear.

In Invention No. 8, from his *Two-Part Inventions*, during the first 8 bars of the leading voice (upper part after the theme ends), the first 2 bars fall into the triple repetition of an insignificant melodic pattern lasting one and one-half times longer than the entire theme.

Figure 9. J. S. Bach, Two-Part Inventions, No. 8.

Using the method of geometrical inversion (even with a compromise of the recurrence of the original position), we obtain the following version of thematic continuity.

Figure 10. Inversion of J. S. Bach, Two-Part Inventions, No. 8 (continued).

Figure 10. Expansions of J. S. Bach, Two-Part Inventions, No. 8 (concluded).

In another case, that of Fugue No. 8 from Bach's *Well-Tempered Clavichord*, Volume I, if we compare the first 12 bars of the original with the version evolved from this same theme by means of geometrical inversion, we cannot fail to see the esthetic advantage of this method of composition over the more casual one derived partly from dogmatic and partly from intuitive channels.

Figure 11. J. S. Bach, Vol. 1, Fugue VIII.

Figure 12. J. S. Bach, Well-Tempered Clavichord, Vol. 1, Fugue VIII.

In some cases geometrical inversions of music give new and often more interesting character to the original. When a composer feels dissatisfied with his theme, he may try out some of the inversions—and he may possibly find them more suitable for his purpose, discarding the original. Such was the case when George Gershwin* wrote a theme for his opera *Porgy and Bess*, where position ⓒ was used instead of the original which was not as expressive and lacked the character of the latter version.

*In the *Musical Courier* of Nov. 1, 1940 Leonard Liebling, editor, wrote: "After George Gershwin had written over 700 songs, he felt at the end of his inventive resources and went to Schillinger for advice and study. He must have valued both, for he remained a pupil of the theorist for four and a half years." *Porgy and Bess*, which took Gershwin more than two years of work under his teacher's supervision, was composed according to the Schillinger System. (Ed.)

An analysis of well-known works of the composers of the past often throws new light upon them, revealing hidden characteristics that become more apparent in the geometrical inversions. For example, the harmonic minor scale combined with certain rhythmic forms produces an effect of Hungarian dance music. In L. van Beethoven's Piano Sonata No. 8, the first theme of the finale in its position ⓑ reveals a decidedly Hungarian character which is not as noticeable in its original form. This analysis also discloses that position ⓓ of the same theme has a more archaic character than the original, linking Beethoven's music with that of Joseph Haydn.

Figure 13. Geometric inversions of L. van Beethoven, Piano Sonata No.8, Finale.

In composing continuity through geometrical inversions, it is important to attend to the rhythmic structure of time elements in the original theme. According to the principles of this theory, whenever rhythmic groups assume natural forms, i.e., have an axis of symmetry, the quality of the melody will not be debased in the ⓑ and ⓒ positions of the original, and the rhythmic resultants as well as the permutation-groups are reversible.

While the principle of inversion does not interfere in any way with the intonation, it may produce undesirable, lasting durations which become exaggerated when the forward and backward moving positions are adjacent. If the original has a long duration at the end and position ⓑ follows immediately, this duration will be doubled. In such a case a rhythmic readjustment is desirable and the elimination of one of the long durations becomes necessary. Complete elimination of the final points having excessive durations may produce, in some cases, even a more satisfactory melodic continuity, as in the example below. (The melody is taken from Figure 4).

ⓐ+ⓑ+ⓒ+ⓓ

Figure 14. Adjusting rhythms in geometric inversion.

Thus, it is possible to plan in advance the composition of melodic continuity through combining geometrical inversions of the original material with a rhythmic group pre-selected for the coefficients of recurrence of the different positions.

Rhythm of Coefficients: $r_{4 \div 3}$
Geometrical Positions: ⓐ, ⓓ, ⓑ
Continuity: 3 ⓐ + ⓓ + 2 ⓑ + 2 ⓐ + ⓓ + 3 ⓑ

The actual technique of transcribing music from one position to another may be worked out in three different ways. The student may take his choice.

1. Direct transcription of the inverted positions from the *graph* into musical notation.
2. Direct transcription from a complete manifold of chromatic tables representing ⓐ and ⓓ positions for all the 12 axes.

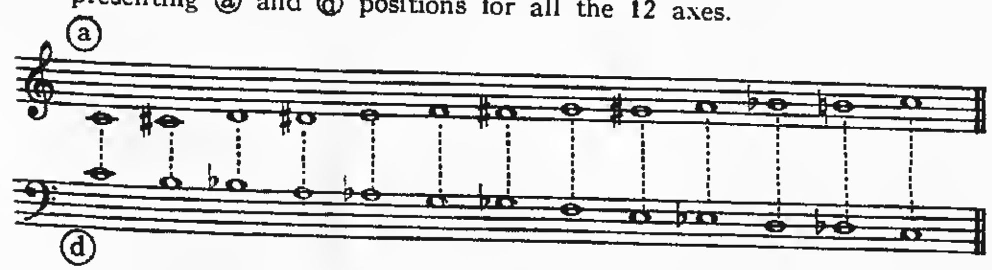


Figure 15. Manifold of chromatic tables for ⓐ and ⓓ.

3. Step by step (melodic) transcription from the original.

The unconscious urge toward geometrical inversions was actually realized in music of the past through those backward and contrary motions of the original pattern which may be found in abundance in the works of the contrapuntalists of the 16th, 17th and 18th centuries. As they did not do it geometrically but tonally, they often *misinterpreted* the tonal structure of a theme appearing in an upside-down position. They tried to preserve the tonal unity instead of preserving the original pattern. Besides these thematic inversions of melodies, evidence of the tendency toward unconscious geometrical inversions may be observed in the juxtaposition of major and minor as the *psychological* poles. In reality, the commonly used harmonic minor is simply an erroneous geometrical inversion of the natural major scale. The correct position ⓓ of the natural major scale is the Phrygian scale and not the harmonic minor. The difference appears in the 2d and 7th degrees of that scale.

In the following examples, $d_{ⓑ}$ indicates the upward reading of the ⓓ scale.

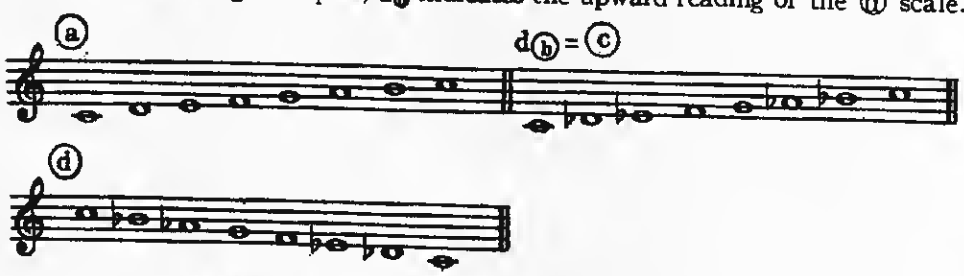


Figure 16. Inversion of natural major.

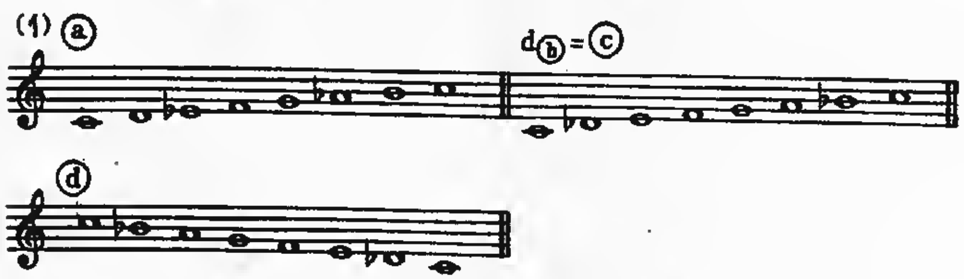


Figure 17. Inversion of harmonic minor.

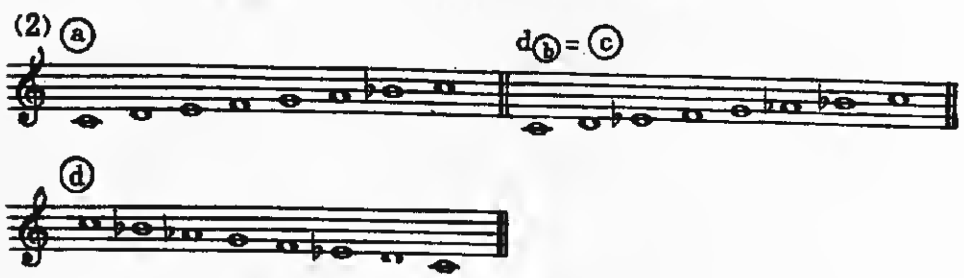


Figure 18. Inversion of Mixolydian.

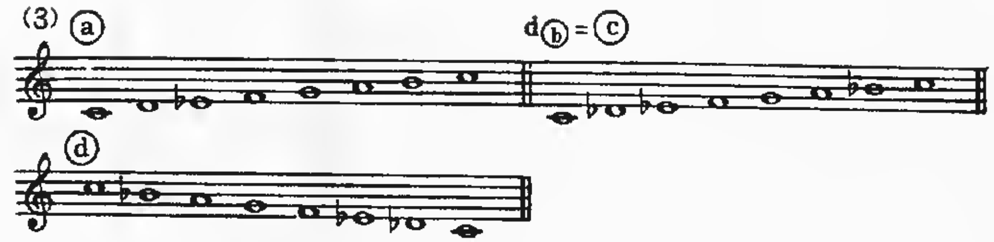


Figure 19. Inversion of melodic minor.

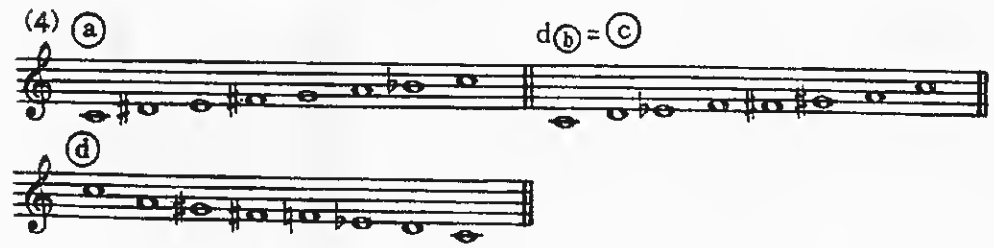


Figure 20. Inversion of Hungarian major.

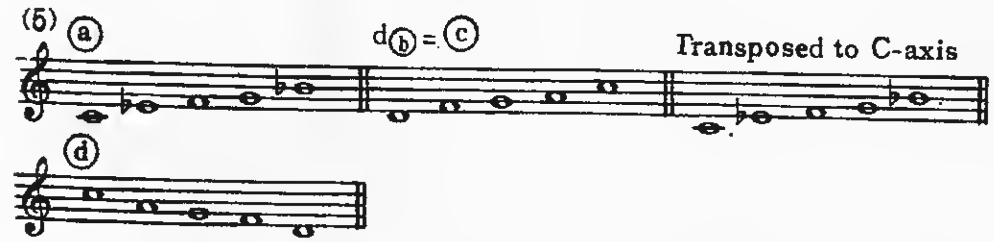


Figure 21. Inversion produces an axis of symmetry.

Thus, we see that the "psychological major" of the harmonic minor is an entirely new scale, figure 17; that the "psychological minor" of the Mixolydian scale is the Aeolian scale, figure 18; that the "psychological major" of the melodic minor scale is d_4 of the melodic major scale, figure 19; that the psychological minor of the Hungarian major scale is not the Hungarian minor scale but a new scale, figure 20; that some of the scales being inverted through their axis of inversion produce an axis of symmetry, i.e., their compensating scales are identical in structure with the original scale, i.e., ⓐ = ⓓ, figure 21.

The transcription of polyphonic continuity into different geometrical inversions must be performed from the pitch-axis of inversion in such a way that not only individual counterparts but also their mutual pitch relations are inverted accordingly. For example, if the pitch-axis of inversion is g and the theme enters on d, the same melody will start on c in position ⓓ—seven semitones in the opposite direction from the axis of inversion as compared with the seven semitones of the original direction. In the following excerpt from a fugue, the theme starts on d and the reply on g; g being the axis of inversion sets the theme on the starting point c and the reply on the starting point g (the invariant of inversion).

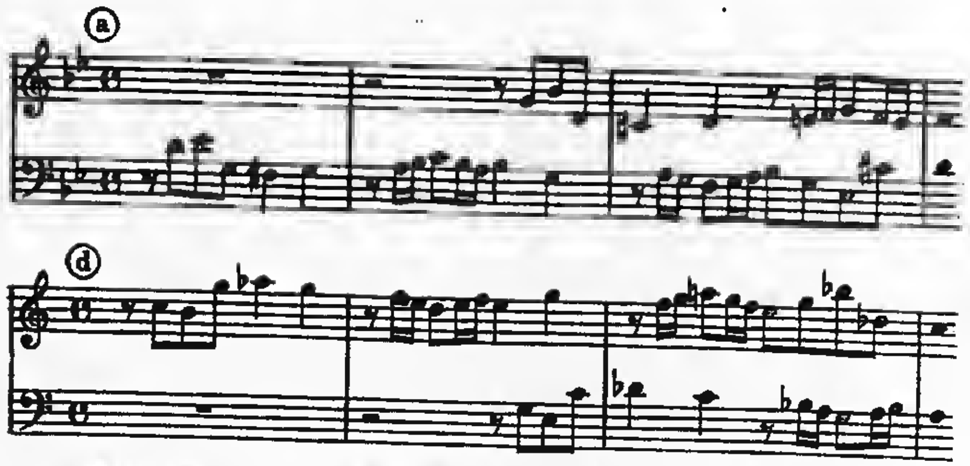


Figure 22. J. S. Bach, Well-Tempered Clavichord, Vol. 1, Fugue XVI.

The effect of psychological contrasts, to which I have referred with regard to scales, takes place with chord structures and their progressions as well. The most obvious illustration is a major triad (c – e – g; 4 + 3) with its reciprocal structure minor triad (c – e♭ – g; 3 + 4). When such a chord is to be inverted from c as an axis, all pitch-units take corresponding places in the opposite direction, i.e., c remains constant (the invariant of inversion), e becomes a♭, and g becomes f. Here is a comparative chart of positions ⓐ and ⓓ of the chords commonly known as triads [S(5)] and 7th chords [S(7)].

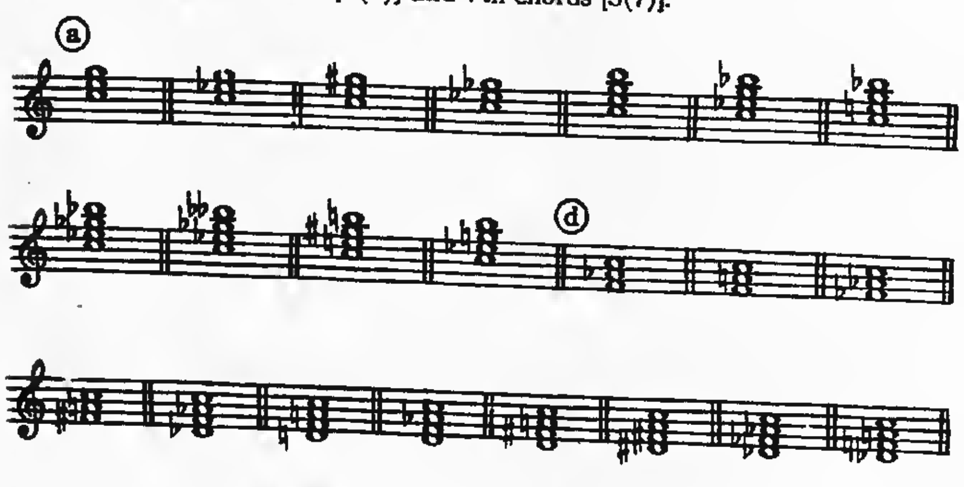


Figure 23. ⓐ and ⓓ positions of the triads.

This method of inverting chords as well as scales in order to find the psychological reciprocal is particularly useful in cases *where there is doubt* as to what the reciprocal chord structure or progression may be. It also provides an exact way of finding the reciprocal structures and progressions in those cases in which the latter are entirely unknown—and the trial and error method does not bring

any satisfactory result. For example, the reciprocal of the structure in the following example may sound quite surprising—yet the above chart shows that position ⓓ does not distort the original structure but merely changes its position.

Figure 24.

From all this, it is easy to see that not only an individual melody or a group of melodies (counterpoint), but also a melody with harmonic accompaniment, may be transcribed via various geometrical inversions. The melody of the earlier example is offered here with an accompaniment of harmony and its inversion into position ⓓ.

Figure 25.

Geometrically, a melody appearing above harmony in the original appears below harmony in position ⓓ . It may also be rewritten, without any damage to the music, by being placed *above* the harmony.

The technique of transcribing any harmonic continuity into different geometrical positions can be greatly simplified by using the method of *enumeration of each voice* of the harmony. Each voice becomes a melody and it is only necessary to know the entire chord, (i.e., the starting-points of such melodies) for the starting-point, after which all voices may be transcribed horizontally (as melodies).

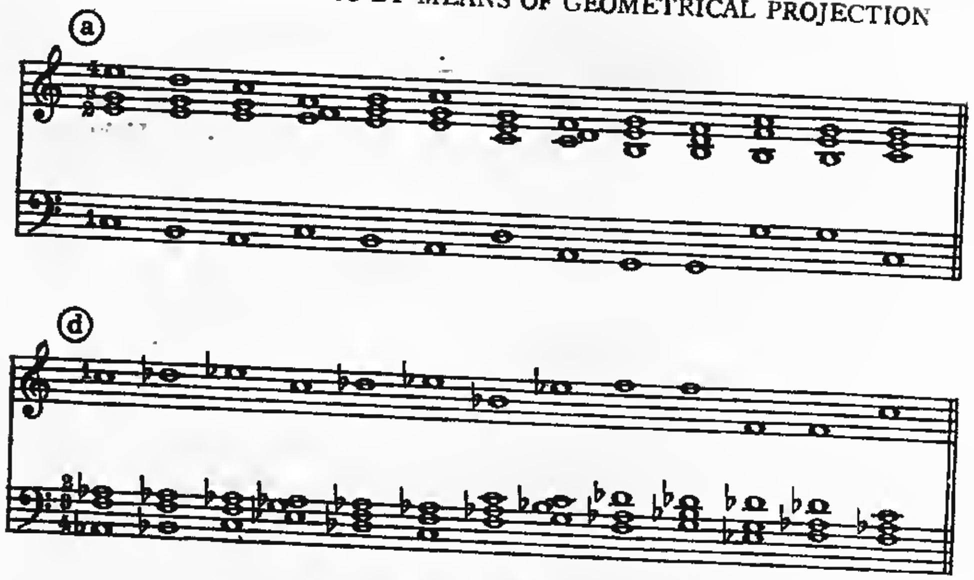


Figure 26. ⓐ *and* ⓓ *of melody with chords.*

If position ⓓ makes the upper voice appear in the bass, the opposite must be true: i.e., the placement of the bass of the original above all other voices.

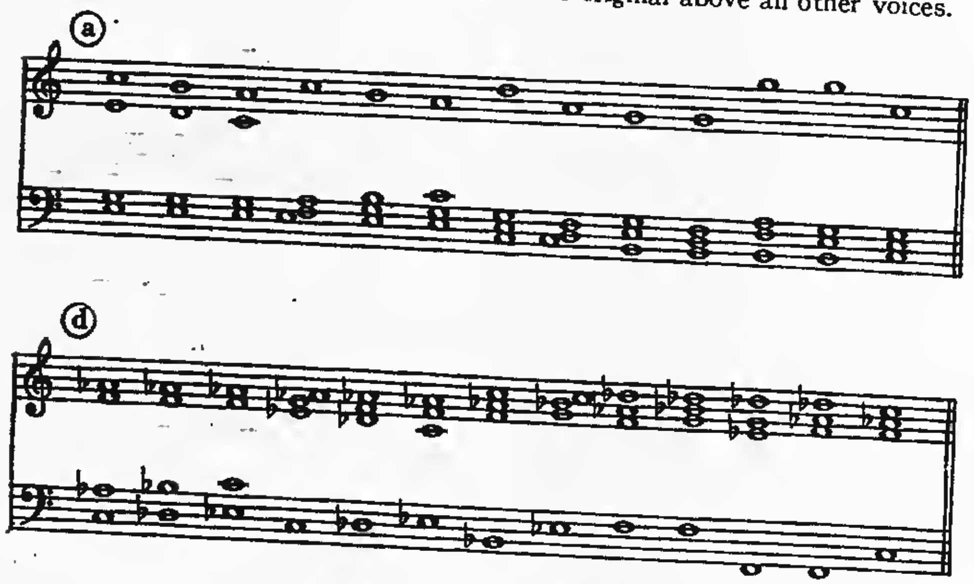


Figure 27. ⓐ *and* ⓓ *of melody with chords.*

The above-mentioned operations make it clear that any of the variations in the original distribution of voices of a chord may serve as a starting point for any harmonic continuity. Thus, a 4-part harmony offers 24 versions in each of the four geometrical positions. This device is superior to the ingenuity of any composer using an intuitive method in order to achieve variety of instrumental forms of the same harmonic continuity.

The following chart represents 24 original forms of distribution of the starting chord (according to the 24 permutations of 4 elements), for the harmonic continuity offered in the preceding figures 26 and 27. When the starting chord has the same structure but different distribution, the resulting sonority of each version also becomes different.

d c c c	d b b b	d b b b	d c c c	d a a a	d a a a
c d b b	b d c c	b d a a	c d a a	a d c c	a d b b
b b d a	c c d a	a a d c	a a d b	c c d b	b b d c
a a a d	a a a d	c c c d	b b b d	b b b d	c c c d
4 3 3 3	4 2 2 2	4 2 2 2	4 3 3 3	4 1 1 1	4 1 1 1
3 4 2 2	2 4 3 3	2 4 1 1	3 4 1 1	1 4 3 3	1 4 2 2
2 2 4 1	3 3 4 1	1 1 4 3	1 1 4 2	3 3 4 2	2 2 4 3
1 1 1 4	1 1 1 4	3 3 3 4	2 2 2 4	2 2 2 4	3 3 3 4

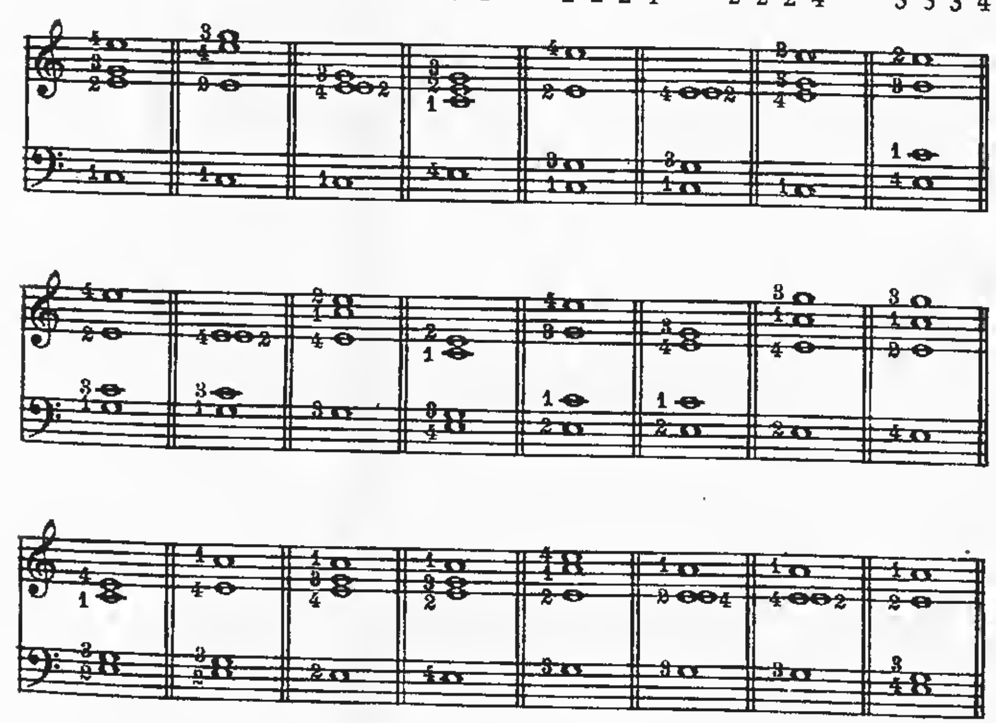


Figure 28. Twenty-four original forms of distribution of starting chord.

When portions of harmonic continuity which are short enough to be retained in the memory are used in different geometrical positions, the result is natural contrasts between adjacent sections of such continuity. In the following example, the preceding harmonic content was used in groups of three chords to one geometrical position. Each group of three chords is followed by its own position ⓓ of the same group. The continuity appears as follows:

[① + ② + ③] ⓐ + [① + ② + ③] ⓓ +
+ [④ + ⑤ + ⑥] ⓐ + [④ + ⑤ + ⑥] ⓓ . . .

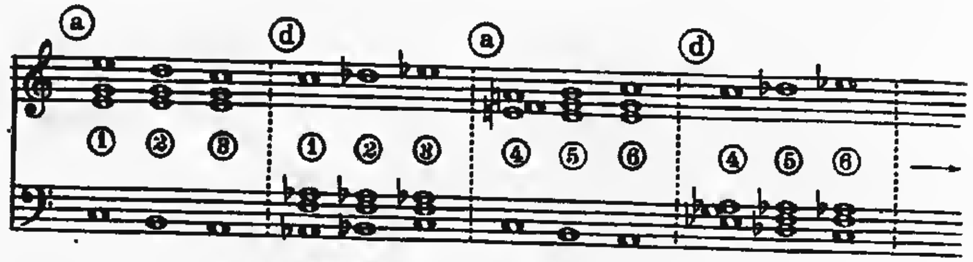


Figure 29. Contrasting harmonic continuity.

Another form of contrasting harmonic continuity derived from the same material may be evolved through consecutive progressions from chord to chord, emphasizing every three chords for one geometrical position.

[① + ② + ③] ⓐ + [④ + ⑤ + ⑥] ⓓ +
+ [⑦ + ⑧ + ⑨] ⓐ + [⑩ + ⑪ + ⑫] ⓓ + . . .

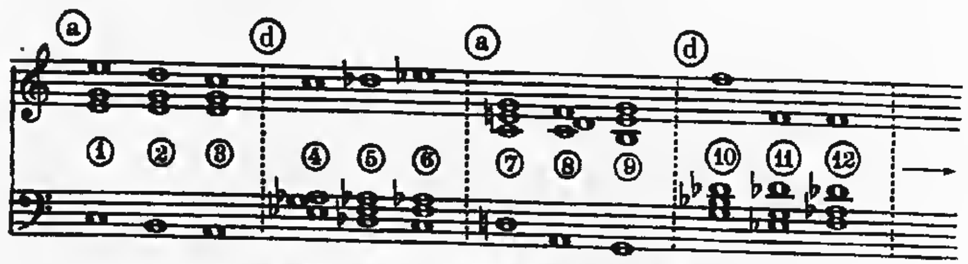


Figure 30. Contrasting harmonic continuity.

The two preceding forms of harmonic continuity are satisfactory only when different orchestration or different registration is applied to each individual group of inversion. When it is desirable to get a mixture of different geometrical positions forming one harmonic continuity and containing contrasts, then every movement of transition to a new geometrical position must be readjusted with regard to voice-leading. It may be accomplished by those students who are not as yet familiar with the theory of harmony *by connecting the two adjacent chords belonging to two different geometrical positions* through their nearest tones. Thus, Figure 29 takes the following appearance:

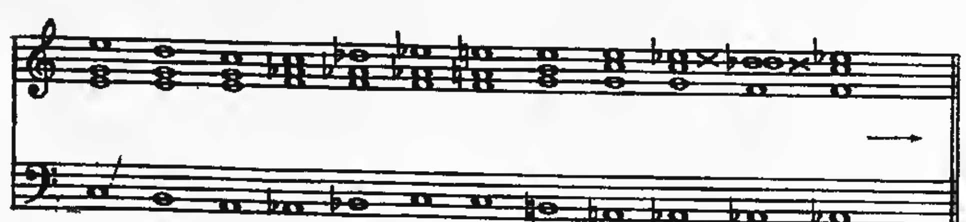


Figure 31. Connecting adjacent chords through nearest tones.

Figure 30 takes the following form when connecting tones are added:

Figure 32. Adding nearest tones to connect adjacent chords of different geometrical positions.

It is possible to create compositions of harmonic continuity from any original chord-progression, where different geometrical positions may appear in any desirable order and with any desirable coefficients of recurrence. In order to obtain a clear presentation of the scheme of progressions, it is necessary to take the entire progression in position ⓐ and to enumerate all chords in the order of their appearance. In order to enumerate the same progression in position ⓑ, it is necessary to start the numbers reading backward. The next step is to enumerate the entire position ⓓ of the chord progression starting at the beginning, and position ⓒ starting at the end, proceeding backwards.

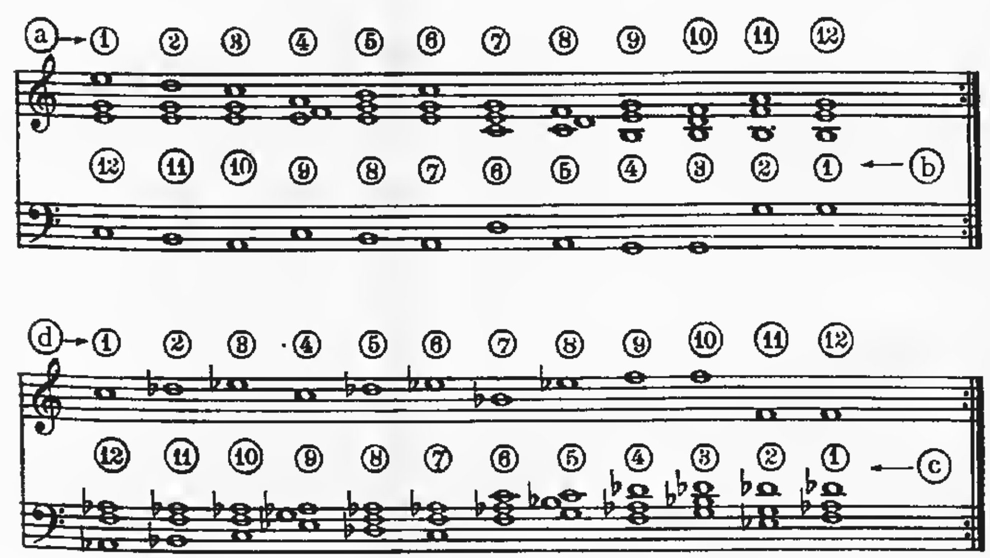


Figure 33. ⓐ ⓑ ⓒ *and* ⓓ *enumerated.*

The following is an example of the composition of a continuity containing all geometrical inversions and based on the rhythm of coefficients of $r_{5 \div 4}$:

Rhythm: $r_{5 \div 4}$

4 ⓑ + ⓒ + 3 ⓓ + 2 ⓐ + 2 ⓑ + 3 ⓒ + ⓓ + 4 ⓐ

Figure 34. Composition of a continuity containing all geometric inversions.

In the above example, the direction of position changes with each coefficient The entire scheme starting in position ⓑ includes the 1, 2, 3 and 4 moving from the end towards the beginning in the original harmonic setting. The next chord moves in the same backward direction but it is a pitch inversion of the preceding progression. Thus, this chord becomes 5. The succeeding three chords are in position ⓓ which means that the time direction changes to forward. As the last chord was the 5 in backward motion, it corresponds to the 8 in forward motion. Therefore, the first chord to be obtained in position ⓓ is 9. As this inversion consists of three chords, they include 9, 10 and 11. Proceeding in a similar fashion, one can evolve any number of harmonic progressions from a limited group of chords.

The last case—harmonic continuity, Figure 34, being adjusted through voice-leading to the nearest positions for adjacent chords belonging to different geometrical inversions—takes the following appearance.

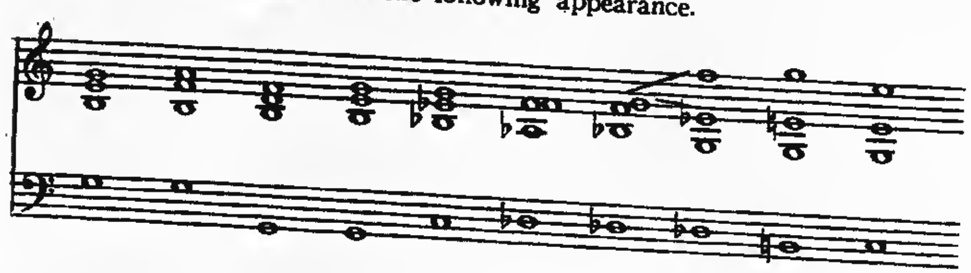


Figure 35 (continued)

Figure 35. Adjusting harmonic continuity of Figure 34 through voice leading. (concluded).

In its final form as above, such a continuity may be assigned to a homogeneous orchestration or registration.

CHAPTER 2

GEOMETRICAL EXPANSIONS

HAVING DISCUSSED the technique of geometrical *inversions*, we may now consider an additional set of techniques, those leading to geometrical *expansions*. Tonal expansions, as distinct from geometrical expansions, were discussed in the *Theory of Pitch-Scales*, Book II.*

On an ordinary graph, the unit of measurement is equivalent to $\frac{1}{12}$ of an inch, and it represents, in this system of notation, the standard pitch-unit, i.e., $\sqrt[12]{2}$ (a semitone). Such units are expressible in arithmetical integers as logarithms to the base of $\sqrt[12]{2}$. Thus, a semitone consists of one unit, a whole tone of two units, etc., along the ordinate.

A melodic graph may be translated into different absolute pitch values by substituting different coefficients for the original p.

To translate a musical graph into $\sqrt[6]{2}$ we would simply use double units on the ordinate for the original single units, while preserving all the other relations within a given melodic continuity. In this case, p = 2p. By using greater coefficients such as 3, 4, 5, 6 or 7 ($\sqrt[4]{2}, \sqrt[3]{2}, \sqrt[12]{2^5}, \sqrt{2}, \sqrt[12]{2^7}$), we obtain the respective units for the pitch intervals.

This form of projection is known as an *optical projection through extension of the ordinate.* It is one of the natural tendencies in visual arts. When artists attempt to produce a distortion (variation) of the original proportions, they are unconsciously attempting to achieve one or another form of geometrical projection.

These variations, when executed geometrically and in accordance with optics, give a greater amount of esthetic satisfaction because they are more natural.

On the next page you will find an example of the translation of one system of proportions into another, as applied to linear design.

*Schillinger describes various methods of tonal expansion in Chapter 5 of Book II, pp. 133-7. In tonal expansion, as contrasted with geometrical expansion, the original pitch-units are not altered; they are merely rearranged. In the first tonal expansion (E_1) of c—d—e—f—g, for example, these pitch-units reappear in the following order: c—e—g—d—f. The student will recall that this new form is obtained by circular permutation in which alternate units are skipped. In geometrical expansion, however, the original pitch-units are not retained. The process, as the student will learn, is one of extending the semitone to a full tone, or more. Thus, c—d—e—f—g would become c—e—g♯—a♯—d. The unit of measurement may be extended so that p, instead of equalling 2p, equals 3p, 4p, 5p, etc. (Ed.)

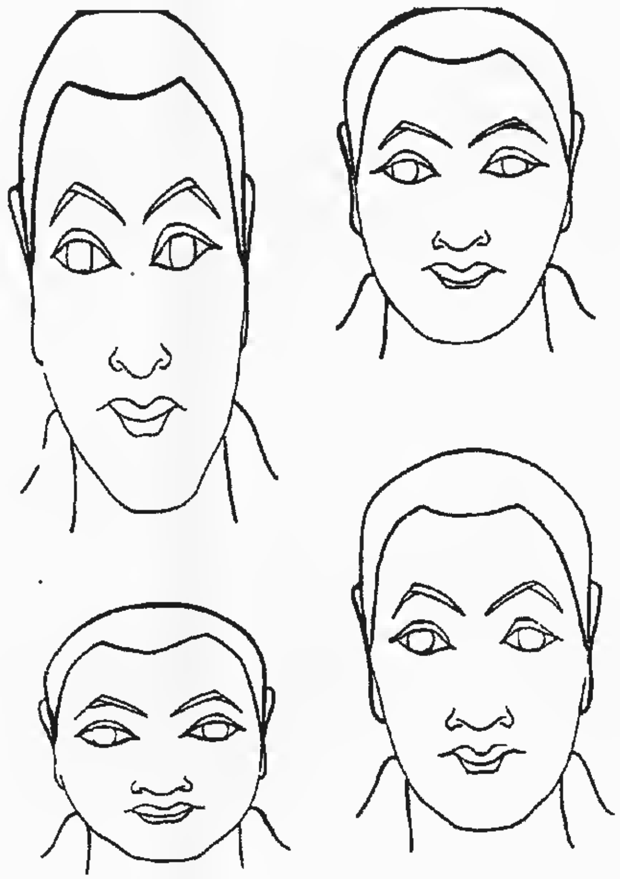


Figure 36. Translating one system of proportions into another.

In the illustration above, the same configuration is presented under different coordinate ratios. The technique of such translation consists of producing a network on the original drawing (with as many units as is desirable with regard to precision) and then transcribing this network into a differently proportioned area, preserving the same number of lines on both coordinates of the network. Then all points of the drawing acquire their respective positions in the corresponding places of the network.

Compare these geometrical projections with the distortions in these and other paintings by El Greco and Modigliani.

Figure 37. El Greco. *

Figure 38. Modigliani. **

*Courtesy of The Metropolitan Museum of Art. **Collection, Museum of Modern Art.

As each coefficient of expansion is applied to music, the original is translated into a different style, a style often separated by centuries. It is sufficient to translate music written in the 18th century by the coefficient 2 in order to obtain music of greater consistency than an original of the early 20th century style. For example, a higher quality Debussy-like music may be derived by translation of Bach or Handel into the coefficient 2.

The coefficient 3 is characteristic of any music based on $\sqrt[4]{2}$ (i.e., the "diminished 7th" chord). Any high-quality piece of music of the past exhibits, *under such projection*, a greater versatility than any of the known samples that would stylistically correspond to it in the past. For the sake of comparing the intuitive patterns with the corresponding forms of geometrical projection, it is advisable to analyze such works as J. S. Bach's *Chromatic Fantasy and Fugue*, Liszt's *B Minor Sonata*, L. Van Beethoven's *Moonlight Sonata*, first movement.

The coefficient 4, being a multiple of 2, gives too many recurrences of the same pitch-units since it is actually confined to but 3 units in an octave. Naturally, such music is thereby deprived of flexibility.

But the 5p expansion is characteristic of the modern school which utilizes the interval of the 4th—such as Hindemith, Berg, Krenek, etc. Music corresponding to further expansions, such as 7p, has some resemblance to the music written by Anton von Webern. Drawing comparisons between the music of Chopin and Hindemith, under the same coefficient of expansion, i.e., either by expanding Chopin into the coefficient 5, or by contracting Hindemith into the coefficient 1, we find that the versatility of Chopin is much greater then that of Hindemith. Such a comparison may be made between any waltz of Chopin and the waltz written by Hindemith from his piano suite, *1922*.

Comparative study of music under various coefficients of expansion reveals that often we are more impressed by the raw material of intonation than by the actual quality of the composition.

The opposite of this procedure of expansion of pitch is *contraction* of pitch. Any pitch interval-unit may be contracted twice, three times, etc., which is expressible in $\sqrt[24]{2}$, $\sqrt[36]{2}$, etc., providing that instruments with corresponding tuning are devised. Those esthetes who usually love to talk about the "economy of material" and "maximum of expression" will perhaps be delighted to learn that an entire 4-part fugue of Bach occupying a range of $3\frac{1}{2}$ octaves would require only one whole tone if the pitch interval-unit were $\frac{1}{18}$ of a tone ($\sqrt[216]{2}$). Applying the same principle to the contraction of the absolute time duration-unit, we could hear this fugue in a few seconds instead of several minutes!

The natural pitch-scale, i.e., the series of harmonics, does not produce uniform ratios but gives a natural logarithmic contraction. The intervals between the pitch-units decrease, while the absolute frequencies increase. This phenomenon is analogous to the *perspective* contraction in space *as we see it*. If music were devised on natural harmonic series, the relative group-coefficients of expansion and contraction could be used. But it seems that the natural harmonic series does not, in fact, provide any flexible material for musical intonation but merely for building up various tone qualities—for the fact is that a group of harmonics sounded at the same intensity produces one saturated unison rather than harmony. This phenomenon is somewhat similar to that of *white light*, in which all spectral hues merge—becoming noticeable only when the beam is broken up. Logarithmic contraction of pitch combined with the logarithmic contraction of time *may* come into existence in the remote future in connection with the development of automatic instruments for composition and execution of music.

The technique of pitch-expansion may be executed directly from a graph or from a corresponding chromatic scale of expansion. In such a case, 2p will produce a whole tone scale progressing through 2 octaves instead of a full chromatic scale progressing through one octave (when p = 1.) While expansion of time extends the graph along the abscissa, the increase of the absolute time unit is not noticeable unless compared with the original. When we hear a musical continuity, we do not know (unless it is extremely exaggerated) whether it is the original velocity or a derivative thereof. The difference becomes apparent only when different coefficients of velocity of the same musical continuity are brought close together. Thus, time extension produces a different pattern on a graph without producing a difference detectable in the absence of comparison.

Pitch expansion works under the same conditions. It is only through comparison that we can learn that a certain musical continuity has been expanded or contracted from its original. This is apparent in the process of *tonal* expansion (which preserves all the pitch-units while the range increases) as it was described in the *Theory of Pitch Scales.**

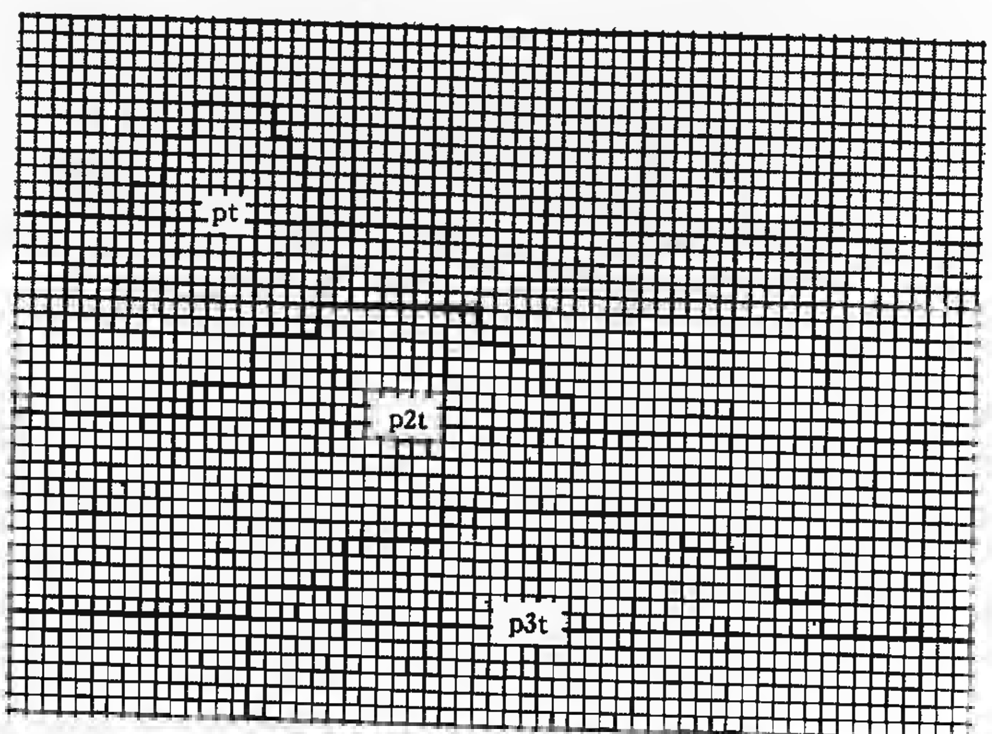


Figure 39. Time and pitch expansions (continued).

*See Book II.

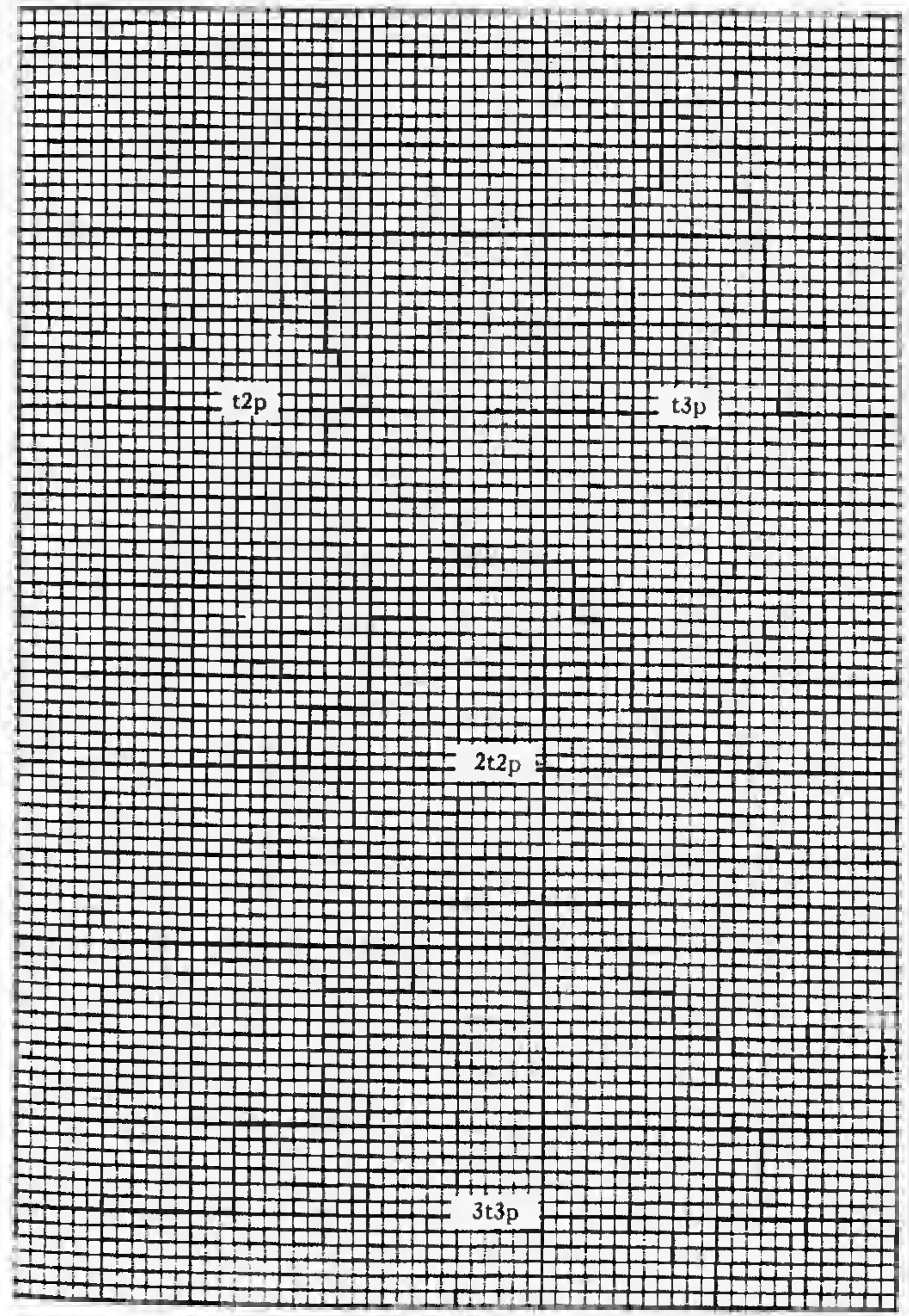


Figure 39. Time and pitch expansions (concluded).

If pt represents the original, 2t and 3t produce the corresponding time expansions. Likewise, 2p and 3p produce the corresponding pitch expansions. The expansion through two coordinates preserves the absolute form of the configuration, merely magnifying it (2p2t and 3t3p).

It might seem at first that the ordinary enlargement or reduction of an original image—such as that effected by any natural optical projection (lantern slide projector, motion picture projector, magnifying glass, etc.)—does not change the appearance of the image. Yet when carried to an extreme, it does in fact transform the image to a great extent. For example, an ordinary close-up of a human head seen on the screen does not change our impression of the image. But when a human head is subjected to a several hundred power magnification, the original image is changed beyond recognition. A photograph of the skin surface of the human arm occupying only 1/100th of a square inch produces an image which is not easily associated with the human arm.

Thus, the difference in the actual sound of music (like the magnification of Haydn into von Webern) is only quantitatively different from the enlarging of visual images. Even with coefficients as low as 5, a melody is transformed beyond recognition. But the magnification of visual images requires at least one-hundred power magnification in order to achieve a similar effect.

It is interesting to note that bizarre effects of optical magnification are often due to the fact that such images are merely *hypothetic* and have no actual correspondences in the physical world of our planet. An image of a chicken can be magnified to the size of the Empire State building (for example, by being projected on an outdoor smoke screen), yet no real live chicken could exist on this planet even the size of an ostrich, because—as the volume grows in cubes—the legs of such a chicken could not support the weight of its body.

The following chart represents pitch expansions of the melody: graphed in Figure 39.

Figure 40. Pitch expansions of the melody of Figure 39. (continued)

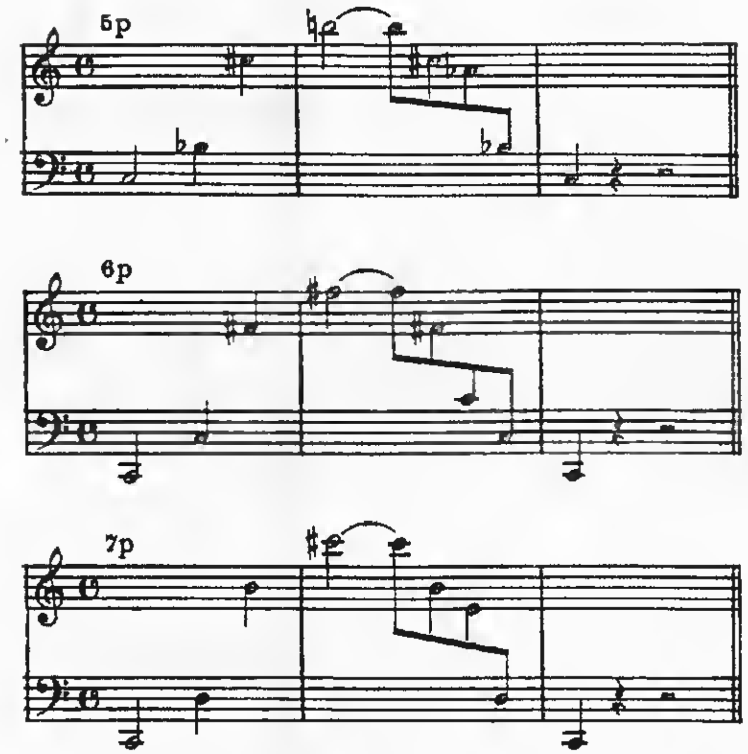
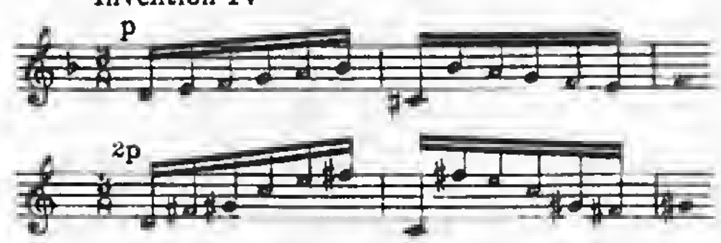


Figure 40. Pitch expansions of the melody of figure 39 (concluded).

For most purposes the lower coefficients are the most practical ones. Examples of geometrical expansion may be seen in the following excerpts from J. S. Bach:

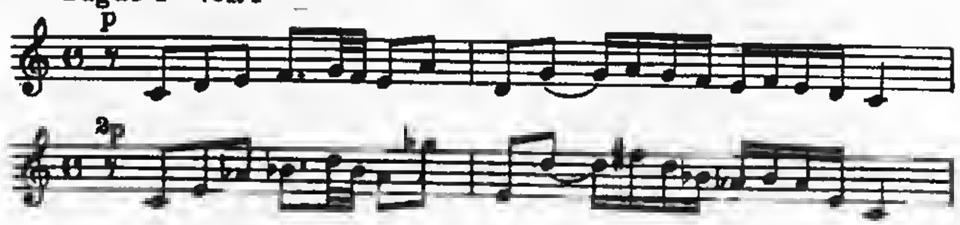
Invention IV



Invention VIII

Figure 41. Geometric expansions of J. S. Bach's Two-Part Inventions.

Fugue I - Vol. 1



Fugue II - Vol. 1

Fugue XII - Vol. 1

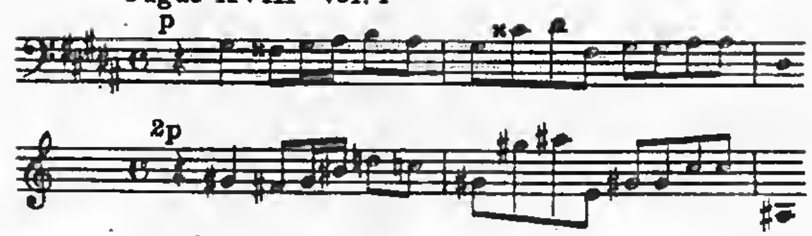
Fugue XIV - Vol. 1

Fugue XV - Vol. 1

Figure 42. Geometric expansions of J. S. Bach's Well-Tempered Clavichord.

Fugue XVI - Vol. 1

Fugue XVIII - Vol. 1



Fugue VII - Vol. 2

Figure 43. More geometric expansions of J. S. Bach's Well-Tempered Clavichord.

Different geometrical expansions may follow one another as elements in a continuity *only* when used in *very short portions*—in order to enable memory to retain the original pattern. When the ear accommodates itself to one coefficient of expansion for a considerable period of time, then a sudden change to a new coefficient produces such a surprising effect that the desirability of the use of the device in one continuity becomes questionable. For this effect is equivalent to a sudden change of style; it may be described as music beginning somewhat like Debussy, suddenly changing to Bach, and then again to Hindemith. Yet tests with various listeners show that in immediately following fragmentary sequences, the device sounds perfectly acceptable.

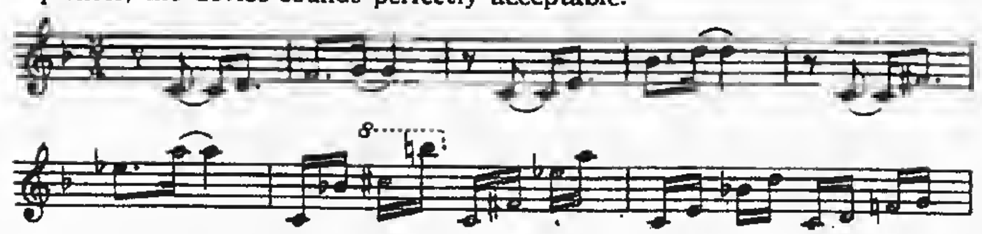


Figure 44. Geometric expansions of George Gershwin's "I Got Rhythm".*

Geometrical expansions of melody may also serve the purpose of modifying motifs through the method of geometrical projection. The original melodic pattern becomes entirely modified—yet the system of pitch-units is the outcome of a consistent translation from one system of pitch relations to another. The technique of such modification is equivalent to the contraction of the general pitch range emphasized by the geometrically expanded form. Some melodies, especially those with big coefficients of expansion, permit several different versions (degrees) of contraction.

The following example presents the exact geometrical expansions with the respective contractions of their ranges:

Figure 45. Geometrical expansions with readjusted (contracted) range.

The process of *range-contraction* often introduces new characteristics into geometrically expanded forms. For example, in the case of 5p in the preceding example: in its readjusted form, it seems to be more "conservative" than in its respective geometrical expansion. In the case of 7p, the contracted form is reminiscent of the music of Prokofief rather than that of von Webern.

Geometrical expansion of the harmony which accompanies melody expanded through the same coefficient (whether with readjusted range or not), must be performed from the pitch axis of the entire system (usually the root-tone).

Figure 46. Geometrical expansion of a harmonized melody.

This translation of harmony may be accomplished either through transcription of a graph or through step by step translation from the original. One may also prepare in advance chromatic scales from the respective pitch axes where all the pitch-units may be found directly in the corresponding expansions.

Figure 47. Scale of pitch units and their corresponding expansions.

Further expansions may be evolved in a similar way. When harmony is to be translated into a geometrical expansion, it is sufficient to find the first chord of its original setting and to proceed horizontally with each voice as melody, thus performing the voice-leading of the original. If after such translation the range seems to be too extreme for any instrumental applications, the above-described range-readjustment may be applied.

Here is an example of a conventional harmonic continuity first translated into 2p and then readjusted into two further contracted forms. In such a case, the extreme upper voice may become one of the inner voices by being placed one octave below.

Figure 48. Harmonic continuity projected into 2 p and readjusted.

Translation of polyphonic continuity into geometrical expansions must be carried out on the same principle. The pitch intervals between the theme and the reply must be doubled or tripled in relation to the pitch axis. For example, if the pitch axis is c and the reply in the original starts on g, when p = 2, the beginning of the reply will be on d, i.e.. the interval 7 becomes 14.

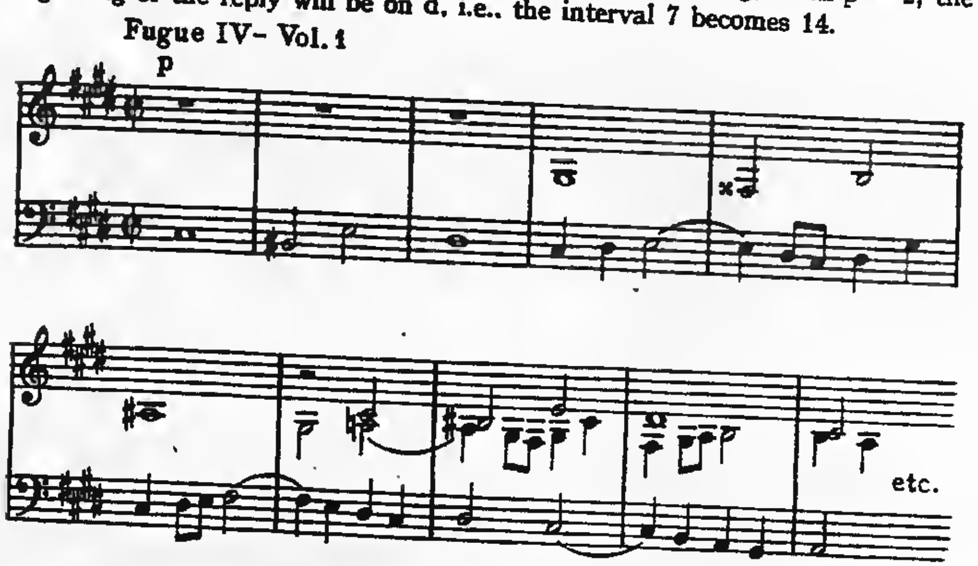


Figure 49. 2p expansion of J. S. Bach, Well-Tempered Clavichord (continued)

Figure 49. 2p expansion of J. S. Bach, Well-Tempered Clavichord. (concluded).

All geometrical *expansions* are subject to geometrical *inversions* as well. A consistent musical continuity may be evolved through the variation of inversions under the same coefficient of expansion. Thus the two methods of mathematical variation of music, based on geometrical projection, bring an effective solution to two very important technical problems:

1. Composition of infinite melodic or harmonic continuity containing organically related contrasts.
2. Translation of music of one epoch into another, "modernization" and "antiquation."

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK IV
THEORY OF MELODY

BOOK FOUR

THEORY OF MELODY

CHAPTER 1

INTRODUCTION

IN ORDER successfully to produce anything out of given material, it is necessary to know the properties of such material as well as all the processes involved. Any material which is to be dealt with must consist of a number of components. Unless all the components required for structural realization are known, the result of such a procedure will be failure. The components of a structural process specify different individual procedures to be coordinated in the whole.

There exists a great deal of scepticism as to the possibility of constructing musical melodies rationally—but no such scepticism exists as to the evolution of chord progressions. This is because it happens that the musical study of harmony is based on a quite developed tradition describing such procedures, while no workable theories of melodic composition have thus far been offered in the civilized world.

Although we hear about such theories existing in Oriental civilizations, such as those of the ancient Hindus and the ancient Chinese, these theories are not available in any form other than the original and therefore are not accessible to anyone not familiar with the respective languages. I may say that there is no evidence among scientific musicological documents which would offer any positive proof of the existence of such theories. Perhaps, it is one of those "Oriental mysteries," like the rejuvenation of an old person or the resurrection of someone buried alive. Naturally, we cannot use such methods—yet we are surrounded by the sceptical attitude of musicians brought up in the romantic traditions of the 18th and 19th centuries, and the intuitive approach to the art of musical composition.* To one associated with the method of engineering musical melodies, however, the possibility of such a creative process is beyond doubt. A buffalo is almost a zoological myth in Europe, but a common reality in America. A zoologist dealing with some rare specimen on the African continent would have to face the same scepticism from people whose scientific criterion is "seeing is believing."

*Although Schillinger refers merely in passing to this development in the history of ideas, it is extremely important to us. Living as we do in the years following the 19th century rise of romanticism, we are heirs of the over-emphasis on intuition and inspiration in the arts. Prior to the romantic reaction, there was no jealous dichotomy between the arts and sciences. In fact, during the 17th and 18th centuries, the role of reason in the arts was widely recognized and accepted. The romanticists revived the Platonic conception of the artist as an inspired madman. "Gie me ae spark o' Nature's fire, That's a' the learning I desire," chanted Robert Burns, the Scotch romanticist. This false dichotomy between knowledge and inspiration has persisted into our time. It has permeated the thinking of composers and the public to such an extent that a scientific approach to the arts is sometimes condemned without thought or investigation. Nevertheless, such an approach is grounded on the best experience of composers through the ages as well as on the most recent discoveries of modern psychology. Schillinger's discoveries were made possible because he began with the idea that the arts could be rationalized and that the process of musical composition was subject, as he demonstrates, to scientific analysis. (Ed.)

Technical experience shows that failure of realization in constructing any instrument is often due to *inefficient engineering* and not to any error in the theoretical assumptions behind it. With all the extraordinary progress that engineering technique has made recently, some people still doubt that a musical instrument for automatic performance may be entirely different in quality from the automatic piano of the present. The difference between the two may be much greater than is the difference between the first biplane of the Wright brothers and the latest air superliner for 120 passengers by Sikorsky. Or consider the first magnifying glass as compared to the new 200-inch telescopic mirror. We have already witnessed an obvious example of the progress which scientific technique has brought into the field of music, for electronic sound production offers any desirable tone quality, intensity and form of attack. Very soon, when performers are relieved of the struggle for tone quality, they will have time to be confronted with the major problem of musical interpretation. This problem of interpretation will be concerned with relative tone colors, dynamics, forms of attack, group distributions, etc. When this becomes a reality, (and that day may come very soon) the composer will lose his present dependence upon the performer. If and when an automatic instrument can carry out the composer's intentions to any desirable degree of subtlety, the composer can celebrate the arrival of a new era that will liberate him from the centuries of slavery imposed upon him by the performer. This, in turn, will call for much greater precision of creative intention on the part of the composer. It will not be sufficient to write the pitch and the time components. It will be necessary to include *all* the technical forms of execution. As our reaction to music changes with different eras and civilizations, with different historic periods, and often varies in trends during one decade, a new profession will emerge in the near future: the members of this new profession will be called *readjusters* of music (not to use the word "arrangers", which would convey a misleading meaning due to its previous connotation). Their duty will be to make the music of primitive civilizations or of any of the remote periods of our civilization comprehensible to the listeners of their own time. This will require various forms of technical readjustment and rejuvenation of the media which have lost their expressiveness a long time ago. The phenomenon of the transcription of music has been known since time immemorial. However, the new method will be much more radical than any of those used in transcriptions of the past; it will require definite engineering techniques instead of vague intentions.

By studying different aspects of music in different civilizations and by mathematical analysis of various procedures involved in the making of a musical composition, a group of engineering routines may be evolved.

The most important stages in evolving a theory of melody are:

1. Study of the general properties of melody with respect to its convertibility and other forms of geometrical projection (expansions-contractions).
2. Comparative study of the patterns appearing in natural configurations (crystal, vegetable and animal forms).

3. Study of the properties of curves and of statistical records specifically (technology of events).
4. Recording and analysis of the reflex patterns (respiratory, muscular, nervous, etc.)
5. Study of the trajectorial curves evolving linear design in the visual arts.
6. Study of graphs expressing intuitive musical compositions in terms of pitch, time, intensity and tone quality.
7. A comparative study of all the above-mentioned patterns.
8. Deduction of a system of patterns to serve as stimuli of reaction of definite character and intensity.
9. Development of a group of routines leading to efficient artistic creation and providing the testing criteria.
10. Elaboration of a scientific theory of production of musical melodies.

A. Semantics

Music in general—and melody in particular—has been considered, since time immemorial, a supernatural, magical medium. Many great philosophers in different civilizations have given their attention and directed their thoughts toward this elusive phenomenon. The more definitions of music you know, the more you wonder what music really is. It seems to fall into the category of life itself. It seems to have too many "x's"

People did not know much about lightning even ten thousand years ago, and ten millennia make only a one-hundredth in the range of *human* evolution. We tend to ascribe supernatural powers to any phenomenon we cannot explain. Today, we are surrounded by things more miraculous than any of the products of ancient imagination—and when you think of the achievements of modern technique, it seems to be incredible that a toy—as simple as melody—should still remain in the category of the irrational.

Following our method of analysis, however, we may assume that *any phenomenon can be interpreted and reconstructed.* To accomplish this, it is necessary to detect *all* the components and to determine the exact form of their correlation.

There are two sides to the problem of melody: one deals with the sound wave itself and its physical components and with physiological reactions to it. The other deals with the structure of melody as a whole, and esthetic reactions to it.

Further analysis will show this *dualism* is an illusion and is due to considerable quantitative differences. The shore-line of North America, for example, may be measured in astronomical, or in topographical, or in microscopic values. The difference between melody from a physical or musical standpoint is a *quantitative* difference. The differentials of the physical analysis become negligible values for purposes of musical (esthetic) analysis.

Melody is a complex phenomenon and may be analyzed from various standpoints. Physically, it can be measured and analyzed from an objective record, such as a sound track, a phonograph groove, an oscillogram or the like. Melody when recorded has the appearance of a curve. There are various families of curves, and the curves of one family have general characteristics. Melodic curve is a trajectory, i.e., a path left by a moving body or a point. Variation of pitch in time continuity forms a melodic trajectory.

Melody from a *physical* standpoint is a compound trajectory of frequency and intensity. Melody from a *musical* viewpoint is a compound trajectory of pitch, quality, and volume. The components of quality are timbre, attack, and vibrato.

Physically, pitch is an accelerated periodic attack. Physically, the difference between rhythm and melody is purely quantitative. Therefore, time-rhythm in a melody may have two forms.

1. Through periodicity of attacks of low frequency, which is unavoidable when the pitch-frequency is constant;
2. Through variation of frequencies, i.e., through changes in pitch itself.

Frequency constitutes musical *pitch*. Any sound wave of a given frequency (constant or variable) generates its own frequency *subcomponents* (known as "partials" or "harmonics") resulting from purely physical causes. The latter are disturbances which convert a simple wave (known as a sine wave) into a complex one. The sound of a simple wave may be heard on specially made tuning forks and electronic musical instruments.

The intensity of a sound wave is one of the factors of disturbance, and the duration of intensity and its stability in time continuity are others. The latter are musical factors: depend on *form of attack* (or *accentuation*). Finally, the resultant of both components and all the subcomponents, i.e., the interaction of all component frequencies and intensities in a sound wave, constitutes the musical component of *timbre and character* (quality) of sound.

The relative importance of musical components and subcomponents has already been measured, so to speak, by agreement among musicians and music lovers. The conclusion has been reached that two melodies are *identical* if their main components (time and pitch) are identical. For instance, a melody played on the piano, or sung, or played loud or soft, or with vibrato or without it, would be considered "the same" melody if rhythm (time) and intonation (pitch) are identical. The subcomponents and the sub-subcomponents pertain to execution, i.e., to the performance of melody, not to its own structural actuality. The very neglect of subcomponents, on the one hand, relieves the composer of a certain amount of responsibility; on the other hand, it leads to loss in esthetic value of melody when the melody is wrongly executed by the performer. For then the performer has to supply the subcomponents without the henefit of any exact indication by the composer and therefore he acts at his own discretion, whether rightly or wrongly.

At this point we may adopt Helmholtz' definition of melody (which satisfies the musical aspect): *melody is a variation of pitch in time.** Is *any* variation of pitch in time a melody? An attempt to answer this question leads into the *semantics of melody*.

*Hermann Ludwig Ferdinand von Helmholtz (1821-1894), the great German physicist and physiologist, sought to devise rules of musical science based on the physical nature of musical sounds. His most significant work is *On the Sensations of Tone as a Physiological Basis for the Theory of Music* published in 1863. (Ed.)

B. Semantics of Melody

The fundamental semantic requirements are that melody must "make sense," it must have (like words) associative power, i.e., it must be able to convey an *idea* or *mood*, to "express something."

But these are also the requirements of language, and yet there is a distinct difference between *word* and *melody* as symbols of expression. The function of *words* is to express the *concept* of actuality, to find its verbal symbol. The function of melody is to express the *structural scheme* of actuality. Words have their origin in thought; melody has its origin in feeling, i.e., originally in the reflexes. *Words generate concepts which may or may not stimulate feelings.* Melody, on the contrary, *stimulates feelings* (emotions, moods) as spontaneous reactions, *which may or may not generate concepts.* Melody expresses actuality *before* the concept is formed for that actuality. This is why, in listening to a melody, one is satisfied with its expression to such an extent that the quest for the concept, "What does it actually express," is never aroused. But, on the contrary, when a melody *does not convey sufficient associative power* (to stimulate reflexes, reactions or moods), then the listener looks for a verbal description of it, or, at least, for a title, a "label," a concept. Melody is *insufficient* whenever it calls for a verbal explanation. When a word does not convince through its own associative power, or in order to increase the latter, one resorts to intonation and gestures.

Words or melody may or may not be self-sufficient. Words that are not self-sufficient call for a specific form of intonation in order to acquire the necessary associative power. We may also state, reciprocally, that melody which is not self-sufficient as intonational form calls for word and often for a symbol in the form of a verbal concept. These two statements can be verified by simply studying the facts.

Here we arrive at the idea that although, in their developed forms, both word and melody are self-sufficient—in their early periods of formation they produce hybrid forms: an intonational form that calls for a concept—and a conceptual form that calls for intonation.

Here are a few of many references. According to the statements of George Herzog, Columbia anthropologist who made some pertinent recordings and demonstrated the phonograms, there are certain Central African tribes whose verbal language is just such a hybrid. A word of the same etymological constitution (spelling) has at least ten different forms of intonation, each attributing a different meaning to the word. In this case *intonation* is an *idiomatic factor*.

In other cases, as in some instances of Chinese music,* melody or even the single units of a scale become symbolic of a concept—i.e., they assume the function of words.

The Stony Indians of Alberta, Canada, try in their songs to express the sound of a brook, the murmur of leaves, etc. Yet as a descriptive means it is not self-sufficient; it calls at least for a title. This is a case in which melody is a bad competitor of poetry.

* See Karl Stork, *History of Music.* (J.S.)

Out of many hypotheses as to the origin of music and word, I select the ***reflexological*** one.* Sound reflexes (of the vocal cords), before they crystallized into relatively distinguishable forms of word and melody, were spontaneous expressions of satisfaction or lack of satisfaction in an animal organism. Any cause of actual or potential disturbance that endangered an organism became a stimulus for the defensive reflex. This is probably the original form of the intonational ***signal***. If such a form was at first an improvised reflex movement of the vocal cords expressing fear—a spontaneous reaction to danger—it may have crystallized later into the etymological form of the concept of "danger."

When an organism is on the verge of struggle for its own survival, it usually resorts to ***intonational signaling*** rather than to an etymological one. Even in our own time, a drowning man does not say: "I am drowning!" He generally shouts: "Help!"

The amount of semantic and acoustical elements in words or melodies varies greatly. There are all gradations from an exclamation to a polyconceptual polysyllabic word of the German language with the relative decrease of the acoustical (intonational) and the relative increase of the semantic element. In many undeveloped forms of speech, an outsider may in fact mistake such speech for melody.**

Melody always contains well-defined acoustical elements, although it may be alien to an ear trained to different systems of intonation. Melody offers also a scale of semantic gradations from imitative descriptive intonations, through symbolic abstractions, to the expression of mechanical forms.

Both imitative and symbolic functions of music tie it closely to verbal semantics. In this stage, melody is the language of a given community only. Tests show that even such commonplace moods as "gayety" and "sadness" cannot be expressed by means of melody that will mean the same thing to all nations. Melody is a ***national*** language or a language of a ***given epoch*** with regard to descriptive or symbolic qualities.

Arabian funeral music sounds anything but "sad" to us because of our association with major scales—which mean gayety, heroism, happiness and satisfaction to us. Gay Arabian dance-songs sound "sad" to us because of our association with harmonic minor scales, which mean exactly the opposite to us. It is similar with the forms of musical harmony. Through previous associations we react to major chords as we react to major scales. Yet we have the curious phenomenon of the Negro-American "blues," which is supposed to express depression, but which, nevertheless, has the richest scale of major chords.

All the controversies ascribing this or that semantic connotation (descriptive or symbolic) to music will vanish when we penetrate the real meaning of music, namely, ***the expression of the forms of movement***. The objectification of

*Here begins, in a partial form, Schillinger's exposition of his theory of the correspondences between music—melody, in particular—and the objective world of life. As such, his theory offers us the means whereby esthetic phenomena can be correlated in a scientific and materialistic way with the rest of human experience; in consequence, even this partial exposition is of the utmost philosophical importance. (Ed.)

**"Program music is a curious hybrid, that is, music posing as an unsatisfactory kind of poetry"—*Oxford History of Music*, Volume 6, Page 3. (J.S.)

this meaning requires only one premise: ***biomechanical, physiological experience, combined with a highly developed sensory system***. The requirement may be satisfied by any normal specimen of the higher animal forms.*

Though commonly unknown and generally repudiated when brought into a discussion, this fundamental form of musical semantics had already been known to Aristotle. Here is his definition: "Rhythms and melodious sequences are movements quite as much as they are actions." This is the first historical instance of penetration into the true nature of musical language.

The meaning of music evolves in terms of physico-physiological correspondences. These correspondences are *quantitative* and the quantities express form. This can be easily illustrated by the following example.

A sound of constant frequency and intensity and made up of a simple wave affects the eardrum and the hearing centers of the brain as an excitor of a simple pattern. Such a pattern may be projected by various means so that its structure becomes apparent to another more developed, and therefore more critical organ of sensation, that of sight. The complexity of reaction (i.e., its form) is equivalent to the complexity of the form of the excitor. The number of components in a wave affects a corresponding number of the arches of the inner ear's Corti's organ, putting them into oscillatory motion. If a sine wave has one component, it will affect only that arch which reacts on the frequency corresponding to that transmitted through the air medium in the form of periodic compressions. When a wave of greater complexity affects the same organ, the reaction becomes more complicated.

It is a known fact that the ear can be trained. Therefore, the pattern of reaction is equivalent to the pattern of excitation with various degrees of approximation. All the components of sound work in similar patterns because these patterns are similar in all sensory experiences. Formation of the patterns is due to (1) *configuration* and (2) *periodicity*. The *configuration* may be simple or complicated in a mathematical sense, i.e., its simplicity or complexity can be expressed in terms of components and their relations. This emphasizes both frequency and intensity in a sound wave, as well as the character of sound which is the resultant of the relations of the two components. *Periodicity* defines the form of recurrence and may be also of different degrees of complexity—for example, the periodicity of recurring monomials as compared to the periodicity of permutable groups.

Our physiological experience, combined with our awareness of that experience through our sensory and mental apparatus, makes it possible for us to understand the meaning of music in terms of "actions." Thus, *regularity* means *stability*, and *simplicity* means *relaxation*. Thus, the satisfied organism at rest is comparable to simple harmonic motion. The loss of stability is caused by powerful excitors affecting the very existence of the organism. Sex and danger are the excitors, and love and fear are the expressions of instability.

*Compare Plato's ideas on the meaning of music in his *Republic* and Ivan Pavlov's experiments with the pitch discrimination of dogs in his *Conditioned Reflexes*. (J.S.)

The awareness of such instability comes through variations in blood circulation sensed through the heart-beat and variations in blood-pressure, resulting in respiratory movements. The whole existence of an organism is a variation of degrees of stability, fluctuating between certain extremes of restfulness and restlessness. The constitution of melody is equivalent to that of an organism. It is a variation of stability in frequency and intensity. Melody expresses those actions we know and feel through our very existence in terms of sound waves.

C. Intentional Biomechanical Processes

We come now to a consideration of *intentional* biomechanical processes. Efficiency of action in relation to its goal is the foundation of evolution. The forms of action by which living organisms adapt themelves to the goal of survival in the existing medium may serve as a fundamental illustration. This efficiency comes about through "instinct" among the lower species, but through the conscious utilization of previous experiences leading to deliberate efficiency among the higher animals. Muscular tension and relaxation constitute the first instruments of such intentional action.

The mechanical constitution of melody varies with times and places, yet its patterns are familiar to us from our own biomechanical experiences.

The "contemplative" and the "dramatic" become two poles of our esthetic reactions. They grow out of the same biomechanical diads: restfulness-restlessness, and stability-instability.

Dramatic patterns themselves evolve out of two sources: the first is fear (defense—dispersed energy) and is caused by danger or aggression; it results in *contraction patterns*. The second is aggression (attack—concentrated energy) and is caused by an impulse or resistance; it results in *expansion patterns*. Confusion of patterns of compression with those of expansion (aberration of perception caused by instability) explains why the very same music sounds "passionate" to one listener but "weary" to another. This is a typical confusion observed by Professor Douglas Moore of Columbia in tests performed on students of non-musical departments at various universities, using Wagner's *Isolde's Love-Death* as material.

All the technical specifications for melodic pattern-making will be given later. The immediate question is: *how does it happen that the physiological patterns are identical with the esthetic patterns?* We can answer this question only hypothetically for we know very little about the technique of pattern formation at present. But as science progresses, we notice more and more correspondences in different fields. We find identical series in such seemingly remote fields as crystal formation, ratios of curvatures in the celestial trajectories, musical rhythms, design patterns, and, finally, in the very molecular structure of matter itself. Modern chemistry shows how by *geometrical* variation of mutual positions of the same group of electrons, entirely different substances are produced. Little as we know for the present about the electro-chemistry of brain-functioning, we may well suspect that all our pattern conception and pattern-making are merely the *geometrical* projection of electro-chemical processes, in the making, that occur in our brain. This geometrical projection is thought itself.

D. Definition of Melody

The summary definition of *melody*:

(1) *Physiological definition:* Melody is an excitor existing in the form of a sound-wave which affects the organ of hearing. The latter being a receiver and a transmitter transfers it to the biomechanical pattern-making center of the brain.

(2) *Semantic definition:* Melody is an expression of biomechanical experiences in the sound medium.

(3) *Musical definition:* Melody is a variation of pitch in time, wherein pitch units follow a preselected scale of frequencies and express a relative stability of each individual unit.

The summary definition of *word*:

(1) *Physiological definition:* Word is an excitor existing in the form of a sound-wave which affects the organ of hearing. The latter, a receiver and a transmitter, transfers it to the concept-making center of the brain.

(2) *Semantic definition:* Word is an abstraction of biomechanical experiences in the sound medium. "Poetic image" is a variation of the original biomechanical abstraction.

(3) *Musical (tonal) definition:* Word is a variation of pitch in time, wherein pitch units express a relative instability of each individual unit and do not necessarily follow a preselected scale of frequencies.

It follows from these definitions: (1) that in symbolic notation (though different patterns are used)—printed letters or musical notes—both word and melody are identical; (2) a poem recited in a foreign or unknown language becomes an undeveloped form of music.

CHAPTER 2

PRELIMINARY DISCUSSION OF NOTATION

BEFORE ENTERING upon the subject of actual computation and construction of melody, there are a number of questions surrounding notational systems that require clarification.

A. History of Musical Notation

The historical evolution of musical notation starts with alphabetical systems of notation of musical pitch. We find this system in ancient Greek notation; the Greeks utilized the characters of the alphabet to indicate intonations. For rhythm they used, among other devices, the rhythmic groups of poetry (*i.e.*, the "foot").

The second step in this evolution brought the use of *neumes*—*i.e.*, indications of musical pitch in the form of graphic symbols. We find evidence of this second step in the Middle Ages. A number of hypotheses have been advanced regarding the source of early medieval notation.*

Byzantine notation from the 10th century to the 15th century evolved a system of ***interval*** indications. This notation, when fully developed, included symbols for an ascending second, a descending second, an ascending third, a descending third, a descending fourth, an ascending fifth and a descending fifth. In the course of a few centuries the symbols gradually modified their appearance, and a new system of representation was evolved.

The first use of horizontal lines (a staff) was devised in the west by Huebald in the 10th century—see his ***De Harmonica Institutione***. The steps on the staff were indicated by the letters "ton" for tone and "sem" for semitone. The words of the text were placed directly on the staff. Only the spaces between the lines of the staff were used.

Guido of Arezzo (who died in 1050) invented the four-line musical staff. Through him we learn also about the origin of most of the present names of musical pitches, these being derived from a hymn to St. John, which the students of a certain monk, Michael, had to sing so that each line would sound one step higher; the first syllables of the lines of the hymn became names for six of the steps

*According to one theory Latin notation was taken from the Hebrew cantillation signs, according to another, from the Byzantine ekphonetic notation. But the most convincing evidence points to the hypothesis that Latin notation derived from the transposition of the signs used for accentuation and punctuation (i.e., grammatical accent-signs) from the vocal text to the melody itself. These grammatical accent-signs may be said to be the source of the Byzantine ekphonetic notation also. The latter did not show the size of the musical intervals, and therefore was useful chiefly as a mnemonic guide. A system of neumes originated in Byzantine notation in the 10th century, but this at first offered little improvement over the ekphonetic notation since it still denoted intervals only approximately.

in our present system of solmisation.* Guido of Arezzo also developed a very complicated system of tone nomenclature for the purposes of solmisation (*i.e.*, "Guido's hand").

Hermannus Contractus, who died in 1054, offered a mixed system of Greek and Latin characters indicating, in terms of tones and semitones, all the intervals of the octave except the augmented fourth and the major and minor seventh.

Another important step in this evolution was the development of a system of notation of musical durations.** The first indications of musical duration represented only two relative durations ("long" and "breve"), to which others were soon added. The 13th century classified music into measured and unmeasured music (***musica mensurata*** and ***musica plana***). The notation of rests also goes back to this early period. Ternary time-signatures originated in the 13th century, binary time-signatures in the 14th.

Our present form of "white" musical notation goes as far back as the 15th century. The evolution of the chromatic signs now in use (sharps, double-sharps, flats, double-flats and naturals) occupied eight centuries, from the 11th century to the 18th. Key signatures did not make frequent use of sharps or flats, except in the one-flat key signature, until the late 15th century.

The system whereby we notate dynamics, attacks and phrasing in graphic symbols and words begins to appear gradually towards the end of the 16th century. This system of notation is seen in such indications as legato, staccato, portamento, crescendo, diminuendo, forte, piano.

Indications of speed and character of motion in words (tempo) came into use (except for an isolated 16th-century example) in the early 17th century. Indications of this sort that are now common include largo, andante, moderato, allegro, presto. We also now have metronomic indications. For example: "MM♩= 96" means that by Maelzel's metronome there will be 96 quarter notes per minute.

Clefs had a gradual evolution from the days when one or more lines were used as part of a rudimentary form of staff notation, during which stage each line was preceded by a letter indicating the pitch. The F, C and G clefs, which are now standard, gradually evolved from the corresponding Latin characters.

Observing the evolution of the notation systems of the past in different musical civilizations, we notice the *casual* character of this evolution. Through continuous trial-and-error attempts, certain forms were improved as compared to their original state; the forms grew more practical. The general trend of this improvement lies in the direction of greater precision of notation.

Early forms of intonation dealt with large groups symbolized by one sign which could be deciphered only by the performers familiar with the conditions

*Ut, re, me, fa, sol and la. (*Ut* is still used in many countries instead of *Do.*)

**Although various tentative methods had been designed to remedy the rhythmic indefiniteness of the neumes, possibly as early as the 8th century, it was not until shortly before 1200 that rhythmic values were definitely established in notation.

and conventionalities of a given musical locality or style. The final forms used today offer an abominable conglomeration of languages, systems and symbols, often mutually exclusive and contradictory. Various innovations which piled up on top of the old forms, symbols and systems, produce extreme confusion.

The evolution of musical notation started on the wrong track from the very beginning. As music was closely associated with words, linguistic forms became the more influential in determining the system of musical notation. While the *neumes* have a directional association with a pitch line the early specimens, unfortunately, are too vague to define such a line with the precision necessary if the line is to be universally deciphered. Music written in such *neumes* could be read only by people who knew their exact intonational content in advance.

The forms of musical notation in Europe developed in correspondence with ecclesiastic forms of music and not with the secular music of a dance-song; the notation of durations evolved in reference to music of the utmost rhythmic simplicity. We are even today handicapped by the arithmetical crudeness and inefficiency of our notation of durations. Any music which does not derive from the ecclesiastic forms of European music looks forbiddingly complicated on our musical staff.

It is easy to demonstrate the inefficiency of our musical notation whenever we have to perform a rhythmic group new to our ear. Few performers can read it by sight; only by discovering a *familiar* rhythmic pattern in the musical notation can one execute it without delay. Since we face difficulties in overcoming the conservatism of our muscular and sensory habits, the best we can do is to use an efficient system of notation.

Our present system of notation is entirely fictitious with regard to pitch. The only fields in which it gives the true correspondence of the intonation of the music performed is when the music is for piano or organ. The actual differences of intonation in *a cappella* singing or in instrumental chamber music playing *leave no trace* whatever in our musical notation. The pitch discrimination ability of humanity has always been in advance of the present system of musical notation.

With the development of greater refinement in the execution of music as to intonation, rhythm and other forms of expression, and with the development of greater precision of thought by composers as to all the detailed specifications for the performance of music—a reliable, precise and versatile system of notation becomes an utmost necessity. In musicological research and comparative musicology, study is hindered and made difficult by the system of musical notation now in use. An efficient system of musical notation must be universal enough to express *any* time duration in *any* form and relationship, *any* form of intonation pertaining to *any* type of tuning system, *any* form of relative intensity, attack and variation of tone quality. Such a system may be developed only on a strictly geometrical basis, the foundation of which lies in the graphic projection of physical phenomena.

The scientific system of recording known as *nomography* deals with different methods of graph notation. While various forms of recording events scientifically exist in all statistical fields, music continues its semi-happy existence in a state of affairs in which nothing can be too wrong—and nothing can be too right! Centuries of the isolation of music from science brought about this unfortunate and chaotic situation. It is about time to acknowledge the inefficiency of our system of musical notation and take a grown-up attitude towards a field which is now unfortunately a back-yard of human thought.

B. Mathematical Notation, General Component

1. Notation of Time.

Measuring musical time from a minimum standard unit was known both to the Greeks (*chronos protos* = primary time) and the Romans. For the reasons presented in the preceding section, this usual way of direct measurement adopted in various other fields did not survive in musical notation.

Assuming that the *shortest* duration of any given musical continuity is to be the standard unit of measurement, any degree of precision may be achieved. In terms of musical notation, this means that if musical continuity includes musical halves, quarters, eighths and sixteenths, one should then express *all* the durations as sixteenths. The standard unit of measurement is the common denominator of various durations occurring in one musical context. We shall express such a unit as "t". When still greater subtlety is required, we shall use "τ" (tau) to mean a unit of deviation. This symbol will be useful in expressing somewhat unaccountable durations, such as individual grace-notes and groups of grace-notes. There also may be a need for applying τ to various forms of unbalancing—groups customarily designated by means of so-called "rubato." Every bar representing a group will always correspond to unity and will be expressed through

$$T = \frac{t}{t} = 1$$

If we want to achieve musical performances that possess in reality the subtlety they claim to have, the expression of the bar would require the inclusion of τ. The latter corresponds to the infinitesimal (dx) of the calculus.

In the new system of notation every time value becomes a rational fraction, the numerator of which expresses the period of duration, and the denominator of which expresses the standard unit of measurement ($\frac{1}{t}$). As problems of musical duration involve not only the values but also their mutual position and distribution in time continuity, it is necessary to introduce a nomenclature which will also take care of the distributive characteristics of durations. For example: ♩. ♪♪♪ $= 3 + 1 + 1 + 1$. As the common denominator in this case is $\frac{1}{6}$, the values acquire the following expression: $\frac{3}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$. If we wish to refer to the third term of this group, such a quantitative indication is not entirely sufficient. The last three terms have uniform values, so they must acquire a special system of indications as to their *place* in the time continuity. As each "t" represents a term, the consecutive enumeration of such terms will be expressed by $t_1, t_2 \ldots t_n$ to indicate their relative position, and the coefficients will express the actual time values. The preceding example will thus acquire the following appearance:

$$\frac{6}{6} = \frac{3t_1}{6} + \frac{t_2}{6} + \frac{t_3}{6} + \frac{t_4}{6}$$

This system of notation permits any form of distributive variations in a given continuity without running the risk of losing any corresponding placement of any given term. For example, $\frac{t_3}{6}$ may be placed in the first position in a new variation group. The previous continuity will acquire the following appearance:

$$\frac{6}{6} = \frac{t_3}{6} + \frac{3t_1}{6} + \frac{t_2}{6} + \frac{t_4}{6}$$

This means that the original third term now appears in the first place in the variation group.

The study of durations and the composition of continuity from the latter is, of course, the subject of my theory of rhythm.*

C. SPECIAL COMPONENTS

2. Notation of Pitch.

Systems of intonation used as material for music constitute the *primary selective systems.* Such systems may be uniform or non-uniform. Non-uniform systems are characterized by variable ratios between the adjacent pitch units. The series of natural harmonics is a non-uniform selective system; the intervals between the pitches contract progressively, producing a natural logarithmic series. This corresponds to the perspective contraction in optics. The systems of uniform ratio in music are known as ***equal temperament***, and they express different forms of symmetry in the range of one octave. One octave is merely the ***simplest*** ratio; analogous systems of pitch symmetry might be evolved from any other ratio conceived as a range of emphasis. Our musical civilization deals with pre-arranged selective system of pitch known as the equal temperament of twelve ($\sqrt[12]{2}$).

The ***secondary selective systems*** constitute all the distributive scales *within* a primary system. The standard unit of pitch measurement, "p", becomes the symbol of a pitch unit within the equal temperament of twelve. Thus, "p" expresses a semi-tone and acquires the value of the unit expressing the logarithm to the base $\sqrt[12]{2}$. The limit integer within the octave (12p) is 11. All other intervals are expressed through respective integers. Any initial pitch represents a starting point of zero. Thus, a semi-tone from 0 equals 1, a whole tone equals 2, etc.

As this does not specify the direction or pitch from the 0 (zero) point, an additional system of indications is required. In view of this need, two methods may be offered: one, to consider all points above 0 as positive and all points below 0 as negative; two, to introduce a system of axes so that with the specification of an axis a certain direction becomes positive or negative. Moving in one direction produces either positive or negative values only. Movement in both

*See Book I.

directions produces an alternation of positives and negatives. For example, the progression c – d – f – g acquires the following notation: 2p + 3p + 2p. The figure c – f – d – g – c has the following notation: 5p – 3p + 5p – 7p.

As T expresses a time group-unit in relation to t, which is the common denominator of the group, P expresses a pitch group-unit (pitch range) in relation to p, the standard unit of pitch measurement in a given primary selective system.

Pitch ranges become important when they are treated as sections of the total range emphasis of a given musical continuity. In such a case, each pitch range corresponds to a certain axis, and the total value of the pitch units within one axis depends un the total value of all axes within the entire range. For example, if a melody evolves in a range of 15p (c – e'♭) and three axes are required, then each axis will emphasize $\frac{15p}{3} = 5p$, i.e., the partial ranges of the total range will be $P_1 = 5p$, $P_2 = 5p$, $P_3 = 5p$ (c – f; f – b♭, b♭ – e♭).

3. Notation of intensity.

In order to establish any system of notation for intensity, we must consider a fundamental fact basic to the psycho-physiological law of Weber-Fechner: that the intensity of reaction does not vary as the intensity of stimulus; for it grows with an increase of about 15 percent in relation to the physical intensity of the stimulus. For example, when we double the amplitude of a sound wave we obtain a reaction in the ear that is only 15 percent and not 100 percent greater.

This means that the difference between very low and medium intensities appears to be much greater to our ear than does the difference between medium and high intensities. For instance, the difference between 5 and 40 decibels seems to be much greater to our ear than the difference between 40 and 75 decibels. This, obviously, is one of the psycho-physiological limitations developed for the protection of the species.

In the future, with the appearance of instruments performing music automatically, any precise mathematical specification will be possible and could be offered in any desirable type of physical correspondence.

For the present, our system of notation has to be limited in exactly the same fashion as it is limited for the expression of durations and pitch. We have to establish a certain *range* of loudness, as conceived musically, (in a given epoch and locality), and assume the lowest degree of it as one unit of intensity. Thus, if we would like to establish three important points of intensity and enumerate them as i_1, i_2 and i_3, their respective values of intensity will be i, 2i, 3i. $i_0 = 0i$. The rests (periods of silence) will be expressed by $i_0 = 0i$. These three degrees of intensity may correspond to *piano*, *mezzo-forte* and *forte* respectively.

The method of denoting intensity by minimum units is more precise, for we can establish a scale of dynamic marks of greater subtlety and precision than by using the method which expresses all this in Italian words. Thus, a selective scale of 2 degrees of intensity ($i_1 = i$, $i_2 = 2i$) may correspond to *piano* and *forte*. A scale of 3 degrees of intensity ($i_1 = i$, $i_2 = 2i$, $i_3 = 3i$) may correspond to *piano*, *mezzo-forte* and *forte*. A scale of 5 degrees of intensity ($i_1 = i$, $i_2 = 2i$, $i_3 = 3i$, $i_4 = 4i$, $i_5 = 6i$) may correspond to *pianissimo*, *piano*, *mezzo-forte*, *forte*, *fortissimo*.

One may devise scales with many more degrees of intensity when the complexity of indications through i remains the same. If, however, we used the Italian words or their musical abbreviations, it would all become quite confusing. The main reason for this confusion is not only the quantity of words employed to indicate the various degrees of intensity, but the lack of an objective scale of intensity. For instance, it is very far from obvious just what relation of intensity *mezzo-forte* has to pianissimo; but in the scale of 5 intensity units, it conveys purely quantitative $i_3 = 3i$ through $i_1 = i$ associations.

3. Notation of Quality.

Musical conception of tone *quality* emphasizes physically such different factors as harmonic saturation (density), duration of tone, form and intensity of attack, etc. In denoting quality we shall consider only the first factor. A one-component wave is the minimum limit of harmonic saturation—and the oboe-like quality (with predominant 5th, 7th and 13th harmonics) is the maximum limit of harmonic saturation, as it appears to our ear. The entire range emphasizes qualities from the flute stops of an organ ("tibia clausa") up to pure reed stops (like the "English Horn"). The intermediate quality zone embraces such tone qualities as clarinet and violin. Tone quality may also be illustrated by means of vowels: the minimum limit is "oo", the maximum, "ee", with the intermediate zone around "o" and "a".

Using the same system of notation as previously (time, pitch and intensity) and indicating quality through "q", we obtain scales with a different number of quality points. Thus, a 2 unit scale—$q_1 = q$, and $q_2 = 2q$—can express relative harmonic saturation in relation to the limits selected. A 3 unit scale consists of $q_1 = q$, $q_2 = 2q$, $q_3 = 3q$. A 5 unit scale: $q_1 = q$, $q_2 = 2q$, $q_3 = 3q$, $q_4 = 4q$, $q_5 = 5q$.

These instrumental media for achieving variation of tone color are the subject of my later discussion of the acoustical basis of orchestration.

D. Relative and Absolute Standards

Notation methods similar to the method applied to the differential deviations in recording musical time must be used when *subtle* deviations from the established scales of pitch, intensity and quality are to be handled.

It is impractical, in the present state of music, to deal with differential equations so long as performers are human beings confined in their interpretation to crude arithmetical limitations. When greater subtlety of performance is required, the respective differential values for the time, pitch, intensity and quality components will be: dt, dp, di and dq. Thus, the existing state of music and of musical notation, and the conception of any recording of a musical composition by means of notation—all this presupposes both scalewise (discontinuous) and differential (continuous) constitution of music. For example, in a "gradual" increase of intensity on the piano, the physical reality of it is a group of intensified points with quick drops, with every succeeding attack greater than

the preceding one and every attack having the characteristic of a constant form of fading (decrease of intensity). On the contrary, when they deal with pitch—which in ordinary musical notation is always discontinuous—the performers of our civilization as well as of the Oriental cultures, actually produce a *differential curve* of frequencies very often.

The actual difference between Chinese singing and Hungarian violin playing is the quantitative difference of time that elapses between the stabilized pitch points of the scale—the time period is longer and the attack is stronger with the Chinese. Minute variations of tone quality are often beyond the control of a performer. Often even a very experienced performer, in trying to produce one tone quality, actually obtains another. How many *unintentional* harmonics break through on account of a wrong angle or wrong pressure of the bow over the strings in violin playing! How many performers on stringed bow instruments produce the jumping effect (saltando) instead of the intended smoothly repeated attacks (portamento) because of mere nervousness! The adoption of the manifold of mathematical resources for musical notation would seem ridiculous for the present when the requirements are so low that deviation from a proper set of time durations and proper intonation is a very common sin.

All forms of musical notation must deal with the expression of *relative* quantities only. Absolute standards of pitch, intensity and quality vary during different epochs. Even the absolute speed of time is somewhat affected. The general tendency toward faster *tempi*, as compared with those used in the 17th century, becomes quite apparent. This is most probably influenced by the general acceleration of vehicular motion and general development of engineering technique. Many performers now interpret some of the classical music of the 18th century and 19th century at much faster *tempi* than would have been desirable or even possible at the time this music was written.

Pitch standards have also had a tendency to accelerate. At the time of Haydn and Mozart, *a* of the middle octave generally had 422 vibrations per second, while the concert tuning of the corresponding tone of today ("American concert pitch") is 440.6, a change of more than one and a half semitones. These variations of frequencies, with regard to absolute pitch, are decided at various international conferences of acousticians and manufacturers of musical instruments. The application of pitch *ranges* also grows. Take, for example, the range of the violin, where g of the small octave being the lower limit, we notice a constant extension of the upper limit: it was c of the third octave during Beethoven's time, e above it with the early Wagner, g# above it with the later Wagner, and b above it with Rimsky-Korsakov (as in *Kitezh*).

One century produced a gain of one complete octave in extending the range of the violin. The desire to obtain greater *tension* effects leads to the employment of higher frequencies; it implies a growth of virtuosity in playing musical instruments. Paganini was about the only person in his time who was capable of playing some of his most difficult works for the violin. Nowadays, however, any capable student of a violin department in the conservatories is able to play them. Today in America, we witness an extraordinary virtuosity in extending the range of such instruments as trumpet and trombone; sometimes the gain is a whole octave beyond the standard range.

The range of intensity also grows because of a desire for production of greater dynamic contrasts and a desire to obtain extreme intensities. One of the causes may be the amount of noise in the big cities of today; it is necessary to be loud in order to stand out amidst noisy surroundings. At the time of Bach few dynamic marks existed. At the time of Beethoven the dynamic expression had to be guided by the conductor's discrimination—the dynamic marks referred ***not to the performers***, but to the listeners. When Beethoven used one or another dynamic mark (***piano*** or ***forte***), he meant that the corresponding degree of intensity would be ***heard by the audience***. Nowadays, however, dynamic marks are used ***for the performer***. The composer now assumes the responsibility of producing the ***total of dynamic balance*** by marking the individual parts in a score in a different way. For example, in order to balance trumpets with clarinets in music which must sound quite ***loud***, the trumpets may have an indication of ***mezzo-forte***—while the clarinets are marked ***fortissimo***. This trend toward producing very loud sounds, so fashionable with the Italian ***bel canto*** in the 19th century, was transferred to the instrumental field in the 20th century.

As to tone quality, there is also a noticeable tendency towards the increase of the quality range. There are many new mutes devised for the brass, producing a considerable variety of tone quality variations; various semi-mechanical instruments (organs) have been built with a tremendous number of stops, which provide different tone qualities. The development of electronic sound production leads to a variety of tone qualities that would have been beyond the imagination of any composer of the past. Some of the models of today place ***more than a billion*** different tone qualities at the disposal of the composer and the performer.

Thus we notice a definite tendency towards the expansion of the range within different sound components, as well as a definite tendency of acceleration. As the absolute standards for any established average tempo, average range for a melody, average range for intensity and tone color are not invariants, all this necessarily affects the system of relations within the above-mentioned component. The notation of relative, not absolute, values is the more important one for the purposes of composition and reproduction of music.

E. Geometrical (Graph) Notation

The adoption of the ***graph*** method for the recording of musical composition and performance has obvious advantages over the present system of musical notation. In the first place, it offers as much precision as is desired; in the second, it stimulates direct associations with the pattern of a given component.

A physical record of what is audible, such as an oscillogram or a photogram of a sound track, is too complicated to be used as musical notation. But the geometrical notation offered in this theory is the general method of graphs, the same as that commonly used for the statistical recording of events, i.e., a record of the variation of special components in time continuity (general component).

Graph notation records individual components through individual curves. The special components of sound are frequency, intensity, and quality—and they may be recorded through the corresponding individual graphs. By means of such notation the composer can define his intentions with the utmost precision; the performer can then decipher the desires of the composer to the latter's full satisfaction.

In the future, with the elimination of the living performer, the graph method will still be valid for use with automatically performing musical instruments. Curves of composition and curves of execution will then merge into one.

The horizontal direction (the *abscissa*, read from left to right) expresses time in all graphs; the vertical direction (*ordinate*) expresses variation of some special component: pitch, intensity, quality. The graph method is an objective one and is therefore a general method. Any wave motion records itself automatically.

The units on cross section paper to be used for a graph recording of music represent the standard units of measurement with respect to the units of preselected pitch, intensity and quality scales. The best graph paper to use is that ruled 12 x 12 per square inch; the reason for this is the versatility of the number 12 with respect to divisibility and the definition of an octave of the equal temperament of twelve for the pitch.

The scales referring to different components may be different in quantity. For example, a scale of pitch may conform to 7 units while the scale of intensity in the same music may conform to 3 units, and the scale of quality to 2 units. This will be reflected respectively in the complexity of the corresponding graphs.

Graduality or suddenness of transition from one stabilized point to another is expressible by a definite degree of curvature. Variation of pitch in the asymmetric tuning system may be recorded on logarithmic graph paper. The logarithmic contractions of abscissa and ordinate were described in Book Three.* Notation of pitch variation of actual violin playing assumes hyperbolic curvatures. The pitch-graphs of piano or organ are rectangular.

The customary conception of melody in the Western World is based on a *rectangular* conception of pitch, i.e., all the sliding between the stabilized tempered pitch-points is left to the discrimination of the performer. Because of this fact, different styles of interpretation reveal different degrees of curvature of a melodic line. Continuous uniform sliding, without any stabilized pitch-points, may be observed in the sound of a siren or of a fire alarm signal; extreme abruptness (rectangular graph) is found on the piano or organ; intermediate forms (hyperbolic graph) are found on stringed bow instruments, woodwind instruments, in the human voice and in the space-control theremin.

In the following exposition only rectangular graphs are used, as the different degrees of curvature refer to the performance and not to the composition of music; such curvatures are to be discussed in my theory of interpretation.

One vertical segment on the graph paper expresses a unit in the corresponding equal temperament, a $\sqrt[12]{2}$ in the case of the present-day system. The intensity curves may be used either as continuously sliding curves or as rectangular curves with the stabilized levels expressing definite predetermined degrees of intensity. The same refers to tone quality graphs: for those instruments which are capable of producing continuous quality variation and are controlled by a graduated scale (such as some of the electronic instruments), the graph is curvilinear; but for those instruments capable only of abrupt (discontinuous) transitions, only a rectangular graph is necessary.

*See pp. 211-2.

CHAPTER 3

THE AXES OF MELODY

WITH THE conclusion of the foregoing discussion of the philosophical setting of the problem of melody and of the notational problems of all music, we are now in a position to approach the actual technology of making melodies.

We are concerned first with two kinds of axes: primary axis and secondary axis.

A. Primary Axis of Melody

Definition: Primary axis is a pitch-time maximum.*

In order to determine what is the pitch-time maximum in any given melodic continuity, sum up all the pitch levels occurring in the continuity; then establish the pitch which has the greatest number value as the *primary axis of the melody.*

Our ear and our auditory consciousness apprehend music in *portions* of continuity. When the preceding portion fades out we hear the new one evolving. The time values absorbed by the memory while one is listening to music varies with the structural constitution of the melody. Rests, ties, accents, and other signs of musical punctuation—all dissociate the portions of musical continuity in our consciousness.

The location of the *primary axis* is therefore relative to the amount of continuity retained by our memory. While concentrating our attention on melody-in-the-making within, let us say, two seconds of emphasis, we detect one primary axis; but during twenty seconds of the same continuity, our attention may be directed towards an entirely different pitch center.

Our musical memory selects the primary axis through its reactions to a quantity of repeated excitations produced by certain frequencies.

Musical orientation is based on the relations of a melodic configuration to its primary axis, without which melody does not produce any musical reality. A melody without an axis seems "not to hold together," to have no comprehensible structural constitution. When some of our contemporary composers reject the idea of the primary axis (consciously or unconsciously), they revolt not only against musical traditions but against the laws of nature as well. So-

*As with other Schillinger concepts, the idea of the primary axis serves a double function. From the standpoint of musical composition, it is the point of reference around which the construction of melodies fluctuates. Without the concept of a primary axis as a starting point, no scientific approach to melody making would be possible. In addition to the technological function, the concept of the primary axis also serves a critical function. It can be used in the analysis of music. As Schillinger indicates, melodies lacking a clearly defined primary axis lack certain qualities, just as melodies containing rapidly changing axes possess others. Thus, the concept of the primary axis is of practical value, and not merely a verbalism. This is true of all Schillinger concepts, such as the secondary axes of melody, forms of trajectorial motion, etc. (Ed.)

called "atonality", i.e., the *neutral* distribution of pitch units within a given tuning system in various arrangements in order to produce a melody, does not make any "musical", i.e., *organic*, sense. Listeners usually object to such music and they are perfectly justified in doing so.

Inasmuch as counting pitch units in their time continuity does not present any difficulties, no properly constructed melody will ever leave room for any doubt as to where its primary axis is located. In analyzing music on a geometrical basis, one comes across a number of inconsistencies even in the well-known themes of important composers of the past. If they had known the mechanical specifications of melody, their intentions would have been clearer both to themselves and to their listeners. Such partly deficient melodies create certain difficulties when analyzed.

Another case—which may seem doubtful only at first (when a student has not acquired enough analytical experience)—is that in which the primary axis is *variable*, i.e., when a melody, after being centered on a definite primary axis, deviates from it for a considerable period of time and establishes a *new* primary axis from which it may proceed further on in a similar fashion. Such a case involves modulation. As a winding plant, such as ivy, stretches from one branch to another, winds around its coils—and, when it grows out the length of the respective branch, stretches to the new one, so in music an analogous case would be the use of modulation as an outcome of excessive tonal stability.

In geometrical notation, primary axis is the abscissa itself, i.e., a continuous horizontal extension. *The primary axis is the only axis which actually sounds.* All the secondary axes merely represent *directional* lines.

We shall analyze now the three characteristic types of melodic structures with respect to their primary axes. The three melodic themes are taken from Ludwig van Beethoven's Piano Sonata No. 8, the *Pathétique.*

B. Analysis of Three Examples

1. The first 8-bar melody is the beginning of the First Allegro. The graph of this melody, as shown in figure 1 on the following page, has its primary axis on the c located on the third space in the treble, where it accumulates a total of 18t. All the other levels do not withstand the competition, as the greatest number value on them does not exceed 6t. Musical analysis does not provide such precision. In the case of this melody, there would be three competing pitch levels: middle c, third space c, and c on the second leger line. Geometrically the lower one accumulates 6t and the upper one 4t, leaving the middle one (18t) without doubt. Musically, all the three c's are the official tonics of the scale. According to the key signature, the melody is written in c minor. Thus, the importance of the axis is greater than that of the tonic.

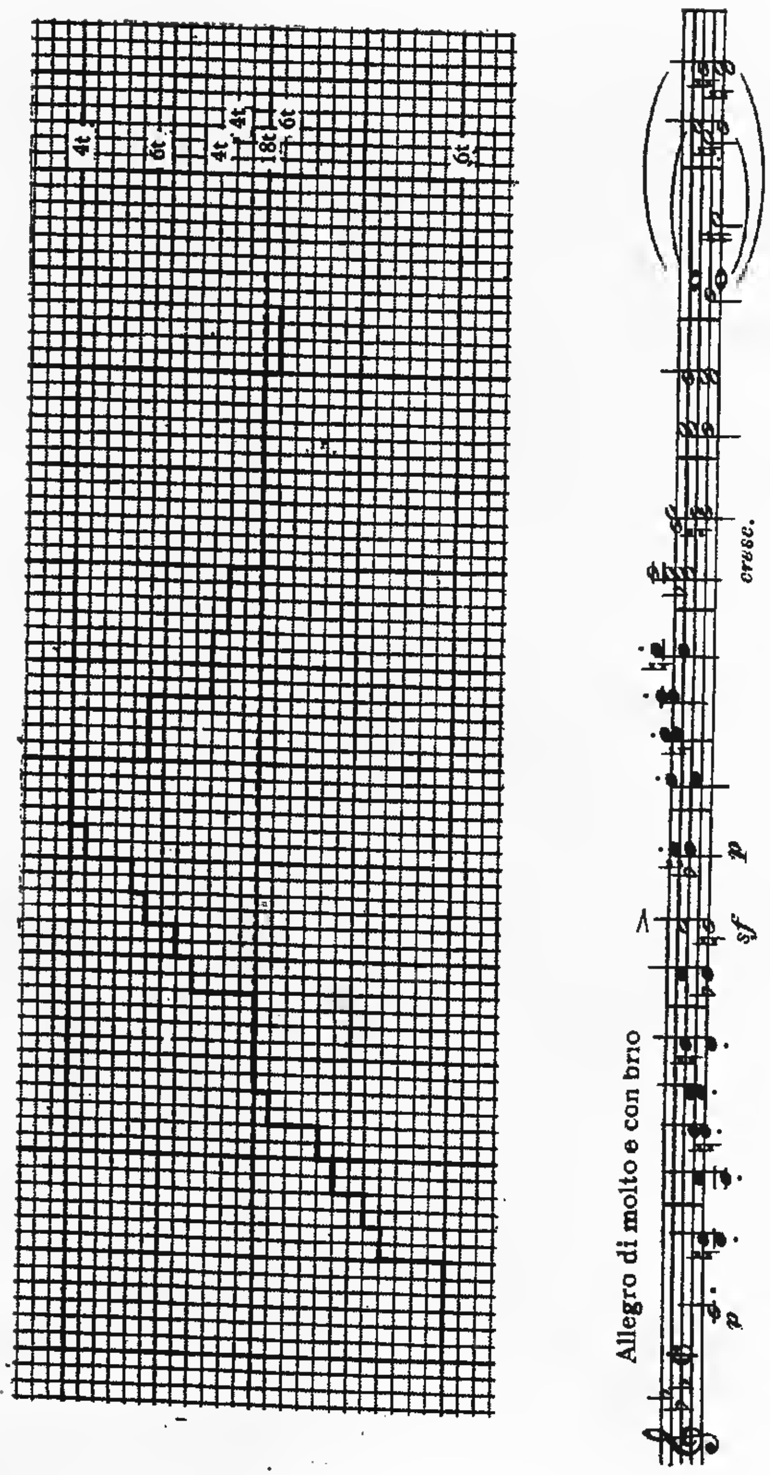


Figure 1. Graph of Beethoven's Pathétique, 1st Movement (Allegro).

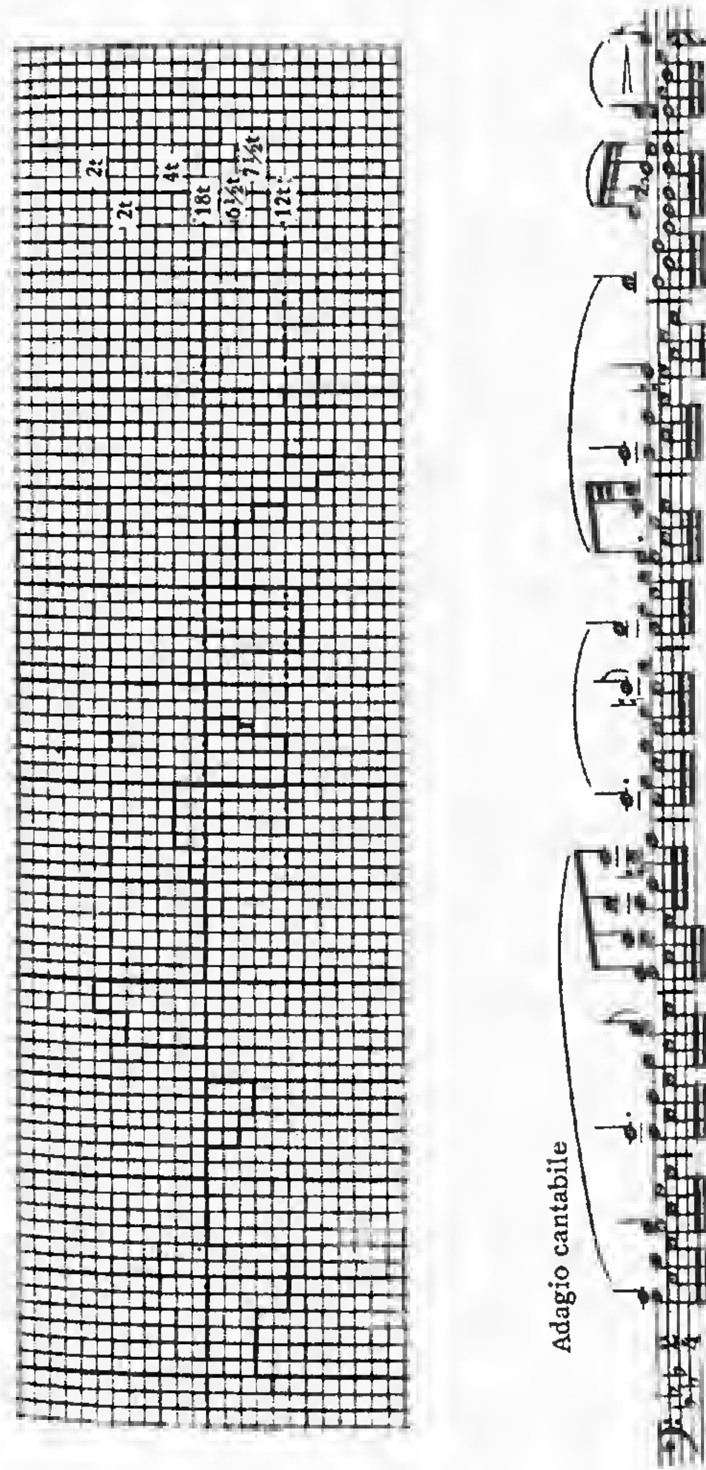


Figure 2. Beethoven's Pathétique, 2nd Movement (Adagio).

2. The first 8-bar melody of the Second Movement of the same Sonata (see figure 2 on preceding page) serves as an illustration of modulation. This melody evolves along the two consecutive primary axes. The first one has 18t, the second—12t. As the melody does not return to the first level, modulation and the establishment of a new primary axis become necessary. If we subtract from the first primary axis the total duration which appears on it after the melody makes a transition to the second primary axis, we obtain 18t − 4t = 14t for the first primary axis. Subtracting the total duration of the second primary axis while the melody adheres to the first primary axis, we obtain 12t − 4t = 8t for the second primary axis. Thus, the first axis level amounts to a total of 14t, and the second primary axis amounts to a total of 8t. From a musical viewpoint this melody, according to its key signature, is written in A♭major. After detection of the two primary axes, we find that in reality this melody evolves in the Mixolydian scale in its first portion, then modulates and establishes itself in the Dorian scale. (See page 249)

3. The first 8-bar melody of the Final Movement of the same Sonata (see figure 3 on following page), illustrates a case of wrong proportions, which may look doubtful to the beginner. There are two competing pitch levels, each amounting individually to 15t. Observing more closely the form of this melody, one finds that the lower axis deviates a number of times from its level, appearing near the beginning and at the very end (besides the four other points). The upper axis appears on the same level in two portions, both near the center of the entire graph. This construction reveals that the melody is actually "attached" to the lower axis, which thus becomes the primary axis. The upper axis being equally important but not important enough to retain the melody centered around it, represents an hypertrophied climax. This melody can be improved by being deprived of part of its overgrown climax. By shifting the entire graph 4t to the right, we would acquire the beginning of the climax on the downbeat of the 4th bar; by taking out the last 4t of the same level, we would improve this melody and still let it end at the same time moment as in the original, i.e., on the downbeat of the 8th bar. (See page 251)

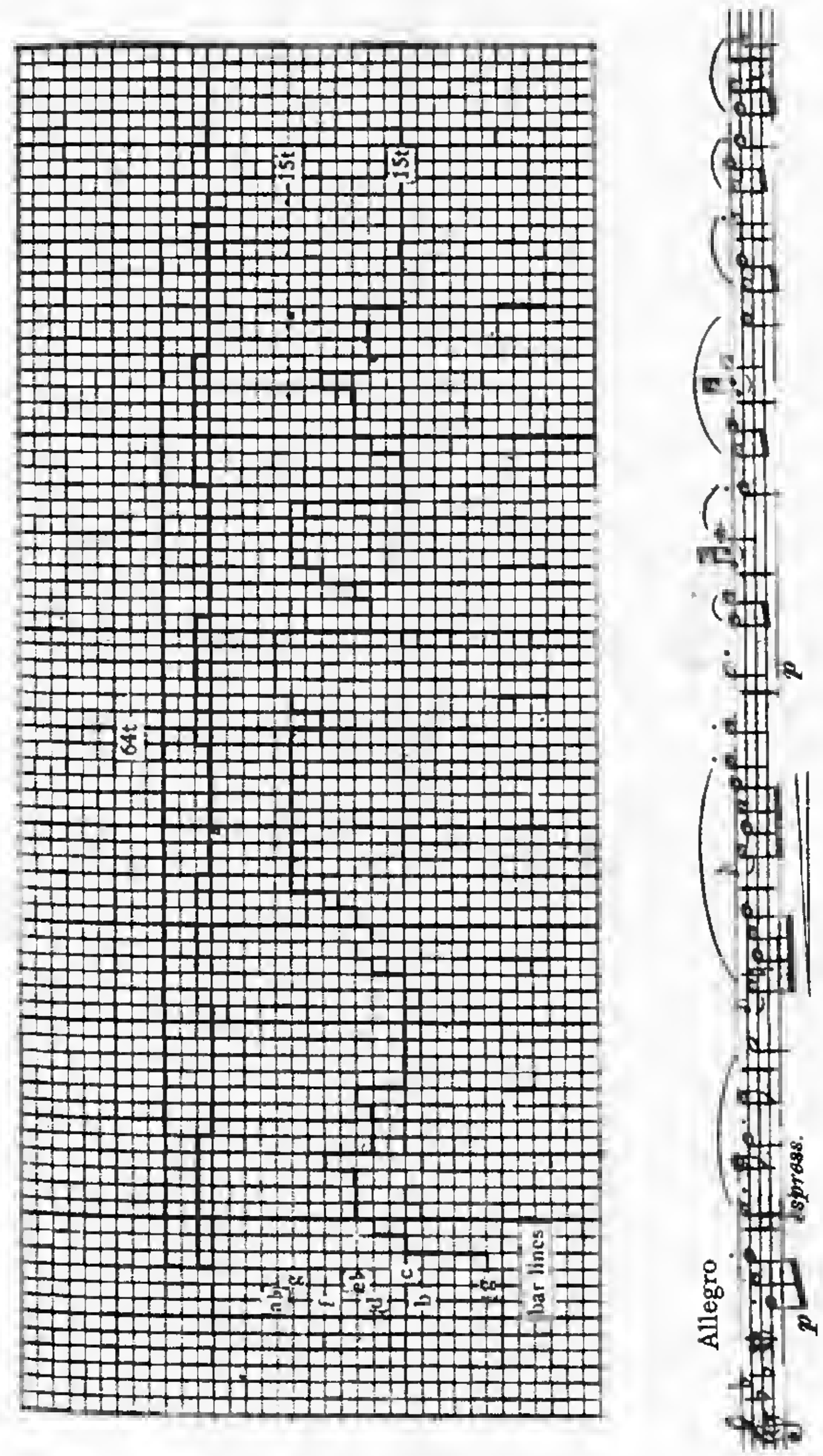


Figure 3. Beethoven's Pathétique, 3rd Movement (Rondo).

C. Secondary Axes

Definition: Secondary axes are the directional axes with respect to the primary axis.

1. The zero axis (0)
2. The "a" axis (a)
3. The "b" axis (b)
4. The "c" axis (c)
5. The "d" axis (d)

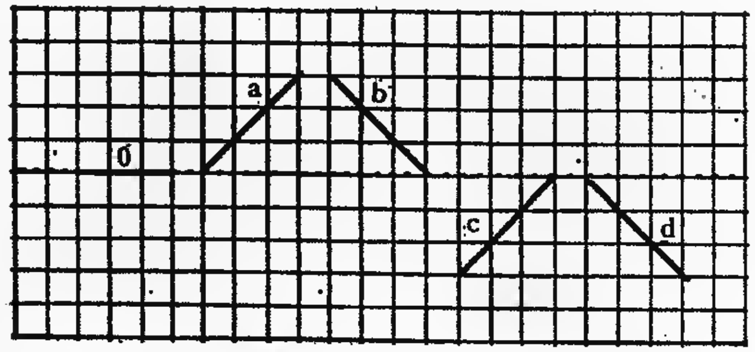


Figure 4. Secondary axes.

The zero axis is the direction of motion along abscissa. The a axis is the ascending direction from the primary axis. The b axis is the descending direction toward the primary axis. The c axis is the ascending direction toward the primary axis. The d axis is the descending direction from the primary axis. The a, b, c and d axes are mutual geometrical inversions obtained by revolving the a axis through the quadrants around the ordinate and the abscissa in an 180° angle. Thus, b represents the backward motion of a; c the backward upside-down of a; d the forward upside-down of a.

The zero axis represents an absolute balance. The balancing axes (leading toward balance) are b and c. The unbalancing axes (leading away from balance) are a and d.

Balancing Axes *Unbalancing Axes*

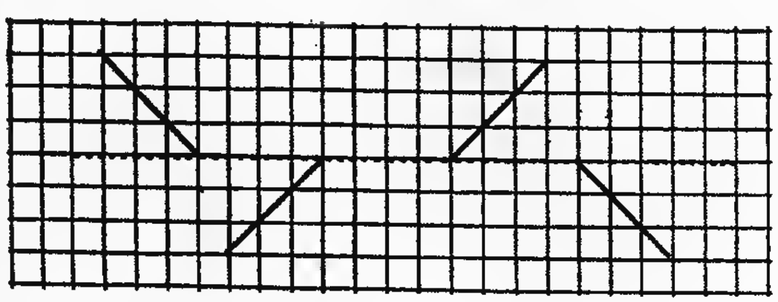


Figure 5. Balancing and unbalancing axes.

The unbalancing axes are characteristic of beginnings. The balancing axes are characteristic of endings. The zero axis is characteristic of the beginning before the motion acquires inertia, or of the ending when all the inertia is exhausted.

Every melody represents a combination of different directions as expressed by 0, a, b, c and d axes. Various combinations of axes produce various forms of melodic continuity. The unbalancing axes produce the effect of tension, the balancing axes produce the effect of release. As the zero level represents zero tensions, the increase of tension grows with the increase of distance from the primary axis.

Composition of melodic continuity, with respect to pitch and time, may be based on monomial, binomial, trinomial and polynomial combinations of the secondary axes.

D. Examples of Axial Combinations

1. Monomials

0 + . . .
a + . . .
b + . . .
c + . . .
d + . . .

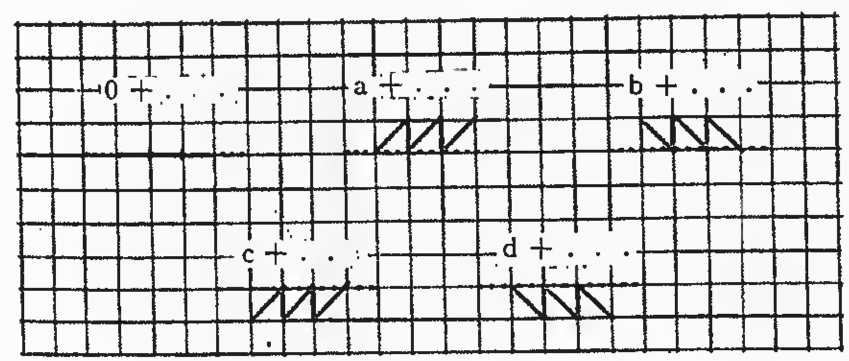


Figure 6. Monomial axial combinations.

2. Binomial Combinations

0 + a

0 + b
0 + c
0 + d

a + b
a + c
a + d

b + c
b + d

c + d

10 combinations, 2 permutations each.
Total number of cases: 10 × 2 = 20.

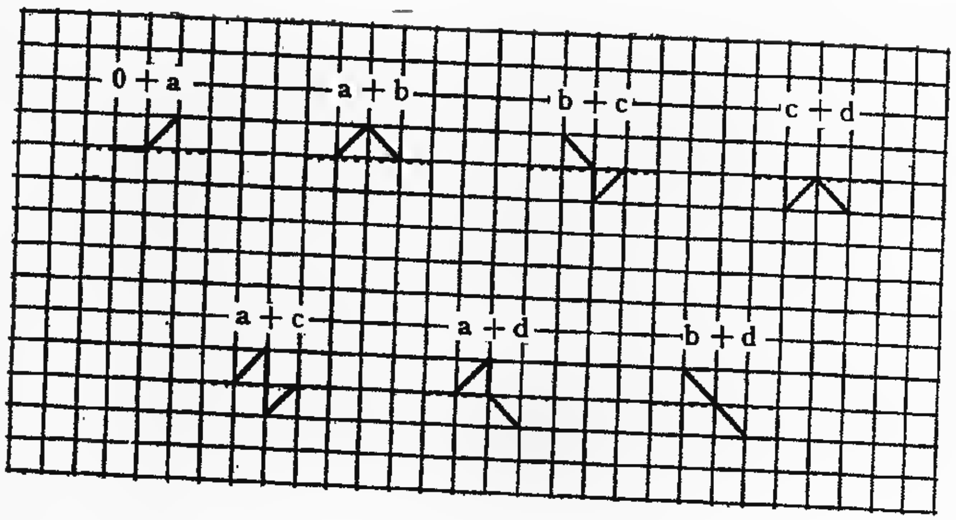


Figure 7. Binomial axial combinations.

3. Trinomial Combinations. Two identical terms.

0 + 0 + a
0 + 0 + b
0 + 0 + c
0 + 0 + d

a + a + 0
a + a + b
a + a + c
a + a + d

b + b + 0
b + b + a
b + b + c
b + b + d

c + c + 0
c + c + a
c + c + b
c + c + d

d + d + 0
d + d + a
d + d + b
d + d + c

20 combinations, 3 permutations each.
Total number of cases: 20 × 3 = 60.

3 different terms:

0 + a + b	a + b + c
0 + a + c	a + b + d
0 + a + d	a + c + d
0 + b + c	b + c + d
0 + b + d	
0 + c + d	

10 combinations, 6 permutations each.
Total number of cases: 10 × 6 = 60.

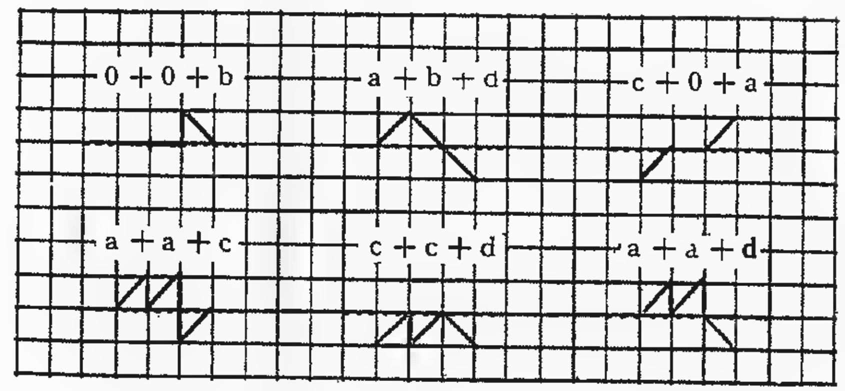


Figure 8. Trinomial axial combinations.

4. Quadrinomial Combinations

4 places with 3 identical terms:

0 + 0 + 0 + a
0 + 0 + 0 + b
0 + 0 + 0 + c
0 + 0 + 0 + d

a + a + a + 0
a + a + a + b
a + a + a + c
a + a + a + d

b + b + b + 0
b + b + b + a
b + b + b + c
b + b + b + d

c + c + c + 0
c + c + c + a
c + c + c + b
c + c + c + d

(continued)

d + d + d + 0
d + d + d + a
d + d + d + b
d + d + d + c

20 combinations, 4 permutations each.
Total number of cases: $20 \times 4 = 80$.

4 Places with 2 identical pairs:

0 + 0 + a + a
0 + 0 + b + b
0 + 0 + c + c
0 + 0 + d + d

a + a + b + b
a + a + c + c
a + a + d + d

b + b + c + c
b + b + d + d

c + c + d + d

10 combinations, 6 permutations each.
Total number of cases: $10 \times 6 = 60$.

4 Places with 2 identical terms:

0 + 0 + a + b
0 + a + a + b
0 + a + b + b
0 + 0 + a + c
0 + a + a + c
0 + a + c + c
0 + 0 + a + d
0 + a + a + d
0 + a + d + d

0 + 0 + b + c
0 + b + b + c
0 + b + c + c
0 + 0 + b + d
0 + b + b + d
0 + b + d + d

0 + 0 + c + d
0 + c + c + d
0 + c + d + d

a + a + b + c
a + b + b + c
a + b + c + c
a + a + b + d
a + b + b + d
a + b + d + d
b + b + c + d
b + c + c + d
b + c + d + d

a + a + c + d
a + c + c + d
a + c + d + d

30 combinations, 12 permutations each.
Total number of cases: $30 \times 12 = 360$.

4 different terms:

0 + a + b + c
0 + a + b + d
0 + a + c + d

0 + b + c + d

a + b + c + d

5 combinations, 24 permutations each.
Total number of cases: $5 \times 24 = 120$.

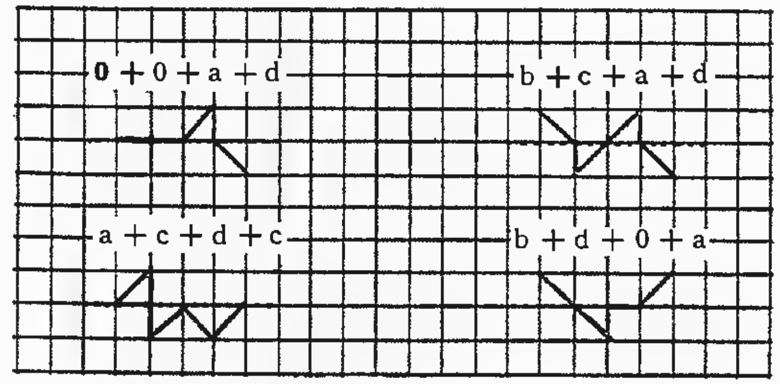


Figure 9. Quadrinomial axial combinations.

5. Quintinomial Combinations

5 Places with 4 identical terms:

0+0+0+0+a
0+0+0+0+b
0+0+0+0+c
0+0+0+0+d

a+a+a+a+0
a+a+a+a+b
a+a+a+a+c
a+a+a+a+d

b+b+b+b+0
b+b+b+b+a
b+b+b+b+c
b+b+b+b+d

c+c+c+c+0
c+c+c+c+a
c+c+c+c+b
c+c+c+c+d

d+d+d+d+0
d+d+d+d+a
d+d+d+d+b
d+d+d+d+c

20 combinations, 5 permutations each.
Total number of cases: $20 \times 5 = 100$.

5 places with 3 identical terms and 2 identical terms:

0+0+0+a+a
0+0+0+b+b
0+0+0+c+c
0+0+0+d+d

a+a+a+0+0
a+a+a+b+b
a+a+a+c+c
a+a+a+d+d

b+b+b+0+0
b+b+b+a+a
b+b+b+c+c
b+b+b+d+d

c+c+c+0+0
c+c+c+a+a
c+c+c+b+b
c+c+c+d+d

d+d+d+0+0
d+d+d+a+a
d+d+d+b+b
d+d+d+c+c

20 combinations, 10 permutations each.
Total number of cases: $20 \times 10 = 200$.

5 Places with 2 identical pairs:

0+0+a+a+b	0+0+b+b+c	0+0+c+c+d
0+0+b+b+a	0+0+c+c+b	0+0+d+d+c
a+a+b+b+0	b+b+c+c+0	c+c+d+d+0

0+0+a+a+c	0+0+b+b+d
0+0+c+c+a	0+0+d+d+b
a+a+c+c+0	b+b+d+d+0

0+0+a+a+d
0+0+d+d+a
a+a+d+d+0

a+a+b+b+c	a+a+c+c+d
a+a+c+c+b	a+a+d+d+c
b+b+c+c+a	c+c+d+d+a

a+a+b+b+d
a+a+d+d+b
b+b+d+d+a

b+b+c+c+d
b+b+d+d+c
c+c+d+d+b

30 combinations, 30 permutations.
Total number of cases: 30 × 30 = 900.

5 Places with 2 identical terms:

0+0+a+b+c	a+a+b+c+d
0+a+a+b+c	a+b+b+c+d
0+a+b+b+c	a+b+c+c+d
0+a+b+c+c	a+b+c+d+d

0+0+a+b+d
0+a+a+b+d
0+a+b+b+d
0+a+b+d+d

0+0+b+c+d
0+b+b+c+d
0+b+c+c+d
0+b+c+d+d

16 combinations, 60 permutations each.
Total number of cases: 16 × 60 = 960.

5 different terms:

0 + a + b + c + d

1 combination, 120 permutations.
Total number of cases: 1 × 120 = 120.

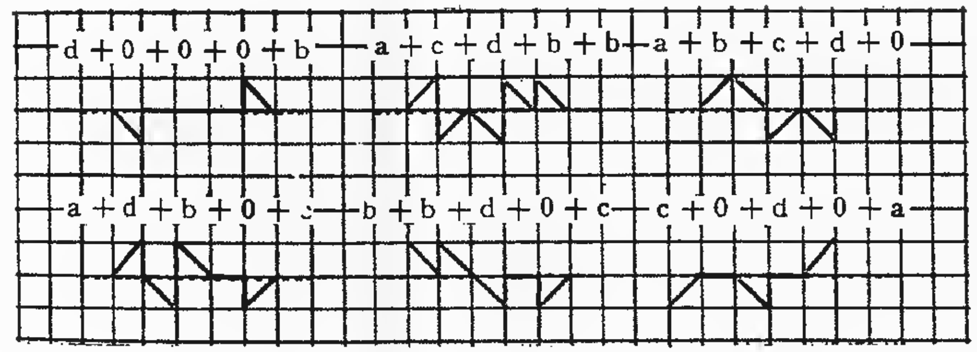


Figure 10. Quintinomial axial combinations.

E. Selective Continuity of the Axial Combinations.

In order to make a preferential selection of recurrence of the secondary axes in composing continuity, coefficients must be used. The following cases are possible.

1. Monomial axis, monomial coefficient

Example:

2a; 3a; 5a; . . .

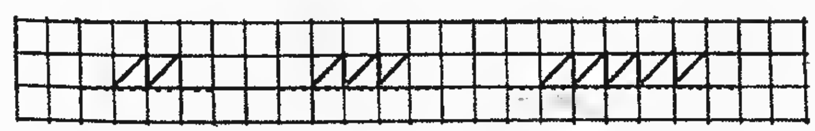


Figure 11. Monomial axis, monomial coefficient.

2. Binomial axial combination, binomial coefficient.
Binomial axial combination, quadrinomial coefficient.
Binomial axial combination, polynomial group-coefficient with even number of terms.

Example:

2a + b; 3a + 2b; 2a + b + a + 2b; 3a + b + 2a + 2b + a + 3b

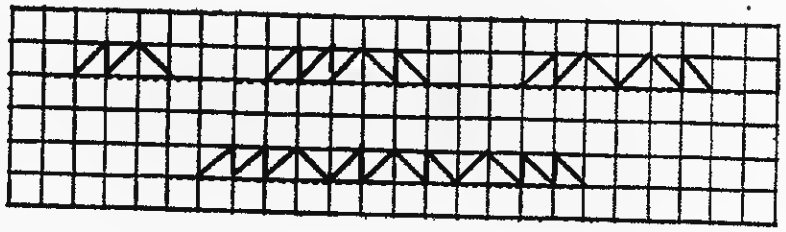


Figure 12. Binomial axial combinations.

3. **Trinomial axial combination, trinomial coefficient.**
Trinomial axial combination. Polynomial group-coefficient with the number of terms divisible by 3.

Example:

3a + b + c; 3a + b + 2c + 2a + b + 3c

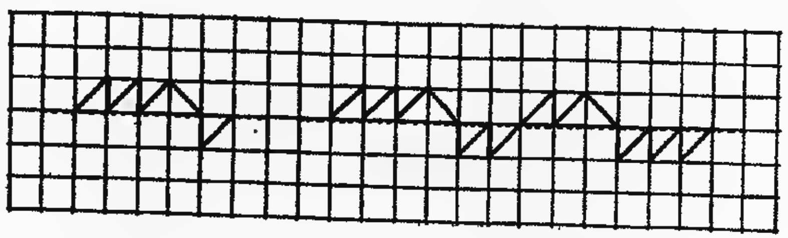


Figure 13. Trinomial axial combinations.

4. **Quadrinomial axial combination, quadrinomial coefficient.**
Quadrinomial axial combination. Polynomial group-coefficient with the number of terms divisible by 4.

Example:

3a + b + 2c + 2d; 4a + b + 3c + 2d + 2a + 3b + c + 4d

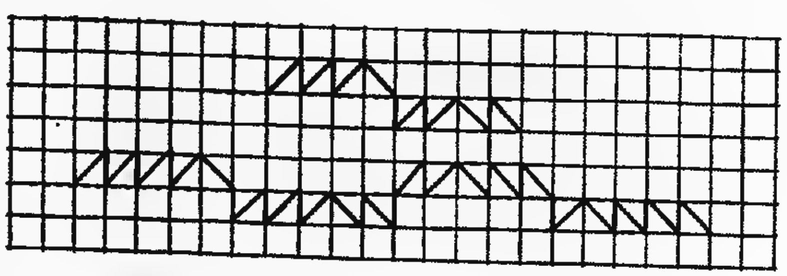


Figure 14. Quadrinomial axial combinations.

5. **Quintinomial axial combination, quintinomial coefficient.**
Quintinomial axial combination. Polynomial coefficient with the number of terms divisible by 5.

Example:

5 (0) + a + 4b + 2c + 3d;
5 (0) + a + 4b + 2c + 3d + 3 (0) + 2a + 4b + c + 5d.

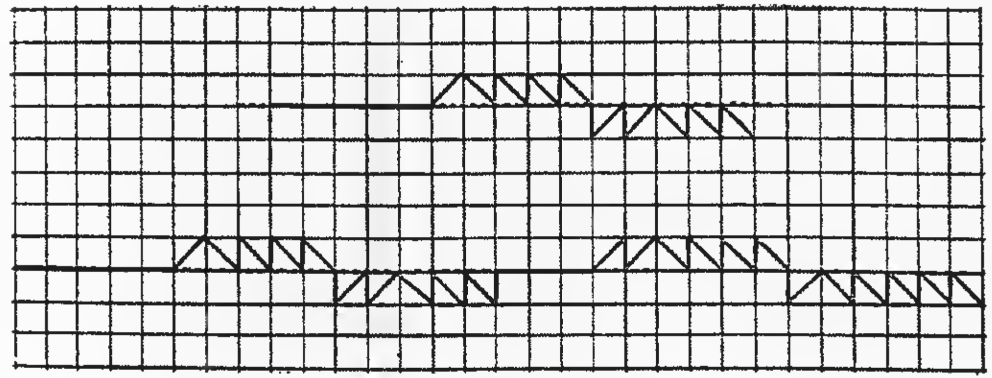


Figure 15. Quintinomial axial combinations.

When the number of terms in a coefficient-group does not coincide with the number of terms in the axial-group, or does not offer common divisors, then interference between the number of places in both groups will occur.

Example:

Binomial axial combination: a + b
Trinomial coefficient group: 3 + 2 + 1
The product: $3 \times 2 = 6$
The complementary factor: 2(3), 3(2)
The resultant of interference: 3a + 2b + a + 3b + 2a + b

F. Time Ratios of the Secondary Axes

Various axial combinations assume various time ratios. Identical axial combinations produce an infinite variety of patterns through different time-ratios. A melody derived from one or another axial pattern is influenced in different degrees by the mutual relations of the balancing and unbalancing axes. An effect of gradual deviation from balance with quick return to balance is entirely changed when the time-ratio is inverted. Deviation from balance on one side of the primary axis produces a different degree of tension when the balancing axis appears on the opposite side of the primary axis than when each combination occurs on one side of the primary axis. With these facts in view, the selection of time coefficients for the secondary axes must be guided by the type of melody, with respect to its tranquility or lack of it. Detailed information on *tension relations* produced by means of axes will be presented at a later point.

It would be correct in most cases to assume a time group unit (T) to be the unit of duration for the secondary axes. Naturally any multiple thereof may serve as a unit; but one bar gives a clear association to minds accustomed to musical ways of thinking.

Here are a few illustrations of the typical axial combinations in relation to time ratios:

1. Monomial Axial Combination

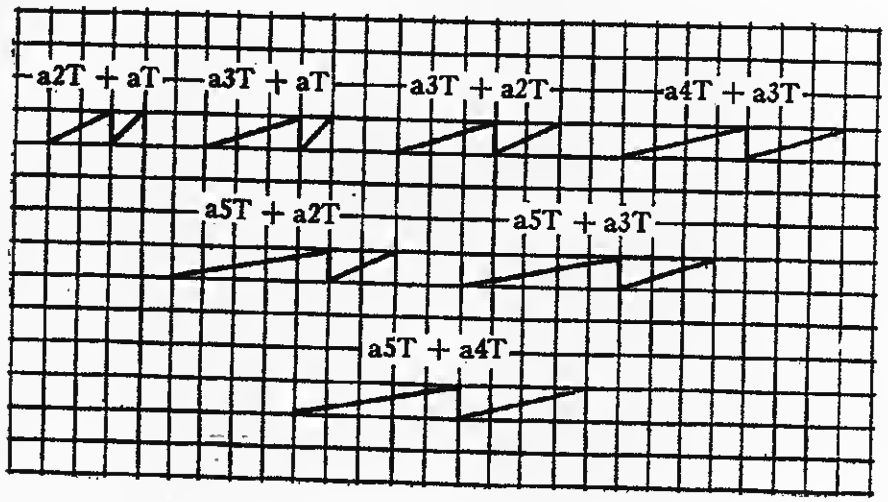


Figure 16. Binomial time-ratio.

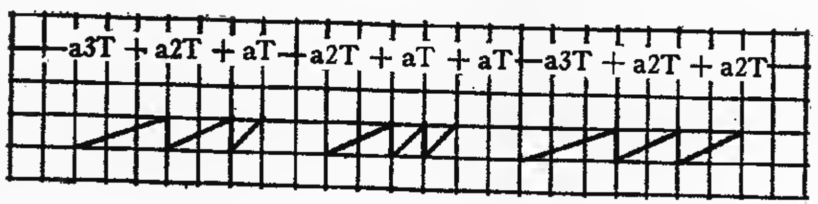


Figure 17. Trinomial time-ratio.

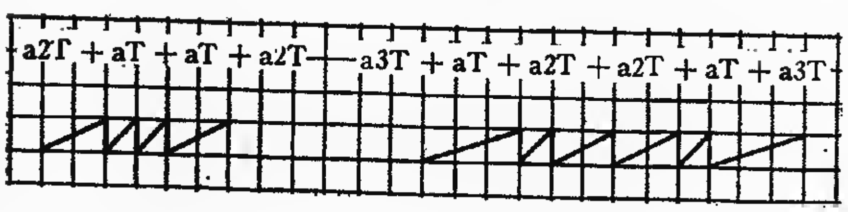


Figure 18. Polynomial time-ratio.

2. Binomial Axial Combination

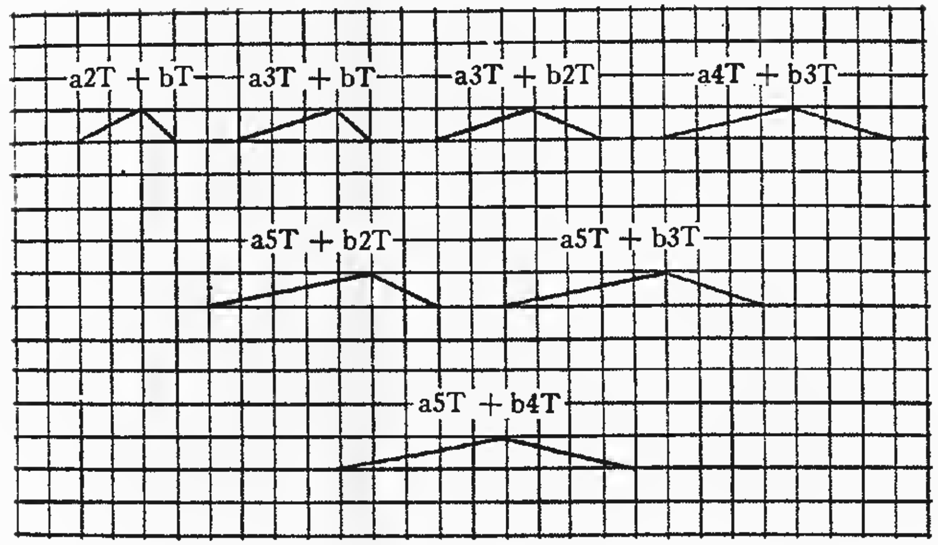


Figure 19. Binomial time-ratio.

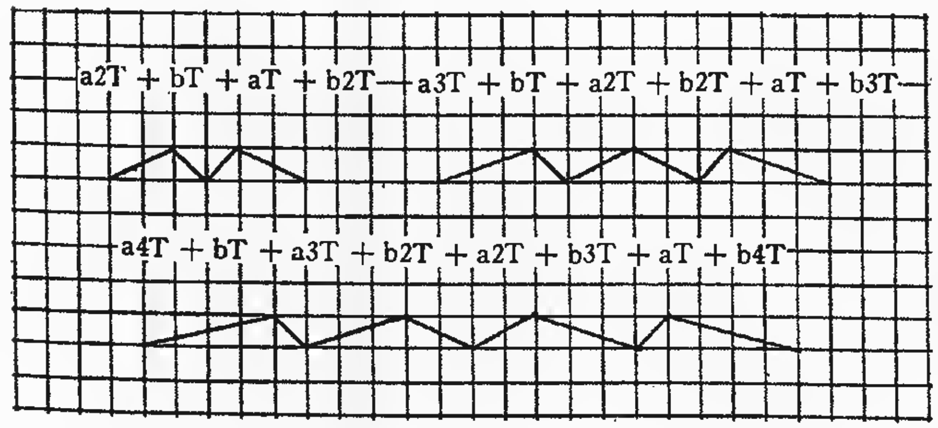


Figure 20. Polynomial time-ratio with the number of terms divisible by 2.

Axes: a, b Time ratio: 3 ÷ 2 ÷ 1

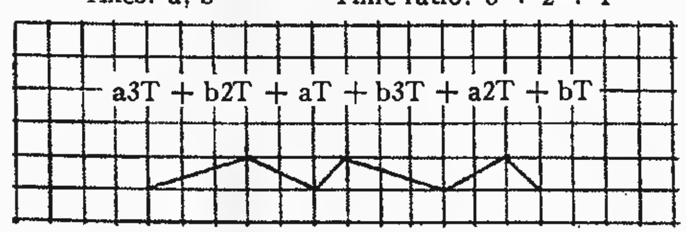
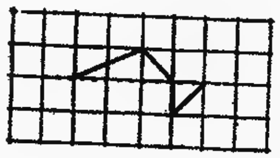


Figure 21. Interference time-ratio.

3. Trinomial Axial Combination

Axes: a, b, c
a2T + bT + cT
Time ratio: $2 \div 1 \div 1$



Axes: a, b, c
a3T + b3T + c2T
Time ratio: $3 \div 3 \div 2$

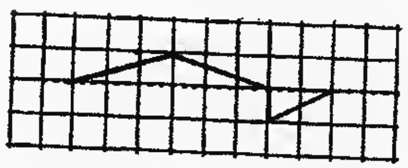
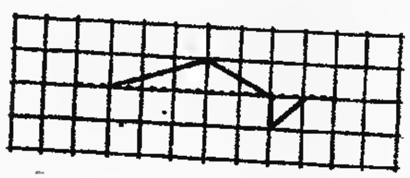


Figure 22. Trinomial time-ratio with two identical terms.

Axes: a, b, c
a3T + b2T + cT
Time ratio: $3 \div 2 \div 1$



Axes: a, b, c
a4T + bT + c3T
Time ratio: $4 \div 1 \div 3$

Figure 23. Trinomial time-ratio with three different terms.

Axes: a, b, c
a3T + bT + c2T + a2T + bT + c3T
Time ratio: $r_{4 \div 3}$

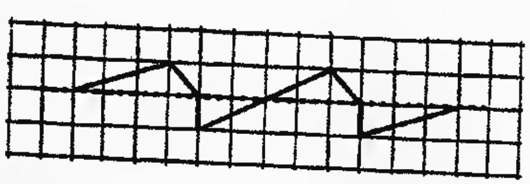
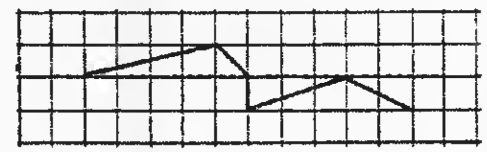


Figure 24. Polynomial time-ratio with the number of terms divisible by 3.

4. Polynomial Axial Combination

Polynomial time-ratio with the number of terms corresponding to the number of terms in the axial group or any multiple thereof.

Axes: a, b, c, d
a4T + bT + c3T + d2T
Time ratio: $4 \div 1 \div 3 \div 2$



Axes: a, b, c, d
a4T + bT + c3T + d2T + a2T + b3T + cT + d4T
Time ratio: $r_{5 \div 4}$

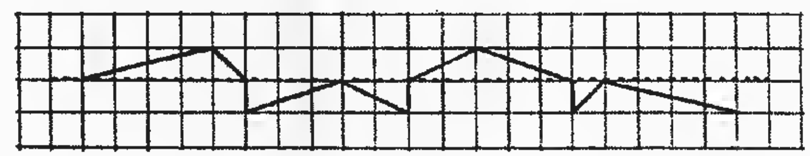


Figure 25. Polynomial time-ratio.

The number of variations for each axial combination, with a selected time-ratio, depends on the number of terms in the axial group and the number of terms in the time-ratio.

A *monomial axial combination* with a binomial time-ratio produces 2 variations:

a2T + aT　　　aT + a2T

A monomial axial combination with trinomial time-ratio having 2 identical terms produces 3 variations:

a2T + aT + aT
aT + a2T + aT
aT + aT + a2T

A monomial axial combination with trinomial time-ratio having all 3 terms different:

a3T + a2T + aT
a3T + aT + a2T
aT + a3T + a2T
a2T + a3T + aT
a2T + aT + a3T
aT + a2T + a3T

A monomial axial combination with polynomial time-ratio produces a number of variations equivalent to the number of permutations of terms in the time-ratio:

a3T + aT + a2T + a2T + aT + a3T
6 elements, 90 permutations.

A *binomial axial combination* with binomial time-ratio. The number of variations equals $2^2 = 4$.

$$\begin{aligned} &a2T + bT \\ &aT + b2T \\ &bT + a2T \\ &b2T + aT \end{aligned}$$

A binomial axial combination with polynomial time-ratio produces a number of variations equivalent to the product of the number of permutations in the axial group by the number of permutations in the time-ratio.

$$a2T + bT + aT + b2T$$

4 terms with 2 identical pairs produce 6 permutations. In this case the axial group and the time-ratio have identical structure. The number of variations: $6^2 = 36$.

Generalization

In order to compute the total number of permutations for any axial combination and any time ratio, it is necessary to synchronize the numbers of terms of both groups.

Let T be the original time ratio, or duration-group, and let Ax be the original axial combination, or axial group. Then let T′ and Ax′ be the synchronized forms of the respective groups. Then synchronization (S) occurs as follows:

$$\begin{aligned} S &= \frac{T}{Ax} \\ T' &= Ax\,(T) \\ Ax' &= T\,(Ax) \end{aligned}$$

The fraction $\frac{T}{Ax}$ must be reduced, if reducible. T′ expresses the total number of terms in the synchronized duration-group and Ax′, the total number of terms in the synchronized axial group.

Let the number of permutations be P and P′ for each respective group. Then *the final number of permutations* (P″) *equals the product of the permutations of both groups in synchronization.*

$$P'' = P \cdot P'$$

Example:

Binomial axial combination $= a + c$

Trinomial time-ratio: $3 + 2 + 1$

$Ax = 2$

$T = 3$

Synchronization:

$$\begin{aligned} S &= \frac{3}{2} \\ T' &= 2(3) \\ Ax' &= 3(2) \\ S &= 3a + 2c + a + 3c + 2a + c \end{aligned}$$

T′ has 6 terms with three identical pairs, (two three's, two two's and two ones). The number of permutations (P) in such a group equals: factorial six (6!) divided by factorial two (2!), by factorial two (2!), by factorial two (2!).

$$P = \frac{6!}{2!\,2!\,2!} = \frac{720}{8} = 90.$$

Ax′ has 6 terms with two groups of three identical elements (three a's and three c's). The number of permutations (P′) in such a group equals: factorial six (6!) divided by factorial three (3!), by factorial three (3!).

$$P_1 = \frac{6!}{3!\,3!} = \frac{720}{36} = 20$$

The total number of permutations (P″) equals P by P_1.

$$\begin{aligned} P'' &= P \cdot P' \text{ or} \\ P'' &= 90 \cdot 20 = 1800 \end{aligned}$$

Time ratios for the axial combinations must be selected according to rhythm families (factorial continuity). In classical music of the 18th century type, the family is $\frac{4}{4}$ series. The binomial ratios of this family are $3 + 1$ and $1 + 3$. Any axial combination selected will assume such ratio when a binomial variation of time (factorial periodicity) is required. For example, a trinomial axis, a, b, c, combined with one of the above binomials produces the following combinations:

$$a3T + bT + c3T + aT + b3T + cT$$

The trinomials of this series are:

$$2 + 1 + 1,\quad 1 + 2 + 1,\quad 1 + 1 + 2$$

With the same selection of axes it would give:

$$a2T + bT + cT$$

Each of these cases offers a corresponding number of variations.

Melody evolving in $\frac{8}{8}$ series will assume the factorial forms of the $\frac{8}{8}$ series. For example, in order to construct a trinomial axial combination for 8-bar continuity, we may choose a, d, b combination of the axes and the time-ratio of $3 \div 3 \div 2$. This will result in:

$$a3T + d3T + b2T$$

This method permits the construction of factorial continuity by means of the secondary axes with any desirable consistency of style. By conforming the selection of the time-ratios to one series of continuity, we achieve the utmost unity of style.

When a *hybrid* style is required, any of the non-corresponding series may be chosen. Such a case would be music evolved in $\frac{8}{8}$ series in its fractional continuity and in $\frac{4}{4}$ series in its factorial continuity.*

*Fractional and factorial continuity are discussed fully in Book I, Chapter 12. (Ed.)

G. Pitch Ratios of the Secondary Axes

As T expresses a time group unit in relation to t, which is the common denominator of the group, so does P express a pitch group unit (pitch range) in relation to p, which is the standard unit of pitch measurement in a given primary selective system.

Pitch ranges become important when they are treated as sections of the total range emphasis of a given musical continuity. In such a case each pitch range corresponds to a certain axis and the total value of the pitch units within one axis depends on the total value of all axes within the entire range. For example, if a melody evolves in a range of 15p (c – e′♭) and three axes are required, each axis will emphasize $\frac{15p}{3} = 5p$, i.e., the partial raoges of the total range will be $P_1 = 5p$, $P_2 = 5p$, $P_3 = 5p$ (c – f; f – b♭; b♭ – e♭).

When the quotient has a remainder, the nearest integer must be taken. For example, if the entire range equals 12p and 2 pitch ranges are required, each of the 2P's equals $\frac{12}{2} = 6$. Diatonic scales not containing such intervals will offer the nearest points to 6. For example, in major or minor scales thc nearest points produce 7p or 5p, thus offering 2 pitch ranges.

$P_1 = 5$ (c – f)
$P_2 = 7$ (f – c′)
or
$P_1 = 7$ (c – g)
$P_2 = 5$ (g – c′)

In symmetric systems of pitch, pitch ranges are between the adjacent tonics. Any scale of the third or the fourth group, whether used in its original or contracted form, produces a number of pitch ranges corresponding to the number of tonics. For example, a scale of the fourth group with 3 tonics in its original form offers 3 pitch ranges:

$P_1 = 8$ (c – a♭)
$P_2 = 8$ (a♭ – e′)
$P_3 = 8$ (e_1 – c″)

As time is infinite and pitch is limited to a few thousand cycles per second, the general conception of coefficients pertaining to P must be handled with a certain amount of discrimination. The type of melody in its general range depends upon the pitch scale from which it is evolved. Thus, scales with one octave range naturally require P's which consist of a very limited number of p's, while scales belonging to the second or the fourth group* naturally lend themselves to widespread P's. The classical average on major and minor scale is about one-half of one octave, the usual binomials being 5 + 7 or 7 + 5. Modern composers have a tendency to write melodies with an enormous pitch range emphasis. In such cases P's may include as many as 11p.

In the following description of various axial combinations in relation to various pitch-ratios, we shall assume a uniform time-ratio.

*See Book II, Chapters 5 and 8. (Ed.)

1. Monomial Axial Combinations

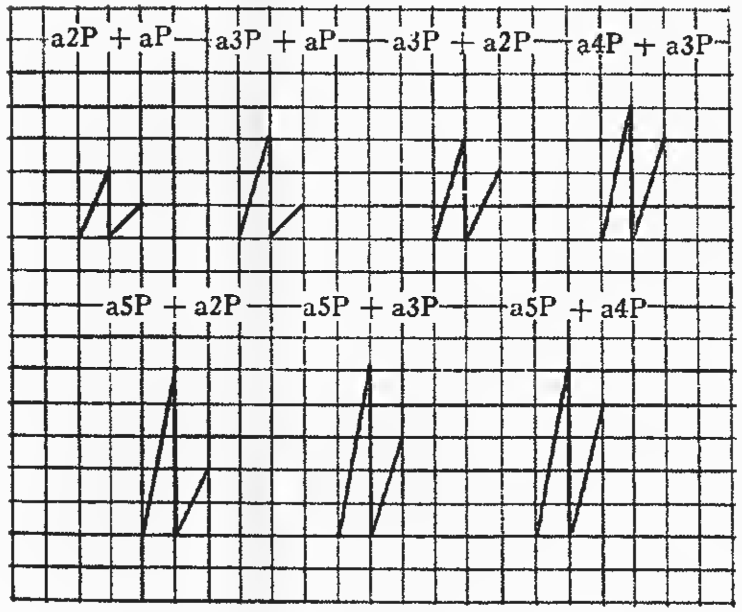


Figure 26. Binomial pitch-ratio.

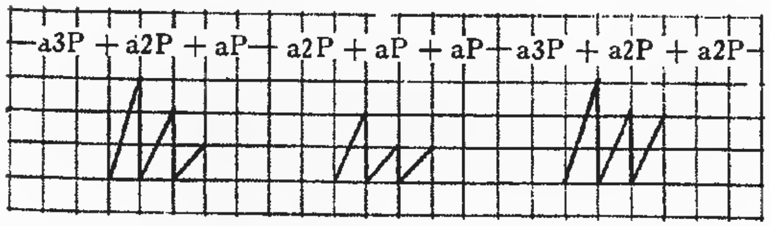


Figure 27. Trinomial pitch-ratio.

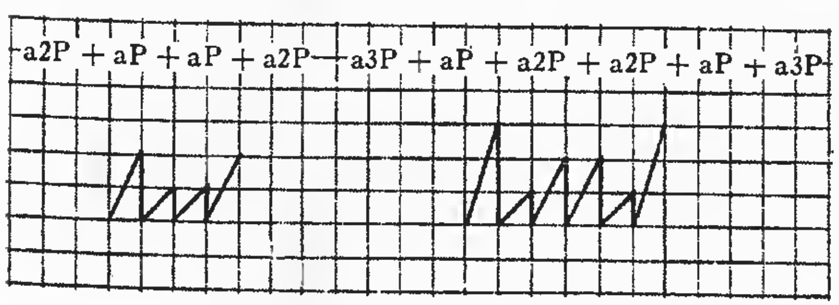


Figure 28. Polynomial pitch-ratio.

2. Binomial Axial Combination

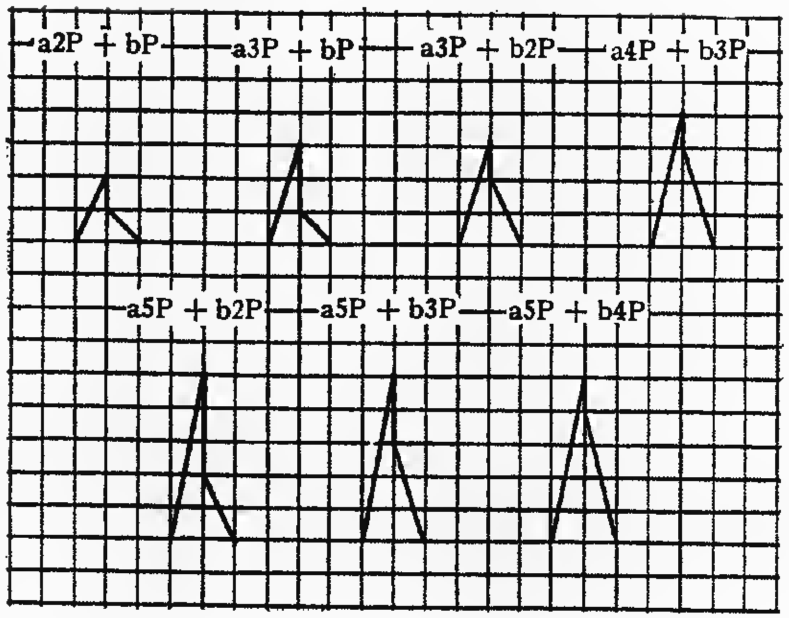
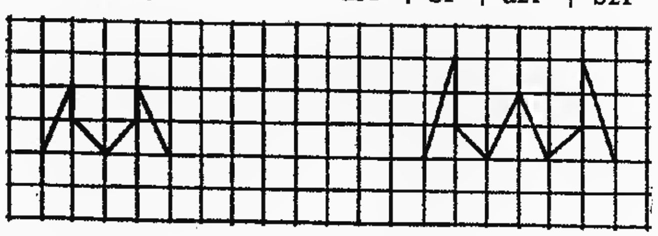


Figure 29. Binomial pitch-ratio.

a2P + bP + aP + b2P   a3P + bP + a2P + b2P + aP + b3P



a4P + bP + a3P + b2P + a2P + b3P + aP + b4P

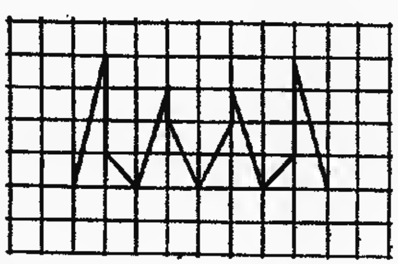


Figure 30. Polynomial pitch-ratio.

Axes: a, b   Pitch-ratio: 3 ÷ 2 ÷ 1

a3P + b2P + aP + b3P + a2P + bP

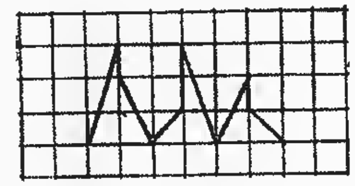


Figure 31. Interference pitch-ratio.

3. Trinomial Axial Combination

Axes: a, b, c   Pitch-ratio: 2 ÷ 1 ÷ 1

a2P + bP + cP

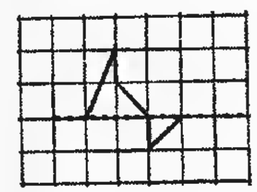


Figure 32. Trinomial pitch-ratio with 2 identical terms.

Axes: a, b, c   Pitch-ratio: 3 ÷ 3 ÷ 2

a3P + b3P + c2P

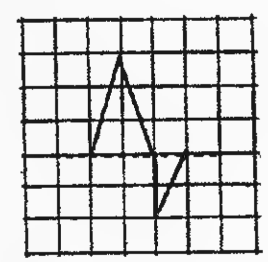
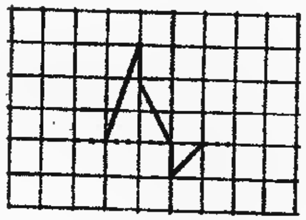


Figure 33. Trinomial pitch-ratio with 2 identical terms.

Axes: a, b, c Pitch-ratio: $3 \div 2 \div 1$

$a3P + b2P + cP$



Axes: a, b, c Pitch-ratio: $4 \div 1 \div 3$

$a4P + bP + c3P$

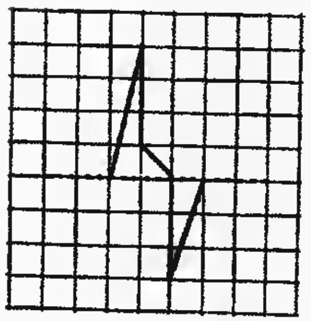


Figure 34. Trinomial pitch-ratio with 3 different terms.

Axes: a, b, c Pitch-ratio: $r_{4 \div 3}$

$a3P + bP + c2P + a2P + bP + c3P$

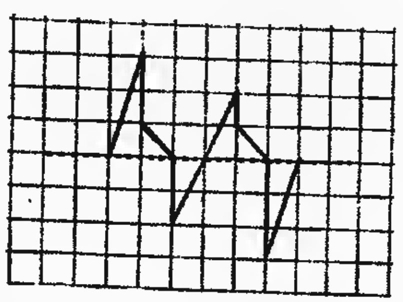


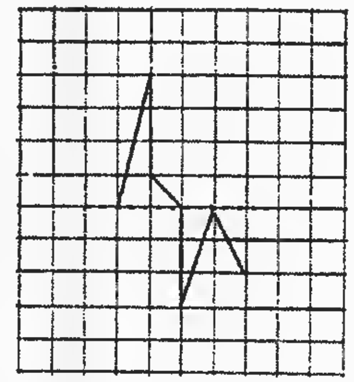
Figure 35. Polynomial pitch-ratio with the number of terms divisible by 3.

4. Polynominal Axial Combination

(a) Polynomial pitch-ratio with (1) the number of terms corresponding to the number of terms in the axial group, or (2) any multiple thereof.

Axes: a, b, c, d Pitch-ratio: $4 \div 1 \div 3 \div 2$

$a4P + bP + c3P + d2P$



Axes: a, b, c, d Pitch-ratio: $r_{5 \div 4}$

$a4P + bP + c3P + d2P + a2P + b3P + cP + d4P$

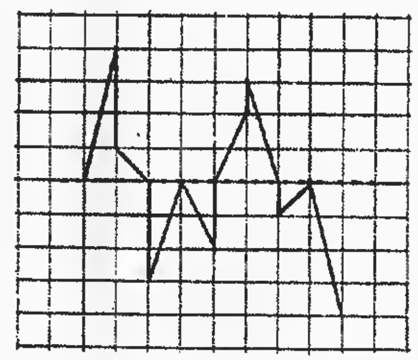


Figure 36. Polynomial pitch-ratio.

(b) Polynomial pitch-ratio with a number of terms which does not correspond to the number of terms in the axial group (interference pitch-ratio).

Axes: a, b Pitch-ratio: $3 \div 2 \div 1$

a3P + b2P + aP + b3P + a2P + bP

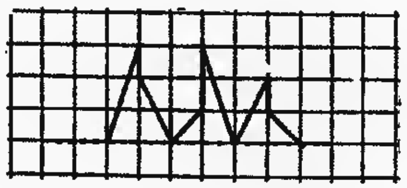


Figure 37. Polynomial pitch-ratio.

The number of variations of each axial combination with a selected pitch-ratio depends on the number of terms in the axial group and the number of terms in the pitch-ratio.

A ***monomial axial combination*** with a binomial pitch-ratio produces 2 variations:

a2P + aP Var. aP + a2P

A monomial axial combination with trinomial pitch-ratio having 2 identical terms produces 3 variations:

a2P + aP + aP
aP + a2P + aP
aP + aP + a2P

A monomial axial combination with trinomial pitch-ratio having all 3 terms different:

a3P + a2P + aP
a3P + aP + a2P
aP + a3P + a2P
a2P + a3P + aP
a2P + aP + a3P
aP + a2P + a3P

A monomial axial combination with polynomial pitch-ratio produces a number of variations equivalent to the number of permutations of terms in the pitch-ratio:

a3P + aP + a2P + a2P + aP + a3P
6 elements, 90 permutations.

A ***binomial axial combination*** with binomial pitch-ratio. The number of variations equals $2^2 = 4$:

a2P + bP
aP + b2P
bP + a2P
b2P + aP

A binomial axial combination with polynomial pitch-ratio produces a number of variations equivalent to the product of the number of permutations in the axial group by the number of permutations in the pitch-ratio:

a2P + bP + aP + b2P

Four terms with 2 identical pairs produce 6 permutations. In this case the axial group and the pitch-ratio have identical structure.

The number of variations: $6^2 = 36$.

Computation of the total number of permutations for the synchronized axial combinations (axial groups) and the synchronized pitch-ratios (pitch-range groups) follows the same formulae as in the case of axial groups synchronized with duration groups.

Pitch ratios for the axial combinations must be selected according to the total range to be emphasized and the type of pitch-scale to be used.

H. Correlation of Time and Pitch Ratios of the Secondary Axes

The correspondence of the number values expressing pitch and time-ratios of the secondary axes is entirely immaterial for reasons mentioned above, i.e., the different nature of the limitations pertaining to time and pitch in our sensory continuum.

But the form of relations between time and pitch that is essential refers to different forms of correspondences between the two, with only a certain amount of influence on the actual ratios—and, in most cases, with no influence at all on the actual number values. The forms of correspondence are:

(1) Parallel (direct)
(2) Contrary (inverse)
(3) Oblique (indirect)

There are two forms of oblique correspondences:

(a) circumstantial
(b) intentional

(1) Parallel Correspondences

a2T2P + bTP aTP + b2T2P

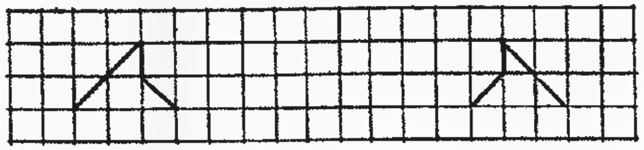
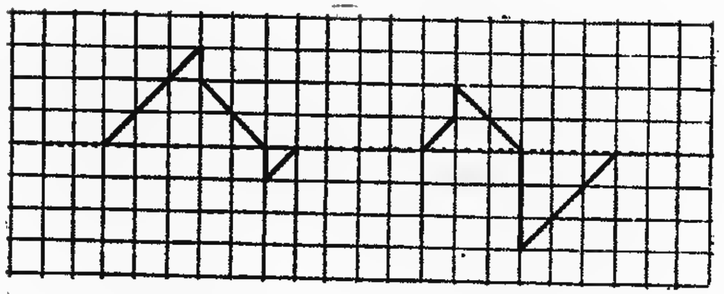


Figure 38. Parallel Correspondences (continued).

a3T3P + b2T2P + cTP   aTP + b2T2P + c3T3P



a4T4P + bTP + c3T3P + d2T2P

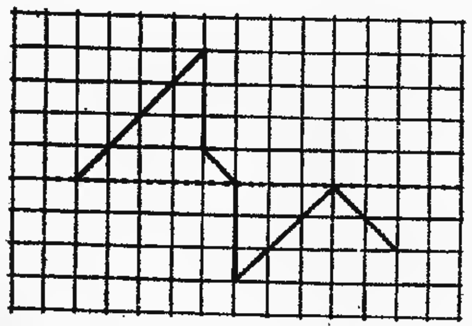
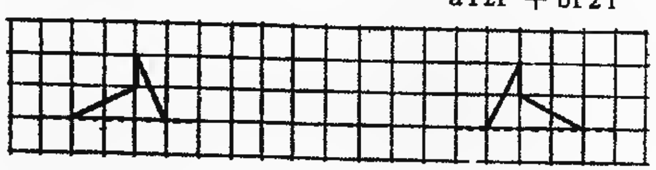


Figure 38. Parallel correspondences. (concluded).

(2) Contrary Correspondences.

aP2T + bT2P   aT2P + bP2T



aT4P + c2T3P + b3T2P + dP4T   aP4T + c3T2P + b2T3P + dT4P

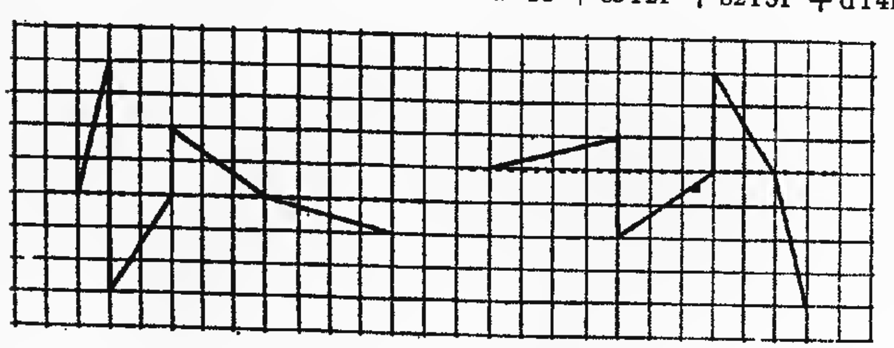
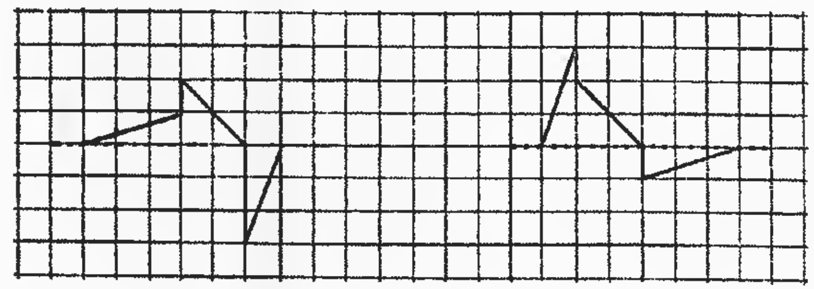


Figure 39. Contrary correspondences.

(3) Oblique Correspondences.

(a) Circumstantial: when the axial combination has an uneven number of terms (this produces a coincidence of both coefficients on the middle term).

aP3T + b2T2P + cT3P   aT3P + b2T2P + cP3T



aP5T + b4T2P + c3T3P + b2T4P + aT5P

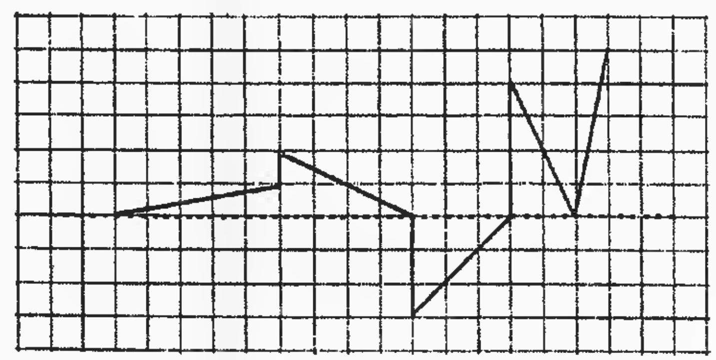


Figure 40. Oblique correspondences. Circumstantial.

(b) Intentional: when partial coincidence is desired regardless of the number of terms.

Axes:	a	b	c	d
T	4	1	3	2
P	2	3	1	2

a4T2P + bT3P + cP3T + d2T2P

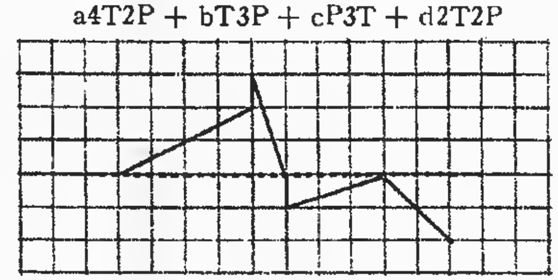


Figure 41. Oblique correspondences. Intentional.

The general effect of parallel correspondences is one which is expected; it may be associated with stability and common sense. The path of melody through time and pitch appears under the conventional mechanical conditions, i.e., the greater the pitch range to be covered, the greater the time required. The smaller the pitch range to be covered, the less the time required.

The contrary correlation produces an effect of tension or surprise. It has an attractive and often dramatic quality. Greater pitch-ranges are achieved in shorter time (greater velocity) and smaller pitch-ranges are covered in a longer period of time (smaller velocity, resistance, delays).

The oblique correlation produces intermediate effects offering more of the surprise element when the coefficients are different, and bringing it back to more conventional effects when there is such a coincidence.

All the problems of the actual relationship of patterns through the character of reaction, resulting as a response to such patterns, are discussed in the following chapters.

CHAPTER 4

MELODY: CLIMAX AND RESISTANCE

THE PROJECTION of melody is a mechanical trajectory. Its kinetic components are balance, impetus and inertia. Resistance produces impetus, leading either towards the *climax*, which is a *pt* (pitch-time) *maximum with respect to the primary axis*, or towards balance. The impetus is caused by resistance which results from rotation. The geometrical projection of rotation is a circle which extends itself in time projection into a cylindrical or spherical spiral, or ultimately (through time extension) into wave motion (plane projection).

The kinetic result of rotary motion is *centrifugal* energy. The discharge of accumulated centrifugal energy is equivalent to a *climax*. A heavy object attached to a string and put into rotary motion about an axis-point develops considerable energy—enough to move it a long distance when detached from the string.

Overcoming inertia increases mechanical efficiency (gain of kinetic energy). Any body set in motion acquires its ultimate possible speed in a certain period of time. The shorter the period from the moment of the application of the initial force (impetus) till the moment when the body acquires its ultimate speed, the greater is the mechanical efficiency of such motion.

Motion is expressible in wave amplitudes; the projection of kinetic climax is the maximum amplitude. Inert matter does not acquire its maximum amplitude instantaneously when starting from balance just as the maximum cannot recede to balance (rest) instantaneously. This is true both of velocities (frequencies) and amplitudes.

Mechanical experiences, whether instinctive or intentional, are known to all types of zoological species and are inherited and perfected in the course of evolution. A grown animal has a perfect judgment of distances, of directions, and of the amount of muscular energy necessary in leaps or flights, without any theoretical knowledge of the law of gravity or mechanics in general. There is no misjudgment in the monkey's flights from tree to tree; there is none for a gazelle leaping over a creek, or for an eagle falling on its prey. A certain amount of intentional mechanical efficiency and psycho-physiologic coordination is inherent with every surviving species of the animal world. The relativity of the standards of mechanical efficiency corresponds to the relativity of reflexes, reactions and judgments.

The leap of a human being over a 14 foot rod was the highest achievement in the International Olympics for 1936, and this with the aid of a pole. The mechanical efficiency of an ordinary flea is fifty times greater. The leap of a human being over a rod 50 feet high would seem supernatural, while the same kind of leap by a flea would be far below the standards of flea efficiency—the flea leaps about one hundred times its own size.

Standards of mechanical efficiency vary with ages and places, even among human beings. They also vary with different races as well as with different ages. The development of athletic qualities and forms of locomotion implies the raising of the requirements necessary for mechanical efficiency.

The geometrical conception of mechanical and bio-mechanical trajectories necessitates the analysis of the corresponding trajectories of nervous impulses and muscular reactions. There are correspondences between the two, and the knowledge of such correspondences leads to scientific production of excitors capable of stimulating the intended reactions (in this case, esthetic excitors: music in general, or melody in particular). Simple reflexes and reactions project themselves into simple trajectorial patterns; on the other hand, excitors having the form of simple trajectories stimulate reactions of a corresponding simplicity. Likewise, this correspondence occurs with complex patterns.

The intensity-interdependence between the excitor and the reaction was formulated in Weber's and Fechner's psycho-physiological law. Both as to configurations (patterns) and as to amplitudes (intensities), there are correspondences between the excitors and the reactions. Judgment based on mechanical experience and mechanical orientation leads higher animals and human beings to certain expectations. In the case of an absolute correspondence between the realization of a mechanical process and the expectation, the resulting reaction is balance (normal satisfaction). A result above expectation stimulates the intensification of activity (positive reaction) and at its extreme, ecstasy. On the other hand, the result of a mechanical process which is below expectation stimulates passivity (negative reaction) and at its extreme, depression. The two opposite poles of reactions, brought to their absolute limit, stimulate astonishment (irrational or zero reaction).

Geometrical projection of a scale of psychological adjectives on a circumference produces the poles of the two rectangular coordinates (the diameters of the circle): 1. normal—absurd; 2. depressing—ecstatic. Producing four new poles on the intermediate arcs of the circumference through the addition of another pair of rectangular coordinates (under 45° to the original pair) we obtain nine poles altogether (including both 0° and 360°). These nine poles, through the application of the method of evolving concept series, become expressible in adjectives standing for the psychological categories.

Scale of psychological categories as represented through geometrical projection on a circumference. (*See Figure 41A on next page*).

The circumference is divided, by the poles of the coordinates, into 8 arcs, 45° each. The geometrical poles correspond to the psychological poles. Arcs represent the transition zones, and the poles—their absolute expression.

Zero or 360° — abnormal	180° — normal
90° — infranormal	270° — ultranormal
135° — subnormal	315° — supernatural
45° — subnatural	225° — supernormal

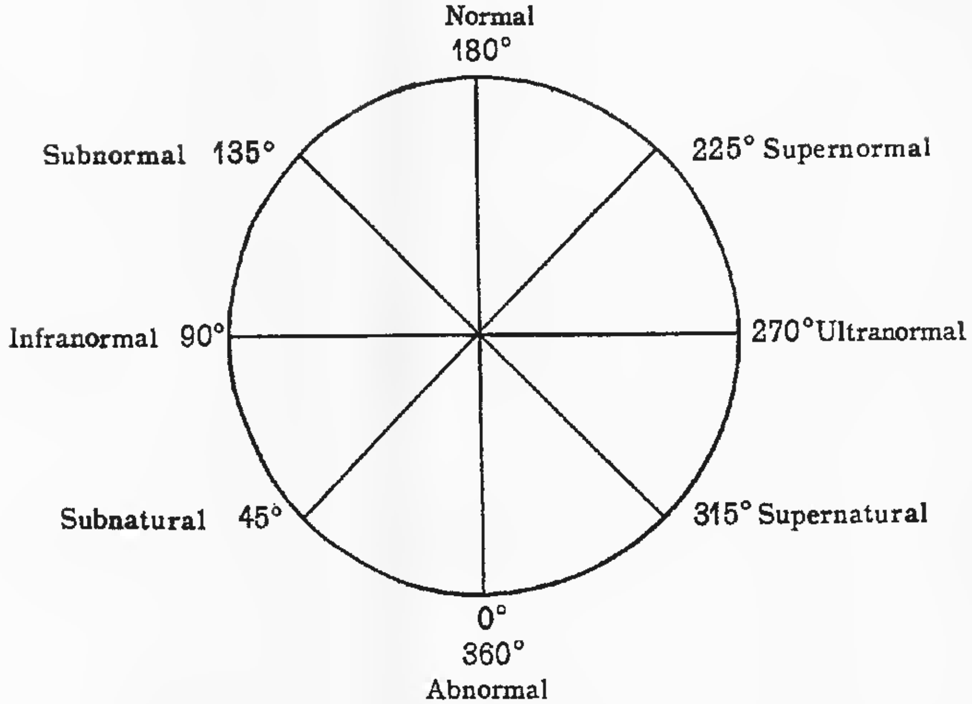


Figure 41A. Scale of psychological categories.

The zone around 0° or 360° stimulates astonishment (zero reaction or delayed reaction). The zone around 45° stimulates either pity or humor. The zone around 90° stimulates depression (pessimism). The zone around 135° stimulates the sense of lyricism (regret, melancholy, pleasant sadness, joyful sadness, controllable, self-imposed sadness; close to positive zone: joy of self-destruction, self-sacrifice.) The zone around 180° stimulates the sense of quiet contemplation, full psychological balance and satisfaction. The zone around 225° stimulates the sense of heroism and admiration. The zone around 270° stimulates the sense of exaltation, ecstasy and worshipping. The zone around 315° stimulates either the sense of the fantastic or the sense of fear (unfavorable surroundings, uncontrollable, unaccountable forces, fear for existence, struggle for survival).

A discus thrower participating in the Olympics and reaching the previous year's record would stimulate a reaction corresponding to 180° point. The actual reflexes of the spectators would be polite applause. Throwing beyond the expected range would stimulate a reaction corresponding to the zone between 180° and 225°, culminating in ultimate ecstasy when it reached 270°—and this would be evidenced in the audience by shouting, stamping and whistling, the reactions increasing not only in intensity, but in quantity as well—i.e., the maximum conceivable limit. The clapping reflexes would grow accordingly from 180° to 270°. If the disc does not reach the range expected, the reaction would be disappointment, increasing toward 135° with the sympathetic spectators.

With the range reaching only 90°, it would lead ultimately toward depression. The spectators will not applaud when the effect of the disc throwing is near the 90° point. It is natural to assume that certain groups of spectators, influenced by their sympathy for opponents of the first discus thrower, would display exactly opposite reactions. These considerations cover the semicircle above the horizon.

The lower zone, on the negative side, i.e., between the 0° and 90°, stimulates the reaction of laughter. In the case of the discus thrower, it would amount to a range of perhaps only a few yards from his position after a long and arduous preparation for the throw. When the spectators see a husky, muscular athlete deprived of mechanical efficiency, they unquestionably react to it as if the episode seemed decidedly humorous.

On the positive side of the lower semicircle, between 315° and 360°, lies the zone of the supernatural, where the range of throw of a disc would be beyond any biomechanical possibility. For example, if the range of throw amounted to three miles. In such cases the presence of a trick or a supernatural force would be a necessary ingredient for the logical comprehension of the phenomenon. The usual reaction would be that of a smile or laughter moving toward astonishment in the direction of the zero point.

The 360° point when reached from the positive side would amount to the absurd caused by an impossible mechanical over-efficiency. Such would be the case when the disc being thrown would never come back, would never fall anywhere on the ground, but vanish in interstellar space, thus overcoming the law of gravity.

When zero is reached from the negative side, it would mean an impossible mechanical inefficiency. In the case of a disc thrower, it would happen if the disc were to slip out of the athlete's hands before he actually threw it.

A trajectory expressing a mechanically efficient kinetic process, whether that of a pendulum or a musical melody, will have mechanical fundamentals in common. *A pendulum cannot start instantaneously at its maximum amplitude; neither can a melody.* A pendulum cannot stop instantaneously from its maximum amplitude; neither can a melody. The corresponding effects in both cases will be either *supernatural* or *humorous.*

The actual quantitative specifications serving different purposes and expressing different forms of mechanical efficiency vary with times and places. To satisfy any esthetic requirement, one has to know the style in which such requirements have to be carried out—also beyond what specifications the entire kinetic process, whether efficient or not, will become meaningless. As standards vary, the coordinates on the circle described above change their absolute positions, i.e., the zero point may move with the entire system, either clockwise or counter-clockwise. If we would assume, with regard to athletic standards, 180° to be a limit of certain mechanical operations—when the achievement of the succeeding epoch increases the quantitative value of normal, placing the point of normality on what is 225° on our diagram, the opposite pole of the coordinate will occupy respectively the 45° position.

Referring to music in general and melody in particular, we find that certain standards become old-fashioned and we begin to feel that although they may be charming yet they are entirely inadequate for the purposes of a more mechanically efficient epoch. We feel it in every field concerned with motion,* i.e., mechanics.

One has a humorous or a pitying reaction toward the 1900 "horseless carriage"—and it becomes still more humorous when there is an accumulation of quantities of the symbols of inadequacy, such as the prerequisites of travel required by a horseless carriage: dusters, goggles, safety belts. We have exactly the same picture (i.e., if we are people representing our epoch rather than living anachronisms), in melodies composed by a Verdi or a Bellini; the mechanical efficiency is so low that it makes us smile, if not laugh. The same melodies stimulate entirely different reactions among octogenarians surviving in our epoch of 400 miles per hour.

In order to achieve an efficient climax, it is necessary to accumulate energy that will be effectively discharged in such a climax. The means for accumulating energy, as was described above, are achieved through rotary motion developing centrifugal energy. Trajectories expressing musical pitches of various frequencies are heard by listeners in relation to the entire trajectory. It is possible not only to show the range of frequencies (such as a form of direct transition from one frequency to another), but also to show in what way this variation of frequency was achieved.

The portion of a melodic trajectory leading toward the climax, without resistance preceding such a climax, does not produce any dramatic effect. It is resistance that makes the climax appear dramatic. A portion of melodic trajectory leading from a climax (maximum amplitude) towards balance (minimum amplitude) must be performed in accordance with natural mechanical laws, i.e., it must contain resistance before it reaches the balance (compare with pendulum). Inefficiency, or excess of the forms of resistance (rotary motion), leads to a mechanical abnormality. Abnormal melody stimulates the sense of dissatisfaction or humor. The forms of resistance leading toward climax acquire centrifugal form (increasing amplitude). The forms of resistance leading toward balance acquire centripetal form (decreasing amplitude). The relative period of rotary motion and amplitudes produces various forms and gradations of resistances. For example, the period of rotation may be long, with the amplitude remaining constant; or the period of rotation may be short with rapidly increasing amplitudes. The period of rotation may be short with correspondingly increasing amplitude. The duration of the rotary period may be in inverse proportion to the amplitude—and often the law of squares takes its place.

*The practical value of Schillinger's work in correlating music and motion—sound and the mathematical laws of motion—appears in each section of his System. In *Theory of Melody* this correlation takes a most interesting form, and yields insights of inestimable value to the composer. The effort to relate psychological categories and music is almost as old as music itself. But most such efforts have been impressionistic, critical and intuitive—and therefore so subjective as to be useless both to the composer and critic. Schillinger's procedure in projecting a melody on a graph and correlating this melodic trajectory with the scale of psychological categories offers a scientific and objective approach to the problem. With intelligent application composers now can unerringly evolve a melody to produce a given psychological effect. Music critics likewise have an instrument—which does not depend on how they feel at a given concert—for judging the success of the composer's effort. (Ed.)

A. Forms of Resistance Applied to Melodic Trajectories

The corresponding forms of resistance as applied to melodic trajectories are:

1. **Repetition** (correspondences: aiming, rotary motion with infinitesimal amplitudes, affirmation of the axis level as a starting point). Musical form: repeated attacks of the same pitch discontinued by rests or following each other continuously.

Physical Form *Musical Form*

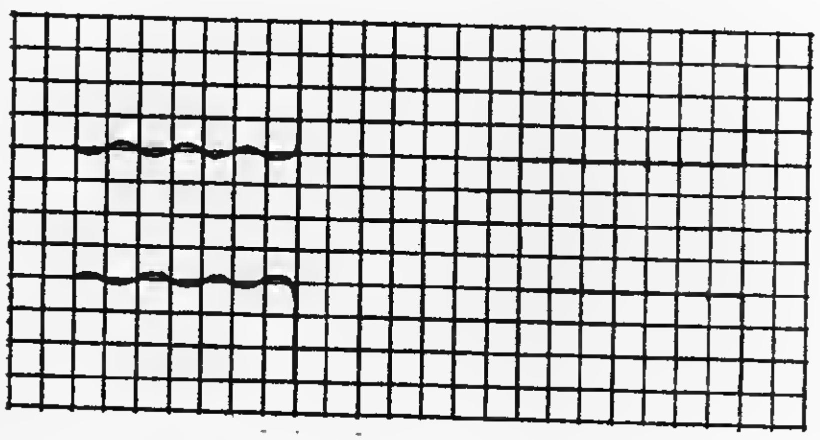


Figure 42. Repetition as form of resistance.

2. **One phase rotation** (correspondences: preliminary contrary motion, initial impulse in archery, artillery, springboard diving, baseball pitching, tennis service, etc.) Musical form: a movement or a group of movements in the direction opposite to the succeeding leap.

Physical Form

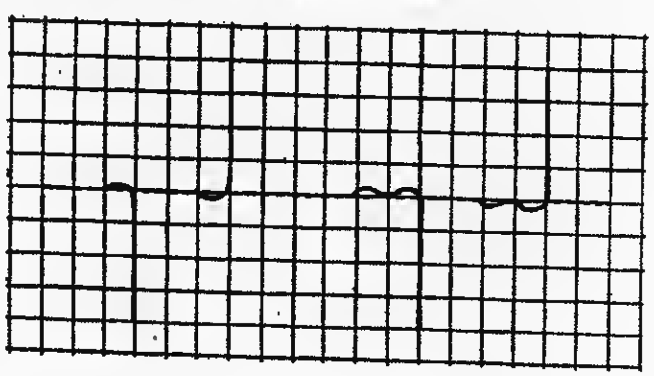


Figure 43. One phase rotation (continued).

Musical Form

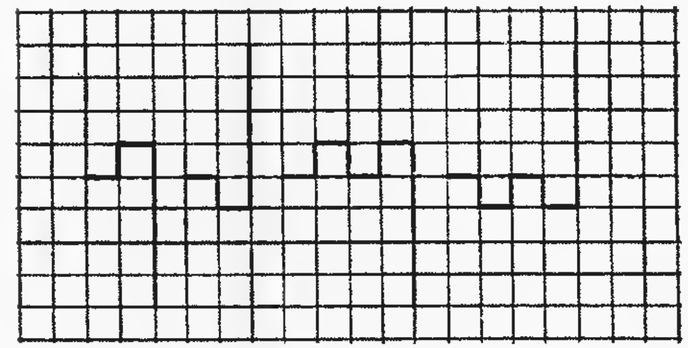
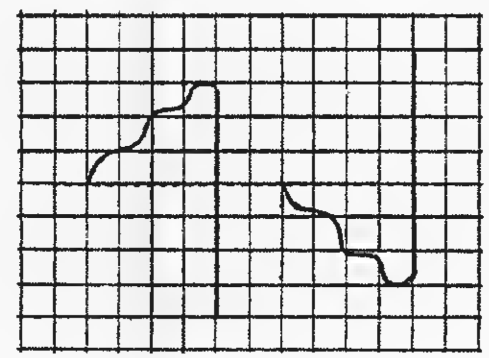


Figure 43. One phase rotation (concluded).

This form often acquires more than one phase following in one direction which intensifies the resistance.

Physical Form



Musical Form

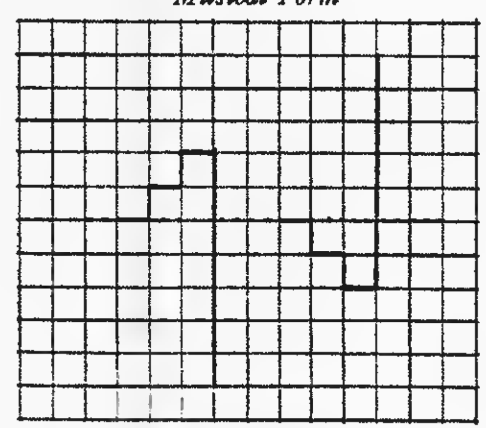


Figure 44. More than one phase rotation.

3. **Full periodic rotation** (one or more periods).

   a. **Constant amplitude (correspondences: rotation around a stationary point, a top, somersaults—with diving and without—lasso, axis and orbit rotation of the planets, Dervish dances).**

Musical Form: mordents, trill, tied tremolo, gruppetto.

Physical Form *Musical Form*

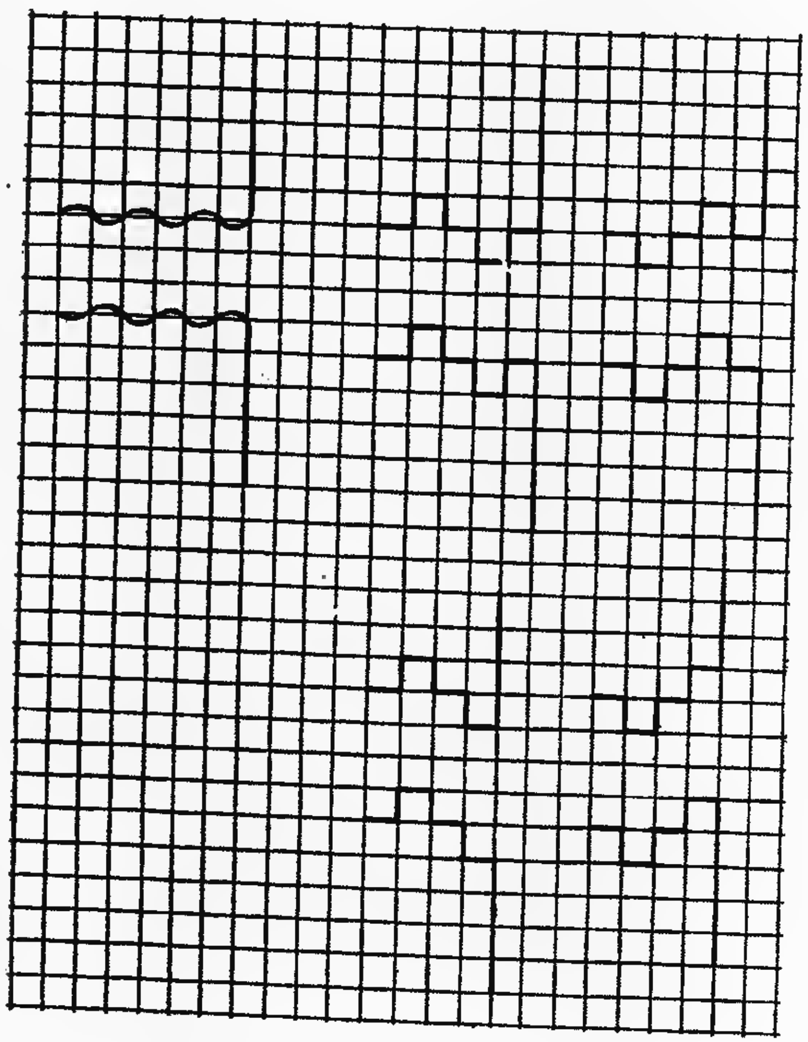


Figure 45. Full periodic rotation: constant amplitude.

Variable amplitude (correspondences: gyroscope, spiral motion, tornado, expansion, contraction). Musical form: expanding and contracting, simple and compound motion.

Whereas the preceding forms of resistance require only one of the secondary axes, the variable amplitude rotation requires a simultaneous combination of two or three secondary axes. In this case the axis leading towards climax or balance will be considered *fundamental* and the other axes—*complementary*.

Simultaneous combinations of two axes:

(a) Centrifugal (expanding):

$\frac{a}{o}$; $\frac{o}{a}$; $\frac{d}{o}$; $\frac{o}{d}$;

Physical Form *Musical Form*

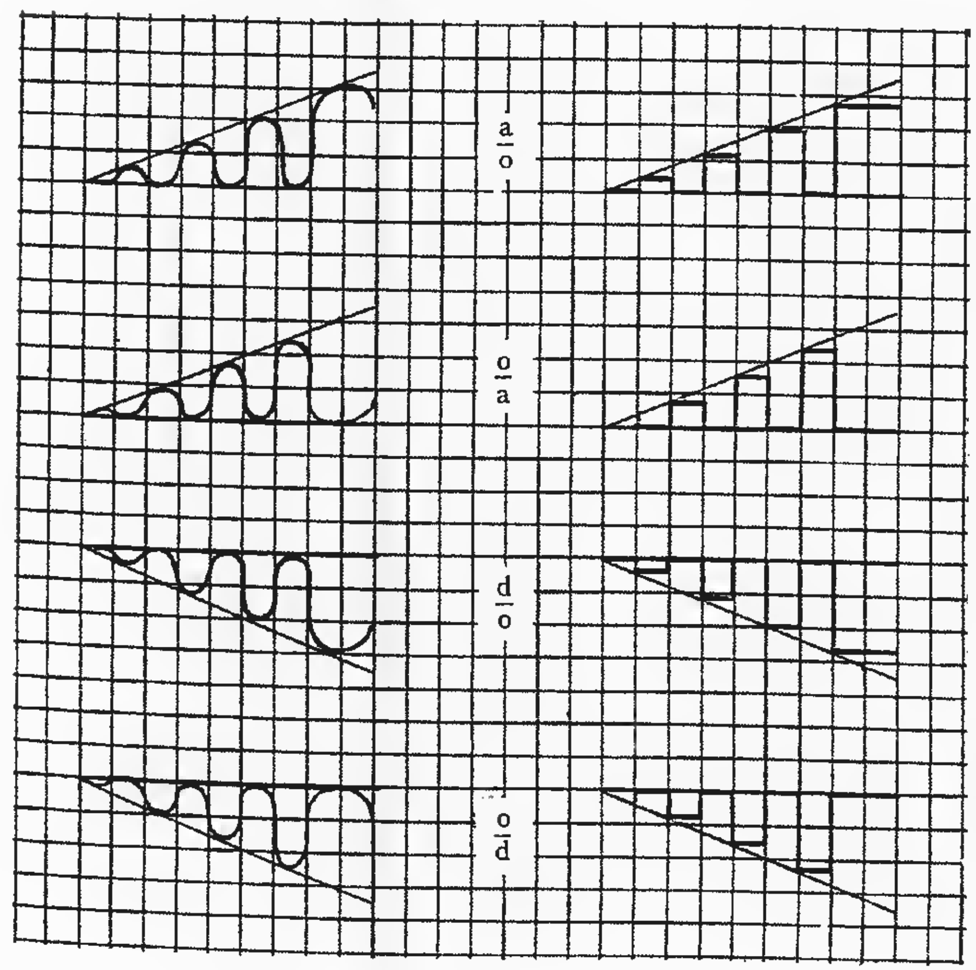


Figure 46. Centrifugal combination of two axes.

(b) Centripetal (contracting)

$\frac{b}{o}$; $\frac{o}{b}$; $\frac{c}{o}$; $\frac{o}{c}$;

Physical Form *Musical Form*

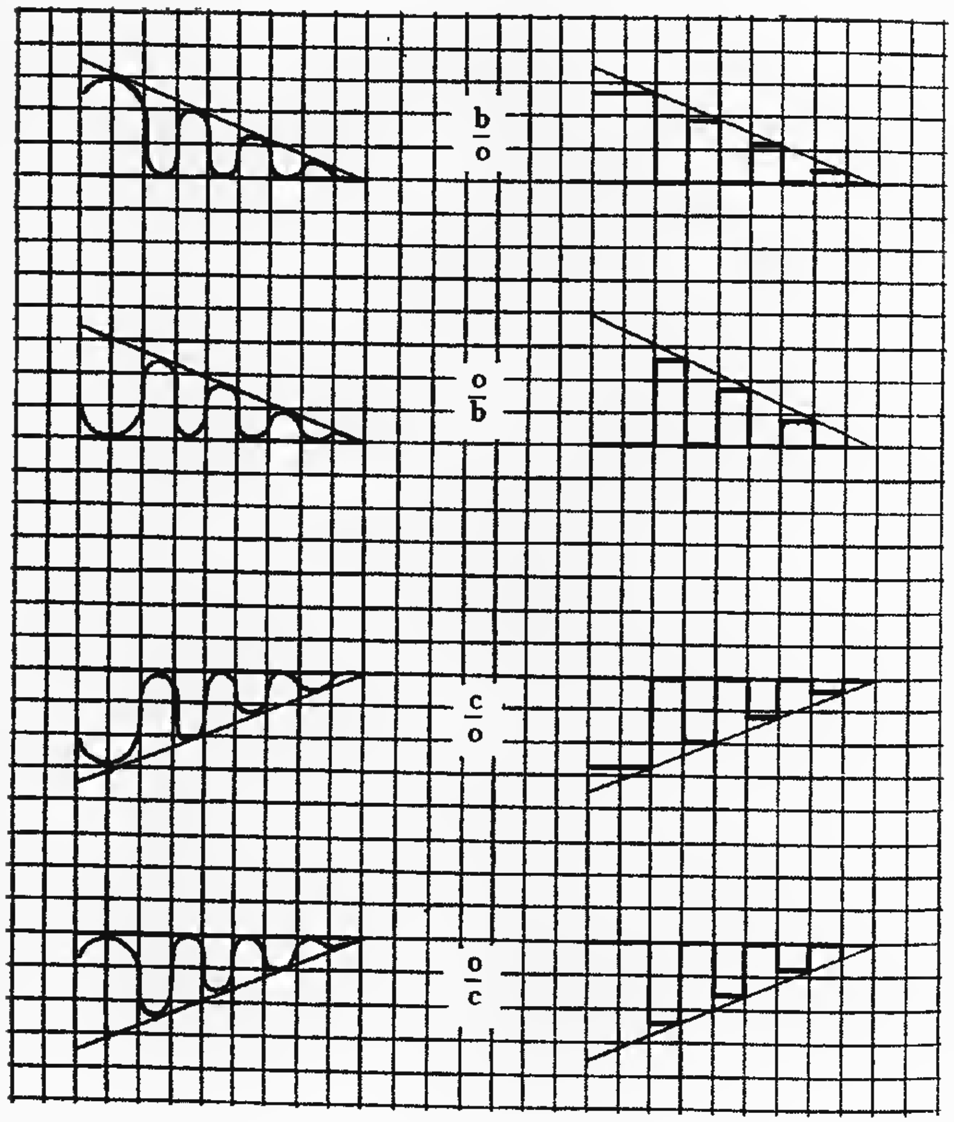


Figure 47. Centripetal combination of two axes.

Simultaneous combinations of three axes:

(a) Centrifugal (expanding):
a ÷ 0 ÷ d; d ÷ 0 ÷ a
(b) Centripetal (contracting):
b ÷ 0 ÷ c; c ÷ 0 ÷ b

Physical Form *Musical Form*

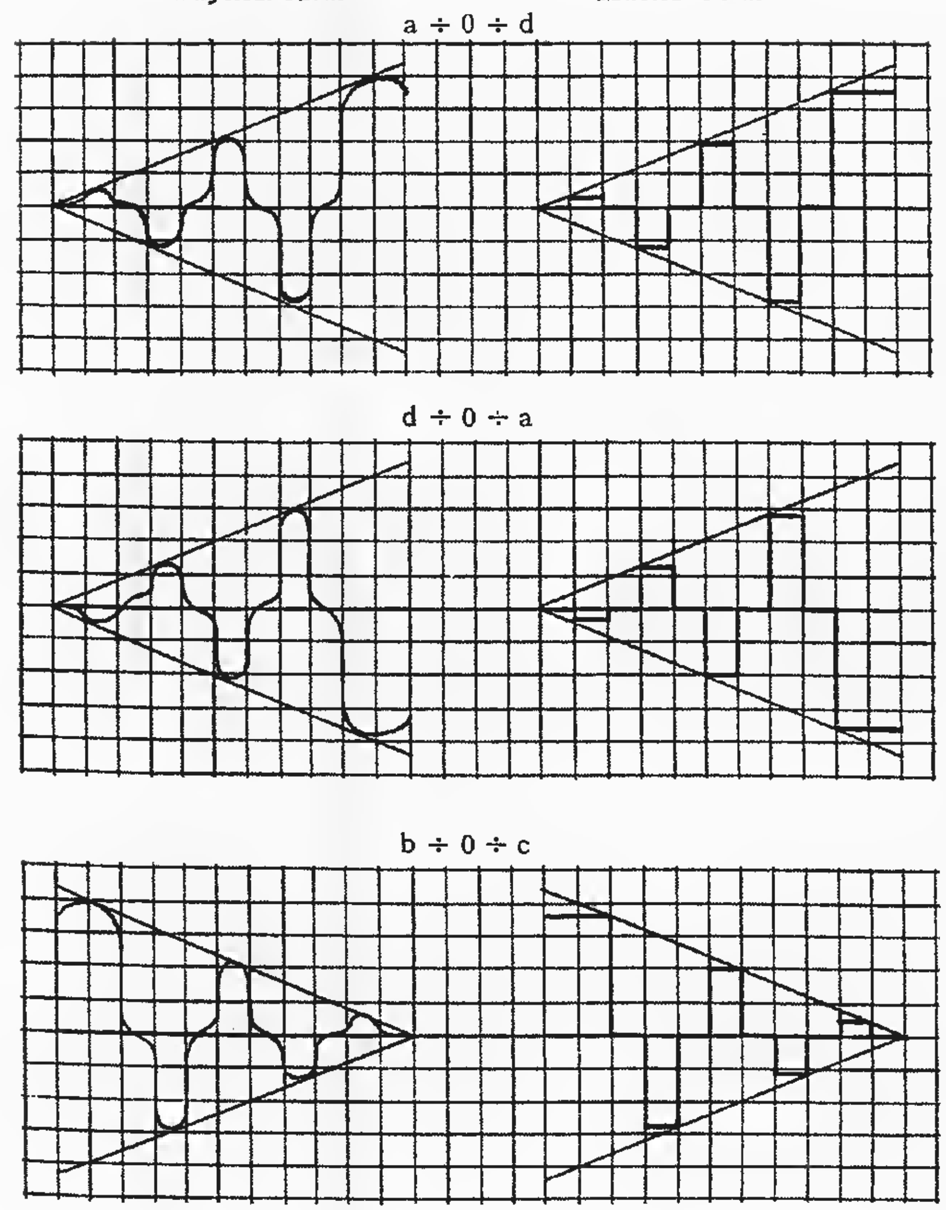


Figure 48. Simultaneous combination of three axes (continued).

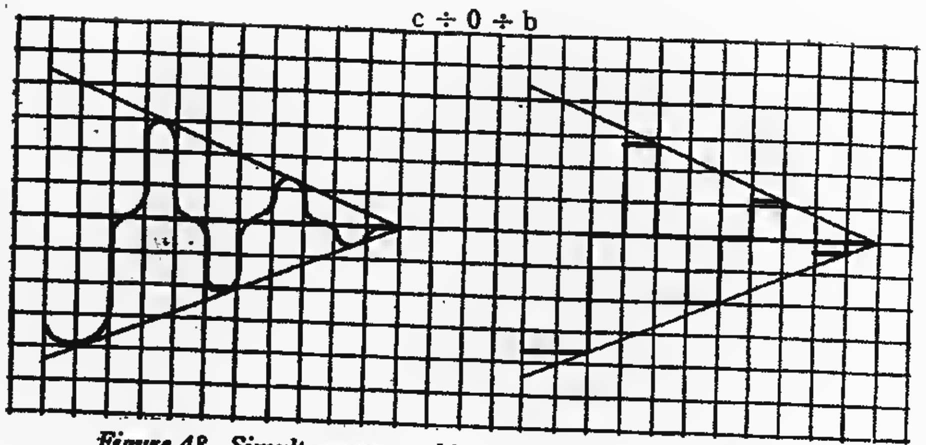


Figure 48. Simultaneous combination of three axes (*concluded*).

As the interval of a pitch level from the primary axis affects tension (gravity effect where P.A. is a gravitational field), resistance may also result from two parallel secondary axes. The complementary parallel axis may be placed either above or below the fundamental axis. The effect of motion through a pair of parallel axes is that of an extended trajectory (delayed, forced inefficiency). In reality it is the usual rotary motion only evolving between the two axis-boundaries.

The correspondences of such motion are: rising and falling, zigzag ascending and descending. Musical form: revolving around alternately progressing points (ascending or descending).

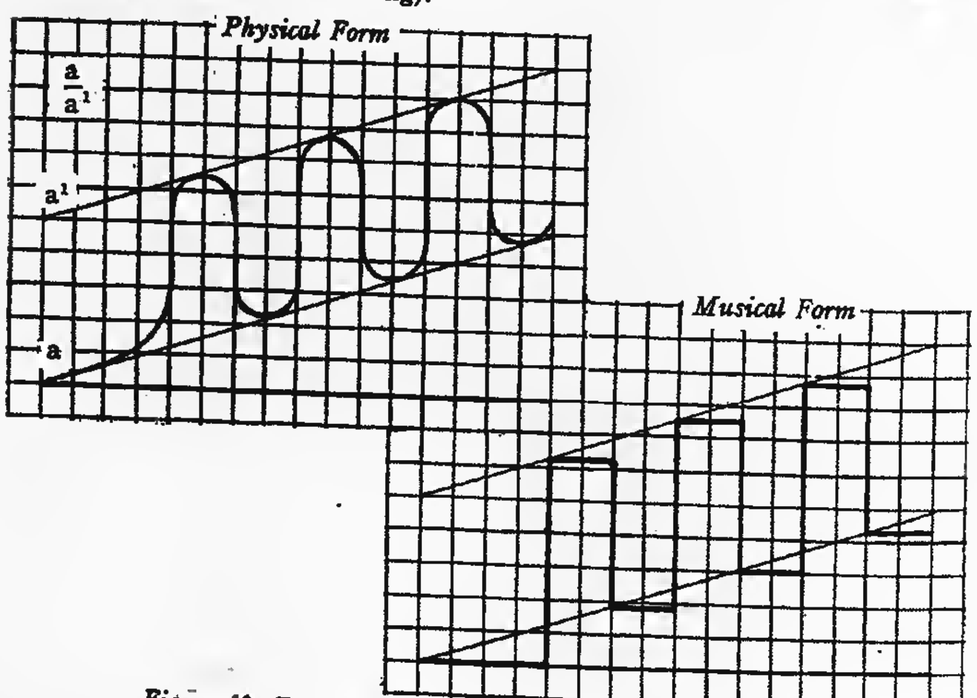


Figure 49. Two parallel secondary axes $\frac{a}{a^1}$ (*continued*).

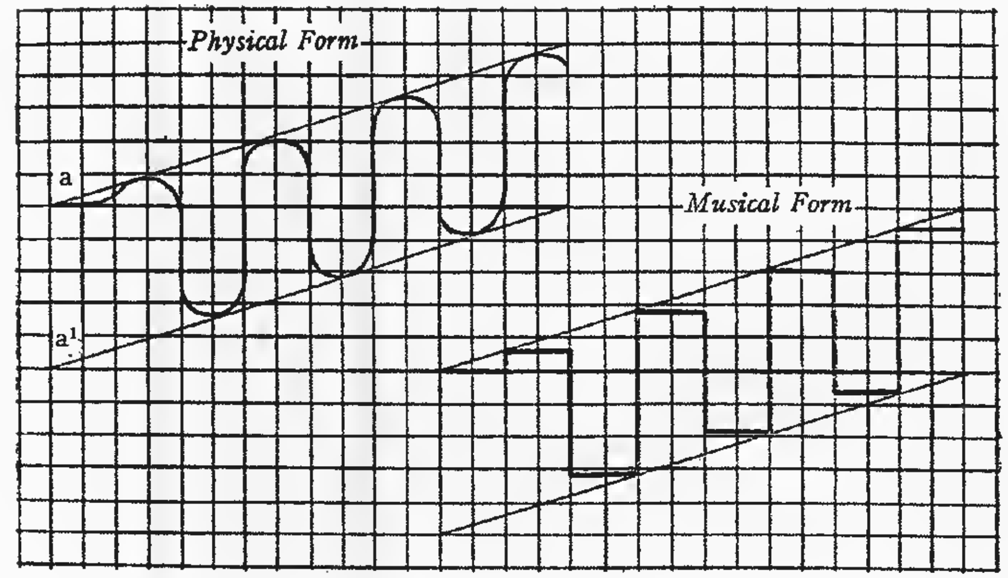


Figure 49. Two parallel secondary axes $\frac{a}{a^1}$ (*concluded*).

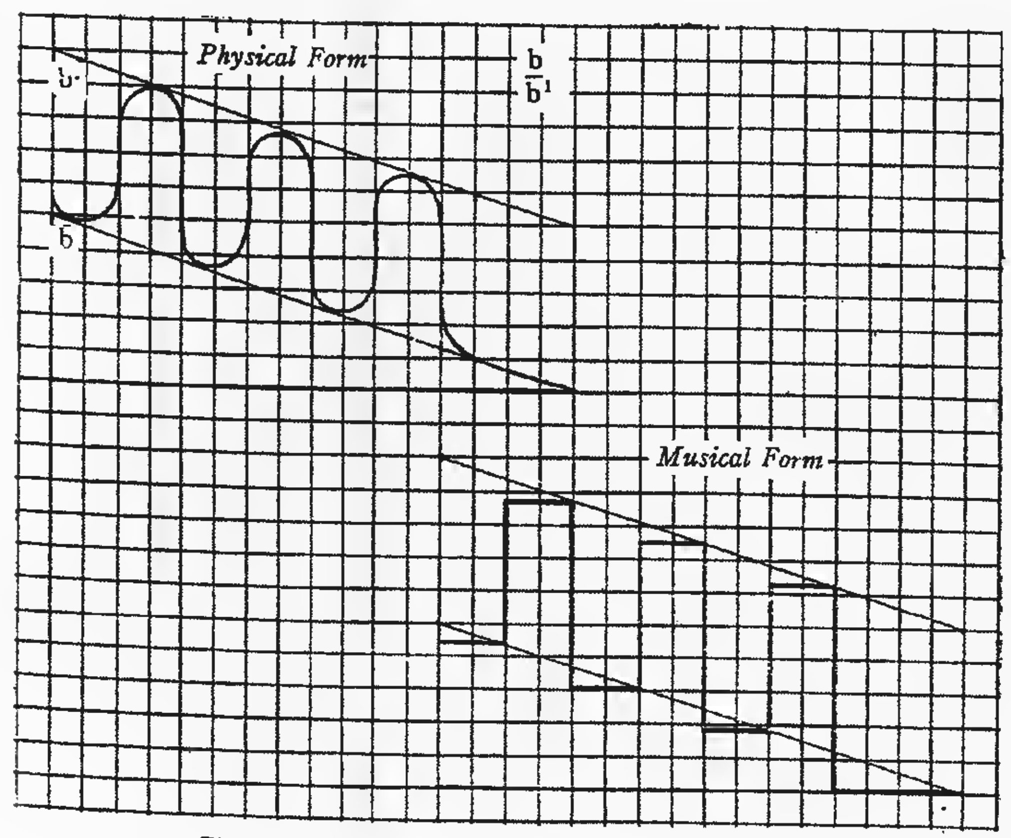
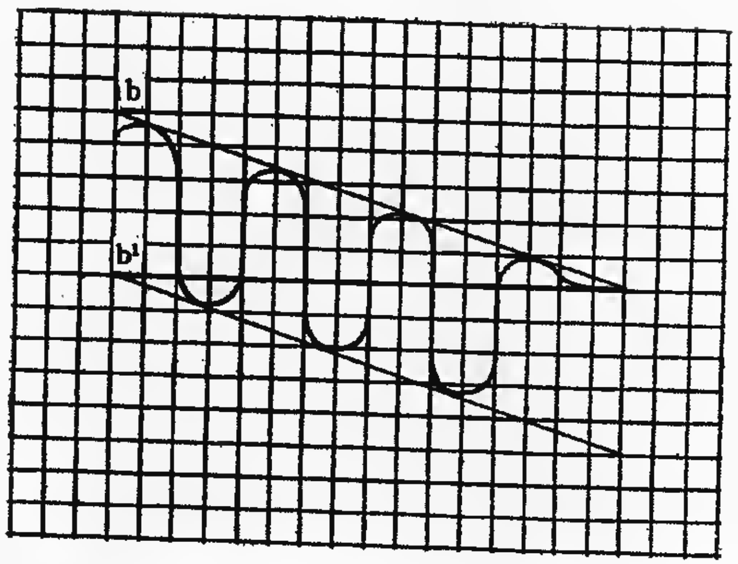


Figure 50. Two parallel secondary axes $\frac{b}{b^1}$ (*continued*).

Physical Form



Musical Form

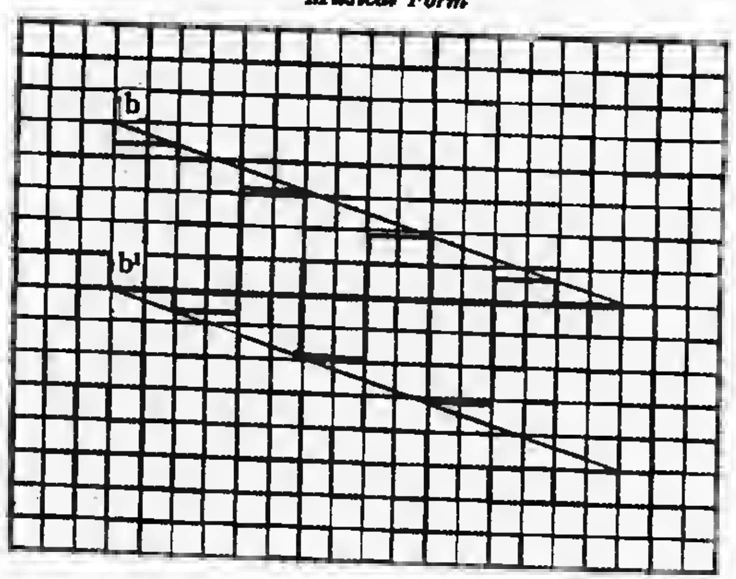


Figure 50. Two parallel secondary axes $\frac{b}{b^1}$ (concluded).

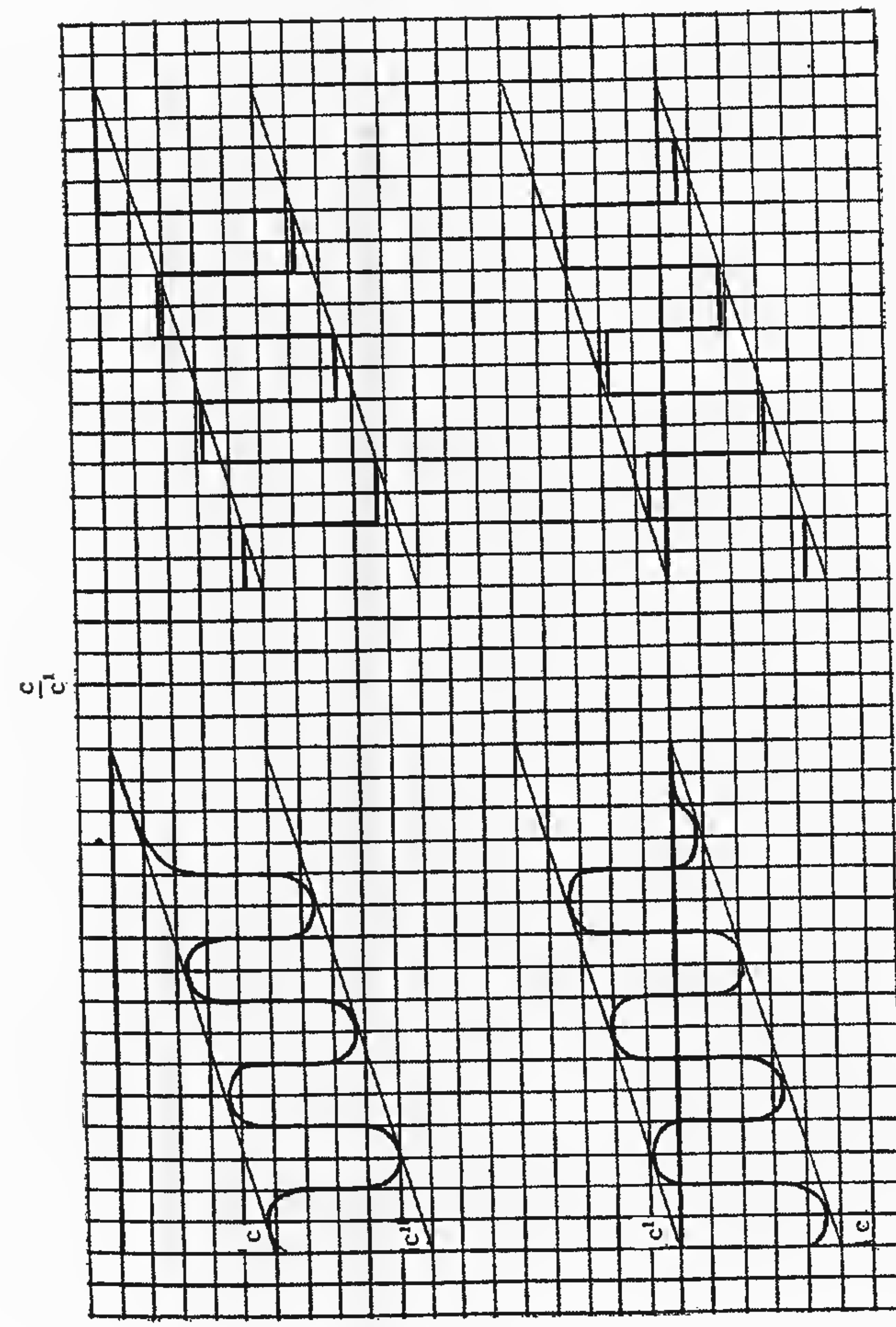


Figure 51. Two parallel secondary axes $\frac{c}{c^1}$.

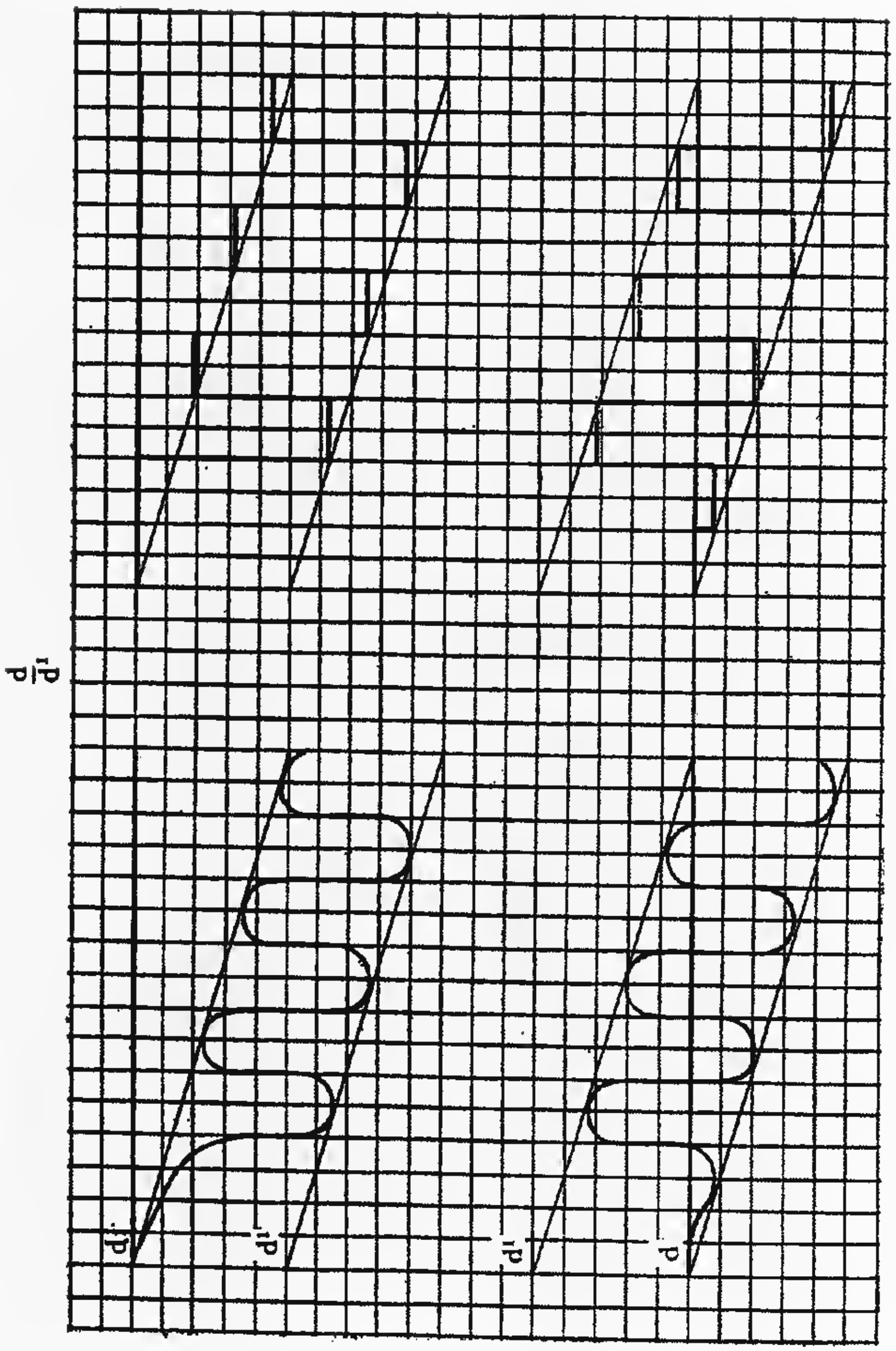


Figure 52. Two parallel secondary axes $\frac{d}{d^1}$.

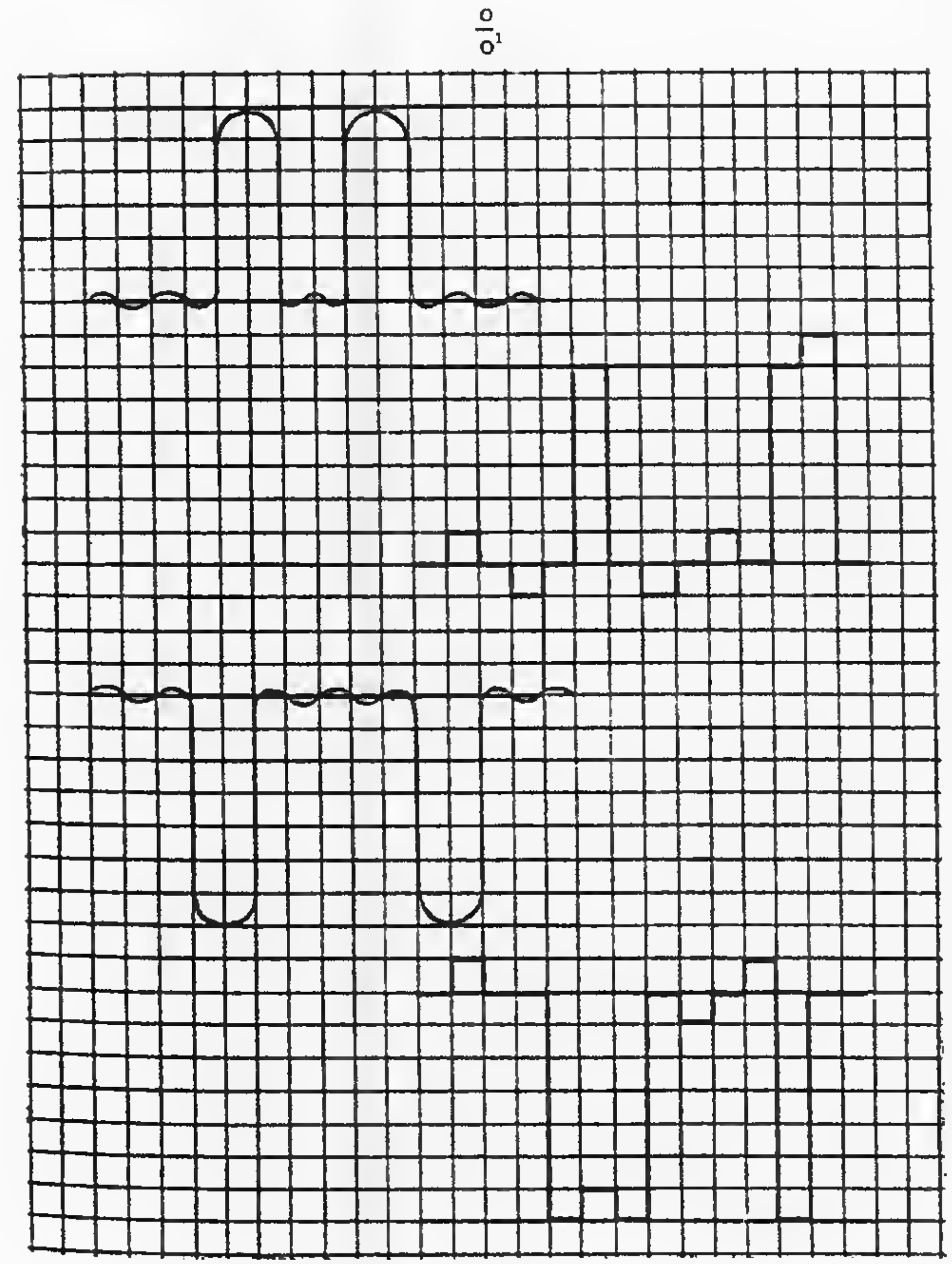


Figure 53. Two parallel secondary axes $\frac{o}{o^1}$.

The 1, 2 and 3 forms of resistance produce the respective degrees of resistance. When more than one form is used in successive portions of melodic continuity, they must follow one another in *increasing* degrees. The opposite arrangement is mechanically inefficient and therefore produces an effect of weakness.

Resistances lead either toward climax or toward balance.

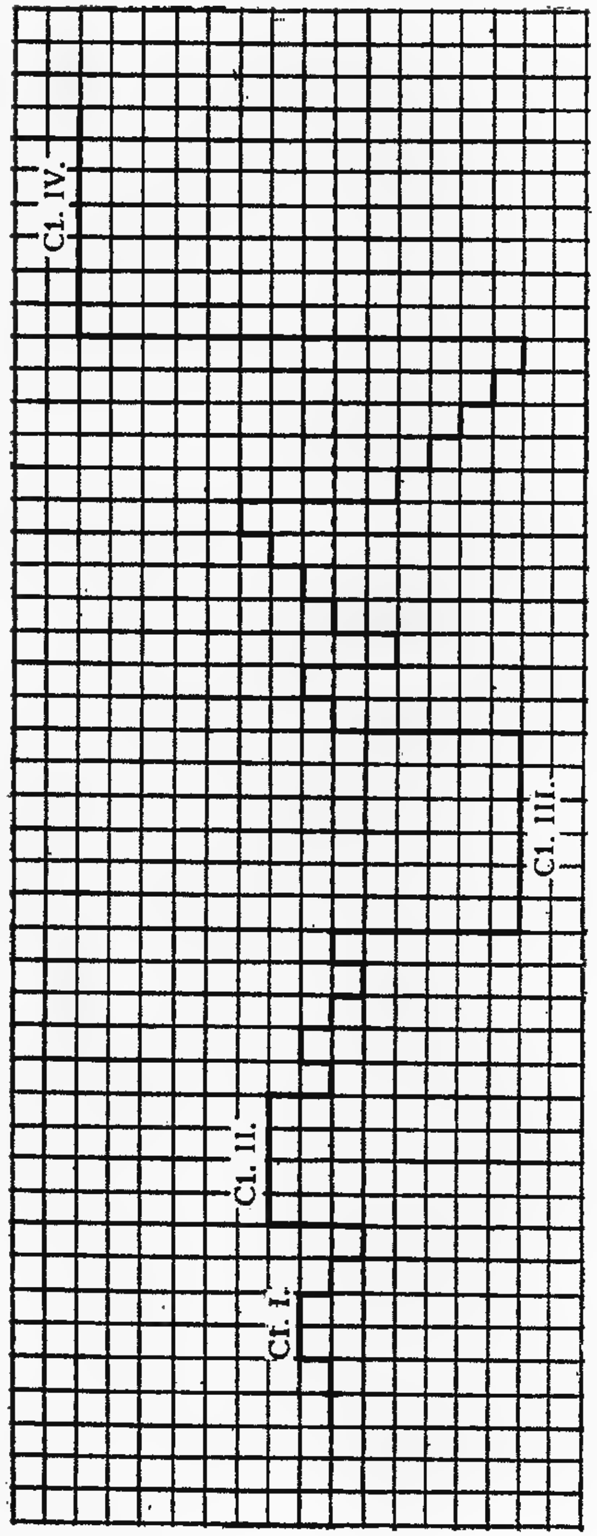


Figure 54. From balance.

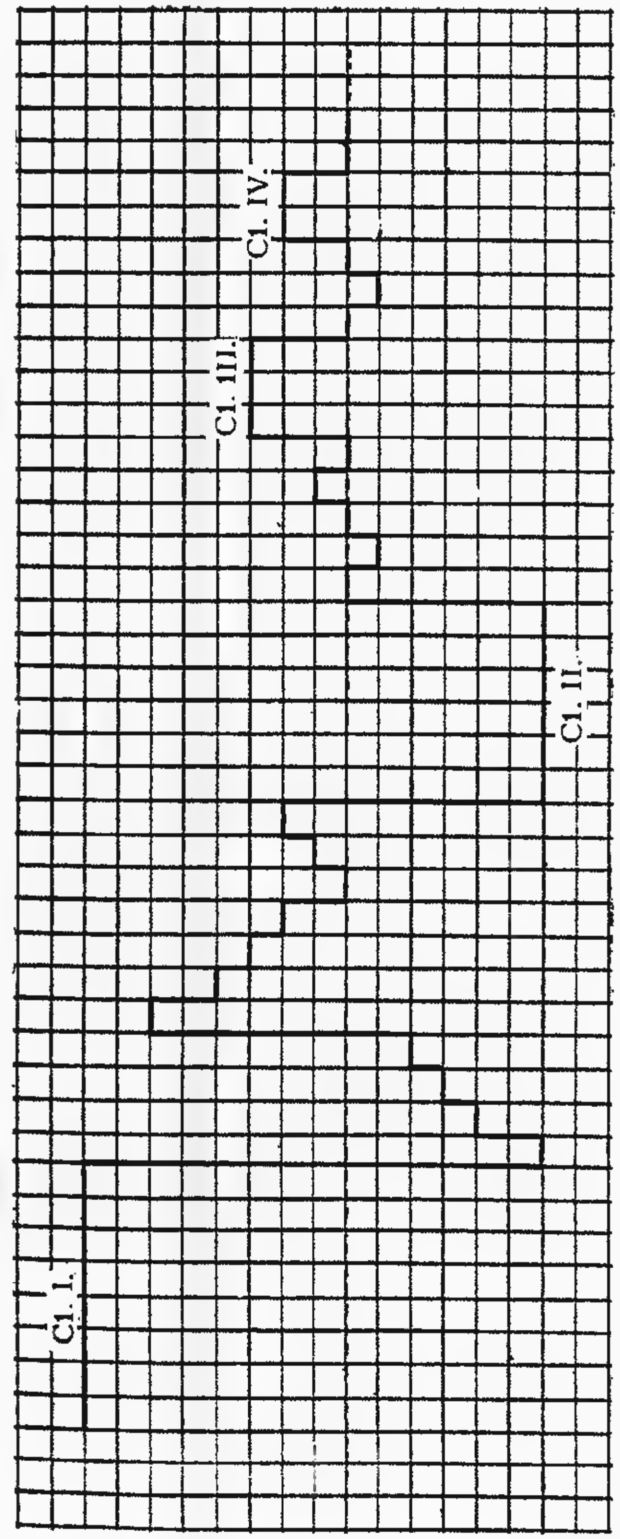


Figure 55. Toward balance.

B. Distribution of Climaxes in Melodic Continuity

The distribution of climaxes in melodic continuity must be arranged with respect to the total duration of such continuity. The relative intensity of climaxes depends on both time and pitch ratios leading toward the respective climaxes. A natural tendency is the expansion of pitch and the contraction of time. These two components mutually compensate each other.

The climactic gain between the two adjacent climaxes takes place when:

1. The pitch-ratio is *increasing* and the time-ratio is *constant;*
2. The time-ratio is *decreasing* and the pitch-ratio is *constant.*

The climactic gain reaches its mechanical maximum when both forms are combined (*increasing* pitch-ratio and *decreasing* time-ratio).

It is practical to save the last effect for the main climax of the entire melodic continuity; use it only when the extreme of exuberance has to be attained.

As a decreasing time-ratio is characteristic of continuity with a group of climaxes, rhythmic material which would appropriately distribute the climaxes must belong to the decreasing series of growth, such as the summation or power series. Smaller number values and in inverse correlation serve as material for the distribution of the pitch ratios for a group of successive climaxes.

This description refers to a trajectory moving towards the main climax and must be inverted for a trajectory moving in the opposite direction.

CHAPTER 5

SUPERIMPOSITION OF PITCH AND TIME ON THE AXES

WHAT IS called "beauty" is the resultant of harmonic relations. In order to obtain a "beautiful" (esthetically efficient) melody, it is necessary to establish harmonic relations between its factorial and its fractional rhythms. This may be achieved by means of a homogeneous series of factorial-fractional continuities.

Rhythm of time durations occurring within the bars must belong to the same series as rhythm of the secondary axes. Naturally, there are hybrid melodies in which fractional and factorial rhythm belong to different series; a homogeneous series is merely an expression of stylistic consistency.

Melodies with structural consistency may be found in nearly every folklore, as well as in the works of composers who synthesized and crystallized the efforts of their predecessors. Beethoven crystallized the melodic style of the "Viennese" school, which at its time was a revolt against counterpoint and polyphonic writing. Bach, in his melodic themes, (in many cases with an odd number of bars), crystallized the efforts of several centuries.

Different styles have different evolutionary velocities. "Jazz" has a very high one, like some specimens of Alpine flora with a very short life-span; jazz has already crystallized its homogeneity. Examples are numerous and may be found more in "swing" *playing* than in the printed copies of the songs.

After the series has been selected, the actual composition of the fractional continuity may be accomplished in two ways:

(1) by using the resultants or the power groups,
(2) by composing freely from the monomials, binomials, trinomials and quintinomials of a given family.

Here is an example of composing fractional continuity in $\frac{4}{4}$ series:

Suppose we have a trinomial of the secondary axes, $a2T + bT + cT$. In this case, $4T = 16t$. To satisfy $16t$ we may use $r_{\underline{4 \div 3}}$, or $\left(\frac{2+1+1}{4}\right)^2$, or any of the variations, i.e., the permutations or the resultants.

A free composition according to (2) may give results identical with some of the variations.

The groups of the $\frac{4}{4}$ series are:

monomial . . . 4
binomials . . . 3 + 1 and 1 + 3
trinomials 2 + 1 + 1, 1 + 2 + 1 and 1 + 1 + 2
the uniform quadrinomial . . 1 + 1 + 1 + 1

Having decided on a2T as (3+1) + (2+1+1), bT as 1+1+2 and cT as 1+3, we obtain $r_{\underline{4 \div 3}}$. By selecting freely various recurrences of the same binomial, like 3+1, we obtain: a2T = (3+1) + (3+1), bT = 3+1, cT = 3+1, or various recurrences of the same trinomial with variations like: a2T = (2+1+1) + (2+1+1), bT = 1+2+1, cT = 1+1+2, we obtain groups that are not identical with the resultants or the power groups.

When a climax is desired, the maximum time value must be placed at the corresponding point of a secondary axis (in *a* at the end, in *b* at the beginning, in *c* at the beginning and in *d* at the end). For instance, if a climax is desired on a2T, it must be the last term of a rhythmic group of this axis. In the $\frac{4}{4}$ series it would be:

a2T = (2+1+1) + (1+3)
or (2+1+1) + (1+1+2)
or (2+1+1) + 4, and the like.

To superimpose a fractional rhythmic group on a factorial group of the secondary axes, means to distribute the points of attack on a pitch trajectory (the path of a moving point).

Let us assume that a group of secondary axes has been constructed with no reference to any particular logarithmic (tuning) system. Placing the pre-selected fractional group above the axes and dropping perpendiculars from the points of attack, we accomplish the distribution of the points of attack (which become the moments of attack) along the pitch trajectory of a hypothetic tuning system.

Example

a2T + bT+cT = (2+1+1) + (1+1+2) + (1+2+1) + (1+3)

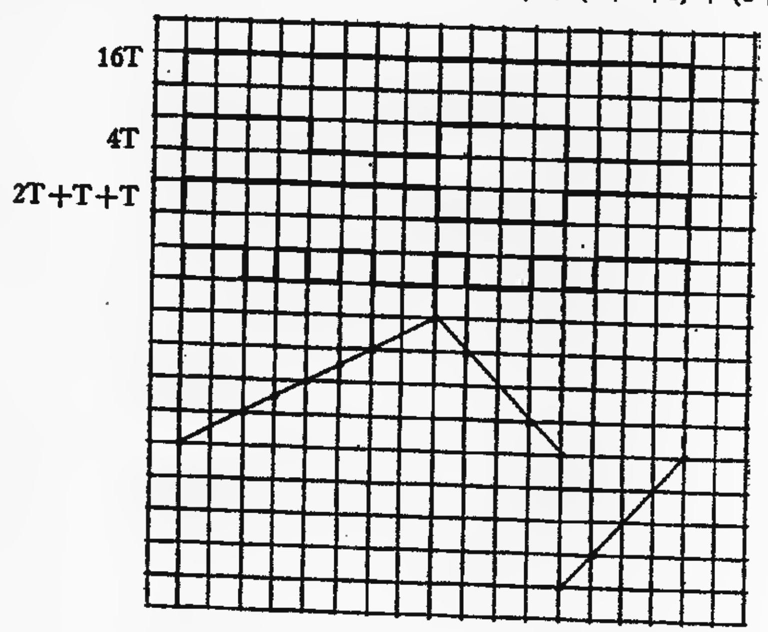


Figure 56. Superimposing a fractional rhythmic group on a factorial group.

Thus, the intersections of dotted lines with secondary axes are the moments of attack on this pitch trajectory.

Here we arrive at the following definition of melody: *melody is the resultant trajectory of the axis-group moving through the points of attack.* Melody, in the academic sense, i.e., with sudden pitch variations within a tuning system, is a *rectangular* trajectory. Melody, in the Oriental conception as well as in any musical actuality, is a *curvilinear* trajectory, i.e., contains a certain amount of pitch-sliding. We shall deal with composition of a melody in the academic sense, as our musical culture leaves the bending of a rectangular trajectory to the instrumental performer.

As the secondary axes form triangles (with respect to the primary axis), two forms of rectangular motion through the points of attack are possible:

(1) ascribed (sine phases).
(2) inscribed (cosine phases).

Although in composing melody a free choice of the two may take place, in balancing melody at its end on *b* or *c* axes, the *ascribed* motion produces an incomplete (i.e., unbalanced) cadence, while the *inscribed* motion produces a complete (i.e., balanced) one. The first one is a device for deviating from balance, i.e., for accumulating tension, *a stimulus for the new recapitulation.*

Examples of rectangular trajectories evolved through the axes of the previous example:

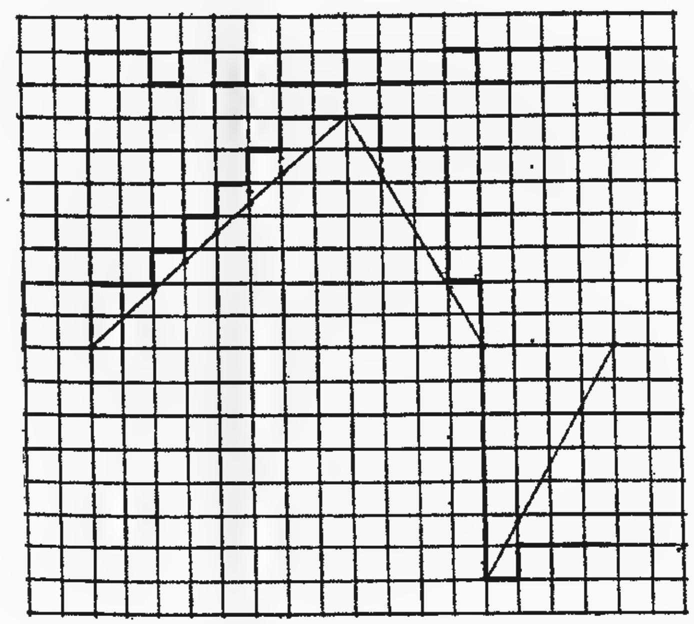


Figure 57. Ascribed motion.

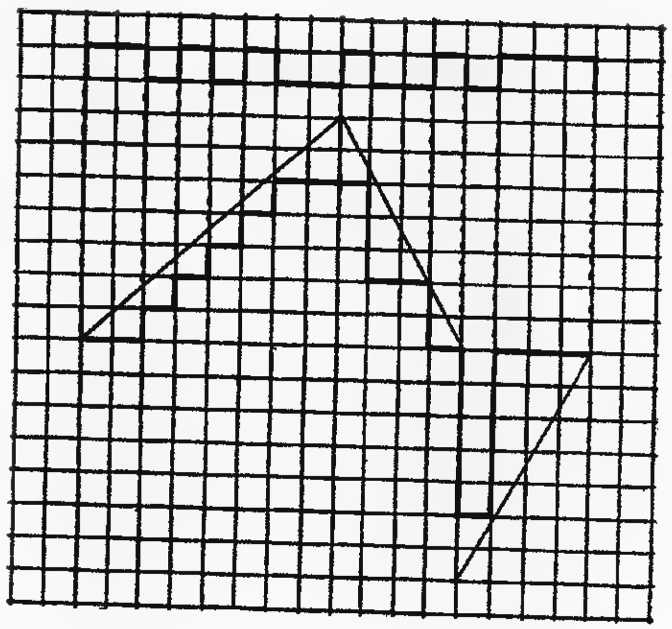


Figure 58. Inscribed motion.

These two potential melodies are totally different as to their pitch progressions. The usual, commonplace composition of pairs varies with respect to the cadence only. Such pairs may be either inscribed or ascribed, but must be identical otherwise; the ending of the first one is ascribed, while the ending of the second is inscribed.

A. Superimposition of Pitch-Rhythm (Pitch-Scale) on the Secondary Axes

Uniform time-intervals (durations) when geometrically projected produce space-intervals, (extensions). Such uniform time scales are *primary selective* systems when $T = r_{a+1}$. When $b \neq 1$ (*i.e.*, is not equal to 1) they become *secondary selective systems* (rhythm-scales).

Uniform pitch-intervals of our tuning system produce logarithms to the base of $\sqrt[12]{2}$ (semitones). The chromatic scale is the primary selective system of pitch in our intonation. Geometrical projection of such a scale is uniformity along the ordinate. Any other pitch-scale within the same tuning system is a *secondary selective system*, (i.e., a derivative of the primary selective system).

It is easy to see that a pitch-time trajectory moving in either ascribed or inscribed form of motion through the *points of intersection* of time (abscissa) and pitch (ordinate) uniformities (primary selective systems), is structurally the simplest form of melody, i.e., a chromatic scale in uniform rhythm.

Here we arrive at the following definition of melody: *melody is a pitch-time trajectory resulting from the intersection of the points of intonation (pitch-units) with the points of attack (time-points) in a specified axis-system.*

When the geometrical points of intersection do not coincide with the pitch-units of a scale, pitch-units nearest to the coincidence-points must be used.

Let us superimpose an Aeolian scale $(2+1+2+2+1+2)$ on the axis-group illustrated in the preceding example. Let us assume $a2P + bP + cP$, i.e., a parallel PT correlation. And let $P = 5$, which in this case gives a symmetric distribution. Further, let pitch *c* be the primary axis. Then a2P extends from *c* to *bb*, bP from *f* to *c*, and cP from *g* to *c*.

Here is the final construction of the axis group:

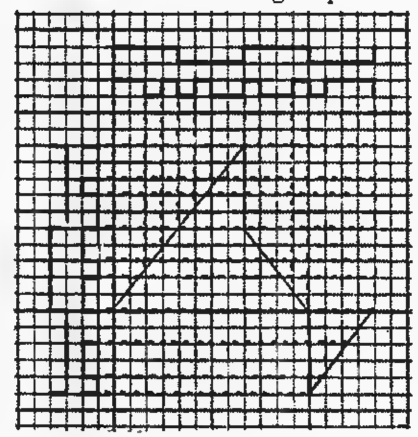


Figure 59. Scheme of the points of geometrical intersection.

This diagram produces a slight deviation from the description given in the text, because of the fact that the scale is so small that it gives deviations. However, this is not essential, as further adjustments follow the scale.

The next step is to adjust the points of intersection to the Aeolian scale. Let us analyze point by point.

If the first point of intersection is *c*, the nearest pitch-unit to the second point of intersection on the Aeolian scale is *d*. Next, we select *eb* as the nearest to the third intersection-point. The fourth falls exactly on *f*. The fifth falls on *f♯* which is not in the scale. In this case either the repetition of *f*, or *g* is available. The next point is nearest to *g*. Through ascribed motion the entire axis *a* would start on *d* and end on *bb*.

As in inscribed motion, *pitch-levels* move *toward* the points of intersection; the first pitch-unit on *b*- axis will be either *f* or *eb*, as the geometrical intersection coincides with *e♮*. The next intersection-point is nearer to *d*. In order to complete *b*– axis through inscribed motion, it will be necessary to consider *c* as the last intersection point. C– axis through the inscribed motion gives points of intersection at *ab* and *c*.

We shall reconstruct now the axis-group with respect to the Aeolian scale, as just described, and draw an inscribed trajectory. This trajectory is the most elementary form of an *actual melody*.

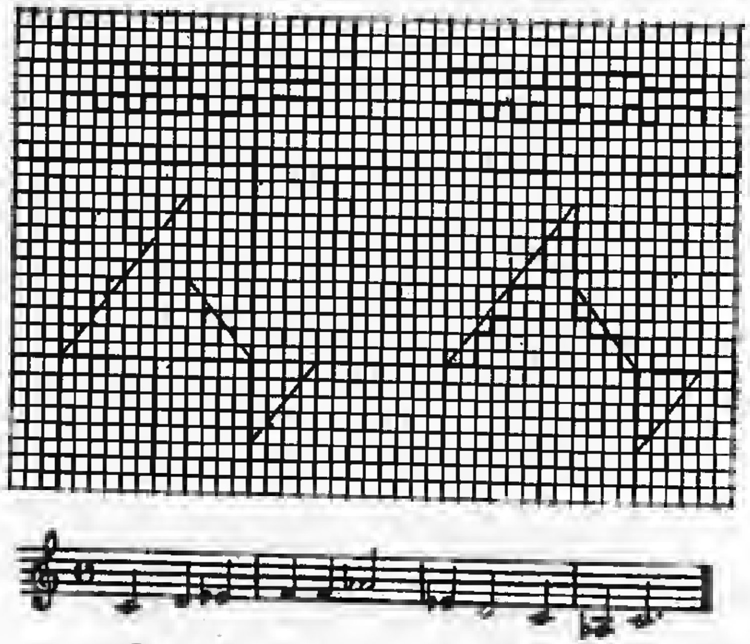


Figure 60. Trajectory of an actual melody.

It would not be difficult to find all other versions, i.e., the ascribed trajectory and the trajectories where either axis may be realized in ascribed or inscribed motion.

Here is a chart of combinations:

Axes:

a	b	c
ascribed	ascribed	ascribed
ascribed	ascribed	inscribed
ascribed	inscribed	ascribed
inscribed	ascribed	ascribed
inscribed	inscribed	inscribed
inscribed	inscribed	ascribed
inscribed	ascribed	inscribed
ascribed	inscribed	inscribed

There are eight versions altogether. After obtaining an actual melody, such melody becomes subject to scale variation, tonal and geometrical expansions and inversions. For instance, the same melody in a "blue" scale would sound:

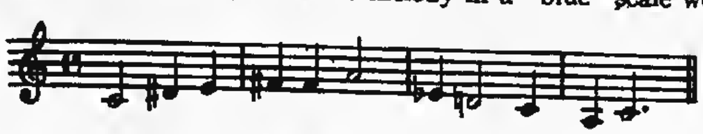


Figure 61. Same melody in a "blue" scale.

Or in a Chinese (2+3+2+2) scale (through translation of the corresponding degrees):

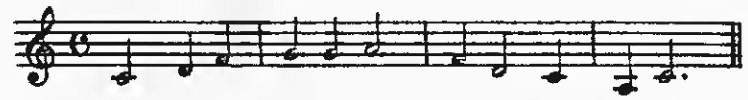


Figure 62. Same melody in a Chinese scale.

Here an allowance has to be made on the first note of the last bar, as the VI does not exist in the Chinese scale (the last degree of the scale, i.e., V, which is *a* substituted).

B. Forms of Trajectorial Motion

The trajectory obtained above was called "the most elementary form of an actual melody" because its form of motion is *simple harmonic* (*i.e.*, motion within the scale). As noted earlier, such a melody cannot be too expressive or dramatic. In order to obtain an expressive melody, it is necessary to build resistances. This cannot be realized without introducing more complex forms of motion.

We shall present now all the trajectorial forms with respect to the zero axis.

(1) Sin (sine) motion with constant amplitude:

Figure 63.

(2) Cos (cosine) motion with constant amplitude:

Figure 64.

(3) Combined sin + cos motion with constant amplitude:

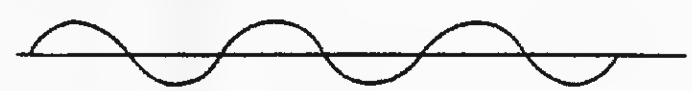


Figure 65.

(4) Combined cos + sin motion with constant amplitude:

Figure 66.

(5) Sin motion with increasing amplitude:

Figure 67.

(6) Sin motion with decreasing amplitude:

Figure 68.

(7) Sin motion with combined increasing-decreasing amplitude:

Figure 69.

(8) Sin motion with combined decreasing-increasing amplitude:

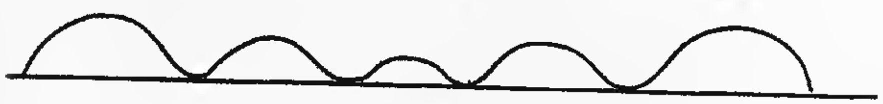


Figure 70.

(9) Cos motion as (5):

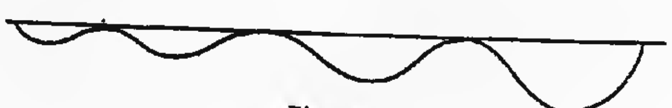


Figure 71.

(10) Cos motion as (6):

Figure 72.

(11) Cos motion as (7):

Figure 73.

(12) Cos motion as (8):

Figure 74.

(13) Combined sin + cos motion with combined amplitude as (5):

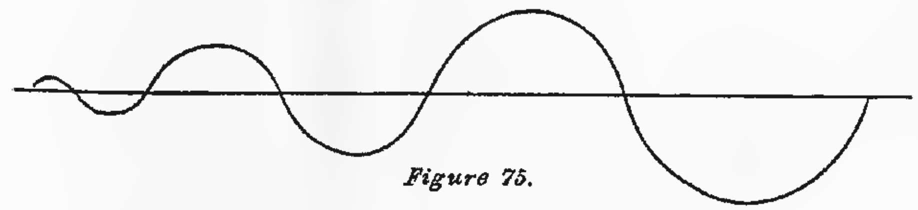


Figure 75.

(14) Combined sin + cos motion with combined amplitude as (6):

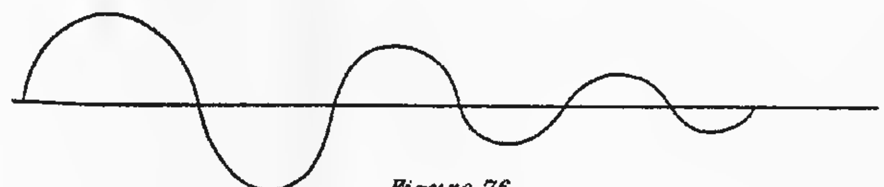


Figure 76.

(15) Combined sin + cos motion with combined amplitude as (7):

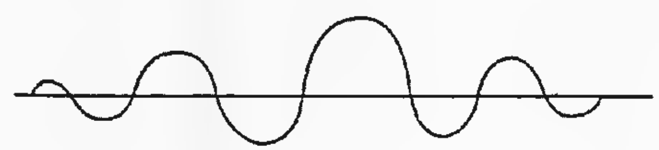


Figure 77.

(16) Combined sin + cos motion with combined amplitude as (8):

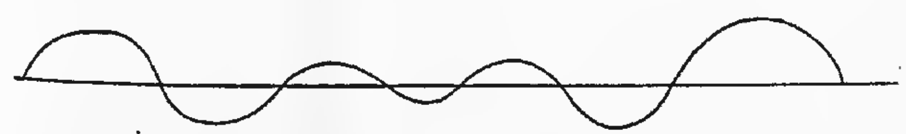


Figure 78.

(17) Combined cos + sin motion with combined amplitude as (13):

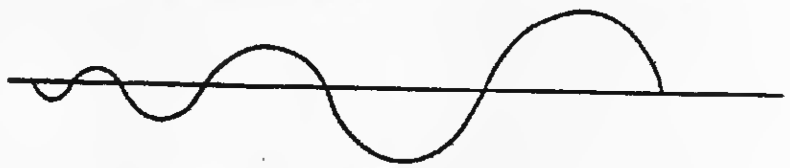


Figure 79.

(18) Combined cos + sin motion with combined amplitude as (14):

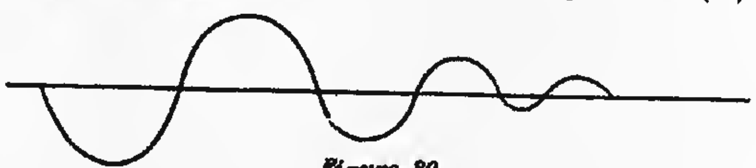


Figure 80.

(19) Combined cos + sin motion with combined amplitude as (15):

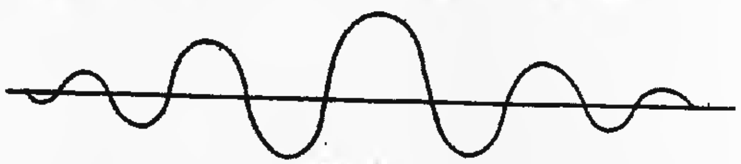


Figure 81.

(20) Combined cos + sin motion with combined amplitude as (16):

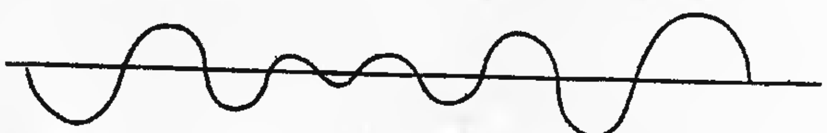


Figure 82.

These twenty versions are merely variations of the two original forms, i.e., (1) and (5). Every cos is ⓓ* of the sin and every decreasing amplitude is ⓑ** of the increasing amplitude.

Further development of these trajectorial forms may be obtained through application of the coefficients of recurrence of the sin, the cos and the growth of amplitudes. For instance, 3 sin + cos + 2 sin + 2 cos + sin + 3 cos on constant amplitude:

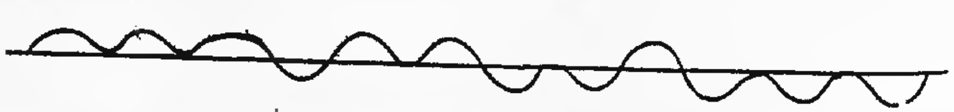


Figure 83.

*The reference is to the fourth position in geometric inversion: the forward-upside down inversion with regard to the original. See Book III. (Ed.)

**The reference is to the second position in geometric inversion: the backward inversion with regard to the original. See Book III. (Ed.)

The same case on increasing amplitude:

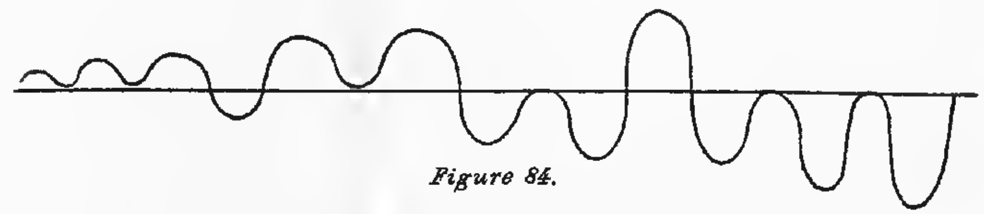


Figure 84.

All these forms being transformed into rectangular trajectories, with respect to a definite intonation (tuning) system, become actual intonation-groups, i.e., melodic forms. For example, a *gruppetto* is sin + cos with constant amplitude.

Including the zero of pitch variation, (absolute zero-axis trajectory), we have the following forms of trajectorial motion:

(1) constant pitch trajectories (repetition on extension).
(2) sin or cos trajectories (one phase motion).
(3) combined trajectories (full period motion or rotation).

Application of various trajectorial forms to *a*, *b*, *c* and *d* axes gives the following correspondences: All the sin of 0 remain sin on all other axes. All the cos of 0 remain cos on all other axes. All the combined forms of 0 with respect to sin, cos and the constancy of amplitude remain respectively the same on all other axes. Zero axis is the only one to be heard. The rest are merely hypothetic lines.

Here are examples of the corresponding translations of a curvilinear sin trajectory into rectangular trajectories of the *0*, *a*, *b*, *c* and *d* axes:

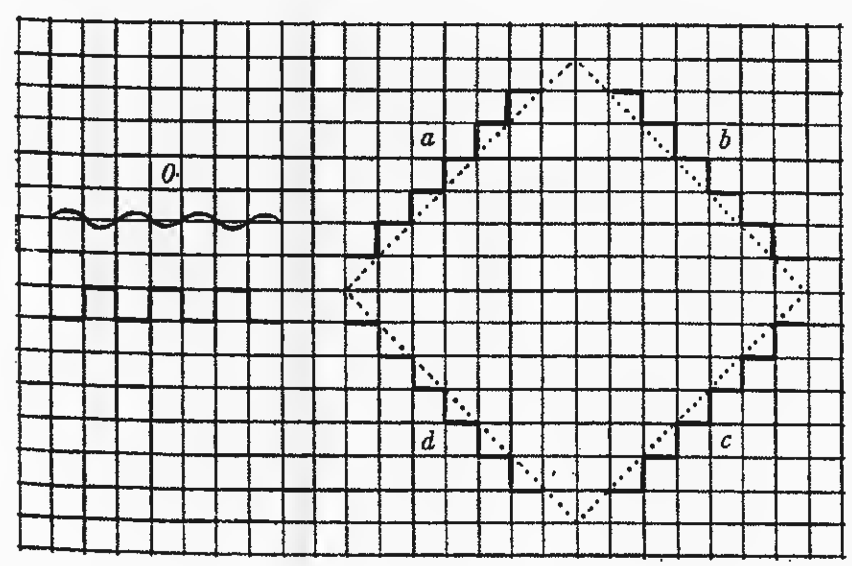


Figure 85. Translation of a curvilinear sin trajectory into rectangular trajectories.

Translations of the cos trajectory:

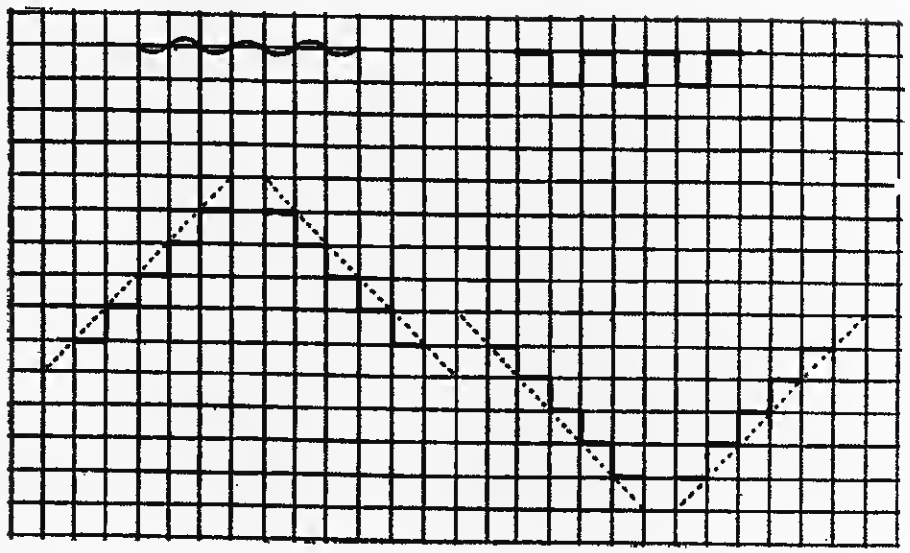


Figure 86. Translation of cos trajectory into rectangular trajectories.

Translations of the combined trajectory:

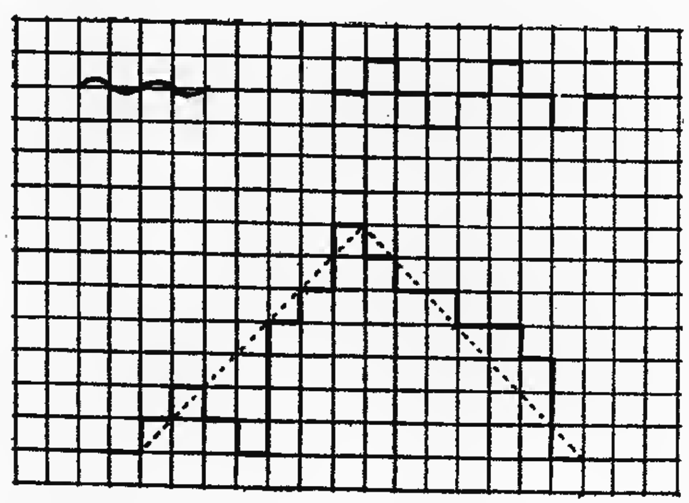


Figure 87. (a) With continuous tangency.

Translation of the combined trajectory:

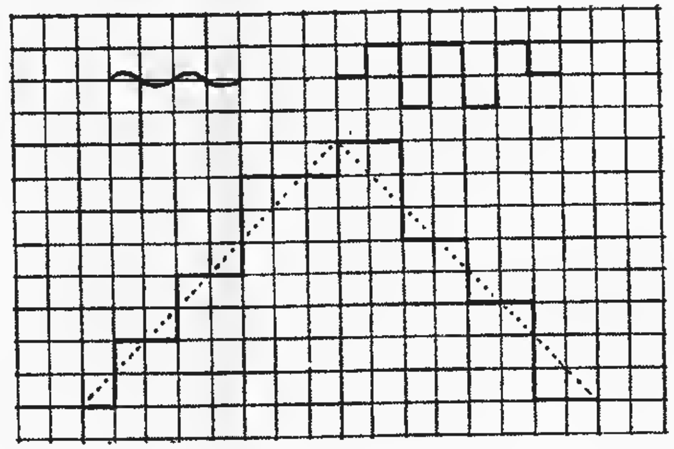


Figure 87. (b) Without continuous tangency.

Figure 87. (a) may be called revolving trajectories.
Figure 87. (b) may be called crossing trajectories.

Deviation of a rectangular trajectory from its corresponding axis signifies inconsistency and lowers the esthetic value of a melody.

An esthetically efficient melody must display, besides consistency, a variety of the forms of motion.

When a trajectory is controlled by the two simultaneous axes (fundamental and complementary), the points of attack may fall on either axis according to the form of alternation.

Example:

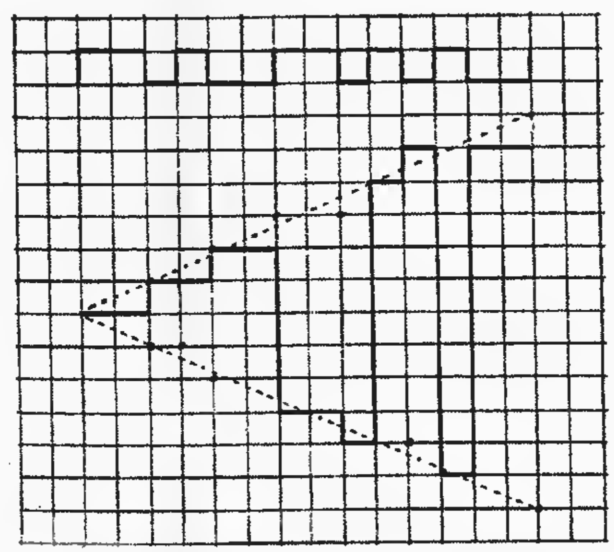


Figure 88. Trajectory controlled by two simultaneous axes.

The form of alternation is subject to distribution, i.e., rhythm.

An example of analysis of the trajectorial motion in J. S. Bach's Two-Part Invention, No. 8:

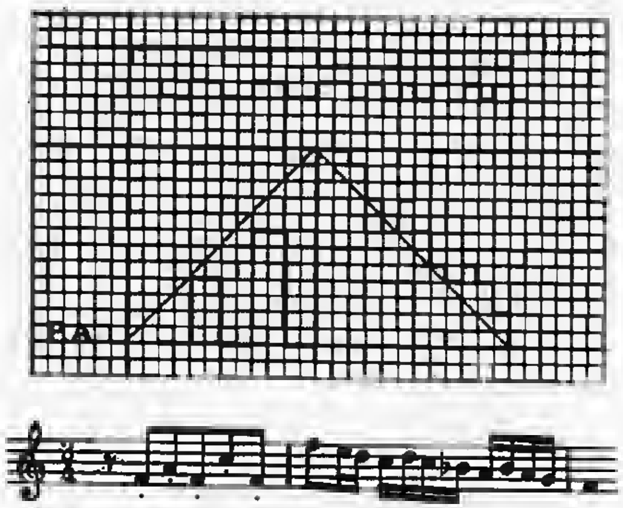


Figure 89. Trajectorial motion in Bach's Two-Part Invention, No. 8.

This trajectory has a primary axis defined by its first, last and two intermediate attacks. The group of the secondary axes is: $\frac{a}{o} + b$. The pitch and time ratios are uniform, ie., $\frac{a}{o}PT + bPT$. The first attack of *b* is a climax. The form of motion on $\frac{a}{o}$ is sin motion with increasing, (centrifugal), amplitude. The alternation of the points of attack on the two conjugated axes is uniform. The form of motion on *b* is combined (sin + cos) and has a constant amplitude. It is ascribed with respect to *b*. The effect of revolving due to the combined form produces a resistance and delays the balance. This melody would lose most of its esthetic value if the o-axis were eliminated (loss of resistance moving toward the climax), and the b-axis were to have one-phase motion.

At this point it would be very advisable for the reader to make a thorough analysis of the outstanding, as well as of the deficient, themes taken from existing music. This procedure must follow all sections of the theory of melody. A precise statement must be made on each item regarding the form and measurement.

Although a theme of any dimension (duration) may be constructed to full satisfaction, it is more practical in most cases to compose continuity out of a short original structure. Memory is very limited and the latter will produce an effect of greater unity.

After having acquired enough experience in analysis, one may start composing melodies according to this theory. Success depends upon thorough knowledge of all the preceding material—and the ability to think!

CHAPTER 6

COMPOSITION OF MELODIC CONTINUITY

WHEN MELODIES are constructed, i.e., plotted, according to the techniques described in earlier chapters of this discussion of the theory of melody, the melodies will be found to have such properties as render them susceptible to the following treatments and techniques:

1. Permutability of the secondary axes with their respective melodies in time continuity.
2. Permutability of the individual pitch-units (preferably through circular permutations) pertaining to one individual secondary axis.
3. Geometrical convertibility of the entire melody.
4. Geometrical convertibility of portions of melody pertaining to the individual secondary axes or any groups thereof.
5. Tonal expansion of the entire melody.
6. Tonal expansion of portions of melody pertaining to any individual secondary axis or portions thereof. In this case different axes may appear with different coefficients of expansion.
7. Combined variations of geometrical inversions and tonal expansions applied to the entire melody.
8. Combined application of geometrical inversions and tonal expansions applied to the portions of melody pertaining to individual secondary axes or any combinations thereof. In this case different coefficients of expansion may be combined with different geometrical inversions.

Consequently, melodic continuity may be composed through any of the above-mentioned forms of variation or any combination thereof.

Here is an example of the quantitative development of melodic continuity from the original theme:

Let us take a trinomial axial combination, a, b, c. Each of the individual axes has four geometrical inversions. Thus, the number of combinations of the three axes that may be used in identical or different geometrical inversions equals $4^3 = 64$. This number refers to one constant E. If any of the individual axes appears in three forms of tonal expansion, the entire quantity will be $64^3 =$ $= 262,144$.

The following is a method of indicating a secondary axis where the geometrical positions and the coefficients of expansion are specified. For example, an axis *a* in position ⓒ in the second expansion (E_2) may be expressed like this:

$$a©E_2$$

A trinomial axial-combination consisting of a, b and c axes, with specified time and pitch ratios, and the geometrical positions and coefficients of expansion, assumes the following appearance:

Time ratios: $2 + 1 + 1$
Pitch ratios: $1 + 2 + 3$
Geometrical positions: ⓓ, ⓐ, ⓑ
Coefficients of expansion: E_0, E_3, E_1
$aP2T_{ⓓ}E_0 + bT2P_{ⓐ}E_3 + cT3P_{ⓑ}E_1$

This method of indication emphasizes not the axial structure alone, but the pitch-units (intonation) as well. For example, a melody in its third displacement, on axis a, in position ⓓ, in the third expansion, may be expressed as follows:

$$a_{ⓓ}E_3d_3$$

When this method is systematically applied, the sequence of the different displacement phases, with regard to consecutive secondary axes, may assume different forms of distribution. For example, it may start with the first phase within the first axis, with the second phase within the second axis, with the third phase within the third axis, etc. It may follow a rhythm of any resultant or any of the series of growth:

(a) $d_3 + d_1 + d_2 + d_2 + d_1 + d_3$
(b) $d_0 + d_1 + d_3 + d_6 + d_{11} + \ldots$

Naturally, the rhythm for such variations of motif depends upon the number of pitch-units within the motif.

Ability to produce expressive melodies (themes) does *not* make a great composer, but ability to produce an organic continuity out of original thematic material *does!*

Going as far back as the strict style of counterpoint written to a *cantus firmus*, we find that the composition of continuity is based on *uniform factorial periodicity:* the theme regularly appears in different voices and that keeps the music moving.

In all elementary homophonic forms, continuity is based on a composition of *biners*, $(a_1 + a_2)$, usually similar structures with different endings, consisting of $4 + 4$ or $8 + 8$ bars. Next comes the method of *terners*, i.e., $a_1 + b + a_2$, involving the introduction of new material in the center term.

The most advanced forms in the past were offered first by J. S. Bach; he used a sequence of biners in contracting geometrical progressions (see Fugue V, Vol. II, *The Well-Tempered Clavichord*). In his case, it meant that a greater overlapping (stretto between the theme and the reply) occurred with each succeeding announcement. In Beethoven's case, it meant a continuous breaking up of the original biner.

All these forms of continuity, (in the past), are rigidly attached to the $\frac{2}{2}$ series.

Richard Wagner built his continuity according to the script, i.e., the operatic libretto. Although he wrote these libretti himself and although he was quite skilful at it, his musical continuity suffered greatly from this syntactic dominance.

Wagner's faults were then adopted as virtues by Scriabine and by others. Literary influence, together with linguistic, logic and syntactic (propositional) technique were the factors that delayed, if they did not prevent, the sound development of the forms of musical continuity.*

Forms of musical continuity are purely quantitative and pertain to motion. They are biomechanical, i.e., they are forms of growth. When they grow normally, they survive better. It is like pure Darwinism: the struggle for existence, the survival of the fittest. A star-fish is not "just a pretty pentagon" but an organic form evolved through the necessity of efficient functioning.

Many an unpretentious melody is appealing, i.e., esthetically efficient, due to the fact that within the eight-bar structure certain processes evolve in a very consistent manner. It happens quite often that the efficiency of structure is greater in smaller portions and smaller in greater portions.

These bio-mechanical forms are primarily concerned with three basic factors:

(1) Symmetric development, i.e., the axis-inversion.
(2) The ratio of growth, such as summation.
(3) Movement with respect to tension and release resulting in balance, i.e., an arithmetical or a geometrical mean.

Growth along the axis of symmetry (compare the case of the human body with its growth along the spinal cord) is a continuity formed by geometrical inversions of the original structure or of its portions (melody) along the primary axis. The regularity of recurrence of the different inversions is subjected to rhythm. Pitch expansions (tonal and geometrical), combined with their geometrical inversions, may be used as components of musical continuity.

The most fluent form of continuity results from symmetric growth along the time-axis. This is the most complete form of continuity as it exemplifies birth, growth, maturity, decline and death—all in one process. To accomplish this in melody it is necessary to split the original structure into a number of elements (such as bars or secondary axes), to show these elements in their gradual addition, and then in their gradual subtraction.

Suppose we have a three-bar structure and split it into a, b, c elements. Gradual addition of the elements will give: $a + ab + abc$. Gradual subtraction of the elements will give: $abc + bc + c$. The combination of the two forms offers—the time-axis on abc. The entire continuity will be this:

$$a + ab + abc + bc + c$$

Examples:

The original structure split into three elements:

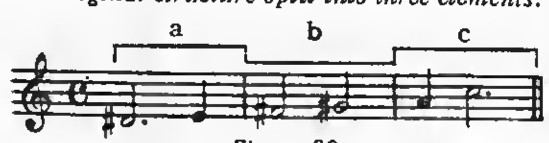


Figure 90.

*See the definition of program music in the *Oxford History of Music*, Vol. 6, page 3, which reads: "Program music is a curious hybrid, that is, music posing as an unsatisfactory kind of poetry." (J.S.)

Continuity composed through the time-axis.

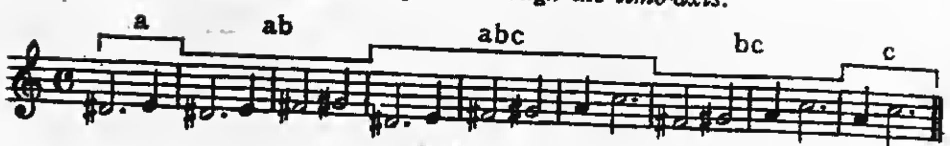


Figure 91.

The process of summation may pertain to the preceding procedure, as well as to factorial ratios of the secondary axes, or the number of individual attacks.

As an example of summation through the first summation series based on the time axis, let us take an eight-bar structure and split it into *a b c d e f g h* elements. The continuity will have this form:

$$a + ab + abc + abcde + abcdefgh + defgh + fgh + gh + h.$$

The entire structure moves across itself through its own axis, while time goes on.

The next point is obvious. Using the same series for the T of the secondary axes, we obtain:

$$T + 2T + 3T + 5T + 8T + 5T + 3T + 2T + T$$

whatever axis (o, a, b, c or d) each term may represent.

Summation through the number of individual attacks may be found in many melodies. Take the popular song by Arthur Johnston *One, Two, Button Your Shoe** for instance.

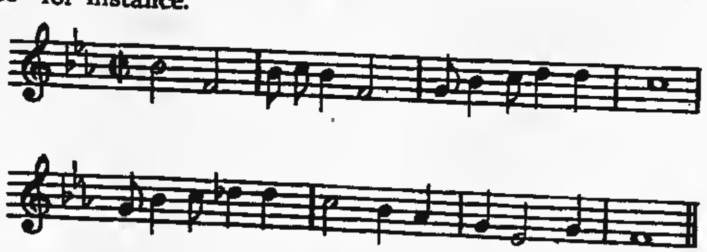


Figure 92

The first eight bars give the following summation of attacks: $2 + 4 + 6 + 12$, i.e., $2 + 4 = 6$, and $6 + 6 = 12$. It means that there are four distinct sub-structures, each containing the number of notes in this particular summation, carried out with absolute precision.

It is important not to confuse the *rhythm of attacks* with the *rhythm of durations.*

The method of summation is very flexible, and with a little initiative one may secure a great deal of variety.

In the song, *But I Only Have Eyes for You*, you find the following scheme of attacks: $6 + 9 + 6 + 3$. This is an *incomplete* form of $3 + 6 + 9 + 6 + 3$, where the central term is the result of summation $3 + 6 = 9$. At the same time, the central term becomes an axis of time symmetry.

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With respect to tension and release, movement resulting in balance may refer to factorial or fractional time-rhythm as well as to the rhythm of the number of individual attacks. Use of the arithmetical mean is the most common device in this case.

An *arithmetical mean* is the quotient of the division of the sum by the number of elements. With two elements, a and b for example, it equals $\frac{a+b}{2}$. Musical intuition has a certain amount of precision, and in some cases these arithmetical means come out with a very good approximation. For example, in the first $3\frac{1}{4}$ bar structure of the song *Stormy Weather*, the first sub-structure has three attacks, the second has seven, and the third has four. The *exact* number for the last sub-structure would be $\frac{3+7}{2} = 5$, not 4. This is a very good approximation, for there is only 20 percent of error; yet you get a greater satisfaction by adding one more attack. Try it by making a triplet out of the two eighths at the beginning of the third bar.

This procedure is analogous mechanically to underbalancing-overbalancing-balancing; or to overbalancing-underbalancing-balancing.

The following graphs and music serve as examples of composition of melodic continuity. Each example given is a complete musical composition written for an unaccompanied instrument. This art has been greatly neglected today. In the 17th and 18th centuries, composers possessed enough technique to accomplish such tasks. J. S. Bach wrote many outstanding works, even sonatas, for violin or viola da gamba alone. Today only a very few high-ranking composers—such as Paul Hindemith (*Suite for Viola* alone) or Wallingford Riegger, an American, (*Suite for Flute* alone, in four movements)—have dared to write a whole opus for an unaccompanied instrument.

The three compositions I offer here are constructed from the scales of the first group. Each graph represents a theme originally plotted. Musical examples are complete compositions developed by means of variation.

The notation is as follows:

M—the entire melody

a, b, c, d—portions of melody pertaining to the respective axes

$\frac{a^1}{a}, \frac{b^1}{b}, \frac{c^1}{c}, \frac{d^1}{d}$

or

$\frac{a}{a^1}, \frac{b}{b^1}, \frac{c}{c^1}, \frac{d}{d^1}$ —parallel binary axes

ⓐ, ⓑ, ⓒ, ⓓ —geometrical positions of M or of the respective axes

$p_0, p_1, p_2, \ldots$—permutations of pitch-units of M or of the respective axes

$E_0, E_1, E_2, \ldots$—tonal expansions of M or of the respective axes

In this form of notation, each original melody (the theme of the composition) appears as $M_{ⓐ}\, p_0 E_0$.

It is advisable to be conservative in planning a complete melodic continuity, as application of too many variations at a time (i.e., p, E and the geometrical positions) may increase the complexity of the entire composition beyond the listener's grasp.

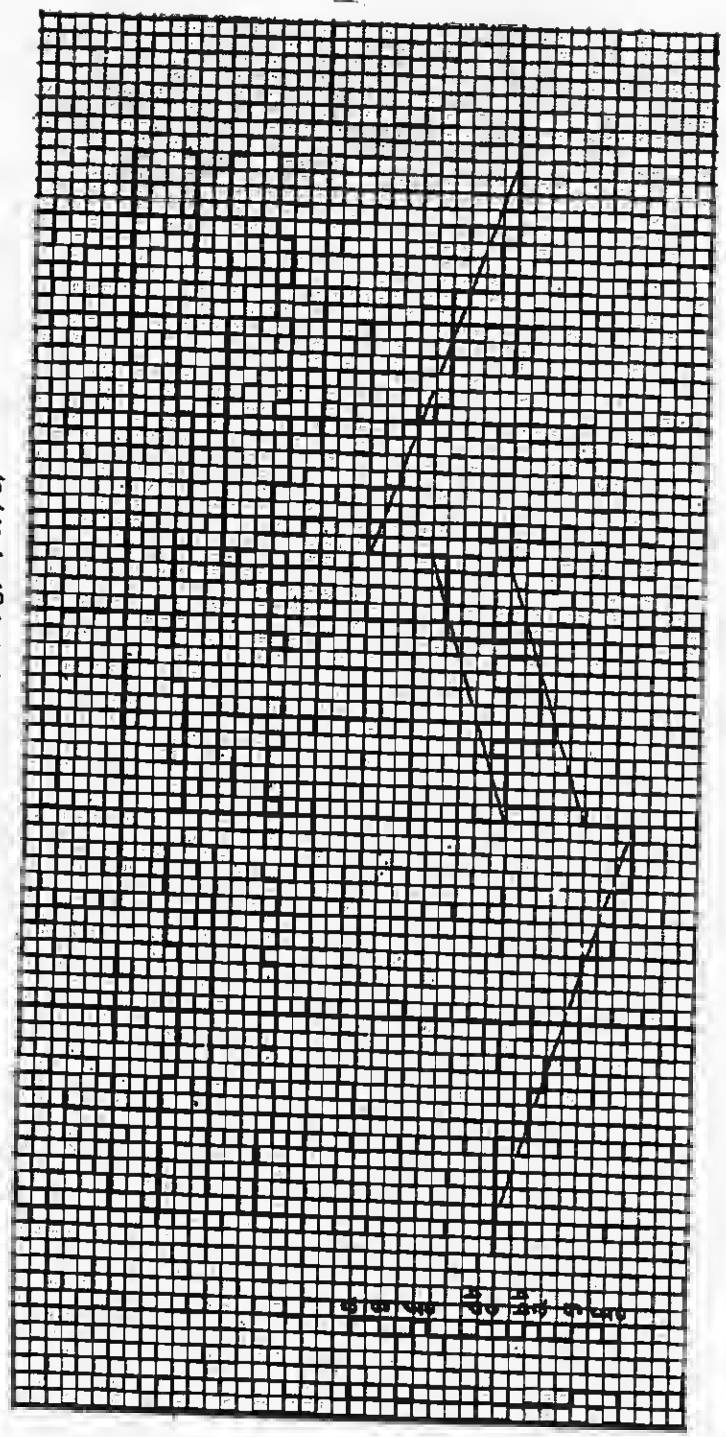


Figure 93. Plotted melody.

$[M@\ E_0p_0] + [\frac{c^1}{c}@\ E_0p_0] + [d'@\ E_0p_0] + [b@\ E_0p_0] + [d@\ E_0p_1] +$
$[b'@\ E_0p_2] + [\frac{c^1}{c}@\ E_0p_3] + [M@\ E_0p_0] + [\frac{c^1}{c}@\ E_0p_4] + [d@\ E_0p_5] +$
$[d'@\ E_1p_0] + [M@\ E_0p_0]$

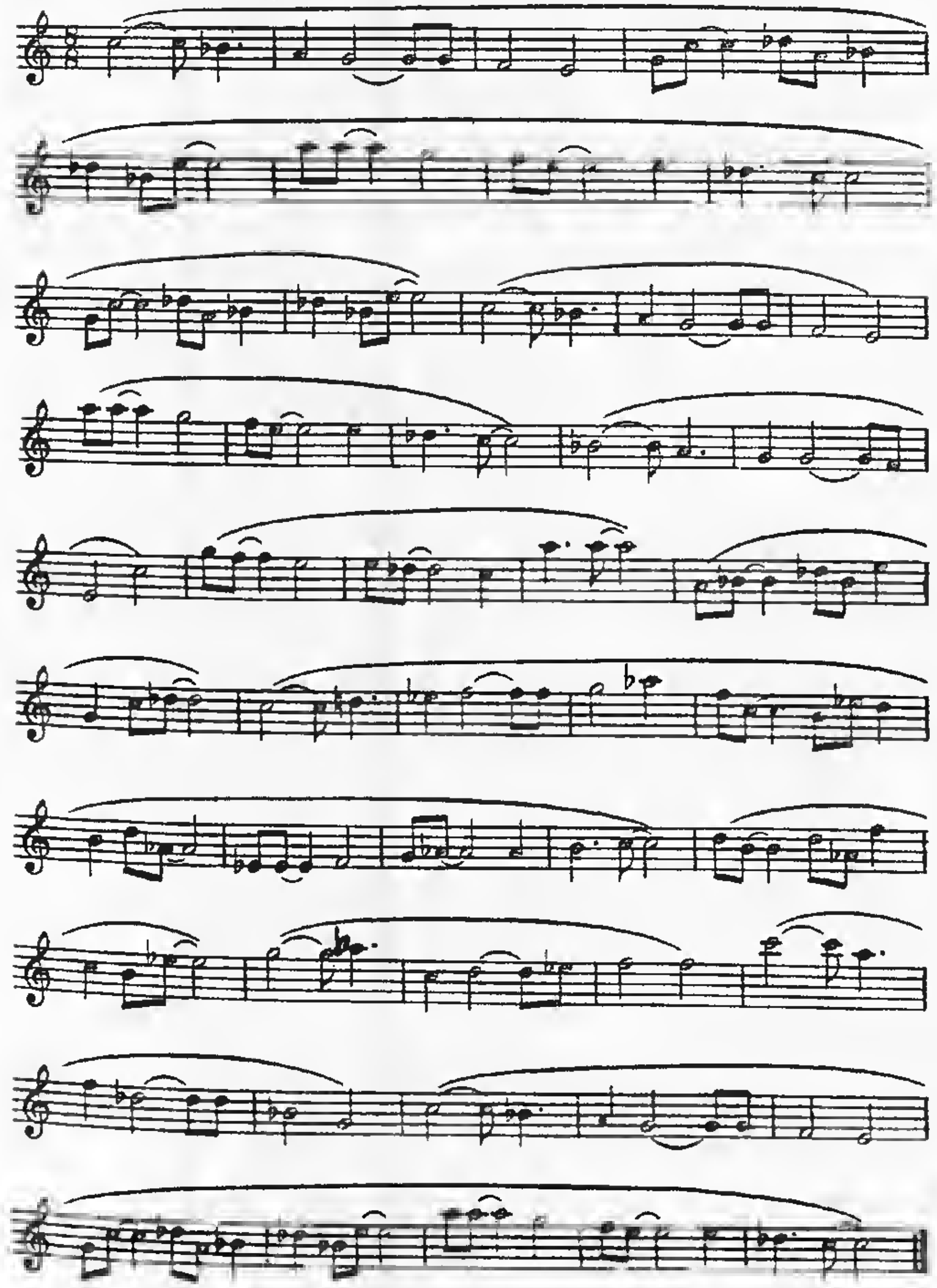


Figure 94. The melody of Figure 93.

c, d, e♭, f♯, g, a, b♭, c, d, e♭, f♯, g, a, b♭, c, d, e♭, f♯, g

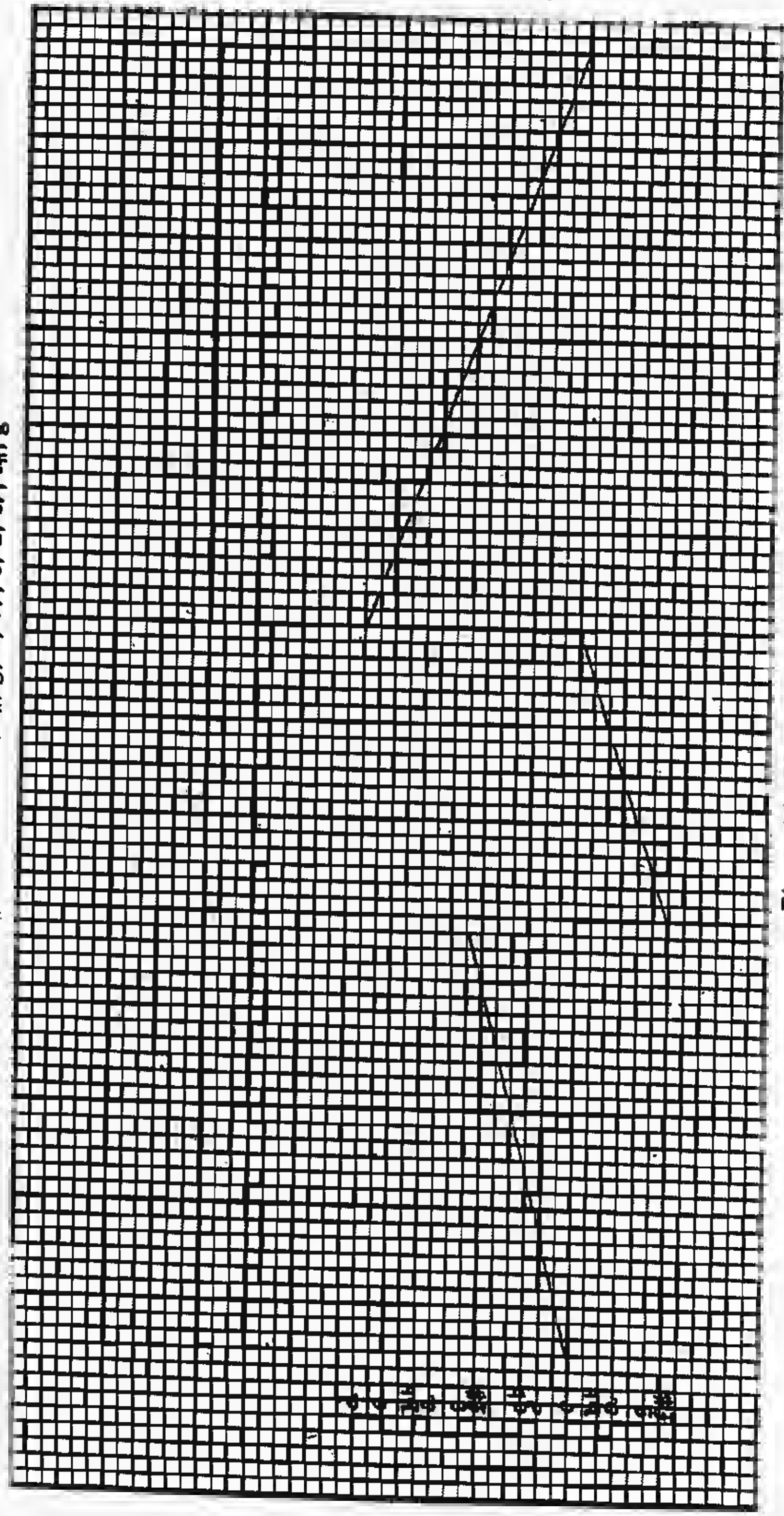


Figure 95. Plotted melody.

$[M_{a}\ E_0p_0] + [M_{d}\ E_0p_0] + [c_{a}\ E_1p_1] + [b_{b}\ E_0p_2] + [a_{a}\ E_1p_2] +$
$[b_{c}\ E_0p_4] + [a_{a}\ E_0p_5] + [b_{a}\ E_0p_0]$

Typos
Some notes should be "G"

Figure 96. The melody of Figure 95.

CHAPTER 7

ADDITIONAL MELODIC TECHNIQUES

IN THIS chapter I have grouped a number of brief discussions of other facets of the process of building melodies, facets which will be of use to the practical composer.

To begin with, there is the question of the use of symmetric scales in melody-making.

A. Use of Symmetric Scales

First: the intervals between the tonics in all settings (i.e., the original, the first contraction and the final contraction, the latter being an equivalent of the scales of the third group) determine the pitch-ranges. The first tonic corresponds to the primary axis.

Secondly: in using the first contraction, we acquire an overlapping of the secondary axes.

Thirdly: the following correspondence of the secondary axes takes place in the original setting.

The a-axis of the lower section is the c-axis of the adjacent upper section. The b-axis of the lower section is the d-axis of the upper. The c-axis of the lower section is the a-axis of the section below the lower section. The d-axis of the lower section is the b-axis of the section below the lower section.

Considering this, it is practical to conceive the axial group in such an arrangement that the first tonic is flanked by other tonics (still referring to the original setting). This permits a unified reading of the axes.
For example:

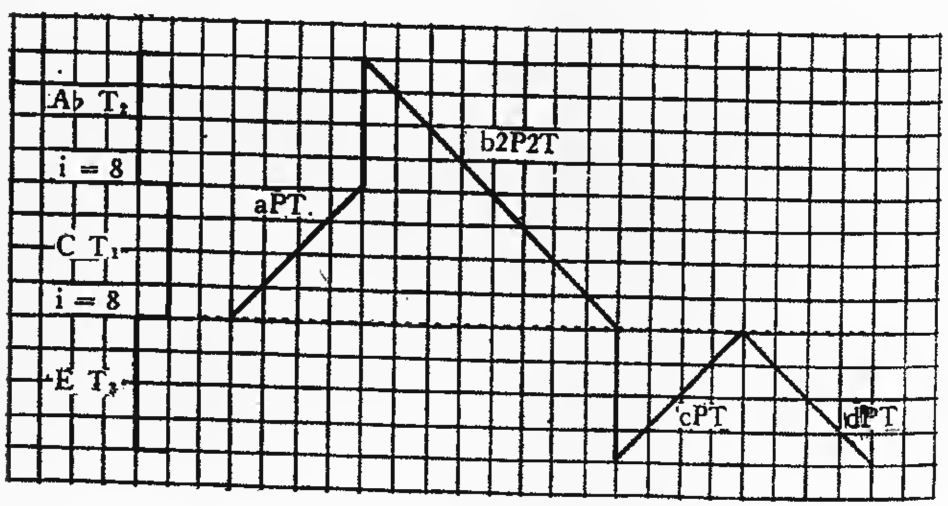


Figure 97. Surrounding first tonic with other tonics.

For the same reason it is practical to surround the first tonic by other tonics (making the first tonic a primary axis) in the settings of the first contraction.
For example:

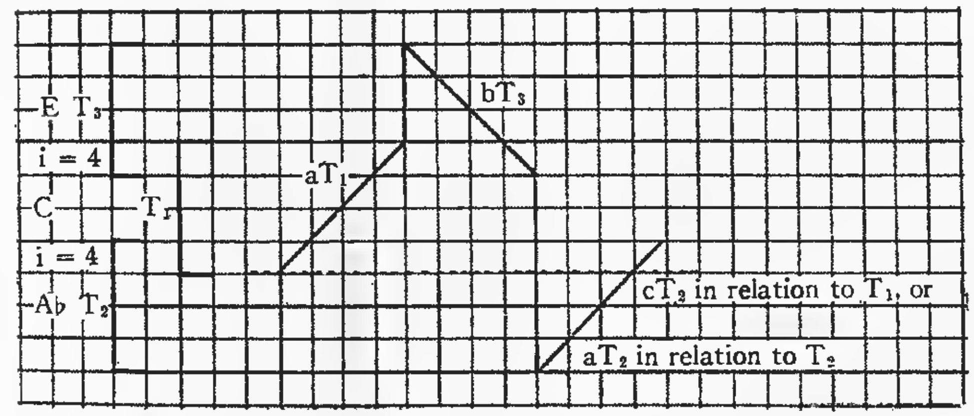


Figure 98. Another illustration of first tonic surrounded with other tonics.

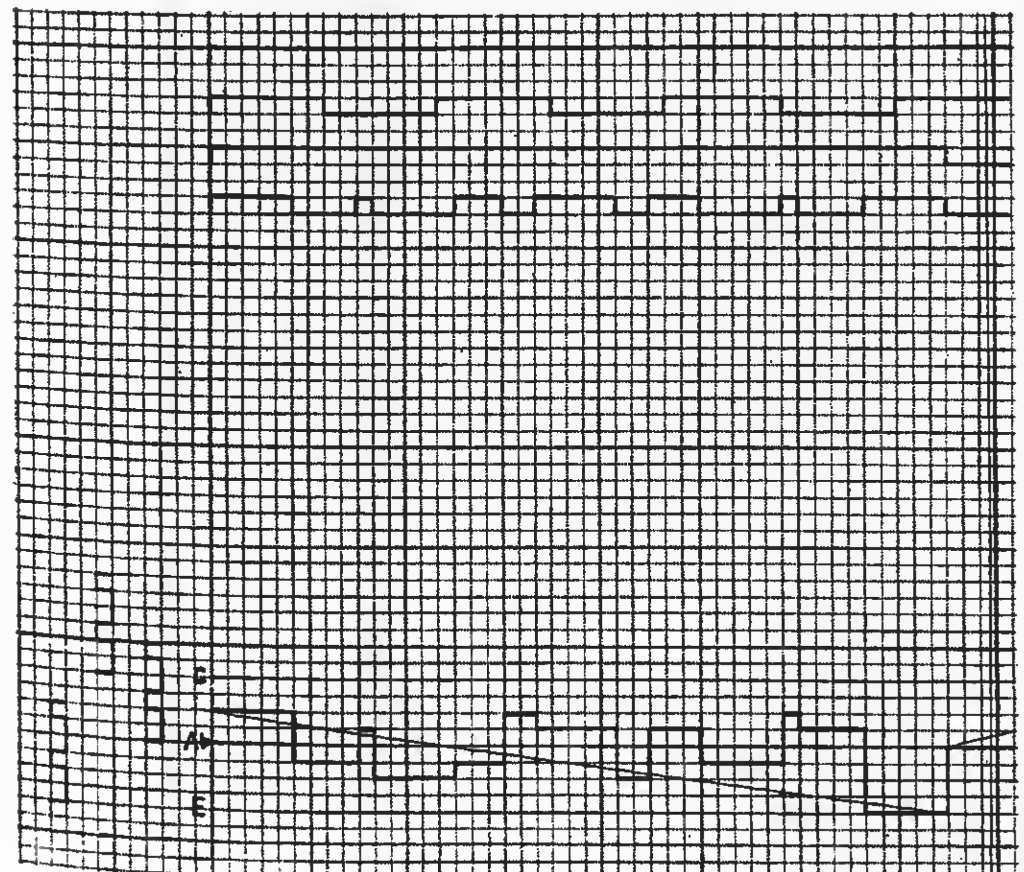


Figure 99. Geometric projection of preceding figure (continued).

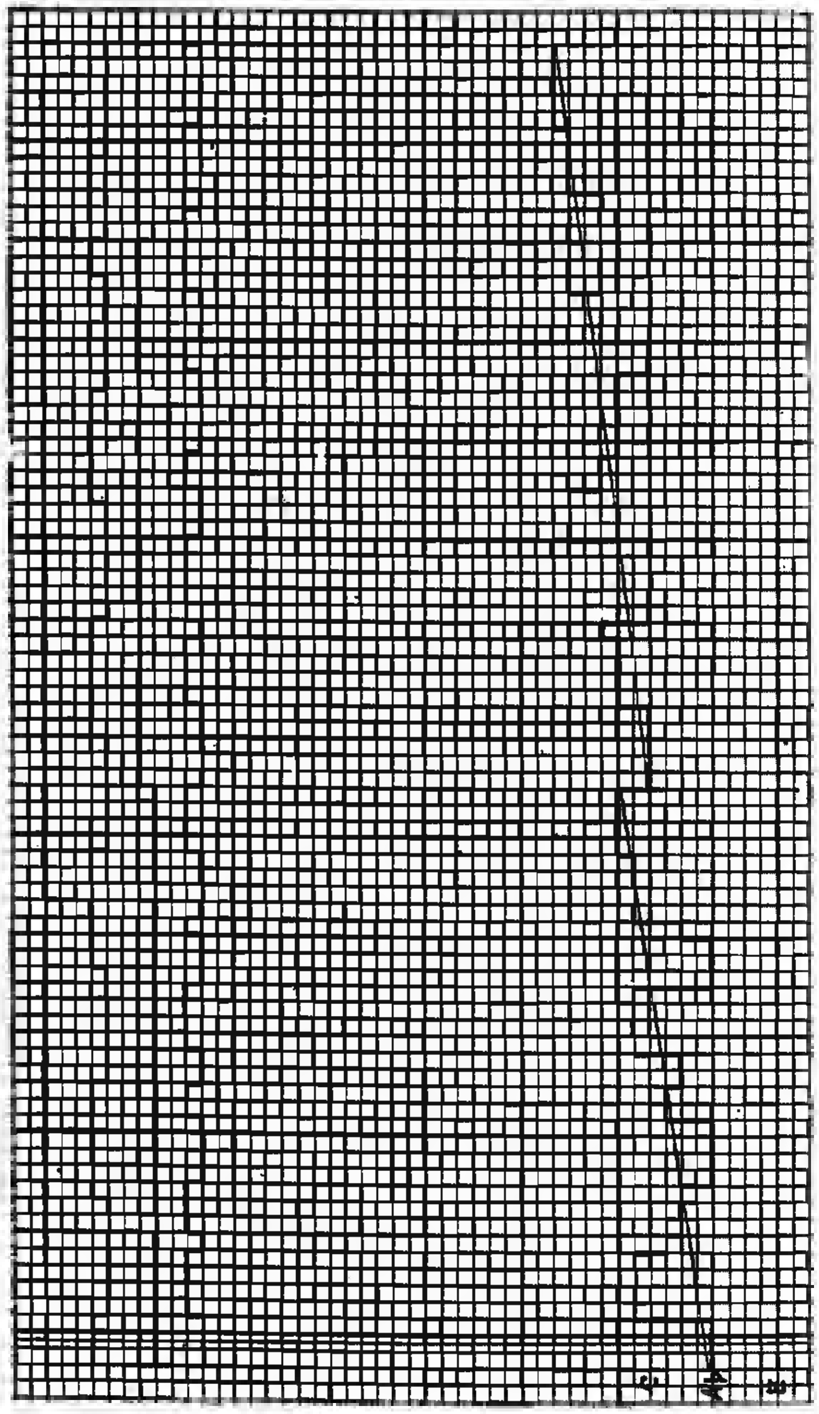


Figure 99. Geometric projection of preceding figure (concluded).

$[M_{ⓐ}\ E_0p_0]+[a_{ⓐ}\ E_0p_1]+[b_{ⓐ}\ E_0p_2]+[c_{ⓐ}\ E_0p_3]+[b_{ⓑ}\ E_0p_4]+$
$[c_{ⓑ}\ E_0p_5]+[M_{ⓓ}\ E_0p_0]+[a_{ⓐ}\ E_1p_0]+[M_{ⓐ}\ E_0p_0]$

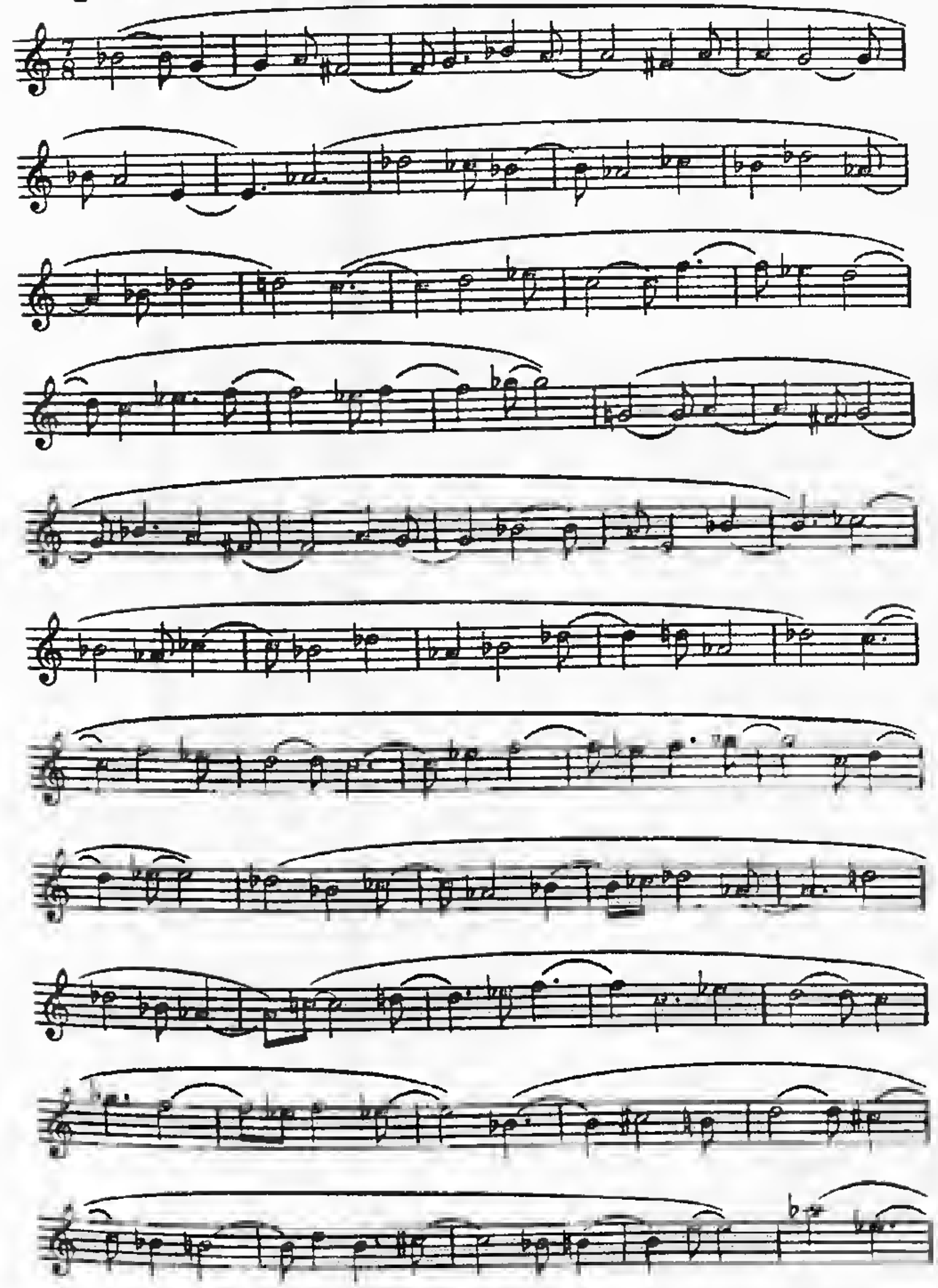


Figure 100. The melody of Figure 99 (continued).

Figure 100. The melody of Figure 99 (concluded).

B. Technique of Plotting Modulations

First: a modulation through common tones. Plot the scales of all keys in which the melody will appear on the left side of the graph. Draw all the pitch-levels which appear in common for any two of the adjacent keys at the corresponding period of time assigned for such modulation. Composition of durations for the modulation must be made in advance, i.e., at the time the entire continuity of time rhythm is planned. The final step is to drop perpendiculars from the points of attack upon the common pitch levels.

No selection of secondary** axes for the period of modulation is necessary.* You are free to choose the trajectorial phases. If the portion of melody appearing in the succeeding key starts on the primary axis, it is desirable to select the phases which will permit the use of leading tones. As modulation means a transition from one primary axis to another, it is necessary to plan the axial schemes for the adjacent keys ***before plotting the modulation.

A modulation through ***chromatic alterations*** must be plotted first rhythmically (i.e., by using long durations) and then by dropping perpendiculars on adjacent tones not common to the two keys.

The technique of *identical motifs* requires first a rhythmic identity of adjacent groups and, secondly, imitation of the first configuration (motif belonging to the preceding key) carried out through pitch-levels of the following key. Both configurations must be in the same pitch-range.

Here is an example of melody plotted with all three types of modulations:

Melody in $\frac{8}{8}$ Series

Theme: $(5+3) + (3+2+3)$; Theme (factorial): $aTP + bT2P$

First Modulation (common tones): $(2 + 1 + 2 + 1 + 2)$

Second Modulation: (chromatic alterations): $(3 + 3 + 3 + 3 + 4)$

Third Modulation (identical motifs):

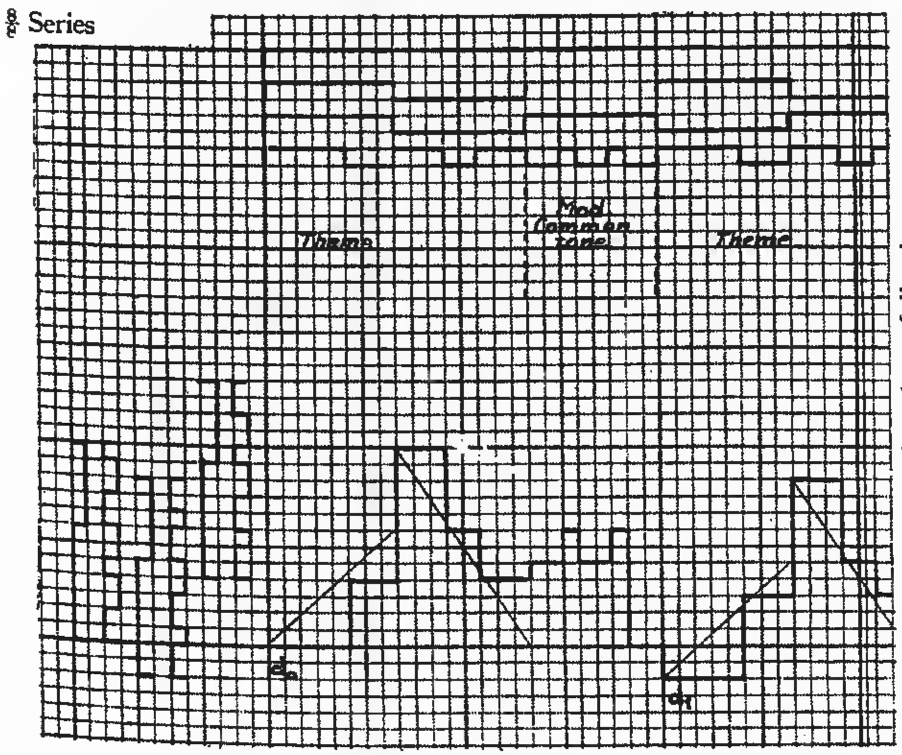
$$(3 + 2 + 1 + 1 + 1) + (3 + 2 + 1 + 1 + 1)$$

The Sequence of Keys and Scales:

(1) C maj. natural: P.A. d_0
(2) A♭ " " P.A. d_1
(3) G " " P.A. d_6
(4) C " " P.A. d_0

Modulating Melody Graphically Composed

$\frac{8}{8}$ Series



Graph continued on following page

Figure 101. Plotted melody with three modulations (continued).

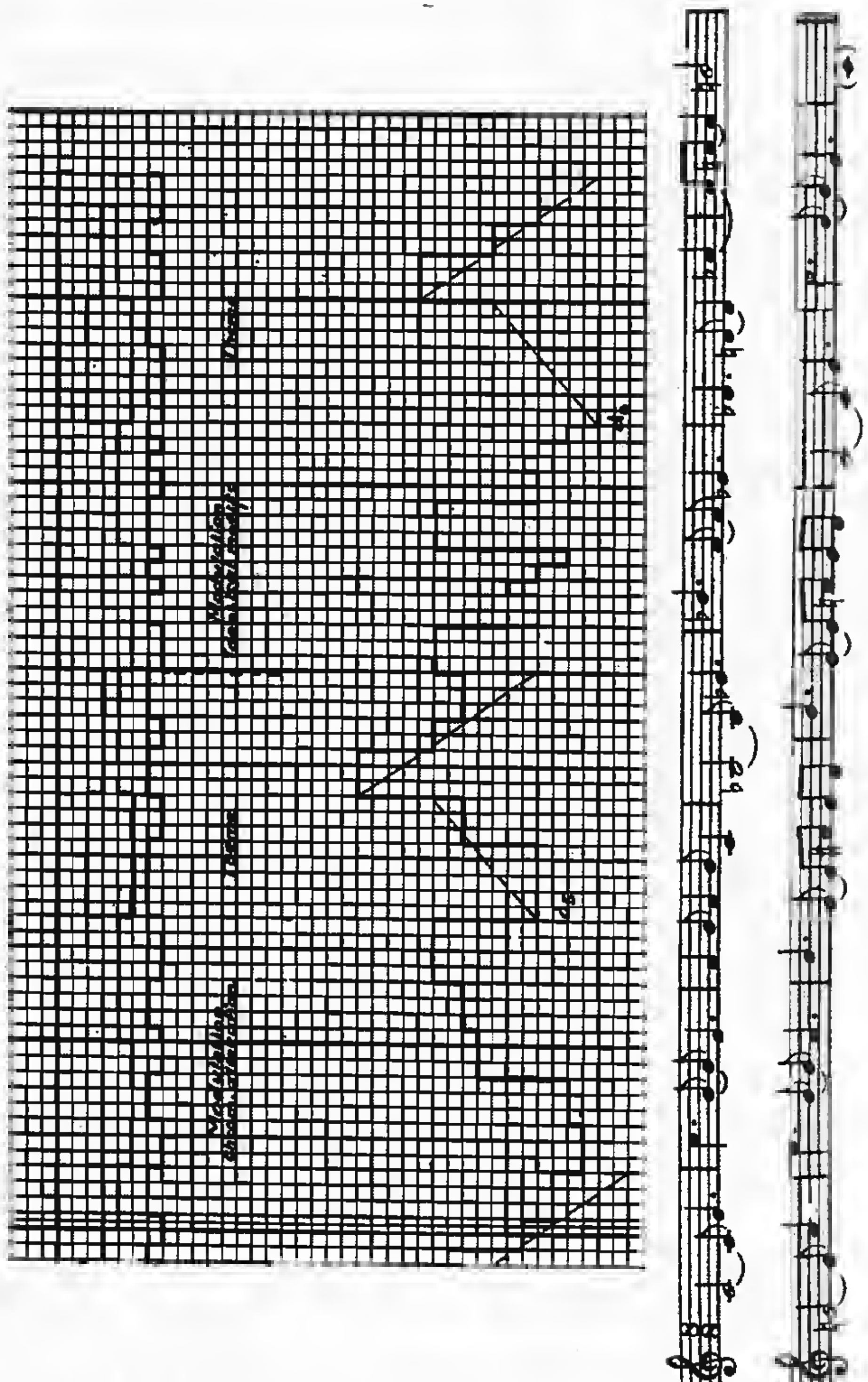


Figure 101. Plotted melody with three modulations (concluded).

Graph concluded from preceding page.

CHAPTER 8

USE OF ORGANIC FORMS IN MELODY

THE TERM ***organic*** is usually associated with living matter. The most obvious forms of organic existence manifest themselves in growth. Different rates of growth have been observed in different fields. Even the ancient Egyptians and Greeks had stumbled upon different forms of regularity, which they discovered as geometrical proportions of a rectangle. This discovery led to the development of a system of proportions expressing a harmonic relation between the preceding and the succeeding link. Numerical values arranged in an increasing order on the basis of this form of proportional growth became known in the 13th Century as ***summation series***. It was formulated by the Italian mathematician, Fibonacci, and became known as the ***Fibonacci series***.

This is how summation series were deduced on a purely geometrical basis. Take a square, and use the diagonal of it as a radius. From any of the four possible points of origin, draw an arc. Extend one of the sides which does not intersect the arc until the arc intersects it. Erect a perpendicular at this point of intersection and extend the opposite side of the square until it intersects the perpendicular. This newly formed rectangle possesses proportions which develop the Fibonacci series.

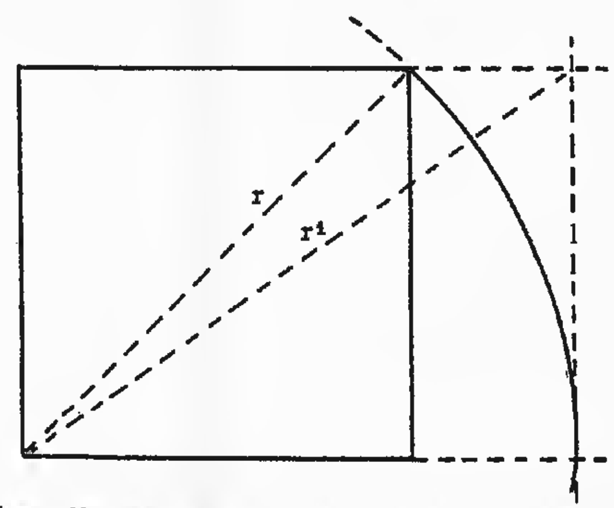


Figure 102. Deducing the Fibonacci series on a geometrical basis.

The Fibonacci series is based on the principle of adding every two consecutive numbers in a series to obtain the third.* Thus, starting with 1, we obtain 1, 2. By adding 1 and 2 we acquire the third number of this row: $1 + 2 = 3$.

*The Fibonacci series is a series in which the first term is one, the second term is two, and every term thereafter is the sum of the two immediately preceding terms. Other related series can, of course, be obtained by using some other number than two as the second term, thereafter proceeding to arrive at each term by adding the two immediately preceding terms. (Ed.)

The following numerical values are obtained in exactly the same way. This summation series developed through eleven terms acquires the following appearance: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144. These numerical values can be obtained purely geometrically, i.e., without computation and directly from the rectangle in Figure 102.

By drawing the diagonal of a rectangle, indicated in Figure 102 as r^1, we subdivide the entire area into two triangles known as "pyramid triangles". Let us consider the lower pyramid triangle for the development of the proportions representing the summation series.

Consider point V in Figure 103 a vertex of the triangle. Drop the perpendicular from point V on the base of the triangle. This produces the line p_1V. Now we have acquired a new triangle, Vp_1p_n. Dropping a perpendicular from the vertex p_1 on the base Vp_n, we acquire a new triangle, $p_1p'p_n$. Continuing this procedure further, we obtain a group of triangles which become partials of the original pyramid triangle. The lines p_1p' p_2p'' etc., produce the extensions which in turn represent the numerical values of the summation series.

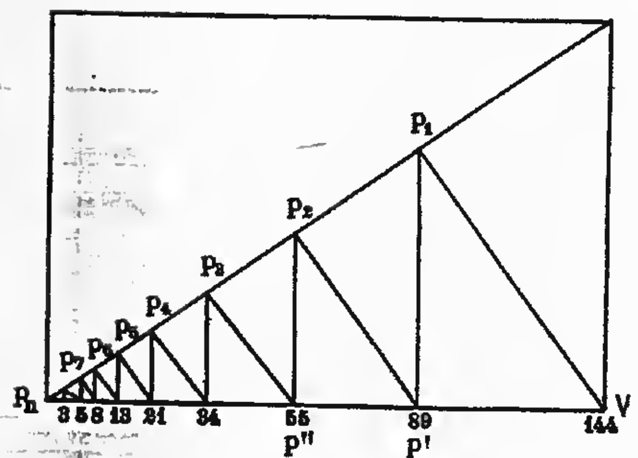


Figure 103. Partials of the pyramid triangles.

A clear realization of the principle of summation series as a foundation of beautiful proportions was presented by Luca Pacioli in his treatise, *De Divina Proportione* (1509). The principle of the "divine proportion" is derived from the ratios of the summation series. It is also known as "Gold Section," "Gold Cut" and "Golden Mean."

This particular proportion is $\frac{b}{a} = \frac{a}{a+b}$. This expression can be read: the short segment is related to the long segment as the long segment is related to the sum of both segments. The usual presentation is in the form of a subdivision of a given line through the "Golden Cut," that is, dissecting a line into two

segments so that the short segment is related to the long segment as the long segment is related to the whole original line. Michelangelo, a friend of Pacioli, applied the "Gold Section" ratio to proportions of the human body.

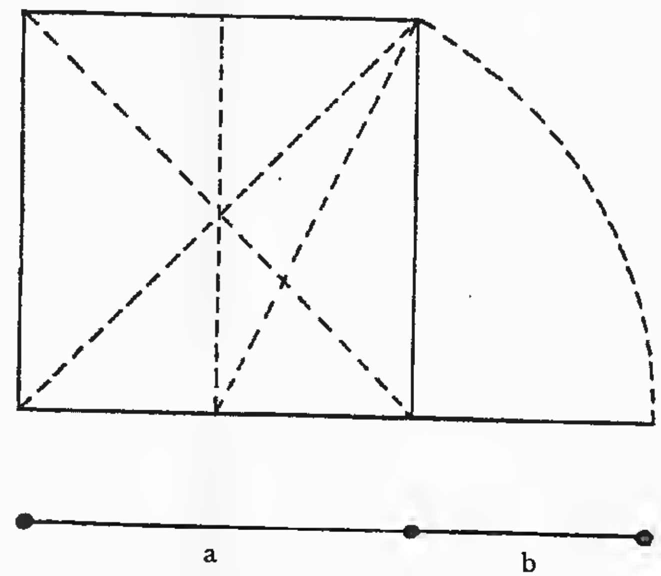


Figure 104. The Golden Mean.

Later, Leonardo da Vinci, while studying plant structures, discovered that the arrangement of leaves on a stem, or of various members of a plant, follows the spiral whose radii grow through the summation series. This study was followed up in the 20th century by A. H. Church in his *Principles of Phyllotaxis* (Oxford University Press).

Artists, and more particularly sculptors since ancient Greece, have devoted themselves to the subject of applying the ratios of the summation series to bodily symmetry. The first known contributor to this analysis was Polykleitos (5th century B.C., Greece). Professor Church has demonstrated that the formation of seeds in a sun flower, the tangent to a maple leaf, and other botanical patterns of growth follow the summation series. Artists and art theorists have tried to develop these principles to serve their purpose. An exhaustive study of spiral formations as they appear in plant and animal life was completed by Theodore A. Cook in his *Curves of Life.**

Renewed interest in summation series was stimulated by the publication of Jay Hambidge's *Dynamic Symmetry,*** in which he tried to apply this principle to pictorial composition. Jay Hambidge and Howard Giles have developed and applied these principles in their teaching of art in New York City. It met with great success and today has become so common that the principle of dynamic symmetry is applied even in the construction of such articles as radio cabinets.

*New York, 1914. **New York, 1920.

A thorough survey and analysis of the whole problem has been accomplished in very extensive research by Wilford S. Conrow, a New York artist, in his *The Ratios of Bodily Symmetry and Growth in Relation to Sculpture and Medical Science.** Some further developments of the Hambidge theory were made by one of his collaborators, Edward B. Edwards, in his *Dynamarhythmic Design*.

A property of the summation series known as Fibonacci series is that it contains symmetry throughout. The word *symmetry* emphasizes the equality of two measured ratios, according to an authority on the subject, Dr. William Churchill. Thus the adjacent portions of any structure following the summation series produces equality of ratios.

Summation series spirals can be constructed through a group of 90° arcs so that the value of the radius grows after every 90° through the summation series.

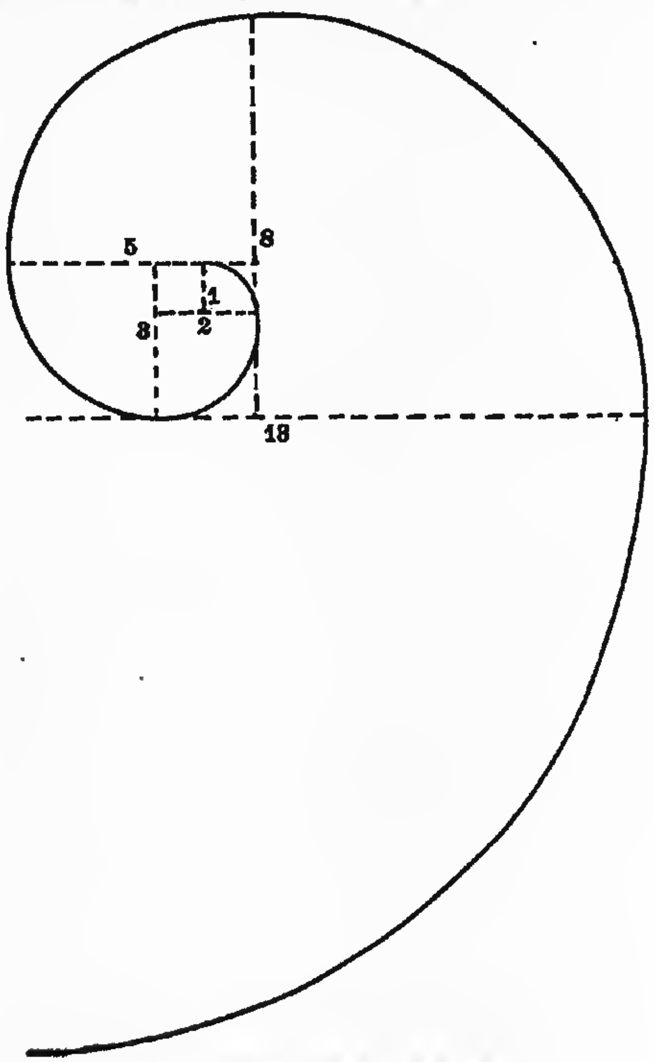


Figure 105. Summation series spirals.

*New York, 1937.

The values of the summation series may be applied to intonations as well. Portions of musical melody appealing to us as organic are based on identical principles of expanding intervals. In music, the unit of measurement for the intervals between the pitch units of an octave is expressed in semitones. The growth of semitones through the summation series in unilateral and bilateral symmetry develops motifs, i.e., melodic forms, which are truly organic as they exhibit the processes of growth of intervals. Such melodic forms can be often found as the outstanding themes of recognized composers as well as in folklore.

Historians and musicologists have an accepted term for such motifs, calling them "traveling" or "wandering" motifs. These motifs have such a universal appeal that, whether they appear in folk music or in the work of an individual composer, they become universally accepted as definite crystallized symbols of musical expression. It is interesting to note that "tonality" is an outcome of organically related number values and is not a "musical" quality a priori.

The unilateral symmetry of the Fibonacci series, applied to semitones, produces the following sequence:

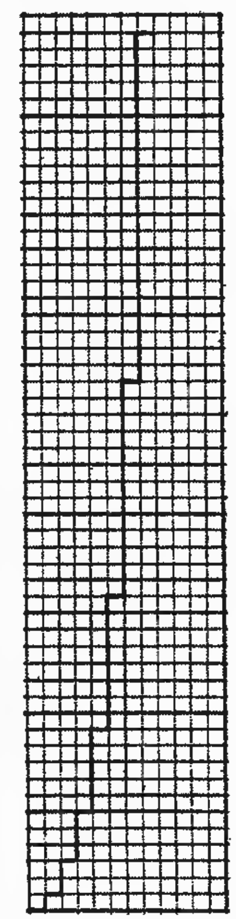


Figure 106. Unilateral symmetry of Fibonacci series (continued).

Unilateral Summation

Figure 106. Unilateral symmetry of Fibonacci series (*concluded*).

As in every spiral, it is only in using a few successive links that we can achieve what we term "beauty." Beyond this the form becomes too extreme; and the same is true in music, too. Thus a melody seems more melodious if it emphasizes only the first few steps of the summation series. Beyond this point the intervals become so great that our conditioned perception of melody, as melody of a vocal type, is disturbed by such extreme dimensions. Some contemporary composers, however, use such intervals, being guided by a purely intuitive urge. Their ears are pleased and satisfied by such wide intervals. The most representative extremist in this field is the Austrian composer, Anton von Webern.

After a melody is constructed through summation series in the unilateral form, it is possible to produce any number of derivative melodies through *readjustment of the range*, i.e., by means of octave transposition of the corresponding pitch units. In such a case any spiral may be confined to a very limited range, yet produce intonations which originally were organically related. The following is range readjustment of the scale in Figure 106.

Range readjustment:

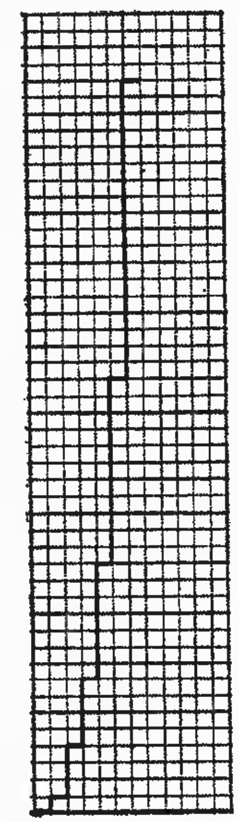
Figure 107. Range readjustment of scale in figure 106.

In addition to the Fibonacci series, a number of *other* summation series of the same class can be developed. We shall call the Fibonacci series the *first* summation series. In order to obtain the *second* summation series of the same class, i.e., by the addition of every two consecutive numbers, we have to start with 1 and add 3 instead of 2; thus we obtain: 1, 3, 4, 7, 11, 18, 29......... The third summation series introduces 4 after 1; thus we obtain 1, 4, 5, 9, 14, 23.........

It is easy to see that the number of summation series in this class is infinite. Other classes of summation series can be developed by obtaining every *fourth* value as the sum of the *three* preceding values. For example: 1, 2, 3, 6, 11, 20...... Further classes represent the addition of a greater quantity of numbers, and there is always an infinite number of series in each class.

My own applications of the various summation series to *design* as well as *music* (not only to pitch, but also to the development of durations) show that such groups of lines or durations or pitches affect us as *organic formations*.

Application of the second summation series to melody producs the following scale:



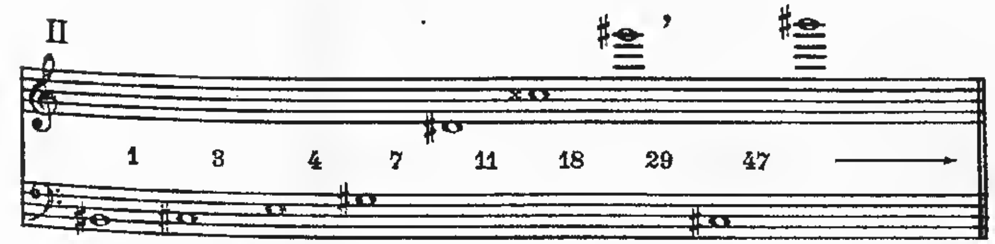


Figure 108. Second summation series applied to melody.

After readjustment of pitch ranges through octave inversion we obtain the following melodic forms:

Figure 109. Re-adjustment of pitch ranges in figure 108.

The above melodic forms are naturally only a few of the basic ones. The following figure represents the third summation series in unilateral symmetry and is followed by examples of readjusted ranges.

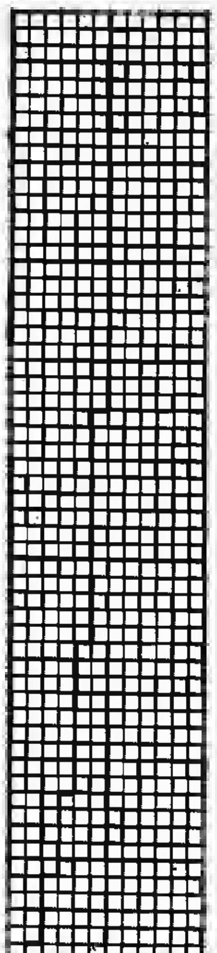


Figure 110. Third summation series in unilateral symmetry (continued).

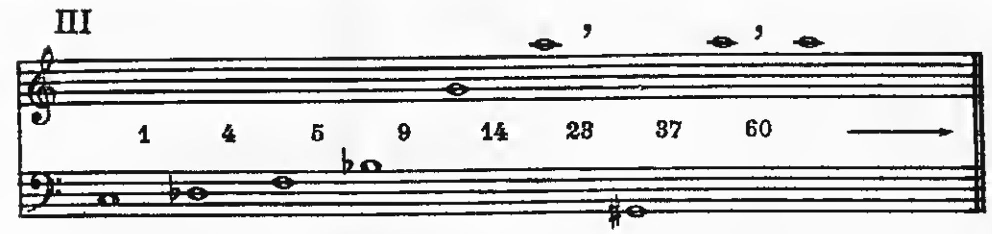


Figure 110. Third summation series in unilateral symmetry.

Range readjustment

Figure 111. Readjusted range in figure 110.

Forms of bilateral symmetry can be devised from summation series in a similar fashion. The values of a summation series follow the directions of an alternating spiral. Thus, if the first number represents an ascending interval, the second number represents a descending interval from the origin. Using the three summation series we obtain the following two fundamental forms.

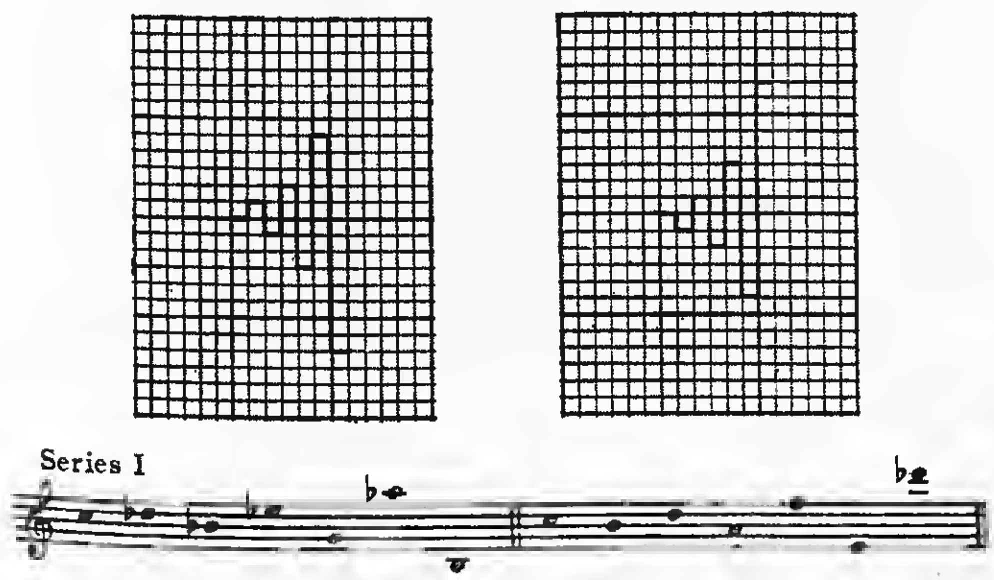
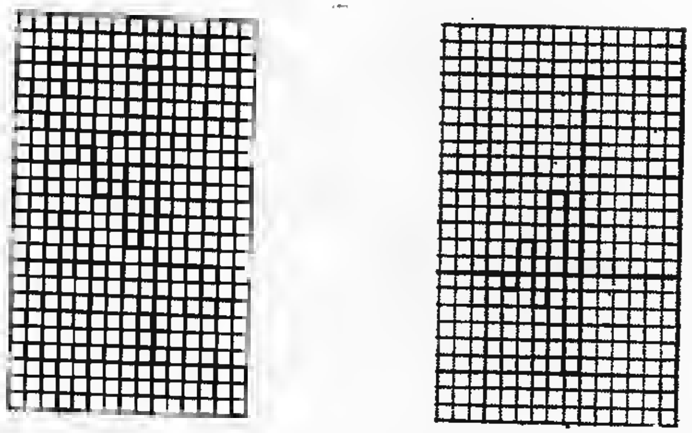
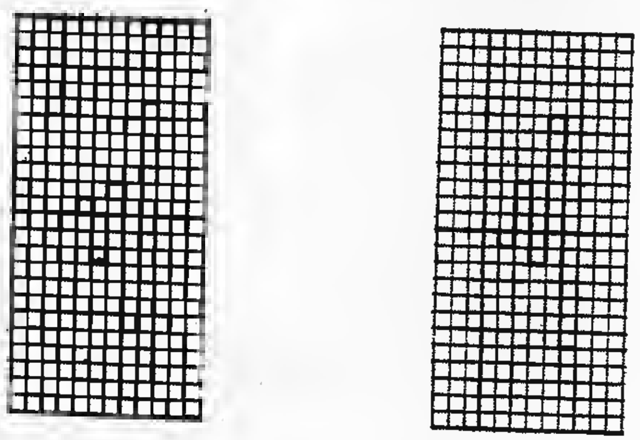


Figure 112. Bilateral symmetry in first summation series.



Series II

Figure 113. Bilateral symmetry in second summation series.

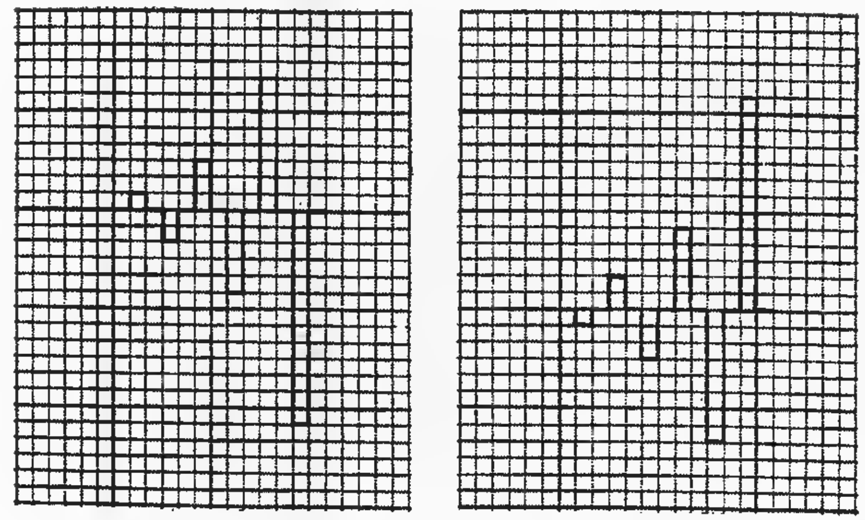


Series III

Figure 114. Bilateral symmetry in third summation series.

The readjustment of range is not necessary in the case of alternate spirals as it is more limited than in the case of unilateral spirals.

Another group of spirals can be developed with the use of bilateral structures and the inclusion of the axes. Such melodic forms being played at a relatively great speed produce effects of three parts moving in rapid alternation. From a musical standpoint, it is similar to a rapid arpeggio with one alternately repeated tone in the center.



Series I

1-1-2+2+3-3-5+5+8-8-13+13 -1+1+2-2-3+3+5-5-8+8+13-13

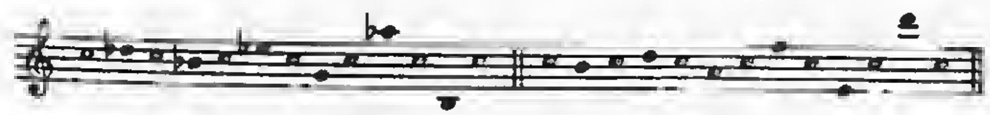
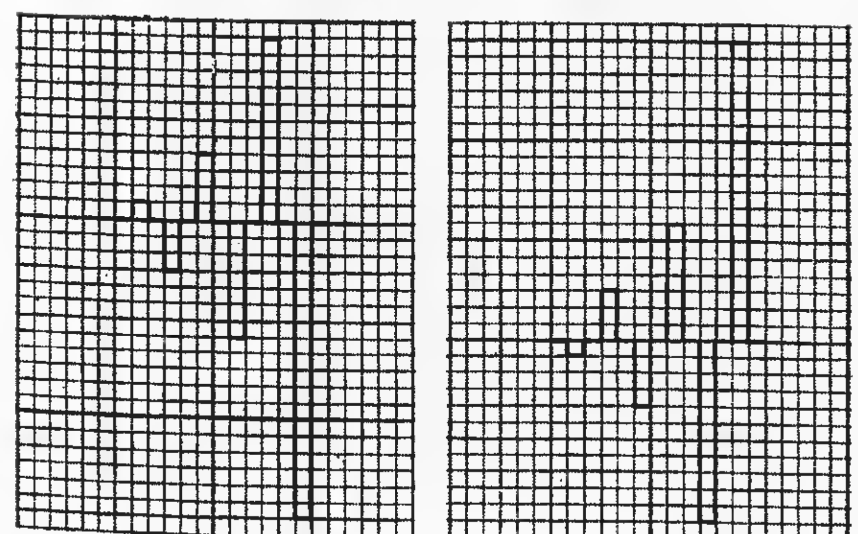


Figure 115. First summation series and alternating axes.



Series II

1-1-3+3+4-4-7+7+11-11-18+18 -1+1+3-3-4+4+7-7-11+11+18-18

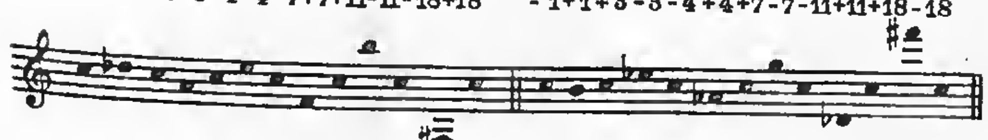
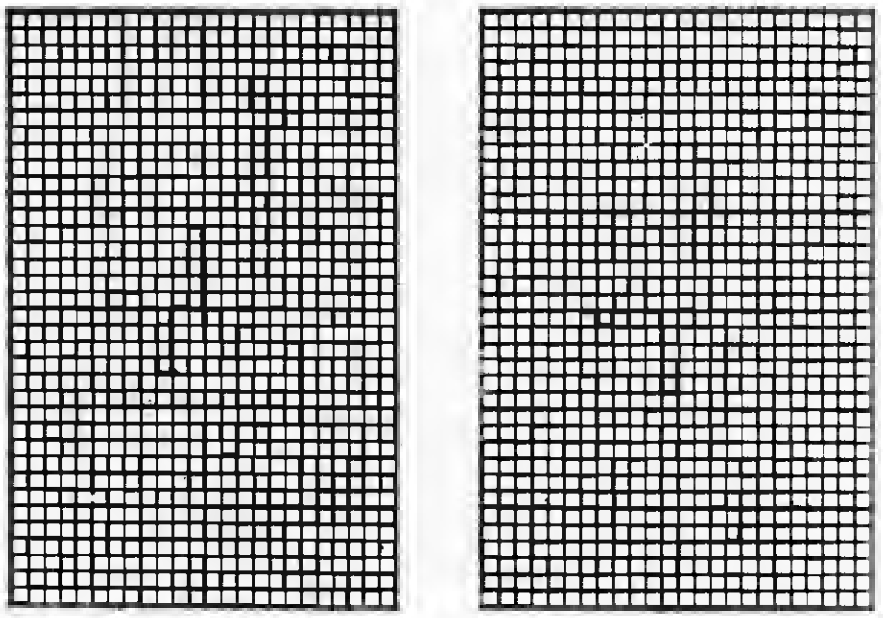


Figure 116. Second summation series and alternating axes.



Series III

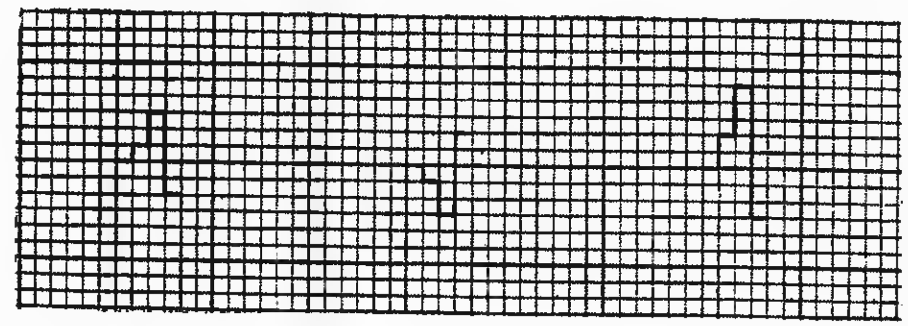
Figure 117. Third summation series and alternating axes.

The most satisfactory melodies from the viewpoint of their organic development are the spirals whose successive links involve movement in the opposite direction. The most common type of crystallized melodic forms usually corresponds to the following formula:

$$\overrightarrow{S} = t_1 + t_2 - t_4.$$

Assuming the ascending steps as the positive and the descending as the negative, we can transcribe the above formula as follows: the spiral sequence consists of the following terms of a summation series: the first term (t_1), followed by the second term (t_2), the *omission* of the third term and the appearance of the following term (t_4) with the opposite sign. The following organic forms of

melody may start at different points of one summation series and be carried out to any desirable limit. The following represents the application of this principle to the three summation series:



Summation Series

I

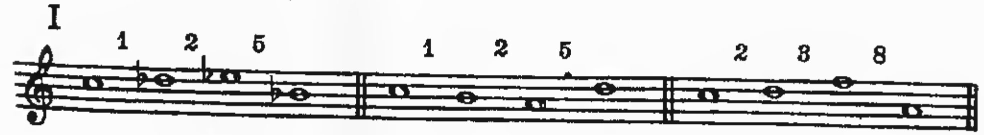
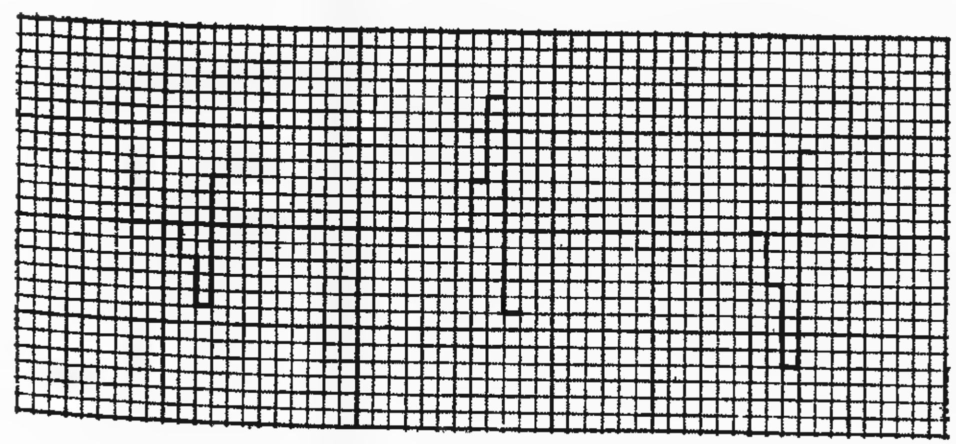


Figure 118. Spiral sequence of first summation series.



II

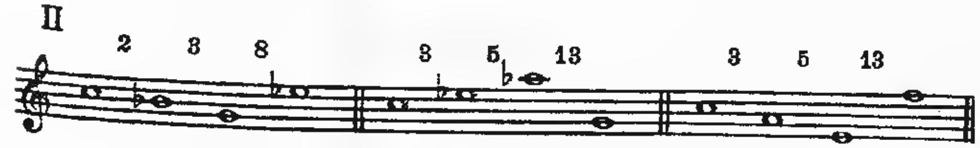


Figure 119. Spiral sequence of first summation series.

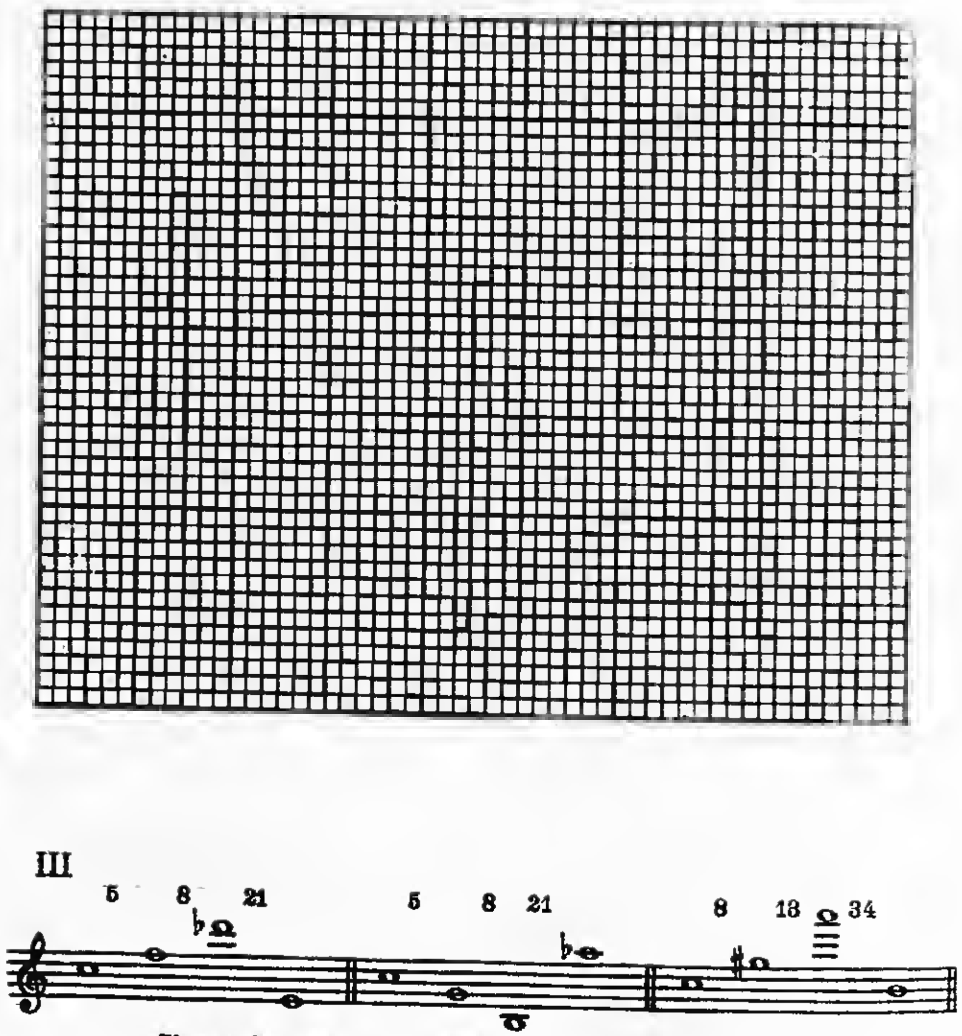


Figure 120. Spiral sequence of first summation series.

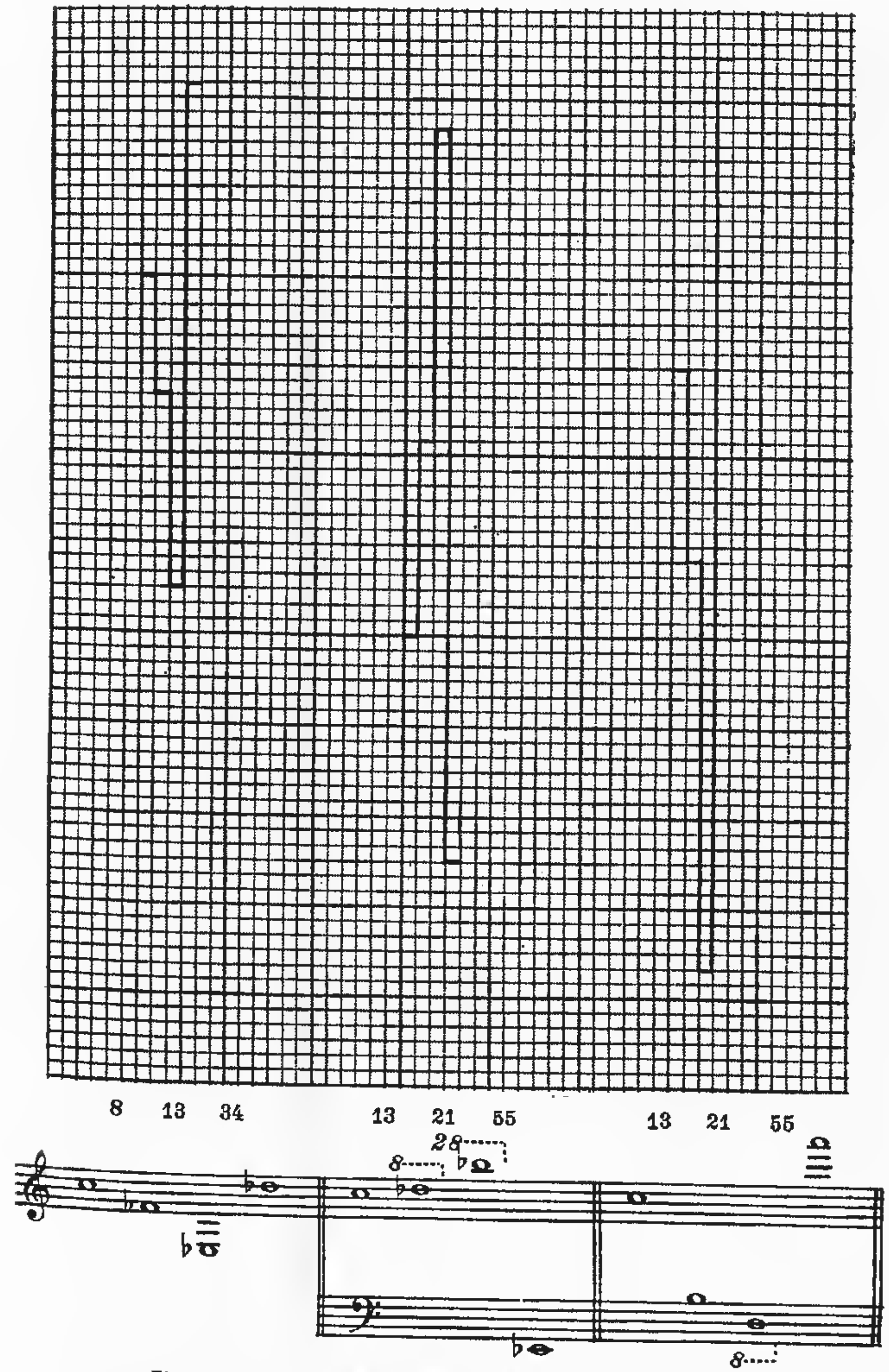


Figure 121. Spiral sequence of first summation series.

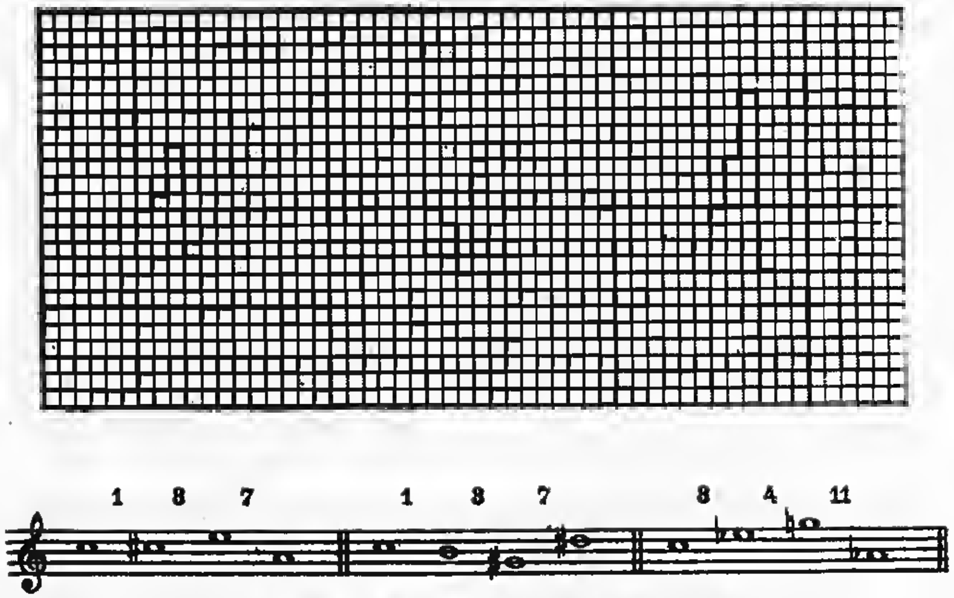


Figure 122. Spiral sequence of second summation series.

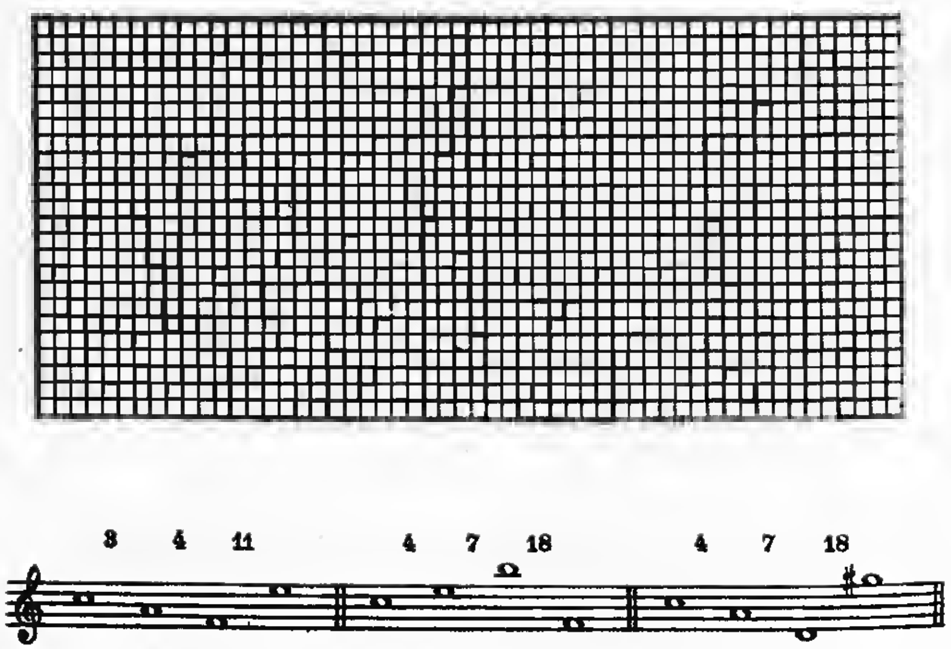


Figure 123. Spiral sequence of second summation series.

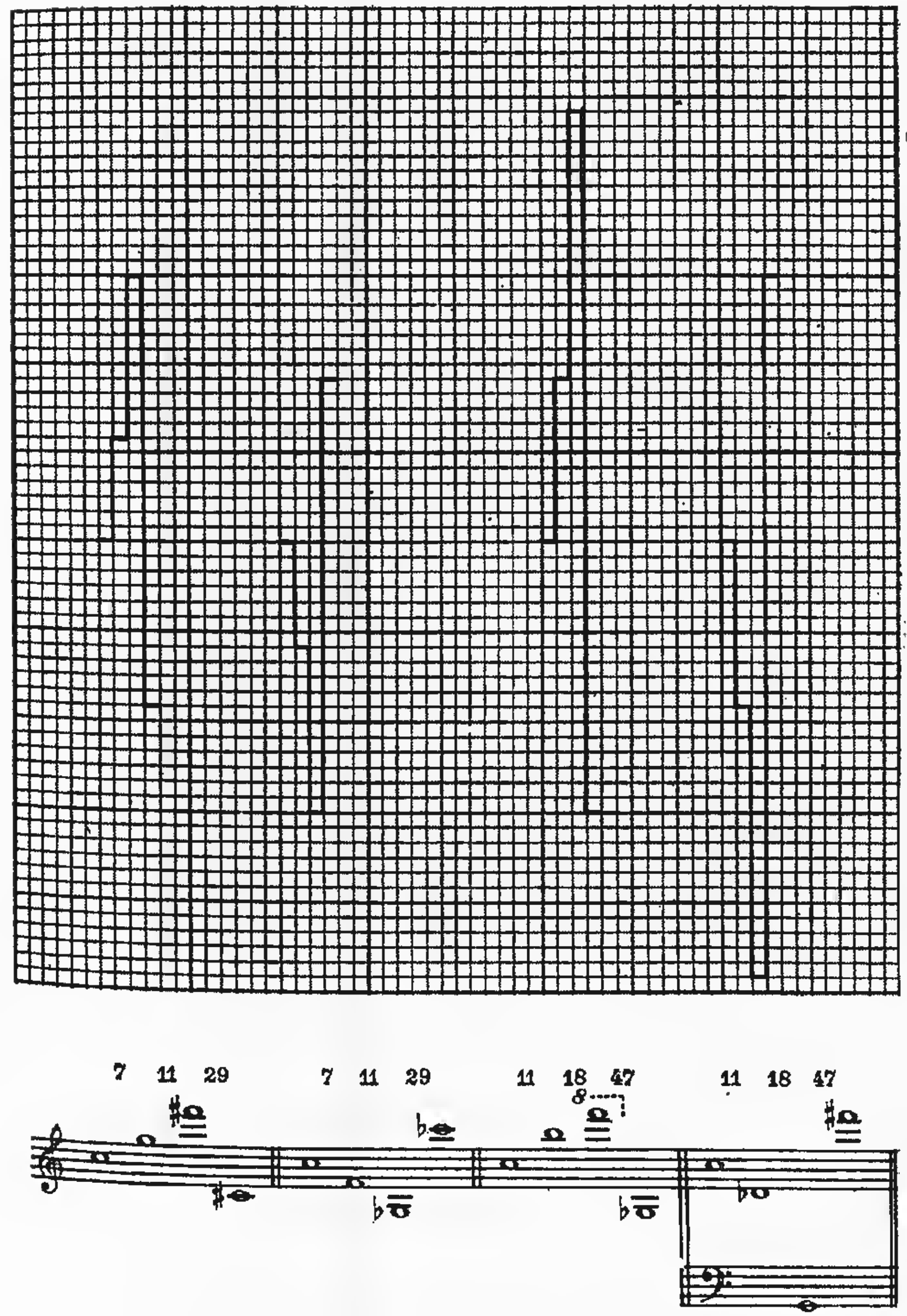


Figure 124. Spiral sequence of second summation series.

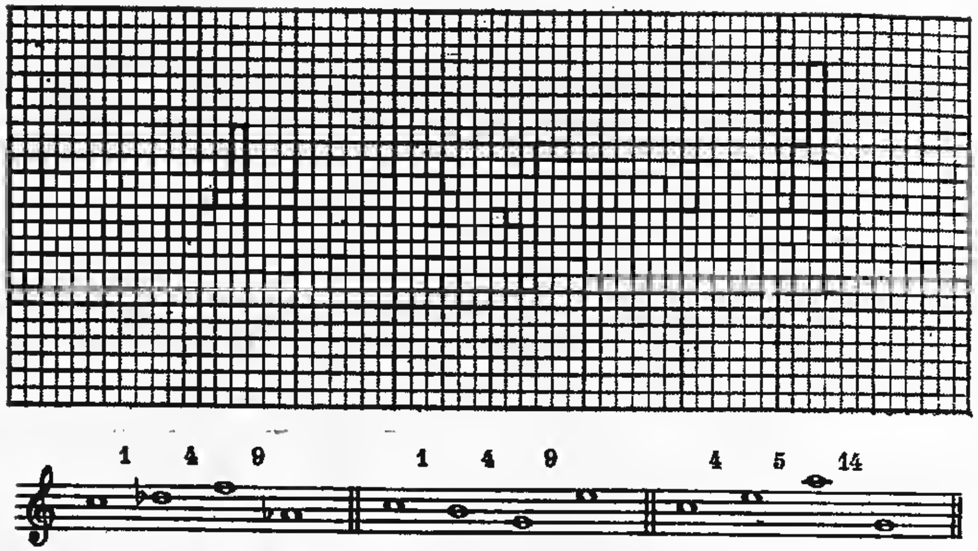


Figure 125. Spiral sequence of third summation series.

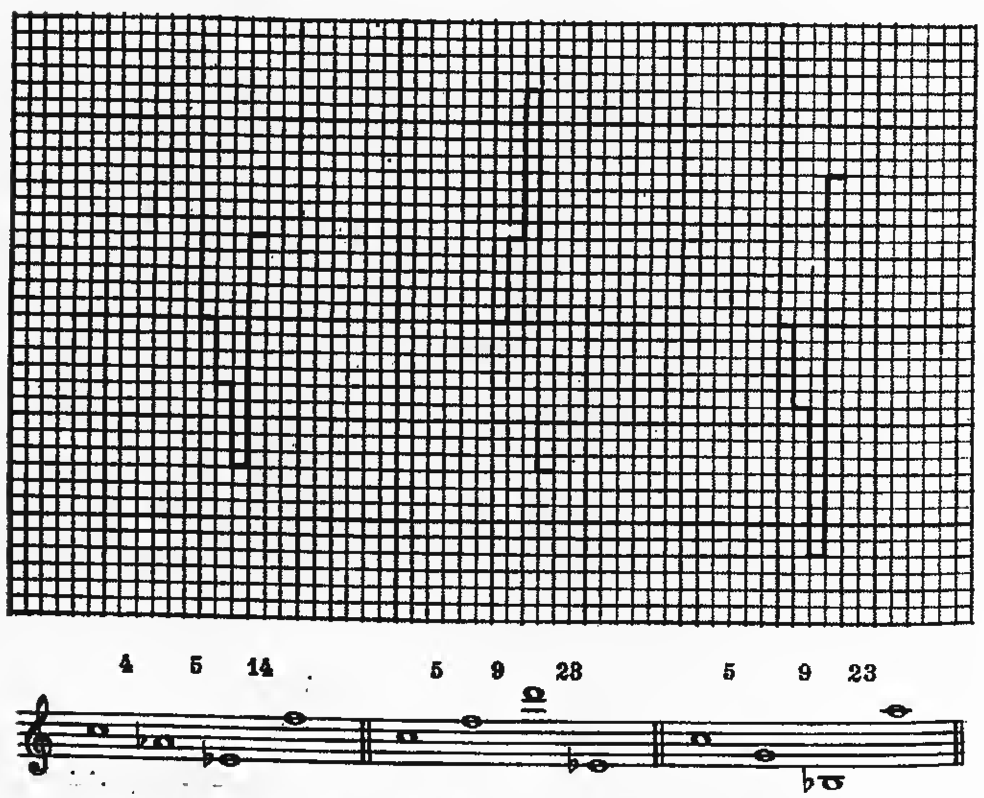


Figure 126. Spiral sequence of third summation series.

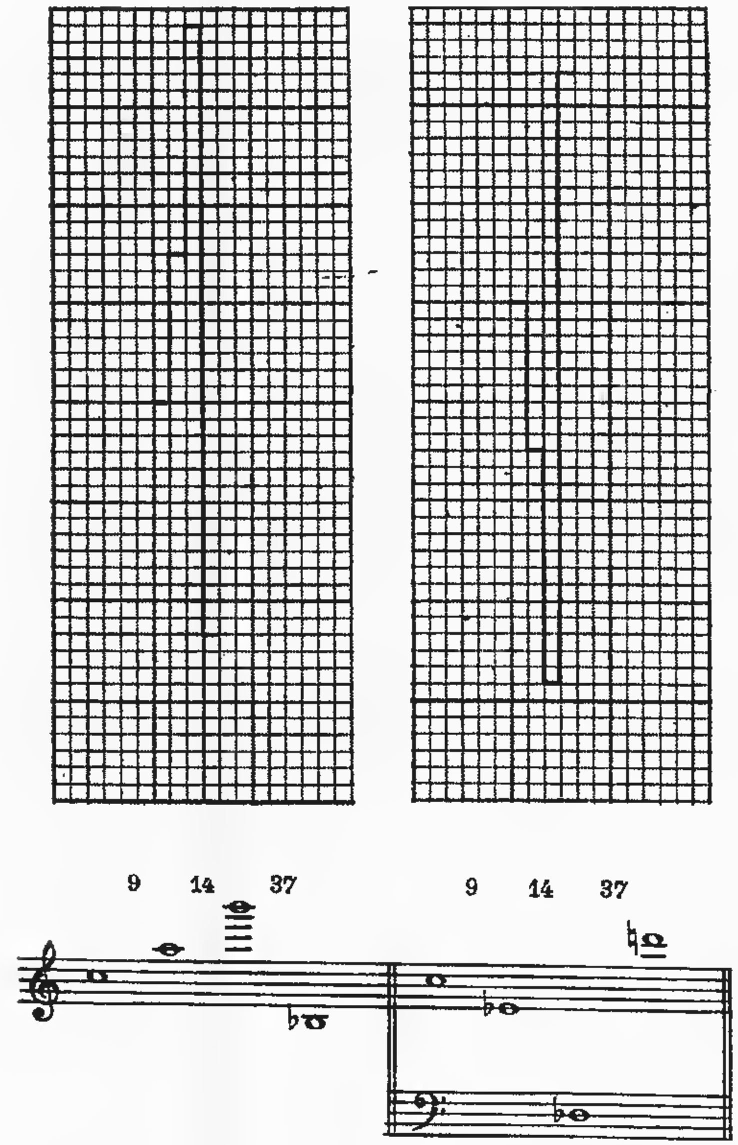


Figure 127. Spiral sequence of third summation series.

All the above forms contain four pitch units and three intervals. More developed forms of organic motifs can be obtained through the addition of three successive terms, the *omission* of one term, and the addition of the next term with the opposite sign:

$$S^{\rightarrow} = t_1 + t_2 + t_3 - t_5.$$

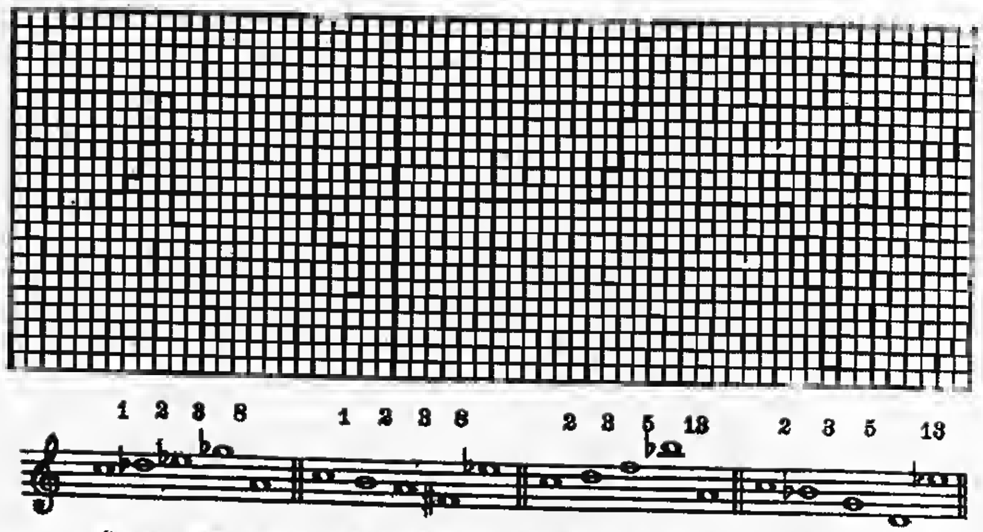


Figure 128. Spiral sequence of five pitch units in first summation series.

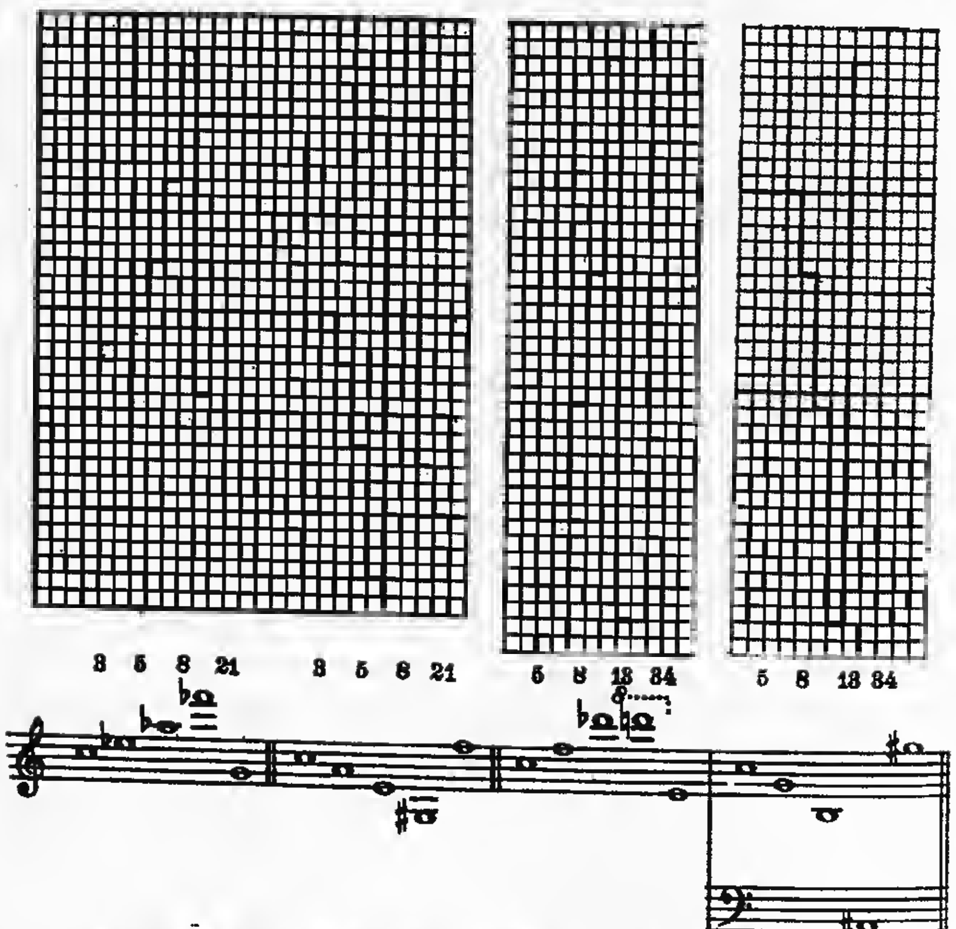


Figure 129. Spiral sequence of five pitch units in first summation series.

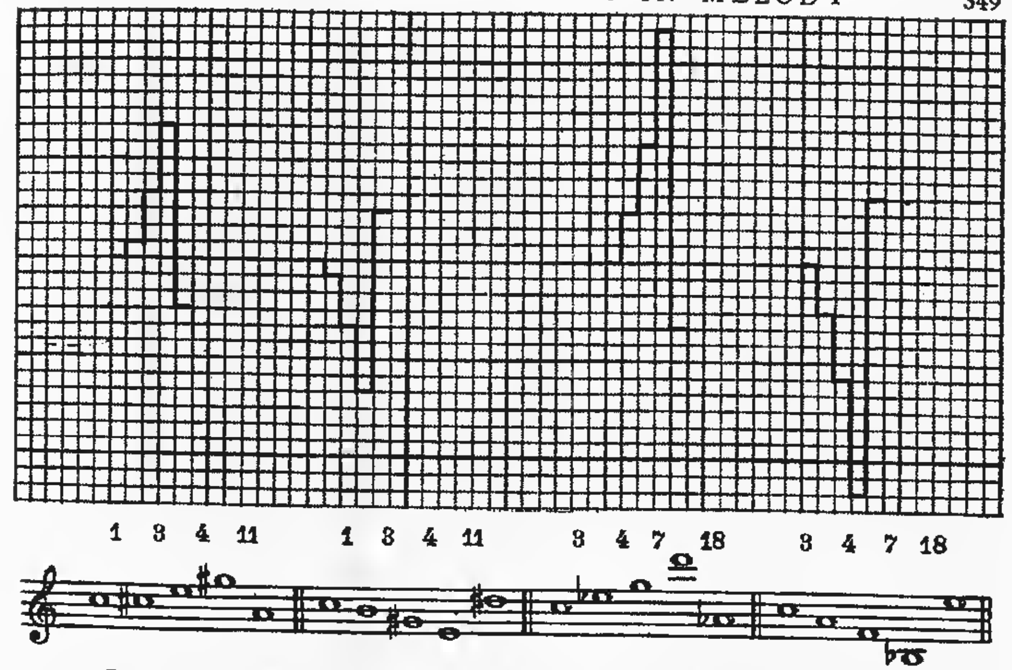


Figure 130. Spiral sequence of five pitch units in second summation series.

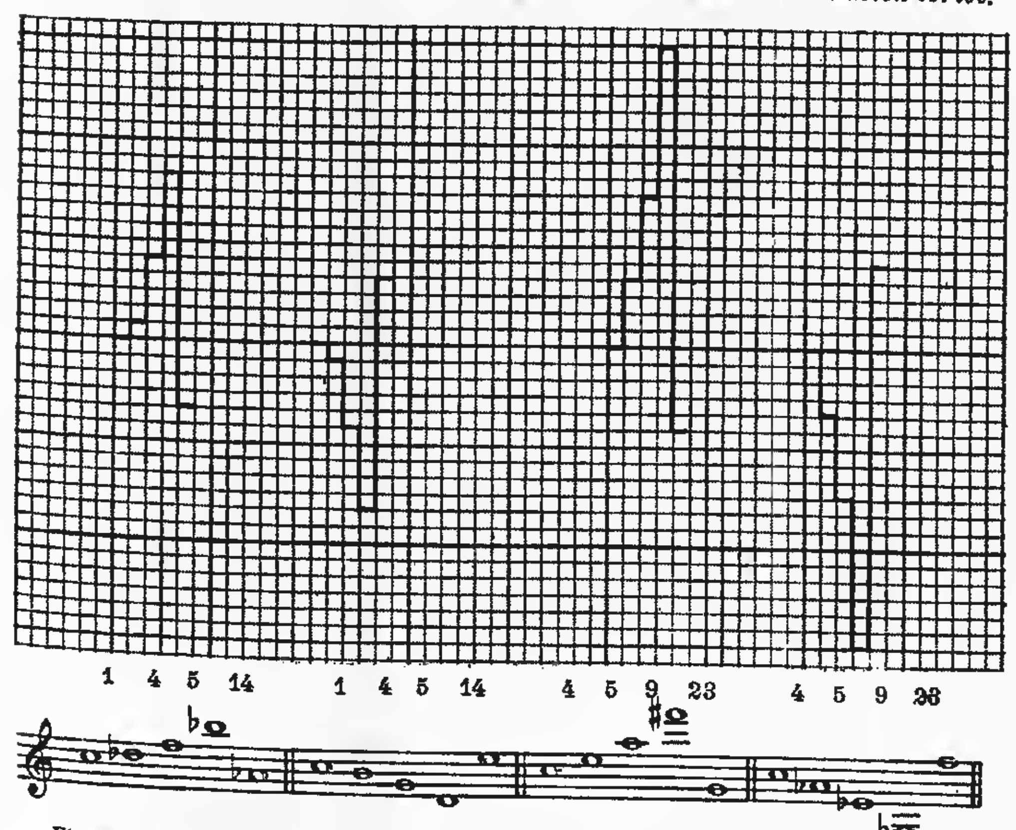


Figure 131. Spiral sequence of five pitch units in second summation series.

Another form of melodic spiral without the change of the original direction can be obtained through the omission of two terms after the summation of three terms and the appearance of the last term with the opposite sign: $S^{\rightarrow} + t_1 + t_2 + t_3 - t_4$.

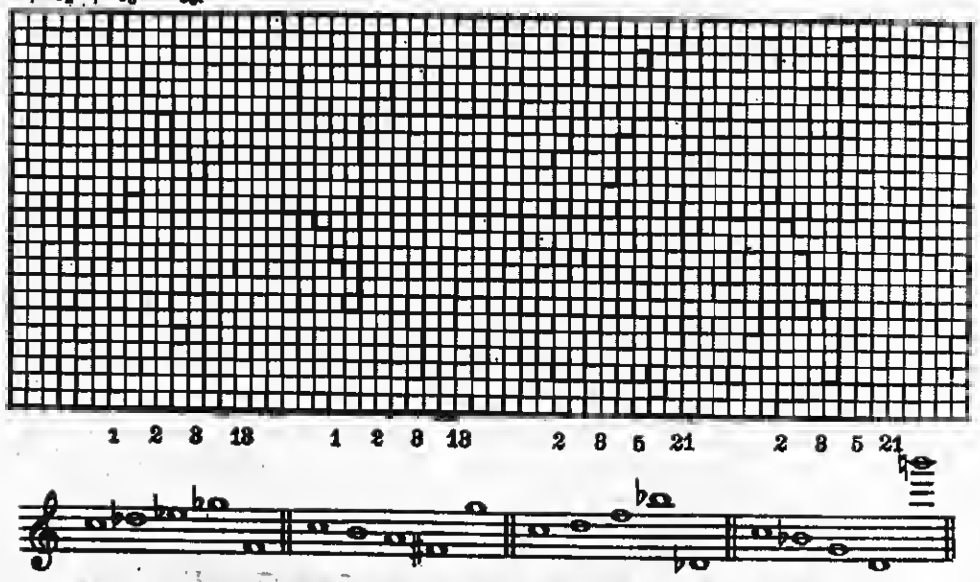


Figure 132. Another type of spiral sequence in first summation series.

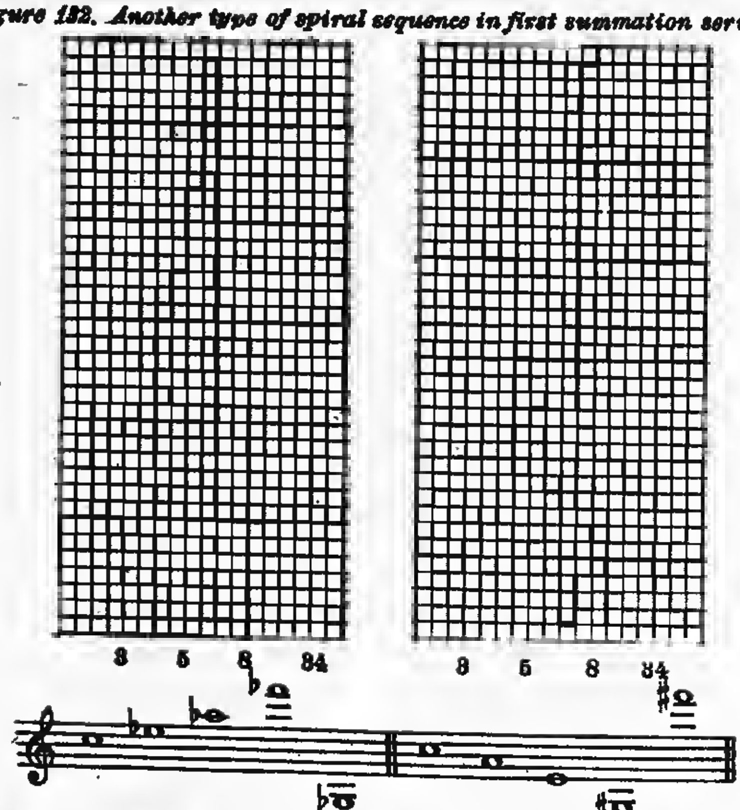


Figure 133. Another type of spiral sequence in first summation series.

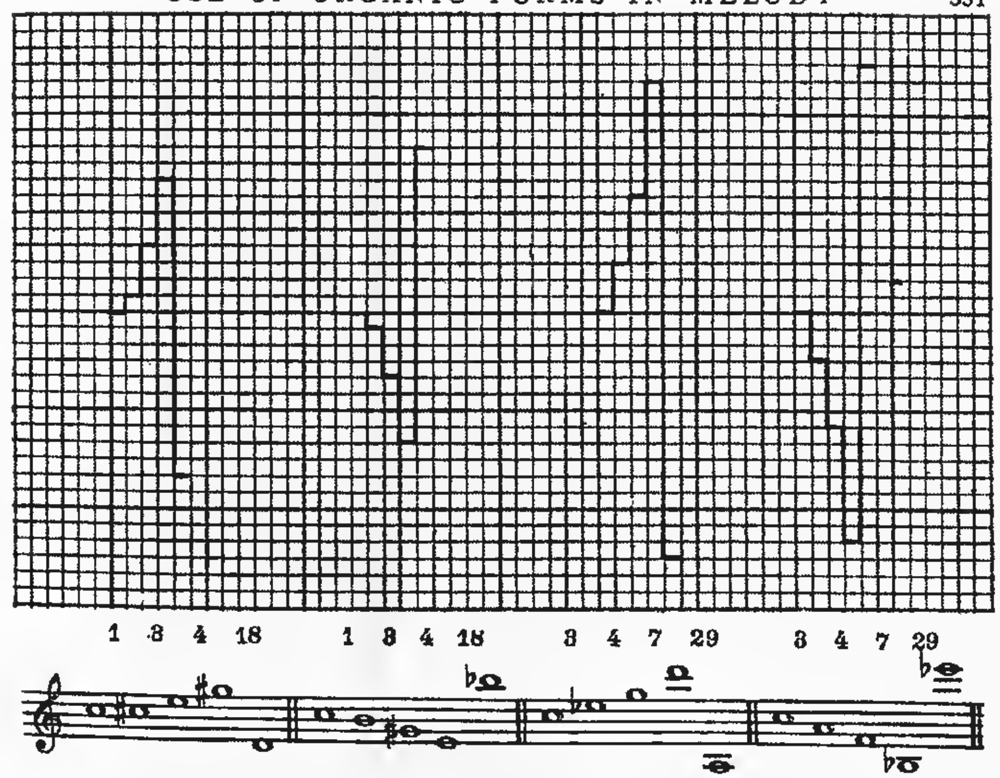


Figure 134. Another type of spiral sequence in second summation series.

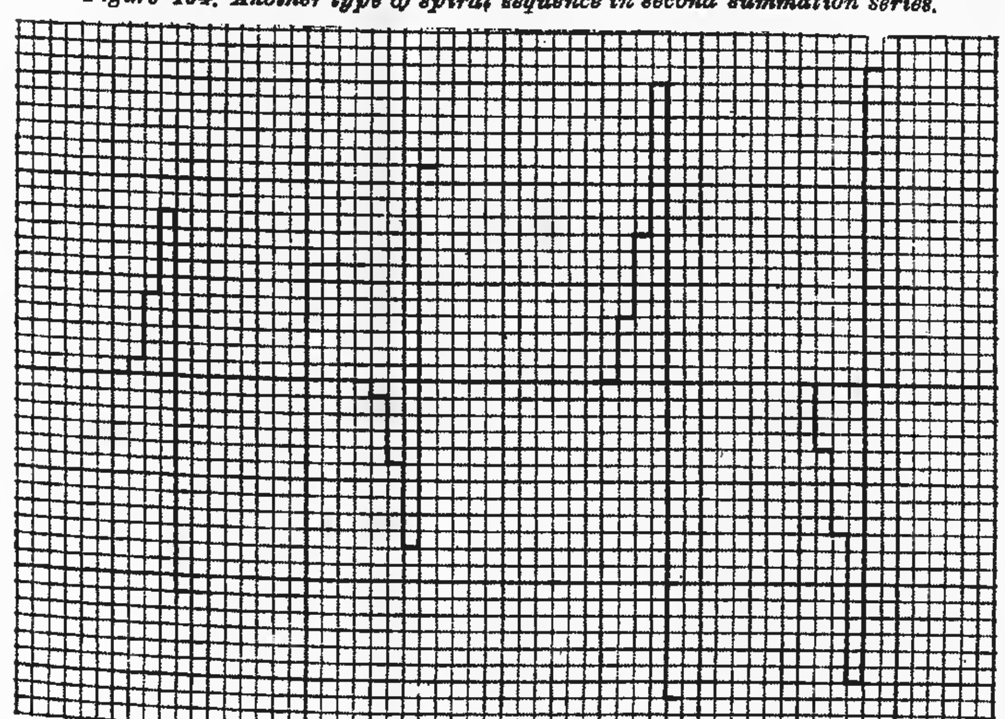


Figure 135. Another type of spiral sequence in second summation series (continued).

Figure 135. Another type of spiral sequence in second summation series.

Many other forms of the harmonic arrangement of numbers produce an *organic effect* upon the listener when such harmonic relations underlie the structure of melodic intervals.

Among such harmonic relations I will mention only the most fundamental ones:

1. Natural harmonic series.
2. Arithmetical progressions.
3. Geometrical progressions.
4. Involution series.
5. Various logarithmic series.
6. Progressive additive series.
7. Prime number series.
8. Arithmetical mean.
9. Geometrical mean.

These series of constant or variable ratios with harmonic arrangement of number values, when translated into an art medium, produce organic or nearly organic effects. Spiral formation as revealed through Summation Series affects us as being organic because there is *an intuitive interdependence of man and surrounding nature.* The patterns of growth stimulate in human beings a definite response which is more powerful than many other similar but casual formations.

Thus we see that the *forms of organic growth associated with life, well-being, self-preservation and evolution appeal to us as a form of beauty when expressed through an art medium.* Intuitive artists of great merit are usually endowed with great sensitiveness and intuitive knowledge of the underlying scheme of things. This is why a composer like Wagner is capable of projecting spiral formations through the medium of musical intonations without any analytical knowledge of the process involved. On the other hand, scientific analysis shows that the efforts of greatly endowed and creative persons could have been accomplished without any waste of time, introspection, or over-sensitiveness. Once the laws underlying certain structures have been disclosed, anyone can develop any number of structures in a class through the use of a formula. This does not prevent an artist, who makes an individual selection (whatever the value of such selection may be), from operating under the illusion of as great a freedom as that he imagines he possesses when he creates through the channels of vague intuition and nebulous notions.

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK V

SPECIAL THEORY OF HARMONY

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

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BOOK V

SPECIAL THEORY OF HARMONY

BOOK FIVE

SPECIAL THEORY OF HARMONY

CHAPTER 1

INTRODUCTION

MY SPECIAL theory of harmony is confined to E_1 of the first group of scales, which contains all musical names (c, d, e, f, g, a, b) and without repetition. There are 36 such scales in all. The total number of seven-unit scales equals 462.*

The uses of E_1 refer both to structures and progressions in the diatonic system of harmony. The latter can be defined as a system which borrows all its pitch units for both structures and progressions from any one of the 36 scales. When the structures are limited to the above scales but the progressions develop through all the semitonal relations of equal temperament, the latter comprises all the symmetric systems of pitch, *i.e.*, the third and the fourth group.

Chord-structures, contrary to common notion, do *not* derive from harmonics. If the evolution of chord-structures in musical harmony had paralleled the evolution of harmonics, we would never have acquired the developed forms of harmony we now possess.

To begin with, a group of harmonics when simultaneously produced at equal amplitudes, sounds like a saturated unison, not like a chord. In other words, a perfect harmony of frequencies and intensities does not result in musical harmony but rather in a unison. This means that through the use of harmonics, we would never have arrived at musical harmony. But actually, we do get harmony and for exactly the opposite reason. The relations of the sounds we use in equal temperament are not simple ratios (harmonic ratios).

When acousticians and music theorists advocate "just intonation", that is, the intonation of harmonic ratios, they are not aware of the actual situation. On the other hand, the ratios they give for certain familiar chords, like the major triad $(4 \div 5 \div 6)$, the minor triad $(5 \div 6 \div 15)$, the dominant seventh-chord $(4 \div 5 \div 6 \div 7)$, do not correspond to the actual intonations of equal temperament. Some of these ratios, like $\frac{7}{4}$, deviate so much from the nearest intonation, like the minor seventh which we have adopted through habit, that it sounds to us out of tune.

Habits in music, as well as in all manifestations of life, are more important than natural phenomena. If the problem of chord-structures in harmony were confined to the ratios nearest to equal temperament, we could have offered $16 \div 19 \div 24$ for the minor triad, for example, as that ratio in fact approaches the tempered minor triad much more closely than $5 \div 6 \div 15$. But, if accepted, this would discredit the approach commonly used in all textbooks on harmony, for

*Special theory is used, of course, in distinction to general theory. Schillinger's special theory confines itself to structures built in thirds, whereas in the general theory—which is set forth at a later point in the work—the possibilities of harmonies which construct chords in fourths, fifths, etc., are discussed. (Ed.)

the following reason: if such high harmonics as the 19th are necessary for the construction of a minor triad, what would chords of superior complexity, which are in use today, look like when expressed through ratios? When a violinist plays b as a leading tone to c and raises the pitch of b above the tempered b, his claims for higher acoustical perfection are nonsense, as the nearest harmonic in that region is the 135th.

Facing facts, we have to admit that all the *acoustical* explanations of chord-structures—to the effect that they are developed from the simple ratios—are pseudo-scientific attempts to rehabilitate musical harmony and to give the latter a greater prestige. Though the original reasoning in this field resulted from the honest spirit of investigation of Jean Philippe Rameau (*Generation Harmonique*, Paris, 1737), his successors overlooked the development of acoustical science. Their inspiration was Rameau—plus their own mental laziness and cowardice.

The whole misunderstanding in the field of musical harmony is due to two main factors:

(1) underrating habit;

(2) confusion of the term "harmonic" in its mathematical connotation—i.e., pertaining to simple ratios—with "harmony" in its musical connotation—i.e., simultaneous pitch-assemblages varied in time sequence.

Thus, musical harmony is not a "natural phenomenon," but a highly conditioned and specialized field. It is the material of musical expression, for which we, in our civilization, have an inborn inclination and need. This need is cultivated and furthered by existing trends in our music and musical education.

CHAPTER 2

THE DIATONIC SYSTEM OF HARMONY

CHORD structures and chord progressions in the diatonic system of harmony have a definite interdependence: ***chord-structures develop in a direction opposite to their progressions.***

This statement brings about the practical classification of the diatonic system into two forms: the ***positive*** and the ***negative***.

As the term *diatonic* implies, ***all pitch-units of a given scale*** constitute both structures and progressions, without the use of any other pitch-units (those not existing in a given scale) whatsoever.

In the form which we shall call ***positive***, all chord structures (S) are the component parts of the entire structure (Σ) emphasizing all pitch-units of a given scale in their ***first tonal expansion*** (E_1) and in position ⓐ. In the same form, chord progressions derive from the same tonal expansion but in position ⓑ.

In the ***negative*** form of the diatonic system, it works in the opposite manner. Chord-structures derive from the scale in E_1 and in position ⓑ, while the progressions develop from E_1ⓐ.

By reason of the personal qualities we have inherited and developed, the positive form produces an effect of greater tonal stability upon us. It is chronologically true that the negative form is an ***earlier*** one. It predominates in the works where the effect of tonality, as we know and feel it today, is rather vague. Such is 14th and 15th century ecclesiastic music, developed on contrapuntal, not harmonic, foundations.

Many theorists confuse the negative form of the diatonic system with "modal" harmony. Since to them diatonic tonality generally means natural major or harmonic minor scales moving in the positive form, they notice the lack of tonal stability when harmony moves backwards. Losing tonal orientation, they mistake such ***progressions*** for modes—and modes are merely derivative scales, and may also have the positive, as well as the negative, form. But—as we have seen in the *Theory of Pitch Scales**—modes can be acquired from any original scale through the introduction of accidentals (sharps and flats).

In the following table, MS represents "melody scale" (pitch-scale), and HS represents "harmony scale" (i.e., the fundamental sequence of chord progressions).

Diatonic System

Positive Form	*Negative Form*
$\Sigma = MS_{E_1 \text{ⓐ}}$	$\Sigma = MS_{E_1 \text{ⓑ}}$
$HS = MS_{E_1 \text{ⓑ}}$	$HS = MS_{E_1 \text{ⓐ}}$

*See Book II, Chapter 3.

Example (Natural Major)

Figure 1. Diatonic system, negative and positive forms.

In the positive form, chords are constructed upward; in the negative, on the contrary, downward. The matter is greatly simplified by the fact that any progression, originally written as positive, becomes negative when read backwards. *All the principles of structures and motion involved are therefore reversible.* No properly constructed harmonic continuity can be wrong in backward motion.

Some composers without training in harmony (for example, Modest Moussorgsky)—as well as beginners because of inadequate study—confuse the positive and the negative forms in writing their harmonic progressions. The resulting effect of such music is a vague tonality. The admirers of Moussorgsky consider such style a virtue (in Moussorgsky's case it is about half-and-half positive and negative) and do not realize that all the incompetent students of a harmony course incompetently taught possess full command over such a style.

A. Diatonic Progressions (Positive Form)

Expansions of the original harmony scale produce the derivative harmony scales. The original HS and its expansions form the *diatonic cycles.* Diatonic (or tonal) cycles represent all the fundamental chord progressions.

There are three Tonal Cycles in the positive form for the seven-unit scales. The first cycle, or *cycle of the third* (C_3), corresponds to HS_{E_0}; the second cycle, or *cycle of the fifth* (C_5), corresponds to HS_{E_1}; the third cycle, or the *cycle of the seventh* (C_7), corresponds to HS_{E_2}. Beyond these expansions of HS lie the negative forms of the diatonic progression.

In addition to both forms of progressions, there may be changes in a chord pertaining to the same root (axis). Connections of an S with its modified S of the same root will be considered a *zero cycle* (C_0).

In the following table, notes are used merely for convenience; they indicate the sequence of roots; their octave position is dictated by purely melodic considerations and by the necessity of moderating the range.

The respective intervals representing cycles must be *constructed* downward for the positive form regardless of their actual position on the musical staff.

Diatonic Cycles (Positive Form)

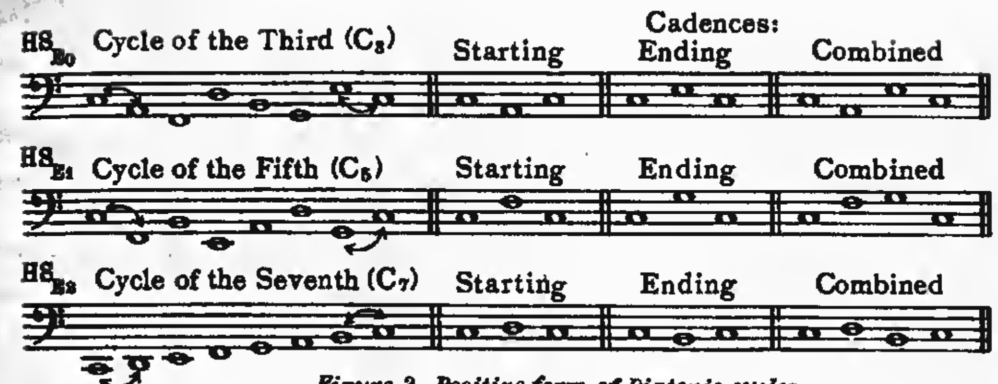


Figure 2. Positive form of Diatonic cycles.

In the above table, arrows indicate *cadences* of the respective cycles. Cadences consist of the axis-chord moving into its adjacent chord and back. It is interesting to note that what are usually known as *plagal cadences* are the *starting cadences* of the cycle, and that *cadences* known as *authentic* are the *ending cadences.* The immediate sequence of starting and ending cadences produces *combined cadences* (the axis-chord is omitted in the middle).

Progressions of constant tonal cycles (C_3, or C_5, or C_7 const.) produce a sequence of seven chords each appearing once and none repeating itself. The repetition of the axis-chord either completes the cycle or starts a new one. The addition of cadences to the cycles is optional as cycles are self-sufficient.

Considering constant cycles as a form of *monomial progression,* we can devise *binomial* and *trinomial progressions* by assigning a sequence of two or three cycles at a time.

In binomial progressions *each chord* appears *twice* and in a different combination with the preceding and the following chord. Thus, a complete *binomial cycle* in a seven-unit scale consists of ($2 \times 7 =$) 14 chords.

Binomial Cycles

$C_3 + C_5$	$C_5 + C_3$	$C_7 + C_3$
$C_3 + C_7$	$C_5 + C_7$	$C_7 + C_5$

Figure 3. Binomial cycles (continued).

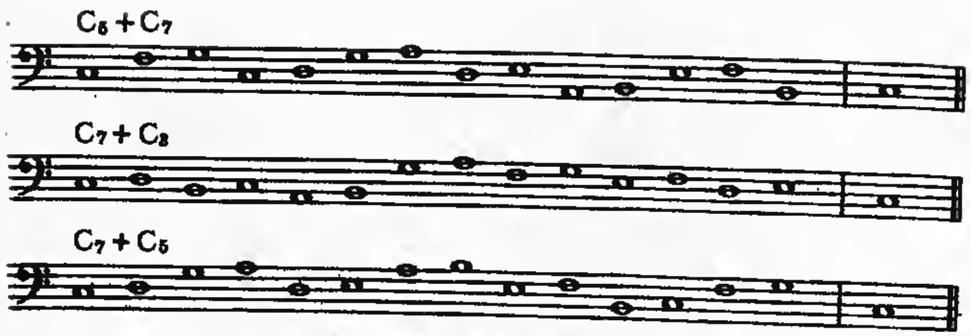


Figure 3. Binomial cycles (concluded).

In trinomial progressions *each chord* appears *three times* and in a different combination from the preceding and the following chord. Thus, a complete *trinomial cycle* in a seven-unit scale consists of $(3 \times 7 =)$ 21 chords.

Trinomial Cycles

$C_3+C_5+C_7$	$C_7+C_3+C_5$	$C_5+C_7+C_3$
$C_3+C_7+C_5$	$C_5+C_3+C_7$	$C_7+C_5+C_3$

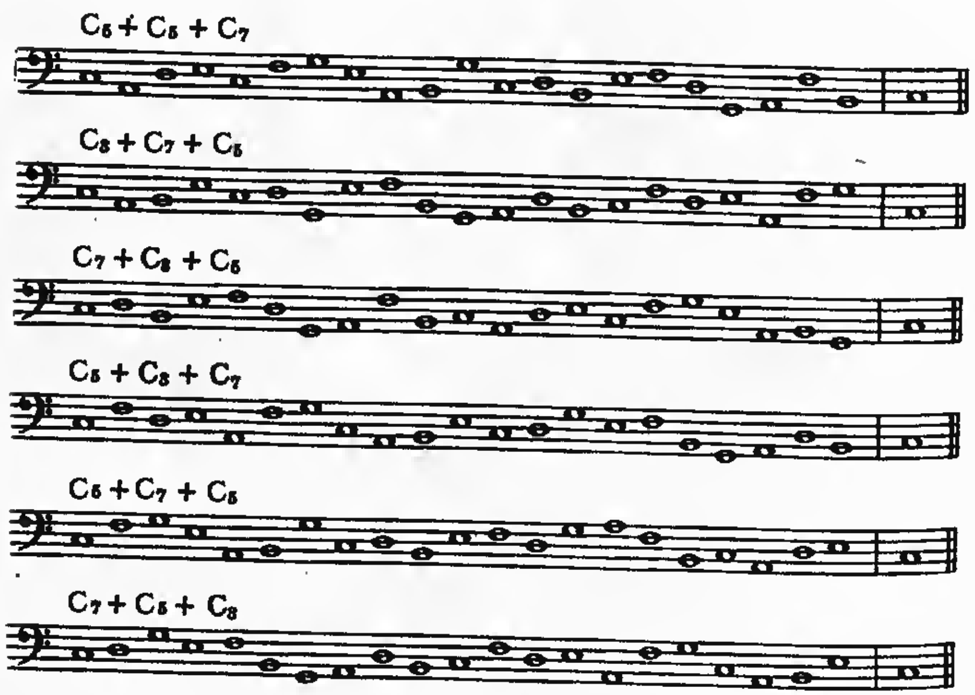


Figure 4. Trinomial cycles.

Both binomial and trinomial cycles produce marked *variety* combined with absolute *consistency* of character (style) of harmonic progression. Being *perfect* in this respect they are of little use when a personal selection of character becomes a paramount factor.

In order to produce an individual style of harmonic progression, it is necessary to use a *selective continuity* of cycles. This can be accomplished by means of the *coefficients of recurrence* applied to a selected combination of cycles. A combination of cycles can be either *binomial* or a *trinomial.* Groups producing coefficients of recurrence can be *binomial*, *trinomial* or *polynomial*. The materials for these are presented in the *Theory of Rhythm.** Rhythmic resultants of different types and their variations provide various groups which can be used as coefficients of recurrence. Distributive power-groups, as well as the different series of growth and acceleration,** can be used for the same purpose.

Binomial Cycles, Binomial Coefficients

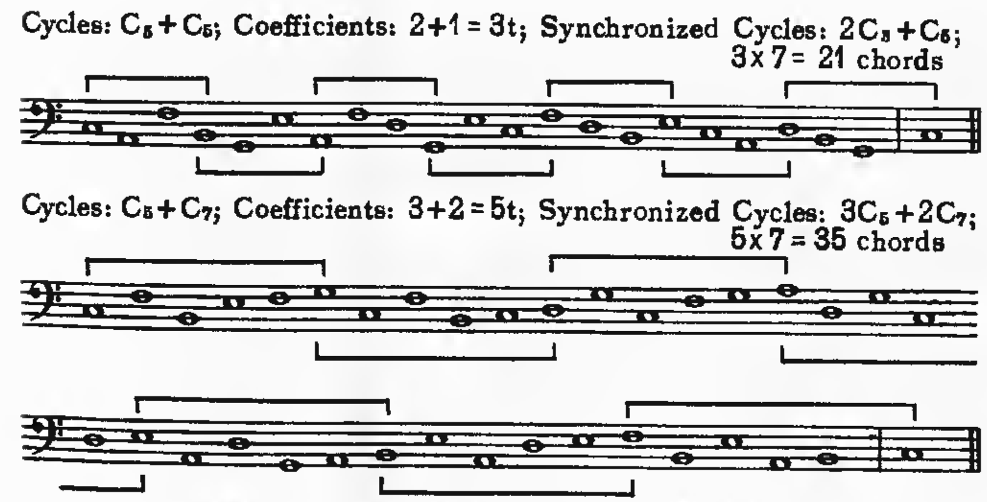


Figure 5. Binomial cycles, binomial coefficients.

*See Book I.

**The distributive power-groups are discussed by Schillinger in Chapter 12 of Book I. Examples of the different series of growth and acceleration are presented in Chapter 14 of the same book. Among the useful series of acceleration for musical purposes are the following:

1) natural harmonic series: 1, 2, 3, 4, 5, 6, 7, 8, 9, etc.;

2) arithmetical progressions: 1, 3, 5, 7, 9, etc.;

3) geometrical progression: 1, 2, 4, 8, 16, etc.;

4) power series: 2, 4, 8, 16, 32, etc.;

5) summation series: 1, 2, 3, 5, 8, 13, 21, etc.;

6) arithmetical progressions with variable differences: 1^{+1}, 2^{+2}, 4^{+3}, 7^{+4}, 11^{+5}, 16^{+6}, 22^{+7}, etc.

7) prime number series: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc. (Ed.)

Binomial Cycles, Coefficient-Groups with the number of terms divisible by 2

Cycles: $C_7 + C_3$; Coefficients: $r_{4 \div 3} = 3+1+2+2+1+3 = 12t$

Synchronized Cycles: $3C_7 + C_3 + 2C_7 + 2C_3 + C_7 + 3C_3$; $12 \times 7 = 84$ chords

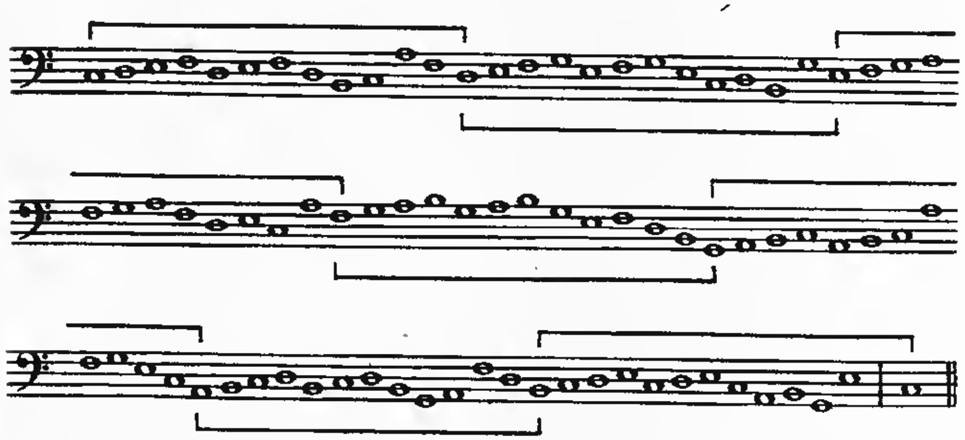


Figure 6. Binomial cycles, coefficient-groups with number of terms divisible by 2.

Binomial Cycles, Coefficient-Groups producing interference with the cycles (not divisible by 2)

Cycles: $C_5 + C_3$ Coefficients: $3+1+2 = 6t$

Synchronized Cycles: $3C_5 + C_3 + 2C_5 + 3C_3 + C_5 + 2C_3$

Synchronized coefficients: $6t \times 2 = 12t$; $12 \times 7 = 84$ chords

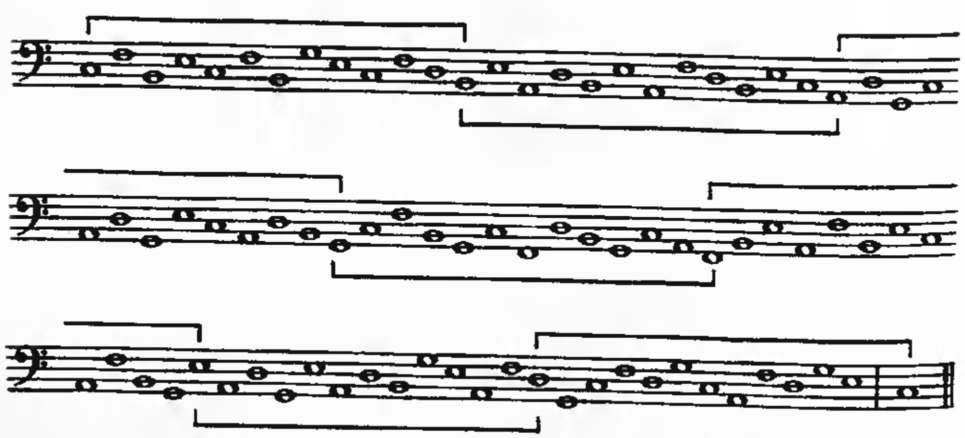


Figure 7. Binomial cycles, coefficient-groups producing interference with the cycles.

Trinomial Cycles, Trinomial Coefficients

Cycles: $C_3 + C_5 + C_7$ Coefficients: $4+1+3 = 8t$

Synchronized Cycles: $4C_3 + C_5 + 3C_7$; $8 \times 7 = 56$ chords

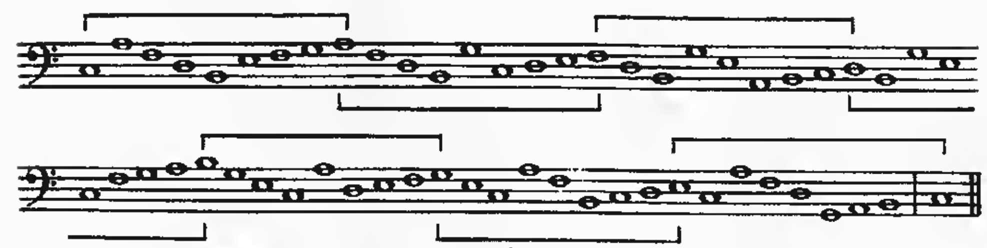


Figure 8. Trinomial cycles, trinomial coefficients.

Trinomial Cycles, Coefficient-Groups with the number of terms divisible by 3

Cycles: $C_7 + C_3 + C_5$; Coefficients: $r_{5 \div 2} = 2+2+1+1+2+2 = 10t$

Synchronized Cycles: $2C_7 + 2C_3 + C_5 + C_7 + 2C_3 + 2C_5$; $10 \times 7 = 70$ chords

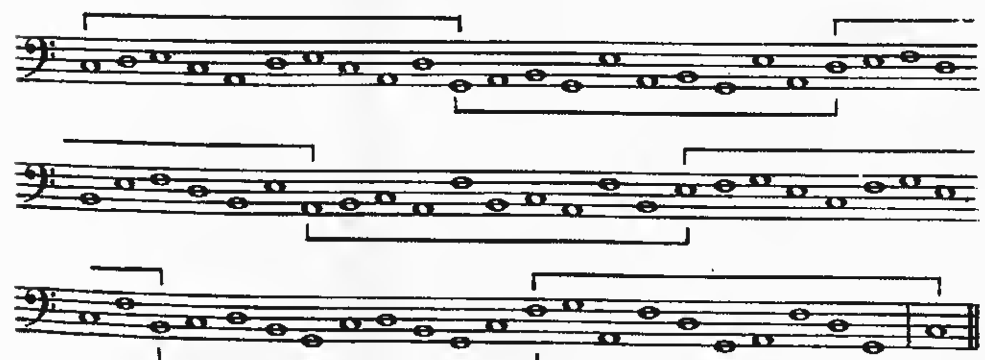


Figure 9. Trinomial cycles, coefficient-groups with number of terms divisible by 3.

Trinomial Cycles, Coefficient-Groups producing interference with the cycles (not divisible by 3)

Cycles: $C_7 + C_5 + C_3$; Coefficients: $3+1 = 4t$

Synchronized Cycles: $3C_7 + C_5 + 3C_3 + C_7 + 3C_5 + C_3$

Synchronized Coefficients: $4t \times 3 = 12t$; $12 \times 7 = 84$ chords

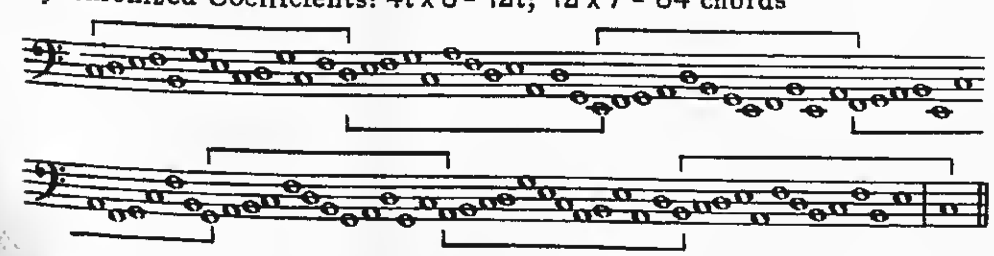


Figure 10. Trinomial cycles, coefficient-groups not divisible by 3.

The *style* of harmonic progressions depends entirely on the form of cycles employed. No composer confines himself to one definite cycle, yet it is the predominance of a certain cycle over others that makes his music immediately recognizable to the listener. In one case it may be that the beginning of a progression is expressed through the cadences of a certain cycle; in another case it may be a prominent coefficient group that makes such music sound distinctly different from other music.

The style of harmonic progressions can be defined as: a definite form of *selective cycles*. Both the combination of cycles (their sequence) and the coefficient group determining their recurrence are the factors of a *style* of harmonic progressions.

B. Development of Cycle Styles

There is much that needs to be said about the development of the cycles, for there are already some wrong notions established in this field.

Though the common belief is that progression from the tonic to the dominant and back to the tonic (ending cadence in C_5) is the foundation of diatonic harmony, historical evidence, as well as mathematical analysis, prove the contrary.

During the course of centuries of European musical history, parallel to the development of counterpoint, there was an awakening of harmonic consciousness. The latter can be traced, in its current forms, back to the 15th century A.D. At that time *harmony* meant *concord*—an agreeable, consonant, stabilized sonority of several voices simultaneously sustained.

Concordant progressions could be achieved through consonant chords moving in consonant relations. Obviously such progressions require common tones; these can be expressed as C_3. As *tonality*—i.e., an organized progression of tonal cycles—was at that time in a state of fermentation, it is natural to expect that the cycle of the third would appear in both positive (C_3) and negative (C_{-3}) forms.

The following are a few illustrations taken from the music of the 15th and 16th centuries.

Opening of "Ave Regina Coelorum"_ Leonel Power, c. 1460

Figure 11. Cycle of the third (continued).

"Benedicta Tu" MS. Pepysian 1236, Madrigal Collection, Cambridge, c. 1460

"Deutsches Lied"_ Adam von Fulda (1470)

Giulio Caccini (1550–1618)

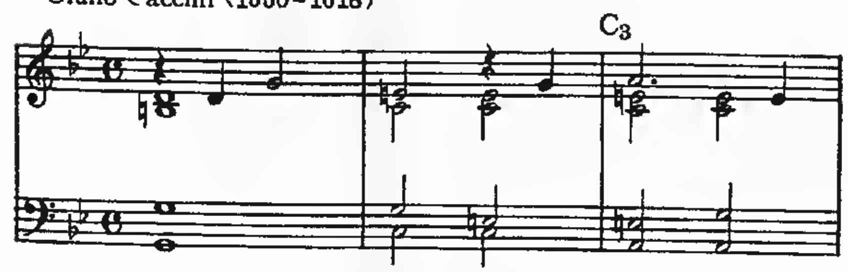


Figure 11. Cycle of the third (concluded).

The cycle of the seventh (C_7), on the other hand, has a purely *contrapuntal* derivation. When the two leading tones (the upper and the lower) move in a cadence into their respective tonics (like b→c and d→c) by means of contrary motion in two voices, we obtain the ending cadence of C_7. Further development of the third part was undoubtedly necessitated by the desire for fuller sonorities. This introduced an extra tone (f in a chord of b) with which the remaining tones form S(6), i.e., a third-sixth-chord or a sixth-chord, the first inversion of the root-chord: S(5).

Figure 12. Cycle of seventh (C_7).

It is only natural to expect the predominance of the C_7 in contrapuntal music. Cadences—such as in *Figure 12*—are most standardized in 13th and 14th century European music; see Guillaume de Machault's (1300-1377) *Mass for the Coronation of Charles V.**

The appearance of the cycle of the fifth occurred at a later date, by which time C_3 and C_7 were already in use. I offer the following hypothesis of the origin of C_5. The positive form might have occurred as a pedal-point development where, by sustaining the tonic and changing the remaining two tones to their leading tones, the sequence would represent C_5. Another interpretation of the origin of C_5 is the one on which the present system of harmony is based, i.e., *omission of intermediate links in a series.* (This principle ties up musical harmony with the harmonic structure of crystals as used in crystallographic analysis.)

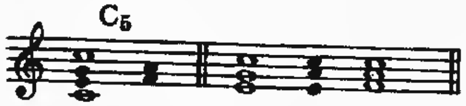


Figure 13. Cycle of the fifth (C_5).

The origin of the negative form of the cycle of the fifth (C_{-5}) is due to the desire to acquire a concord supporting a leading tone. Let b be a leading tone in the scale of c. The most concordant combination of tones in pre-Bach times, i.e., in the *mean temperament* tuning system,** which harmonized the tone b was the G-chord (g, b, d). But, in the movement from G-chord to C-chord, the form of the cycle is positive. In reality both forms, the positive and the negative, are the beginning and the ending cadences. Compare *Figure 14* with *Figure 13*.

Figure 14. Cycle of the fifth (C_5). (See pp. 363 and 386).

1. Richard Wagner

The development of harmonic progressions in the European music of the last three centuries can be easily traced back to its sources. The style of every composer is *hybrid*, yet the quantitative predominance of certain ingredients (like the cycles appearing with the different coefficients of recurrence) produces individual characteristics.

In the followiog exposition, I will *confine* the concept of "style" to harmonic progressions in the diatonic system.

Richard Wagner was the greatest representative of C_3 in the 19th century. This statement is backed by actual statistical analysis of tonal cycles in his works, as compared to those of his contemporaries and predecessors. C_5 was the universal vogue of the whole century preceding Wagner. In fact, it is not even

*Recording issued by Gramophone Shop of New York City.

**Officially recognized in Europe before the advent of *equal temperament* tuning. (Ed.)

necessary to analyze all the works of Wagner; the most characteristic progressions may be found at the *beginning* of his preludes to music-dramas and also in the various cadences.

The beginnings of the major works of any composer are important for the reason that they cannot be casual: the beginning is the "calling card" of a composer. The importance of cadences as determinants of harmonic styles was stressed by our contemporary Alfredo Casella in a paper *Evolution of Harmony from the Authentic Cadence.*

Wagner, being German and being an intentionally Germanic composer, undoubtedly had done some research into earlier German music, for he intended to deal with the subjects of German mythology in which he was well versed. Fifteenth century German music discloses such an abundance of C_3 that it is only natural to expect there would be strong influence by such an authentic source of Germanic music on Wagner's creations. In his time, Wagner's harmonic progressions sounded revolutionary because many things had been forgotten in four hundred years, and the archaic acquired a flavor of the modernistic. So far as the development of diatonic progressions in Wagner's music is concerned, it appears to the unbiased analyst that the whole mission of Wagner's life was to develop a consistent combined cadence in C_3.

Starting with an early work like *Tannhäuser*, we find that the very beginning of the overture is typical in this respect.

Figure 15. Opening of Tannhäuser.

Later on, we find more extended progressions of C_3, as in the aria of Wolfram von Eschenbach (the scene of the Minnesingers' contest):

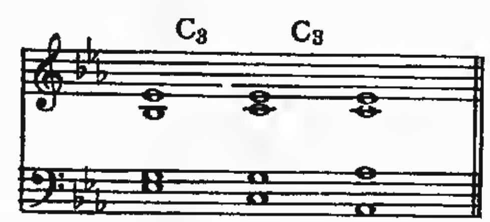


Figure 16. Aria of Wolfram von Eschenbach.

Lohengrin abounds even more in C_3 than *Tannhäuser*. In the "Farewell to the Swan", as in many other passages in the same opera, we find the characteristic back-and-forth fluctuation: $C_3 + C_{-3}$.

Figure 17. "Farewell to the Swan".

In forming his cadences, Wagner sometimes paid tribute to the dominating "dominant" of Beethoven (C_5). This produced combined hybrid cadences, which are characteristic of *Lohengrin*. The first part of such a cadence is the beginning cadence in C_3 while the second part is the ending cadence in C_5: I — IV — V — I.

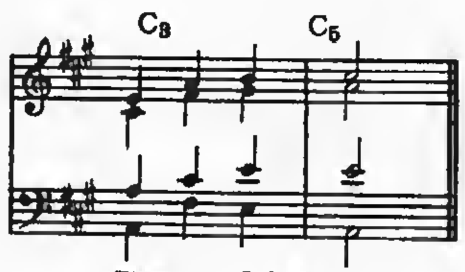


Figure 18. Lohengrin.

Though he dealt with types of progression other than diatonic in the course of his career, Wagner came back to diatonic purity in its most complete and consistent form in his last work, *Parsifal*. The beginning of the prelude to Act I reveals that the composer came to a realization of the combined cadence of C_3: I — VI — III;

Figure 19. Beginning of Parsifal.

The more extensive sequences of C_3 are: I — VI — IV — II;

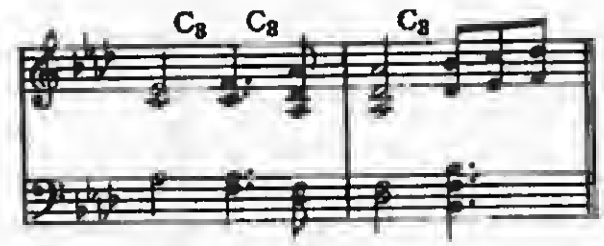


Figure 20. Parsifal *motif.*

The complete combined cadence appears in the "Procession of the Knights of the Grail": I — VI — III — I.

Figure 21. Procession of The Knights of The Grail.

2. Hegemony of C_5, 1750-1850

The second half of the 18th century and the first half of the 19th century are the period of the hegemony of the dominant and C_5 in all its aspects in general. The latter are: continuous progressions of C_5; starting, ending and combined cadences (I — IV — I; I — V — I; I — IV — V — I). The main bodies of music possessing these characteristics are the Italian opera and the Viennese School.

To the first belong Monteverdi, Scarlatti, Pergolesi, Rossini, and Verdi. The second is represented by Dittersdorf, Haydn, Mozart, Beethoven, and Schubert. Today this style has disintegrated into the least imaginative creations in the field of popular music. Nevertheless, it is the stronghold of harmony in educational music institutions.

Here are a few illustrations of C_5 style in the early sonatas for piano by Ludwig van Beethoven: Sonata Op. 7, Largo; Sonata Op. 13, Adagio Cantabile.

Figure 22. C_5 style in early Beethoven sonatas.

Any number of illustrations can be found in Mozart's and Beethoven's symphonies, particularly in the conclusive parts of the last movements.

3. C_7 in Bach

Assuming that the historical origin of the cycle of the seventh can be traced back to contrapuntal cadences, it would be only logical to expect to find evidence of C_7 in the works of the great contrapuntalists. I choose for the illustration of C_7, as characteristic starting progressions, some of the well-known Preludes to Fugues taken from the First Volume of the *Well-Tempered Clavichord* by Johann Sebastian Bach: Prelude I; Prelude III; Prelude V.

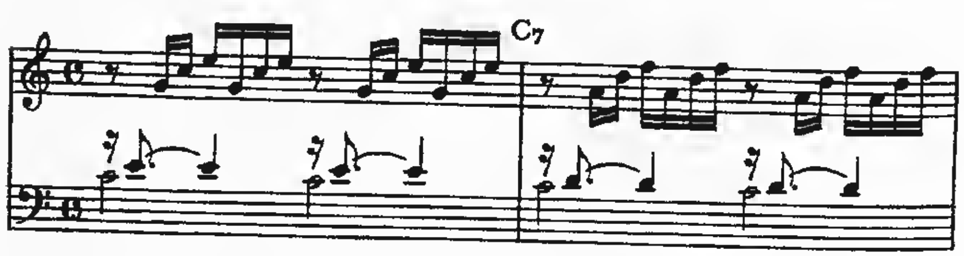


Figure 23. C_7 in Bach (continued).

Figure 23. C_7 in Bach (*concluded*).

Bach's famous *Chaconne in D-minor* for violin discloses the same characteristics: the first chord is d, and the second chord is e—which makes C_7.

A consistent and ripe style of diatonic progression corresponds to a consistent use of one form, either positive or negative, and *not* to an indiscriminate mixture of both. Many theorists confuse the hybrid of positive and negative forms with *modal progressions*, which these theorists have never defined clearly. In reality, *modal progressions* are in no respect different from *tonal progressions* except for scale structure. Both types (tonal and modal) can be either positive, or negative, or hybrid. Modes can be obtained by the direct change of key signatures, as set forth in my theory of pitch-scales (transposition to one axis).*

Here is an example, which is typical of Moussorgsky from the opera *Boris Godounov*.

Figure 24. Hybrid of positive and negative forms

In the above example, the mode (scale) is Cd_6, the fifth derivative scale of the natural major in the key of C, known as the Aeolian mode; the progression of tonal cycles is a hybrid of positive and negative forms.

*See Book II.

C. TRANSFORMATIONS OF S(5).

In traditional courses in harmony the problems of progressions and voice-leading are treated as inseparable. Each pair of chords is described as a sequence and as a form of voice-leading. Thus each case becomes an individual case where the movement of voices is described in terms of melodic intervals—like: "a fifth down", "a second up", "a leap in soprano", "a sustained tone in alto", etc. No person of normal mentality can ever memorize all the rules and exceptions offered in such courses. In addition to this unsatisfactory form of presentation of the subject of harmony, one finds out very soon that the abundance of rules covers very limited material, mostly the harmony of the second-rate 18th century European composers.

The main defect of existing theories of harmony is in the use of the *descriptive method*. Each case is analyzed apart from all other cases and without yielding any general underlying principles. But the mathematical treatment of this subject discloses the *general properties* of the positions and movements of the voices in terms of *transformations of the chordal functions.*

Any chord, no matter of what structure, is from a mathematical standpoint an *assemblage of pitch units*, or a *group of conjugated functions* (elements). These functions are the different pitch-units distributed in each group, assemblage, or chord, according to the different number of voices (parts) and the intervals between the latter.

In groups with three functions, known as three-part structures ($S = 3p$), the functions are a, b and c. These functions behave through *general forms of transformation* and not through any musical specifications.

As in this branch we are dealing with so-called four-part harmony, we have to define the meaning of this expression more precisely.

When an S(5) constitutes a chord-structure, the functions of the chord are: *the root, the third and the fifth* or 1, 3 and 5. In their general form they correspond to a, b and c, i.e., $a = 1$, $b = 3$, and $c = 5$. The bass of such harmony is a constant root-tone, i.e., const. 1 or const. a.

Thus the transformation of functions affects all parts except the bass. Here, therefore, we are dealing with groups consisting of three functions.

Such groups have two fundamental transformations: (1) clockwise (↻) and (2) counterclockwise (↺)

The clockwise transformation is:

a
c b

The counterclockwise transformation is:

a
c b

Each of these transformations has two meanings: the first meaning is to be read—

a is followed by b
b " " " c
c " " " a,
for the ↻; and

a is followed by c
c " " " b
b " " " a
for the ↺

discloses the mechanism of the *position of a chord.*

The second meaning is to be read—

a transforms into b
b " " c
c " " a,
for the ↻; and

a transforms into c
c " " b
b " " a,
for the ↺.

These constitute *the forms of voice-leading.*

1. Positions

The different positions of $S(5) = 1, 3, 5$ can be obtained by constructing the chordal functions downward from each phase of the transformations.

$$\begin{array}{ccc} a & b & c \\ b & c & a \\ c & a & b \end{array}\downarrow \ \circlearrowright \quad \text{and} \quad \begin{array}{ccc} a & c & b \\ c & b & a \\ b & a & c \end{array}\downarrow \ \circlearrowleft$$

Substituting 1, 3, 5 for a, b, c, we obtain

$$\begin{array}{ccc} 1 & 3 & 5 \\ 3 & 5 & 1 \\ 5 & 1 & 3 \end{array}\downarrow \ \circlearrowright \quad \text{and} \quad \begin{array}{ccc} 1 & 5 & 3 \\ 5 & 3 & 1 \\ 3 & 1 & 5 \end{array}\downarrow \ \circlearrowleft$$

The clockwise positions are commonly known as "open", and the counter-clockwise as "closed."

Here are the positions for $S(5) = 4 + 3 = c - e - g$. The bass is added to double the root.

Positions

Figure 25. Open and close positions of S5.

D. Voice-Leading

The movement of the individual voices follows the groups of transformation in this form: a of the first chord transforms into b of the following chord; b of the first chord transforms into c of the following chord; c of the first chord transforms into a of the following chord. The above three forms constitute *clockwise* voice-leading.

For *counterclockwise* voice-leading, the reading must follow this order: a of the first chord transforms into c of the following chord; c of the first chord transforms into b of the following chord; b of the first chord transforms into a of the following chord.

↻		↺
a → b		a → c
b → c	and	c → b
c → a		b → a

Applying the above transformations to 1, 3, 5 of the S(5), we obtain:

↻		↺
1 → 3		1 → 5
3 → 5	and	5 → 3
5 → 1		3 → 1

Clockwise form: the root of the first chord becomes the third of the next chord; the third of the first chord becomes the fifth of the next chord; the fifth of the first chord becomes the root of the next chord.

Counterclockwise form: the root of the first chord becomes the fifth of the next chord; the fifth of the first chord becomes the third of the next chord; the third of the first chord becomes the root of the next chord.

Both forms apply to all tonal cycles. Let us take C_3 in the natural major, for example. The first chord is $C = c - e - g$ and the next chord is $A = a - - c - e$.

Clockwise form gives the following reading:

c → c
g → a
e → e

Figure 26. Clockwise transformation of C_3.

Counterclockwise form gives the following reading:

c → e
g → c
e → a

Figure 27. Counterclockwise transformation of C_3.

Let us take C_5 in the same scale. The chords are: $C = c - e - g$ and $F = f - a - c$.

↻

c → a
g → f
e → c

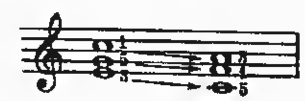


Figure 28. Clockwise transformation of C_5.

↺

c → c
g → a
e → f

Figure 29. Counterclockwise transformation of C_5.

Let us take C_7 in the same scale. The chords are: $C = c - e - g$ and $D = d - f - a$.

↻

c → f
g → d
e → a

or

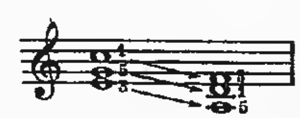


Figure 30. Clockwise transformation of C_7.

Both forms of ↻ are acceptable in this case, as the intervals in both directions are nearly equidistant.

↺

c → a
g → f
e → d

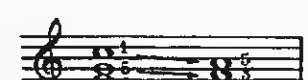


Figure 31. Counterclockwise transformation of C_7.

Each tonal cycle permits a continuous progression through one form of transformation. In the following table const. 1 in the bass is added. The commas indicate an octave variation introduced when the extension of range becomes impractical.

In C_7 both directions are combined, offering the most practical form for the range.

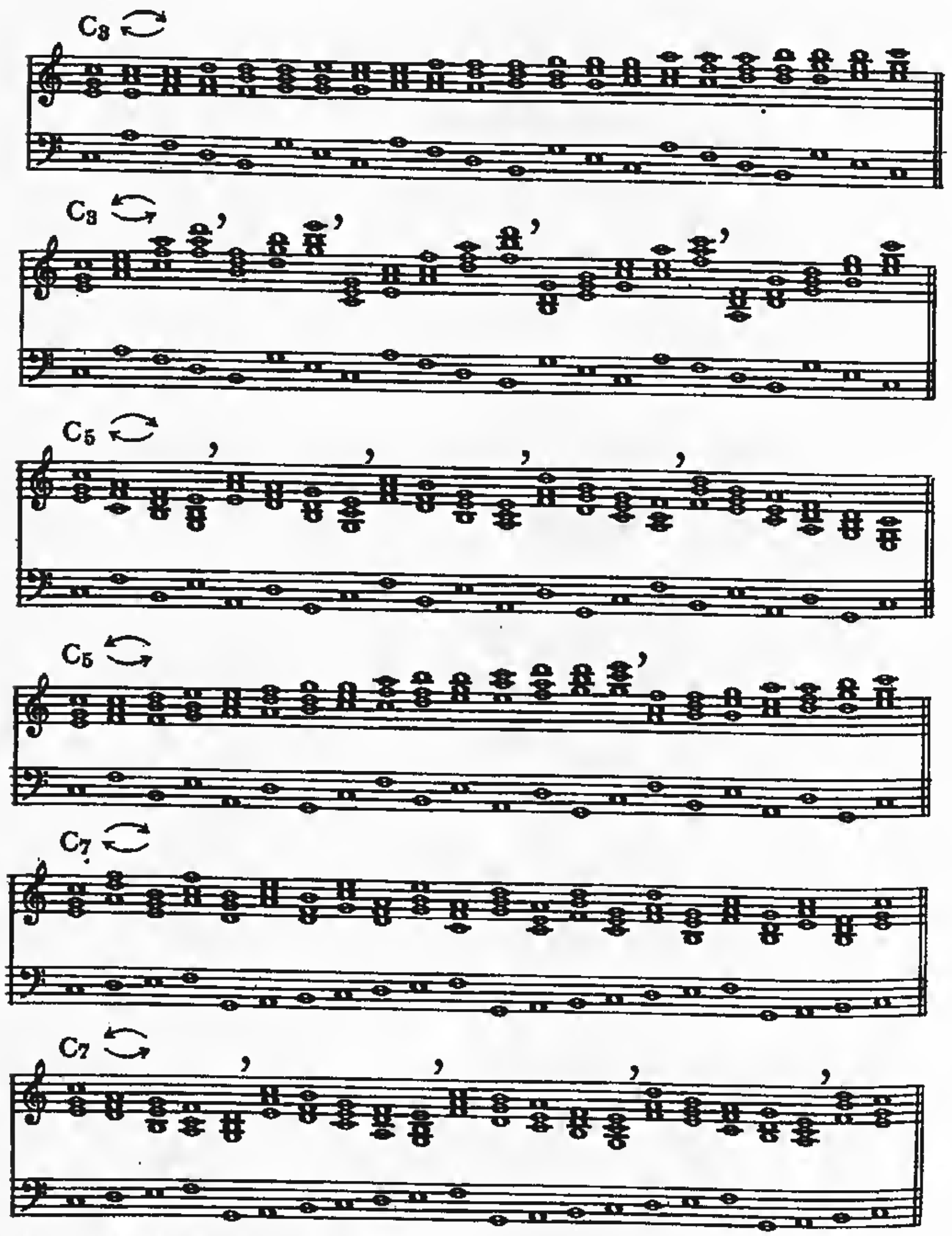


Figure 32. Clockwise and counterclockwise transformations of C_3, C_5, C_7.

Both clockwise and counterclockwise transformations are applicable to *all* positions for the starting chord. When the first chord is in the ↻ (open) position, the entire progression remains automatically in such a position. When the first chord is in ↺ (close) position, the entire progression remains in that position. This constancy of position (open or close) is not affected either by the constancy of the tonal cycles or by the lack of such constancy.

The transition from close to open position and vice-versa can be accomplished through the use of the following formula:

Constant b transformation

	Const. 3
a → c	1 → 5
b → b	3 → 3
c → a	5 → 1

It is best to have 3 in the upper voice for such purposes, as in some positions voices will otherwise cross. Function 3 from close to open position moves upward to function 3 of the following chord. Reverse the procedure from open to close.

Figure 33. Transitions from open to close positions and vice-versa.

Continuous application of const. 3 transformation produces a consistent variation of the ↻ and the ↺ positions, regardless of the sequence of tonal cycles.

The following table offers continuous progressions through const. cycles and const. 3 transformation.

Constant 3 Transformations

Figure 34. Continuous progressions through constant cycles and constant 3 transformations (continued).

C_5 Const. 3

C_7 Const. 3

Figure 34. Continuous progressions through constant cycles and constant 3 transformations (concluded).

E. How Cycles and Transformations are Related

There are four forms of relationship between cycles and transformations with regard to the variability of both:

(1) const.-cycle, const.-transformation;
(2) const.-cycle, variable transformation;
(3) variable cycle, const.-transformation;
(4) variable cycle, variable transformation.

The forms of transformation produce their own periodic groups which may be superimposed on the groups of cycles.

Monomial forms of transformations (const. transformations):

(1) ↻, (2) ↺, (3) const. 3.

Binomial forms of transformations:

(1) ↻ + ↺, (2) ↺ + ↻

Here const. 3 is excluded because of the crossing of inner voices.

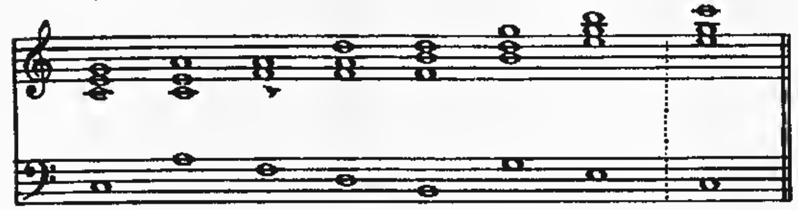
When coefficients of recurrence are applied to the forms of transformations, *selective transformation-groups* are produced.

For example: 2 ↻ + ↺; 3 ↺ + 2 ↻; 2 ↻ + ↺ + ↻ + 2 ↺; 4 ↻ + ↺ + + 3 ↻ + 2 ↺ + 2 ↻ + 3 ↺ + ↻ + 4 ↺; ↺ + 2 ↻ + 3 ↺ + 5 ↻ + 8 ↺; 4 ↻ + 2 ↺ + 2 ↻ + ↺.

Although the groups of tonal cycles, as well as the forms of transformations, may be chosen freely with the writing of each subsequent chord, nevertheless rhythmic planning of both cycles and transformations guarantees a greater regularity and, therefore, greater unity of style.

Here are examples of variable transformations applied to constant tonal cycles.

C_3 const. 2 ↻ + ↺ + ↻ + 2 ↺; ↻ added for the ending



C_7 const. 4 ↻ + 2 ↺ + 2 ↻ + ↺

C_5 const. 3 ↺ + ↻ + 2 ↺ + ↻

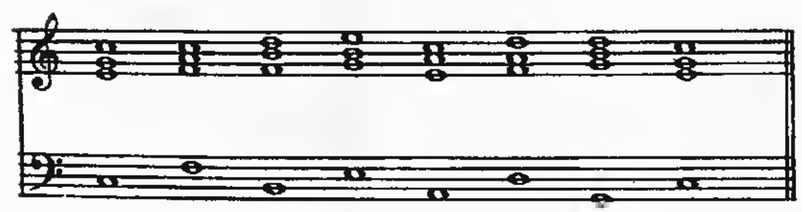


Figure 35. Variable transformations of constant tonal cycles.

Examples of variable transformations applied to variable tonal cycles.

$C_5 + C_7 + C_3$; 2 ↺ + ↻

Figure 36. Variable transformations of variable tonal cycles (continued).

$2C_7 + C_8 + 3C_5$; $4 \circlearrowleft + 2 \circlearrowright + 2 \circlearrowleft + \circlearrowright$

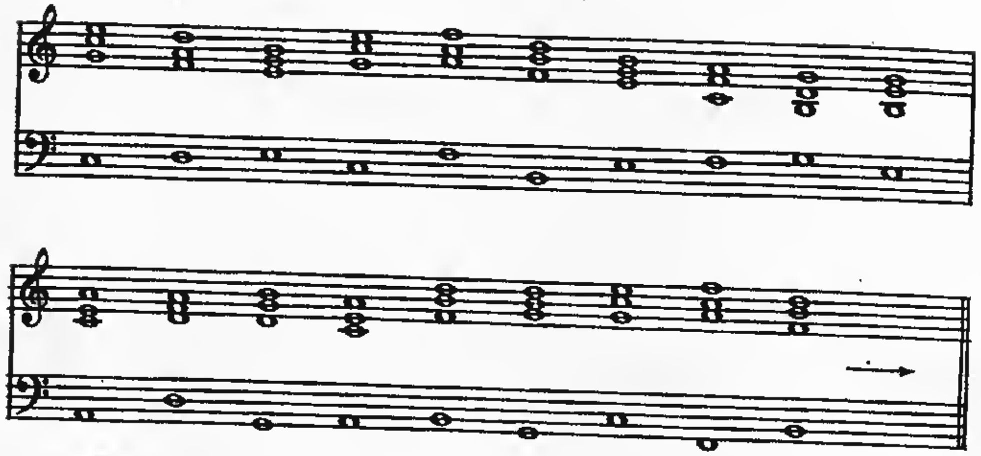


Figure 36. Variable transformations of variable tonal cycles (concluded).

All forms of harmonic continuity, because of their properties of redistribution, modal variability and convertibility, are subject to the following modifications:

(1) Placement of the voice representing constant function, and originally appearing in the bass, in any other voice, i.e., tenor, alto or soprano. There are four forms of such distribution:

s	s	s	S
a	a	A	a
t	T	t	t
B	b	b	b

Capital letters represent the voice functioning as const. 1.

Original

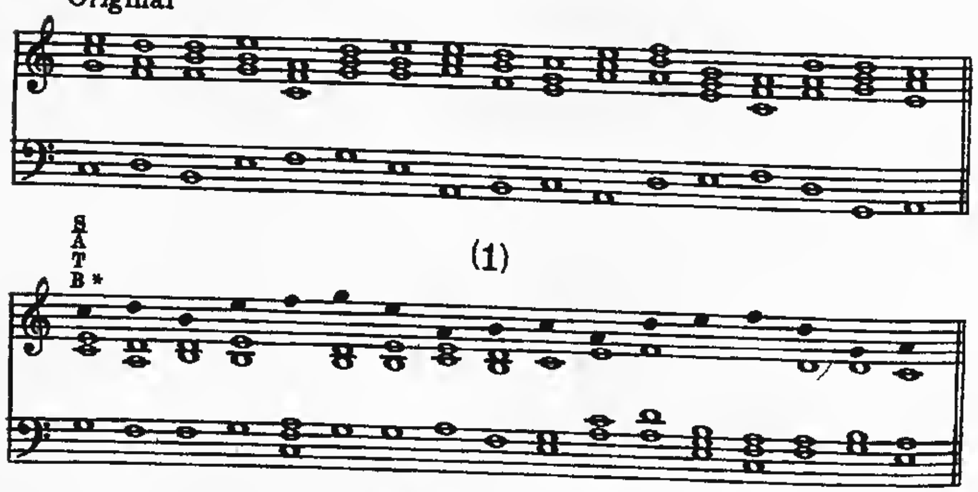


Figure 37. Varying position of constant voice.

*In Schillinger's original M.S., the constant voice is indicated by notes in red; here, however, they are indicated by quarter-notes, to avoid the complication of printing a second color. (Ed.)

(2) General redistribution (vertical permutations) of all voices according to the 24 variations of 4 elements.

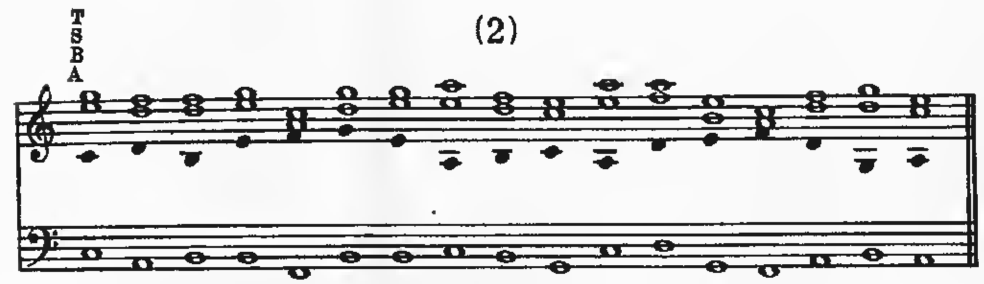


Figure 38. General redistribution of all voices.

(3) Geometrical inversions: ⓐ, ⓑ, ⓒ and ⓓ for any or all forms of distribution of the four voices.

Original ⓓ (3)

Figure 39. Geometrical inversion, position ⓓ.

(4) Modal variation by means of *modal transposition*, i.e., direct change of key signature, without relocating the notes on the staff.

Original + 3♭ (4)

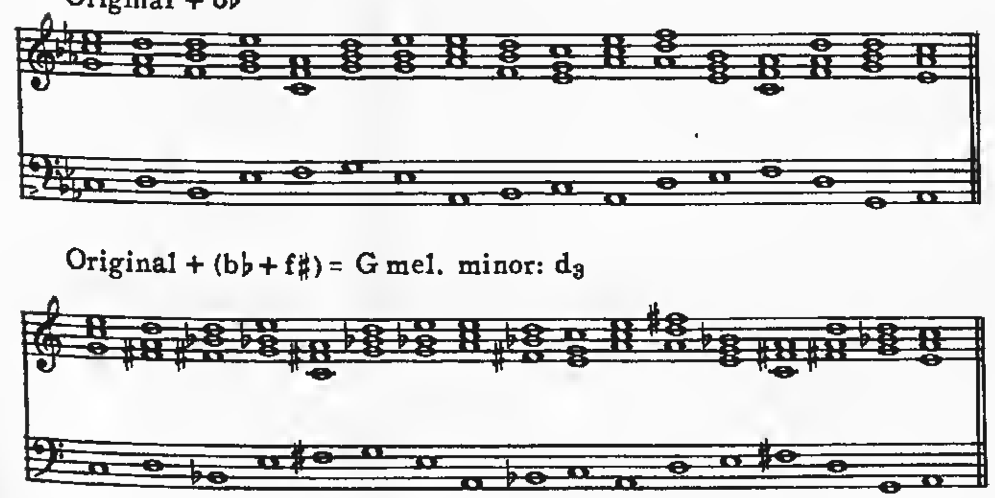


Figure 40. Modal variation by modal transposition.

E. The Negative Form

As previously indicated, the negative form of harmony can be obtained by direct reading of the positive form in position ⓑ.

Here, for the sake of clarity, I offer some technical details which explain the theoretical side of the negative form.

According to the definition given of the harmony scale in the negative form, we obtain the latter by means of further expansions of HS. In the positive form we use: HS_{E_0} (= C_3), HS_{E_1} (= C_5) and HS_{E_2} (= C_7).

By further expanding HS, we acquire the cycles of the negative form: HS_{E_3} (= C_{-7}), HS_{E_4} (= C_{-5}), HS_{E_5} (= C_{-3}).

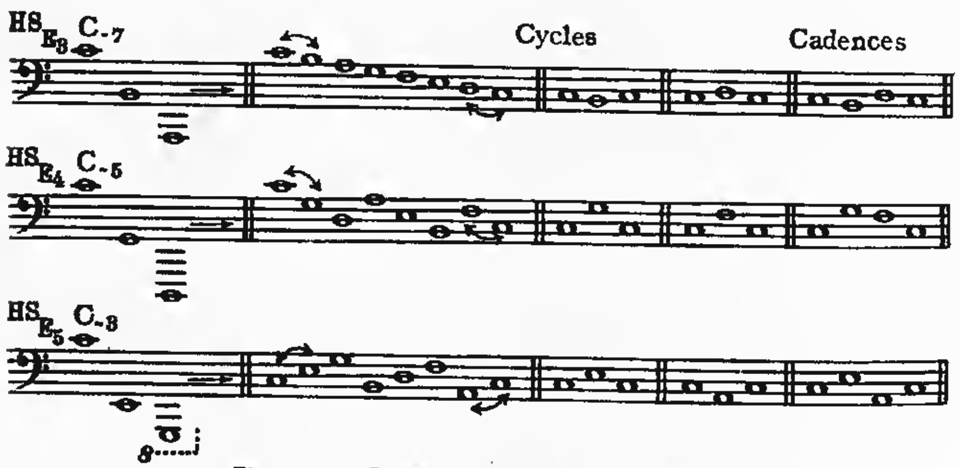


Figure 41. Cycles of the negative form.

As in this negative form, the chord-structures are built downward from a given pitch unit, such a pitch unit becomes the root-tone of the negative structure: *the negative root* (− 1). All chord-structures of the negative form, according to the previous definition, derive from HSⓑ. Thus, in order to construct a *negative S* (5), it is necessary to take the next pitch-unit downward, which becomes *the negative third* (− 3), and the next unit downward from the latter, which becomes *the negative fifth* (− 5).

For example, if we start from c̲ as a − 1, we obtain a negative S (5) where a̲ is − 3 and f̲ is − 5.

Figure 42. Negative C-major.

Positions of chords, as they were expressed through transformation, remain identical in the negative form, providing that they are constructed upward. In such a case, the addition of a const. 1 in the bass must be, strictly speaking, transferred to the soprano.

Here is how a negative CS (5) would appear in its four-part settings.

Figure 43. Four-part setting of negative CS (5).

Under such conditions, if the chord is constructed downward, the reversal of ↻ and ↺ reading takes place.

Transformations as applied to voice-leading possess the same reversibility: if everything is read downward, the ↻ and the ↺ transformations correspond to the positive form, while in the upward reading the ↻ becomes the ↺, and vice-versa.

Let us connect two chords in the negative cycle of the third: CS (5) + C_3 + + ES (5).

CS (5) = −1, −3, −5 = c − a − f.
ES (5) = −1, −3, −5 = e − c − a.

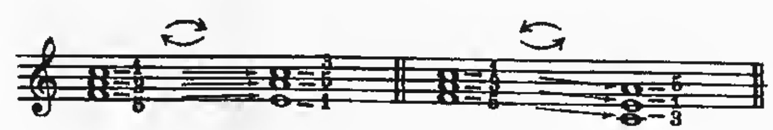


Figure 44. CS (5) + C_3 + ES (5).

It is easy to see that in the upward reading, chord C corresponds to F while chord E corresponds to A. Transposing this upward reading to C, we find that this progression is C → E. This proves the reversibility of tonal cycles and the correctness of reading the positive form of progressions in position ⓑ when the negative form is desired.

The mixture of positive and negative forms in continuity does not change the situation, but merely reverses the characteristics of voice-leading with regard to positive and negative forms. For example, C_3 in ↻ in the positive system produces two sustained common tones. In order to obtain an analogous pattern of voice-leading in C_{-3}, it is necessary to reverse the transformation, i.e., to use the ↺ form in this case.

CHAPTER 3

THE SYMMETRIC SYSTEM OF HARMONY

DIATONIC harmony can be best defined as *a system in which chord-structures as well as chord-progressions derive from a given scale.* The structural constitution of pitch assemblages, known as chords, as well as the actual intonation of the sequences of root-tones, known as tonal cycles, are entirely conditioned by the structural constitution of the scale, which is *the source of intonation.*

Symmetric harmony is *a system of pre-selected chord-structures and pre-selected chord progressions*, one independent of the other. In the symmetric system of harmony, *scale* is the result; scale is the *consequence of chords in motion.* The selection of intonation for structures is independent of the selection of intonation for the progressions.

A. Structures of S(5)

In this part of my treatment of harmony only such three-part structures will be used as satisfy our definition of the *special* theory of harmony. The ingredients of chord-structures here are limited to 3 and 4 semitones. Under such limitations only *four forms of S*(5) are possible. It should be remembered, however, that the number of all possible three-part structures amounts to 55, which is the general number of three-unit scales from one axis.

Table of S(5)

$S_1(5) = 4 + 3$, known as a major triad;
$S_2(5) = 3 + 4$, known as a minor triad;
$S_3(5) = 4 + 4$, known as an augmented triad;
$S_4(5) = 3 + 3$, known as a diminished triad.

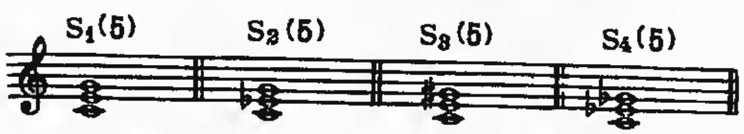


Figure 45. Table of S(5) structures.

Inasmuch as S(5) will be the only structure treated at present, we shall simplify the above expressions to the following form:

$S_1;\ S_2;\ S_3;\ S_4.$

Regardless of what the chord-progression may be, the structural constitution of chords appearing in such a progression may be either constant or variable. Constant structures will be considered as monomial progressions of structures, while the variable structures will be considered as binomial, trinomial and polynomial structural groups.

1. Monomial Forms of S(5)

$S_1 + \ldots$
$S_2 + \ldots$
$S_3 + \ldots$
$S_4 + \ldots$
Total: 4 forms

2. Binomial forms of S(5)

$S_1 + S_2$	$S_2 + S_3$	$S_3 + S_4$
$S_1 + S_3$	$S_2 + S_4$	
$S_1 + S_4$		

6 combinations, 2 permutations each.
Total: 12 forms

3. Trinomial forms of S(5)

$S_1 + S_1 + S_2$	$S_2 + S_2 + S_3$	$S_3 + S_3 + S_4$
$S_1 + S_1 + S_3$	$S_2 + S_2 + S_4$	
$S_1 + S_1 + S_4$		
$S_1 + S_2 + S_2$	$S_2 + S_3 + S_3$	$S_3 + S_4 + S_4$
$S_1 + S_3 + S_3$	$S_2 + S_4 + S_4$	
$S_1 + S_4 + S_4$		

12 combinations, 3 permutations each.
Total: 36 forms

$S_1 + S_2 + S_3$	$S_2 + S_3 + S_4$
$S_1 + S_2 + S_4$	
$S_1 + S_3 + S_4$	

4 combinations, 6 permutations each.
Total: 24 forms.

The total of all trinomials: $36 + 24 = 60$.

4. Quadrinomial forms of S(5)

$S_1 + S_1 + S_1 + S_2$	$S_2 + S_2 + S_2 + S_3$	$S_3 + S_3 + S_3 + S_4$
$S_1 + S_1 + S_1 + S_3$	$S_2 + S_2 + S_2 + S_4$	
$S_1 + S_1 + S_1 + S_4$		
$S_1 + S_2 + S_2 + S_2$	$S_2 + S_3 + S_3 + S_3$	$S_3 + S_4 + S_4 + S_4$
$S_1 + S_3 + S_3 + S_3$	$S_2 + S_4 + S_4 + S_4$	
$S_1 + S_4 + S_4 + S_4$		

12 combinations, 4 permutations each.
Total: 48 forms.

$S_1 + S_1 + S_2 + S_2$	$S_2 + S_2 + S_3 + S_3$	$S_3 + S_3 + S_4 + S_4$
$S_1 + S_1 + S_3 + S_3$	$S_2 + S_2 + S_4 + S_4$	
$S_1 + S_1 + S_4 + S_4$		

6 combinations, 6 permutations each.
Total: 36 forms

$S_1 + S_1 + S_2 + S_3$ $S_2 + S_2 + S_3 + S_4$
$S_1 + S_1 + S_2 + S_4$
$S_1 + S_1 + S_3 + S_4$

$S_1 + S_2 + S_2 + S_3$ $S_2 + S_3 + S_3 + S_4$
$S_1 + S_2 + S_2 + S_4$
$S_1 + S_3 + S_3 + S_4$

$S_1 + S_2 + S_3 + S_3$ $S_2 + S_3 + S_4 + S_4$
$S_1 + S_2 + S_4 + S_4$
$S_1 + S_3 + S_4 + S_4$

12 combinations, 12 permutations each.
Total: 144 forms.

$S_1 + S_2 + S_3 + S_4$

1 combination, 24 permutations.
Total: 24 froms.

The total of all quadrinomials: $48 + 36 + 144 + 24 = 252$.

In addition to all these fundamental forms of the groups of S(5), which represent a ***neutral harmonic continuity of structures***, there are groups with coefficients of recurrence, which represent a ***selective harmonic continuity of structures***. The latter are subject to individual selection.

Any rhythmic groups* may be used as coefficients of recurrence.

Examples

(1) $2S_1 + S_3$
(2) $3S_3 + S_2$
(3) $3S_1 + 2S_3 + S_2$
(4) $2S_2 + S_1 + S_2 + 2S_1$
(5) $2S_1 + S_2 + S_3 + 2S_4$
(6) $3S_1 + S_2 + 2S_1 + 2S_2 + S_1 + 3S_2$
(7) $3S_1 + S_3 + 2S_2 + 2S_1 + S_3 + 3S_2$
(8) $4S_3 + 2S_2 + 2S_3 + S_2$
(9) $2S_1 + S_2 + S_1 + S_2 + S_1 + S_2 + 2S_1 + 2S_2 + S_1 + S_2 + S_1 + S_2 +$
$+ S_1 + 2S_2$
(10) $4S_1 + 2S_2 + 2S_4 + 2S_1 + S_2 + S_4 + 2S_1 + S_2 + S_4$
(11) $S_1 + 2S_2 + 3S_4 + 5S_3$
(12) $S_2 + 3S_1 + 4S_2 + 7S_1$

*In this brief sentence, Schillinger reminds the reader that all of his basic rhythmic procedures, set forth in great detail in the earlier section of rhythm, may be applied as a means of determining the pattern of coefficients of structures. (Ed.)

B. Symmetric Progressions. Symmetric Zero Cycle (C_0)

A group of chords with a common root-tone but with variable positions and variable structures produces a symmetric zero cycle (C_0).

Such a group may be an independent form of harmonic continuity as well as a portion of other symmetric forms of harmonic continuity.

Coefficients of recurrence in the groups of structures, when used in a continuity of C_0, acquire the following meaning: a structure with a coefficient greater than one changes its positions until the next structure appears. The change of structure requires the preservation of the position of the chord.

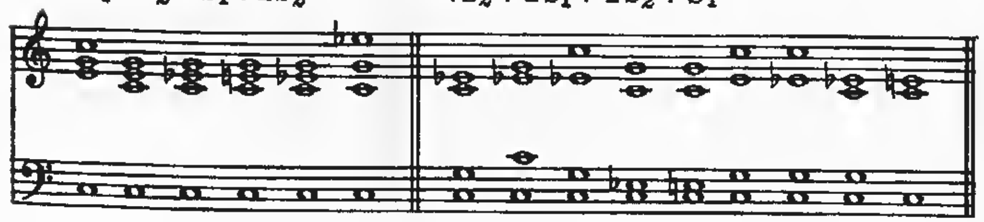
This can be expressed as *a form of interdependence of structures and their positions in the C_0*:

S const. ——————position var.
S var. .——————position const.

For instance, in a case of $3S_1 + S_3 + 2S_2 = S_1 + S_1 + S_1 + S_3 + S_2 + S_2$, the constant and variable positions appear as follows:

var. var. const. const. var.
$S_1 + S_1 + S_1 + S_3 \quad + \quad S_2 + S_2$

$2S_1 + S_2 + S_1 + 2S_2$ $4S_2 + 2S_1 + 2S_2 + S_1$



$4S_1 + S_3 + 3S_2 + 2S_4 + 2S_3 + 3S_2 + S_4 + 4S_1$

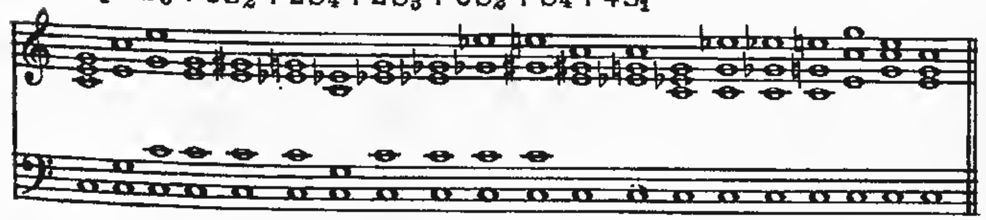
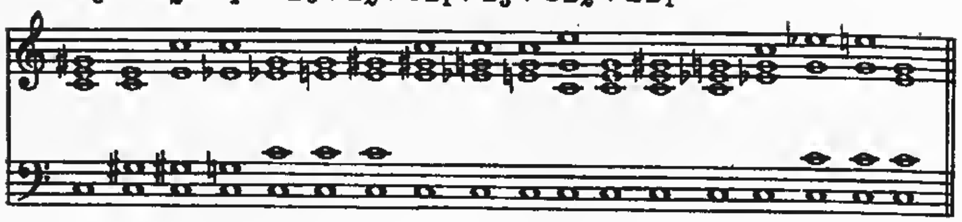


Figure 46. Harmonic continuity in C_0 (continued).

$3S_3 + 2S_2 + S_1 + 2S_3 + S_2 + 3S_1 + S_3 + 3S_2 + 2S_1$



$S_4 + 3S_1 + 4S_3 + 7S_3$

Figure 46. Harmonic continuity in C_0 (concluded).

CHAPTER 4

DIATONIC-SYMMETRIC SYSTEM OF HARMONY
(Type II)

THE diatonic-symmetric system of harmony must satisfy two requirements:

(1) all root-tones of the diatonic-symmetric system must belong to one scale of the First Group;

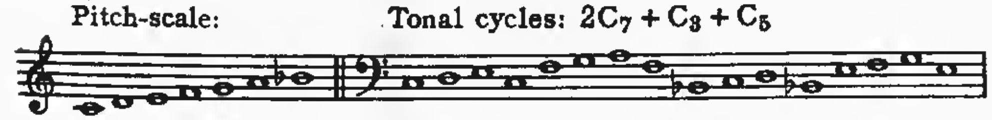
(2) all chord structures must be pre-selected; they are not affected by the intonation of scale formed by the root-tones.

In this system of harmony, structural groups must be superimposed upon the progressions of the root-tones belonging to one scale. This form of harmony has advantages over the diatonic system itself, to which I refer as Type I. Like the diatonic system, the diatonic-symmetric system produces a united tonality, which is due to the structural unity of the scale. Unlike the diatonic system, the diatonic-symmetric system is not bound to use the structures which are considered defective in the equal temperament [like $S_4(5)$, for example], as the individual structures and the structural groups are a matter of free choice.

Unlike the diatonic system, the diatonic-symmetric system has a greater variety of intonations, as the pre-selected structures unavoidably introduce new accidentals (alterations), which implies a modulatory character without destroying the unity of the tonality.

Examples of Harmony Type II.

Pitch-scale: Tonal cycles: $2C_7 + C_3 + C_5$



Structural group: S_1 (5) const.

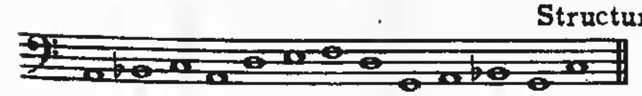


Figure 47. Diatonic-symmetric system (continued).

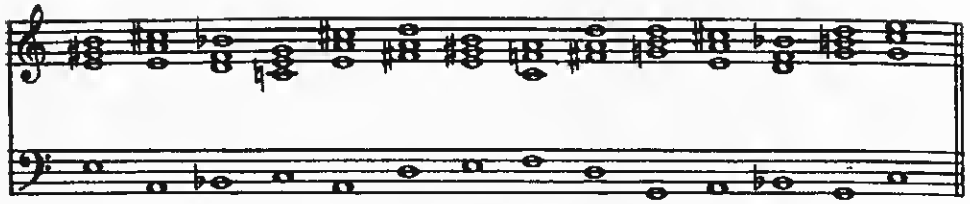


Figure 47. Diatonic-symmetric system (concluded).

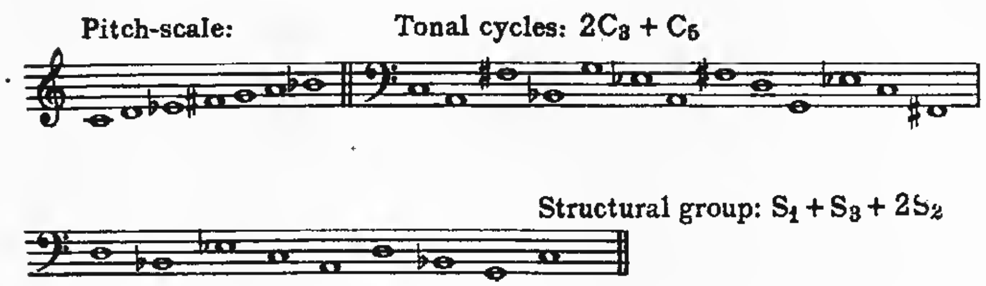


Figure 48. Diatonic-symmetric system (continued).

Figure 48. Diatonic-symmetric system (concluded).

CHAPTER 5

THE SYMMETRIC SYSTEM OF HARMONY

(Type III)

THE symmetric system of harmony of the third type must satisfy the following requirements:

(1) the root-tones and their progressions are the roots of two (i.e., $\sqrt{2}, \sqrt[3]{2}, \sqrt[4]{2}, \sqrt[6]{2}, \sqrt[12]{2}$), that is, the points of symmetry of an octave.

(2) chord structures are pre-selected.

As a consequence of motion through symmetric roots, each voice of harmony produces one of the pitch-scales of the third group.

Symmetric C_0 represents one tonic;
$\sqrt{2}$ represents two tonics;
$\sqrt[3]{2}$ represents three tonics;
$\sqrt[4]{2}$ represents four tonics;
$\sqrt[6]{2}$ represents six tonics;
$\sqrt[12]{2}$ represents twelve tonics.

The correspondences of the tonal cycles and the symmetric roots are as follows:

One tonic: C——————C
C_0

Two tonics: C————F♯————C
C_5 C_{-5}

Three tonics: C————A♭————E————C
C_3 C_3 C_3
C————E————A♭————C
C_{-3} C_{-3} C_{-3}

Four tonics: C————A————F♯————E♭————C
C_3 C_3 C_3 C_3
C————E♭————F♯————A————C
C_{-3} C_{-3} C_{-3} C_{-3}

Six tonics: C——D——E——F♯——A♭——B♭——C
C_7 C_7 C_7 C_7 C_7 C_7
C——B♭——A♭——F♯——E——D——C
C_{-7} C_{-7} C_{-7} C_{-7} C_{-7} C_{-7}

Twelve tonics: C——D♭——D♮——E♭——E♮ . . .
C_7 C_7 C_7 C_7
C——B——B♭——A——A♭
C_{-7} C_{-7} C_{-7} C_{-7}

Transformations with regard to positions and voice-leading remain the same as in the diatonic system. In case of doubt, cancel *all* the accidentals and test the leading of voices that way.

A. Two Tonics

Two tonics break an octave into two uniform intervals. The second tonic (T_2) being the $\sqrt{2}$ produces the center of an octave. This property makes the *two-tonic system reversible.* All points of intonation in the ↻ as well as in the ↺ transformation are identical, that is, both clockwise and counterclockwise voice-leading produce the same pattern of motion. This is true only in the case of two tonics.

Two tonics form a *continuous system*, i.e., the recurring tonic does not appear in its original position. Two tonics produce a *triple recurrence-cycle* before the original position falls on the first tonic (T_1) for the ↻ and the ↺. Const. 3 produces a closed system.

Figure 49. S_1 const. and const. 3.

The upper voice of harmony produces the following scale: c – d♭ – e – f♯ – g – a♯ – (c) = (1+3) + 2 + (1+3) + 2. All other voices of the above progression produce the same scale starting from its different phases.

It is easy to see that this scale belongs to the third group and is constructed on two tonics.

By selecting other structures and structural groups of S(5), one can get other scales of the third group. For example, the use of S_2 const. produces the following scale: c – d♭ – e♭ – f♯ – g – a – (c) = (1+2) + 3 + (1+2) + 3.

Structural groups may be used in two ways:

(1) S changes with each tonic;

(2) the groups of S produce C_0 on each tonic.

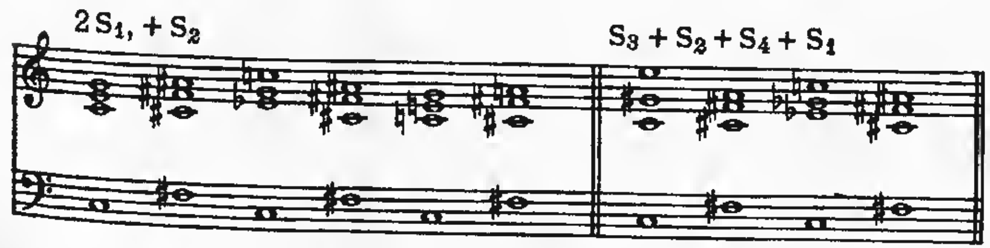


Figure 50. S changes with each tonic.

Figure 51. S produces C_0 on each tonic.

Combinations of the preceding two methods in the structural selection of each tonic of any one symmetric system are applicable to all symmetric sytems.

Figure 52. Structural selection of each tonic.

Longer progressions may be obtained through the use of longer structural groups, such as rhythmic resultants, power-groups, series of growth, etc.

In some cases, the number of terms in the structural group produces interference against the number of tonics in the symmetric system.

Example:

$T_1,\ T_2;\ 2S_1 + S_2 + S_1 + S_2 + S_1 + S_2 + 2S_1.$

$(S_1T_1 + S_1T_2 + S_2T_1 + S_1T_2 + S_2T_1 + S_1T_2 + S_2T_1 + S_1T_2 + S_1T_1) + (S_1T_2 + S_1T_1 + S_2T_2 + S_1T_1 + S_2T_2 + S_1T_1 + S_2T_2 + S_1T_1 + S_1T_2).$

B. Three Tonics

Three tonics produce a closed system for $\circlearrowright$ and $\circlearrowleft$, and a continuous system (two recurrence-cycles) for const. 3.

Figure 53. Three tonics.

C. Four Tonics

Four tonics produce a continuous system (three recurrence-cycles) for $\circlearrowright$ and $\circlearrowleft$, and a closed system for const. 3.

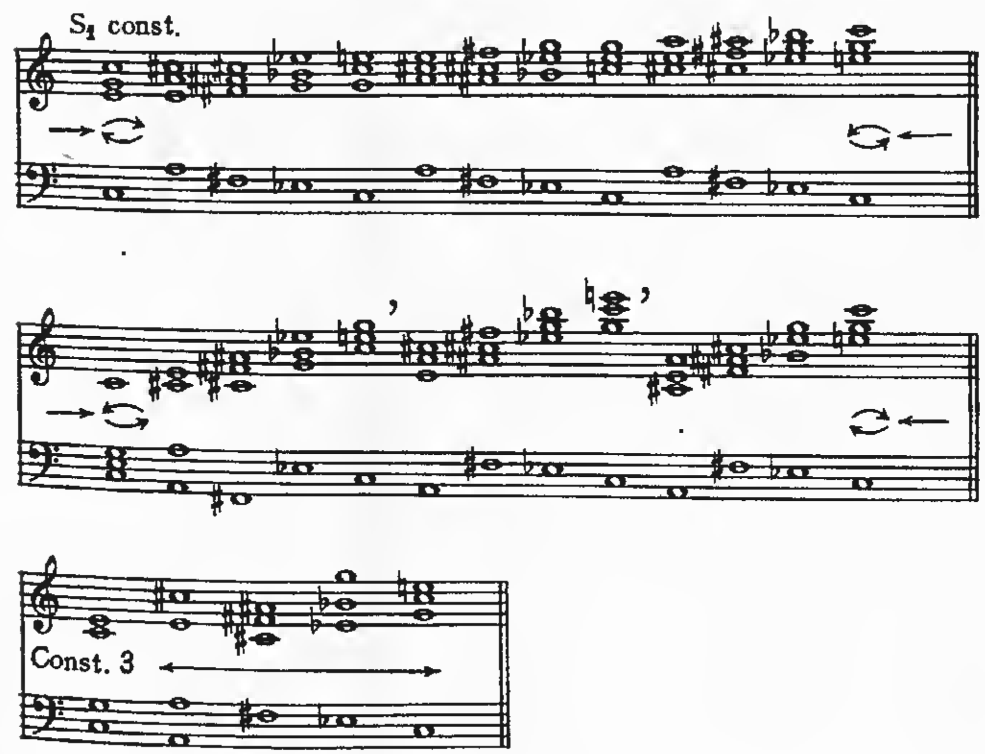


Figure 54. Four tonics.

D. Six Tonics

Six tonics produce a closed system for $\circlearrowright$ and $\circlearrowleft$, as well as for the const. 3.

S_1 const.

Figure 55. Six tonics.

E. Twelve Tonics

Twelve tonics produce a closed system for $\circlearrowright$ and $\circlearrowleft$, as well as for the const. 3.

S_1 const.

Figure 56. Twelve tonics.

CHAPTER 6

VARIABLE DOUBLINGS IN HARMONY

HARMONY, in many cases conceived as an accompaniment, may be given a self-sufficient character by means of *variable doublings*. This device gives to chord progressions a *greater versatility of sonority and voice-leading* than the one usually observed.

Variable doublings comprise the three functions of S(5). Thus, the root, the third or the fifth can be doubled. The notation to be used is: $S(5)^{\textcircled{1}}$, $S(5)^{\textcircled{3}}$ and $S(5)^{\textcircled{5}}$.

When the root-tone remains in the bass, $S(5)^{\textcircled{1}}$ is the only case of doubling where all three functions (1, 3, 5) appear in the upper three parts.

The following represents a comparative table of functions in the three upper parts under various forms of doubling.

$S(5)^{\textcircled{1}} = 1, 3, 5$
$S(5)^{\textcircled{3}} = 3, 3, 5$
$S(5)^{\textcircled{5}} = 3, 5, 5$

In cases $S(5)^{\textcircled{3}}$ and $S(5)^{\textcircled{5}}$, only three positions are possible for each case. Black notes represent variants where unison is substituted for an octave.

Positions

Figure 57. Various forms of doubling in 3 upper parts.

Transformations

S(5)① ←——————→ S(5)③

5 ↔ 5	5 ↔ 3	5 ↔ 3
3 ↔ 3	3 ↔ 5	3 ↔ 3
1 ↔ 3	1 ↔ 3	1 ↔ 5

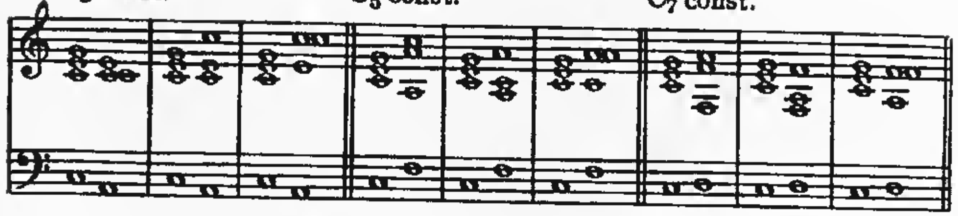


Figure 58. Transformation S(5)① ←——→ S(5)③.

S(5)③ ←——————→ S(5)③

5 ↔ 3
3 ↔ 5
3 ↔ 3

Figure 59. Transformation of S(5)③ ←——→ S(5)③.

S(5)① ←——————→ S(5)⑤

5 ↔ 5	5 ↔ 3	5 ↔ 5
3 ↔ 5	3 ↔ 5	3 ↔ 3
1 ↔ 3	1 ↔ 5	1 ↔ 5

Figure 60. Transformation of S(5)① ←——→ S(5)⑤.

S(5)⑤ ←——————→ S(5)⑤

5 ↔ 3
5 ↔ 5
3 ↔ 5

Figure 61. Transformation of S(5)⑤ ←——→ S(5)⑤.

S(5)③ ←——————→ S(5)⑤

5 ↔ 5
3 ↔ 5
3 ↔ 3

Figure 62. Transformation of S(5)③ ←——→ S(5)⑤

In reading these tables, consider *identical directions of the arrows* for the sequence of structures and for the corresponding transformations.

Note that there always are three transformations when S(5)① participates and only one when it does not.

Musical tables in the above figures are devised from an initial chord in the same position. Similar tables can be constructed from all positions as well as in reverse sequence; also in the cycles of the negative form.

Variable doublings are subject to distributive arrangement and can be superimposed on any desirable cycle-group.

Example: $2C_3 + C_5 + C_7$; $S(5)^{\textcircled{1}} + 2S(5)^{\textcircled{3}} + S(5)^{\textcircled{5}}$.

$$H^{\rightarrow} = S(5)^{\textcircled{1}} + C_3 + S(5)^{\textcircled{3}} + C_3 + S(5)^{\textcircled{3}} + C_5 + S(5)^{\textcircled{5}} + C_7 + S(5)^{\textcircled{1}}.$$

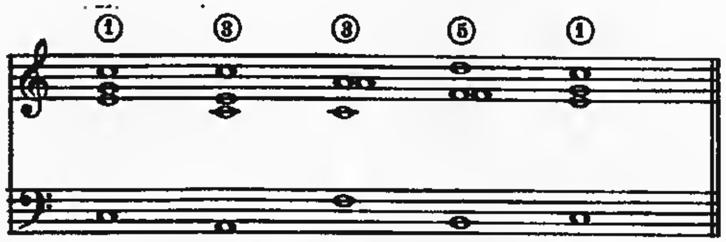


Figure 63. Variable doublings superimposed on a cycle-group.

Example: $2C_5 + C_3 + C_5 + 2C_7$; $S(5)^{\textcircled{3}} + S(5)^{\textcircled{1}} + S(5)^{\textcircled{3}} + S(5)^{\textcircled{5}}$.

$$H^{\rightarrow} = S(5)^{\textcircled{3}} + C_3 + S(5)^{\textcircled{1}} + C_5 + S(5)^{\textcircled{3}} + C_3 + S(5)^{\textcircled{5}} + C_5 + S(5)^{\textcircled{3}} + C_7 + S(5)^{\textcircled{1}} + C_7 + S(5)^{\textcircled{3}} + C_3 + S(5)^{\textcircled{5}} + C_3 + S(5)^{\textcircled{3}} + C_3 + S(5)^{\textcircled{1}} + C_5 + S(5)^{\textcircled{3}} + C_7 + S(5)^{\textcircled{5}} + C_7 + S(5)^{\textcircled{3}}.$$

Figure 64. Variable doublings superimposed on a cycle-group.

Variable doublings are applicable to all types of harmonic progressions, thus including types II and III.

Type II (diatonic-symmetric).
$H^{\rightarrow}$ as in the preceding example.
$S^{\rightarrow} = 2S_2 + S_3 + S_1$

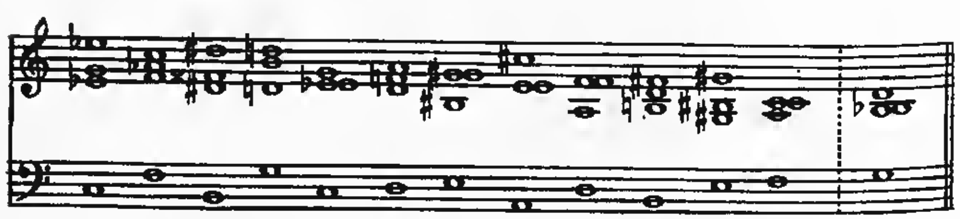


Figure 65. Variable doublings are applicabe to type II.

Type III (symmetric).

$$H^{\rightarrow}\ (6T) = T_1S_1^{\textcircled{5}} + T_2S_2^{\textcircled{3}} + T_3S_1^{\textcircled{1}} + T_4S_3^{\textcircled{3}} + T_5S_4^{\textcircled{5}} + T_6S_1^{\textcircled{3}} + T_1S_2^{\textcircled{5}}.$$

Figure 66. Variable doublings are applicable to type III.

CHAPTER 7

INVERSIONS OF THE S(5) CHORD

THE usual technique of inversions, strictly speaking, is unnecessary to a composer. The reason is that through vertical permutation of the positions of parts in any harmonic continuity of S(5), the inversions appear automatically when inner or upper parts become the bass parts. This technique was fully described in Book Three, *Variations of Music by Means of Geometrical Projection*, in the section on continuity of geometrical inversions.*

For an *analyst* or a *teacher*, however, a thorough systematization of the classical technique of inversions is a necessity, for there is no other branch of harmony where the confusion is greater and the information less reliable.

The first inversion of S(5) is known as a "sixth-chord" or a "third-sixth-chord" and is expressed in this notation by the symbol S(6). The only condition under which S(5) becomes an S(6) is when the *third* (3) appears in the bass. The positions of the upper voices are not affected by such a change, but the forms of doublings are affected. Which doublings are appropriate in each case, will be discussed later.

Assuming that any S(6) may be either S(6)①, or S(6)③, or S(6)⑤, we obtain the following Table of Positions:

Figure 67. Positions of S(6).

It is easy to memorize the above table, as S(6)① and S(6)⑤ positions are systematized through the following characteristics: (1) the doubled function appears above the remaining functions; (2) the doubled function surrounds the remaining function; (3) the doubled function appears below the remaining function.

*See Book III, Chapter 1, pp. 200-203.

S(6)③ is identical with S(5) positions, except that the bass has constant 3.

Harmonic progressions ($H^{\rightarrow}$) consisting of S(5) and S(6) are based on the following combinations by two:

1. S(5) → S(5); 2. S(5) → S(6); 3. S(6) → S(5); 4. S(6) → S(6).

As the first case is covered by the previous technique, we are concerned, for the present, with the last three cases.

All the following transformations, being applied to voice-leading, are reversible, as in the case of variable doublings of S(5). Tonal cycles are always measured through *root-tones*.

S(5) ⟵⟶ S(6)①

5 ↔ 5	5 ↔ 1	5 ↔ 1
3 ↔ 1	3 ↔ 5	3 ↔ 1
1 ↔ 1	1 ↔ 1	1 ↔ 5

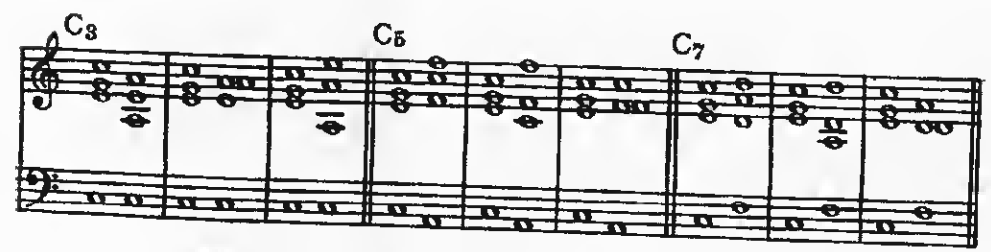


Figure 68. Transformations of S(5) ⟵⟶ S(6)①.

S(5) ⟵⟶ S(6)⑤

5 ↔ 1	5 ↔ 5	5 ↔ 5
3 ↔ 5	3 ↔ 1	3 ↔ 5
1 ↔ 5	1 ↔ 5	1 ↔ 1

Figure 69. Transformation of S(5) ⟵⟶ S(6)⑤.

S(5) ⟷	S(6)③
5 ↔ 3	5 ↔ 1
3 ↔ 1	3 ↔ 3
1 ↔ 5	1 ↔ 5
	Const. 3
	Const. 3

Figure 70. Transformation of S(5) ⟷ S(6)③.

S(6)① ⟷ S(6)①

5 ↔ 1
1 ↔ 5
1 ↔ 1

Figure 71. Transformation of S(6)① ⟷ S(6)①.

S(6)① ⟷ S(6)⑤

5 ↔ 5
1 ↔ 1
1 ↔ 5

Figure 72. Transformation of S(6)① ⟷ S(6)⑤.

S(6)⑤ ⟷ S(6)⑤

5 ↔ 1
5 ↔ 5
1 ↔ 5

Figure 73. Transformation of S(6)⑤ ⟷ S(6)⑤.

S(6)① ⟷		S(6)③
5 ↔ 5	5 ↔ 1	5 ↔ 3
1 ↔ 3	1 ↔ 5	1 ↔ 1
1 ↔ 1	1 ↔ 3	1 ↔ 5

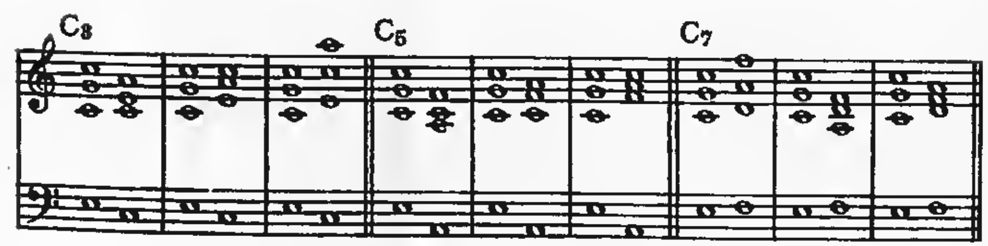


Figure 74. Transformation of S(6)① ⟷ S(6)③.

S(6)③ ⟷	S(6)③
5 ↔ 1	5 ↔ 3
3 ↔ 5	3 ↔ 1
1 ↔ 3	1 ↔ 5

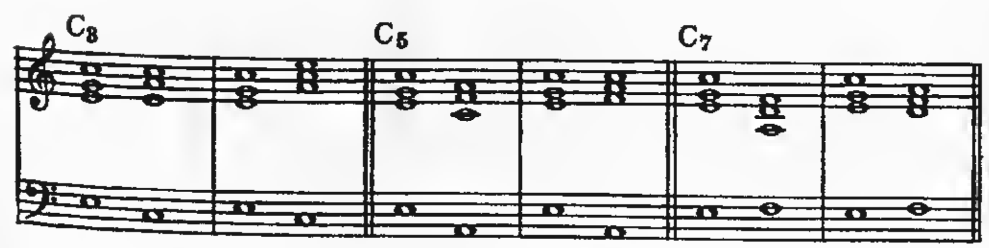


Figure 75. Transformation of S(6)③ ⟷ S(6)③

S(6)⑤ ←———→		S(6)③
5 ↔ 5	5 ↔ 1	5 ↔ 3
5 ↔ 3	5 ↔ 5	5 ↔ 1
1 ↔ 1	1 ↔ 3	1 ↔ 5

Figure 76. Transformation of S(6)⑤ ←——→ S(6)③

Any variants which conform to identical transformations (like the black notes in some of the preceding tables) are as acceptable as those in the tables.

A. Doublings of S(6)

Musical habits are formed comparatively rapidly. Once they assume the form of natural reactions, they influence us more than the purely acoustical factors. This is particularly true in the case of doublings of S(6). The mere fact that identical doublings in the different musical contexts affect us in a different way, shows that our auditory reactions in music are not natural but conditioned.

The principles offered here are based on a comparative study of the respective forms of music.

There are two technical factors affecting the doubling in an S(6):

(1) the structure of the chord;

(2) the degree of the scale (on which the chord is constructed).

These two influences are ever-present, regardless of the type to which the respective harmonic continuity belongs.

While in harmonic progressions of type II and III, the structure of the chord is the most influential factor—in the diatonic progressions (type I) it is exactly the reverse. The influence of a constant pitch-scale is so overwhelming that each chord becomes associated with its definite position in the scale. Thus, one chord begins to sound to us as a dominant and another as a tonic, a mediant or a leading tone. This hierarchy of the various chords calls for the different forms of doubling, particularly when the respective chords appear in the different inversions.

The following is most practical for use in diatonic progressions.

Strong Factor			Weak Factor		
The degree of the scale	Regular Doubling	Irreg. Doubling	The structure of the chord	Regular Doubling	Irreg. Doubling
I, IV, V, VI	①, ⑤	③	$S_1(6)$	①, ⑤	③
II, III, VII	③	①, ⑤	$S_2(6)$	③	①, ⑤
			$S_3(6)$	③	—
			$S_4(6)$	①, ③, ⑤	—

Figure 77. Table for doubling in diatonic progressions.

Regular doublings are *statistically predominant*. Irregular doublings, in most cases, are the result of *melodic* tendencies.

In reading the above table, give preference to the strong factor, except in the case of $S_3(6)$ and $S_4(6)$. It is customary to believe that an $S_1(6)$ must have doubled root or fifth. But in reality it seldom happens when such a chord belongs to II, III or VII. Naturally, all our habits with regard to doublings are formed on the more customary major and minor scales. The above table will work perfectly when applied to such scales. There will be no discrepancy when $S_3(6)$ and $S_4(6)$ are compared with the data on the left side of the table, as such structures do not occur on the main degrees of the usual scales.

In using less familiar scales, however, one or another type of doubling will not make as much difference. Yet in such cases the structure may become a more influential factor, though the sequence is diatonic.

In types II and III the most practical forms of doublings are:

Structure	Regular Doubling	Irregular Doubling
$S_1(6)$	①, ⑤	③
$S_2(6)$	①, ⑤	③
$S_3(6)$	③	—
$S_4(6)$	①, ③, ⑤	—

Figure 78. Forms of doubling for types II and III.

B. Continuity of S(5) and S(6)

The comparative characteristic of S(5) is its stability, due to the presence of the root-tone in the bass. The absence of the root-tone in the bass of S(6) deprives this structure of such stability.

Composition of continuity consisting of S(5) and S(6) results in an interplay of stable and unstable units or groups. The following fundamental forms of continuity utilizing the above-mentioned structures are possible:

(1) S(5) const. ——— stable

(2) S(6) const. ——— unstable

(3) [S(5) + S(6)] + . . . alternate

(4) 2S(5) + S(6) + S(5) + 2S(6)

3S(5) + S(6) + 2S(5) + 2S(6) + S(5) + 3S(6)

4S(5) + S(6) + 3S(5) + 2S(6) + 2S(5) + 3S(6) + S(5) + 4S(6)

——→ increasing instability

increasing stability ←——

(5) 4S(5) + 2S(6) + 2S(5) + S(6)

——→ proportionately decreasing ratios

proportionately increasing ratios ←——

(6) S(5) + 2S(6) + 3S(5) + 5S(6) + 8S(5) + 13S(6)

——→ progressive over-balancing of unstable elements

S(6) + 2S(5) + 3S(6) + 5S(5) + 8S(6) + 13S(5)

——→ progressive over-balancing of stable elements

Many other forms of the distribution of S(5) and S(6) may be devised on the basis of my theory of rhythm.*

Diatonic

S(6) Const.; $2C_7 + 2C_5 + C_3 + C_5$

5 3 3 1 3 3 1

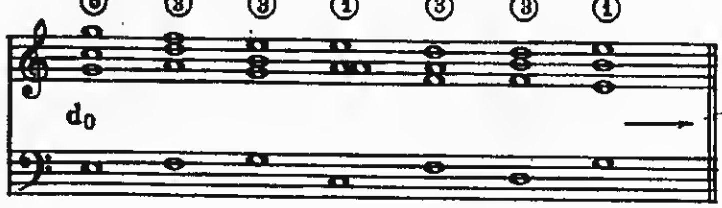


Figure 79. Progressions in diatonic, diatonic-symmetric and symmetric (continued).

*See Book I.

5 1 3 1 1 3 3

Diatonic-Symmetric

$2S_2(6) + S_1(6) + S_3(6) + S_1(6) + S_4(6)$; $2C_5 + C_7 + C_5 + 2C_7$

1 5 1 3 1 5 1

Symmetric

$S_3(6) + S_2(6) + S_4(6) + 2S_1(6)$; Six tonics

3 1 3 1 5 3

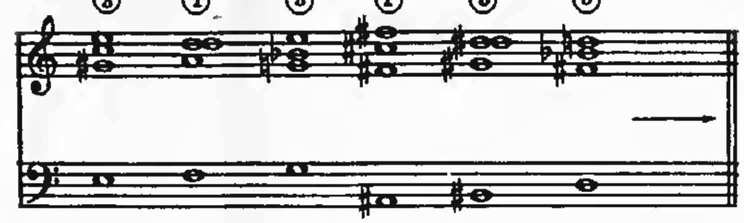


Figure 79. Progressions in diatonic, diatonic-symmetric and symmetric. (concluded).

Diatonic

$3S(6) + S(5) + 2S(6) + 2S(5) + S(6) + 3S(5)$; $2C_5 + C_7$

Figure 80. Progressions in diatonic, diatonic-symmetric and symmetric (continued).

Diatonic-Symmetric

$2S_2(6)+S_1(5)+S_4(6)+2S_2(5)$; $2C_7+C_5$; Scale of roots: Aeolian

Symmetric

$\{[S_1(5)+S_2(6)]\,T_1+[S_4(6)+S_1(5)]\,T_2\}+\ldots$ Four tonics

Figure 80. Progressions in diatonic, diatonic-symmetric and symmetric. (*concluded*).

CHAPTER 8

GROUPS WITH PASSING CHORDS

A. Passing Sixth-chords

A GROUP with a passing S(6) is a *pre-set combination* of three chords: namely, S(5) + S(6) + S(5). *Every passing chord occupies the center of its group, appears on a weak beat* and has a *doubled bass.* The complete expression for a group (G) with passing sixth-chord is:

$$G_6 = S(5) + S(6)^{③} + S(5).$$

This formula is not reversible in actual intonation. The relationship between the extreme chords of G_6 is C–5. This relationship remains constant in all cases of classical music.

We shall extend this principle to all cycles. Under such conditions G_6 retains the following characteristics:

(1) The transformation between the extreme chords of the group is *always clockwise* for both the positive and the negative cycles.
(2) The bass progression is: $1 \rightarrow 3 \rightarrow 1$, which necessitates the first condition.

In the classical form of G_6, the bass moves by the thirds. Thus, 3 in the bass under S(6) is a third above its preceding position under the first S(5), and a third below its following position under the last S(5).

In order to obtain G_6, it is necessary to connect S(5) with the next S(5) through C–5 and add the intermediate third of the first chord in the bass, without moving the remaining voices.

$$G_6 = \underbrace{S(5) + S(6)^{③} + S(5)}_{\text{C–5}}$$

Figure 81. Passing sixth chords.

There are three melodic forms for the bass movement.

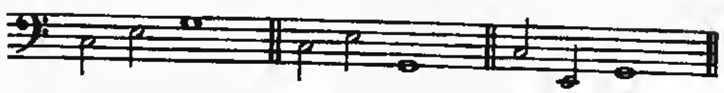


Figure 82. Melodic forms for bass.

Combinations of these three forms in sequence produce a very flexible bass part and, being repeated with one G_6, make expressive cadences of a Mozartian flavor.

Figure 83. Combination of figures 81 and 82.

B. Continuity of G_6.

Continuity of such groups can be obtained by connecting them through the tonal cycles.

Connecting by C_5 closes the sequence, while C_3 and C_7 produce a progression of $7G_6$.

Figure 84. Continuity of G_6

Further versatility of G_6 progressions can be achieved by varying the cycles between the groups. Any time a decisive cadence is desirable, C_5 must be introduced, as this cycle closes the progression.

$$H^{\rightarrow} = G_6 + C_7 + G_6 + C_7 + G_6 + C_3 + G_6 + C_7 + G_6 + C_3 + G_6 + C_3 + G_6 + C_5$$

Figure 85. Varying cycles between G_6.

C. Generalization of G_6

In addition to the classical form of G_6, other forms can be developed through the use of other than C–5 cycles within the group. Of course, each cycle produces its own characteristic bass pattern.

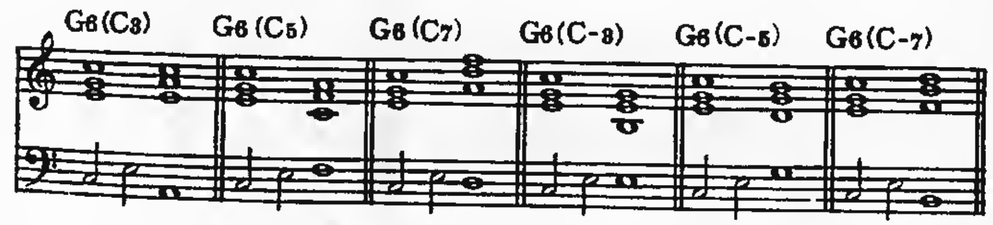


Figure 86. Various forms of G_6.

The respective variations of the bass pattern will be as follows:

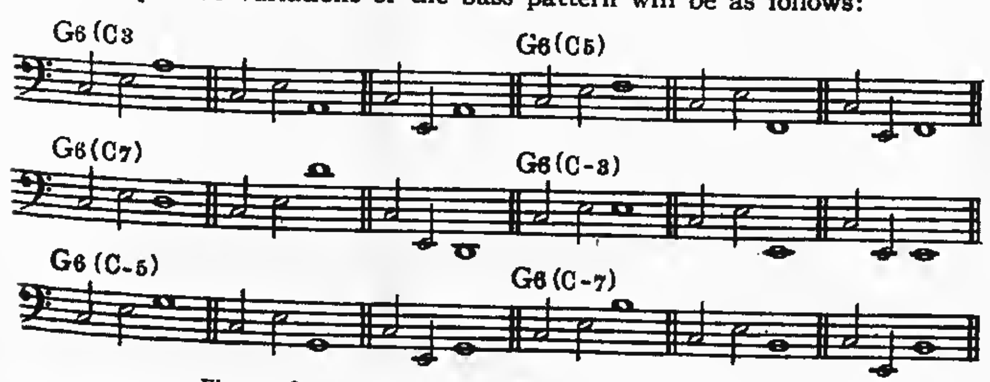


Figure 87. Variations of bass pattern for figure 86.

D. Continuity of the Generalized G_6

Such a continuity can be developed through the selective progressions of the various forms of G_6 combined with the various cycle connections between the groups.

Example:

$$H^{\rightarrow} = G_6(C\text{-}5) + C_7 + G_6(C_3) + C_5 + G(C\text{-}7) + C_3 + \\ + G(C\text{-}5) + C_5 + G(C_7) + C_5$$

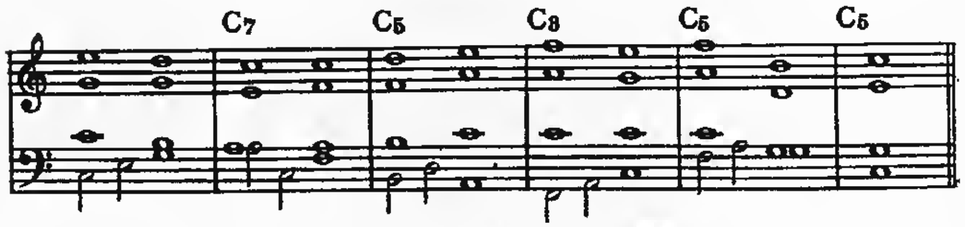


Figure 88. Continuity of generalized G_6.

E. Generalization of the Passing Third

It follows from the technique of groups with a passing sixth-chord that the first two chords, i.e., S(5) and S(6)③, belong to C_0, and that the position of the three upper parts does not change until the last chord of the group appears. This last chord, S(5), can be in any relation but C_0 with the preceding chord.

If we think of the appearance of the third in the bass during S(6)③ merely as a passing third, it is easy to see that this entire technique can be generalized. The passing 3 can be used after any S(5), providing the transformation between the latter and the following S(5) is *clockwise* for all the cycles. Such a device can be applied to any progression of S(5) with the root-tones in the bass.

Example:

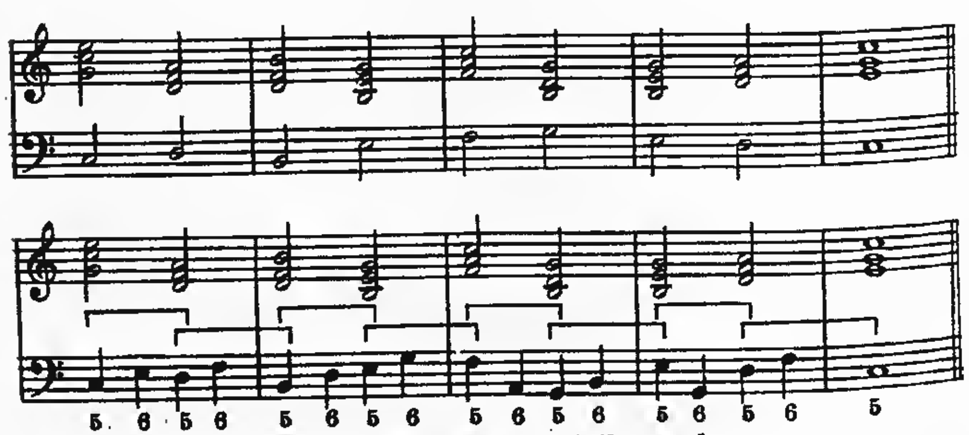


Figure 89. Passing third illustrated.

The effect of such harmonic continuity is one of overlapping groups of G_6, as marked in the preceding figure.

F. Applications of G_6 to Diatonic-Symmetric (Type II) and Symmetric (Type III) Progressions

The use of structures of S(5) and S(6)③ in the groups with a passing sixth-chord must satisfy the following requirement: *the adjacent S(5) and S(6)③ of one group must have identical structures.*

This requirement does not affect the form of the last S(5) of a group; neither does it influence the selection of the forms of S(5) in the adjacent groups.

As each G_6 consists of three places, two of which are identical, the number of structural combinations for the individual groups equals $4^2 = 16$.

$S_1 + S_1$	$S_2 + S_1$	$S_3 + S_1$	$S_4 + S_1$
$S_1 + S_2$	$S_2 + S_2$	$S_3 + S_2$	$S_4 + S_2$
$S_1 + S_3$	$S_2 + S_3$	$S_3 + S_3$	$S_4 + S_3$
$S_1 + S_4$	$S_2 + S_4$	$S_3 + S_4$	$S_4 + S_4$

Thus, we obtain 16 forms of G_6 with the following distribution of structural combinations:

$G_6 = S_1(5) + S_1(6)③ + S_1(5)$
$G_6 = S_1(5) + S_1(6)③ + S_2(5)$
$G_6 = S_1(5) + S_1(6)③ + S_3(5)$
$G_6 = S_1(5) + S_1(6)③ + S_4(5)$

$G_6 = S_2(5) + S_2(6)③ + S_1(5)$
$G_6 = S_2(5) + S_2(6)③ + S_2(5)$
$G_6 = S_2(5) + S_2(6)③ + S_3(5)$
$G_6 = S_2(5) + S_2(6)③ + S_4(5)$

$G_6 = S_3(5) + S_3(6)③ + S_1(5)$
$G_6 = S_3(5) + S_3(6)③ + S_2(5)$
$G_6 = S_3(5) + S_3(6)③ + S_3(5)$
$G_6 = S_3(5) + S_3(6)③ + S_4(5)$

$G_6 = S_4(5) + S_4(6)③ + S_1(5)$
$G_6 = S_4(5) + S_4(6)③ + S_2(5)$
$G_6 = S_4(5) + S_4(6)③ + S_3(5)$
$G_6 = S_4(5) + S_4(6)③ + S_4(5)$

As the melodic interval in the bass, while moving from the root (1) in $S(5)$ to the third (3) in $S(6)^{\textcircled{3}}$, is identical for the forms S_1 and S_3, as well as S_2 and S_4, the total quantity of intonations in the bass part for one type of G_6 is $\frac{4}{2} = 2$.

$$S_1 + S_1$$
$$S_1 + S_2$$
$$S_2 + S_1$$
$$S_2 + S_2$$

As each intonation has 3 melodic forms and there are two different intonations, the total number of intonations combined with melodic forms in the bass part is $2x\ 3 = 6$.

Figure 90. Six melodic forms in the bass part.

1. Progressions of Type II.

Example:

Forms of S: $S_2(5) + S_2(6)^{\textcircled{3}} + S_1(5)$
$H^{\rightarrow} = G_6(C\text{–}5) + C_3 + G_6(C\text{–}5) + C_7 + G_6(C\text{–}5) + C_5$.

Figure 91. Progression of type II.

Example:

Forms of S: $[S_1(5) + S_1(6)^{\textcircled{3}} + S_2(5)] + [S_3(5) + S_3(6)^{\textcircled{3}} + S_2(5)]$
$H^{\rightarrow}$ = as in the preceding example

Figure 92. Progression of type II.

Example:

Forms of S: $S_2(5) + S_2(6)^{\textcircled{3}} + S_2(5)$
$H^{\rightarrow}$ = as in Figure 88.

Figure 93. Progression of type II.

Generalization of the passing third is applicable to this type of harmonic progression as well. The following is an application of the structural group $2S_1 + S_2 + 2S_1 + S_2 + 2S_1$ to Figure 89.

Figure 94. Generalization of passing third in type II.

2. Progressions of Type III.

Applications of G_6 to symmetrical systems of tonics disclose many unexplored possibilities, among which the two-tonic system deserves particular attention. As intervals forming the two tonics are equidistant, the passing tones of $S(6)^{\textcircled{3}}$, which in turn may also be equidistant from T_1 and T_2, produce, in the bass movement, diminished seventh-chords in symmetric harmonization—a device heretofore unknown.

The justification for the use of G_6 in the symmetrical systems of tonics is based on the following deductions from the original classical form, i.e., $G_6(C\text{–}5)$.

(Diatonic) (Symmetric)

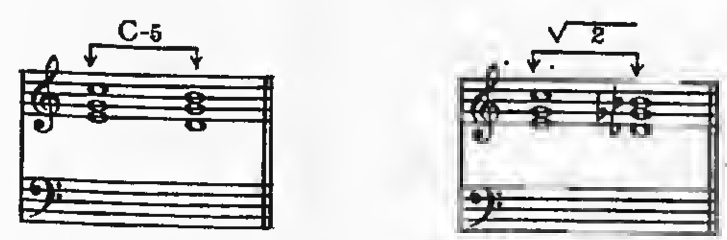


Figure 95. Justification for use of G_6 in symmetric systems of tonics.

The above-mentioned equidistancy of the two tonics permits retention of $H^{\rightarrow} = 3G_6$ until the cycle closes. Selecting S_1 for the entire G_6, we obtain:

Figure 96. Progression of type III.

The overlapping of groups, indicated by the brackets in the above Figure, is an invariant of the symmetrical systems. Thus the passing third can be considered a general device for progressions of type III.

The number of bass patterns for the cycle of the *two tonics* equals: $2^2 = 4$.

The number of intonations in each cycle of the two tonics equals: $2^2 = 4$. The latter is due to the use of the different forms of S(5). The interval between 1 and 3 equals 4, and is identical for $S_1(5)$ and $S_3(5)$. The interval between 1 and 3 equals 3, and is identical for $S_2(5)$ and $S_4(5)$. Thus, by distributing the different structures through two tonics, we obtain the following combinations:

$S_1(T_1) + S_1(T_2)$
$S_1(T_1) + S_3(T_2)$
$S_3(T_1) + S_1(T_2)$
$S_3(T_1) + S_3(T_2)$
identical intonations in the bass part

$S_2(T_1) + S_2(T_2)$
$S_2(T_1) + S_4(T_2)$
$S_4(T_1) + S_2(T_2)$
$S_4(T_1) + S_4(T_2)$
identical intonations in the bass part

$S_1(T_1) + S_2(T_2)$
$S_1(T_1) + S_4(T_2)$
$S_3(T_1) + S_2(T_2)$
$S_3(T_1) + S_4(T_2)$
identical intonations in the bass part

$S_2(T_1) + S_1(T_2)$
$S_2(T_1) + S_3(T_2)$
$S_4(T_1) + S_1(T_1)$
$S_4(T_1) + S_3(T_2)$
identical intonations in the bass part

The following is a table of intonations and melodic forms in the bass part on two tonics. Total: $4^2 = 16$.

Figure 97. Intonations and melodic forms in bass part on two tonics.

The above combinations can be incorporated into a versatile continuity of G_6 on two tonics.

Example:

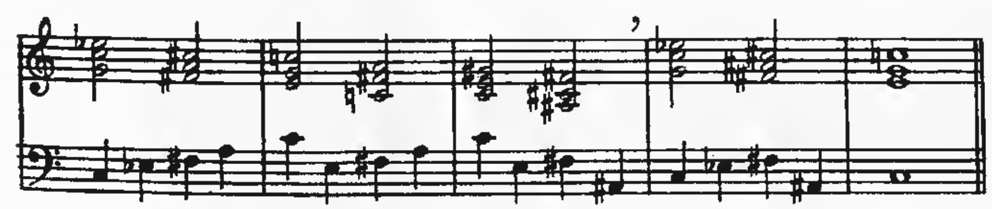


Figure 98. Continuity of G_6 on two tonics.

Application of G_6 to *three tonics* produces 8 melodic forms in the bass part: $2^3 = 8$.

$T_1a_2 + T_2a_2 + T_3a_2$
$T_1b_2 + T_2a_2 + T_3a_2$
$T_1a_2 + T_2b_2 + T_3a_2$
$T_1a_2 + T_2a_2 + T_3b_2$
$T_1b_2 + T_2b_2 + T_3a_2$
$T_1b_2 + T_2a_2 + T_3b_2$
$T_1a_2 + T_2b_2 + T_3b_2$
$T_1b_2 + T_2b_2 + T_3b_2$

Figure 99. Application of G_8 to three tonics.

The number of distributions of the different S through three tonics is $4^3 = 64$, while the number of non-identical intonations is $2^3 = 8$.

Non-identical intonations:

$S_1(T_1) + S_1(T_2) + S_1(T_3)$	$S_2(T_1) + S_2(T_2) + S_1(T_3)$
$S_1(T_1) + S_1(T_2) + S_2(T_3)$	$S_2(T_1) + S_1(T_2) + S_2(T_3)$
$S_1(T_1) + S_2(T_2) + S_1(T_3)$	$S_1(T_1) + S_2(T_2) + S_2(T_3)$
$S_2(T_1) + S_1(T_2) + S_1(T_3)$	$S_2(T_1) + S_2(T_2) + S_2(T_3)$

The total number of different intonations and melodic forms in the bass part is $8^2 = 64$.

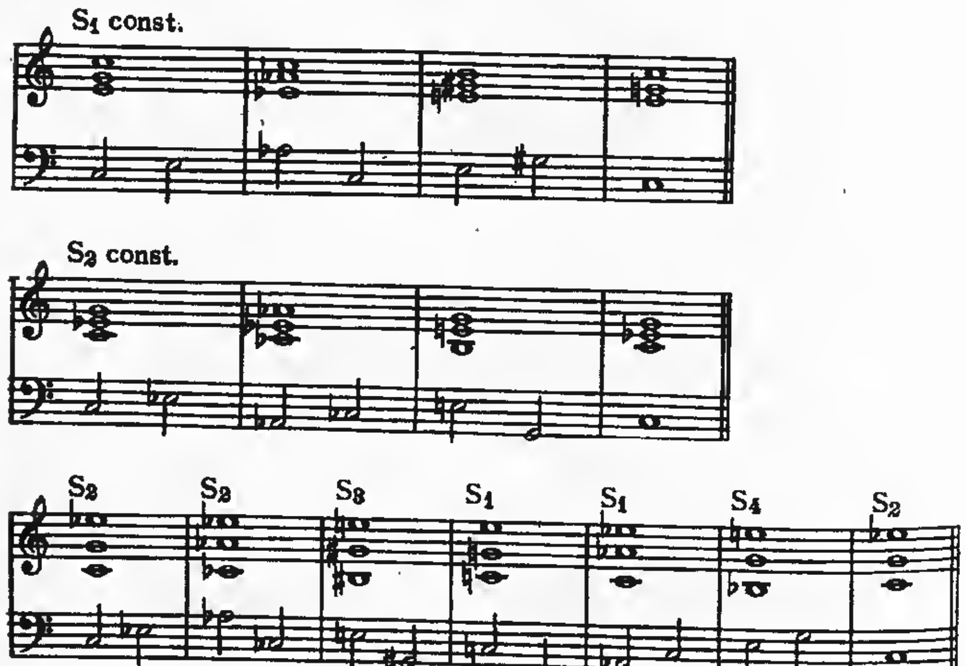


Figure 100. Examples of continuity of G_8 on three tonics.

Application of G_8 to *four tonics* produces $2^4 = 16$ melodic forms in the bass part. The number of distributions of the four forms of S through four tonics produces $4^4 = 256$ intonations. The number of intonations in the bass part is limited to $2^4 = 16$. Thus the total number of intonations and melodic forms in the bass part is $16^2 = 256$.

Figure 101. Continuity of G_8 on four tonics.

Application of G_6 to *six tonics* produces $2^6 = 64$ melodic forms in the bass part. The number of distributions of the four forms of S through four tonics produces $4^6 = 4096$ intonations. The number of intonations in the bass part is $2^6 = 64$. The total number of intonations and melodic forms in the bass part is $64^2 = 4096$.

Figure 102. Examples of continuity of G_6 on six tonics.

Application of G_6 to *twelve tonics* produces $2^{12} = 4096$ melodic forms in the bass part. The number of distributions of the four forms of S through four tonics produces $4^{12} = 16,777,216$. The number of intonations in the bass part is $2^{12} = 4096$. The total number of intonations and melodic forms in the bass part is $4096^2 = 16,777,216$.

Figure 103. Continuity of G_6 on twelve tonics (continued).

Figure 103. Continuity of G_6 on twelve tonics (concluded).

G. Passing Fourth-Sixth Chords: $S(\frac{6}{4})$

The second inversion of S(5) is a *fourth-sixth chord*: $S(\frac{6}{4})$. This name derives from the old *basso continuo* or *generalbass*, where intervals were measured from the bass.

Figure 104. Passing $S(\frac{6}{4})$.

$S(\frac{6}{4})$ has a fifth (5) in the bass while the three upper parts have the six usual arrangements.

The use of $S(\frac{6}{4})$ in classical music is a very peculiar one. This chord appears only in definite pre-set combinations. One of them is the *group with a passing fourth-sixth chord*: G_4^6.

As in the case of G_6, the passing chord itself appears on a weak beat, being surrounded by the two other chords, and has a doubled fifth: $S_4^6$⑤. The two other chords of G_4^6 are: S(5) and S(6). The latter can have two forms of doubling (regardless of the chord-structure): S(6)① and S(6)③.

The group with a passing fourth-sixth chord, contrary to G_6, is *reversible.*

$$G^6_4 = S(5) + S(^6_4) + S(6).$$

This property being combined with the choice of two possible doublings produces four variants.

$$G^6_4\uparrow\textcircled{1} = S(5) + S(^6_4) + S(6)\textcircled{1}$$
$$G^6_4\downarrow\textcircled{1} = S(6)\textcircled{1} + S(^6_4) + S(5)$$
$$G^6_4\uparrow\textcircled{5} = S(5) + S(^6_4) + S(6)\textcircled{5}$$
$$G^6_4\downarrow\textcircled{5} = S(6)\textcircled{5} + S(^6_4) + S(5)$$

The arrows in the above formulae specify the directions of the bass pattern which is always scalewise, and therefore can be either ascending or descending.

The bass pattern is developed on *three adjacent pitch-units,* which correspond to the three chords of G^6_4.

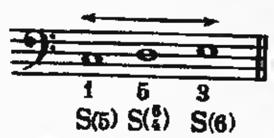


Figure 105. Bass pattern.

Arabic numerals represent the respective chordal functions.

Transformations between S(5) and $S(^6_4)$ in the G^6_4: *as the bass moves from 1 to 5,* when read in upward motion, *the three upper voices must move clockwise* in order to get the transformation of 1 into 3.

Figure 106. Transformations between S (5) and S $(^6_4)$.

The transition from $S(^6_4)$ into $S(6)\textcircled{1}$ or $S(6)\textcircled{5}$ follows the forms of transformations, where two identical functions participate, as in the cases of $S(5) \leftrightarrow S(6)\textcircled{1}$ and $S(5) \leftrightarrow S(6)\textcircled{5}$.

However, classical technique adopted definite routines concerning this transition:

(1) *one part* must carry out a melodic form *reciprocal to the bass* (i.e., position ⓑ of the bass melody);

(2) it is *this reciprocal part* that deviates from its path in order to *supply the doubling of the fifth* in an S(6).

Under such conditions G^6_4 acquires the following appearances:

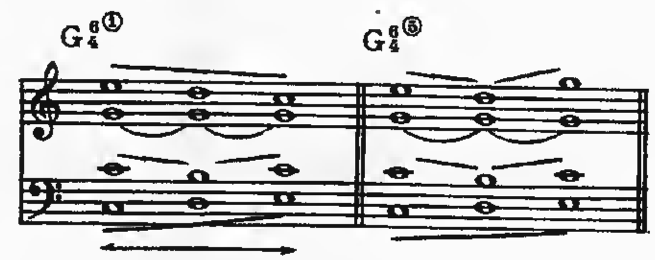


Figure 107. G $^6_4$ *transformations.*

In the sequence of operations the following items should be considered in the order indicated:

(1) bass
(2) part reciprocating the bass
(3) common tone
(4) part supplying the third for $S(^6_4)$

The relations between the chords of G^6_4 are as follows:

$$S(5) + C_{-5} + S(^6_4) + C_5 + S(6) \quad (C_0)$$

$$S(6) + C_{-5} + S(^6_4) + C_5 + S(5) \quad (C_0)$$

Each group can be carried out in 6 positions which depend on the starting position.

The following is the table of all four forms of G^6_4 in one position.

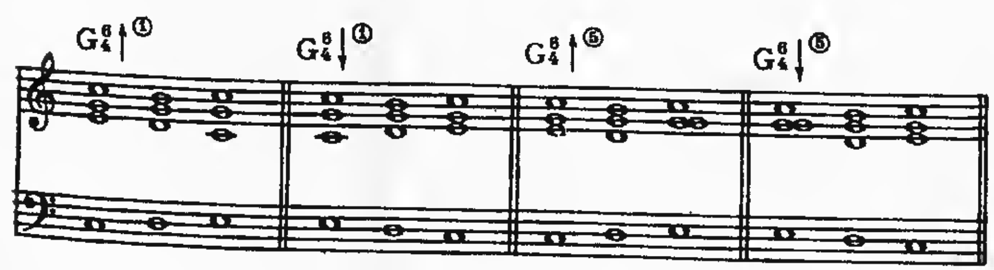


Figure 108. Forms of G $^6_4$ *in one position.*

The different forms of G^6_4 can be connected by means of tonal cycles and their coefficients of recurrence can be specified.

It is desirable to make the following tables:

(1) $G^6_4\uparrow$① const.; C_3 const., C_5 const., C_7 const.
(2) $G^6_4\downarrow$① const.; C_3 const., C_5 const., C_7 const.
(3) $G^6_4\uparrow$⑤ const.; C_3 const., C_5 const., C_7 const.
(4) $G^6_4\downarrow$⑤ const.; C_3 const., C_5 const., C_7 const.
(5) $G^6_4\uparrow$① const.; $C^{\rightarrow} = C_3 + C_5 + C_7$
(6) $G^6_4\downarrow$① const.; $C^{\rightarrow} = C_3 + C_5 + C_7$
(7) $G^6_4\uparrow$⑤ const.; $C^{\rightarrow} = C_3 + C_5 + C_7$
(8) $G^6_4\downarrow$⑤ const.; $C^{\rightarrow} = C_3 + C_5 + C_7$
(9) $G^6_4\uparrow$① + $G^6_4\downarrow$⑤ + $G^6_4\uparrow$⑤ + $G^6_4\downarrow$①; C_3 const.
(10) " " " " ; C_5 const.
(11) " " " " ; C_7 const.
(12) " " " " ; $C^{\rightarrow} = C_3 + C_5 + C_7$

$C^{\rightarrow}$ is the symbol of a group of cycles (cycle continuity).

Continuity of G^6_4, when connected through a constant tonal cycle, consists of seven cycles: $C^{\rightarrow} = 7C$.

Example:

$G^6_4\uparrow$① const. $C^{\rightarrow} = C_3$ const.

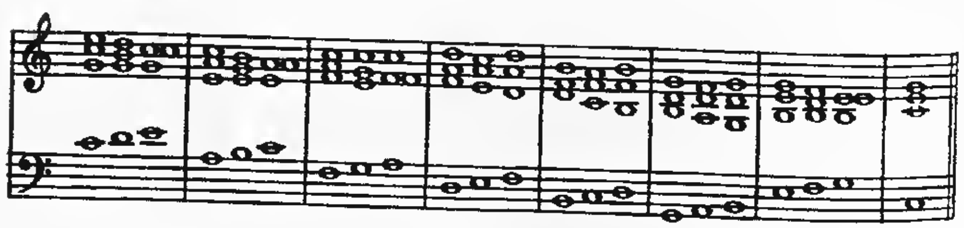


Figure 109. Continuity of G^6_4.

Continuity of G^6_4 of different forms and connection through different cycle-groups can be applied in its present form to diatonic progressions.

G^6_4 in symmetric progressions of types II and III requires *identical structures for the two extreme chords of one group.* This requirement does not affect the middle chord of the group, i.e., $S(^6_4)$, nor does it influence the selection of structures for the following groups.

Examples of continuity with G^6_4
in progressions of types I and II.

$$H^{\rightarrow} = 2G^6_4\uparrow + G^6_4\downarrow + G^6_4\uparrow + 2G^6_4\downarrow;\ C^{\rightarrow} = C_5 + 2C_7 + 2C_3 + C_5.$$

Figure 110. Continuity with G^6_4 in progressions of type I.

$H^{\rightarrow}$ and $C^{\rightarrow}$ as in the preceding example.
$S^{\rightarrow} = 2(S_1 + S_2) + (S_3 + S_2)$.

Figure 111. Continuity with G^6_4 in progressions of type II.

Application of G^6_4 to symmetric systems requires the following sequence of tonics:

$$GH^{\rightarrow} = (T_1 + T_2 + T_1) + (T_2 + T_3 + T_2) + (T_3 + T_4 + T_3) + \ldots$$

For example, the three-tonic system must be distributed as follows:

$$GH^{\rightarrow} = (T_1 + T_2 + T_1) + (T_2 + T_3 + T_2) + (T_3 + T_1 + T_3).$$

The number of tonics in the respective system specifies the cycle. Each group may begin with either S(5) or S(6).

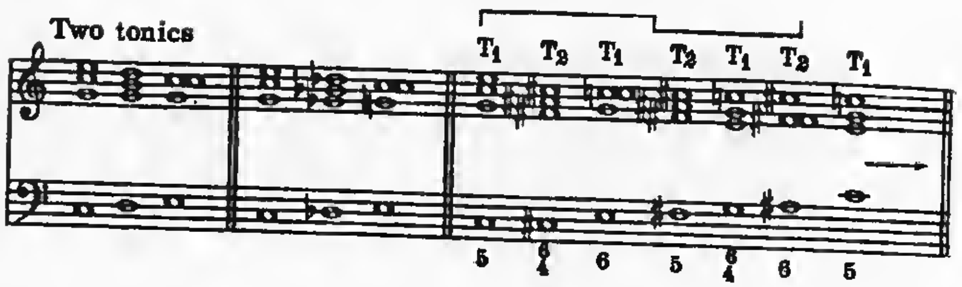
Each group acquires the following distribution of inversions:

$$G^6_4 = T_1S(5) + T_2S(^6_4) + T_1S(6)$$

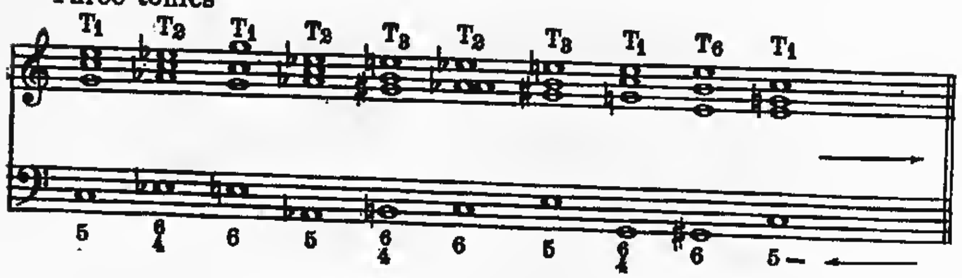
Under such conditions, each tonic appears in all the three inversions.

Table of G^6_4 applied to all symmetric systems

Two tonics



Three tonics



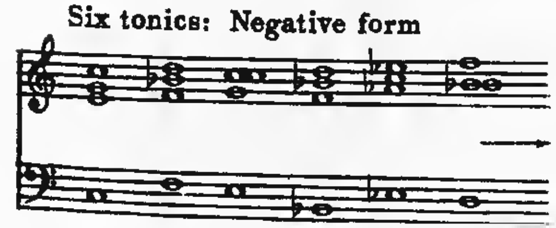
Four tonics

Six tonics

Figure 112. G^6_4 applied to symmetric systems (continued).

Twelve tonics

Six tonics: Negative form



Twelve tonics: Negative form

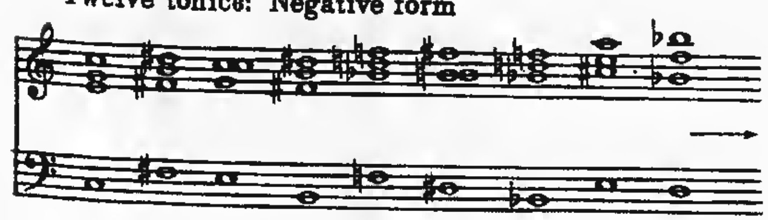


Figure 112. G^6_4 applied to symmetric systems (concluded).

Other negative forms are not as practical: inversions weaken tonality.

Example of variation of structures and directions.

Four tonics.

$$GH^{\rightarrow} = [S_1(5) + S_2(\tfrac{6}{4}) + S_1(6)] + [S_2(6) + S_1(\tfrac{6}{4}) + S_2(5)] +$$
$$+ [S_3(6) + S_4(\tfrac{6}{4}) + S_3(5)] + [S_2(5) + S_3(\tfrac{6}{4}) + S_2(6)]$$

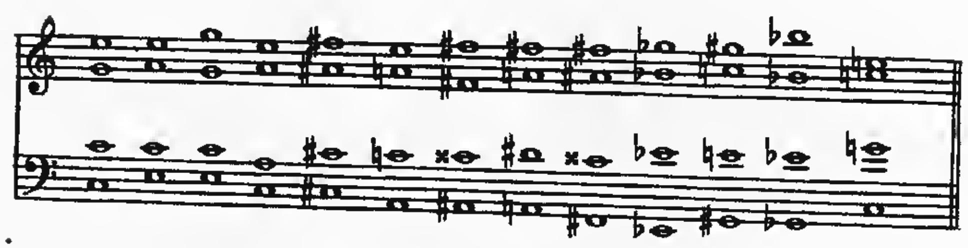


Figure 113. Variation of structures and directions.

H. Cycles and Groups Mixed

Tonal cycles can be introduced into the continuity of groups, and groups can be introduced into the continuity of cycles.

It is convenient to plan the mixed form of cycle-group continuity by bars (T).

Bars of cycles and bars of groups can be assigned to have different coefficients of recurrence.

When planning such a continuity in advance, it is important to remember that there is always a cycle-connection *between the bars.*

Examples:

$$H^{\rightarrow} = 2TC + TG + TC + 2TG = (C_5 + C_3) + C_7 + (C_3 + C_7) +$$
$$+ C_5 + G_6 + C_7 + (C_3 + C_3) + C_5 + G^6_4|① + C_7 + G^6_4|⑤ + C_3.$$

Type I

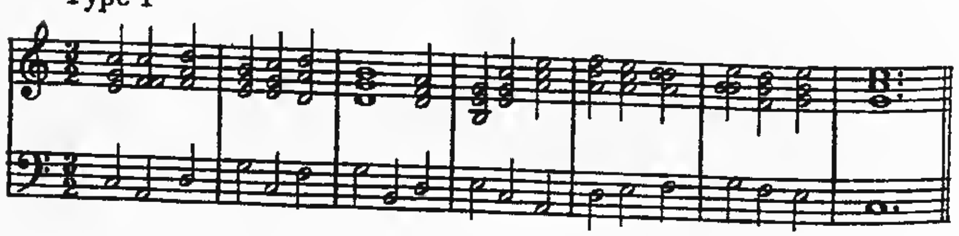


Figure 114. Cycles and groups mixed (continued).

Type II

Figure 114. Cycles and groups mixed (concluded).

CHAPTER 9

THE SEVENTH CHORD

THE seventh chord, in the diatonic as in other systems, has the following positions:

A. Diatonic System

Figure 115. Inversions of the seventh chord.

A seventh-chord, including all of its inversions, has 24 positions altogether. The classical system of harmony is based on the *postulate of resolving seventh: the seventh moves one step down.*

Figure 116. Resolving the seventh

This postulate provides a means for the continuous progression of S(7); in addition, it is the basis of the entire system of diatonic continuity (cycles).

One movement is required to produce C_3: the movement of the seventh alone. This results in a clockwise transformation.

Figure 117. Producing C_3

Two movements are required to produce C_5: the movement of both the seventh and of the fifth, each moving one step down. This results in a crosswise transformation.

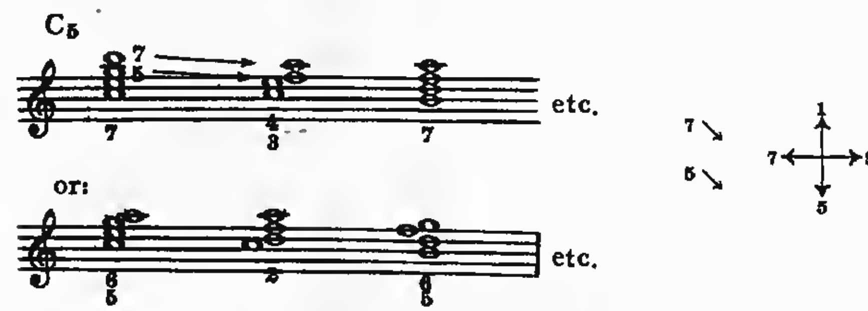


Figure 118. Producing C_5.

Three movements are required to produce C_7: the movement of the seventh, of the fifth, and of the third, each moving one step down. This results in a counter-clockwise transformation.

Skipping two chords in C_3, we obtain:

Figure 119. Producing C_7.

This type of music may be found among contrapuntalists of the 17th and 18th centuries. Palestrina, Bach and Händel obtained similar results by means of suspensions.

Assigning a system of cycles, we can produce a continuity of S(7). The starting chord may be taken in any position.

Example: $C_5 + C_3 + C_5 + C_7 + C_3 + C_3 + C_5$

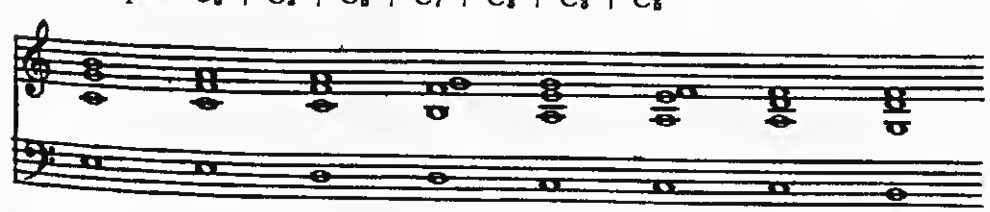


Figure 120. Continuity of S(7).

This continuity—being entirely satisfactory harmonically—may prove in some cases to be unsatisfactory melodically because of the continuous downward movement of all voices. When it is desirable to do so, this characteristic may be eliminated by means of two devices:

(1) exchange of the common tones
(2) octave inversion of the common tones

The same continuity of cycles assumes the following form:

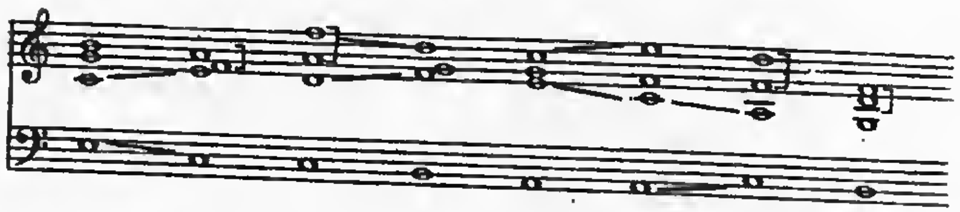


Figure 121. The same continuity with a more satisfactory melody.

Since C_7 does not provide at least two common tones, the use of the above devices in C_7 is precluded.

As continuity of the second type offers better melodic forms for all voices, it may be desirable to pre-set certain melodic forms in advance. For example, it is possible to obtain, by means of continuous C_5, the following form of descent through two parallel axes (b) or (d)—as in the music of Frederic Chopin.

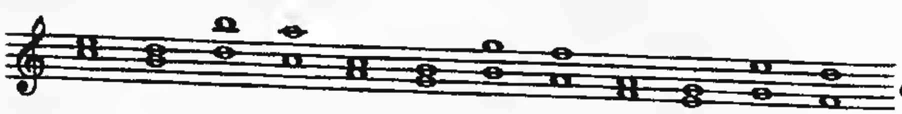


Figure 122. Continuous C_5 through two parallel axes.

This may be harmonized as follows:

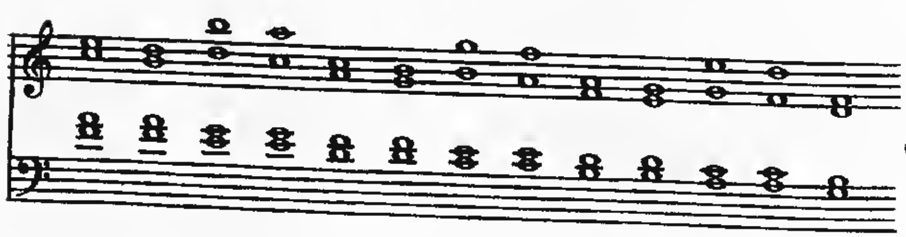


Figure 123. Harmonizing the continuum of figure 122.

Diatonic C_0 becomes a necessity in order to avoid that excess of saturation typical of the continuity of S(7) with variable cycles.

The principle of moving continuously through C_0 is based on the exchange and inversion of common tones.

The exchange and inversion of adjacent functions brings the utmost satisfaction. Nevertheless, it is not desirable to use the two extreme functions for such a purpose since they produce a certain amount of harshness.

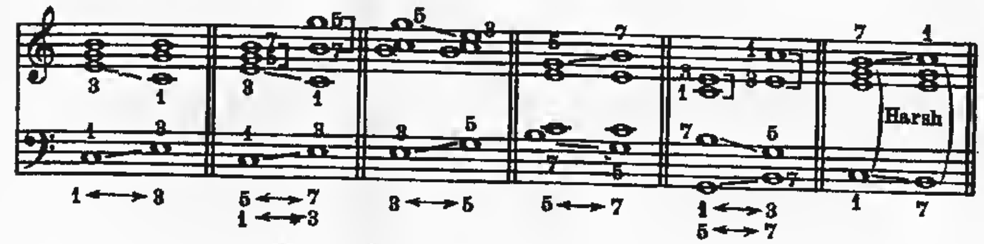


Figure 124. Inversion of adjacent functions.

An example of continuity of C_0:

Figure 125. Continuity of the C_0.

The final form of continuity of S(7) consists of mixtures of all cycles (including C_0) based on a rhythmic composition of the coefficients of recurrence.

Example: $2C_5 + C_0 + 2C_3 + C_0 + 2C_7 + C_0$

Figure 126. Final form, continuity of S(7).

B. The Resolution of S(7).

Resolution of an S(7) into an S(5) in all positions and inversions may be defined as a *transition from four functions to three functions.*

S(5) in four-part harmony and with a normal doubling (doubled root) consists of:

1, 1, 3, 5

And S(7) consists of:

1, 3, 5, 7

Thus, when a transition occurs, the root takes the place of the seventh. Therefore the resolution is provided through the ***motion of S(7) → S(7) and the substitution of one for the seven,*** i.e., the function which would otherwise have become a seventh in the continuity of seventh-chords now becomes a root-tone in order to achieve a resolution.

7 → 1
5 → ⑦ → 1
3 → 5
1 → 3

Note: Do not move S(7) → S(5) in C_0

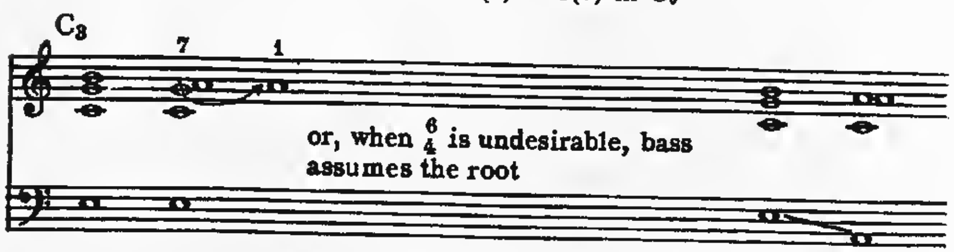


Figure 127. Resolutions in diatonic cycles.

This case provides an explanation of why a tonic triad acquires a tripled root and loses its fifth:

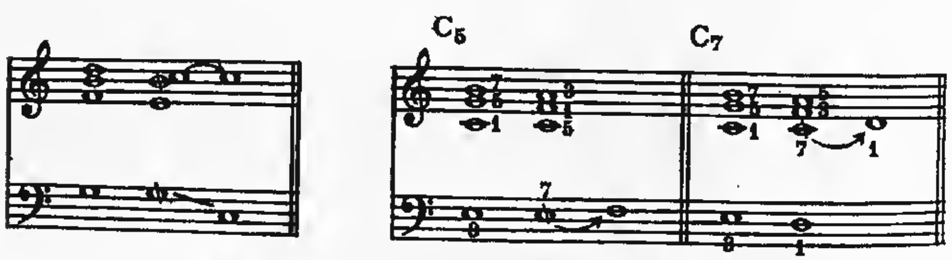


Figure 128. Tonic triad acquires a tripled root.

1. Preparation of S(7)

There are three methods of preparing an S(7), i.e., of transition from S(5) to S(7):

(1) suspending
(2) descending
(3) ascending

The first method is the only one producing the *positive* (C_3, C_5, C_7) cycles. Methods (2) and (3) are the outcome of the *intrusion of melodic factors* into harmony. These are obviously in conflict with the nature of harmony (like those groups with passing chords we have already studied) as they produce

negative cycles, and these in turn contradict the postulate of the resolving seventh universally observed in classical music.

The technique of preparing the seventh consists of assigning a certain consonant function (1, 3, 5) to become a dissonant function (7) and of either sustaining the assigned function of the S(5) over the bar line, or moving it one step downward or upward.

The last two forms of a seventh conventionally occur on a *weak* beat.

Here are different positions, inversions and cycles of the S(5) → S(7) transition.

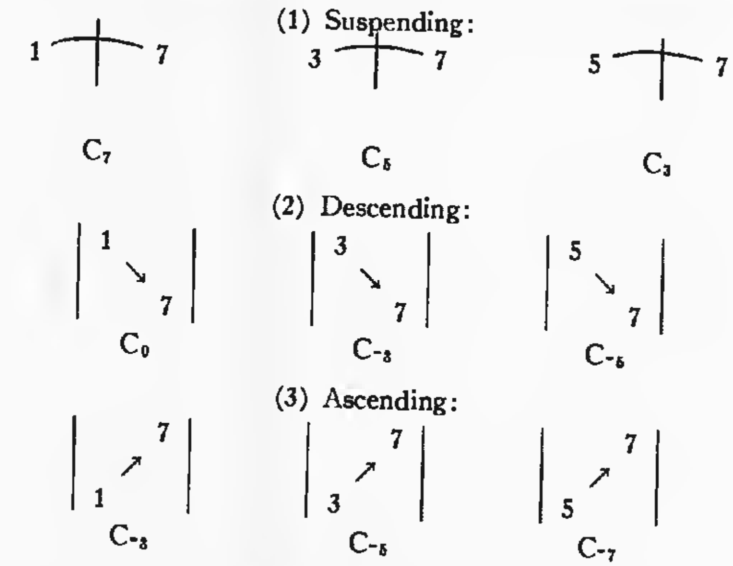


Figure 129. Different positions of transition of S(5) → S(7)

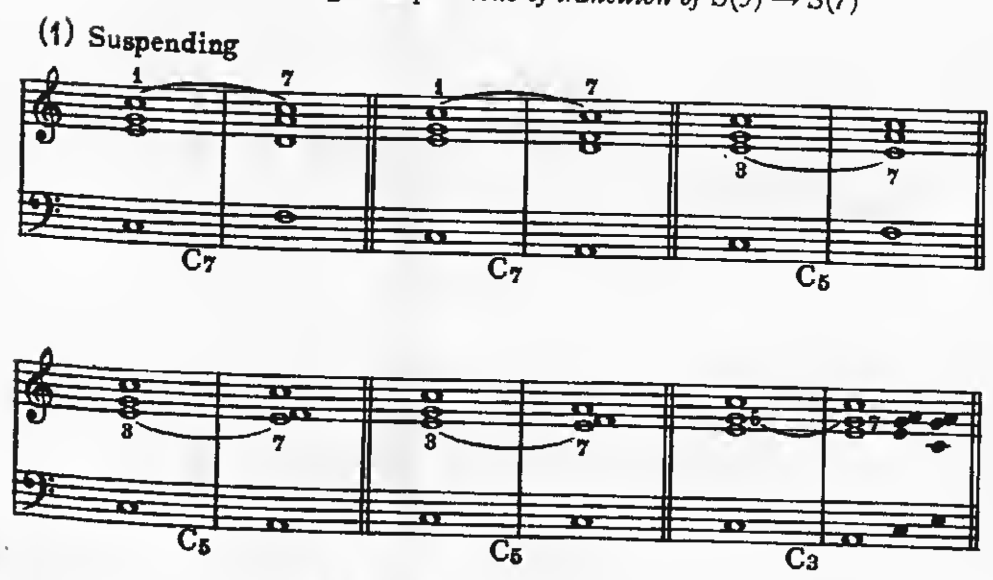
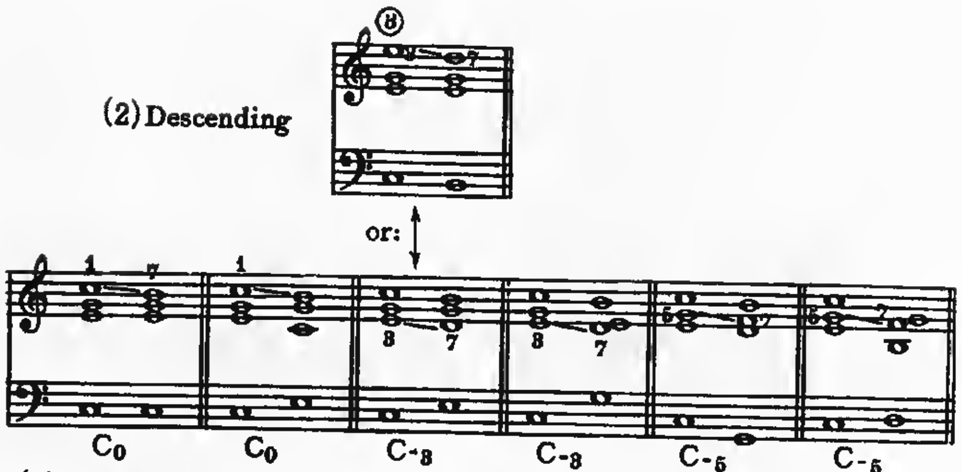


Figure 130. Preparation of S(7) (continued).

(2) Descending



(3) Ascending

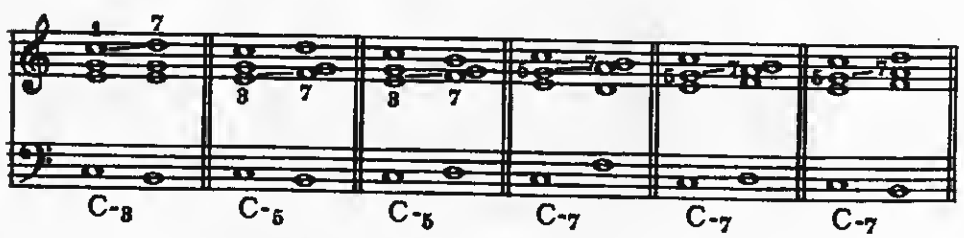


Figure 130. Preparation of S(7) (concluded).

A mixture of zero, positive and negative cycles, provides the final form of continuity based on S(5) and S(7).

For more efficient planning of such continuity, use bar lines for the layout. The preparation of S(7) may be either positive or negative; the resolution is always positive.

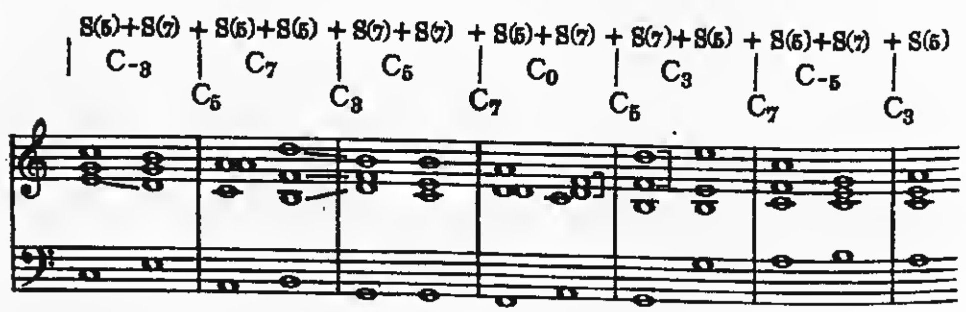


Figure 131. Preparation of S(7)

C. With Negative Cycles.

The *negative* system of tonal cycles may be used as an independent system. The negative system is in reality a geometrical inversion of the positive system. Every principle, rule or regulation of the positive system thus becomes its own converse in the negative.

Chord structures become E_1⑬ of the original scale. Chord progressions are based on $E_1$⑥ which forms the C_{-3}. Clockwise transformations become counterclockwise and vice versa.

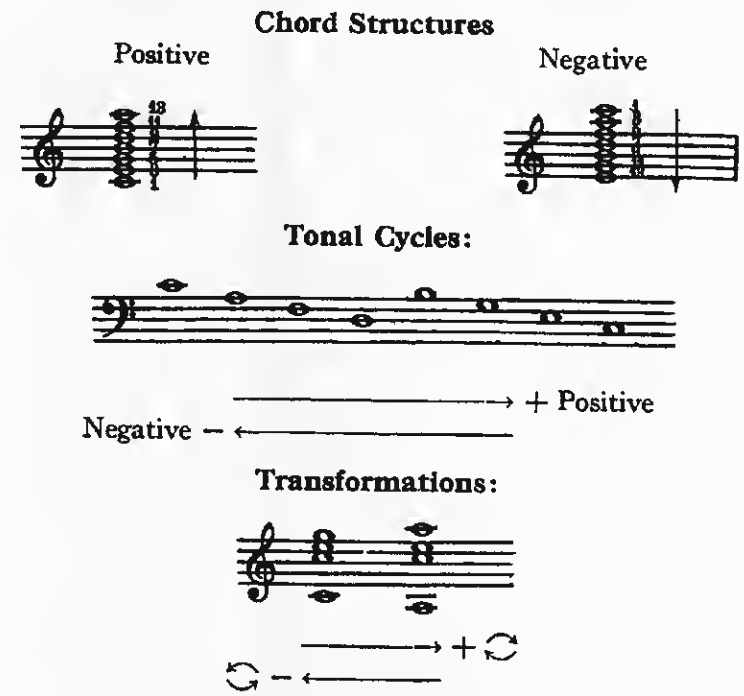


Figure 132. Negative Cycles.

The postulate of resolving seventh for the negative system must be read: *the negative seventh moves one step up*. The C_{-5} requires the negative seventh and negative fifth to move one step up. The C_{-7} requires all tones except the root to move up. This system may be of great advantage in building climaxes.*

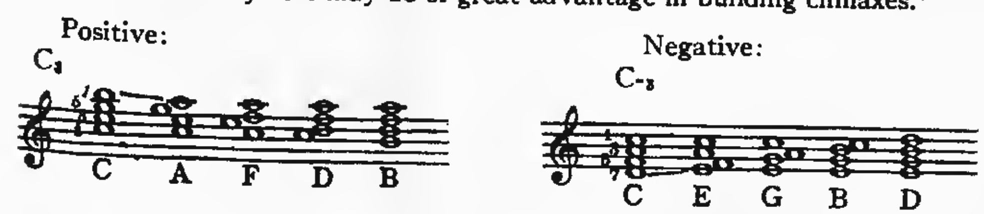


Figure 133. Positive (C_3) and negative (C_{-3}).

*Passing observations of this character, though made casually by Schillinger, should be noted carefully by the student. They offer valuable ideas and techniques which may be successfully exploited in composition and arranging. (Ed.)

The root-tone of the negative system is the seventh of the positive, and *vice-versa*.

It is easy to see how the other cycles would operate.

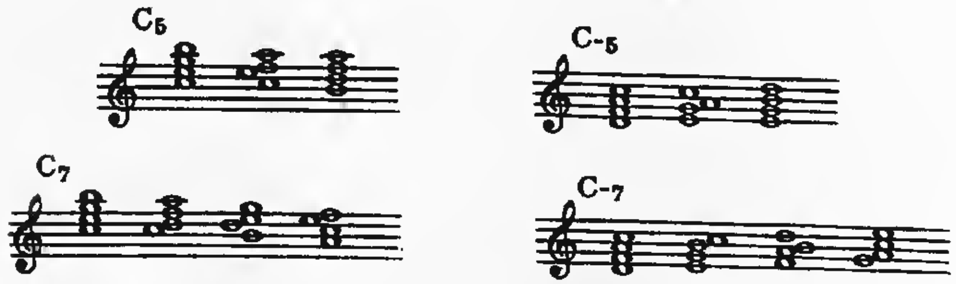


Figure 134. Positive (C_5, C_7) and negative (C_{-5}, C_{-7}).

If one wishes to read the negative system as if it were positive, the technique must be changed as follows:

The C_{-3} requires the ascending of 1
The C_{-5} " " " " 1 and 3
The C_{-7} " " " " 1, 3 and 5

1. Special Applications of S(7)

S(7) finds its application in $G_{\frac{6}{4}}$, either as the first or the last chord of the group.

The following forms are possible:

$\underleftrightarrow{S(5) + S(\frac{6}{4}) + S(\frac{6}{5})}$	$\underleftrightarrow{S(7) + S(\frac{6}{4}) + S(6)}$
$\underleftrightarrow{S(5) + S(\frac{6}{4}) + S(\frac{4}{3})}$	$\underleftrightarrow{S(7) + S(\frac{6}{4}) + S(\frac{4}{3})}$
$\underleftrightarrow{S(5) + S(\frac{6}{4}) + S(2)}$	$\underleftrightarrow{S(7) + S(\frac{6}{4}) + S(2)}$

Figure 135. Forms of S(7).

The cycle between the extreme chords of $G_{\frac{6}{4}}$ may be either C_0, or C_3, or C_5.

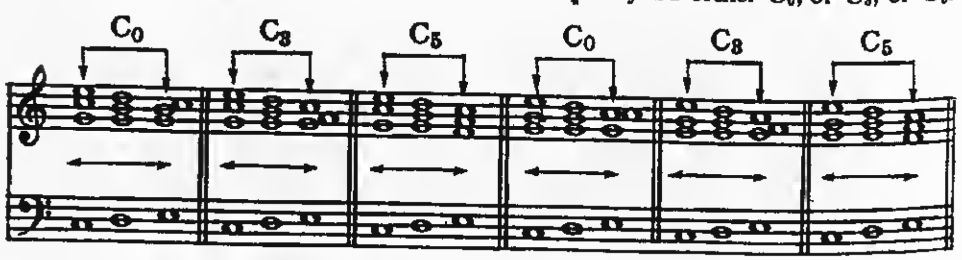


Figure 136. Cycle between extreme chords of G^6_4.

Besides $G_{\frac{6}{4}}$ there is a special group in which $S(\frac{4}{3})$ is used as a passing chord. There are two forms of this group.

(A) $G_{\frac{4}{3}(5)} = \underrightarrow{S(5)^{\textcircled{8}} + S(\frac{4}{3}) + S(7) \text{ or } S(5)}$

(B) $G_{\frac{4}{3}(6)} = \underrightarrow{S(6)^{\textcircled{5}} + S(\frac{4}{3}) + S(7) \text{ or } S(5)}$

Figure 137. $S(\frac{4}{3})$ used as a passing chord.

These two forms may be used in one direction only. All positions are available.

The rule of voice-leading is: the bass and one of the voices of doubling move stepwise down; common tones are sustained.

The cycle between the extreme chords in the first form is C_3; in the second form it is C_0.

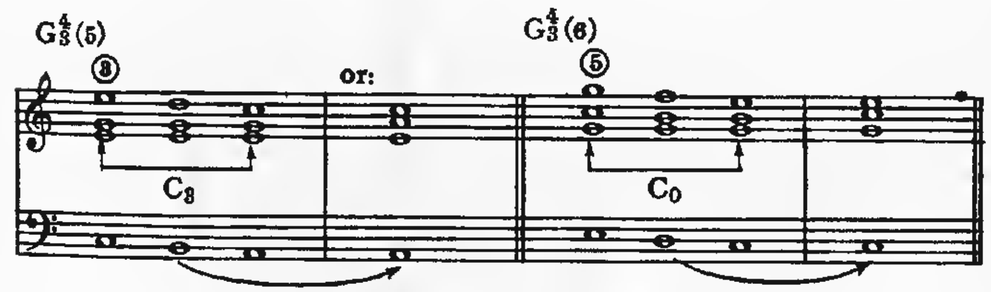


Figure 138. Voice-leading exemplified.

2. Cadences

The following applications of S(7) are commonly known:

(1) IV_7 $I_{\frac{6}{4}} — V_7$ I_5
(2) $IV_{\frac{6}{5}}$ " " "
(3) $II_{\frac{6}{5}}$ " " "
(4) $II_{\frac{4}{3}}$ " " "

In addition to this, the following forms may be offered:

(5) Any of the previous forms $I_{\frac{6}{4}} — III_{\frac{6}{5}}$ I_5
(6) " $I_{\frac{6}{4}} — VII_{\frac{6}{5}}$ I_5

Besides these, there are two ecclesiastic forms:

(1) $I_5 — IV^{\textcircled{5}}_{(II\frac{4}{3})}$ I_5
(2) $I_5 — IV^{\textcircled{5}}_{(VII\frac{4}{3})}$ I_5

Figure 139. Applications of S(7).

D. S(7) IN THE SYMMETRIC ZERO CYCLE (C_0).

Symmetric C_0 exhibits extraordinary versatility with S(7): seven structures of the latter have been in use.

If the forms of S(7) had been evolved scientifically, they would have been obtained in the following order:

Taking c − e − g − b♭ (4 + 3 + 3) as the most common form and producing variations thereof, we obtain two other forms:

c − e♭ − g − b♭ (3 + 4 + 3)
and c − e♭ − g♭ − b♭ (3 + 3 + 4)

Taking another form, c − e − g − b (4 + 3 + 4), we obtain two other forms:

c − e − g♯ − b (4 + 4 + 3)
and c − e♭ − g − b (3 + 4 + 4)

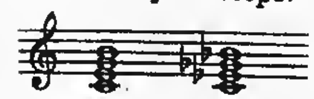
These two groups of three are distinctly different; but inasmuch as music has made use of them for some time, our ears accept mixing all of them in one harmonic continuity.

Besides these six forms there is a c − e♭ − g♭ − b♭♭ (3 + 3 + 3); and there might have been c − e − g♯ − b♯ (4 + 4 + 4), if it were not for the fact that c − b♯ is an enharmonic octave.

A continuity on symmetric C_0 of all seven structures offers 5,040 permutations. Thus a c-chord alone can move (without changing its position and without coefficients of recurrence being applied) for 5,040 × 7 = 35,280 chords.

The method of selecting the best of the available progressions must be based on the following principle: **the best progressions on symmetric C_0 are due to identity of steps or to contrary motion.**

(1) Identity of Steps:



[all semitones]

(2) Contrary motion:

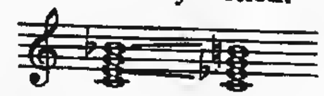


Figure 140. Best progressions on symmetric C_0.

The principle of variation of chord-structures and their positions remains the same as in S(5):

Structure		*Position*
Constant	————	Variable
Variable	————	Constant

S(7) in the following table has a dual system of indications: letter symbols and adjectives.* The adjectives are chosen so that they do not pertain to degrees of a particular scale but to structure alone. Thus, so common an adjective as "dominant" must be abandoned.

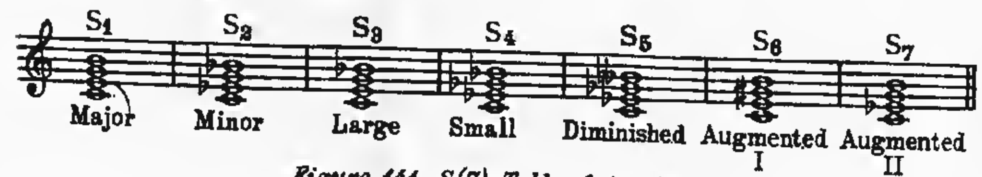


Figure 141. S(7) Table of structures

1. An example of Continuity in C_0:

Structures: $S_2 + S_7 + S_4 + S_1$
Coefficients $(r_{5\div4})$: $4S_2 + S_7 + 3S_4 + 2S_1 + 2S_2 + 3S_7 + S_4 + 4S_1$

Figure 142. Continuity in C_0

As in S(5), any combination of the forms of S(7) by 2, 3, 4, 5, 6 and 7 may be used.

*By *adjective*, Schillinger means the descriptive word used to indicate the *shape* of an S(7). (Ed.)

2. S(7) in Type III (Symmetric).

As in previous cases, in dealing with symmetrical tonics, we may apply C_0 either to any of the tonics or with a continuous change of chord structures, a change occurring with each tonic.

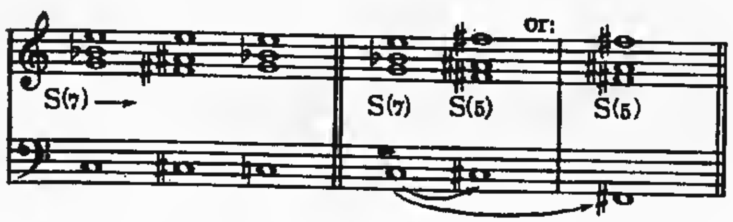
When structures of S(5) and S(7) have to be specified in one continuity, they must have full indications:

$S_1(5)$; $S_2(5)$; $S_3(5)$ $S_4(5)$ and
$S_1(7)$; $S_2(7)$; $S_3(7)$; $S_4(7)$; $S_5(7)$; $S_6(7)$; $S_7(7)$

3. Two Tonics ($\sqrt{2}$)

As the $\sqrt{2}$ forms the center of the octave, the progression $1 \rightarrow \sqrt{2}$ (C→F♯) is positive and $\sqrt{2} \rightarrow 2$ (F♯→C) is negative.

The system of Two Tonics which was continuous on S(5) becomes closed on S(7). Transformations correspond to C_6.



Example of continuity

Figure 143. Continuity on two tonics.

4. Three Tonics ($\sqrt[3]{2}$)

Continuous system: moves four times; transformations correspond to C_3. To obtain S(7) after an S(5), use the position which would correspond to a continuous progression of S(7).

An example of continuity:

Figure 144. Three tonics $\sqrt[3]{2}$

5. Four Tonics ($\sqrt[4]{2}$)

A closed system: transformations correspond to C_3; S(7) after S(5) as in the three-tonic system.

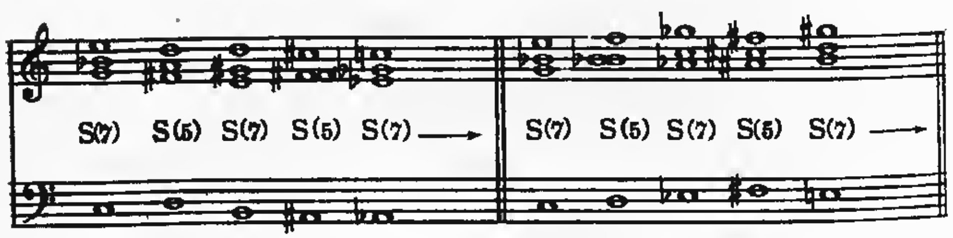
Figure 145. Four tonics (continued).

Example of continuity:

Figure 145. Four tonics (concluded).

6. Six Tonics ($\sqrt[6]{2}$)

A continuous system: moves two times; transformations correspond to C_7; S(7) after S(5) as in previous cases; both positive and negative progressions are fully satisfactory; to obtain the negative progressions, read the positive ones backwards.



Example of continuity

Figure 146. Six tonics.

7. Twelve Tonics ($\sqrt[12]{2}$)

A closed system: all specifications and applications as in the six-tonic system.

Example of continuity

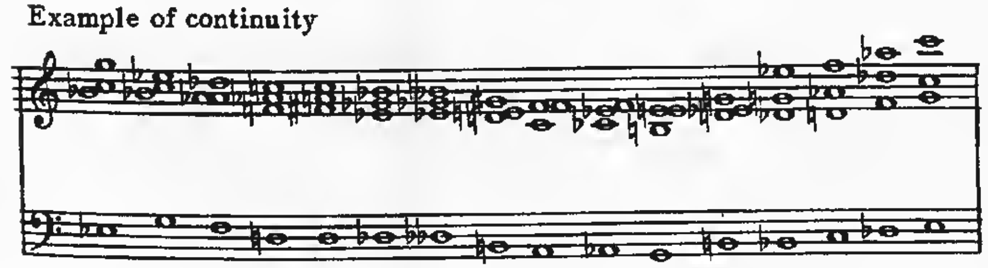


Figure 147. Twelve tonics.

E. Hybrid 5-Part Harmony

The technique of continuous S(7) makes it possible to evolve a hybrid five-part harmony, in which the bass is a constant root tone and the four upper functions assume variable forms of S(7) with respect to the bass.

By placing an S(7) on either the root, or the third, or the fifth, or the seventh of the bass root, we obtain all forms of S in five-part harmony. An S(5) has to be represented with the addition of 13th (the so-called "added sixth").

Forms of Chords in Hybrid Five-Part (4 + 1) *Harmony*

The 4 Upper Parts	5 3 1	7 5 3	9 7 5	11 9 7	13 11 9
S	13	1	3	5	7
The Bass	1	1	1	1	1
The forms of Tension	S(5)	S(7)	S(9)	S(11)	S(13)

Figure 148. Chord forms in hybrid five-part harmony.

It is possible to move either forms or any of the combinations of forms continuously in any rhythmic form of continuity.

Note that the tonal cycles do *not* correspond in the upper four parts to the tonal cycles in the bass when the forms of tension are variable. For example, f — a — c — e may be 3 — 5 — 7 — 9 in a DS(9) or 7 — 9 — 11 — 13 in a GS(13). In such a case, a progression C_5 for the bass with S(9) → S(13) produces C_0 for the upper four parts.

The principle of exchange and octave-inversion of the common tones holds true.

Three forms of harmonic continuity will be used in the following illustrations (these forms of continuity are applicable in four-part harmony as well). When chord structures of greater tension are desired, and also when compensation for the diatonic system's deficiencies is required, it is often desirable to use pre-selected *forms* of chord-structures which nevertheless *move diatonically*. Such a system has a bass belonging to one definite diatonic scale, while the chord structures acquire various accidentals in order to produce a definite sonority. In the general classification of harmonic progressions, this latter type is known as *diatonic-symmetric*.

1. Three Types of Harmonic Progressions

I. Diatonic
II. Diatonic-Symmetric
III. Symmetric

The following examples will be worked out in all three types of harmonic continuity. Constant and variable forms of tension will be offered.

In order to select a desirable form or forms of structure for the different forms of tension, it is advisable to select a scale first, as such a scale offers the manifold of forms of tension. For example, if the scale selected is c — d — e — f♯ — g — a — b♭; S(5) = c — e — g — a; S(7) = c — e — g — b♭; S(9) = c — e — g — b♭ — d; S(11) = c — g — b♭ — d — f♯; S(13) = c — b♭ — d — f♯ — a.

Though the same scale would be *ideal* for the progression, it is not impossible and not undesirable to use some other scale for the chord-progressions.

2. Tables and Examples

(a) *Continuity of S*(5) [*monomials*]
Scale: c — d — e — f♯ — g — a — b♭

Type I

Type II

Type III

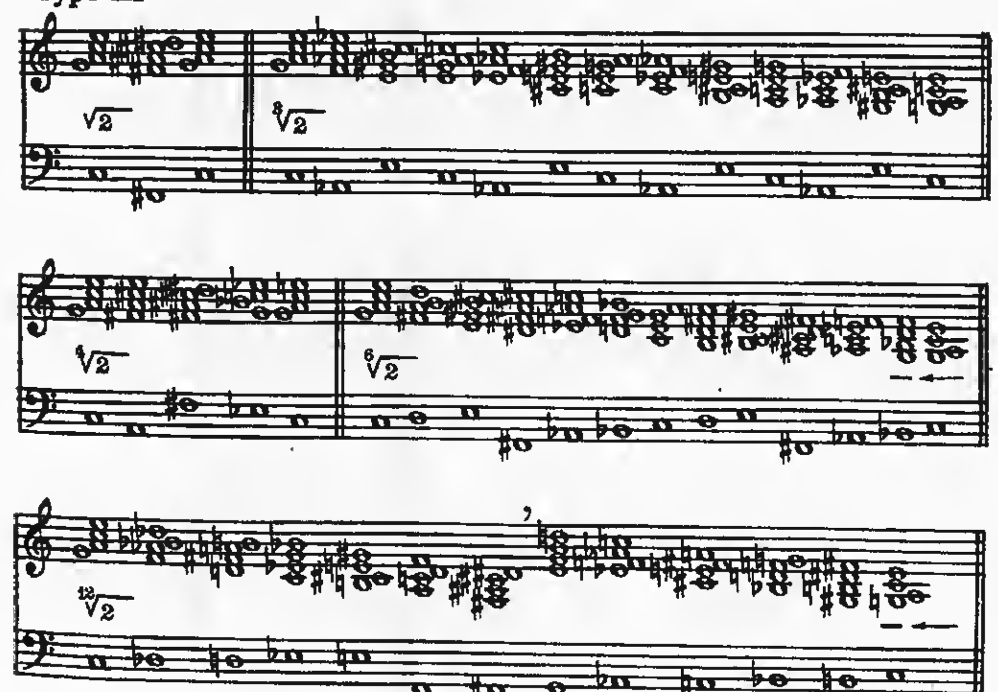


Figure 149. Continuity of S(5) monomials.

(b) *Continuity of S(7) [monomials]*

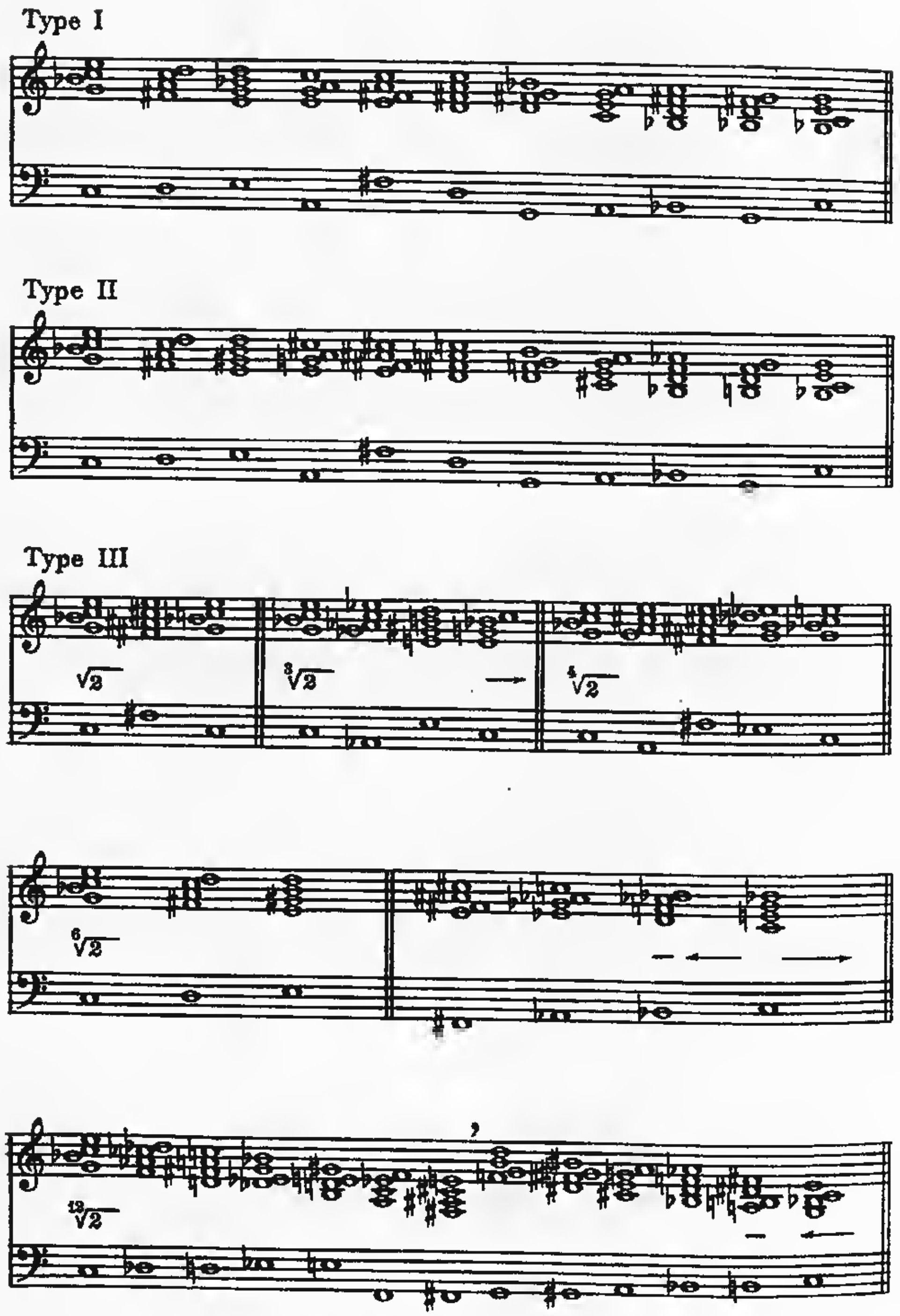


Figure 150. Continuity of S(7) monomials.

(c) *Continuity of S(9) [monomials)*

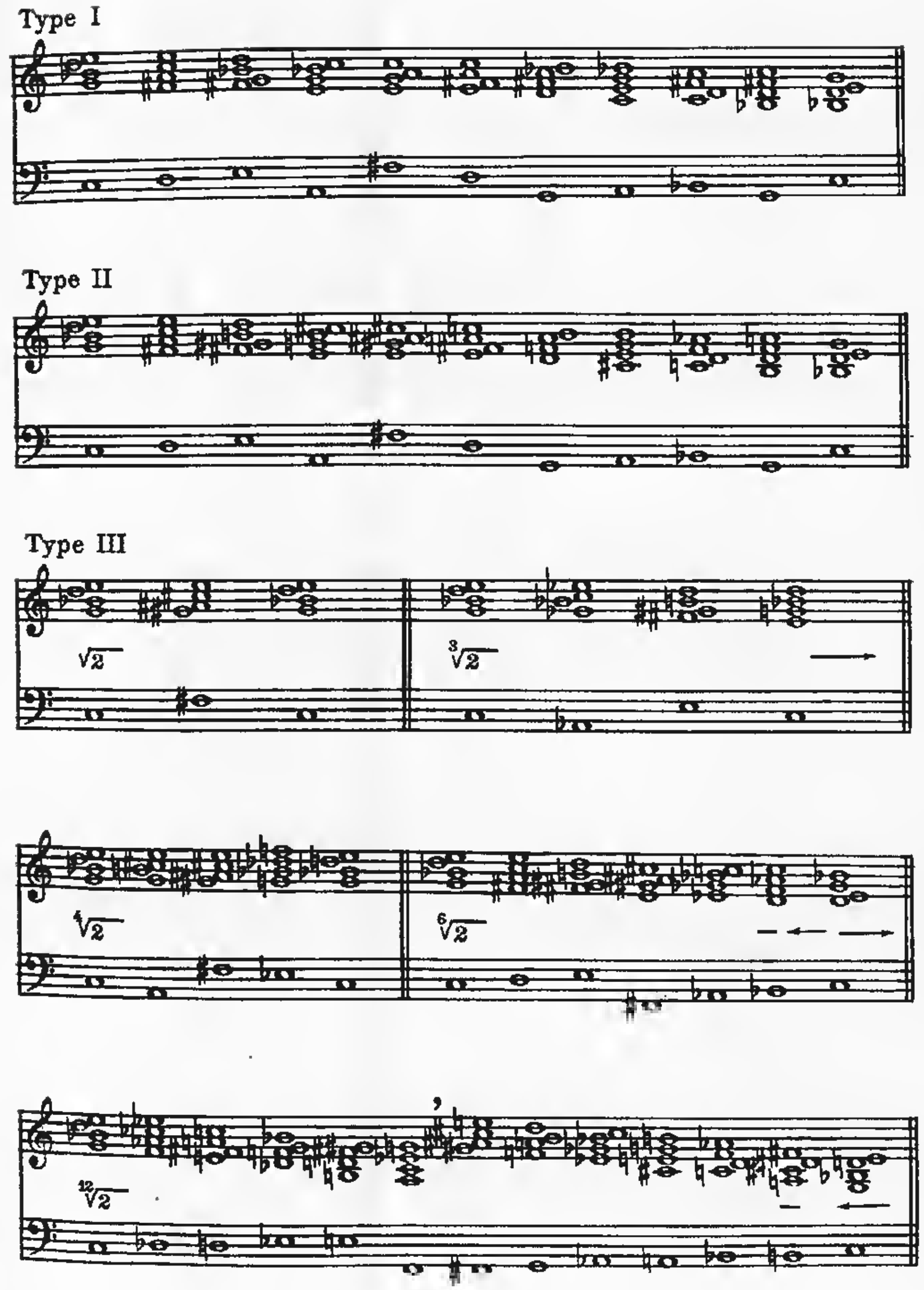


Figure 151. Continuity of S(9) monomials.

(d) *Continuity of S(11) [monomials]*

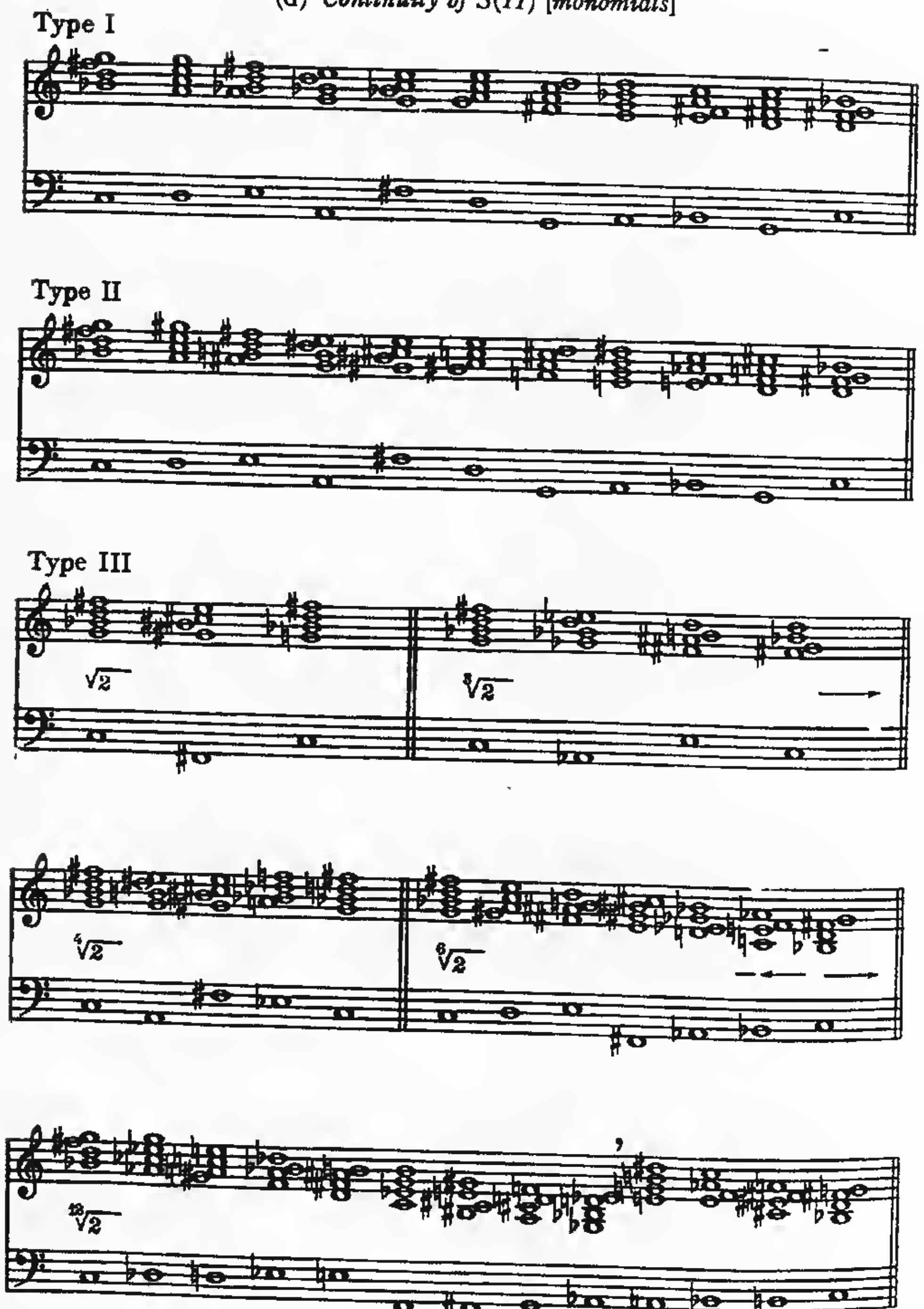


Figure 152. Continuity of S (11) monomials.

(e) *Continuity of S(13) [monomials)*

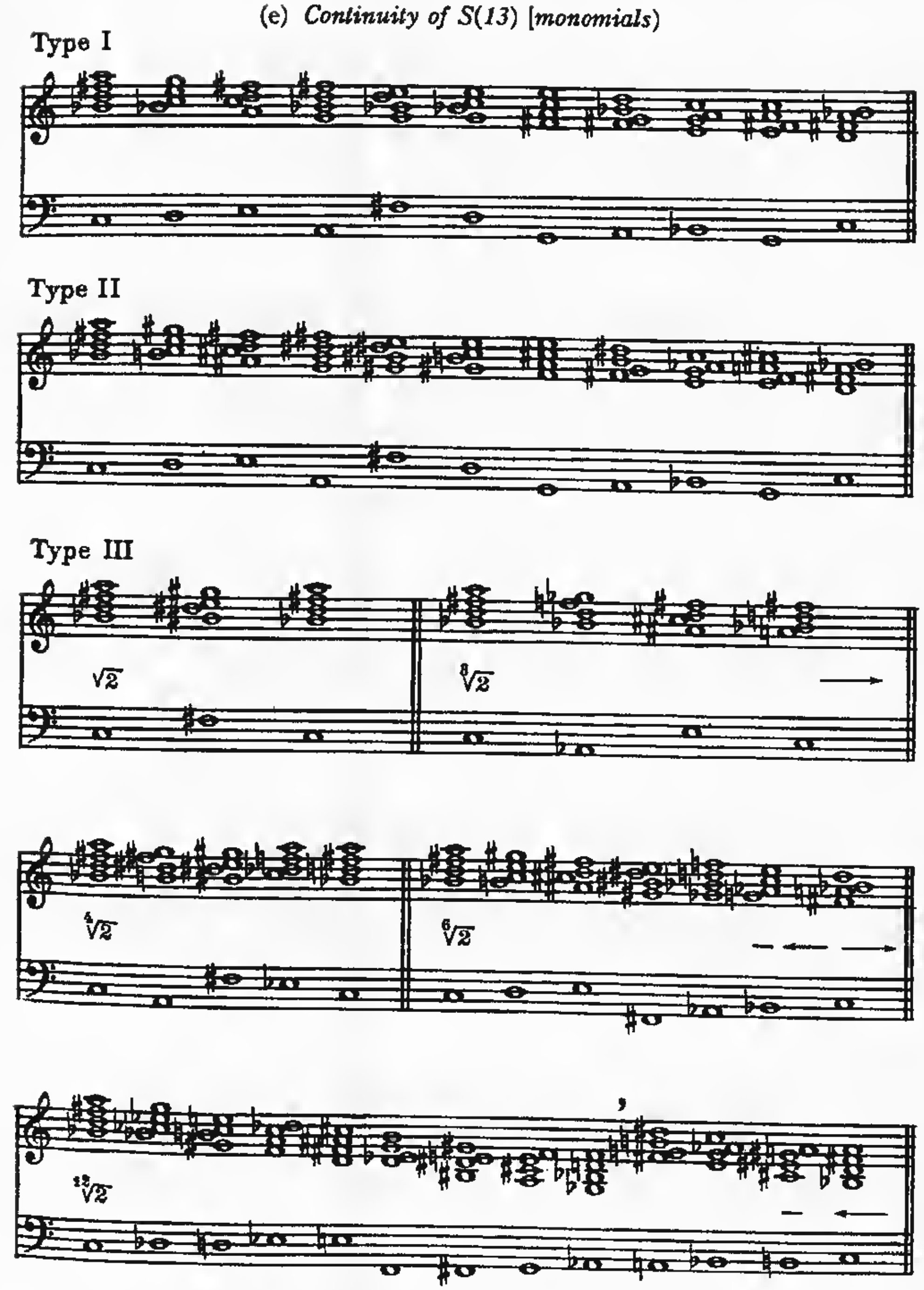


Figure 153. Continuity of S (13) monomials.

Combinations by two (binomials), three (trinomials), four (quadrinomials) and five (quintinomials) may be devised in a similar way.

3. Table of Combinations

The Arabic numerals in the following tables represent the chord structures (S):

a. *Combinations by 2*

5 + 7	7 + 9	9 + 11	11 + 13
5 + 9	7 + 11	9 + 13	
5 + 11	7 + 13		
5 + 13			

10 combinations, 2 permutations each
Total: 10 × 2 = 20

b. *Combinations by 3*

5 + 7 + 9	7 + 9 + 11	9 + 11 + 13
5 + 7 + 11	7 + 9 + 13	
5 + 7 + 13	7 + 11 + 13	
5 + 9 + 11		
5 + 9 + 13		
5 + 11 + 13		

10 combinations, 6 permutations each
Total: 10 × 6 = 60

c. *Combinations by 4*

5 + 7 + 9 + 11	7 + 9 + 11 + 13
5 + 7 + 9 + 13	
5 + 7 + 11 + 13	
5 + 9 + 11 + 13	

5 combinations, 24 permutations each
Total: 5 × 24 = 120

d. *Combinations by 5*

5 + 7 + 9 + 11 + 13

1 combination, 120 permutations
Total: 1 × 120 = 120

All other cases of trinomial, quadrinomial, quintinomial and larger combinations are treated as coefficients of recurrence.

Example: $S^{\rightarrow} = 2S(5) + S(7) + 2S(9) = S(5) + S(5) + S(7) + S(9) + (S(9)$, i.e., a quintinomial with two identical pairs.

Coefficients of recurrence may be applied to the composition of continuity consisting of the forms of variable tension.

Examples:

Type I. $2S(5) + S(9) + S(13) + 2S(7)$

Type II

Type III $\sqrt[3]{2}$

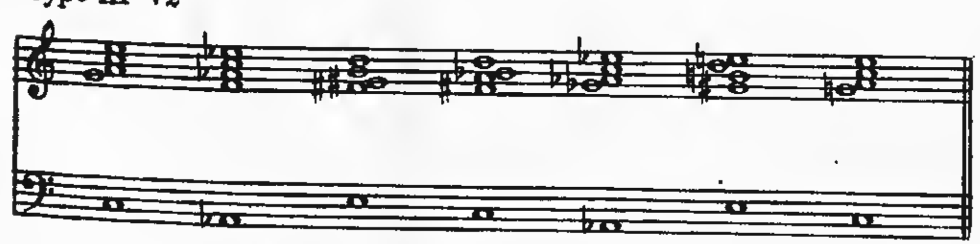


Figure 154. Coefficients of recurrence applied to variable tension continuity.

CHAPTER 10

THE NINTH CHORD

A. S(9) IN THE DIATONIC SYSTEM

NINTH-CHORDS in four-part harmony are used ***with the root-tone in the bass only***, thus operating as a hybrid four-part harmony—like S(5) with the doubled root. The three upper parts are 3, 7 and 9. The 7 and the 9 are subject to resolution through stepwise downward motion.

If one function resolves at a time, it is always the higher one (the ninth). A resolution of one function at a time produces C_0. Other cycles derive from the simultaneous resolutions of two functions (the ninth and the seventh). No consecutive S(9)'s are possible through this particular type of system for S(9) alternates with S(7) and S(5).

The reason for first resolving the 9th rather than the 7th in C_0 is that the latter procedure would result in a chord-structure alien to the usual seven-unit diatonic scales; the intervals in the three upper voices are fourths.

Figure 155. Resolving the ninth.

1. Positions of S(9)

As the bass remains constant, the three upper voices are subject to six permutations resulting in corresponding distributions.

Figure 156. Table of positions of S(9).

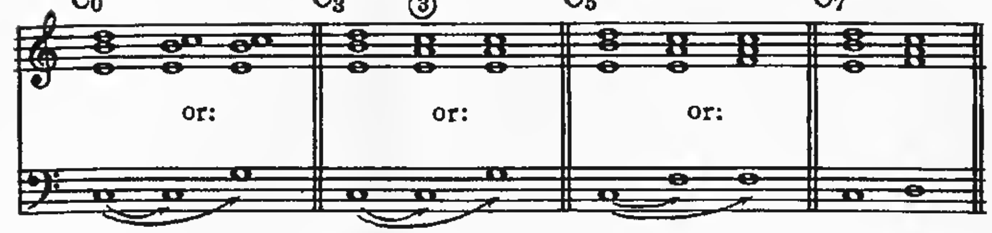


Figure 157. Resolutions of S(9).

The resolutions (except in C_0) produce positive cycles only. In C_3 they are characteristic of Mozart, Clementi and others of the same period. C_5 (the second resolution) is the most commonly known, especially with b♭ in the first chord (making a "dominant chord" of F-major).

C_7 is characteristic of Bach and contrapuntalists who developed such progressions from the idea of two pairs of voices moving in thirds in contrary motion. Read the last measure with b♭ and f♯ and add S(5) g-minor. All these cases of resolution were known to the classics through *melodic manipulations* (i.e., as a part of their contrapuntal heritage) and not through the idea of those independent structures we call S(9).

Preparation of S(9) bears a great similarity to the preparation of S(7). There is even an absolute correspondence in the cycles with respect to technical procedures.

The same three methods constitute the technique of preparation (suspending, descending, ascending).

2. Table of Preparations

(1) *Suspending:*

3 — 9	5 — 9	7 — 9
1 — 7	3 — 7	5 — 7
C_7	C_5	C_3

(2) *Descending:*

3 ↘ 9	5 ↘ 9	7 ↘ 9
1 ↘ 7	3 ↘ 7	5 ↘ 7
C_0	C_{-3}	C_{-5}

(3) *Ascending:*

3 ↗ 9	5 ↗ 9	7 ↗ 9
1 ↗ 7	3 ↗ 7	5 ↗ 7
C_{-3}	C_{-5}	C_{-7}

Figure 158. Preparations of S(9).

It follows from the above chart that some of the preparations of S(9) require an S(5); some require S(7); and some allow both. It is practical to have S(5) or S(7) preparing S(9) with the root in the bass.

The first form of preparation was known to the classics as a double suspension.

Figure 159. Double suspension.

A similar cadence was used in major.

Here is another example of a characteristic classical cadence:

Figure 160. Characteristic classical cadence.

*Figure 161. Example of continuity containing S(9)**

*At this point in the original manuscript, Schillinger writes: "If the student is to grasp all the implications of the foregoing material, he should work out the following instructions as home-work:

(1) Make complete tables of preparations and resolutions from all positions.

(2) Write diatonic continuity containing S(9).

(3) Make some modal transpositions of the examples thus obtained.

(4) Write continuity containing S(9) in the second type (diatonic-symmetric) of harmony. Select chord-structures from the examples of hybrid five-part harmony."

(Ed.)

B. S(9) in the Symmetric System

The classical (preparation-resolution) technique just described—and commonly used in the diatonic system—is also applicable to the symmetric system. Symmetric roots correspond to the respect cycles: C_5, to $\sqrt{2}$; C_3, to $\sqrt[3]{2}$ and $\sqrt[4]{2}$; C_7, to $\sqrt[6]{2}$ and $\sqrt[12]{2}$. With this in view, a continuity consisting of S(5), S(7) and S(9), and operated through classical technique, may be offered.

Symmetric C_0 is quite fruitless when S(9) alone is used, for the upper three functions (3, 7, 9) produce an incomplete seventh-chord, the permutations of which ($3 \leftrightarrow 7$, $3 \leftrightarrow 9$) sound awkward. There is one exception: $7 \leftrightarrow 9$.

As S(9) in hybrid four-part harmony is an incomplete structure—5 is omitted—the adjectives descriptive of chord structure may be applied only with a certain allowance for the 5th.

There are two distinctly different families of S(9), not to be mixed except when in C_0:

(1) *The minor seventh family.*
(2) *The major seventh family.*

The **minor 7th family** includes the following structures:

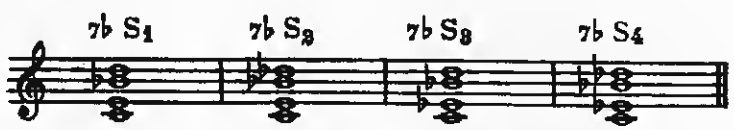


Figure 162. Minor 7th family.

To these the following adjectives may be applied in their respective order:

$7\flat S_1$ — large.
$7\flat S_2$ — diminished.
$7\flat S_3$ — minor.
$7\flat S_4$ — small.

The **major 7th family** includes the following structures:

Figure 163. Major 7th family.

The respective adjectives are:

$7\natural S_1$ — major.
$7\natural S_2$ — augmented I.
$7\natural S_3$ — augmented II.

These are the only possible forms.

It seems that all combinations of the two families, except those producing consecutive sevenths ($7\flat S_4 \leftrightarrow 7\natural S_1$; $7\natural S_2 \leftrightarrow 7\natural S_3$; $7\flat S_3 \leftrightarrow 7\natural S_2$; $7\flat S_1 \leftrightarrow 7\flat S_4$), are satisfactory when in C_0. On the different roots, the forms of S(9) must belong to one family.

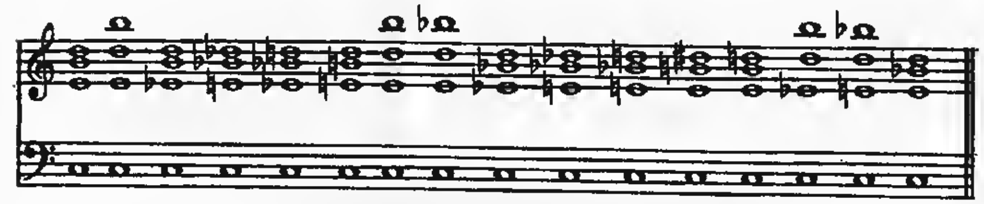


Figure 164. Example of C_0 continuity.

Full indication for S(9) when used in combinations with S(5) and S(7):

$7\flat S_1(9)$; $7\flat S_2(9)$; $7\flat S_3(9)$; $7\flat S_4(9)$
$7\natural S_1(9)$; $7\natural S_2(9)$; $7\natural S_3(9)$

Two tonics ($\sqrt{2}$). The technique corresponds to C_5.

Figure 165. Two tonics ($\sqrt{2}$)

To resolve the last chord of the preceding table, use position ⓑ of the resolution technique.

Figure 166. Resolution of last chord of figure 165.

Figure 167. Example of continuity.

Three tonics ($\sqrt[3]{2}$). The technique corresponds to C_3.

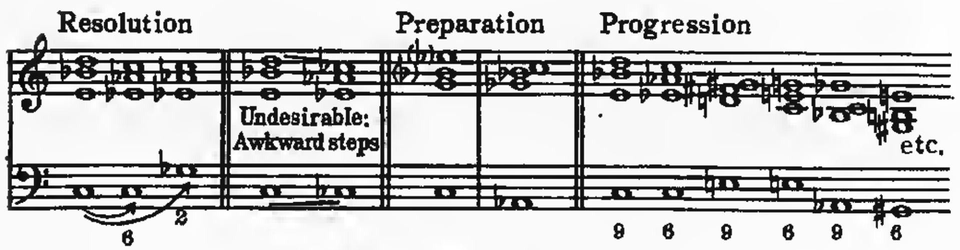


Figure 168. Three tonics.

In order to acquire a complete understanding of voice-leading in the preceding table of progressions (9 — 6 — 9 — 6 etc.), one should construct mentally an S(7) instead of an S(6). Then the first two chords will appear in the following positions:

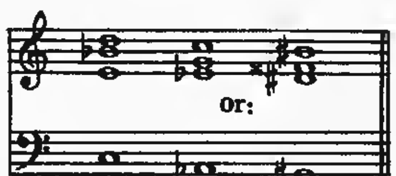


Figure 169. Positions of S(9) to S(6).

It is clear now that d# and f^x are the necessary 7 and 9 of the following chord.

Figure 170. Example of continuity (continued).

Figure 170. Example of continuity (concluded).

Four tonics ($\sqrt[4]{2}$). The technique corresponds to C_3.

Figure 171. Four tonics.

Six tonics ($\sqrt[6]{2}$). The technique corresponds to C_7.

Figure 172. Six tonics $\sqrt[6]{2}$.

The above consecutive sevenths are unavoidable with *this* technique.

The position of every S(9) is based on the assumption that the preceding chord was S(5) and not S(7).

Figure 173. Continuity: S(9) + S(7) + S(5).

The negative system which may be obtained by reading the above tables in position ⓑ is not as desirable with these media as the positive. The same concerns the following $\sqrt[12]{2}$. More plastic devices (general forms of transformations) will be offered later.

Twelve tonics ($\sqrt[12]{2}$). The technique corresponds to C_7.

Figure 174. Twelve tonics $\sqrt[12]{2}$

Figure 175. Continuity: S(9) + S(7) + S(5).*

*In the original manuscript, Schillinger suggests that the implications of this material be studied through the following: "Exercises in the different symmetric systems containing S(5), S(7) and S(9) with application of different structures and the C_0 between the roots." (Ed.)

CHAPTER 11

THE ELEVENTH CHORD

A. S(11) IN THE DIATONIC SYSTEM

IN FOUR-PART harmony, eleventh chords [S(11)] are used with the root-tone *in the bass only*, thus forming a hybrid four-part harmony [like that formed by S(5) with the doubled root]. The three upper parts consist of 7, 9, 11. An S(11) has an advantage over S(9) in that the upper functions form a complete S(5). All three upper functions are subject to resolution through stepwise downward motion. Resolutions of fewer than the three upper functions produce C_0.

No consecutive S(11)'s are possible in *this* particular system. They alternate with the other structures.

For reasons explained in the previous chapter, the C_0 resolutions must follow in the direction of the decreasing functions: if only one is resolved, 11 must be resolved first; then, 9; then, 7. When two functions resolve simultaneously, they are 11 and 9. An S(11) allows a continuous chain of resolutions:

S(11) ↘
S(9) 9 ↘
S(7) 7 ↘
S(6)③

An eleventh-chord through resolution of the eleventh becomes a ninth-chord; a ninth-chord through resolution of the ninth becomes an incomplete seventh-chord (without a fifth), or a complete $S(\frac{4}{3})$ as in the corresponding resolutions of S(9); an incomplete seventh-chord through resolution of the seventh becomes a sixth-chord with doubled third.

1. Positions of S(11)

As the bass remains constant, the three upper voices are subject to six permutations. Seventh, ninth and eleventh form a triad corresponding to a root, a third and a fifth while the bass corresponds to the pitch-unit one degree higher than the root of the triad.

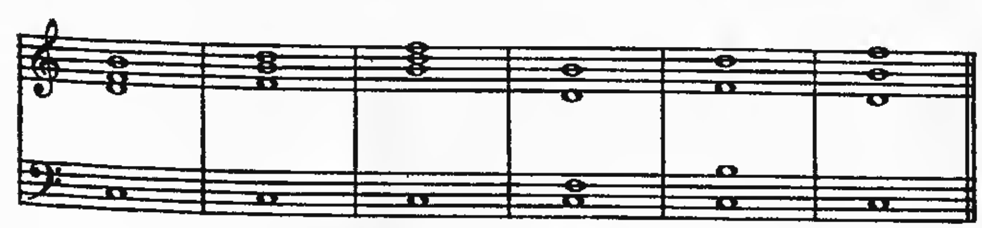


Figure 176. Positions of S(11).

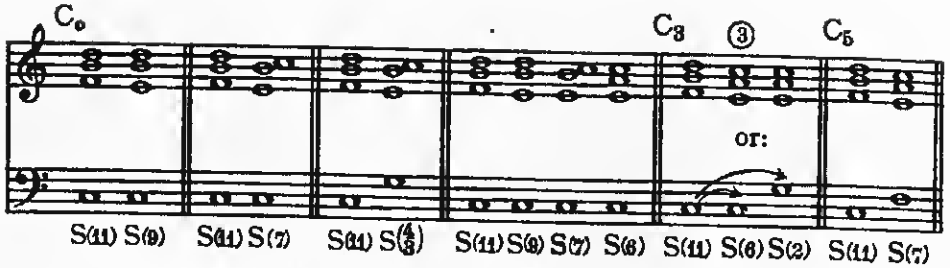


Figure 177. Resolutions of S(11).

As it follows from the above table, when S(11) resolves into S(9) in C_0, S(9) has its proper structural constitution (i.e., 1, 3, 7, 9). The C_7 resolution does not appear on this table for the reason that the structural constitution of S(9) into which S(11) would resolve is 1, 5, 7, 9, and this does not sound satisfactory, according to *our* musical habits.

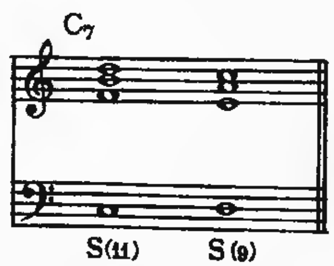


Figure 178. S(11) → S(9).

The above resolutions correspond to the classical resolutions of the triple suspensions.

B. Preparation of S(11)

Preparation of S(11) in the positive cycles has a cyclic correspondence to the preparation of S(7) and S(9) through suspensions. Nevertheless, the manner of reasoning is somewhat different in this case.

As S(11) has an appearance of an S(5) with a bass corresponding to the pitch-unit one degree higher than the root of the triad, the most logical assumption is: take S(5)①, move its bass one step up and this will produce an S(11) with a proper structural constitution. In such a case, the relation of the three stationary upper functions is C_0. The tones being common tones, may be inverted or exchanged.

The first case gives a clue to the preparation of other cycles (positive and negative as well). The method of preparation implies merely the more gradual transformation (↻ or ↺) of the three upper functions.

To prepare S(11) after an S(5) in C_0, move all upper functions down scalewise and leave the bass stationary (which is the converse of the first proposition).

Preparations of S(11)

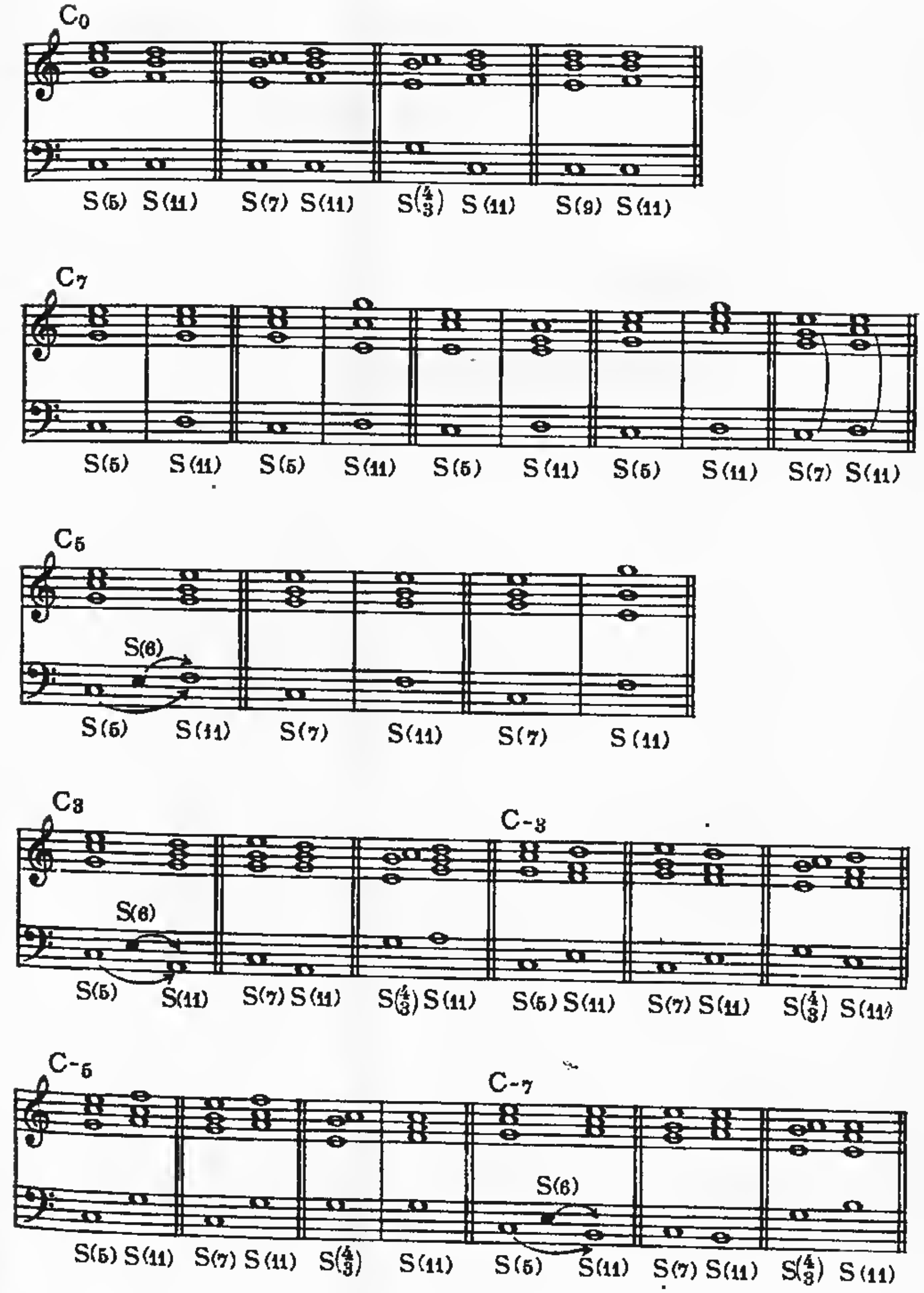


Figure 179. Preparations of S(11).

When all tones are held in common in the three upper parts, it is advisable to use the over-the-bar suspension method. (See page 462.)

When some of the upper parts move and some remain stationary, either the within-the-bar or the over-the-bar preparation may be used.

Characteristic progressions and cadences, in which all forms of tension [from S(5) to S(11)] are applied, would be:

Figure 180. Characteristic progressions and cadences employing S(5) to S(11).

Example of Continuity Containing S(11)

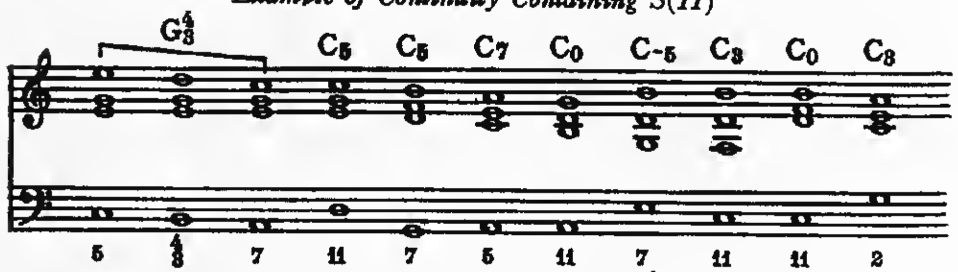


Figure 181. Continuity containing S(11).

C. S(11) IN THE SYMMETRIC SYSTEM

The above technique of diatonic progressions containing S(11) is applicable to the symmetric system as well. The cyclic correspondence previously used remains the same. Thus, preparations of S(11) are possible in all systems of the symmetric roots, whereas resolutions can be performed only when the acting cycle is C_3 ($\sqrt[3]{2}$ and $\sqrt[4]{2}$) and C_6 ($\sqrt{2}$). There is no difficulty with any preparation of S(11) after a resolution, as the latter always consists of 1, 3, 5 and therefore may be connected with the following chord through the usual transformations.

Unlike S(9), S(11) produces a highly satisfactory C_0, due to the presence of all functions without gaps in the three upper parts.

As with the ninth-chords, there are two distinctly different families of S(11) which are not to be mixed, except when in C_0. The distinction becomes even greater than before and the mixing becomes still more "dangerous."

The structural constitution of S(11) permits the classification of such structures as S(5) with regard to their upper functions.

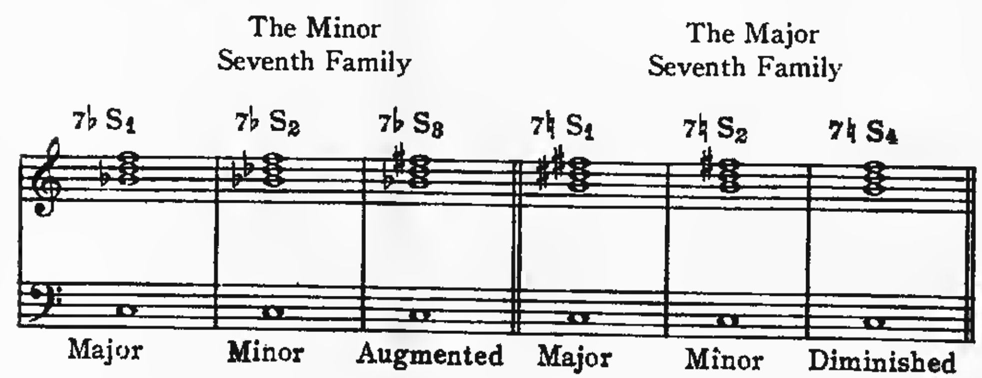


Figure 182. Forms of S(11).

There are two less common forms. The diminished in the first group and the augmented in the second group.

Figure 183. 7♭ S_4 (11); 7♮ S_3 (11).

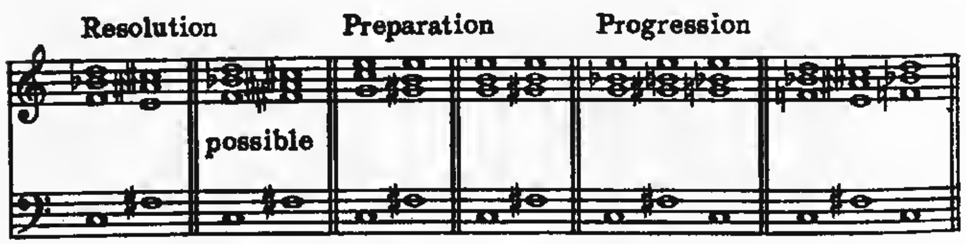
The selection of better progressions in C_0 for the continuity of S(11) must be analogous to the selection of forms for S(5). If desired, consecutive sevenths may be avoided by permutations.

Figure 184. Example of C_0 continuity.

Full indications for S(11) when used in combination with other structures:

$7\flat S_1(11)$; $7\flat S_2(11)$; $7\flat S_3(11)$
$7\natural S_1(11)$; $7\natural S_2(11)$; $7\natural S_3(11)$

Two tonics ($\sqrt{2}$). The technique corresponds to C_5; clockwise or counterclockwise transformations for continuous S(11).



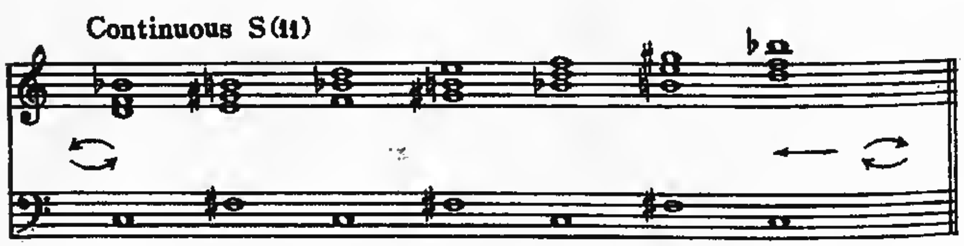


Figure 185. Two tonics $\sqrt{2}$.

You may consider the upper three parts either as 7, 9, 11 in ↻ and ↺ transformations or as 1, 3, 5 with a displaced bass.

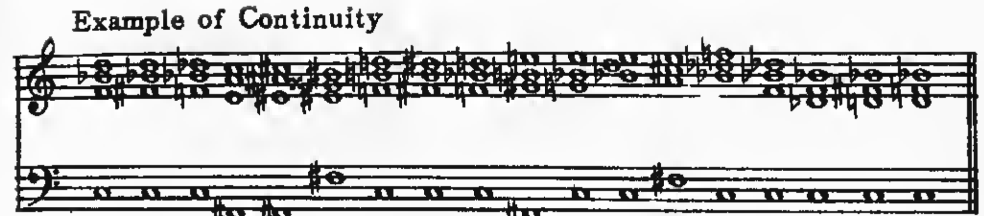


Figure 185. Two tonics (concluded)

Three tonics ($\sqrt[3]{2}$). The technique corresponds to C_3 or to the ↻ and ↺ transformations.

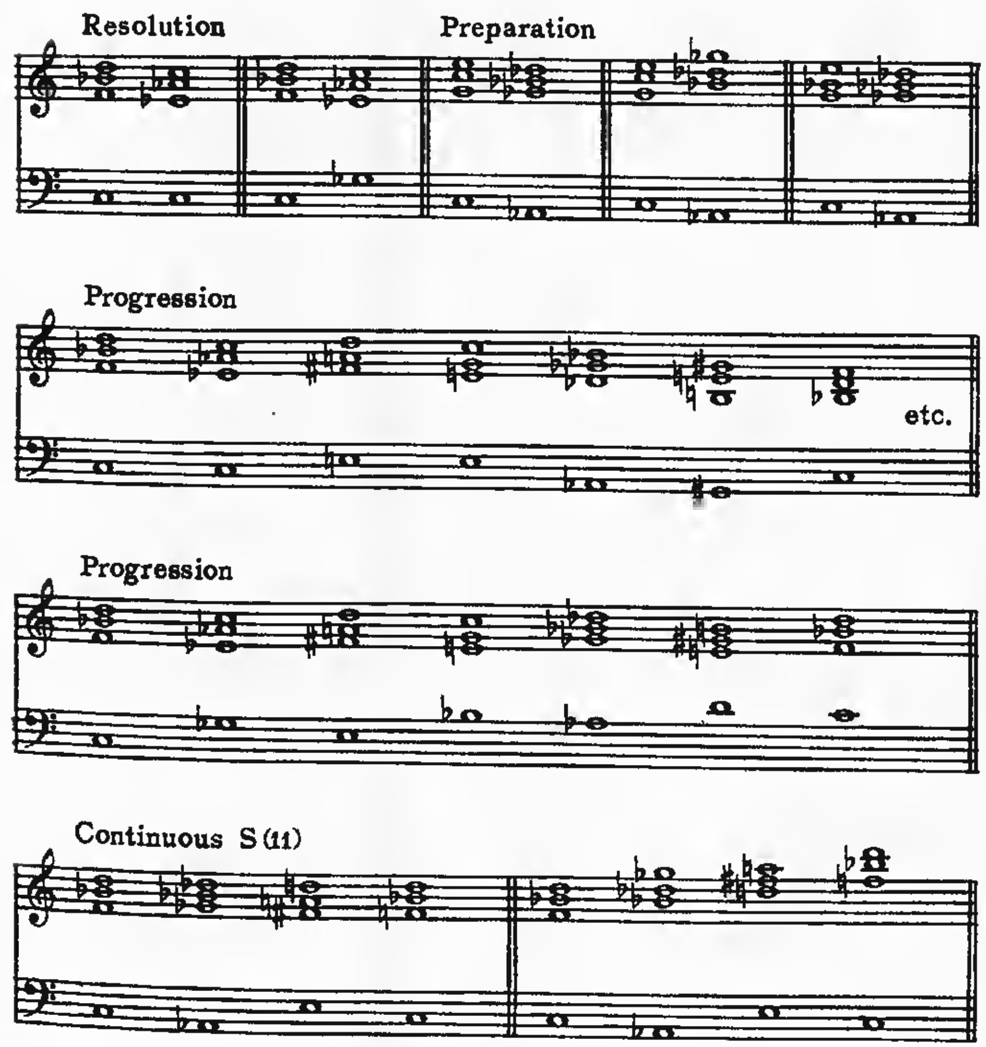
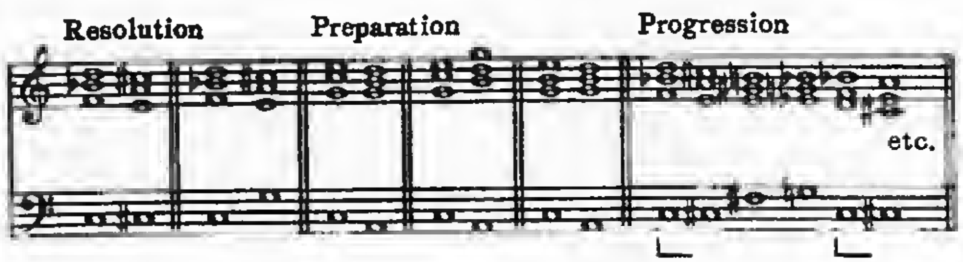


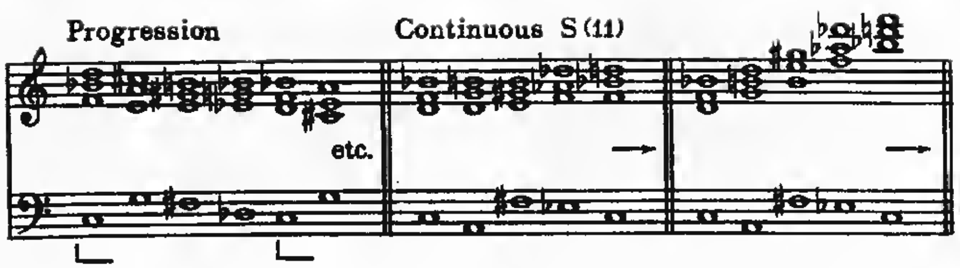
Figure 186. Three tonics $\sqrt[3]{2}$.

Example of Continuity

Figure 186. Three tonics (concluded).

Four tonics ($\sqrt[4]{2}$). The technique corresponds to C_3 or to the ⟳ and ⟲ transformations.





Example of Continuity

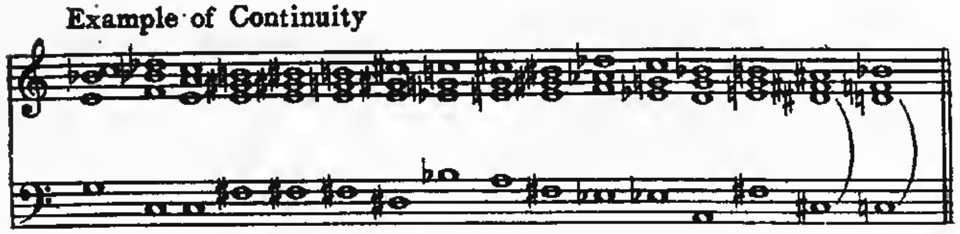


Figure 187. Four tonics $\sqrt[4]{2}$.

With the complexity of the harmony above, the consecutive ninths (if they are both major and move on a whole tone) are perfectly admissible.

Six tonics ($\sqrt[6]{2}$). Use ⟳ and ⟲ transformations only.

Continuous S(11)

Example of Continuity

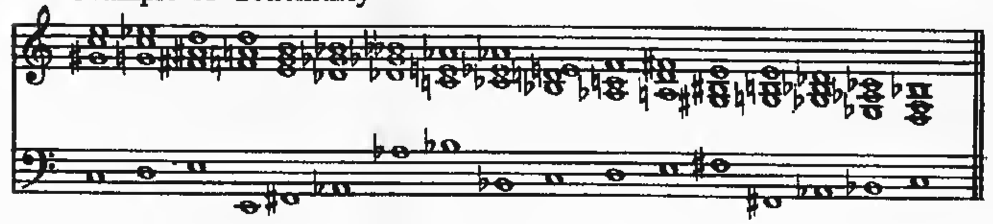
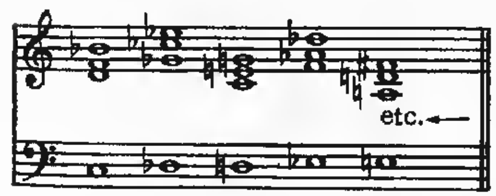


Figure 188. Six tonics.

Twelve tonics ($\sqrt[12]{2}$). Use ⟳ and ⟲ transformations only.

Continuous S(11)



Example of Continuity

Figure 189. Twelve tonics $\sqrt[12]{2}$ *.

*In the original manuscript, Schillinger suggests that the following work be done: "As with S(9), utilize various structures, forms and progressions on S(11). The transformation technique is applicable to diatonic and diatonic-symmetric progressions as well." (Ed.)

D. In Hybrid Four-Part Harmony

The general technique of transformations for groups with three functions may now be adopted for a generalization of the forms of voice-leading in hybrid four-part harmony. The three upper parts perform the transformations corresponding to the groups with three functions, and the bass remains constant.

The following technique is applicable to any type of harmonic progression: diatonic, diatonic-symmetric, or symmetric. The specifications for the following forms of S are chosen *with respect to their sonority*. Those marked with an asterisk in the following tables are less commonly used than the unmarked ones. The charts of transformations for the latter are worked out; the reader may easily substitute them for those marked with the asterisk.

Forms of Hybrid Four-Part (3 + 1) Harmony

The Three upper parts.	5 3 1	5 3 13	7 5 3	7 3 1	9 7 3	9 7 1	11 9 7	13 9 7	13 11 7
The bass.	1	1	1	1	1	1	1	1	1
Forms of tension.	S(5)	* S(5)	S(7)	* S(7)	S(9)	* S(9)	S(11)	S(13)	* S(13)

Figure 190. Forms of hybrid four-part harmony.

When the numerals expressing the functions in a group are identical with the numerals of the succeeding group, certain forms of transformation—such as constant abc—may be eliminated because of their complete parallelism. When the numerals in the two allied groups are partly identical, some of the forms (constant a, constant b, constant c) give either favorable or unfavorable partial parallelisms. The partial parallelisms are favorable when the parallel motion forms desirable intervals with the bass. They are unfavorable when the motion causes a consecutive motion of the seventh or ninth with the bass (consecutive seventh, consecutive ninth).

Inasmuch as the actual *quality* of voice-leading depends on the structures of the two allied chords, the student will be able—upon completion of all these charts in musical notation—to make his own preferential selection.

When the numerals in the two allied groups are either partly or totally different, often the *constant abc* transformation becomes the most favorable form of voice-leading. There is a natural compensation at work in this case. *Homogeneous* structures are compensated by *heterogeneous* transformations—and heterogeneous structures are compensated by homogeneous transformations. For example, if the allied groups are both S(5), the constant abc transformation

would be unconventional: 1 → 1, 3 → 3, 5 → 5, which gives consecutive octaves and fifths. On the contrary, when the functions have different numerals, the smoothest voice-leading results from this particular transformation.

When two allied groups have different or partly different numerals for their functions, the first group becomes the *original* group and the succeeding group beomes the *prime* group. When a transformation between two such groups is performed, the prime group in turn becomes the original group for the next transformation.

The Original Group	The Prime Group
a c b	a^1 c^1 b^1

For example, by connecting S(5) + S(9) + S(13) we obtain the following numerals in their corresponding order:

S(5)	S(9)	S(13)
1 5 3	3 9 7	7 13 9

When the functions of S(5) are connected to the functions of S(9), the first group is the original group; the second is the prime group. When the functions of S(9) are connected to S(13), the functions of S(9) form the original group, and the functions of S(13) form the prime group.

Here is a complete table of transformations.

Forms of Transformations in the Homogeneous Groups

		Const. a	Const. b	Const. c	Const. abc
a c b	a c b	a c b	a c b	a c b	a c b
a → b b → c c → a	a → c c → b b → a	a → a b → c c → b	a → c b → b c → a	a → b b → a c → c	a → a b → b c → c

Figure 191. Transformations in the homogeneous groups.

Forms of Transformations in the Heterogeneous Groups

The Original Group.	The Prime Group.
a c b	a^1 c^1 b^1

↻	↺	Const. a	Const. b	Const. c	Const. abc
$a \rightarrow b^1$ $b \rightarrow c^1$ $c \rightarrow a^1$	$a \rightarrow c^1$ $c \rightarrow b^1$ $b \rightarrow a^1$	$a \rightarrow a^1$ $b \rightarrow c^1$ $c \rightarrow b^1$	$a \rightarrow c^1$ $b \rightarrow b^1$ $c \rightarrow a^1$	$a \rightarrow b^1$ $b \rightarrow a^1$ $c \rightarrow c^1$	$a \rightarrow a^1$ $b \rightarrow b^1$ $c \rightarrow c^1$

Figure 192. Transformations in the heterogeneous groups.

Here are all the combinations for the two allied groups, applied to all forms of tension.

Binomial Combinations of the Original and the Prime Groups

S(5) ↔ S(7)	S(7) ↔ S(9)	S(9) ↔ S(11)	S(11) ↔ S(13)
S(5) ↔ S(9)	S(7) ↔ S(11)	S(9) ↔ S(13)	
S(5) ↔ S(11)	S(7) ↔ S(13)		
S(5) ↔ S(13)			

10 Combinations, 2 permutations each.
Total number of cases: $10 \times 2 = 20$.

Figure 193. Binomial combinations.

The following pages contain tables of transformations for the 20 binomials consisting of one original and one prime group. Each S tension is represented in this table by one structure only. The sequence of the forms of transformations in this table remains the same for all cases: (1) ↻; (2) ↺; (3) Const. a; (4) Const. b; (5) Const. c; (6) Const. abc.

1. Table of transformations for the twenty binomials.

		S(5) ⟶ S(7)			
1 → 5 3 → 7 5 → 3	1 → 7 3 → 3 5 → 5	1 → 3 3 → 7 5 → 5	1 → 7 3 → 5 5 → 3	1 → 5 3 → 3 5 → 7	1 → 3 3 → 5 5 → 7

		S(7) ⟶ S(5)			
3 → 3 5 → 5 7 → 1	3 → 5 5 → 1 7 → 3	3 → 1 5 → 5 7 → 3	3 → 5 5 → 3 7 → 1	3 → 3 5 → 1 7 → 5	3 → 1 5 → 3 7 → 5

		S(5) ⟶ S(9)			
1 → 7 3 → 9 5 → 3	1 → 9 3 → 3 5 → 7	1 → 3 3 → 9 5 → 7	1 → 9 3 → 7 5 → 3	1 → 7 3 → 3 5 → 9	1 → 3 3 → 7 5 → 9

		S(9) ⟶ S(5)			
3 → 3 7 → 5 9 → 1	3 → 5 7 → 1 9 → 3	3 → 1 7 → 5 9 → 3	3 → 5 7 → 3 9 → 1	3 → 3 7 → 1 9 → 5	3 → 1 7 → 3 9 → 5

Figure 194. Transformations of binomial combinations.

		S(5) ⟶ S(11)			
1 → 9 3 → 11 5 → 7	1 → 11 3 → 7 5 → 9	1 → 7 3 → 11 5 → 9	1 → 11 3 → 9 5 → 7	1 → 9 3 → 7 5 → 11	1 → 7 3 — 9 5 → 11

		S(11) ⟶ S(5)			
7 → 3 9 → 5 11 → 1	7 → 5 9 → 1 11 → 3	7 → 1 9 → 5 11 → 3	7 → 5 9 → 3 11 → 1	7 → 3 9 → 1 11 → 5	7 → 1 9 → 3 11 → 5

		S(5) ⟶ S(13)			
1 → 9 3 → 13 5 → 7	1 → 13 3 → 7 5 → 9	1 → 7 3 → 13 5 → 9	1 → 13 3 → 9 5 → 7	1 → 9 3 → 7 5 → 13	1 → 7 3 → 9 5 → 13

		S(13) ⟶ S(5)			
7 → 3 9 → 5 13 → 1	7 → 5 9 → 1 13 → 3	7 → 1 9 → 5 13 → 3	7 → 5 9 → 3 13 → 1	7 → 3 9 → 1 13 → 5	7 → 1 9 → 3 13 → 5

Figure 195. Transformations of binomial combinations.

		S(7) ⟶ S(9)			
3 → 7 5 → 9 7 → 3	3 → 9 5 → 3 7 → 7	3 → 3 5 → 9 7 → 7	3 → 9 5 → 7 7 → 3	3 → 7 5 → 3 7 → 9	3 → 3 5 → 7 7 → 9

		S(9) ⟶ S(7)			
3 → 5 7 → 7 9 → 3	3 → 7 7 → 3 9 → 5	3 → 3 7 → 7 9 → 5	3 → 7 7 → 5 9 → 3	3 → 5 7 → 3 9 → 7	3 → 3 7 → 5 9 → 7

		S(7) ⟶ S(11)			
3 → 9 5 → 11 7 → 7	3 → 11 5 → 7 7 → 9	3 → 7 5 → 11 7 → 9	3 → 11 5 → 9 7 → 7	3 → 9 5 → 7 7 → 11	3 → 7 5 → 9 7 → 11

		S(11) ⟶ S(7)			
7 → 5 9 → 7 11 → 3	7 → 7 9 → 3 11 → 5	7 → 3 9 → 7 11 → 5	7 → 7 9 → 5 11 → 3	7 → 5 9 → 3 11 → 7	7 → 3 9 → 5 11 → 7

Figure 196. Transformations of binomial combinations.

		S(7) ⟶ S(13)			
3 → 9	3 → 13	3 → 7	3 → 13	3 → 9	3 → 7
5 → 13	5 → 7	5 → 13	5 → 9	5 → 7	5 → 9
7 → 7	7 → 9	7 → 9	7 → 7	7 → 13	7 → 13

		S(13) ⟶ S(7)			
7 → 5	7 → 7	7 → 3	7 → 7	7 → 5	7 → 3
9 → 7	9 → 3	9 → 7	9 → 5	9 → 3	9 → 5
13 → 3	13 → 5	13 → 5	13 → 3	13 → 7	13 → 7

		S(9) ⟶ S(11)			
3 → 9	3 → 11	3 → 7	3 → 11	3 → 9	3 → 7
7 → 11	7 → 7	7 → 11	7 → 9	7 → 7	7 → 9
9 → 7	9 → 9	9 → 9	9 → 7	9 → 11	9 → 11

		S(11) ⟶ S(9)			
7 → 7	7 → 9	7 → 3	7 → 9	7 → 7	7 → 3
9 → 9	9 → 3	9 → 9	9 → 7	9 → 3	9 → 7
11 → 3	11 → 7	11 → 7	11 → 3	11 → 9	11 → 9

Figure 197. Transformations of binomial combinations.

		S(9) ⟶ S(13)			
3 → 9	3 → 13	3 → 7	3 → 13	3 → 9	3 → 7
7 → 13	7 → 7	7 → 13	7 → 9	7 → 7	7 → 9
9 → 7	9 → 9	9 → 9	9 → 7	9 → 13	9 → 13

		S(13) ⟶ S(9)			
7 → 7	7 → 9	7 → 3	7 → 9	7 → 7	7 → 3
9 → 9	9 → 3	9 → 9	9 → 7	9 → 3	9 → 7
13 → 3	13 → 7	13 → 7	13 → 3	13 → 9	13 → 9

		S(11) ⟶ S(13)			
7 → 9	7 → 13	7 → 7	7 → 13	7 → 9	7 → 7
9 → 13	9 → 7	9 → 13	9 → 9	9 → 7	9 → 9
11 → 7	11 → 9	11 → 9	11 → 7	11 → 13	11 → 13

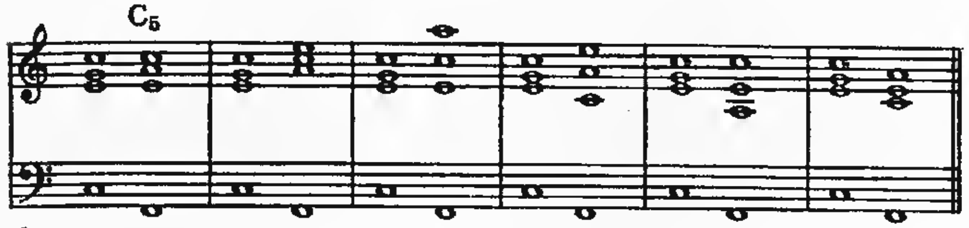
		S(13) ⟶ S(11)			
7 → 9	7 → 11	7 → 7	7 → 11	7 → 9	7 → 7
9 → 11	9 → 7	9 → 11	9 → 9	9 → 7	9 → 9
13 → 7	13 → 9	13 → 9	13 → 7	13 → 11	13 → 11

Figure 198. Transformations of binomial combinations.

S(5) ⟶ S(7)

C_3

C_5

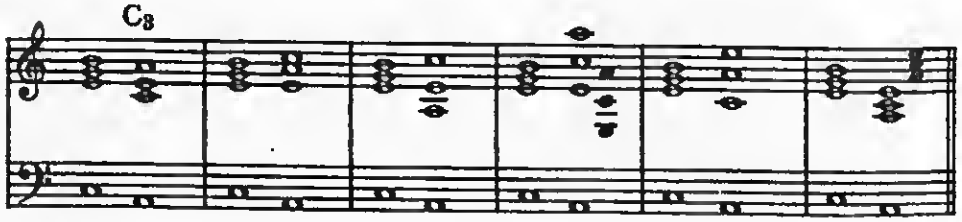


C_7

Figure 199. S(5) ⟶ S(7).

S(7) ⟶ S(5)

C_3



C_5

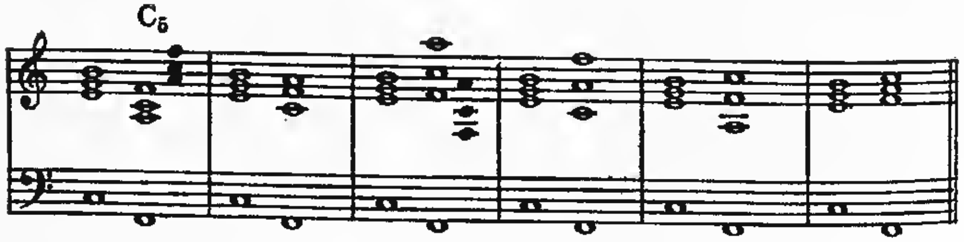


Figure 200. S(7) → S(5) (continued).

C_7

Figure 200. S(7) → S(5) (concluded).

S(5) ⟶ S(9)

C_3

C_5

C_7

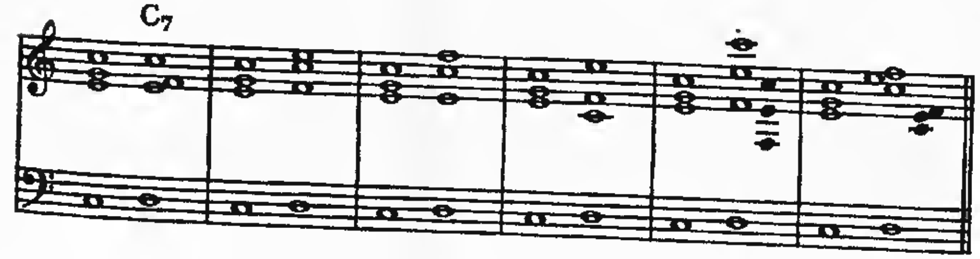


*Figure 201. S(5) ⟶ S(9)**

*In the original manuscript, Schillinger suggests that the student make additional tables for: S(5) → S(5); S(7) → S(7); S(9) → S(9); S(11) → S(11); S(13) → S(13). (Ed.)

It is easy to work out all cases in musical notation by applying each case to all three tonal cycles.

As in previous cases, continuity may be composed in all three types of harmony (diatonic, diatonic-symmetric and symmetric). Structures of different tension may be selected for the composition of continuity. Different individual styles depend upon the coefficients of recurrence applied to structures of differing tensions.

The first of the following two examples of continuity is produced through structures of constant form and tension [S(13)]; the second illustrates a continuity of variable forms and variable tensions distributed through $r_{3\div2}$.

Continuity of Groups with Identical Functions

S(13) → Type II. Scale: b♭-harm., d_1.

Figure 202. Structures of constant form and tension S(13).

Continuity of Groups with Different Functions

2S(9) + S(7) + S(11) + S(13) + 2S(11); Type III. $\sqrt[4]{2}$

Figure 203. Variable forms and tensions through $r_{3\div2}$.

CHAPTER 12

GENERALIZATION OF SYMMETRIC PROGRESSIONS

THE forms of symmetric progressions heretofore used in this portion of my discussion of harmony were based on a ***monomial*** symmetry of the uniform intervals of an octave.

But in order to obtain other mixtures (binomials, trinomials and polynomials) of the original forms of symmetry within an octave, it is necessary to establish a general nomenclature for all intervals of an octave. As all intervals are special cases of the twelve-fold symmetry, any diatonic form may be considered a special case of symmetry as well.

The system of enumeration of intervals may follow the upward or downward direction from any established axis point. As both directions include *all* intervals (which means both positive and negative tonal cycles), the matter of preference must be determined by the quantitative predominance of the type of intervals generally used. It seems that the descending system is the more practical, for smaller numbers can then be used to express the positive steps on three and four tonics; the negative, on six and twelve tonics.

In the following exposition, the descending system will be used exclusively. This need not prevent one from using the ascending system.

Scales of Intervals within one Octave Range:

Descending System:	*Ascending System:*
c → c = 0	c → c = 0
c → b = 1	c → d♭ = 1
c → b♭ = 2	c → d = 2
c → a = 3	c → e♭ = 3
c → a♭ = 4	c → e = 4
c → g = 5	c → f = 5
c → f♯ = 6	c → f♯ = 6
c → f = 7	c → g = 7
c → e = 8	c → a♭ = 8
c → e♭ = 9	c → a = 9
c → d = 10	c → b♭ = 10
c → d♭ = 11	c → b = 11
c → c_1 = 12	c → c^1 = 12

Monomials

Two Tonics:	6 + 6
Three Tonics:	4 + 4 + 4 or 8 + 8 + 8
Four Tonics:	3 + 3 + 3 + 3 or 9 + 9 + 9 + 9
Six Tonics:	2 + 2 + 2 + 2 + 2 + 2 or 10 + 10 + 10 + 10 + 10 + 10
Twelve Tonics:	1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1
or	11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11

Figure 204. Intervals and tonics within one octave.

So approached, each constant system of tonics becomes a form of monomial periodicity of a certain pitch-interval, expressible in the form of a constant number-value, which in turn expresses the quantity of semitones from the preceding pitch-unit.

In the framework of this system, the problem of mixing various tonics (or any interval-steps in general) becomes reduced to the process of composing binomials, trinomials or any more extended groups (such as rhythmic resultants, their modifications through permutations and powers, series of growth), i.e., to the rhythmic distribution of steps.

The vitality of such groups, i.e., the periodicity of their recurrence until the completion of their cycle, depends upon the divisibility-properties of the sums of their interval-quantities. The total sum of all number-values expressing the intervals becomes a divisor of 12, or any multiple thereof. This signifies the motion of a certain group through an octave (or octaves).

For example, a binomial 3 + 2 has 12 recurrences until it completes its cycle, as 3 + 2 = 5, and the smallest multiple of 12, divisible by 5 is 60. This is true of all prime numbers when used as divisors.

C – A – G – E – D – B – A – F♯ – E – C♯ – B

B – G♯ – F♯ – D♯ – C♯ – A♯ – G♯ – F – E♭

E♭ – C – B♭ – G – F – D – C

Figure 205. Binomial 3 + 2.

This property makes mixtures of three and four tonics very desirable when a long harmonic span is necessary without a variety of steps.

The process of division serves as a testing tool of the vitality of compound symmetric groups.

Two tonics close after two cycles, as 6 + 6 = 12, or $\frac{12}{6} = 2$;

$r_{4\div3}$ closes after one cycle, as 3 + 1 + 2 + 2 + 1 + 3 = 12, and $\frac{12}{12} = 1$;

$r_{5\div4}$ closes after three cycles, as 4 + 1 + 3 + 2 + 2 + 3 + 1 + 4 = 20, and $\frac{60}{20} = 3$.

Greater variety without deviating from a given style may be achieved by means of permutations of the members of a group. For example, a group with a short span may be revitalized through permutations:

(3+1+2) + (3+2+1) + (2+3+1) + (1+3+2) + (1+2+3) + (2+1+3)

or: C – A – G♯ – F♯ – E♭ – D♭ – C – B♭ – G – F♯ – F♮ – D – C

C – B♮ – A – F♯ – E♮ – E♭ – C

Figure 206. Permutating a group with a short span.

The selection of number values is left to the composer's discretion; if he wants to obtain the tonic-dominant character of classical music, the only thing he needs is an excess of the value 5.

Anyone equipped with this method can dodge extremities of style by a cautious selection of the coefficients of recurrence. For instance, in order to produce that style of progressions which lies somewhere between Wagner and Ravel, it is necessary to have the 5, the 3, and the 10 in a certain proportion—such as: 2_{3+5+10}, i.e.,

C – A – F♯ – C♯ – D♯ – C – A – E – F♯, etc.

Naturally, selection of the *tensions* and of the *forms* of structures in definite proportions is as important as selection of the forms of progressions when a certain definite style must be produced.

On the other hand, this method offers a fascinating pastime, as one can produce chord progressions from any number combinations. Thus, a telephone directory becomes a source of inspiration.

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5 + 7 + 5 + 7 + 3 is equivalent to
C – G – C – G – C – A.

This progression closes after 4 cycles:

C – G – C – G – C – A – E – A – E – A – F♯ –

F♯ – C♯ – F♯ – C♯ – F♯ – D♯ – A♯ – D♯ – A♯ – D♯ – C

Figure 207. Chord progressions from a telephone number.

When zeros occur in a number-combination, they represent zero-steps, i.e., zero cycles (C_0). Then the form of tension, the structure, or the position of a chord has to be changed.

Example of Continuity:

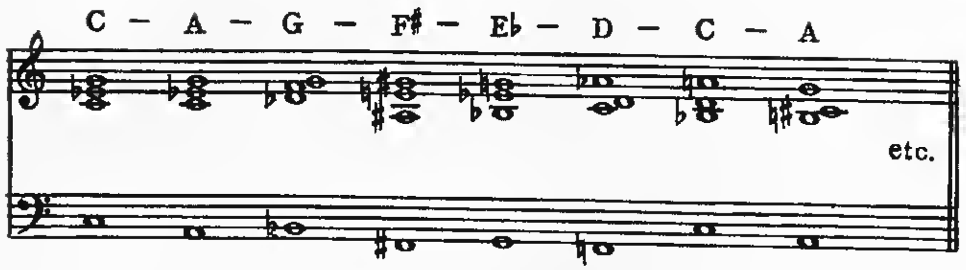


Figure 208. Progression: $r_{5\div3}$.

A. Generalized Symmetric Progressions as Applied to Modulation Problems

The *rhythm of chord progressions* expressed in number-values may serve the purpose of transition from one key to another. This procedure can be approached in two ways: (1) as a problem of connecting the tonic chords of the preceding and the following key; or (2) as a problem of connecting any chord of the preceding key to any chord of the following key. The last case requires movement through diatonic cycles in both the preceding and the succeeding keys.

The technique of performing such modulations, based on the rhythm of symmetric progressions, consists of two steps:

(1) detection of the number-value expressing the interval between the two chords, where such connection must be established;

(2) composition of a rhythmic group from the numeral expressing the interval between the above-mentioned chords. For example, if one wants to perform a modulation by means of symmetric progressions from the chord C (which may or may not be in the key of C) to the chord E♭ (which may or may not be in the key of E♭), the first procedure to perform is to compose rhythm from the interval 9. The techniques set forth in the *Theory of Rhythm** offer many ways of composing such groups: composition of binomials, trinomials or larger groups from the original number, or any permutation thereof.

The number of terms in a group will define the number of chords for the modulatory transition. Breaking up number 9 into binomials, we obtain: $8 + 1$, $7 + 2$, $6 + 3$, $5 + 4$, and their reciprocals. When a binomial is used in this sense, the two chords are connected through one intermediate chord. For example, taking $5 + 4$ we acquire: C – G – E♭. If more chords are desired, any other rhythmic group may be devised from number 9. For example, $4 + 1 + 4$, which will give C – A♭ – G – E♭, i.e., two intermediate chords.

*See Book I. (Ed.)

When a number-value expressing the interval between the two chords to be connected through modulation is a *small* number, it is necessary to add the invariant 12. This places the same pitch-unit (or the root of the chord) in a different octave without changing its intonation. For example, if a modulation from a chord of C to the chord of B♭ is required, such an addition becomes very desirable.

C → B♭ = 2
B♭ → B_1♭ = 12
$12 + 2 = 14$

Some rhythms derived from the value 14:

$7 + 7$ = C – F – B♭
$5 + 2 + 2 + 5$ = C – G – F – E♭ – B♭

In cases such as this, rhythmic resultants may be used as well, providing the necessary changes are made.

$r_{4\div3} = 3 + 1 + 2 + 2 + 1 + 3$

Readjustment:

$3 + 1 + 2 + 2 + 1 + 3 + 2$ = C – A – A♭ – F♯ – F♭ – E♭ – C – B♭
Or:
$r_{5\div3} = 3 + 2 + 1 + 3 + 1 + 2 + 3$

Readjustment:

$3 + 2 + 1 + 2 + 1 + 2 + 3$ = C – A – G – F♯ – E – E♭ – D♭ – B♭

All these procedures guarantee the appearance of the desirable B♭ point.

When a modulation of still greater extension is required, the invariant of addition becomes 24 or 36—or even a higher multiple of 12—from which rhythmic groups may be composed.

Many persons engaged in the work of "arranging" find this type of transition more effective than the modulations ordinarily used. Naturally, selection of structures of different tension and form may be made according to the requirements of the general style of harmony used in a particular arrangement. The best modulations will result from use of that symmetry which may be detected in any given piece of music.

Even when the tonic-dominant progression is characteristic of harmonic continuity, this method may be used with success—it simply requires the composition of a rhythmic group in which the original value is 5. In this seemingly limited case there is still a choice of steps: $4 + 1$; $3 + 2$; $2 + 3$; $1 + 4$.

Examples of Modulations
Through Symmetric Groups

(1) Key of C to Key of E♭; i = 9

Symmetric Group: 1 + 3 + 1 + 3 + 1 (r_3 of $\frac{9}{3}$ series)

Figure 209. Modulation through symmetric group: C → E♭.

(2) Key of C to Key of E♭

Chords to be Connected: D——B♭; i = 4; 4 + 12 = 16

Symmetric Group: $\underline{r_{4\div3}}$ = 3 + 1 + 2 + 1 + 1 + 1 + 1 + 2 + 1 + 3

Figure 210. Modulation through symmetric group: D → B♭.

CHAPTER 13

THE CHROMATIC SYSTEM OF HARMONY

THE basis of the chromatic is: *transformation of diatonic chordal functions* into *chromatic* chordal functions and back into *diatonic*. Chromatic continuity evolved on this basis emphasizes various phenomena of harmony which do *not* confine themselves to diatonic or symmetric systems. What are usually known as "modulations" are simply a special case of the whole chromatic system. Chord progressions usually called "alien chord progressions" find their exhaustive explanation in the chromatic system.

Wagner was the first composer to manipulate intuitively this type of harmonic continuity. Not having any basic theoretical principle for handling such progressions, Wagner often wrote them in an enharmonically confusing way. (Note, also, that J. S. Bach made an unsuccessful attempt to move in chromatic systems; see *The Well-Tempered Clavichord*, Vol. 1, Fugue 6, measure 16). It is necessary, for analytical purposes, to rewrite such music in its proper notation, i.e., chromatically rather than enharmonically. A more consistent *notation* of chromatic continuity may be found among such *followers* of Wagnerian harmony as Borodin and Rimsky-Korsakov.

The chromatic system of harmonic continuity is based on progressions of *chromatic groups*. Every chromatic group consists of *three* chords which express the three stages of the following mechanical process: balance—tension—release. These three moments correspond to the diatonic-chromatic-diatonic transformation.

A chromatic group may consist of one or more simultaneous *operations*. Such operations are alterations of diatonic tones into chromatic tones, by raising or lowering them. The initial diatonic tone of a chromatic group and the next alteration have the same *name*, but the ensuing release results in a pitch-unit of a new name.

The two general forms of chromatic operations are these:

(1) x ↗ x♯ ↗ y

(2) x ↘ x♭ ↘ y

In their application to musical names these general forms may become, for instance, g — g♯ — a or g — g♭ — f. Such steps are always semitones. At each such moment of release in a chromatic group a new chordal function (and, in some cases, the same) becomes the starting point of the next chromatic group; thus the whole evolves into an infinite chromatic continuity.

Such a continuity has the following appearance:

d – ch – d
d – ch – d , etc.
d – ch – d

Chromatic continuity in such a form offers a very practical measure distribution, for it permits one to place two chords in each measure. Such a distribution places the release of tension on the downbeat and this sounds satisfactory to our ears, probably because we have acquired the habit of hearing them in such a distribution.

As in diatonic progressions it is the commonness of tones or the resolution of chordal functions—or as in symmetric progressions, it is the symmetric roots—which become the controls of motion, so in chromatic progressions such stimuli are the chromatic alterations of the diatonic tones.

In addition to the form of continuity of chromatic groups presented in the preceding diagram, two other forms are possible. The latter do not necessarily require the technique of the chromatic system. The first of these additional forms of continuity produces an overlapping over one term:

(1) d – ch – d
d – ch – d
d – ch – d

Note that the second part produces the first term of a chromatic group, while the first one produces the second term.

(2) d – ch – d
d – ch – d

Note that two or more parts of harmony coincide in their transformation in time, although the form of transformation may be different in each part.

Any chord acquiring a chromatic alteration becomes more "intense" than the same chord without such alteration. If the middle term of a chromatic group has to be intensified, the following forms of tension may constitute a chromatic group:

S(5)	S(7)	S(5)
S(5)	S(7)	S(7)
S(7)	S(7)	S(5)
S(7)	S(7)	S(7)

The only combination which is undesirable—it produces an effect of weakness—is that in which the middle term is S(5).

Operations in a given chromatic group correspond to a group of chordal functions which may be assigned to any form of alterations. For technical reasons 4-part harmony is here limited to S(5) and S(7) forms with their inversions; so all transformations of functions in the chromatic group deal with the four lower functions, 9, 11 and 13 being excluded.

Numerical Table of Transformations for the Chromatic Groups.*

1–1–1	3–3–3	5–5–5	7–7–7
1–1–3 1–3–1 3–1–1	3–3–1 3–1–3 1–3–3	5–5–1 5–1–5 1–5–5	7–7–1 7–1–7 1–7–7
1–1–5 1–5–1 5–1–1	3–3–5 3–5–3 5–3–3	5–5–3 5–3–5 3–5–5	7–7–3 7–3–7 3–7–7
1–1–7 1–7–1 7–1–1	3–3–7 3–7–3 7–3–3	5–5–7 5–7–5 7–5–5	7–7–5 7–5–7 5–7–7
1–3–5 1–5–3 5–1–3 3–1–5 3–5–1 5–3–1	1–3–7 1–7–3 7–1–3 3–1–7 3–7–1 7–3–1	1–5–7 1–7–5 7–1–5 5–1–7 5–7–1 7–5–1	3–5–7 3–7–5 7–3–5 5–3–7 5–7–3 7–5–3

Figure 211. Transformations for chromatic groups.

Some of these combinations must be excluded because of the adherence of the seventh to the classical system of voice-leading, the descending resolution.

The preceding table offers 16 different versions for each starting function (1, 3, 5, 7). In addition to this, any middle chord of a chromatic group may assume one of the seven forms of S(7); any of the last chords of a chromatic group may have either one of four forms of S(5) or one of seven forms of S(7).

*The table is to be read this way: "1 - 3 - 5" means, for example, that a tone—let us say, C of the major triad, C-E-G—selected for chromatic alteration will be the 1 (root) of the first of the three chords in the chromatic groups; it will be, when altered (say, C#), the 3 of the second chord; and it will be, when the alteration is completed (say, D), the 5 of the third chord. (Ed.)

Thus, each starting point offers either 28 or 49 forms. The total number of starting points for one function equals 16. These quantities must be multiplied by 16 in order to show the total number of cases.

$$28 \times 16 = 448$$
$$49 \times 16 = 784$$

This applies to one initial function only, and, as any group may start with any of the four functions, the total quantity is $4(784 + 448) = 4{,}928$. A number of these cases eventually exclude themselves because of the above-mentioned limitation imposed by the tradition of voice-leading.

Actual realization of chromatic groups must be accomplished on what we may call the two fundamental bases: the major and the minor. This concept of *harmonic basis* refers to any *three adjacent chordal functions*, such as:

5	7	9	11	13
3	5	7	9	11
1	3	5	7	9

Owing to practical limitations this section of my discussion of harmony will deal with the first ($\begin{smallmatrix}5\\3\\1\end{smallmatrix}$) basis only.* The terms major and minor correspond to the structural constitution in the usual sense: major = 4 + 3, and minor = 3 + 4. All fundamental chromatic operations are derived from these two bases.

Major Basis	Minor Basis
1♯	1♭
3♭	3♯
5♯	5♭

These six forms of chromatic operations (3 on each basis) are used independently. Chromatic operations available from the major basis are: raising of the root-tone; lowering of the third; raising of the fifth. Note that they are the opposite of those of the minor basis.

*But observe that in dealing with an S(9), S(11), or S(13) in cases in which any three tones form a triad, the chromatic operation may be performed on these three tones *as if* they were a single triad rather than part of a larger structure. (Ed.)

Examples of Chromatic Groups: One Operation

Table of Transformations.

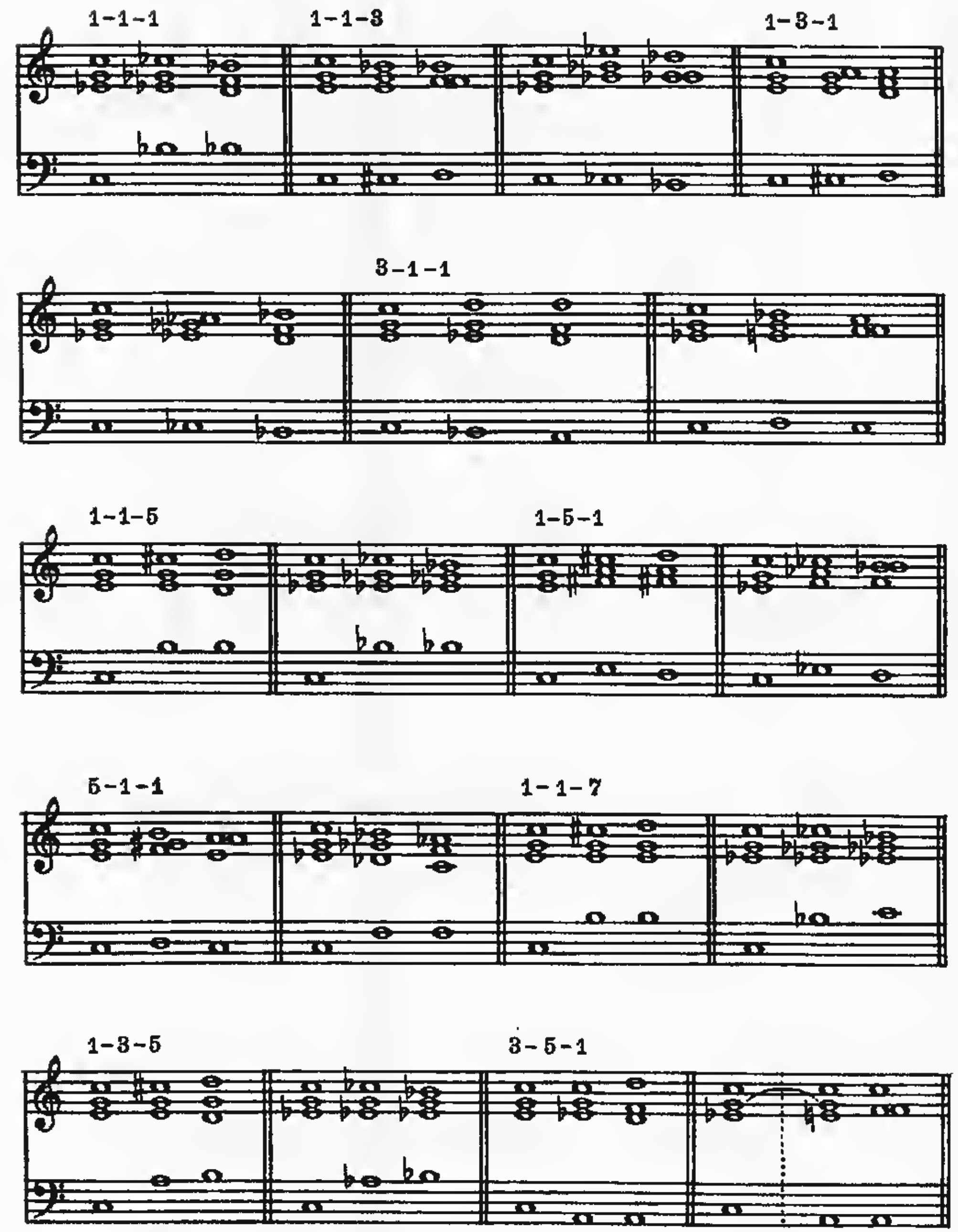


Figure 212. One operation transformations (continued).

Figure 212. One operation transformations (concluded).

The reader will wish to try to find the remaining cases through the table of transformations of the chordal functions, remembering that the classical system of voice-leading (so far as special harmony is concerned) must be carried out through the chromatic continuity: a seventh either descends or remains (as in traditional cadences); it may even go up one semitone because of the chord structure, yet it ***must retain its original name***—as in d – d# in the last case.

Through selection of different chromatic groups (which may be used, of course, with coefficients of recurrence) a chromatic continuity may be composed.

From the explanation offered thus far, it will be obvious that every last chord of the preceding group (and therefore the first chord of the following group) must be of major or minor basis. Operations from other bases will be explained in the following lesson.

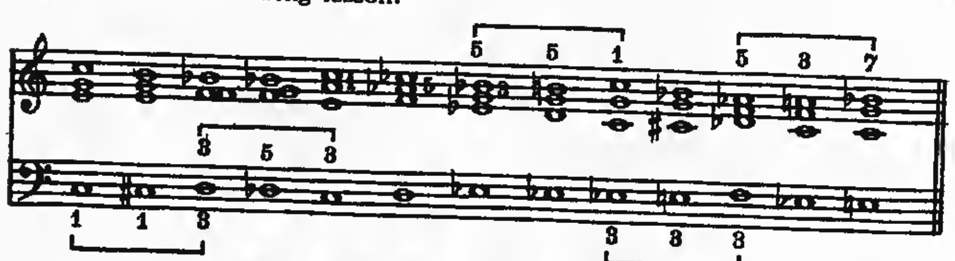


Figure 213. Example of chromatic continuity.

A. Operations from $S_3(5)$ and $S_4(5)$ Bases

As the 3 of $S_3(5)$ is identical with the 3 of $S_1(5)$, the fundamental operations correspond to those for $S_1(5)$. They are:

(1) raising of 1
(2) lowering of 3

Function 5 does not participate in the fundamental operations as it is already altered; and, as the form of the middle chord is pre-selected, the fifth requires rectification* in many cases, although it retains its name. All forms of doublings are acceptable.

As the 3 of $S_4(5)$ is identical with the 3 of $S_2(5)$, the fundamental operations correspond to those for $S_2(5)$. They are:

(1) lowering of 1
(2) raising of 3

The fifth does not participate in the fundamental operations but may be rectified.

Figure 214. Operations from an augmented S_3 (5) basis.

*The meaning of *rectification* in this context is explained on page 503. (Ed.)

Figure 215. Operations from a diminished S_4 (5) basis.

Figure 216. Chromatic continuity including all bases.

B. Chromatic Alteration of the Seventh

Because of the classical tendency toward a downward resolution of the seventh, chromatic alterations in this case conventionally follow the same direction. This lowering of the seventh (both major and minor) can be carried out from all forms of S(7). If the seventh is minor, it is more practical to have it as sharp or natural, since lowering of the flat produces a double-flat. Do *not* operate from a diminished seventh.

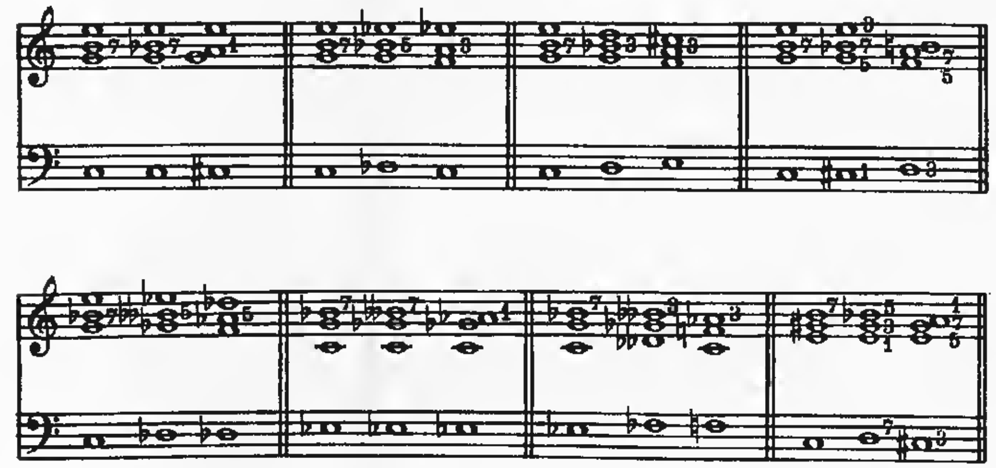


Figure 217. Examples of operations from the seventh.

All the single operations may now be incorporated into a final example of a form of chromatic continuity:

Figure 218. Operations from 1, 3, 5 and 7, all bases.

C. Parallel Double Chromatics

Parallel double chromatics occur when *fundamental operations are performed from an opposite base.* In such a case the *rectification* of the third is required.

If, for example, we decide to lower the 1 of the $S_1(5)$ basis, it becomes necessary to adjust 3 to its proper basis, i.e., in this case to lower it.

We shall consider the alterations of 1 and 5 as *fundamental*; the correction of 3, as *complementary.*

The following table represents all operations.

Parallel Double Chromatics.

$S_1(5)$ basis		$S_2(5)$ basis	
Fundamental	1♭	Fundamental	1♯
Complementary	3♭	Complementary	3♯
Fundamental	5♭	Fundamental	5♯
Complementary	3♭	Complementary	3♯

Figure 219. Parallel double chromatics.

The fundamental chromatics represent the middle term of a complete chromatic group, whereas the complementary chromatics do not necessarily perform the conclusive movement designated by their alterations. Thus, the scheme of chromatic groups for the parallel double chromatics is generalized as follows:

$$(1)\quad x \nearrow x\sharp \nearrow y \quad \text{(fundamental)}$$
$$x^1 \nearrow x^1\sharp \quad \text{(complementary)}$$

$$(2)\quad x \searrow x\flat \searrow y \quad \text{(fundamental)}$$
$$x^1 \searrow x^1\flat \quad \text{(complementary)}$$

For example, if c – c♭ – b♭ is a fundamental operation, the complementary chromatic is e – e♭. The complementary chromatic e♭ does not necessarily move into d. It may remain, or it may even move upward—depending on the chordal function assigned to it.

The same is true of the ascending chromatics. If c – c♯ – d is the fundamental operation, the complementary chromatic is e♭ – e. The complementary chromatic e does not necessarily move to f; it may remain, or even move downward, depending on the chordal function assigned to it.

The assignment of chordal functions must be performed for the two simultaneous operations: fundamental and complementary. It is practical to designate the ascending alterations as: $\frac{3}{1}$ or $\frac{5}{3}$, and the descending—as: $\frac{7}{5}$ or $\frac{5}{3}$.

This protects the resulting harmonic continuity from a wrong direction and sometimes from an excess of accidentals, particularly in reference to the middle term of a chromatic group.

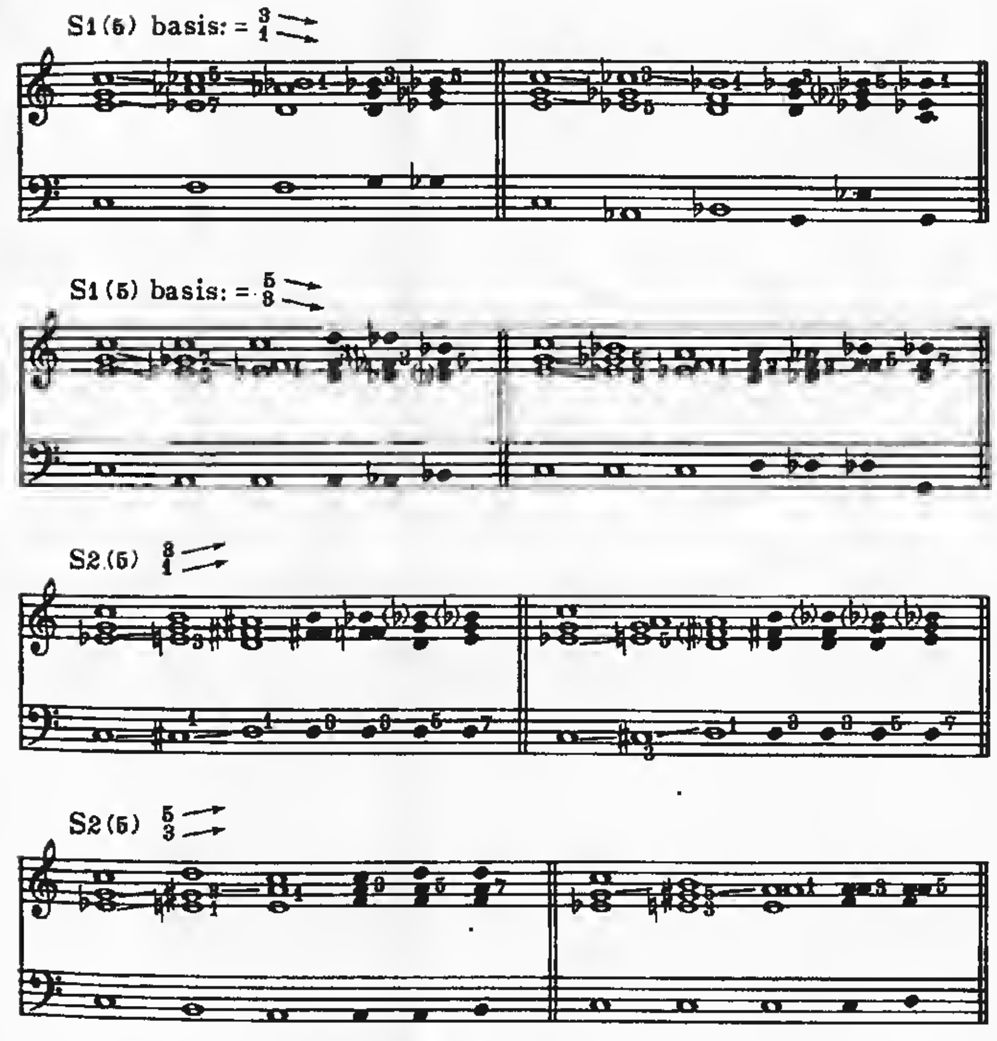


Figure 220. Double parallel chromatics.

By assigning the opposite bases, we can obtain double parallel chromatics at any desirable point in the chromatic continuity.

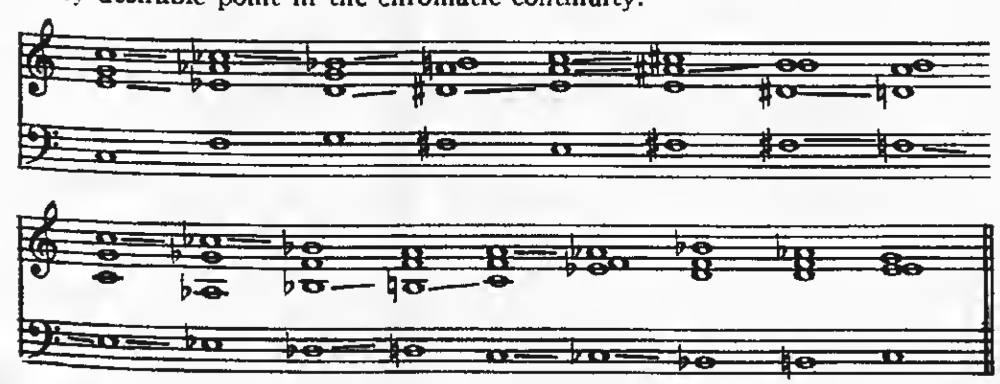


Figure 221. Continuity of double parallel chromatics.

Double parallel chromatics are the quintessence of chromatic style in harmony. It is these chromatics that created the unmistakable character of Wagnerian and post-Wagnerian music. While an analysis of the music of Borodin, Rimsky-Korsakov, Franck or Delius does not present any difficulties to an analyst familiar with my theory, the music of Wagner often requires transcribing into *chromatic* rather than enharmonic notation. One of the progressions typical of Wagner's later period, for example, (we find much of it in his *Parsifal*) is:

Figure 222. Typical Wagner.

But when transcribed into chromatic notation, it has the following appearance:

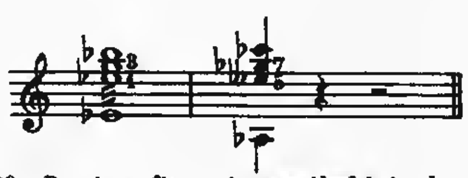


Figure 223. Previous figure transcribed into chromatic notation.

This corresponds to the $S_1(5)$ basis: $\begin{matrix} 3\searrow \\ 1\searrow \end{matrix}$

There are many instances in which double parallel chromatics are evolved on the basis of passing chromatic tones; they are abundant in the music of Rimsky-Korsakov, Borodin and lately have become very common in American popular and show songs (*Cuban Love Song*, *The Man I Love*, for example.) The historical source of passing chromatic tones, however—the technique of which I shall discuss later—is Chopin rather than Wagner or the post-Wagnerians.

D. Triple and Quadruple Parallel Chromatics

Triple parallel chromatics occur when the 1 is raised in $S_4(5)$ basis. This, being the fundamental operation, requires the correction of the third (3♯) and of the fifth (5♯). The triple alterations become:

5		7
3	or	5
1		3

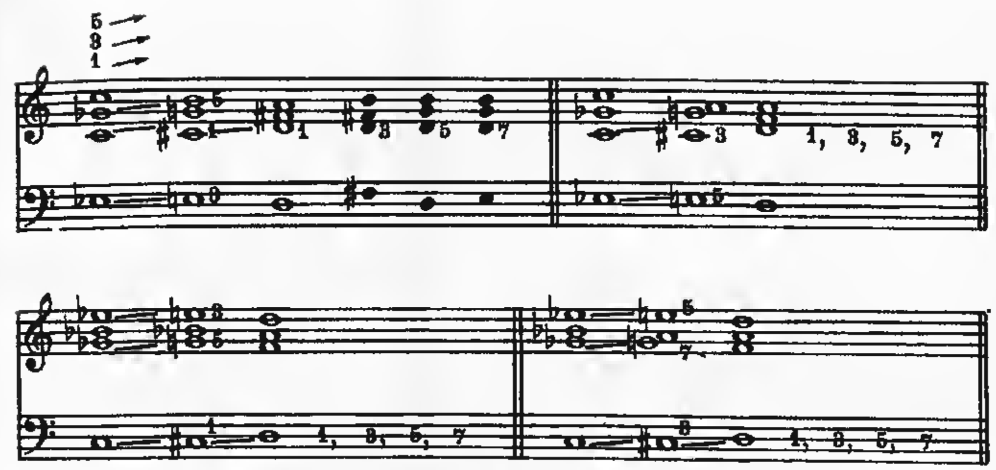


Figure 224. Triple parallel chromatics.

Quadruple parallel chromatics occur when the 1 is raised in $S_6(7)$ basis [diminished seventh-chord]. This requires the alteration of all remaining functions, i.e., 3♯, 5♯ and 7♯. This is the only interpretation satisfying those cases of chromatic parallel motion of the diminished seventh-chords—such as that found in Beethoven's Piano Sonata No. 7, the *largo* movement, (measure 20 from the end and the following five bars in relation to the adjacent harmonic context). Such a continuous chain of quadruple parallelisms takes place when the same operation is performed several times in succession.

As the chromatic system is limited to four functions (1, 3, 5, 7), quadruple parallel chromatics remain with their original assignments (while being altered).

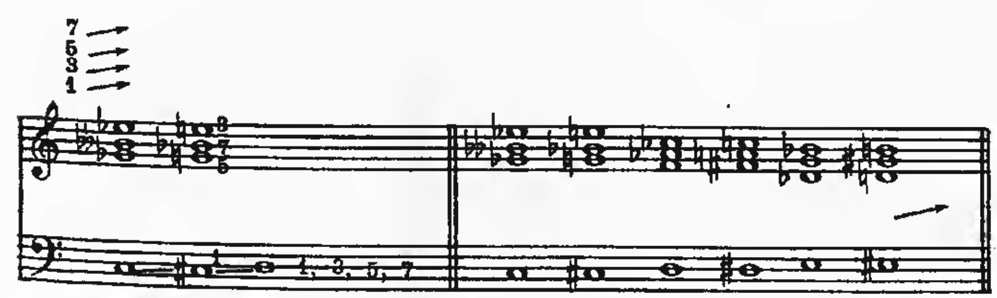


Figure 225. Quadruple parallel chromatics.

By combining all forms of chromatic operations, i.e., single, double, triple and quadruple, we obtain an example of the final form of mixed chromatic continuity. See Figure 226 on the following page.

Figure 226. Continuity of mixed chromatic operations.

E. Enharmonic Treatment of the Chromatic System

By *reversing* the original directions of chromatic operations, we more than double the original resources of the chromatic system.

Enharmonic treatment of chromatic groups consists of the substitution of raising for lowering, and *vice versa*. This changes the original direction of a group and brings the second, or "tension," chord to a new point of release in the third chord.

The following formula expresses all conditions necessary for the enharmonic treatment.

$$(1)\quad x \nearrow x\sharp = y\flat \searrow z\ (1, 3, 5, 7)$$

$$(2)\quad x \searrow x\flat = y\sharp \nearrow z\ (1, 3, 5, 7)$$

Progressions of this kind are characteristic of the post-Wagnerian composers—see Borodin's opera *Prince Igor*, Rimsky-Korsakov's opera *Coq d'Or* and Moussorgsky's *Khovanschina*.

Figure 227. Enharmonic treatment of chromatic system (concluded).

In using double or triple chromatics, all or some of the altered functions can be enharmonized.

Figure 228. Enharmonic treatment of double and triple chromatics.

F. Overlapping Chromatic Groups

The use of overlapping chromatic groups produces a highly saturated form of chromatic continuity. Alterations in the two overlapping groups may be either both ascending, or both descending—or one of the groups can be ascending while the other is descending. The choice of ascending and descending groups *depends on the possibilities presented by the preceding groups during the moment of alteration.*

The general form of overlapping chromatic groups is:

$$
\begin{array}{lllll}
d - ch - d & & \\
 & d - ch - d
\end{array}
$$

This scheme, being applied to ascending and descending alterations, offers 4 variants.

(1) x ↗ x♯ ↗ y
    x ↗ x♯ ↗ y

(2) x ↘ x♭ ↘ y
    x ↗ x♯ ↗ y

(3) $x \nearrow x\sharp \nearrow y$

$x \searrow x\flat \searrow y$

(4) $x \searrow x\flat \searrow y$

$x \searrow x\flat \searrow y$

Thus parallel as well as contrary forms are possible.

Each of the mutually overlapping groups has a *single* chromatic operation.

Figure 229. Examples of overlapping chromatic groups.

The sequence in which such groups can be constructed is as follows: (For purposes of illustration, we use figure 229 just given. The procedure in other cases is similar).

Write the first chord first:

1.

Figure 230. Procedure for constructing overlapping chromatics.

The next step is to make operations in one voice; in this example, 1♯ was chosen in the bass:

2.

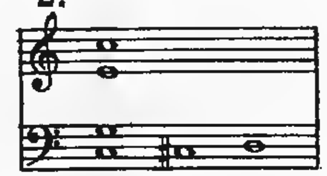


Figure 231. Step 2.

The next step is to construct the middle chord of this group; 1♯ was assumed to remain 1, which yielded the C♯ seventh-chord:

3.

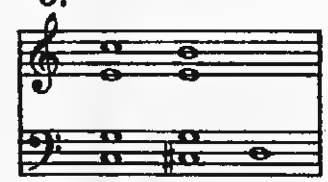


Figure 232. Step 3.

The next step is *to estimate the possibilities of other voices with regard to chromatic alterations.*

The b → b♭ step permits us to construct a chord which necessitates the inclusion of d and b♭. Another possibility might have been to produce g → g♯, which would also permit the use of d in the bass, as in the second example of the figure just given. The third possibility might have been the step e → e♯, in the alto voice, which also permits the use of d. Even such steps as e → e♭ or g → g♭ would be possible, although the latter would require an augmented S(7), i.e. (reading upward) d − g♭ − e♭ − b♭.

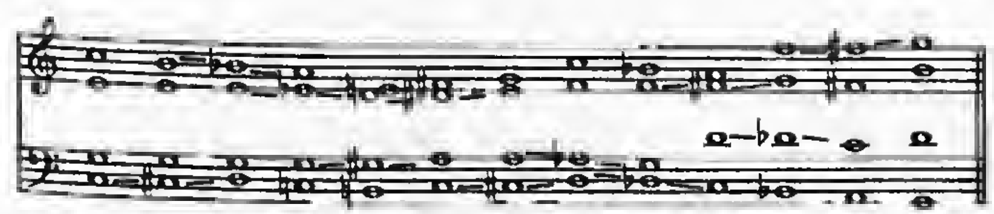


Figure 233. Continuity of overlapping chromatic groups.

G. Coinciding Chromatic Groups

The technique of evolving *coinciding chromatic groups* is quite different from all the chromatic techniques previously described. It is more similar to the technique of *passing chromatic tones,* which we shall discuss later.

Coinciding chromatic groups are evolved as a form of contrary motion in those *two voices which are a doubling of one function of the chord with which the group begins.*

The general form of a coinciding chromatic group is:

d – ch – d
d – ch – d

The contrary directions of the chromatic operations may be either outward or inward:

(1) x ↗ x♯ ↗ y
x ↘ x♭ ↘ y

(2) x ↘ x♭ ↘ y
x ↗ x♯ ↗ y

Assignment of the two remaining functions in the middle chord of a coinciding group can be performed by considering them enharmonically instead of by sonority.

For instance, in a group

c ↘ c♭ ↘ b♭
c ↗ c♯ ↗ d

the $\frac{c\flat}{c\sharp}$ interval can be read enharmonically, i.e., as $\frac{b}{c\sharp}$ in which case it becomes $\frac{7}{1}$ or $\frac{9}{3}$, etc. It is easy then to find the two remaining functions, like 3 and 5. Thus we can construct a chord c♯ – e – g – b.

As coinciding chromatics result from doubling, it is very important to have full control of the variable doubling technique. The doubling of 1, 3, 5 and also 7 (major or minor) must be used intentionally in all forms and inversions of S(5) and S(7)—the latter, naturally, to obtain the doubled 7.

Notation of chromatic operations as in all other forms of chromatic groups).

Figure 234. Coinciding chromatic groups.

It is important to remember in executing the coinciding chromatic groups that the first procedure is to establish the chromatic operations.

1.

Figure 235. Step 1 in constructing a coinciding chromatic group.

—and the second procedure is to add the two missing functions.

2.

Figure 236. Step 2.

After doing this, the final step is to assign the functions in the last chord of the group.

3.

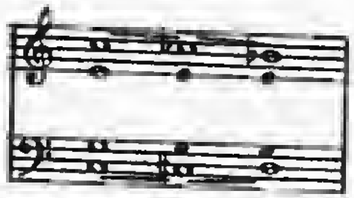


Figure 237. Step 3.

All coinciding groups are reversible. In moving from an octave inward by semitones, the ***last term of the group produces a minor sixth.*** In moving outward from unison or octave, the ***last term of the group produces a major third.***

It is important to take these considerations into account while evolving a continuity of coinciding chromatic groups. Any such group can start from any two voices producing (vertically) a unison, an octave, a major third or a minor sixth.

The following are all movements and directions with respect to c.

(1) c ↗ c♯ ↗ d
c ↘ c♭ ↘ b♭

(2) e ↘ e♭ ↘ d
c ↗ c♯ ↗ d

(3) c ↘ c♭ ↘ b♭
c ↗ c♯ ↗ d

(4) a♭ ↗ a♮ ↗ b♭
c ↘ c♭ ↘ b♭

Figure 238. Continuity of coinciding chromatic groups.

All techniques of chromatic harmony may now be utilized in the mixed forms of chromatic continuity.

CHAPTER 14

MODULATIONS IN THE CHROMATIC SYSTEM

MODULATION—that is, key-to-key (or, more generally, scale-to-scale) transition—may be accomplished by means of chromatic alterations—in addition to the technique previously offered, i.e., modulation by means of Generalized Symmetric Progressions. The theory of modulation is, in other words, a special case of the entire chromatic system of harmony. Viewed in such a way, the present explanation *absorbs all possible cases of modulation* and offers a unified picture of all forms pertaining to key changes.

As our musical system operates with *seven names*, i.e., c, d, e, f, g, a, b, every possibility within the seven-unit scales must be a combination thereof. The actual intonations resulting from such combinations of the seven names are due to various combinations of naturals and accidentals, sharps and flats.

This permits a form of reasoning whereby *each musical name may become the root-tone of a chord linking the preceding to the following key*. As all *names* are common to all keys and all chords within the keys (so far as special harmony is concerned), every chord may be presumed to be a common chord between any preceding and following key.

For example, a transition from the key of C to the key of G may be accomplished by means of seven modulations:

	Preceding Key	*Common Chord*	*Following Key*
(1)	C	C	G
(2)	C	D	G
(3)	C	E	G
(4)	C	F	G
(5)	C	G	G
(6)	C	A	G
(7)	C	B	G

Figure 239. Modulations.

The fifth case is the least practical one; it anticipates the following key and makes the appearance of the latter too obvious.

Combining all forms of modulations from all keys to all keys and assuming that every chord may be a common chord, we obtain the following table of musical names in naturals:

C	D	E	F	G	A	B
D	E	F	G	A	B	C
E	F	G	A	B	C	D
F	G	A	B	C	D	E
G	A	B	C	D	E	F
A	B	C	D	E	F	G
B	C	D	E	F	G	A

Figure 240. Table of musical names.

From the above table it follows that a common chord may be established *on any degree of the preceding key*—which, in turn, corresponds to a *certain degree of the following key*.

Applying this principle to the figure just given, we obtain the following key-chord correspondence:

Key	*Chord*	*Key*
C	C	G
	I = IV	
C	D	G
	II = V	
C	E	G
	III = VI	
C	F	G
	IV = VII	
C	G	G
	V = I	
C	A	G
	VI = II	
C	B	G
	VII = III	

Figure 241. Key-chord correspondences.

The rest of the necessary procedure in key-to-key modulation is simply the ***chromatic readjustment of accidentals***. The following key demands that the tones be adapted to its ***real key signature***. Thus ***all names*** which are not the same in pitch in both the preceding and the following signatures must be altered. When all names of the common chord are common tones (identical pitches), modulation becomes ***diatonic***, i.e., the intonation of the preceding and following keys, in this particular chord, coincides. Thus, ***diatonic modulation is a special case of chromatic modulation***.

The technique includes:

(1) the preceding key has to be developed through diatonic cycles;
(2) the particular common name chord has to be selected;
(3) the corresponding chromatic alterations have to be made;
(4) the correspondence of the degrees has to be established;
(5) after the common chord is repeated with the accidentals of the following key (preferably in the form of a seventh chord), the continuation—and, possibly, completion—has to be performed through the diatonic cycles of the following key..

When a full completion is needed, a cadence may be added. The common chord thus has the significance of the middle chord of a chromatic group. In case the modulation becomes diatonic, there is no repetition of the common chord and there is no need to have the latter in the form of a seventh chord.

Figure 242. Examples of modulation.

There are some cases—mostly those in which the preceding and the following keys are one semitone apart, or in which the common chord is one semitone away from the following key—in which alterations cause ***consecutive sevenths*** or an awkward hidden seventh. If such steps are to be avoided, it is necessary to have the common chord first as S(5), then as S(7). To avoid the hidden seventh, double the fifth in S(5), i.e., use S(5)⑤ for the first common chord.

Figure 243. To avoid consecutive or "hidden" sevenths.

This theory of modulation is applicable to all seven-unit scales in which each musical name appears once and in which none of the seven intonations (pitches) coincide. There are 36 such fundamental scale structures, and each of the 36 has six derivative scales (modes) by pitch permutations, producing a total of $36 \times 7 = 252$ scales.

The ears of our audiences—and often those of composers themselves—are accustomed to modulations dealing with *natural major*, *harmonic major*, *harmonic minor* and *melodic minor*. All other seven-unit scales, even the modes of these scales, sound new and strange. Therefore a free use of 252 scales offers a new and virgin field to a composer who wants to achieve originality without departing from the established trends of musical reasoning.

Examples of original key-scale relations in modulation.

(1) Key of Cd_0: c — d♭ — e♭ — f — g — a — b; modulation to the key of Ed_3: a — b — c♯ — d♯ — e — f — g; common chord: F.
(2) Key of Cd_1: d — e — f♯ — g♯ — a — b — c; modulation to the key of B♭d_6: g — a — b♭ — c — d — e — f♯; common chord: E.

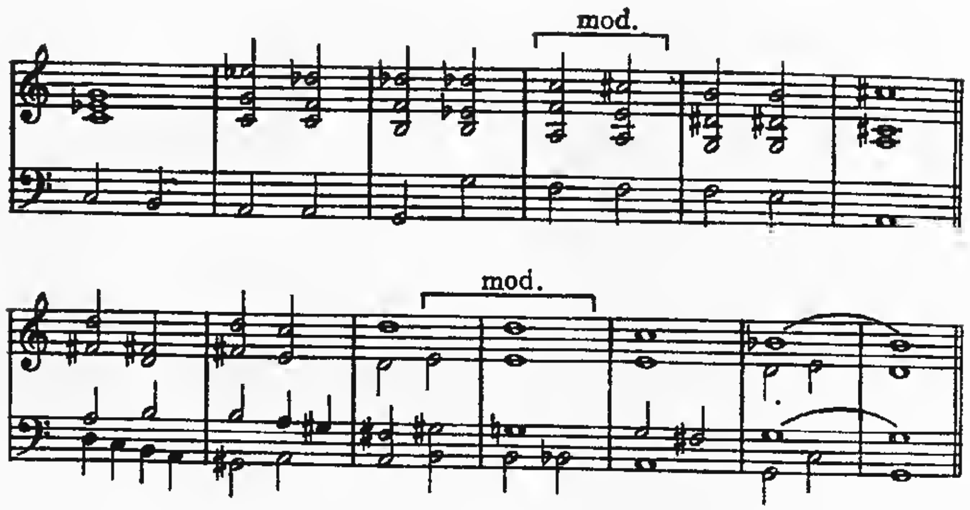


Figure 244. Original key-scale relations in modulation.

The readjustment of accidentals of the following key may be performed gradually (one by one) when an instantaneous change would produce an effect of too abrupt a character, as is usually the case when there is considerable difference between the real signature of the preceding and the real signature of the following key (see the modulation in the preceding example). We shall call such cases *extended modulations.*

Key I: Cd_0 Nat. major;
Key II: G♯d_0 Nat. major;
common chord: A.

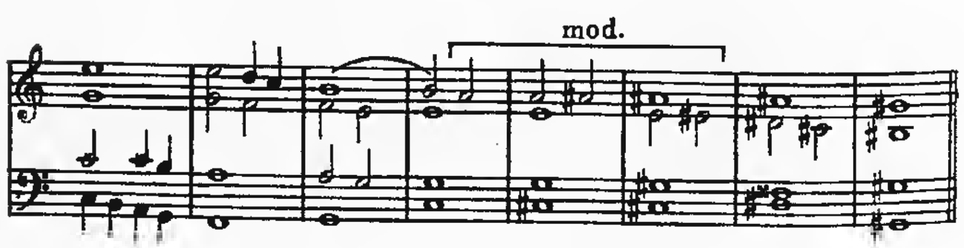


Figure 245. Example of an extended modulation.

The reader is now in a position to work out a systematic tabulation of all modulations, if necessary. Here is offered one example of a table comprising all the modulations between two *keys* and two *scales.* More such tables—between other pairs of keys and scales—may be worked out in a similar way.

Example of all modulations between two keys and two scales.

Key I: Cd_0 Nat. major;
Key II: E♭d_0 Nat. major; common chords: (1) C; (2) D; (3) E; (4) F; (5) G; (6) A; (7) B.

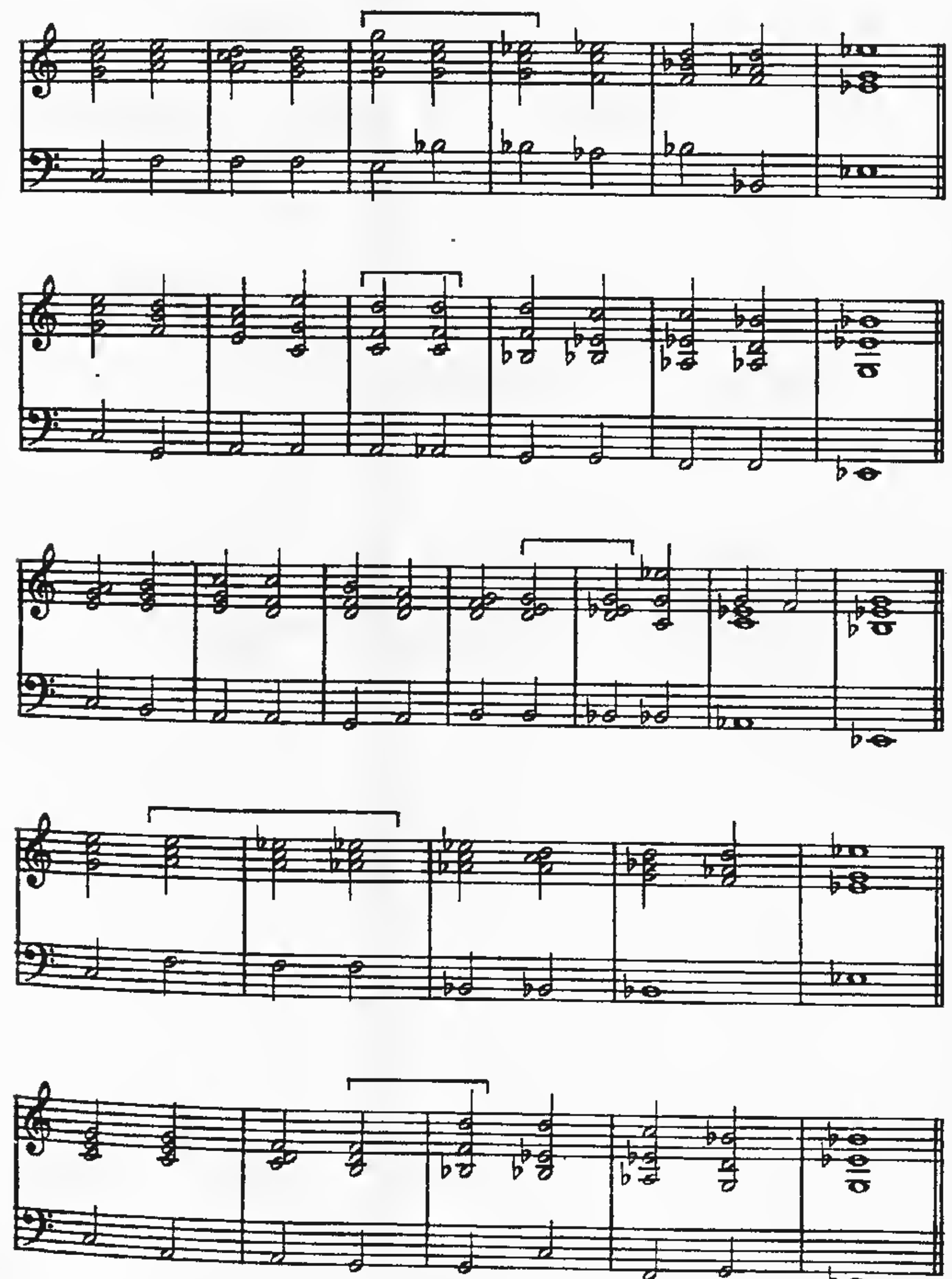


Figure 246. All modulations between two keys and two scales (continued).

Figure 246. All modulations between two keys and two scales (concluded).

A. Indirect Modulation

It happens that the typical, trivial academic modulations into remote key signatures by means of intermediate keys are known to lack musical interest. These academic modulations sound very unimaginative, indeed, especially after one has listened to a symphony by Schubert or by Mendelssohn—not to mention the modulations of such leading composers who came after Schubert and Mendelssohn, as Wagner, Brahms, or Franck.

But if academic modulations are analytically dissected and if the cause of their triviality is found, then, perhaps, the same analysis will lead us to an explanation of the modulatory secrets of Schubert, Mendelssohn, Wagner and others. Such was the reasoning which led me to the discovery of the *theory of indirect modulations.*

The fundamental solution of this problem (i.e., the selection of intermediate keys to link the initial and the terminal key) lies in establishing a *scale of key-signatures.*

Let us take the established key signatures, i.e., those which are real for natural major. Let us next assume the starting point to be "zero accidentals" (i.e., key of C), which will become the axis of symmetry for the reciprocal position of the opposite accidentals. For example, 3 sharps above the axis are equidistant with 3 flats below the axis,

Under such conditions the scale of key-signatures, which we shall here limit to seven accidentals, will assume the following appearance:

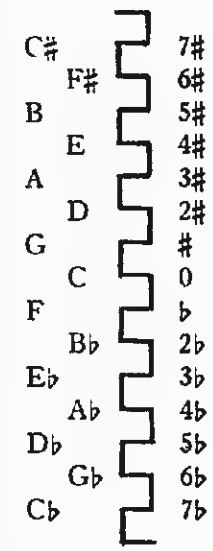


Figure 247. The Scale of Key Signatures for the Natural Major.

The academic method of planning intermediate keys for remote modulations exhibits a definite tendency (even though it is usually expressed through a whole system of complicated rules) which tendency can be formulated as: the *accumulation of sharps* when the preceding key has fewer sharps than the following key, and the *accumulation of flats* when the preceding key has fewer flats than the following key. When such a tendency is carried into practice directly, the outcome of such planning can be graphically represented as: *scalewise motion through the scale of key signatures.* The latter would be one general rule working in all such cases and would exclude all other rules.

Let us, for illustration, apply this rule to the planning of a typical academic modulation. Major and minor keys are frequently alternated—and we shall follow this precedent: let D♭ major be the preceding and E major be the following key; by drawing a scalewise graph between the limits of the preceding and the following key we obtain the key-sequence shown in Figure 248 on the following page.

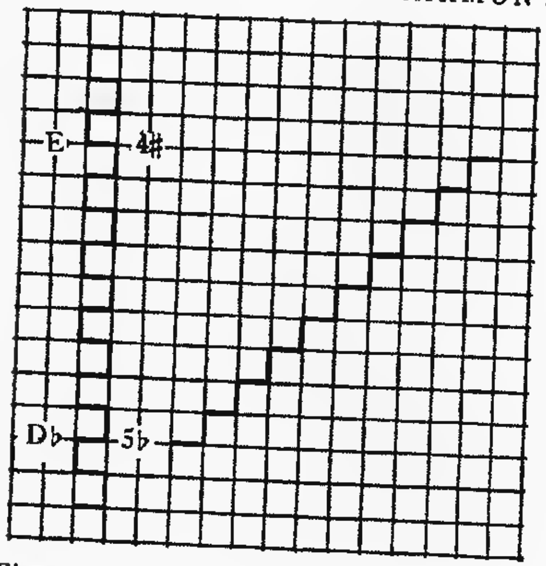


Figure 248. Scalewise graph D♭ major to E major.

The graph above can be read as follows:

D♭ + f + E♭ + g + F + a + G + b + A + E

The capital letters here are used to represent major keys; the small letters, to represent minor keys.

In some cases the academic procedure provides a direct transition through minor sub-dominants of the major and major dominants of the minor keys. In applying this method to the above case we would obtain a shorter scheme:

D♭ − f − E♭ − g − F − a − E.

—where a-minor is the subdominant of the following key, E-major; in such a case the graph would assume the following appearance:

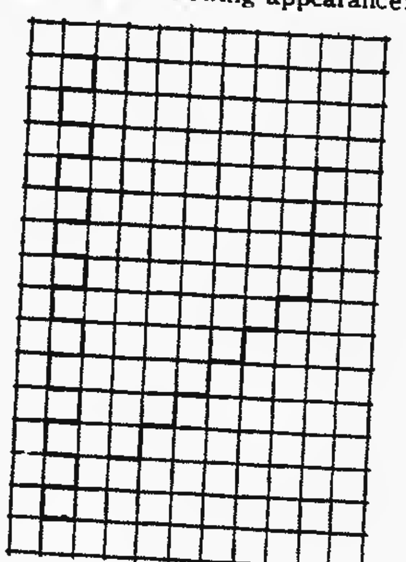


Figure 249. Shortening the modulation.

One could extend this principle still further to make wider gaps on the scale of key signatures; but this would not help, for the trajectory still remains predominantly scalewise—and such a form never produces anything of interest.

Thus we arrive at the conclusion that, by producing more dramatic forms of trajectories on the scale of key signatures, we can obtain more expressive modulations.

The fact is that *any trajectory which is not scalewise produces modulations with musical interest.* Various forms of resistance, binary and ternary axes—as set forth in my earlier discussion of the theory of melody—constitute such material.

Here are a few examples of the planning of such modulations to remote keys.

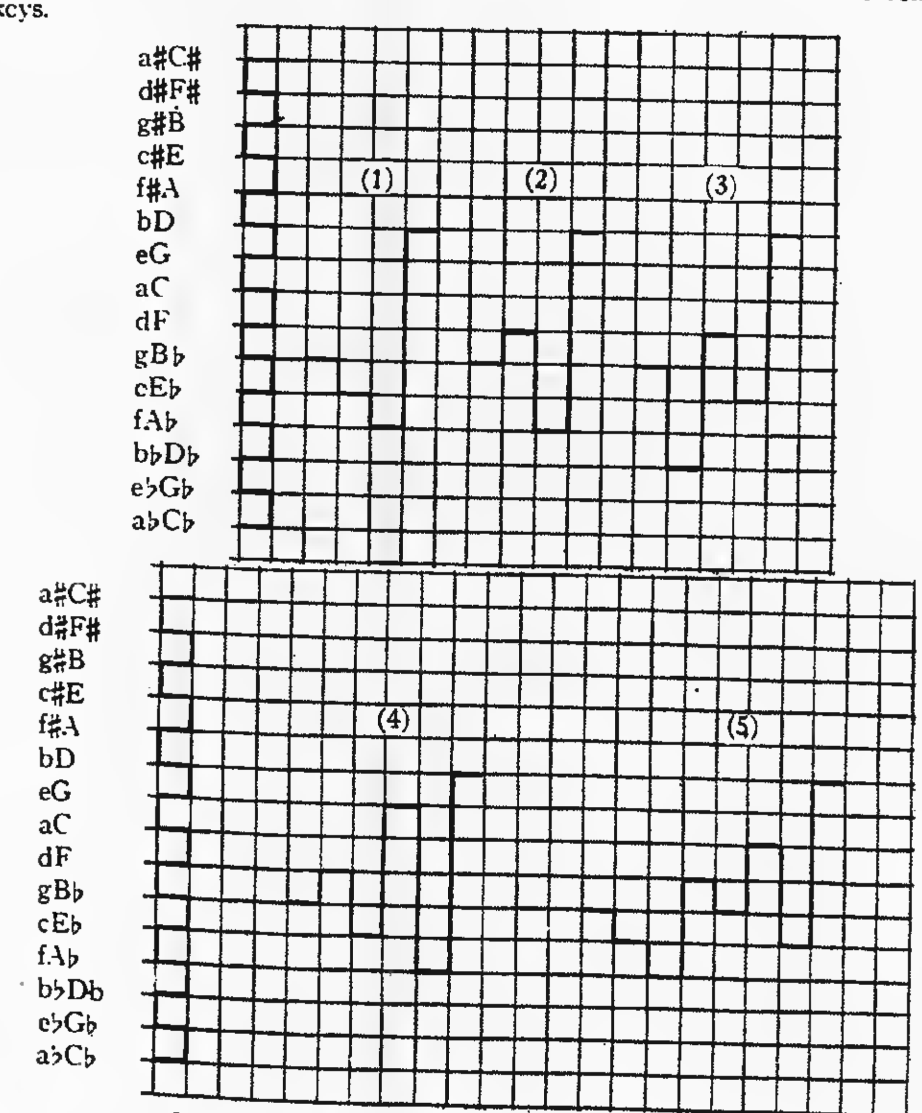


Figure 250. Modulations which are not scalewise.

Deciphering the graph, we get:

(1) B♭ + c + A♭ + D; B♭ + c + A♭ + b;
g + E♭ + f + D; g + E♭ + f + b.

(2) B♭ + d + A♭ + D; B♭ + d + A♭ + b;
g + F + A♭ + D; g + F + A♭ + b.

(3) B♭ + G♭ + d + E♭ + D; B♭ + G♭ + d + E♭ + b;
g + b♭ + F + c + D; g + b♭ + F + c + b.

(4) B♭ + d + E♭ + e + A♭ + D; B♭ + d + E♭ + e + A♭ + b;
g + F + c + G + f + D; g + F + c + G + f + b.

(5) B♭ + c + A♭ + d + g + C + c + D;
B♭ + c + A♭ + d + g + C + E♭ + b;
g + E♭ + f + d + B♭ + a + c + D;
g + E♭ + f + d + B♭ + a + E♭ + b.

Figure 251. The previous figure deciphered.

Indirect modulations can be plotted as ***key sequences with specific time arrangement.*** The amount of time allowed to each intermediate key is a matter of *rhythmic* distribution. The latter can be expressed either in chord-units (H) or in time group units (T)—which is, practically, the same thing when the number of H in each T is constant (i.e., when the number of chords in each bar is the same).

Let us take as an example the first modulation in group (4) of Figure 250.

B♭ + d + E♭ + e + A♭ + D.

As the durations of the first and the last key are specified *a priori*, it is necessary to plan the duration of intermediate keys only. The intermediate keys should present some definite equivalent of time with regard to the preceding and the following keys, as well as to the duration of the entire modulatory group.

Let us assume that music developing in the first and the last key is based on 8T as a structural unit. Then if we want to allow 8T for the entire modulation group, we can easily distribute the four intermediate keys. The simplest solution in this case would be: $\frac{8}{4} = 2$, i.e., 2T per key. Now time-key representation takes the following appearance:

B♭8T + d2T + E♭2T + e2T + A♭2T + D8T.

The above scheme can be plotted as shown in Figure 252 on the following page.

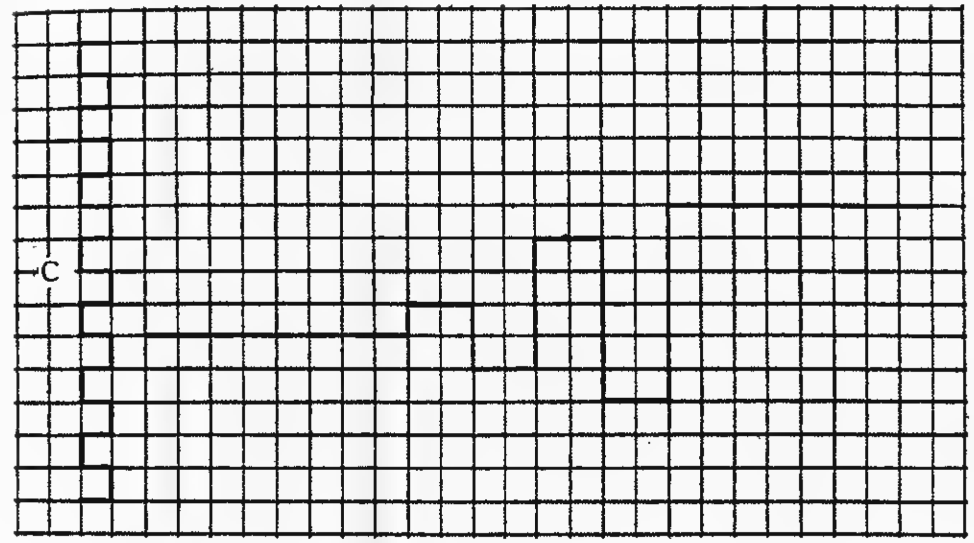


Figure 252. First modulation of figure 250 plotted to 8T structure.

It is easy to see that the same scheme can be represented through the quantity of H. For example, assuming that there are three chords per T, and substituting 3H for each T, we obtain:

B♭24H + d6H + E♭6H + e6H + A♭6H + D24H.

Any non-uniform distribution of intermediate keys must conform to the rhythmic series to which the *factorial continuity* of the theme belongs.

For example, let us assume that the above described 8T-groups belong to $\frac{8}{8}$ series; we want to find a proper form of distribution for the four intermediate keys, and we want to express it in T-units—and this amounts to the construction of a quadrinomial in $\frac{8}{8}$ series. Of the three trinomials of this series, i.e. 3+3+2, 3+2+3 and 2+3+3, we may choose any one. Let us take the first trinomial and evolve a ***binomial split-unit group*** out of the first term: 3 = 2+1. Then the quadrinomial acquires the following form: 2+1+3+2.

Applying this quadrinomial to the modulation group under discussion, we obtain:

B♭8T + d2T + E♭T + e3T + A♭2T + D8T.

In applying such key-time schemes, it is well to carry them out as closely as possible, although there is no need for absolute mathematical precision. Only the *total duration* of the entire modulation group must be carried out *exactly*.

Example of indirect modulation with key-time planning:

(B♭8T) + d2T + E♭T + e3T + A♭2T + (D8T)

Figure 253 Indirect modulation with key-time planning.

CHAPTER 15

THE PASSING SEVENTH GENERALIZED

As we have seen before, the preparation of the seventh in C_0 requires a descending step from the root-tone. In C_3 the seventh while resolving, becomes a new root-tone. This fact permits us to develop a *continuity of the passing seventh* when C_3 is constant. All transformations are applicable. All chords must be S(5).

Figure 254. Passing seventh.

By reading the above figure backwards, we obtain an ascending scale in the bass. The cycle in such a case becomes negative (C_{-3}) and the transformation is $\circlearrowright$.

Figure 255. Previous figure read backwards.

Examples of the other forms of transformation:

Figure 256. Other forms of transformation.

In each case the role of the bass can be transferred to soprano.

Figure 257. Transferring role of bass to soprano.

A great flexibility of the melodic form can be achieved by a leap of a seventh upward for the positive cycle, and a leap of a seventh downward for the negative cycle.

Figure 258. Adding flexibility of melodic form.

Melodic forms and control of the leaps can be accomplished by pre-set forms of distribution of the scalewise steps. For example, a coefficient group can determine the number of pitch-units moving in succession until the leap occurs.

Figure 259. Pre-set form of distribution of scalewise steps (continued).

Figure 259. Pre-set form of distribution of scalewise steps (concluded).

A variety of melodic forms may also be obtained by mixing C_3 and C_{-3}.

$$4C_{-3} + 3C_3$$

Figure 260. Mixing C_3 and C_{-3}

All forms of the ***generalized passing seventh*** are applicable to modal transposition, as well as to progressions in harmony of types II and III. The latter must be confined to $\sqrt[3]{2}$ and $\sqrt[4]{2}$, (three and four tonics), as only these two systems correspond to the C_3 and C_{-3}.

(A) Phrygian (d_2)

(B) Persian

Figure 261. Modal transposition.

In type II all structures must be specified as S(7) and S(9) in such a way as to conform to the seventh of one family.

Example: large + 2 minor + small.

Figure 262. In type II, all structures must be S(7) and S(9).

The use of but one consistent S for the whole progression results in a consistent scale in the moving voice for each half of the entire cycle.

Figure 263. S(7) large constant.

A. Generalized Passing Seventh in Progressions of Type III

The fundamental material for this technique is the progressions based on three and four tonics.

In the case of three tonics the interval between the roots equals 4 semitones. This makes it possible to use *three forms of the seventh*: the major, the minor and the diminished.

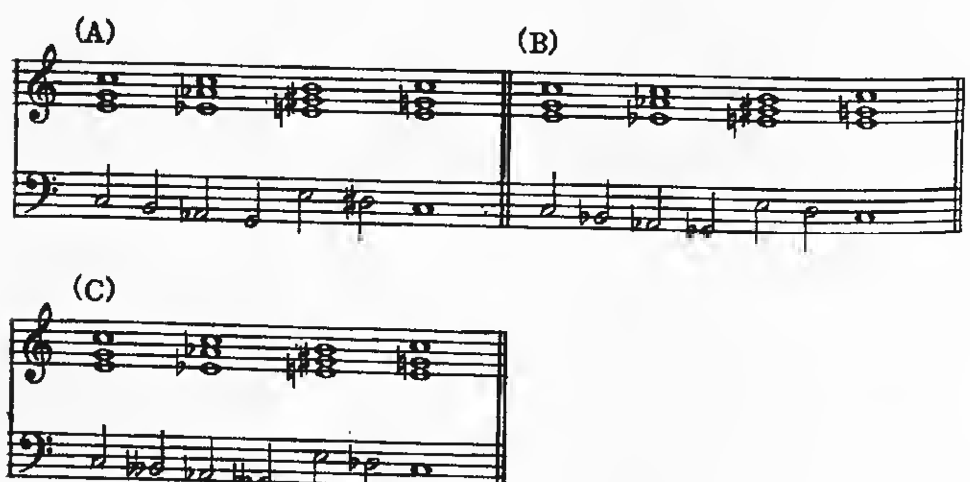


Figure 264. Passing 7th in $\sqrt[3]{2}$.

In this field, by means of symmetric chord progressions and the passing seventh device (which need not always "pass" in the conventional sense), we obtain pitch-scales of the *third group*. The number of tonics in the scales corresponds to the number of tonics in the chord progression. As in previous cases, the bass part can be placed *above* the remaining three voices and, as before, it is subject also to *octave variation* (leap into the adjacent octave).

As the 1, 3 and 5 of the three voices may be $S_1(5)$, $S_2(5)$, $S_3(5)$ or $S_4(5)$, it is possible to obtain automatically some of the *new structures* of S(7), which belong to the category usually known as "altered chords."

In four tonics the interval between the roots equals 3 semitones. This gives us a choice of a *major* and a *minor seventh*.

Figure 265. Passing 7th in $\sqrt[4]{2}$.

The above two cases produce the Arabian scale called "String of Pearls" (*Zer ef Kend*) in its two versions. The ascending forms can be obtained by reversing the cycles:

C – E – A♭ – C for the three tonics
and C – E♭ – F♯ – A for the four tonics.

By *mixing the positive and the negative forms*, we can acquire a more diversified melodic structure.

Figure 266. Mixing positive and negative forms.

This *mixture of structures and of the sevenths* introduces still greater variety into symmetric progressions and results in *mixed scales.*

Figure 267. Mixture of structures and of sevenths.

Further development of this field may be obtained through the assumption that *1 followed by 7 in C_0* can be used in any symmetric system other than C_3.

Figure 268. Passing seventh in $\sqrt{2}$, $\sqrt[6]{2}$, $\sqrt[12]{2}$.

Under such conditions, i.e., without the necessity of resolving the seventh by scalewise downward motion, it is possible to apply the technique of the passing seventh to *generalized symmetric progressions.*

Figure 269. Applying the passing seventh to generalized symmetric progressions.

B. Generalization of Passing Chromatic Tones

As there are three forms of the seventh available in the system of three tonics, we may now incorporate all three into a continuous progression.

Figure 270. Three forms of the seventh in one progression.

Applying the same to the negative form of three tonics, we obtain:

Figure 271. Negative form of three tonics.

Likewise, the system of four tonics offers two forms of the seventh—we shall now use them in succession:

Figure 272. Four tonics offer two forms of the seventh.

The same approach is applicable to the negative form of four tonics:

Figure 273. Negative form of four tonics.

All forms of the passing seventh between the roots of symmetric systems produce ***continuous chromatic passages*** connecting the roots.

The number of pitch-units of the chromatic scale to be harmonized by one chord equals ***12 divided by the number of symmetric roots.*** Thus, the system of two tonics requires $\frac{12}{2} = 6$, i.e., six units of the chromatic scale to be harmonized by one chord.

Figure 274. Two tonics.

The system of six tonics requires $\frac{12}{6} = 2$, i.e., two units of the chromatic scale to be harmonized by one chord.

Figure 275. Six Tonics (continued).

Figure 275. Six Tonics (concluded).

In the system of twelve tonics, each pitch-unit of the chromatic scale must be harmonized by one chord, as $\frac{12}{12} = 1$.

In this way we establish an interrelation between the complete form of symmetry of our tuning system ($\sqrt[12]{2}$) and the sub-systems of this symmetry ($\sqrt{2}$, $\sqrt[3]{2}$, $\sqrt[4]{2}$ and $\sqrt[6]{2}$).

A mathematical representation of the forms of symmetric harmonization of the chromatic scale would be:

Form of symmetry	Number of chromatic units per chord
1	12
$\sqrt{2}$	6
$\sqrt[3]{2}$	4
$\sqrt[4]{2}$	3
$\sqrt[6]{2}$	2
$\sqrt[12]{2}$	1

Figure 276. Symmetric harmonization of chromatic scale.

We see that a *variable quantity of units of the chromatic scale may be harmonized by means of generalized symmetric progressions.*

Example: 6 + 1 + 4 + 2 + 3

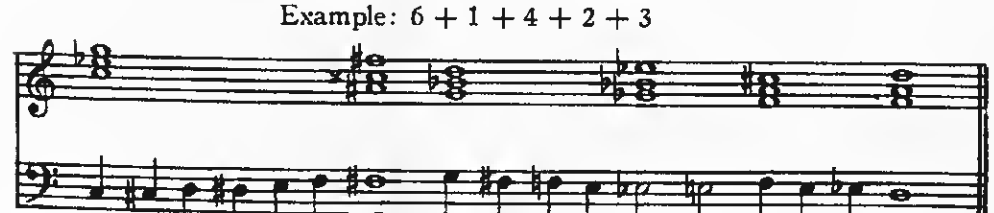


Figure 277. Harmonizing by means of generalized symmetric progressions.

Leaps are applicable to chromatic passages in the same way as they have been applied to the passing seventh. Such leaps can be performed from any point; the structures may be varied.

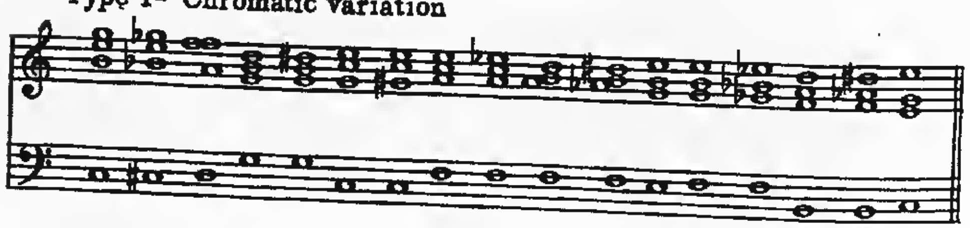
Figure 278. Leaps are applicable to chromatic passages.

When passing chromatics fill the intervals between the symmetric roots, they result in chromatic passages *within* symmetric progressions.

In addition to this technique, there is a possibility of *filling the interval of a whole tone in any type of harmonic progressions, after the voice-leading* related to the given type of progression *is completed.*

Type I= Original

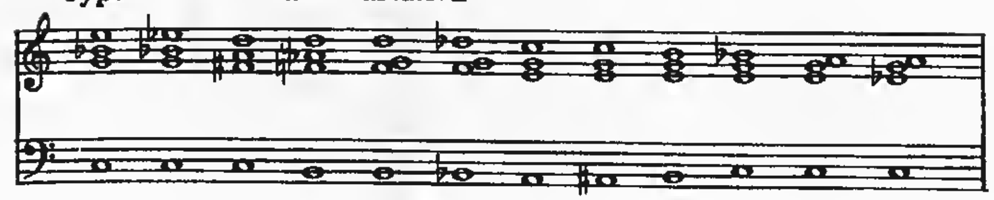
Type I= Chromatic variation



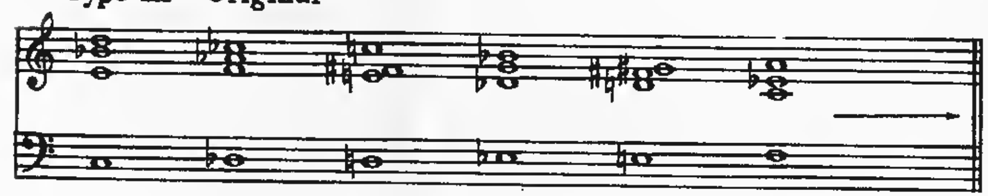
Type II= Original

Figure 279. Passing chromatic tones (continued).

Type II= Chromatic variation



Type III= Original



Type III= Chromatic variation

Figure 279. Passing chromatic tones (concluded).

Although a chromatic harmonic continuity (of any form) offers but limited possibilities for the insertion of passing chromatics, such a procedure can nevertheless be accomplished if and when necessary.

Chromatic= Original

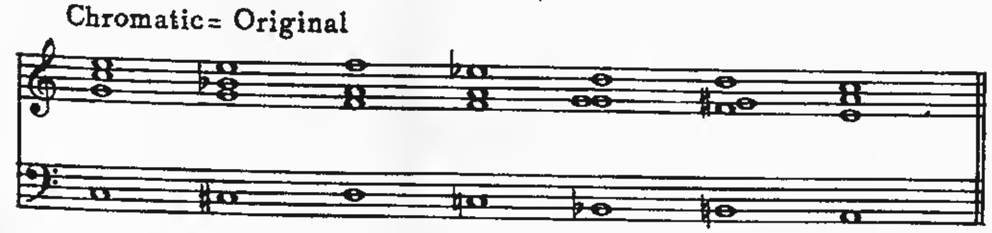


Figure 280. Passing chromatics inserted into chromatic harmonic continuity (continued).

Figure 280. Passing chromatics may be inserted into chromatic harmonic continuity. (*concluded*)

C. Altered Chords

Pitch assemblages produced by one or more simultaneous passing chromatics are usually known under the name of "*altered chords.*"

Whereas the usual academic scope of information regarding "altered chords" is very limited, it becomes virtually *inexhaustible*—and the forms of altered chords become all but *limitless*—when evolved through the technique of passing chromatic tones.

Some of the altered chords, although *different* in their written forms, *correspond* in their *intonation* to forms already studied in this special theory of harmony. These structures, familiar in their intonation, frequently necessitate entirely new progressions (as compared with the familiar types of progressions)

For example, a chord—

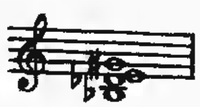


Figure 281.

may be known in the key of harmonic G♯—minor (II) in the following notation:

Figure 282. Same chord in G♯ minor (II).

Yet in the first notation it moves into A-minor $(I)_A$—

Figure 283. Moves into A minor (I) in first notation.

—whereas in the second notation it could move through any cycle in the key of G♯ – minor.

Some other altered chords *do not correspond to any of the structures previously classified*, as they contain an interval 2—and all structures previously classified contain 3 and 4 only.

For example:

Figure 284. Other altered chords.

—where $^{f}_{d\#}$ is an interval of two semitones.

In order to obtain a progression where such altered chords occur, it is necessary to start with a chord produced by passing chromatics and *alternate continuously the altered and the regular chords.*

Figure 285. Alternating altered and regular chords.

CHAPTER 16

AUTOMATIC-CHROMATIC CONTINUITIES

AUTOMATIC chromatic continuity may be devised by means of *semitonal motion* in which one direction is followed by whatever voice or voices happen to be moving.

A. In Three-Part Harmony

In three-part harmony, any form of S(5) may be selected as a starting point; this alone offers 4 forms for one voice moving at a time:

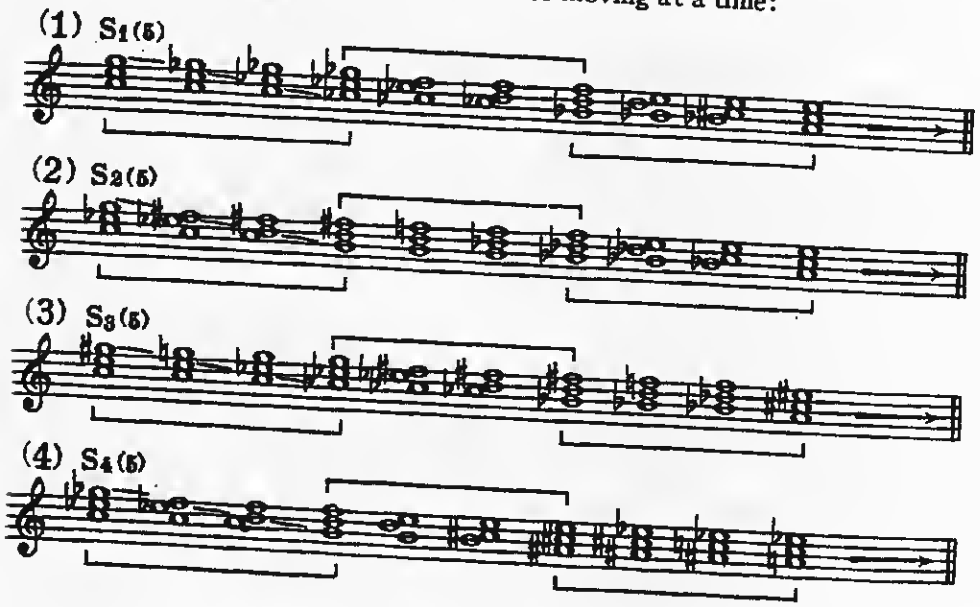


Figure 286. S(5) offers 4 forms for one voice moving at a time.

The above table represents the fundamental progression: SAT. As there are three modifications to each group (H), and each succeeding group starts one semitone lower, we obtain $3 \times 12 = 36$ groups for each case. Each chromatic continuity of this type closes at $36 + 1$, i.e., on the 37th chord of the progression.

Further possibilities develop from variations of the part-sequence, of which there are 6 in this case:

SAT, STA, TSA, AST, ATS, TAS.

Thus, the total number of progressions moving in one direction is $4 \times 6 = 24$, each moving in the same direction. By reversing the direction of each progression, we double the quantity.

Summarizing the total number of possible forms for three-part automatic chromatic continuity, we arrive at the following figures for all cases in which one voice moves at a time.

4S produce 4 forms of intonation
Chromatic scale produces a sequence of 36 chords
SAT produce 6 variations of the sequence for each form of intonation
The 2 directions (upward and downward) double the quantity of all forms of sequence and intonation

Total: $4 \times 6 \times 2 = 48$ forms

(a) $S_2(5)$: ATS ↓ (b) $S_3(5)$: TSA ↑

Figure 287. (a) $S_2(5)$: ATS ↓. (b) $S_3(5)$: TSA ↑

The technique of semitonal motion makes it possible to move two voices at a time. Both voices must move in one direction.

The number of combinations out of three elements, taken two at a time, is mathematically:

$$_3C_2 = \frac{3!}{2!\,(3-2)!} = \frac{6}{2 \cdot 1} = 3$$

In the simultaneous (vertical) arrangement they are:

	S	S
A		A
T	T	

Simultaneous motion of two parts requires the use of all three combinations arranged in any of the 6 possible forms of succession. Otherwise, there would be no way to arrive at the original structure and the entire sequence would become more and more distorted. Some cases produce consecutive seconds, and these, if felt to be undesirable, may be omitted.

An example of two-part chromatic motion in all six permutations of the original combination.

Original Structure: $S_1(5)$
Direction: ↑

(a) $\frac{A}{T} + \frac{S}{T} + \frac{S}{A}$

(b) $\frac{A}{T} + \frac{S}{A} + \frac{S}{T}$

(c) $\frac{S}{A} + \frac{A}{T} + \frac{S}{T}$

(d) $\frac{S}{T} + \frac{A}{T} + \frac{S}{A}$

(e) $\frac{S}{T} + \frac{S}{A} + \frac{A}{T}$

(f) $\frac{S}{A} + \frac{S}{T} + \frac{A}{T}$

Figure 288. Two-part chromatic motion.

Another form of automatic chromatic continuity applies to the variable number of parts participating in simultaneous moves. In three-part harmony, it is possible to alternate the two simultaneous parts and to use the remaining one to produce compensation.

Thus the following forms are available:

(1) S / A / T; (2) S / A / T; (3) S / A / T,

as well as their reciprocals:

(1) S / A / T; (2) S / A / T; (3) S / A / T.

Figure 289. Alternating two simultaneous parts.

The above combinations may be further combined into continuously varying groupings.

(1) (S A T) + (A S T) + (A T S)

(2) (S A T) + (A S T) + (S A T)

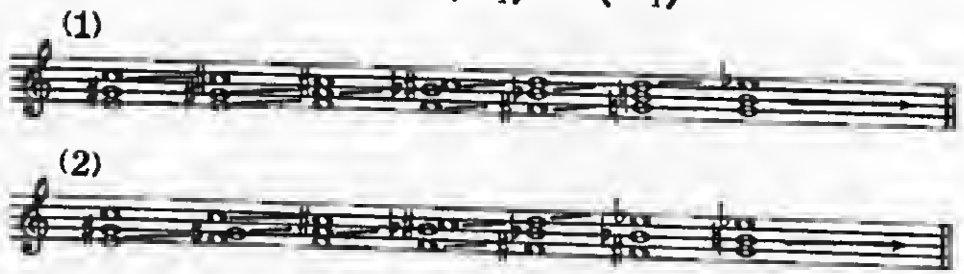


Figure 290. Combining foregoing (figure 289) with continously varying groups.

The final form of three-part continuity consists of variations of the single and double moves, and variations of the sequence in which the latter appear in both descending and ascending directions.

Example:

S↓ A↓ ; A↓ T↓ ; S↓ ; T↑ ; A↑ ; S↑ ; A↓ T↓ ; S↓ A↓ ; T↓ ; T↑ ; S↑ ; A↑

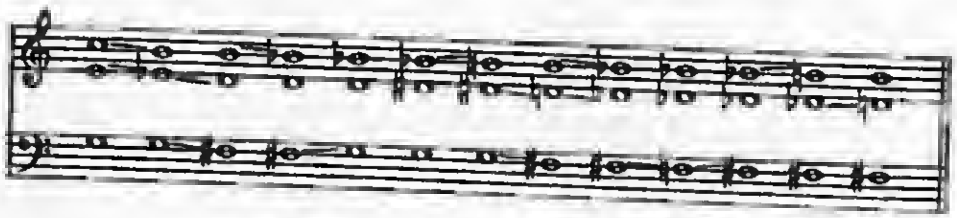


Figure 291. Varying the sequence in ascending and descending directions.

It is important to note that the *direction can be changed only after all three voices have performed their moves.*

A. In Four-Part Harmony

In four-part harmony any form of S(7) may be selected as a starting point. This offers 7 forms for one voice moving at a time. Considering the sequence SATB to be one group, we obtain $4 \times 12 = 48$ chords for the descending and as many for the ascending progressions.

As there are 7 forms of intonation to each progression, we obtain: $7 \times 24 \times 2 = 336$ forms, i.e., 7 forms times 24 variations of the SATB sequence, times 2 for the descending and the ascending directions.

The number of combinations out of four elements taken two at a time is:

$$_4C_2 = \frac{4!}{2!\,(4-2)!} = \frac{24}{4} = 6$$

These combinations by two refer to two simultaneous moves to be used independently, i.e., *without compensation.*

S A ; S T ; S B ; A T ; A B ; T B.

In using these combinations by free choice, remember that the direction may be changed only after the participation of all four voices in equal quantities and regardless of the combination selected.

For example:

S↑ A ; A↑ T ; B↑ ; S↑ T ; B↑ represents a group in which all voices participate twice.

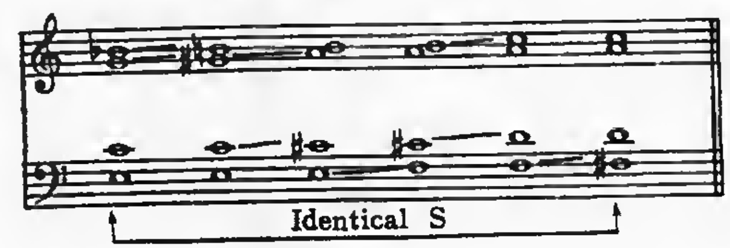


Figure 292. All voices participate twice.

Another form of four-part automatic chromatic continuity is based on the compensation of pairs of the simultaneously moving voices.

The following forms are available:

(1) S A ; T B

(2) S T ; A B

(3) S B ; A T

(4) S T ; A B

(5) S B ; A T

(6) S A . T B

As the first of the two combinations in each group is compensated by another combination *by two*, this includes the reciprocals as well.

Still another form of automatic chromatic continuity is produced by simultaneous motion of three voices (in any combination) followed by motion of the one remaining voice.

The following groups are available:

(1) S A T B ; (2) S A T B ; (3) S A T B ; (4) S A T B .

$${}_4C_3 = \frac{4!}{3!\,(4-3)!} = \frac{24}{6\cdot 1} = 4$$

Single, double and triple movements may be arranged in any desirable sequence, provided that there is no conflict with the principle that the number of moves in the adjacent groups must equalize in one direction.

Figure 293. Combined devices in automatic chromatic continuity.

(In the above, the identical structures are marked.)

In many cases of four-part continuity, especially with the single moves, classical forms of *suspensions* and *anticipations* take place.

(a) Original Structure: S(7) = 4 + 3 + 4.
Sequence: TASB

(b) Original Structure: S = 3 + 3 + 5.
Sequence: TSAB

See the corresponding music examples on the following page.

Figure 294. Four-part continuity with single moves.

CHAPTER 17

HYBRID HARMONIC CONTINUITIES

PURITY of harmonic style is more inherent in the music of those composers who were born at a time when they could crystallize past experiences along identical technical lines. Palestrina, J. S. Bach, Wagner, Chopin, Scriabine, Ravel, Debussy, Hindemith—all have sufficient unity in their harmonic expressions.

For practical purposes, however—especially in the field of "arranging" when re-harmonization of a song is desirable—it is sometimes necessary to produce harmonic styles that are intentionally hybrid.

We shall consider that ***the mixture of diatonic, symmetric and chromatic forms is hybrid.***

This type of harmonic continuity requires quick changes from one type of harmony to another. The reason for this is that our ears get used to one type very quickly; an instantaneous change to another type, when the habit is already formed, often produces an undesirable disturbing effect. The diatonic type conflicts strongly with the symmetric. It becomes necessary to separate the two one from another by means of the chromatic type, which is more neutral in character.

The **first** necessary condition for successful mixing of harmonic types is ***the insertion of the chromatic type between the diatonic and the symmetric.*** This can be expressed by the following diagram:

diatonic — chromatic — symmetric
$\longleftarrow\!\!\longrightarrow$

Hybrid harmonic continuity may be of any desirable length, providing that the diatonic and the symmetric have no immediate contact. For example: di + ch + + sy + ch + sy + ch + di + ch + di.

The **second** requirement for the successful execution of the hybrid continuity concerns the ratios in which the three different types appear. As the chromatic type neutralizes the effect of the preceding type (whether diatonic or symmetric), it is necessary to have more of it.

The most desirable of the simple ratios for this purpose is: di + 2ch + sy.

The **third** requirement concerns the quantities expressing the ratio. In moderate tempo, approximately two or three chords are a desirable unit. In fast tempo the quantity should be increased accordingly. Thus, the average form of hybrid harmonic continuity ($\overrightarrow{Hy}$) can be expressed as follows:

$$\overrightarrow{Hy} = di3H + ch6H + sy3H$$

When the above requirements are actually fulfilled, the resulting music may achieve a very high quality.

The inclusion of one more refinement guarantees the utmost smoothness to such progressions. This becomes particularly important when music is intended for mass consumption—as in dance music, for instance.

The refinement consists of maintaining an ***identical intervallic root-relation*** between ***the last two chords of the preceding chromatic group*** and between ***the last chord of the chromatic group and the first chord of the following symmetric group.*** Further relations of the symmetric group are not influenced by this.

Figure 295. Maintaining identical root relations.

In the above example, identical steps occur at two successive points: between E and F (the last two chords of ch) and between F and F♯ (the last chord of ch and the first chord of sy). As the figure shows, the subsequent relations of the symmetric group (6 + 9 + 9 semitones) are not influenced by the preceding identity of steps.

Figure 296. Hybrid harmonic continuity: di+ch+sy+ch+sy+ch+di+ch+di

CHAPTER 18

LINKING HARMONIC CONTINUITIES

When contrasts between analogous portions of harmonic continuity are desirable, the latter may be bridged by harmonic *connections* of a *different* type. The degree of contrast between the continuity and the connections ("bridges") depends on the type of progressions used in both. One can easily recognize a composer by the type of continuity and connections he uses. For instance, it is typical of Wagner to make *symmetric* connections between the portions of *diatonic* continuity. The starting point of each consecutive section is in the $\sqrt[4]{2}$ relation to the preceding section.

Figure 297. The "Pilgrims' Chorus" from Tannhäuser.

Assuming that any form of harmonic progression is used either as continuity or as connection, we obtain the following nine forms of *combined harmonic continuity*.

(1) diatonic progressions, diatonically connected;
(2) diatonic progressions, symmetrically connected;
(3) diatonic progressions, chromatically connected;
(4) symmetric progressions, diatonically connected;
(5) symmetric progressions, symmetrically connected;
(6) symmetric progressions, chromatically connected;
(7) chromatic progressions, diatonically connected;
(8) chromatic progressions, symmetrically connected; and
(9) chromatic progressions, chromatically connected.

Each form of combined continuity provides a certain amount of variation within its own limitations.

In the first case above, *progressions* may be developed through diatonic cycles (No. 1), whereas the connections will be developed through groups with passing chords. This can be done in reverse as well by connecting one form of group with a cycle *alien* to the group. For instance, G_3^4 (in which C_{-5} and C_5 participate) can be connected to the following G_3^4 through C_3 or through C_7.

When diatonic progressions of cycles are connected through G_6 or G_3^4, there is one extra chord within the connection, i.e., the first chord of the group is the last chord of the preceding progression, the middle chord of the group is the extra chord and the last chord of the group is the first chord of the succeeding progression.

Figure 298. Diatonic progression connected through G_6.

This form of combined continuity requires the *exact recurrence of the cycle group*; however, the *positions* of chords as well as their *forms of tension* may be varied in each subsequent progression of one continuity.

When groups are connected by a cycle, there is no extra chord to be gained.

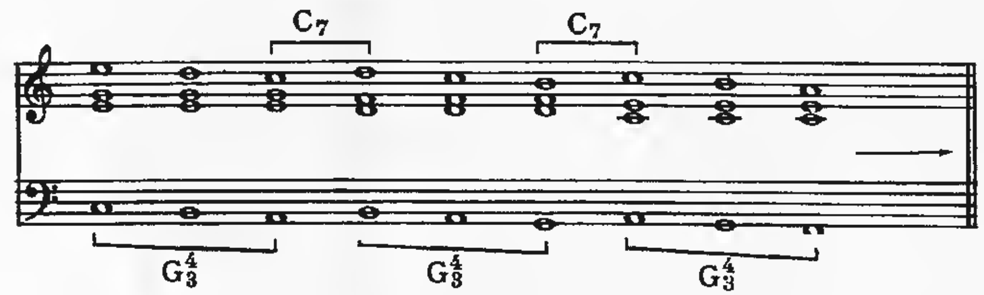


Figure 299. Groups connected by a cycle.

The diatonic connection of symmetric progressions (No. 4) may be accomplished by assuming that *the last chord of the symmetric progression belongs to a certain key*. Thereupon, through one cycle connection the harmony affirms the assumed key in which the subsequent symmetric progression then begins. There is no extra chord appearing during the connection. All forms of symmetry may be used.

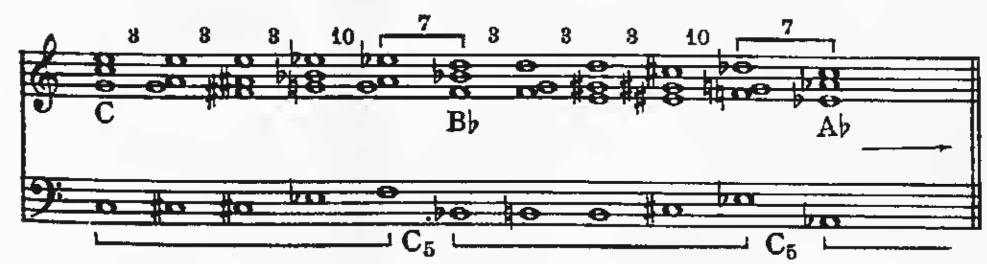


Figure 300. Diatonic connection of symmetric progressions.

Symmetric connection of symmetric progressions (No. 5) must be based on selection of such forms of symmetry as do not appear in the progression itself.

Figure 301. Symmetric connection of symmetric progressions.

A symmetric connection of diatonic progressions (No. 2) does not produce an extra chord, but rather an interval ($\sqrt{2}, \sqrt[3]{2}, \sqrt[4]{2}, \sqrt[6]{2}, \sqrt[12]{2}$). Such a connection may be planned *either* in relation *to the first* or *to the last chord* of the diatonic progression. In Figure 290 the connection through $\sqrt[4]{2}$ referred to the first chord of each diatonic progression.

Figure 302. Symmetric connection of diatonic progressions.

Diatonic progressions may be connected through any form of chromatic group (No. 3) (parallel or contrary). An extra chord is gained through such a connection. This type of combined continuity, incidentally, usually sounds like diatonic harmony with modulations.

Figure 303. Chromatic connection of diatonic progressions.

The chromatic connection of symmetric progressions (No. 6) introduces one extra chord. Any form of chromatics can be used. It is desirable that the interval between the extreme chords of the chromatic connection should not duplicate any steps of the symmetric progression.

Figure 304. Chromatic connection of symmetric progression.

The diatonic connection of chromatic progressions (No. 7) is achieved by assigning the last chord of a chromatic progression to the key in which such a chord may exist. The latter is connected by a diatonic cycle with some other chord in the same key. Thereupon the chromatic progression is resumed. There is no extra chord gained in the cycle connection. Chromatic progressions (consisting of one or more chromatic groups of any type) may be varied after each diatonic connection.

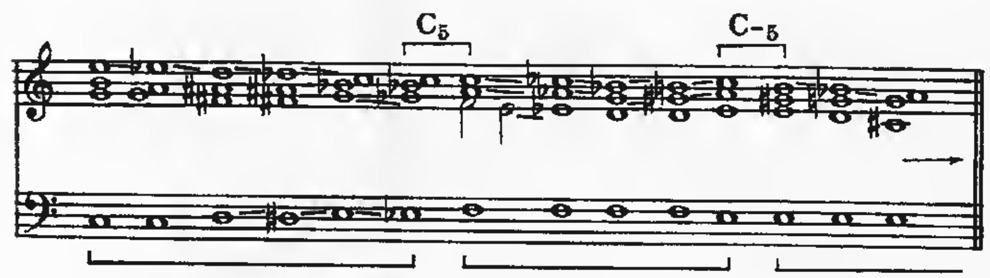


Figure 305. Diatonic connection of chromatic progressions.

Symmetric connection of chromatic progressions (No. 8) is achieved through the selection of a root which does not produce chromatic steps in any voice. There is no gain of an extra chord.

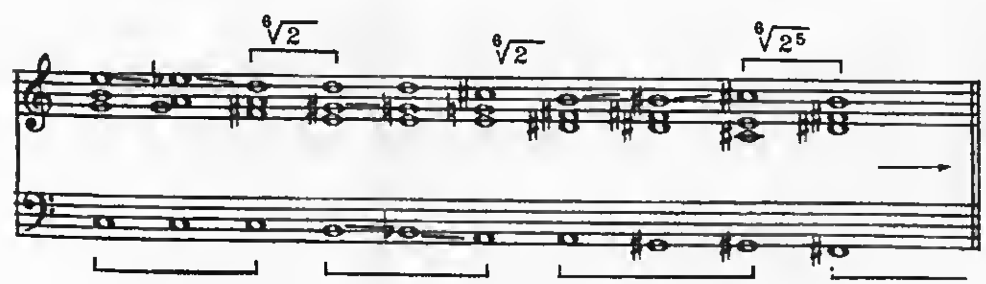


Figure 306. Symmetric connection of chromatic progressions.

The chromatic connection of chromatic progressions (No. 9) may be accomplished by introducing contrasting forms of chromatics. Contrasts may be achieved by the juxtaposition of parallel and contrary chromatics, or by the juxtaposition of chromatics and enharmonics. An extra chord is gained by such connections.

Figure 307. Chromatic connection of chromatic progressions.

The above nine forms of combined harmonic continuity can be further combined ***by varying the forms of connection between each set of progressions of any one particular type.***

CHAPTER 19

A DISCUSSION OF PEDAL POINTS

PEDAL Point or Organ Point (P.P.) is a primary axis about which chord progressions are formed. The various patterns of motion which the remaining voices produce in relation to the pedal point consequently result in different effects corresponding to the axial combinations, including the 0-axis.*

P.P. is primarily conceived as a sustained bass, but by means of vertical rearrangement of parts, one can achieve the appearance of a P.P. in any desirable voice. We shall compose pedal points first as basses.

P.P.$\frac{0}{0}$, i.e., a pedal point with a more or less stationary or a slightly revolving pattern for the motion of upper voices, produces either the effect of *accumulation* or *discharge of energy*—the first resulting from a *crescendo*; the second, from a *diminuendo*. In such a form, P.P. is used either *at the beginning* of a composition, mostly as an *introduction* ("take-off") or at the end of it, mostly as a *coda* ("landing"). The next stage of dramatic expression is obtained through the use of several secondary axes against P.P. The following may be considered as fundamental combinations.

$$\text{P.P.}\ \frac{(a+b)+\cdots}{0} \quad \text{and} \quad \text{P.P.}\ \frac{(b+a)+\cdots}{0}$$

In some cases, the pedal point leads to a climax. Then the entire P.P. serves as a form of accumulation of energy followed by discharge (climax). In such cases P.P. is associated with *crescendo* and requires a *prolonged a-axis* for the upper parts of harmony. The climax itself is the ultimate forte.

$$\text{P.P.}\ \frac{a}{0}$$

After such a vigorous climax, an anti-climax—i.e., *moving toward ultimate balance*—is often necessary. Being usually a coda or an episode preceding recapitulation, this requires a gradual dissipation of energy which can be expressed through the use of an *extended b-axis* of upper voices in relation to P.P. The dynamic form for this is *diminuendo.*

$$\text{P.P.}\ \frac{b}{0}$$

The devices of this theory of harmony supply all the necessary forms by which the patterns of harmonic motion—as expressed through tonal cycles in relation to the quantity of voices—may be obtained at will.

For instance, an a-axis for three parts may be obtained through C_3 const., as well as through some techniques of ascending chromatics. For a gradual ascent, $C_3 \circlearrowright$ and $C_6 \circlearrowleft$ may be used. For ascent in leaps, $C_3 \circlearrowleft$ is the corresponding technique. All the negative forms of continuous S(7) or any other structures in the hybrid five-part harmony produce the same axis.

*The *axes* referred to are those of the theory of melody, i.e., a, b, c, d and 0. (Ed.)

It often happens that the number of parts in harmony determines the patterns of motion under the same cycle. For example, four tonics S(9) const., i.e., C_3, are stationary in five parts but climb quite decisively in four (the three upper parts of the hybrid four).

A vigorous alternating (ascending-descending) motion may be achieved through the use of $C_7 \circlearrowright$ in three parts (see S(5) in C_7).

An extended descent, i.e., b-axis, may be obtained through the use of continuous S(7) in four-part harmony, through the $\circlearrowleft C_7$ (thus including the six and the twelve tonics), through $\circlearrowright C_5$, as well as by means of descending chromatics.

An oscillating pattern, which may be considered to be an 0-axis in that it has but limited amplitude, may be achieved through the alternate use of positive and negative forms in any type of harmony where parts move through limited melodic intervals. The technique of continuous S(7), alternating the positive and the negative forms under limited coefficients, produces such patterns. For example: $3C_5 + 2C_{-5} + 2C_7 + 3C_{-7} + \ldots$ etc.

The *reversal* of all of the above-described considerations may still serve the purpose, *provided that dynamics becomes a more influential factor than harmony.*

The effect of "vanishing" or "flying away" may be accomplished by what we consider dissipation of energy or moving toward balance. However, we have conditioned associations with the quantity of sound and volume. The use of an a-axis combined with a crescendo leads to a climax—yet the very same axis combined with diminuendo associates itself in our perception with an object growing smaller and flying away in an upward direction. This is due to the influence of the concepts "high" and "low" in reference to frequencies, transformed into spatial analogies. Flying away in an upward direction, which to us as human beings is associated with extreme tension (overcoming of gravity, and the effort necessary to accomplish it), corresponds to the increasing tension of our vocal chords, which tension we all mimic *sympathetically* in infinitesimal degrees while listening to music. Thus, vanishing in mid-air through visual association with the gradual diminution of an object corresponds to the *0-axis combined with diminuendo*; whereas, vanishing into the ground from above requires the *b-axis combined with diminuendo*. This analysis throws light on the many different effects achieved in music by means of pedal point.

An obvious example of the effect of "vanishing" in an upward direction is the brief pedal point in Rimsky-Korsakov's *Scheherazade*, accomplished by the most humble harmonic means.

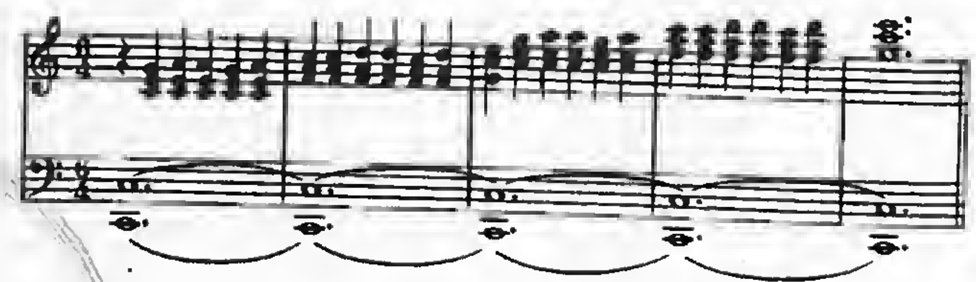


Figure 308. "Vanishing" in upward direction.

In the first movement of Beethoven's Sonata No. 8 (the *Pathetique*), the first four measures of the *allegro con brio* produce a "take-off" by means of $\frac{a}{0}$ axis; this establishes a firm foundation for the music to follow. In the same movement, the third theme begins with a two bar $\frac{0}{0}$ axis pedal point (e♭), accumulating energy for the following diverging texture of broken chords: $\frac{a}{d}$ axis. In the same movement, there is a climactic pedal point preceding a recapitulation. This pedal point on g (the dominant of the key), being of intensely revolving character, accumulates a tremendous energy (due to the crescendo), which energy is dissipated in a duly extended descending (b-axis) passage leading into the recapitulation.

A. Classical Pedal Points

It is regrettable that the manner in which classical composers used the various forms of pedal point is not now generally known to the more prominent contemporary composers; the evidence of this consists of the misplaced pedal points in their own works. In order thoroughly to understand the *classical* approach to this problem, it is first important to classify and define the traditional forms of pedal point.

The two fundamental forms of the classical pedal point are: P.P.T., i.e., *pedal point on the tonic*, and P.P.D., i.e., *pedal point on the dominant.*

P.P.T. affirms the harmonic axis of the composition, i.e., it *establishes the key.* Technically, P.P.T. may usefully be defined as an extension of the ecclesiastic plagal cadence.

The cadence itself consists of *Tonic + Subdominant + Tonic*, which usually appears as $I + IV_{\frac{6}{4}} + I$ $(T + S_T + T)$. Here T is the tonic, S_T—the subdominant with a tonic characteristic. This form has for long been used in many sung prayers of the Christian church of different denominations and is usually associated with the intonation formula for "Amen," although it appears very frequently, too, at the beginnings. The device was undoubtedly used in order to help the singers "to tune-up"; it is most prominent in music of the Russian-Orthodox Greco-Catholic church.

Figure 309. Ecclesiastical plagal cadence.

This form of plagal cadence later developed into the form using II S $\binom{6}{5}$ and VII S$\binom{4}{3}$. It often appears as a pedal point on the tonic.

Figure 310. Classical pedal point on tonic.

P.P.T. is either an *initial* (to be used at the very beginning) or the *final* (to be used at the very end) pedal point. It is a sort of airdrome, we might say, from which the flight has to be begun. Thus the initial P.P.T. corresponds to the "take-off", and the final P.P.T. corresponds to the "landing."

But, P.P.D., on the other hand, is a *climactic* pedal point; it corresponds to the *apex* of a flight. Technically P.P.D. may be defined as an extension of the compound authentic cadence or, more specifically, as *an extension of the cadential* $I\ S\binom{6}{4}$.

The cadence from which such a pedal point derives is as follows:

$$T + S + T_D + D + T$$

Here T is the tonic, S is the subdominant, T_D is the tonic with a dominant characteristic, D is the dominant.

It can be symbolized in the following form:

$$I + \underset{(II)}{IV} + I_{\frac{6}{4}} + \underset{(III)}{V} + I$$

The actual P.P.D. starts on $T_D = I_{\frac{6}{4}}$.

P.P.T. begins with the tonic and ends with a plagal cadence (S + T). P.P.D. is prepared by a subdominant (S), starts with T_D ($I_{\frac{6}{4}}$) and ends with an authentic cadence (D + T). The development of the progressions which occur above P.P. will be discussed later.

As P.P.T. signifies either the beginning or the ending, it usually appears in the introduction or at the beginning of the first theme, or in the conclusive theme or as a coda. P.P.D., which signifies climax, usually appears before the recapitulation.

More than one pedal point can be used in sequence in the course of one composition or one movement. Let us take the most typical scheme of *thematic distribution*—A + A_1 + B + A_1—where A and A_1 are the modified expositions of one theme. B is the contrasting theme (middle strain in a song), and A_1 (at the end) is the recapitulation of the first theme (usually in an abbreviated form).

Consider A to include the introduction and the second A_1 to include the coda. We may then chart all the possible combinations of pedal points to be used in a typical scheme of thematic distribution.

	A	A_1	B	A_1
(1)	P.P.T.			
(2)			P.P.D.	
(3)				P.P.T.
(4)	P.P.T.		P.P.D.	
(5)	P.P.T.			P.P.T.
(6)			P.P.D.	P.P.T.
(7)	P.P.T.		P.P.D.	P.P.T.

Figure 311. Traditional location of pedal points.

In the last two cases, P.P.D. is often immediately followed by P.P.T.

The two predominant types of classical pedal point are the *diatonic* and the *chromatic* pedal points.

B. Diatonic Pedal Point

Diatonic P.P.D. or P.P.T. consists of the free use of the diatonic cycles in both the positive and the negative form. It must satisfy all the requirements as to the proper start and proper cadences.

P. P. D.

P. P. T.

Figure 312. P.P.D. + P.P.T. in diatonic type.

The three parts above the pedal point in the above figure are devised by means of classical and hybrid transformations. It is useful to know the structural specifications for the chords appearing above the pedal point and devised in three parts. They are:

5 3 1	3 1 1	7 3 1	7 5 1	9 7 1
S(5)	S(5)	S(7)	S(7)	S(9)

Figure 313. Structural specifications of chords in figure 312.

As to their transformations, they have to be treated as abc, which corresponds practically to a mixture of classical and hybrid techniques.

If all chords above the P.P. are S(5) in three parts, then the classical transformations cover the field completely (↻, ↺ and const. 3).

Continuous four-part setting above the P.P. corresponds to the technique of S(7) const. Note that this device produces very expressive pedal points reminiscent of those of J. S. Bach and Händel, particularly when modes, harmonic major, melodic major or melodic minor are used. All that is necessary is the addition of a stationary bass to the upper four parts moving as seventh-chords.

Figure 314. Continuous 4-part setting above P.P. corresponds to technique of S(7) const.

C. Chromatic (Modulating) Pedal Point

Classical modulating pedal points (P.P.D. and P.P.T.) consist of a rapid succession of key-to-key transitions. The latter are usually performed by means of the chord on the VII or V of the following key; such a limitation is not necessary, however, and any other intermediate chords may be used.

The most important—and heretofore unsolved—problem is that of the particular key selection to be evolved above the pedal point.

As P.P.D. is associated with the *authentic cadence* ($I_{\frac{6}{4}} \overline{C_{-5}} V$), its natural tendency is to modulate through a chain of dominants, i.e., through the keys in C_{-5} relation—which amounts to moving *toward sharps*.

The natural tendency of P.P.T. is to modulate through the chain of *subdominants*, i.e., through the keys in C_5 relation—i.e., in the direction of *flats*. This is due to the fact that P.P.T. is associated with the plagal cadence ($I \rightarrow IV^{6}_{4}$). (Small letters represent the *minor mediants*: lower and upper).

Table of natural key tendencies for modulations in P.P.T. and P.P.D.

	e —	a —	d —	g —	c —	f —	b♭ —	e♭
P.P.T.:	C —	F —	B♭ —	E♭ —	A♭ —	D♭ —	G♭ —	C♭
	a —	d —	g —	c —	f —	b♭ —	e♭ —	a♭
	e —	b —	f♯ —	c♯ —	g♯ —	d♯ —	a♯ —	e♯
P.P.D.:	C —	G —	D —	A —	E —	B —	F♯ —	C♯
	a —	e —	b —	f♯ —	c♯ —	g♯ —	d♯ —	a♯

Figure 315. Natural key tendencies for modulations in P.P.T. and P.P.D.

However, the natural tendency has only a partial influence in the selection of keys for modulating pedal points.

The main factor, usually neglected in academic musical theories, is *the sonority of the tonic* (*I*) *S*(*5*) of the respective key *in its relation* to pedal point. This can be defined in the form of a requirement, which is: *only those keys may be selected for the classical type of pedal point modulation in which I S*(*5*) *taken together with P.P. produces a crystallized structure acceptable in the established four-part harmony.* For instance, it is *wrong* to modulate to the key of D-minor on a P.P.D. in the key of C-major (or C- minor), for the unit g in the bass produces—together with the upper parts—a structure of 1, 5, 7, 9 for S(9), which is not the accepted form, 1, 3, 7, 9. In this case, even though the key tendency is correctly carried out, the result is not satisfactory in sonority.

On the other hand, a key selection which may be contrary to the natural tendency, such as F-minor above the P.P. on g, is perfectly satisfactory as I S(5) in that key, for together with the bass it produces an accepted form of S(11): 1, 7, 9, 11.

The above structural requirement *excludes* the following keys in relation to a pedal point on *c*:

F♯ — minor
G — major
G — minor
G♯ — minor
A — major

All other keys are fully acceptable. An allowance is made for F♯ major because the sonorous quality of the $\sqrt{2}$ is very desirable. The best sounding position for f♯ $S_1(5)$ above *c* is when the pitch-unit nearest to the bass is *a*♯. The latter produces an equivalent of *b*♭ and furnishes a perfect acoustical support for *c*♯ and *f*♯.

Figure 316. Chromatic (modulating) pedal point.

D. Symmetric Pedal Point

Progressions of types II and III and the generalized symmetric progressions may be included in this group. All these types have more or less the same characteristics when used above the pedal point. Any forms of three and four-part harmony may be used.

The pedal point itself is the main (original) tonic. In cases of the generalized symmetric progressions, it is the root-tone of the chord with which the pedal point begins.

Figure 317. P.P. with harmonic progressions of type II, III and generalized.

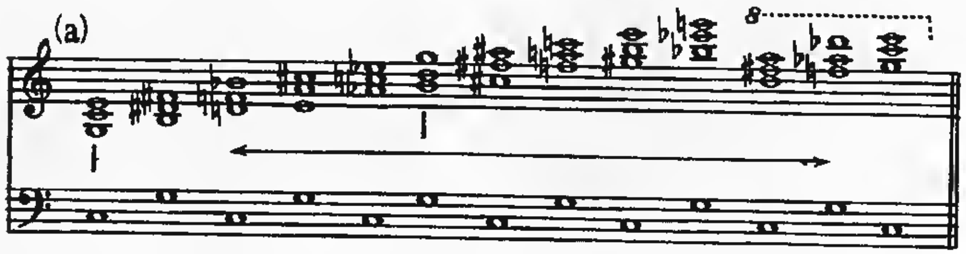
A more extensive form of the symmetric pedal point (type III) may be devised through a group of pedal points, each tonic becoming a pedal point in succession. The remaining parts form progressions through the same system of symmetry.

Figure 318. An extended symmetric (type III) P.P.

Special Case

There is a special case in which diatonic alternate pedal points on the tonic and the dominant merge with the symmetry of the $\sqrt[12]{2}$ on $S_1(5)$. This is an equivalent of the entire chromatic scale versus tonic-dominant. $S_1(5)$ is *the only* satisfactory form of sonority. Note the coincidence of the tonic and the dominant as S(5).

(a)



(b)

Figure 319. Alternate diatonic P.P. on tonic and dominant merge with symmetry of $\sqrt[12]{2}$ in $S_1(5)$.

CHAPTER 20

MELODIC FIGURATION

Preliminary Survey of the Techniques

THE technique of melodic figuration* consists mainly of the process of evolving *leading tones* for chordal tones in a given harmonic continuity. When leading tones move into chordal tones, they produce *directional units.* Melodic figuration can be defined as a process of transforming *neutral* units (chordal tones) into *directional* units.

A. Four Types of Melodic Figuration

There are three types of leading tones satisfying such a definition:

Type one: *suspended tones* (suspensions), i.e., tones belonging to the preceding chord and held over; such tones must be moved into an adjacent chordal tone.

Type two: *passing tones,* i.e., pitch-units inserted between two other pitch-units moving in sequence and constituting chordal tones. Passing tones may, or may not, belong to the same scale as that in which the harmonic continuity has been evolved. In the first case, they are *diatonic* passing tones; in the second, *chromatic* passing tones.

Chromatic passing tones were discussed earlier under their own heading; here, only diatonic passing tones will be discussed.

Type three: *auxiliary tones,* i.e', unprepared leading tones selected with no regard to basic pitch scales. They too, can be either *diatonic* (in which case they have an "ecclesiastic" flavor) or *chromatic* (in which case they add an extreme lyrical expressiveness, due to sudden intensifications, to the music). Chromatic auxiliary tones are one of the most powerful resources of expression in the music of Mozart, Schubert, Chopin, Chaikovsky,** Scriabine and, in some instances, Wagner—as in *Tristan and Isolde.* Contemporary popular songs dealing with love or despair are overloaded with this device.

The fourth type of melodic figuration is based on a technique different from the evolution of leading tones: it introduces certain chordal tones (one or more) of the *following* chord into the *preceding* chord. This device is known as *anticipated tones* or anticipations. It has long been neglected because composers, for some reason, associate anticipations with antiquated harmonies. But it becomes a very important source of harmonic expression when used in harmonic continuity of a more developed type.

*By melodic figuration is meant the process of converting a harmonic continuum into a partially melodic continuum, the melodic characteristics being introduced into the continuum itself—in contrast to the *melodization of harmony,* which means the fabrication of a melody to *go with* a given H⟶. (Ed.)

**Schillinger, Russian-born, preferred this simplified spelling to the more commonly accepted form, "Tschaikowsky." (Ed.)

B. Development of Suspensions

The effect of a suspension is to intensify the chord by means of common tones which, while being suspended, rise in rank as a chordal function after which rise they are then released. Every suspension consists of three consecutive phases: preparation, suspension, and resolution. Our ears, due to heredity and habits, accept a suspension only on a strong beat. The source of this habit is strict counterpoint, in which dissonances were only permitted on weak beats and on strong beats, by suspending ("tying over") a common tone. Classical harmonic structures had not been fully crystallized at the time suspensions were used in counterpoint this way, and so these suspended tones produced antiquated harmonic structures resembling those of the old *organum* type.

One of the most common suspensions was the $7 \uparrow 11$, which, at the moment of suspension, produces the structure: $S(11) = 1, 5, 11$. Naturally, such a structure fails to conform to the later classical form—and, although Mozart had already felt the need of a more nearly perfected structure at the moment of suspension, theories of harmony even today continue to advocate this most antiquated form.

The following figure illustrates the evolution of structure under suspension. It has been gradually realized that it is necessary to support the eleventh by the ninth; and the ninth, by the seventh.

Figure 320. Evolution of structure under suspension.

The historical crystallization of S(7) as an *independent* structure goes back to the 18th century; the crystallization of S(9) goes back to the middle of the 19th century.

This analysis may well lead us to the conclusion that it is essential that the structures under suspension conform to these crystallized forms of S(7), S(9) and S(11).

A single suspension requires an S(7) in which the suspended tone is a prepared 7th.

A double suspension requires an S(9) in which the suspended tones are the prepared 7th and 9th.

A triple suspension requires preparation of the 7th, 9th and 11th in an S(11).

Similar considerations require that passing tones (diatonic) conform to crystallized chord-structures. This means that the single passing tones must be *passing 7ths*, double passing tones must be *passing 7ths and 9ths*, and triple passing tones must be *passing 7ths, 9ths and 11ths*.

In addition, some groups with passing chords give other double passing tones in parallel (G^4_3) or contrary (G^6_4) motion.

All other cases are crude and antiquated; they create harsh and empty-sounding gaps when orchestrated.

It should not be forgotten that the best composers of the 18th century, such as Mozart and Scarlatti, through constant use of correct suspensions, helped to crystallize the structures of the future, such as S(9).

Figure 321. Use of suspensions in the 18th century.

As classical theory offers suspended and passing tones under positive forms which are always descending, it would be important to have as secure a system for devising *ascending* resolutions of suspensions and for ascending passing tones. Such a system of melodic figuration would exist as a normal one under cycles of consistently negative form. For practical purposes, it is expedient to invert the positive form into ⓓ, either the geometrical or the tonal (i.e., without any changes of accidentals of the original), rather than to think of the 1 as "a negative 7th," the 3rd as "a negative 5th," etc.

The techniques of melodic figuration are applicable to all types of harmonic progression in close, open or mixed positions in four- and five-part harmony.

CHAPTER 21

SUSPENSIONS, PASSING TONES AND ANTICIPATIONS

ALL the elements of melodic figuration may be classified according to direction (ascending, descending), according to chordal functions employed (1 - 13), according to their adherence to scale (diatonic, chromatic)—and, in addition, according to the number of elements simultaneously employed.

The elements	Number of elements employed:	Direction:	Chordal functions employed:	Adherence to scale:
Suspended Tones	Single	↘	↑ 7	Diatonic
	Double	↘ ↘	↑ 9 ↑ 7	Diatonic
	Triple	↘ ↘ ↘	↑ 11 ↑ 9 ↑ 7	Diatonic
Passing Tones	Single	↘	↘ 7	Diatonic
	Single, Double and Triple	Parallel and contrary motion ↗ and ↘	Whole tone interval	Chromatic
	Double	↘ ↘	↘9 ↘7 | G_4^6 ↘ ↗ | G_3^4 ↘ ↘	Diatonic
	Triple	↘ ↘ ↘	↘ 11 ↘ 9 ↘ 7	Diatonic
Auxiliary Tones	Single	↗ ↘	any function	Diatonic Chromatic
	Double	↗↗ | ↘↘ | ↗↘ | ↘↗	any functions	Diatonic Chromatic
	Triple	↗↗↗ | ↘↗↗ | ↗↘↗ | ↗↗↘	any functions	Diatonic
		↘↘↗ | ↘↗↘ | ↗↘↘ | ↘↘↘		Chromatic
Anticipated Tones	Single		any function	
	Double		any functions	
	Triple		any functions	

Figure 322. Elements of melodic figuration.

A. Types of Suspensions

Single Suspensions: Single suspended tones may be obtained by making the 1, the 3 or the 5 become a prepared 7. The functions, 1, 3 and 5, serve as the preparation; the 7th, as a suspension; and the nearest function one step lower, as a resolution.

Figure 323. Single suspensions.

Double Suspensions: Double suspended tones may be obtained by making the 1 and 3, or the 3 and 5, or the 5 and 7 become a prepared 7 and 9. In a continuity of double suspensions, one of the suspended voices may appear in the *bass*, thus producing an inversion of S(9). The voice-leading in such a case remains usual, i.e., the remaining two voices must furnish 1 and 3.

Figure 324. Double suspensions.

Triple Suspensions: Suspending the 1, 3 and 5, and the 3, 5 and 7 as 7, 9 and 11 produces triple suspensions.

Figure 325. ~~Triple~~ Triple suspensions.

Mixed Suspensions: By combining single, double and triple suspensions, one can achieve considerable variety:

Figure 326. Mixed forms of suspensions.

Ascending Suspensions: We may also obtain *ascending* suspensions by inverting the positive form into position ⓓ.*

Geometrical Inversion.

Figure 327. Geometric inversion: figure 326 in position ⓓ.

*As explained in Book III concerning geometrical and tonal inversions—position ⓓ, the reader will recall, is the original upside down; a geometrical inversion preserves the exact intervals, whereas a tonal inversion alters the intervals to conform to the original or some other key. (Ed.)

Tonal Inversion: By canceling the accidentals or by readjusting them, we obtain the same, but in the original—or any other—key.

Figure 328. Tonal inversion.

B. Passing Tones

Single Passing Tones: Single passing tones may be produced by moving the 1 downward to 7, stepwise. The particular sequence of voices in which the passing tone will then appear depends upon the particular cycles and the voice-leading.

If passing tones are desirable in one continuous voice, the procedure should be carried out through the procedure I have already described in *generalization of the passing seventh.** A progression of $S(5)C_3$ const. must be written first; the passing sevenths are then inserted afterward. When this procedure has been completed, the bass may then be placed into any other voice (geometrical variation of positions).

Figure 329. Single passing tones.

*See pages 534-6. [Ed.]

Double Passing Tones: Double passing tones are either the passing 7 and 9 obtained by means of downward motion from the 1 and 3, or the 3 and 5, or the 5 and 7—or are parallel and contrary passing tones of the group G^6_4 and G^4_3.

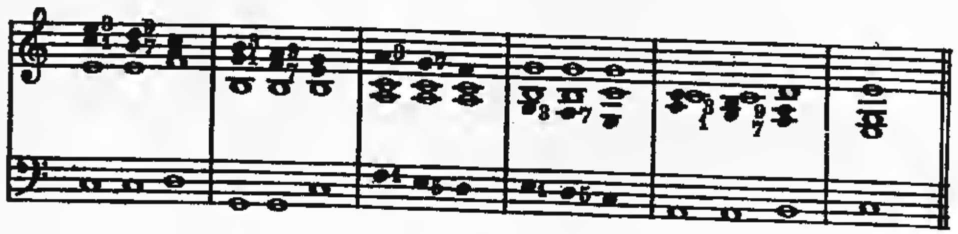


Figure 330. Double passing tones.

Triple Passing Tones: Triple passing tones are 7, 9 and 11 obtained by means of downward motion from the 1, 3 and 5; this corresponds to a preparation of S(11) in C_0.

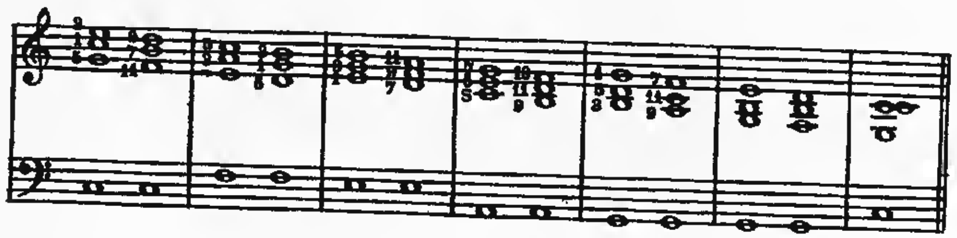


Figure 331. Triple passing tones.

Mixed Forms: Single, double and triple passing tones can be combine in one harmonic continuity.

Figure 332. Mixed forms of passing tones.

Since passing chromatic tones were discussed previously, there is no need for additional illustrations. Chromatic passing tones may be used in combination with diatonic passing tones.

Ascending Passing Tones: Ascending passing tones may be obtained through geometrical or tonal inversions ⓓ. Here are two inversions of Figure 332.

Figure 333. Geometrical inversion of figure 332.

Figure 334. Tonal inversion of figure 332.

Suspended Tones in a Given Chord Progression: So far we have discussed the techniques of suspended and passing tones evolved during the process of composing harmonic continuity—i.e., the $H^{\rightarrow}$ was not already set.

Now, however, we shall develop the technique of producing suspensions in a *given* harmonic continuity. In addition to the standard forms of suspensions, we shall use *delayed resolutions* for this technique—that is, suspensions in which the dissonant (higher numbered) functions become temporarily consonant (lower numbered)—and then resolve in the customary way. Root, third and fifth may also be suspended if the seventh is held. As long as the structures are properly represented during the period of suspension* any functions can be suspended.

Figure 335. Harmonic progression serving as a theme.

*That is, so long as the tones at the moment of suspension comprise altogether an acceptable (S) of some kind. (Ed.)

Figure 336. Variation by means of suspensions.

The simplest way to obtain *ascending* suspensions is to write the harmonic progression first, then to evolve suspensions, and finally to re-write the result into position d. Otherwise, the original harmonic progression must be written in a consistently *negative* form, and the suspensions must be evolved through 1, 1 and 3, or 1, 3 and 5—resolving upward.

Passing Tones in a Given Chord-Progression: By combining diatonic and the chromatic passing tones, a corresponding variation can be evolved. Using Figure 335 again as a theme, we obtain the following:

Figure 337. Variation by means of passing tones (Diatonic and chromatic).

C. Anticipations

Anticipated tones may be evolved from any chordal function of the following chord, provided that such a function is not the same in pitch as the voice in which the anticipation occurs. The nomenclature I use is: anticipated root, → 1; third, → 3; etc.

Single anticipated tones may be evolved to the root, to the third, to the fifth, to the seventh—or to any higher chordal function which is actually present in the following chord. Such forms may be called *anticipations of a constant chordal function.*

In addition to this form, *anticipations of variable chordal functions* may be used, and these may be selected at random. They provide greater variety in the quality of tension, whereas the first form provides a unity of tension. Both forms may be evolved for any harmonic continuity, as shown in the example:

Anticipated 1.

Theme: Type I

Figure 338. Anticipation of a constant chordal function.

Theme: Type II

Figure 339. Anticipation of a constant chordal function (continued).

Variation: → 1

Figure 339. Anticipation of a constant chordal function (concluded).

Theme: Type III

Variation: → 1

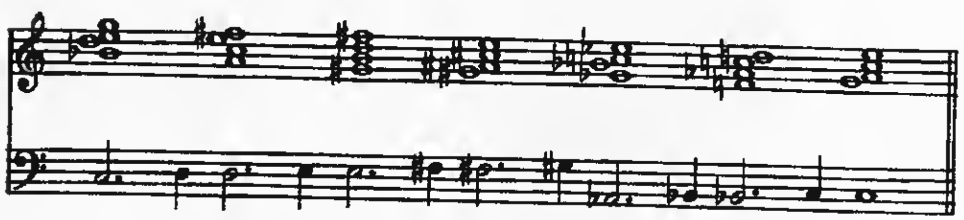
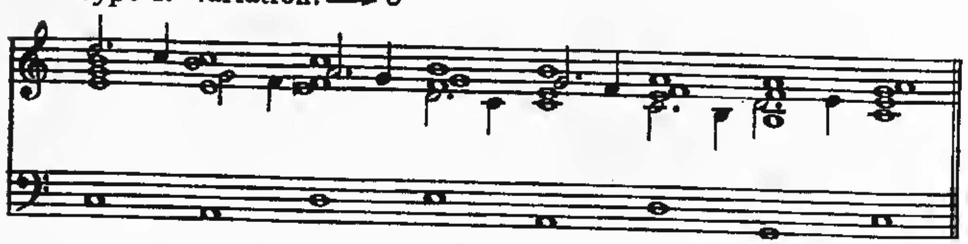


Figure 340. Anticipation of a constant chordal function

Anticipated 3.

Type I: Variation: → 3



Type II: Variation: → 3

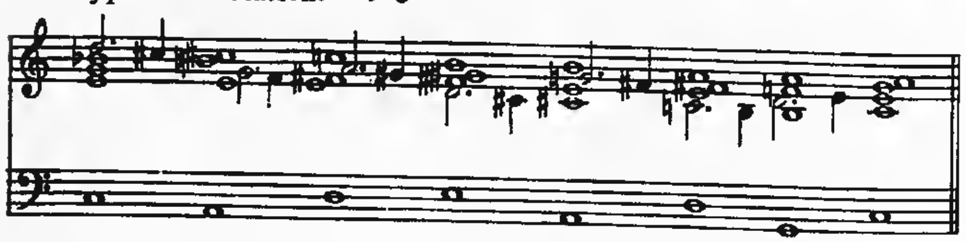


Figure 341. Anticipation of a constant chordal function (continued).

Type III: Variation: → 3

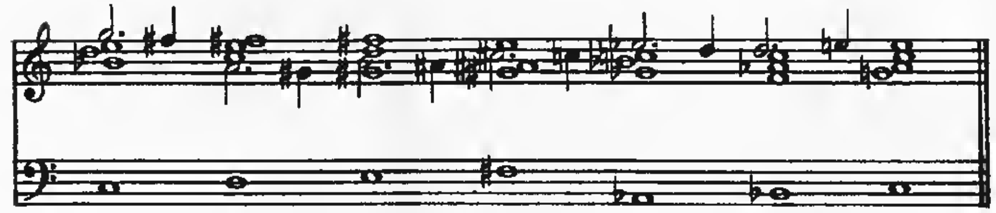


Figure 341. Anticipation of a constant chordal function (concluded).

Anticipated 5.

Type I: Variation: → 5
(common tone) due to cycles and structures

Type II: Variation: → 5
(common tone) due to cycles and structures

Type III: Variation: → 5

Figure 342. Anticipation of a constant chordal function.

Anticipated 7.

Type I: Variation: → 7
(common tone)

Type II: Variation: → 7
(common tone)

Type III: Variation: → 7

Figure 343. Anticipation of a chordal tone.

2. Anticipation of a Variable Chordal Function

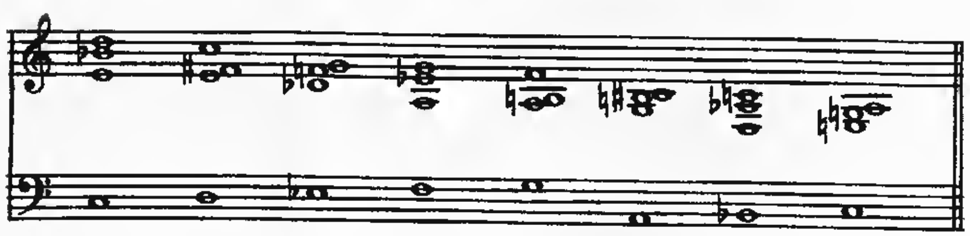
Theme: Figure 335.

Variation: variable anticipated chordal functions

Figure 344. Anticipation of a variable chordal function.

Double and Triple Anticipations

Theme: Type III: $\sqrt[5]{2}$, S(9) const.



Variation: Anticipation of variable chordal functions
(double and triple)

Figure 345. Double and triple anticipations.

Combined devices: suspended, passing and anticipated tones.

Theme: figure 335.

Variation: evolving combined devices to a given harmonic progression.

Figure 346. Combined devices.

CHAPTER 22

AUXILIARY TONES

AUXILIARY tones, being harmonically unmotivated, may be evolved in a given harmonic continuity. Any chordal function may be preceded by an upper or lower auxiliary tone. The interval *between* the auxiliary and the chordal tone depends on the type of harmony.

In diatonic progressions, auxiliary tones may be either diatonic—i.e., based entirely on those pitch units which produced the harmony itself and hence based on one definite diatonic scale—, or chromatic, i.e., arrived at by free selection among the leading tones which do not exist in the given scale.

As I noted earlier, diatonic auxiliary tones accentuate the diatonic character of the harmony and, because of previous associations, produce in us an impression of "ecclesiastic" music. This is particularly true when such scales as natural major are used. At the same time, some of the derivative scales of the same natural major, when they are supplied with diatonic auxiliary tones, may not produce this "ecclesiastic" impression but rather suggest such styles of harmonic writing as are to be found in compositions by Ravel and, particularly, by Debussy when these composers do express themselves diatonically.

Because of our previous experiences, we have established many auditory habits, among them an especially keen, critical perception of auxiliary tones. When chord progressions evolve diatonically from familar scales, we anticipate *a priori* certain definite forms of auxiliary tones. For example, in $S_1(5)$ the $\searrow 3$—i.e., an upper auxiliary tone (descending) to the third of a minor triad—must be $i = 2$, i.e., an interval of two semitones. If, instead, the movement is only one semitone, an ordinary listener will regard such an auxiliary tone as a "wrong note." The same is true for $S_2(5) \searrow 1$; our ears want the i (interval) to equal 2. The real cause of these reactions is the fact that ordinary listeners are familiar with the harmonic minor in which scale such auxiliary tones are diatonic. What seems wrong to the listener when $i = 1$ would sound perfectly natural if he were familiar, by ear, with d_2 of harmonic major; in such a case, in the key of C (c − d − e − f − g − a♭ − b), the root triad would be e − g − b and the discussed auxiliary tones would then be f → e and a♭ → g, with $i = 1$.

In harmonic progression of Types II and III, the auxiliary tones are governed by the master-structure (Σ) of each chord.* This means that the auxiliaries are diatonic during each individual chord. If a certain Σ produces $\nearrow 5$, $i = 1$, and such a Σ is used throughout, then all cases of $\nearrow 5$ must have $i = 1$. On the

*A Σ (the Greek letter, *sigma*) is an expansion of a scale; in Schillinger's *special* harmony, as in the present discussion, it is the first tonal expansion, E_1, of some 7-note scale, as explained by him in his discussion of pitch scales. A Σ of the C major scale would be, for example, C – E – G – B – D – F – A reading upwards. This Σ is the master-structure from which the substructures S(5), S(7), S(9), etc., are derived. The Σ concept is the source of many of the brilliant harmonic and orchestral effects which Schillinger pupils—and *only* Schillinger pupils, so far as we are aware—use in their music and in their arrangements. (Ed.)

other hand, in progressions based on more than one Σ, the respective differences will affect the intervals of the auxiliary tones. For example, if we compare two Σ's—c − d − e − f♯ − g − a − b♭, with c − d − e − f − g − a − b♭—we find that in $\Sigma_1 \nearrow 5$, $i = 1$ (c-chord: f♯ → g), whereas in $\Sigma_2 \nearrow 5$, $i = 2$ (c-chord: f → g).

If chromatic auxiliary tones are used in progressions of Types II and III, they have to be pre-set in definite relations to the pitch-units of the given Σ. For instance, in Σ_1 (c − d − e − f♯ − g − a − b♭) of the preceding example, we may introduce a $\nearrow 13$, $i = 1$ (i.e., g♯ → a of the c-chord), which is not in the Σ. In this case, one would have to transpose such an auxiliary tone to each chord of identical Σ, whenever the auxiliary tone is to be used. (See page 824, footnote.)

A melodic form containing directional units may start either on a chordal or on an auxiliary tone. However, *it must end with a chordal tone.* Taking this into consideration, we may now evolve many melodic forms of different complexity and character.

Figure 347. Melodic forms of auxiliary tones.

Those forms in which the *chordal* tones predominate produce a more restful effect on us than forms in which the auxiliary tones predominate. We might well expect to find a delay in arriving at the final chordal tones in the music of those composers who express (intentionally or unintentionally) "longing," "restlessness" and "dissatisfaction." And, indeed, Chopin and Chaikovsky have each the same style of handling auxiliary tones; the difference between their respective styles in this regard lies mostly in the particular intervals between the chordal tones and in the predominance of the *a* axis in Chaikovsky and the *b* axis in Chopin.* Mozart's music already had developed some of the chromatic auxiliary tones which became prominent later in Chopin, Schumann and Chaikovsky. Beethoven, whose music suggests to us a more masculine character, uses a decided predominance of chordal tones in figures containing auxiliaries; the latter

*The *a* axis is, of course, the secondary melodic axis leading upward *from* the primary axis; the *b* axis is the secondary axis leading downward *to* the primary. (Ed.)

usually conform to those well-known melismatic developments commonly known as *gruppetti*. Scriabine uses delays still more exaggerated than in the music of Chaikovsky or Chopin. And a Wagnerian characteristic is his simple directional units used with chromatic auxiliaries superimposed upon chromatic harmonic continuity.

J. S. Bach

Mozart

Beethoven

Chopin

Figure 348. Melodic forms produced by auxiliary and chordal tones, typical of different composers (*continued*).

Chaikovsky

Scriabine

Wagner

Figure 348. Melodic forms produced by auxiliary and chordal tones, typical of different composers (*concluded*).

The auxiliary tones we wish to use in any case may be pre-selected either as (a) auxiliaries to a definite chordal function, or (b) auxiliaries to a group of chordal functions, and may appear in one or more voices simultaneously. We shall consider such forms to be *thematic*.

We could classify all ascending and descending forms of auxiliary tones for one, two, three and four voices, but practically speaking, we have a choice of direction (ascending or descending) depending on the case.

Classification of single auxiliary tones

↗1	↘1	↗3	↘3	↗5	↘5	↗7	↘7

Classification of double auxiliary tones

↗3	↘3	↗3	↘3	↗5	↘5	↗5	↘5
↗1	↗1	↘1	↘1	↗1	↗1	↘1	↘1
↗7	↘7	↗7	↘7	↗5	↘5	↗5	↘5
↗1	↗1	↘1	↘1	↗3	↗3	↘3	↘3
↗7	↘7	↗7	↘7	↗7	↘7	↗7	↘7
↗3	↗3	↘3	↘3	↗5	↗5	↘5	↘5

This table can, of course, be extended to include higher chordal functions.

Classification of triple auxiliary tones

↗5	↘5	↗5	↗5	↘5	↘5	↗5	↘5
↗3	↗3	↘3	↗3	↘3	↗3	↘3	↘3
↗1	↗1	↗1	↘1	↗1	↘1	↘1	↘1
↗7	↘7	↗7	↗7	↘7	↘7	↗7	↘7
↗3	↗3	↘3	↗3	↘3	↗3	↘3	↘3
↗1	↗1	↗1	↘1	↗1	↘1	↘1	↘1
↗7	↘7	↗7	↗7	↘7	↘7	↗7	↘7
↗5	↗5	↘5	↗5	↘5	↗5	↘5	↘5
↗1	↗1	↗1	↘1	↗1	↘1	↘1	↘1
↗7	↘7	↗7	↗7	↘7	↘7	↗7	↘7
↗5	↗5	↘5	↗5	↘5	↗5	↘5	↘5
↗3	↗3	↗3	↘3	↗3	↘3	↘3	↘3

Figure 349. Single, double, and triple auxiliary tones.

This table can also be extended to include higher chordal functions. A table for four simultaneous functions can be devised in a similar fashion. These tables are to be used as guides in the choice of pre-selected groups of directional units.

For instance: ↗ 1 + ↘ 5 + ↘ 7;
↘ 3 + ↗ 3 + ↘ 5;
↗ 5 + ↘ 3 + ↗ 1; etc.

Coefficients of recurrence of any type and form are also applicable to this problem. Examples:*

$r_{3 \div 2}$ ↗ 1 + ↗ 1 + ↘ 1 + ↗ 1 + ↘ 1 + ↘ 1
$r_{3 \div 2}$ ↗ 1 + ↘ 1 + ↗ 3 + ↘ 1 + ↗ 3 + ↘ 3
$r_{4 \div 3}$ ↗ 1 + ↘ 1 + ↗ 1 + ↗ 3 + ↘ 5 + ↘ 5 + ↗ 7 + ↗ 7 + ↘ 3 + + ↘ 1 + ↗ 1 + ↘ 1

Each directional unit in the example above applies to one chord.

Another way of selecting the sequence of auxiliary tones is by the parts. The sequence of soprano (S), alto (A), tenor (T) and bass (B)—SATB—or any variation thereof (of which there are 24 for four-part harmony) permits us to have full control over the order in which the auxiliary tones appear. When such a harmonic continuum is orchestrated (vocally or instrumentally), the *sequence of definite voices or instruments* as they enter with a certain figure becomes a matter of considerable importance. We shall consider these forms to be *neutral*. A more detailed specification is possible through the assignment of directions to the sequence of parts: for intance: ↗ T + ↘ B + ↘ A + ↗ S.

These groups are, of course, subject to variation by means of permutations or by means of coefficients of recurrence. Example:

SATB + ATBS + TBAS + BSAT
2 SATB + BTAS + SATB + 2 BTAS
4 TSAB + 2 ABTS + 2 TSAB + ABTS

There is still another way of selecting the sequence of auxiliary tones through several parts, following the principle of "reciprocity" or free choice.

Examples of reciprocity:

```
  S      S  S
A  ;   A;     A; . . .
T    T    T
  B  B       B

  S   S     S
A  ; A  ;  A  ; . . .
T    T       T
B      B  B
```

Examples of free selections:

```
S    S S   S    S
A A; A  ;   A;  A A; . . .
  T    B  B B   T T
                  B
```

*In the first example the $r_{3 \div 2}$ (2+1+1+2) is applied in turn to ↗ and to ↘; in the third example, the $r_{4 \div 3}$ is applied to the 1, 3, 5 and 7, in turn, with the ↗ and ↘ following a 1+1+2+2+2+2+2+1+1 pattern. (Ed.)

Application of Auxiliary Tones

Diatonic Progressions,
Diatonic and Chromatic
Auxiliary Tones.

(1) Single auxiliary tones- constant chordal function

↗ 1 const.

↘ 1 const.

↗ 8

↘ 3

Figure 350. Single auxiliary tones; constant chordal function (continued).

↗ 5

↘ 5

↗ 7

↘ 7

↗ 9

↘ 9

Figure 350. Single auxiliary tones; constant chordal function (concluded).

Variable Chordal Functions. (A) Neutral Selection Through the Sequence of Parts.

↗ 3 + ↘ 5 + 1. (B) Thematic Selection through Pre-set Chordal Functions.

Figure 351. Variable chordal function.

(2) Double Auxiliary Tones. (A) Neutral Selection through the Sequence of Parts.

(B) Thematic Selection through Pre-set Chordal Functions. 3 5
1 3 3

Figure 352. Double auxiliary (continued).

Figure 352. Double auxiliary (concluded).

(3) Triple Auxiliary Tones. (A) Neutral Selection through the Sequence of Parts.

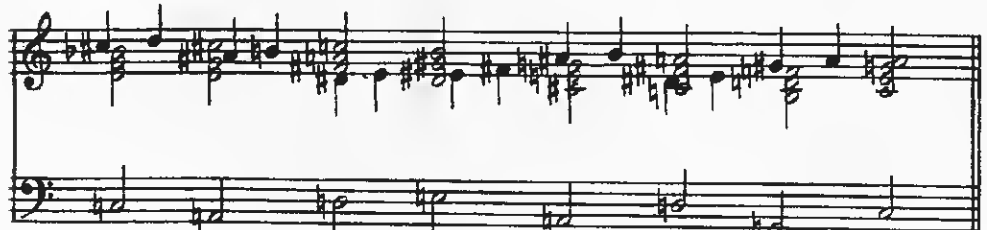
(B) Thematic Selection through Pre-set Chordal Functions. 5 3 1 9 7 3

Figure 353. Triple auxiliary.

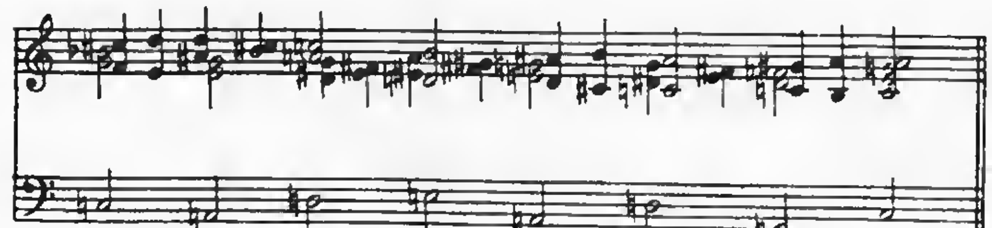
Diatonic-Symmetric Progressions,
Diatonic and Chromatic Auxiliary Tones.

Theme: Type II

Variation: → 9



Variation: < 9 8



Variation: S A T B (Diatonic auxiliaries)

Figure 354. Auxiliary tones in diatonic-symmetric progressions.

Symmetric Progressions,
Diatonic and Chromatic Auxiliary Tones.

Theme: Type III: $\sqrt[6]{2}$ S(9) const.

Variation: $\nearrow$ 5 (Diatonic auxiliary)

Variation: $\nearrow$ 7 ch, $\nearrow$ 5 d, $\nearrow$ 3 ch

Variation: BTASB

Figure 355. Auxiliary tones in symmetric progressions.

CHAPTER 23

NEUTRAL AND THEMATIC MELODIC FIGURATION

By combining all the devices using the *suspended*, the *passing*, the *anticipated*, and the *auxiliary* tones, we attain the final—and fully versatile—form of melodic figuration.

We shall distinguish the two forms of it:

(1) Neutral melodic figuration.
(2) Thematic melodic figuration.

Neutral melodic figuration may be effected in the following forms:

(a) Free development of resources *without* preliminary planning; this corresponds to that technique, the best examples of which are to be found in J. S. Bach's 371 chorals.
(b) Free selection of resources, but with *preliminary planning* of the sequence of parts in which the figuration is to appear.

Theme: Type I

(A)

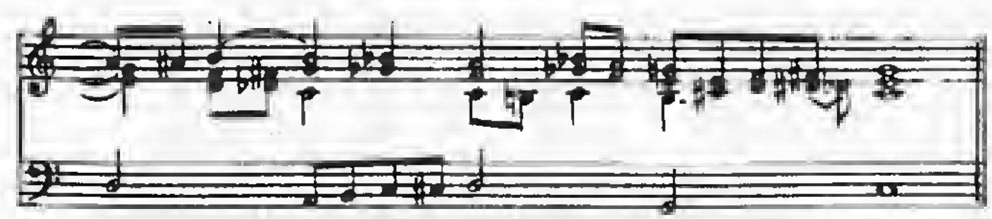


Figure 356. Neutral melodic figuration (continued).

(B) TABS + ABST + BSTA + STAB

(B)

Figure 356. Neutral melodic figuration (concluded).

Thematic melodic figuration, however, presupposes that the motif to be used throughout the different parts of the harmonic continuity will be selected in advance. A motif to be put to such use must be approached, first, as a *group* of both chordal and non-chordal tones. Ascertaining which tones are in fact the chordal tones is a process based on the principle that, in every seven-unit scale, either the *first* or the *second* pitch-unit is a chordal tone. This gives us two possible definitions to any scale—

(1) c d e f g a b

—in which c, e, g, b are selected as the chordal tones;

or—

(2) c d e f g a b c

—in which d, f, a, c are to be the chordal tones.

The non-chordal tones then become either auxiliary or passing tones.

Once the chordal functions are designated, their assignment must be performed from the *axis* of the motif. If this axis is not sufficiently prominent, then any arrangement of units by thirds may suggest the position of chordal tones in the motif.

In other cases, the chordal tones may best be detected by elimination of all the accidentals which do not belong to the (real) key signature. Analysis of an actual motif and assignment of the chordal tones will illustrate this process:

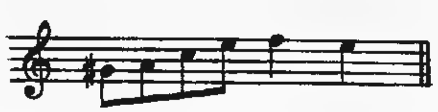


Figure 357. Analysis of a motif.

In this case the grouping of thirds is quite apparent for a, c and è are obviously the chordal functions; g# is the lower auxiliary tone to a, and f is the upper auxiliary tone to e. It is understood, in this example, that the entire motif must be superimposed on one chord. The grouping of chordal tones is as follows:

The next step is to assign any one of the seven possible systems to our reading of chordal functions; we may select from the following:

Figure 358. Assigning chordal functions.

Inasmuch as the axis of this motif obviously falls on e, we have to bear in mind that whatever chordal function we select for the motifs, starting chordal function must also be present in the chord itself. For example, if e becomes assigned to function as the ninth, we must start on the fifth.

This assignment of chordal functions in a motif may be either constant or variable. In the first case, chords of a certain tension are required. If the starting point becomes a ninth, all chords must be S(9), as a minimum form of tension. Assigning the axis, in the above case, to a seventh, the starting point in the chord will be a third.

In the variable assignment of chordal functions, the sequence in which the motif appears in the different parts is controlled by the SATB arrangement (24 fundamental forms).

In the constant assignment of chordal functions, the *sequence* in which the motif appears in the different parts is controlled by voice-leading which will necessitate the appearance of the assigned chordal function in some specified voice.

In using thematic melodic figuration, it is advisable to have open positions of the chords so as to provide sufficient range for the motif to move.

Examples of Thematic Melodic Figuration
(Diatonic Progressions)

(1) Constant assignment of chordal functions.
  (a) 1, 3, 5 (axis placed on the fifth) operation from the root.
  The missing functions are compensated (marked with cross) and the original voice-leading resumed.
  (b) 3, 5, 7 (axis placed on the seventh) operation from the third.
  (c) 5, 7, 9 (axis placed on the ninth) operation from the fifth.

See the corresponding music examples on the following pages.

(1) (a)

(b)

Figure 359. Thematic melodic figuration. Diatonic progressions (continued).

(c)

Figure 359. Thematic melodic figuration (concluded).

(2) Variable assignment of chordal functions

(a) SATB
(b) BTAS
(c) ABST

2 (a)

(b)

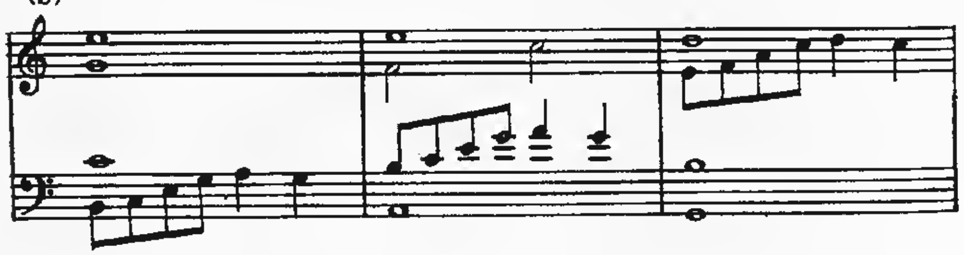


Figure 360. Variable assignment of chordal functions (continued).

(c)

Figure 360. Variable assignment of chordal functions (concluded).

In progressions of Types II and III, *at least* the chordal tones of the thematic motif must conform to the particular Σ (13) to be carried out through the harmony. Example:

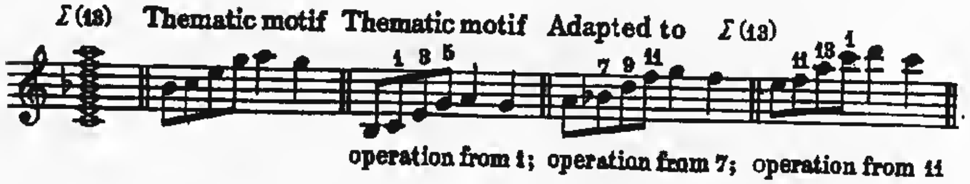


Figure 361. Chordal tones must conform to Σ in progressions of Types II and III.

Any crossing of adjacent voices by the thematic motif is undesirable. Compensation of the missing tones during the period of figuration is desirable but unnecessary, particularly in fast tempi.

The range of some thematic motifs is so great that they are bound to cross adjacent voices; in such a case the harmonic continuity has to be rearranged into *extra-open* position, with the original voice-leading preserved. Example:

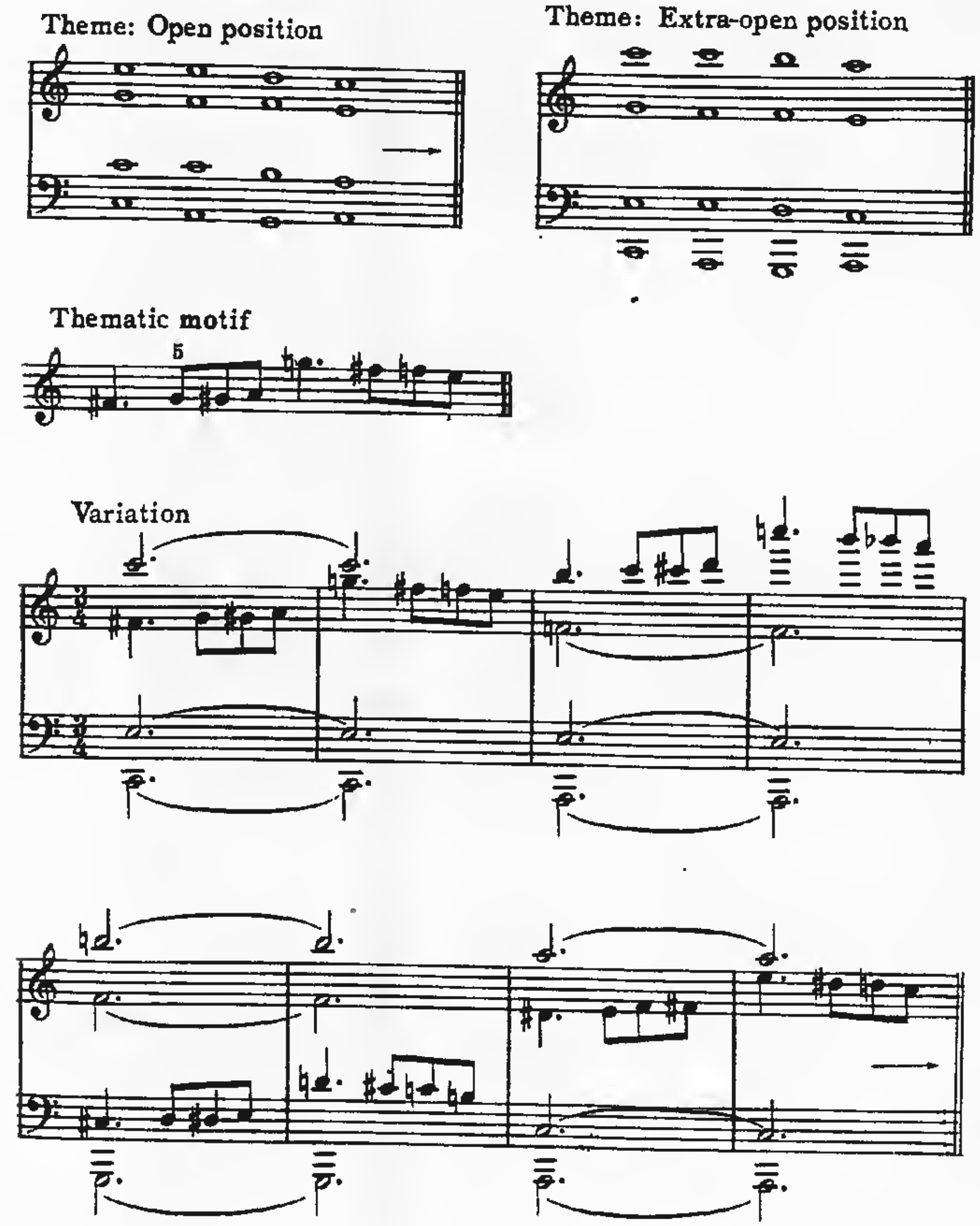


Figure 362. Using extra-open position to avoid crossing of adjacent voices.

CHAPTER 24

CONTRAPUNTAL VARIATIONS OF HARMONY

WHEN different parts of a harmonic continuity enter and drop out at different time intervals, the continuity acquires contrapuntal* characteristics. This effect arises from greater independence of the voices; it can be accomplished by operations upon any type of harmonic continuity. The sequence in which the different parts may enter or drop out is naturally subject to permutations.

Any three-part harmony offers us six variations for either entering or dropping out, making a total of 12 variations:

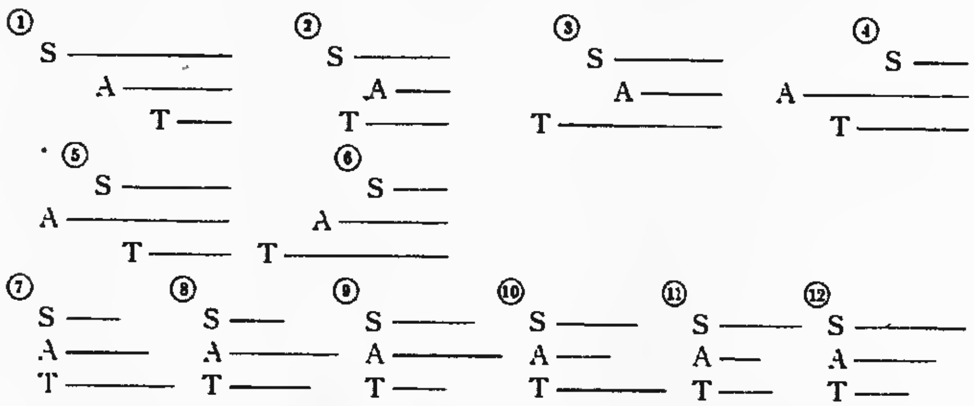


Figure 363. Contrapuntal variations of three-part harmony.

This table can be reduced** to three variations each way (through circular permutations) making a total of 6 variations.

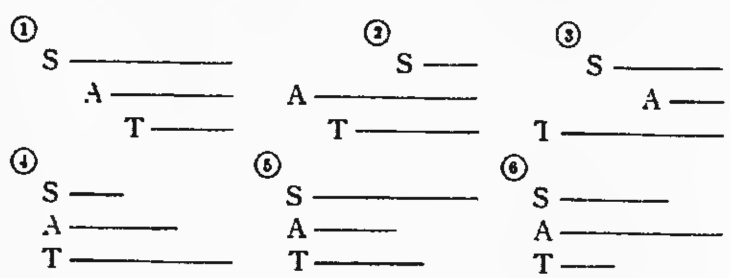


Figure 364. Contrapuntal variations of three-part harmony.

*Indeed, it is Schillinger who gives to this matter of sequence and interval of entrance and dropping out its proper emphasis in counterpoint itself, as will be seen later, in contrast to the customary emphasis on the techniques of simultaneous melodic lines. Consequently, the usefulness of the techniques described in this chapter cannot be overestimated. (Ed.)

**Reduction becomes desirable when the full set of 12 general permutations provide too much raw material, and when some *casual* selection of fewer than 12 would lack the logic of the 6 circular permutations—a lack that would be reflected in the resulting music. (Ed.)

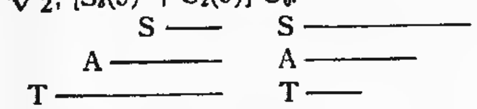
Likewise, any four-part harmony affords 24 permutations each way, making a total of 48 variations. This can be reduced through circular permutations to 4 variations each way, making a total of 8 variations.

In five-part harmony, general permutations produce 120 variations each way, making a total of 240 variations. This can be reduced through circular permutations: 5 variations each way, making a total of 10 variations.

(1) Three-Part Harmony.

(a) Theme: Type III: $\sqrt[4]{2}$, $[S_3(5) + S_2(5)]\ C_0$.

(b) Variation:



(1)(a)

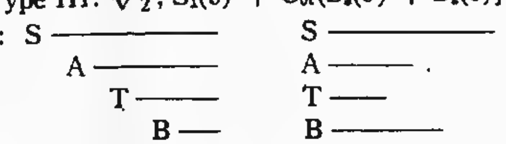
(b)

Figure 365. Contrapuntal variation of three-part harmony.

(2) Four-Part Harmony.

(a) Theme: Type III: $\sqrt[4]{2}$, $S_1(5) + C_0[(S_3(5) + S_2(5)]$.

(b) Variation:



(2)(a)

Figure 366. Contrapuntal variations of four-part harmony (continued).

(b)

Figure 366. Contrapuntal variations of four-part harmony (concluded).

(3) Five-Part Harmony.

(a) Theme: Type II.
(b) Variation:

S ——
A ——
T ——
B_I ——
B_{II} ——

(3) (a)

(b)

Figure 367. Contrapuntal variation of five-part harmony.

Deciding upon the number of attacks after which the next voice will enter or will drop out may be a matter of free selection and distribution. Or the number of attacks for each voice may be rhythmically arranged. Attack-groups may be composed either with or without interference in relation to the part-sequence group. For example, the part-sequence group might be distributed in one-to-one correspondence to the attack-group:

S ——
A ——
T ——
B ——

A = 4a + 3a + 2a + 2a

Then: S4a + S3a + S2a + S2a
A3a + A2a + A2a
T2a + T2a
B2a

Theme: Type I: 11 H.*
Variation.

Theme

Variation

Figure 368. Correlation of part-sequences and attack groups.

*The "11 H" we may read, of course, as "eleven harmonies"—harmonies rather than chords, for each H might be subjected to various attack patterns producing more than one "chord" for each H. (Ed.)

When several entrances produce different part-sequence groups, their interference against the attack-group offers the possibility that each voice may have a different number of attacks at each of its consecutive entrances. Example:

Part-sequence group:

S ——
A ———————— A = 3a + 2a
T ————

The synchronized part-sequence group would then be:

S3a S2a
A3a + A2a + A3a + A2a + A3a + A2a
T2a + T3a + T3a + T2a

(The bass is excluded in the variation)

Theme: 15 H: Chromatic.
Variation.

Figure 369. Synchronizing part-sequences and attack groups.

It is this technique which enables us to obtain vocal or instrumental orchestration comparable to that found in the scores of the best composers of the past (Palestrina, Bach, Händel, Wagner, and others).

When this technique for the contrapuntal variation of harmony is applied to harmony that has already been subjected to melodic figuration (neutral or thematic), many more developed forms of counterpoint (including imitations) may be derived from harmony.

One of the advantages that "contrapuntalized" harmony has over counterpoint proper is that it permits complete control over the style or type of harmony *a priori*. Another advantage lies in the fact that this technique is incomparably easier than any purely contrapuntal technique. Still another advantage comes from the fact that it is possible to use such a contrapuntal variation against its own harmonic theme, the theme functioning as a harmonic background; the latter may take on, by means of patterns of attack, any instrumental form, such as (1) sustained chords, (2) staccato chords, (3) broken chords (arpeggio).

In all these cases, the counterpoint stands out against its own harmonic background (accompaniment), particularly when the background is sounded by instruments (or voices) different from the counterpoint itself. When these devices are applied to arranging (when the thematic motif is a fragment of a given piece), they produce very effective introductions, transitions, and conclusions (codas).

The following techniques for melodic or contrapuntal development of harmonic continuity may be suggested:

A. Neutral or thematic melodic figuration carried out in one voice. This, combined with other voices, produces a melody-with-accompaniment:

Theme: Figure 359.
Variation: Thematic Melodic Figuration in Soprano.

Figure 370. Melodic figuration in soprano.

B. Neutral or thematic melodic figuration carried out through all voices and assigned either to a sequence of chordal functions (Fig. 359, 1) or to a sequence of parts in which the motif appears (Fig. 360).

C. Neutral or thematic melodic figuration (as in B) with gradual entrances or gradual dropping out of voices. When such a form is based on thematic figuration, the result is a *fugato*, i.e., a group of imitations.

SATB

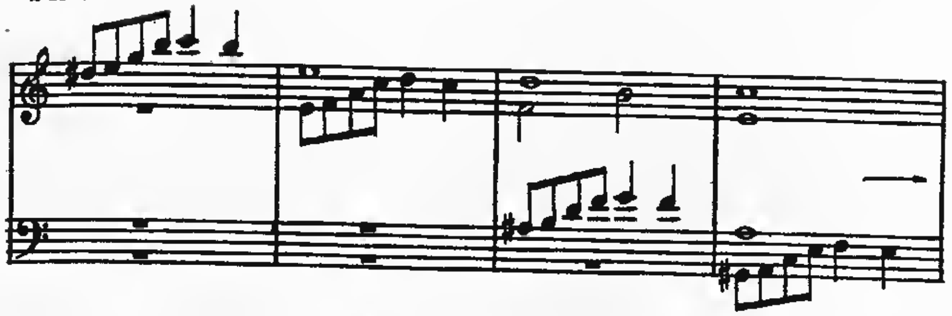


Figure 371. Melodic figuration in all voices.

It is easy to achieve the opposite effect by using the inverted position ⓑ.*

When a fugato is to be used as an introduction which is to have a duration of 4T, all that is necessary is to compose a 4H continuity so that the last chord will lead directly to the following exposition (an equivalent of an exposition in a song is the "chorus"). A fugato used as a modulating interlude between successive expositions (choruses) should be developed from a harmonic continuity that effects the desired modulation. A 4T introduction or interlude may also be constructed by making a three-part fugato with a cadence on the last chord (H_4).

When an 8T introduction or interlude is desirable, the thematic motif emphasizing one chord (H) should occupy a duration rhythmically to 2T.

Likewise, a 6T introduction or interlude may be constructed from 2T-per-H motifs in three-part Fugato with a 2T cadence at the end.

Theme after: *Honey-Suckle Rose,*** by Thomas Waller.

Theme

Var. 1

Figure 372. Introductions or interludes on a given theme (*continued*).

*Inversion ⓑ is the original right side up but backwards. (Ed.)
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Var. 2

Var. 3

Figure 372. Introductions or interludes on a given theme (*concluded*).

D. Accompanied Fugato with constant or variable *density** in the harmonic accompaniment.

(1) Constant density in the harmonic accompaniment. Example shown on the following page.

(2) Variable density in the harmonic accompaniment; decreasing density in the accompaniment. Example shown on the following page.

(3) Variable density in the harmonic accompaniment; decreasing density in fugato, increasing density in the accompaniment. [Reverse the procedure of (2)].

**Density* is a term which will be explained more fully at a later point in the text; it is enough to say that it has to do with the total number of parts sounding, in relation to their distribution over the total practical range of instrumentation. (Ed.)

Figure 373. Accompanied fugato with constant or variable density.

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK VI

THE CORRELATION
OF HARMONY AND MELODY

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

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BOOK VI

THE CORRELATION OF HARMONY AND MELODY

BOOK SIX

THE CORRELATION OF HARMONY AND MELODY

CHAPTER 1

MELODIZATION OF HARMONY

THE composition of melody with its harmonic accompaniment can be accomplished either (a) by correlating the melody with a chord progression, or (b) by composing the melody to such a progression. While the former procedure is the one more commonly known—and attempts have even been made to develop a theory to this effect—it is the second procedure which has in fact brought forth music of unsurpassed harmonic expressiveness; many composers, particularly the operatic ones (among them, Wagner), composed the melodic parts of their music to harmonic progressions.

So far as my theory is concerned, the technique of harmonization of melody can be developed only if the opposite process is known. If melody can be expressed in terms of harmony, i.e., as a sequence of chordal functions and their respective tensions, then a scientific and universal method for the harmonization of melody can well be formulated by reversing the whole system of operations.

The process of composing melody to chord progressions thus becomes what I shall call the ***melodization of harmony***.* The word "melodization" cannot now be found in English dictionaries, but we may be certain it will be found there soon, for the discovery of a new technique necessitates the introduction of a new operational concept.

At this point, I shall apply my theory of melodization to those particular harmonic progressions which satisfy the definition given earlier for the *Special Theory of Harmony*,** as distinct from *general* harmony*** which will be discussed considerably later. According to this definition, all chord-structures are based on E_I, the first expansion of those seven-unit scales which contain seven musical names without any identical intonations. So approached, any pitch unit of melody can be only one of these seven functions: 1, 3, 5, 7, 9, 11, or 13. These seven functions produce that manifold which I call the *scale of tension*. By arranging this scale of tension in a circular fashion, one obtains two harmonic directions: the clockwise and the counterclockwise. See Figure 1 on the following page.

*This phrase, which designates one of Schillinger's most brilliant discoveries, refers to the construction of a melody to go with an $H^{\rightarrow}$ already constructed, in contrast to melodic figuration, which does not add any additional voice to those of which the $H^{\rightarrow}$ (harmonic continuum) is composed. (Ed.)

**See Book V.

***See Book IX.

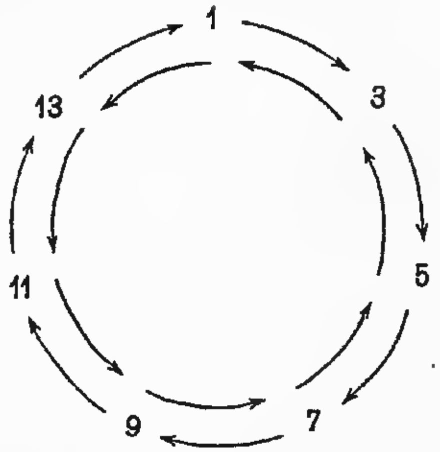


Figure 1. Scale of tension

Clockwise functioning of the consecutive pitch-units of a melody obtains the positive form of tonal cycles.

Counterclockwise functioning of the consecutive pitch-units of a melody obtains the negative form of tonal cycles.

If we assume, for example, that *all pitch-units* of a melody are *stationary and identical* and that we may therefore select any pitch-unit that is stationary, we may choose *c* as such a unit, for illustration. By assigning clockwise functioning to such a unit, the positive form of harmonic progressions is obtained:

	1	3	5	7	9	11	13	1	
Melody:	c	+ c	+ c	+ c	+ c	+ c	+ c	+ c	C_3
Chords:	C	+ A	+ F	+ D	+ B	+ G	+ E	+ C	

By reading the above progression backwards, the negative form is obtained.

Omission of certain of these chordal functions for the consecutive pitch-units of the melody will result in a change of *cycles* but not of direction.

	1	5	9	13	3	7	11	1	
Melody:	c	+ c	+ c	+ c	+ c	+ c	+ c	+ c	C_5
Chords:	C	+ F	+ B	+ E	+ A	+ D	+ G	+ C	

Likewise:

	1	7	13	5	11	3	9	1	
Melody:	c	+ c	+ c	+ c	+ c	+ c	+ c	+ c	C_7
Chords:	C	+ D	+ E	+ F	+ G	+ A	+ B	+ C	

It follows, from the above, that *every chord has seven forms of melodization* insofar as the 1, 3, 5, 7, 9, 11 or 13 can be added as a melodic tone to the chord

itself. Reduction of the scale of tension decreases this quantity accordingly.

Let us consider all the reduced forms of the scale of tension to be the *ranges of tension*. When each chord is melodized by but one attack (or one pitch-unit), the range of tension can be entirely under control.

The minimum range of tension that is possible may be secured by causing but one chordal function to appear in the melody. Let us assume that such a function is the root-tone of the chord. Then, if harmony consists of three parts, the melody so obtained will sound like the bass of progressions of S(5) const.

For example:

$$2C_5 + C_3 + C_5 + 2C_7$$

Melody:	c	+ f	+ b	+ g	+ c	+ d	+ e	+ . . .
Chords:	C	+ F	+ B	+ G	+ C	+ D	+ E	+ . . .

Figure 2. Minimum range of tension: one chordal function in melody.

It is clear that the particular pattern of melody in such a case is conditioned by the cycles through which the roots of the chords move. Predominance of C_7 produces scalewise steps or leaps of the seventh. Other cycles influence the melodic pattern accordingly.

If we assign any *other* chordal function (but still *one* function for the entire progression), the resulting melodic pattern does not change, although the *form of tension* does vary. This time we shall use the 7 to melodize the same chord progression.

Figure 3. Using seventh to melodize chord progression of Figure 2.

Different ranges of tension produce different styles of melodization. Historically, melodization progresses clockwise through the scale of tension. A narrow range, confined to the lower functions, produces the more archaic or more conservative styles, and the resulting melodization may suggest Haydn or other early forms (I say "early," since in most cases such styles later become hackneyed);

but when a narrow range of tension is confined to higher rather than lower functions the result is melodization that suggests stylistically Debussy or Ravel. An intermediate form may produce characteristics of Wagner, or Franck, or Delius. And when the entire scale is used as a range of tension, i.e., from 1 to 13, the resulting melodization becomes highly flexible, indeed, in its expressiveness.

A. Diatonic Melodization

It follows from the preceding exposition that any chordal function may participate in melodization. The only procedure that remains to be effected is to assign chordal functions for melodization with regard to actual chord-structures. Let us denote melody as M and harmony as H. In terms of attacks, when one pitch-unit has been assigned to melodize each chord, the attack formula is $\frac{M}{H} = 1$. Under such conditions it is possible to evolve seven forms of melodization. For example, a C– chord may be melodized by c(1), or by e(3), or by g(5), or by b(7), or by d(9), or by f(11), or by a(13).

Figure 4. Seven forms of melodization when $\frac{M}{H} = 1$.

The majority of these pitch-units of M are satisfactory; two of them (d and f), however, do not result in satisfactory melodization. This is because such high functions, without support from the immediately preceding function in harmony are not ordinarily acceptable. Similarly, the presence of lower functions in the melodization of high-tension chords is equally unacceptable. The 13 is fully satisfactory, however, as melodization of S(5) because by sonority it converts an S(5) into S(7).*

*As a by-product of these circumstances, a special technique devised by Schillinger may be mentioned. It has been shown (1) that *any* triad will harmonize any tone in the same scale except the 9 and 11; (2) that the 9 is acceptable when two or more melody tones occur per H; (3) that a 9 or 11 not preceded by, respectively, a 7 or 9, is statistically rare in any combination of H and M; and (4) that the undesirable effects of an unsupported 9 or 11 are minimized in fast tempi when three or more melodic tones occur to each H. Now it happens to be so that *any* diatonic melody, provided it moves at $\frac{M}{H} = 3$ or more, may be harmonized by *any* progression of triads. S(5)—and, when S(7)'s are used, the results are still better. In this way, a 16-measure diatonic $M^{\rightarrow}$ where $\frac{M}{H} = 3$ may be constructed separately, and a 16-measure $H^{\rightarrow}$ of S(7) may also be constructed independently—and the two will "fit." (Ed.)

We can now construct the table of melodization for the fifth voice above four-part harmony when both melody and harmony are *diatonic.*

Table I: $\frac{M}{H} = 1$.

M	7, 13	9, 13	5, 11, 13	5, 13	5, 11	5, 9
		7	9	11	13	13
	5	5	7	9	9	11
H	3	3	3	7	7	7
	1	1	1	1	1	1
S	S(5)	S(7)	S(9)	S(11)	S(13)	

Figure 5. Table of melodization for fifth voice when M and H are diatonic.

It follows from the above table that:

(1) classical and hybrid four-part harmony can be used for diatonic melodization;
(2) all chordal tones actually participating in the chord as well as the functions designated as M can be used for diatonic melodization;
(3) by diatonic melodization we mean the participation of pitch units of one diatonic scale, from which scale the chord-progression is itself evolved;
(4) the use of 13 in the melody with an S(7) is more conventional when the root of the chord is in the bass (i.e., this would exclude inversions);
(5) the alternatives that exist in the table for selection of functions for the melodization of S(13) arise from *two* forms of structures covered by hybrid four-part harmony.

Assuming that there are, on the average, about *five* practical pitch-units (functions) for the melodization of each chord through the form $\frac{M}{H} = 1$, the number of possible melodizations of one harmonic continuity (under such conditions) equals 5 to that power the exponent of which represents the number of chords. Thus a progression consisting of 8 chords produces $5^8 = 390,625$ possible melodizations!

The two fundamental factors which determine the quality and the character of melodization are:

(a) the range of tension;
(b) the melodic *pattern*, i.e., the axial combinations of melodic structure.

Interest may be concentrated on either one, or on both; attack-interference patterns give additional interest to melodization.

In the following example, R represents the range of tension, and A denotes the axial combination.* All the following examples may be played in any system of accidentals.

Figure 6. Diatonic melodization $\frac{M}{H} = 1$ (*continued*).

*Note that A is used in this portion of the text to denote a quite different aspect of the problem, that of *axial patterns* rather than *axes*. The axes are: *o*, zero axis, unchanging; *a*, *upwards from* the primary axis; *b downward to* the primary; *c*, *upward to* the primary; and *d*, *downward from* the primary axis. (Ed.)

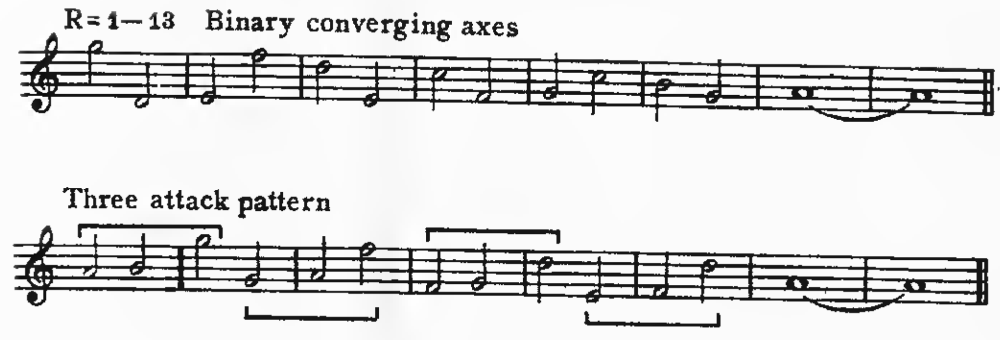


Figure 6. Diatonic melodization $\frac{M}{H} = 1$ (*concluded*).

B. More than One Attack in Melody per H

Increase in the number of attacks of M per H requires a slight remodeling of Table I (Fig. 5). *Any higher function may be supported by the immediately preceding function of immediately preceding rank.* For instance, 9 may be used for melodization of S(5) if two conditions are met:

(1) it must be immediately preceded by 7, *and*

(2) the root of S(5) must be in the bass, a necessary condition for the support of 9. For the same reason, 11 can be used for melodization of S(7) if preceded by 9 *and* if S(7) has a root in the bass.

Additions to Table I:

7 → 9	9 → 11
	7
5	5
3	3
1	1
S(5)	S(7)

Figure 7. Table II: $\frac{M}{H} = 2, 3, 4, \ldots$

Figure 8. Diatonic melodization $\frac{M}{H} = 2$.

With the further growth of the number of attacks of $\frac{M}{H}$, greater allowances (particularly in fast *tempi*) can be made. This is particularly true of the use of "unsuitable" functions for melodization when such functions are used *as auxiliary tones moving into chordal tones*, i.e., chordal tones actually present in the harmonic accompaniment. Such styles of melodization (particularly in harmonic minor) may easily be associated with the music of Mozart, Chopin, Schumann, Chaikovsky and Scriabine, i.e., with the sentimental, romantic, lyrical type of melodization.

Examples of Diatonic Melodization.

$\frac{M}{H} = 3$

Figure 9. Diatonic melodization $\frac{M}{H} = 3$ (continued).

A = b

A = a + b

Binary parallel axes

Binary diverging axes

Binary converging axes

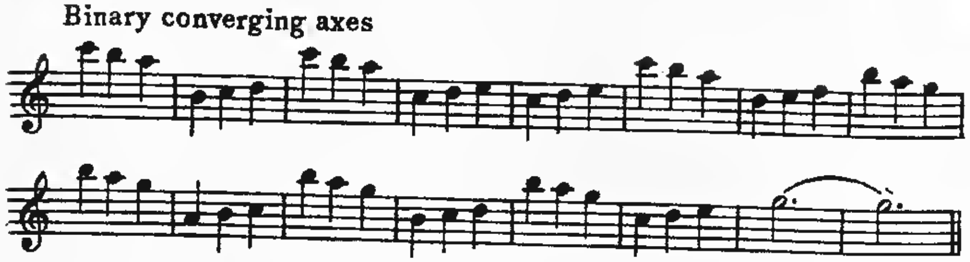


Figure 9. Diatonic melodization $\frac{M}{H}$ = 3 (*continued*).

Two attack pattern

Four attack pattern

Figure 9. Diatonic melodization $\frac{M}{H}$ = 3 (*concluded.*)

Examples of Diatonic Melodization

$\frac{M}{H} = 4$

Figure 10. Diatonic melodization $\frac{M}{H}$ = 4 (*continued*).

A=a

A = b

A = a+b

Binary parallel axes

Binary diverging axes

Figure 10. Diatonic melodization $\frac{M}{H} = 4$ (*continued*).

Binary converging axes

Three attack pattern

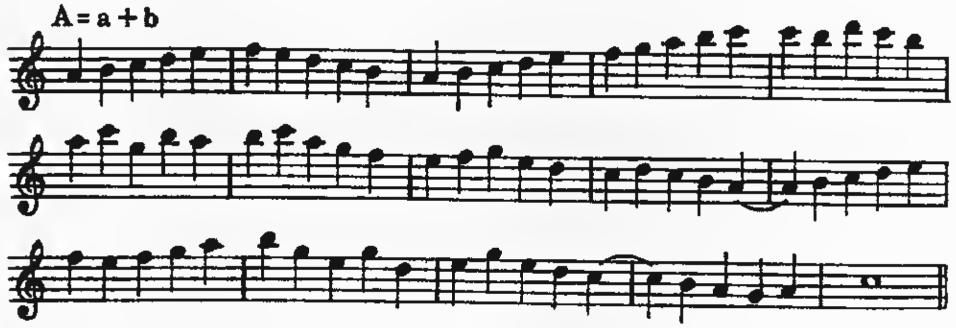
Five attack pattern

Figure 10. Diatonic melodization $\frac{M}{H} = 4$ (*concluded*).

Examples of diatonic melodization, when $\frac{M}{H} = 5$.

Figure 11. Diatonic melodization $\frac{M}{H} = 5$ (*continued*).

A = a + b



Binary parallel axes

Binary diverging axes

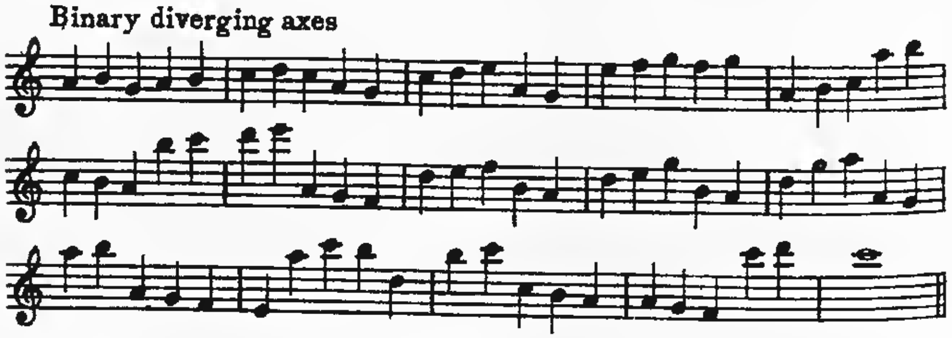


Figure 11. Diatonic melodization $\frac{M}{H} = 5$ *(continued).*

Three attack pattern

Figure 11. Diatonic melodization $\frac{M}{H} = 5$ *(concluded).*

Examples of diatonic melodization, when $\frac{M}{H} = 6$

Compare the following illustrations with Chopin, when the example is played in C – minor.

Figure 12. Diatonic melodization $\frac{M}{H} = 6$ *(continued).*

Binary parallel axes

Binary converging-diverging axes

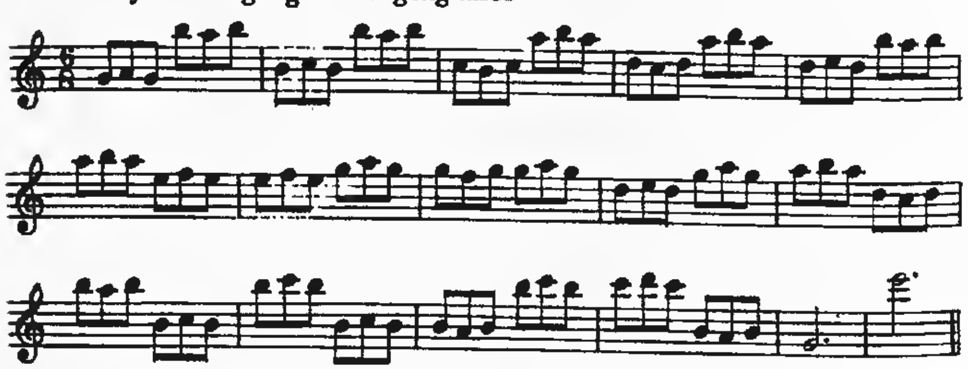


Figure 12. Diatonic melodization $\frac{M}{H}$ = 6 (*continued*).

Attack pattern: 4+2

Figure 12. Diatonic melodization $\frac{M}{H}$ = 6 (*concluded*).

Examples of Diatonic Melodization. $\frac{M}{H} = 7$

Figure 13. Diatonic melodization $\frac{M}{H}$ = 7 (*continued*).

Binary diverging- converging pattern

Figure 13. Diatonic melodization $\frac{M}{H} = 7$ *(continued).*

Attack pattern: (4+3)+(2+3+2)+(3+4)

Figure 13. Diatonic melodization $\frac{M}{H} = 7$ *(concluded).*

Examples of Diatonic Melodization. $\frac{M}{H} = 8$

Compare classical type ($\frac{4}{4}$ series) with jazz ($\frac{8}{8}$ and $\frac{12}{12}$ series) in the following illustrations.

Figure 14. Diatonic melodization $\frac{M}{H} = 8$ *(continued).*

A = b+a

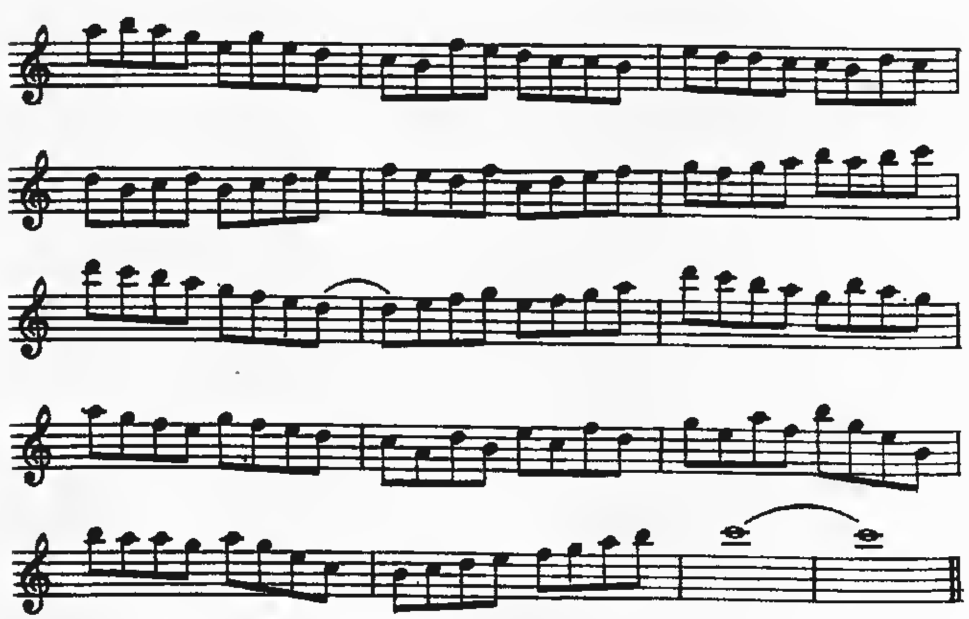


Figure 14. Diatonic melodization $\frac{M}{H} = 8$ (continued).

Binary diverging- converging axes

Attack pattern 8 (3)+8+2+8

Figure 14. Diatonic melodization $\frac{M}{H} = 8$ (concluded).

Examples of Diatonic Melodization. $\frac{M}{H} = 12$

Figure 15. Diatonic Melodization. $\frac{M}{H} = 12$ *(continued).*

Figure 15. Diatonic melodization $\frac{M}{H} = 12$ *(concluded).*

CHAPTER 2

COMPOSING MELODIC ATTACK-GROUPS

IN ALL the forms of melodization previously discussed, the attack-group of M was constant in relation to H. Any preselected quantity of attacks per chord (H) was carried out consistently. The monomial attack group (A) in all these cases was an integer remaining constant throughout $H^{\rightarrow}$. This monomial form of an attack-group can be expressed as $\frac{M}{H} = A$, where A can be any integer from one to infinity.

Now, however, we are to consider binomial attack-groups for the melody. This situation may be expressed as $\frac{M}{2H} = A_1 + A_2$, i.e., the melody covering two successive chords consists of two *different* attack-groups.*

For instance:

$$(1)\ \frac{M}{2H} = 2a + a; \qquad (2)\ \frac{M}{2H} = 3a + 2a;$$
$$(3)\ \frac{M}{2H} = 5a + 3a; \qquad (4)\ \frac{M}{H} = a + 8a; \ldots$$

These expressions can be further deciphered as:

$$(1)\ \frac{M}{H_1} + \frac{M}{H_2} = 2a + a; \qquad (2)\ \frac{M}{H_1} + \frac{M}{H_2} = 3a + 2a;$$
$$(3)\ \frac{M}{H_1} + \frac{M}{H_2} = 5a + 3a; \qquad (4)\ \frac{M}{H_1} + \frac{M}{H_2} = a + 8a; \ldots$$

The main technical significance of a binomial attack-group is that it introduces contrast between the two successive portions of M. Tbe greater the contrast required, the greater the difference between the two number-values of the binomial. This proposition can be reversed as follows: the contrast between the two terms of a binomial decreases when their values approach equality.

Thus, $\frac{M}{2H} = a + 6a$ contrasts more than $\frac{M}{2H} = 2a + 6a$; $2a + 6a$ possesses more contrast than $3a + 6a$; and $3a + 6a$ has more contrast than the *least* contrasting, $5a + 6a$. With further balancing we return to a monomial, $\frac{M}{H_1} + \frac{M}{H_2} = 6a + 6a$ which means that $\frac{M}{H} = 6a$.

If permutation takes place in a binomial attack-group, it results in a *second order* binomial attack group. For instance: $\frac{M}{2H} = 4a + 2a$; in the course of $H^{\rightarrow} = 4H$, this becomes: $\frac{M}{4H} = \frac{M}{H_1} + \frac{M}{H_2} + \frac{M}{H_3} + \frac{M}{H_4} = 4a + 2a + 2a + 4a$.

*$\frac{M}{2H} = A_1 + A_2$ is not the same situation as $\frac{M}{H} = 2$; the latter means two uniform attacks of melody per H, while the former means two *groups* of melody attacks per 2H, and one group need not be the same as the other. (Ed.)

What is true of binomial attack-groups is true of any polynomial; the latter, too, are subject to permutations.

Examples of trinomial attack-groups:

$$(1)\ \frac{M}{3H} = 3a + 2a + a; \quad \frac{M}{H_1} + \frac{M}{H_2} + \frac{M}{H_3} = 3a + 2a + a;$$
$$(2)\ \frac{M}{3H} = 4a + a + 3a; \quad \frac{M}{H_1} + \frac{M}{H_2} + \frac{M}{H_3} = 4a + a + 3a;$$
$$(3)\ \frac{M}{3H} = a + 2a + 4a; \quad \frac{M}{H_1} + \frac{M}{H_2} + \frac{M}{H_3} = a + 2a + 4a;$$
$$(4)\ \frac{M}{3H} = 3a + 5a + 8a; \quad \frac{M}{H_1} + \frac{M}{H_2} + \frac{M}{H_3} = 3a + 5a + 8a.$$

Figure 16. Trinomial attack-groups.

Examples of polynomial attack groups based on the resultants of interference:

(1) $r_{4 \div 3}$:
$$\frac{M}{6H} = \frac{M}{H_1} + \frac{M}{H_2} + \frac{M}{H_3} + \frac{M}{H_4} + \frac{M}{H_5} + \frac{M}{H_6} = 3a + a + 2a + 2a + a + 3a.$$

(2) $r_{3 \div 2}$:
$$\frac{M}{7H} = \frac{M}{H_1} + \frac{M}{H_2} + \frac{M}{H_3} + \frac{M}{H_4} + \frac{M}{H_5} + \frac{M}{H_6} + \frac{M}{H_7} =$$
$$= 2a + a + a + a + a + a + 2a.$$

(3) $r_{9 \div 8}$:
$$\frac{M}{16H} = 8a + a + 7a + 2a + 6a + 3a + 5a + 4a +$$
$$+ 4a + 5a + 3a + 6a + 2a + 7a + a + 8a.$$

Figure 17. Polynomial attack-groups.

The effect produced by such composition of attacks as (3) is that of counterbalancing the original binomial; the melody starts with excessive animation over H_1 (8a) and complete lack of it over H_2(a); it follows into that state which is closest to balance, after which the counterbalancing begins, ultimately reaching its converse: $a + 8a$.

In all cases of $r_{a \div b}$, maximum animation takes place at the beginning and at the end. When the *opposite* effect is desired (minimum animation at the beginning and at the end), use the permutation of binomials (which is possible when the number of terms in the polynomial is even). For instance: (3) can be transformed into $\frac{M}{16H} = a + 8a + 2a + 7a + 3a + 6a + 4a + 5a + 5a +$
$+ 4a + 6a + 3a + 7a + 2a + 8a + a$.

In addition to resultants, involution (power) groups, various series of variable velocities (natural harmonic series, arithmetical and geometrical progressions, summation series), may be used as the forms of attack-groups.

For instance: $(2 + 1)^2$: $\frac{M}{4H} = 4a + 2a + 2a + a$;
$(1 + 3)^2$: $\frac{M}{4H} = a + 3a + 3a + 9a$;
$\frac{M}{\cdot} = 2a + 3a + 5a + 8a + 13a.$

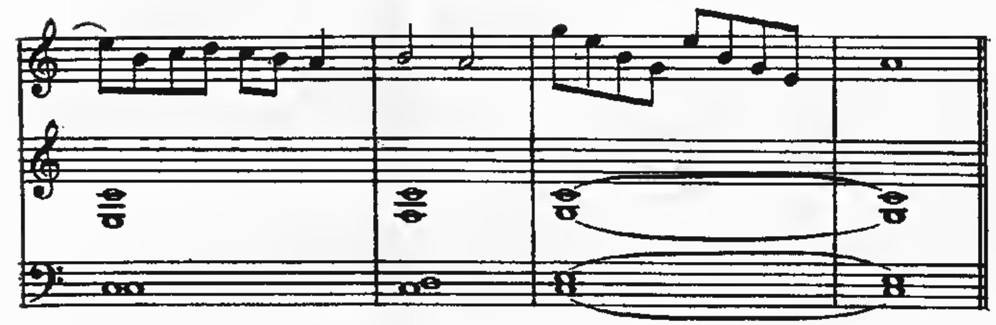
In the present examples, I shall use the simplest duration-equivalents of attacks, as this subject is to be a matter of further analytical investigation later in our text.

Examples of Diatonic Melodization with
Variable Quantity of Attacks of M over H:
$\frac{M}{H}$ = A var.

$\frac{M}{H}$= Var.- a: 1+8+2+7+3+6+4+5+5+4+6+3+7+2+8+1

*after attacks have been planned, ties may be added (as above)

Figure 18. Diatonic melodization with $\frac{M}{H}$ = A variable. (continued).



$\frac{M}{H}$= Var.- a: (1+4+2) p

$\frac{M}{H}$= Var.- a: 4+2+2+1; $H^{\rightarrow}$: Hybrid 4 part harmony

Figure 18. Diatonic melodization with $\frac{M}{H}$ = A variable (concluded).

The ties in the above examples were added after the completion of the melodization.

A. How the Durations for Attack-Groups of Melody are Composed

Durations for the attack-groups of melody may be composed by means of the techniques previously discussed as evolution of style in rhythm.* Every attack-group—monomial, binomial, trinomial, quintinomial, etc.—can be expressed through the different numerical series. For instance, a binomial of $\frac{3}{3}$ series is $2 + 1$, or its converse; a binomial of $\frac{4}{4}$ series is $3 + 1$, or its converse; a binomial of $\frac{8}{8}$ series is $5 + 3$ or its converse. Likewise, a trinomial of $\frac{4}{4}$ series is either $2 + 1 + 1$ or one of its permutations; a trinomial of $\frac{6}{6}$ is $4 + 1 + 1$ or one of its permutations; and a trinomial of $\frac{8}{8}$ series is $3 + 3 + 2$ or one of its permutations.

By selection of the durations for the attack-groups according to the different series, we may *translate* a piece of music from one rhythmic style into another.

When a choice is to be made as to the use of a binomial or a trinomial, the form of balance (unbalancing, balancing) becomes the decisive factor.

Of the two binomials, $3 + 1$ and $1 + 3$, the former is the more suitable at the beginning of melody; the latter, at the end. As to a trinomial in $\frac{4}{4}$ series: we might well use $2 + 1 + 1$ at the beginning, $1 + 2 + 1$ somewhere about the center, and $1 + 1 + 2$ at the end. Likewise, in $\frac{8}{8}$ series, $3 + 3 + 2$ at the beginning, $3 + 2 + 3$ about the center and $2 + 3 + 3$ at the end. Four attacks can be achieved, among other ways, by splitting one of the terms of a trinomial. Splitting the terms serves as a general technique for acquiring more terms for low determinants.

Here are examples of the composition of durations for the attack-groups of melody where each term of an attack-group corresponds to one chord: $\frac{M}{H} = A$.

$$A^{\rightarrow} = A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7$$

$A_1 = a;$

$A_2 = a + b;$

$A_3 = a + b + c;$

$A_4 = a + b + c + d + e;$

$A_5 = a + b + c;$

$A_6 = a + b;$

$A_7 = a$

$$A^{\rightarrow} = a + 2a + 3a + 5a + 3a + 2a + a$$

Series: $\frac{3}{3}$

$$T = 3H_1 + (2+1)H_2 + (1+1+1)H_3 + (\tfrac{1}{2} + \tfrac{1}{2} + 1 + \tfrac{1}{2} + \tfrac{1}{2})H_4 + (1+1+1)H_5 + (1+2)H_6 + 3H_7.$$

*See Book I, Chapter 13.

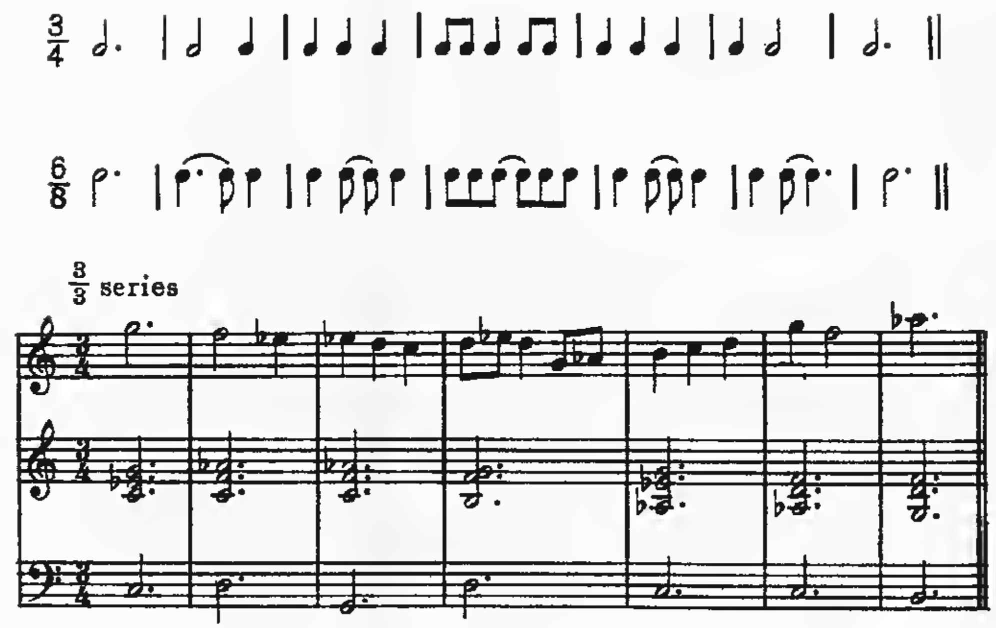


Figure 19. Series $\frac{3}{3}$.

Series: $\frac{4}{4}$

$$T = 4H_1 + (3+1)H_2 + (2+1+1)H_3 + (1+1+\tfrac{1}{2}+\tfrac{1}{2}+1)H_4 + (1+1+2)H_5 + (1+3)H_6 + 4H_7.$$

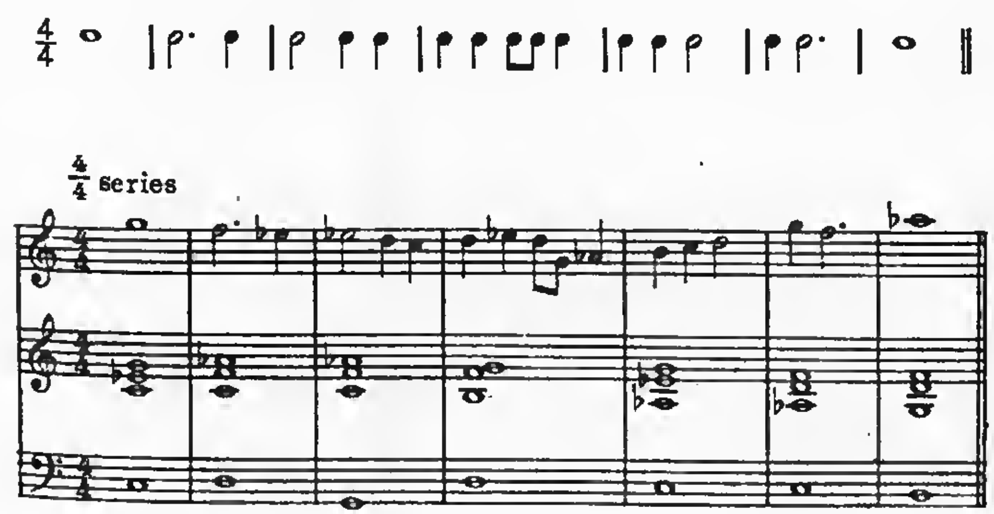


Figure 20. Series $\frac{4}{4}$.

Series: $\frac{6}{8}$

$$T = 6H_1 + (5+1)H_2 + (4+1+1)H_3 + (1+1+2+1+1)H_4 + \\ + (1+1+4)H_5 + (1+5)H_6 + 6H_7.$$

or

(Waltz or Mazurka)

$\frac{6}{8}$ series

Figure 21. Series $\frac{6}{8}$.

Series: $\frac{8}{8}$

$$T = 8H_1 + (5+3)H_2 + (3+3+2)H_3 + (2+1+2+1+2)H_4 + \\ + (2+3+3)H_5 + (3+5)H_6 + 8H_7.$$

(Fox-trot, Rhumba, Charleston)

$\frac{8}{8}$ series

Figure 22. Series $\frac{8}{8}$.

The final and most refined technique of coordination of attack with duration-groups occurs when the attack-groups are constructed independently of T. This results in an interference between the attack-groups and the duration-groups, and the duration of the individual chords coincides neither with the bar-lines nor with their simplest subdivisions.

A simple case for our illustration: let us choose $A = r_{5\div4} = 4a + a + 3a + 2a + 2a + 3a + a + 4a = 20\,a$.

Execute the durations as $T = \underline{r_{4\div3}}$. As T in this case has 10a and A has 20a, the interference is a simple one.

$$\frac{a\ (A)}{a\ (T)} = \frac{20}{10} = \frac{2}{1};\quad \frac{1}{2}\ \frac{(20)}{(10)}$$

Hence, $T' = 16t \cdot 2 = 32t$.
Let $T'' = 8t$, then:

$$N_{T''} = \frac{32}{8} = 4$$

The duration of each consecutive H equals the sum of durations during the time of attacks corresponding to such an H. H_1 corresponds to 4a, the durations of which constitute $3t + t + 2t + t$, and so H_1 will last 7t. Likewise, the next chord, i.e., H_2 will last t—since melodization at this point consists of one attack, and that attack corresponds to one unit of duration.

Here is the final solution* of the case:

(1) $$\frac{a\ (M)}{a\ (H)} = \frac{4}{1} + \frac{1}{1} + \frac{3}{1} + \frac{2}{1} + \frac{2}{1} + \frac{3}{1} + \frac{1}{1} + \frac{4}{1} = \\ = 4aH_1 + aH_2 + 3aH_3 + 2aH_4 + 2aH_5 + 3aH_6 + aH_7 + 4aH_8$$

(2) $$\frac{T\ (M)}{T\ (H)} = \left(\frac{3+1+2+1}{7} + \frac{1}{1} + \frac{1+1+2}{4} + \frac{1+3}{4}\right) + \left(\frac{3+1}{4} + \frac{2+1+1}{4} + \frac{1}{1} + \right. \\ \left. + \frac{1+2+1+3}{7}\right) = \left[\left(\frac{3t+t+2t+t}{7t}\right) H_1 + \left(\frac{t}{t}\right) H_2 + \left(\frac{t+t+2t}{4t}\right) H_3 + \right. \\ \left. + \left(\frac{t+3t}{4t}\right) H_4\right] + \left[\left(\frac{3t+t}{4t}\right) H_5 + \left(\frac{2t+t+t}{4t}\right) H_6 + \left(\frac{t}{t}\right) H_7 + \right. \\ \left. + \left(\frac{t+2t+t+3t}{7t}\right) H_8\right]$$

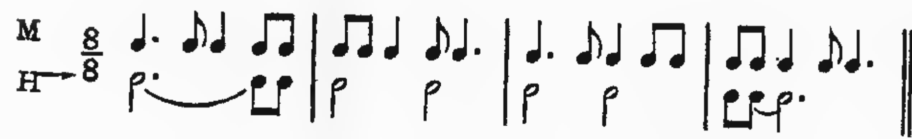


Figure 23. $\frac{a(M)}{a(H)}$ *and* $\frac{T(M)}{T(H)}$ *(continued).*

*This technique is one which the reader who does not remember the antecedent techniques may understand more rapidly if we give this parallel explanation: the number of attacks in the melody for each H is controlled by $r_{5\div4}$, which is $4 + 1 + 3 + 2 + 2 + 3 + 1 + 4$—that means, 4 melody notes against the first H, 1 against the second, 3 against the third H, and so on. But of what *duration* shall each attack of melody be? That is controlled by $\underline{r_{4\div3}}$—which is $3 + 1 + 2 + 1 + 1 + 1 + 1 + 2 + 1 + 3$ — meaning that the first melody note will last for 3 units, the second for one unit, the third for two units, etc. When the end of the pattern is reached, it begins all over again until the two, $r_{5\div4}$ and $\underline{r_{4\div3}}$, come to and end at the same point. Knowing, then, that there are to be 4 melodic attacks for the first H, and that the duration of these attacks is to be, respectively, 3 and 1 and 2 and 1, we see that the duration of the H must be the sum—i.e., 7. This process is carried out until the two resultants, $r_{5\div4}$ and $\underline{r_{4\div3}}$, close. (Ed.)

Figure 23. $\frac{a(M)}{a(H)}$ *and* $\frac{T(M)}{T(H)}$ *(concluded).*

B. Direct Composition of Durations Correlating Melody and Harmony

Time-rhythm of both melody and harmony can be set simultaneously by means of a *proportionate distribution of durations for a constant quantity of attacks of* $\frac{M}{H}$. This can be achieved by synchronizing a polynomial (consisting of the corresponding number of terms, representing attacks) with its square, or by synchronizing the square of a polynomial with its cube, etc. For instance, we might assume that we would like to have 4 attacks per chord with the duration in the style of the $\frac{4}{4}$ series. Let us take a quadrinomial from that series, $3 + 1 + 2 + 2$, and square it.

$$(3+1+2+2)^2 = (9+3+6+6) + (3+1+2+2) + (6+2+4+4) + (6+2+4+4)$$

This distributive square represents T (M). The T (H) is the original quadrinomial, synchronized with the distributive square:

$$8\,(3+1+2+2) = 24 + 8 + 16 + 16$$

We obtain:

$$\frac{T\ (M)}{T\ (H)} = \frac{9t + 3t + 6t + 6t}{24t} + \frac{3t + t + 2t + 2t}{8t} + \frac{6t + 2t + 4t + 4t}{16t} + \frac{6t + 2t + 4t + 4t}{16t}$$

Figure 24. Correlating melody and harmony: direct composition of durations (continued).

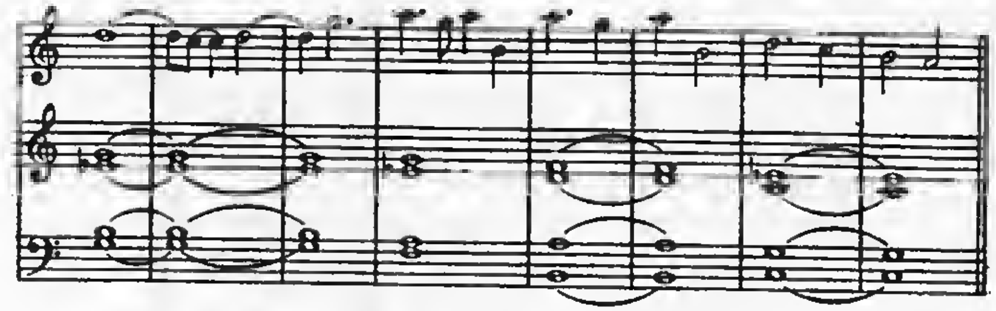


Figure 24. Correlating melody and harmony: direct composition of durations (concluded).

Likewise, a synchronization of the distributive square with the distributive cube of the same polynomial may be used for melodization of harmony. The group arising from the square furnishes durations for the chords; the group arising from the cube furnishes durations for the melody.

$$\frac{T\ (M)}{T\ (H)} = \frac{(2+1+1)^3}{4(2+1+1)^2} = \frac{8t+4t+4t}{16t} + \frac{4t+2t+2t}{8t} + \frac{4t+2t+2t}{8t} + \frac{4t+2t+2t}{8t} + \frac{2t+t+t}{4t} + \frac{2t+t+t}{4t} + \frac{4t+2t+2t}{8t} + \frac{2t+t+t}{4t} + \frac{2t+t+t}{4t}.$$

This produces harmony: $H^{\rightarrow} = 9H$, and melody: $M = 27a$, with constant 3 attacks per chord.

Figure 25. Synchronization of distributive square with distributive cube (continued).

Figure 25. Synchronization of distributive square with distributive cube (concluded).

For still greater contrast in quantity of attacks between M and $H^{\rightarrow}$, use the synchronized first power group for $H^{\rightarrow}$, and use the distributive cube for M.

In addition to distributive powers, coefficients of duration can be used. For instance:

$$\frac{M}{H\rightarrow} = \frac{(3+1+2+1+1+1+1+2+1+3) + (3+1+2+1+1+1+1+2+1+3)}{6+2+4+2+2+2+2+4+2+6}$$

C. Chromatic Variation of Diatonic Melodization

It is expedient to construct a chromatic melody for a diatonic chord progression by using two successive operations:

(1) Diatonic melodization of the harmony; and then

(2) Chromatization of the diatonic melody.

The first technique has been fully covered in the preceding explanation.

The second, chromatization, can be accomplished by means of passing or auxiliary chromatic tones. The most practical way to perform this rhythmically is by means of *split-unit groups*, as discussed earlier in the *Theory of Rhythm* under "Variations."* This splitting does not change the character of durations with respect to their style; it merely increases the degree of animation of the melody.

*See Book I, Chapter 9.

Diatonic melodization

Chromatic variation

Figure 26. Chromatization of diatonic melody.

D. Symmetric Melodization:

The Σ (13) Families

Each style of symmetric harmonic continuity (the Type II, the Type III and the generalized) is governed by the Σ (13) families. Pure styles are controlled by any one Σ(13); hybrid styles are based usually on two, sometimes on as many as three, Σ (13).

The complete manifold of Σ (13) families corresponds to the 36 seven-unit pitch-scales which contain the seven names of non-identical pitches; the Σ (13) is the first expansion (E_1) of such scales.

We shall classify all forms by considering 1, 3, 5 and 7 to be the *lower* structure [as S(7)], with 9, 11 and 13 constituting the *upper* structure [as S(5)], eliminating all enharmonic coincidences and eliminating all those adjacent thirds which do not satisfy $i = 3$ or $i = 4$. These limitations are necessitated by the restricted scope of the special theory of harmony.

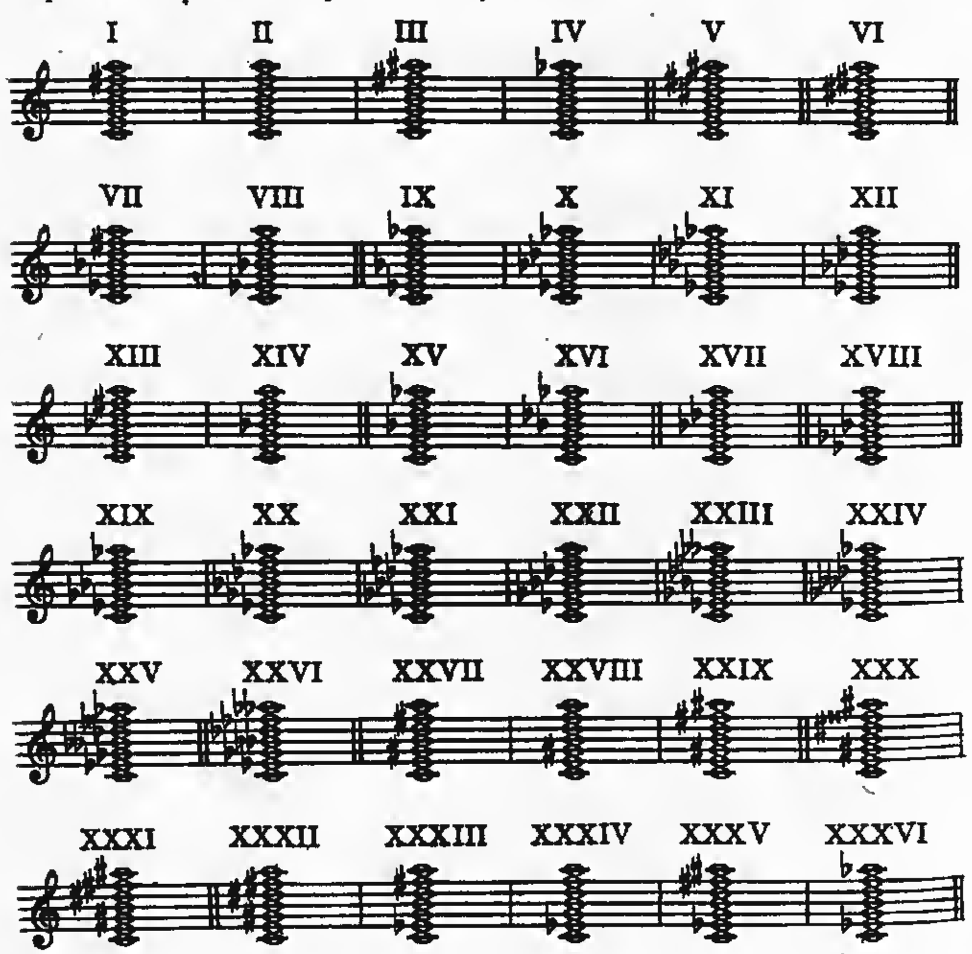


Figure 27. Complete table of Σ 13.

Symmetric melodization provides the composer with resources particularly suitable for equal temperament ($\sqrt[12]{2}$). In the diatonic system some chord-structures, particularly those of high tension, produce harsh-sounding harmonies; but in the symmetric system both the chord-structures and the intonations of the melody are entirely under control—they are subject to choice. The technique of symmetric melodization makes it possible to surpass the refinements of Debussy and Ravel. And whereas it took any important composer many years to crystallize his own original style, this technique of melodization offers us 36 styles to choose from if one Σ (13) is used at a time. The number of possible styles grows enormously with the introduction of blends based on *two* Σ (13). Thereupon the number of styles becomes $36^2 = 1{,}296$. Likewise, by blending *three* Σ (13), which is a reasonable limit of mixing, we acquire $36^3 = 46{,}656$ styles!

We should note, too, that only four of the 36 master-structures have been explored to any extent; the rest are virgin territory, packed with the most expressive resources of melody and harmony.

In offering the following technique, I shall use symmetric progressions of Type II, Type III and the generalized form in four- and in five-part harmony. The main difference between the four- and the five-part type of harmony is *density*. For massive accompaniments, use five; for lighter ones, use four-part harmony.

When all *substructures* [S(5), S(7), S(9), S(11)] derive from *one master-structure* [Σ (13)], they derive all their intonations from that master-structure. The easiest way to acquire a quick orientation in any Σ (13) is to prepare a chromatic table of the master-structure. Taking one Σ (13) [XIII] from Figure 27, we obtain the following table of transpositions:

Figure 28. Substructures of one Σ (13).

Such a table is very helpful; in it one can find all intonations of both melody and harmony for any symmetric progression. Each Σ (13), being E_1 of a seven-unit scale, corresponds to E_0 of the same scale.

The remainder of the procedure of melodization is based on the same *principle of tension* as in diatonic melodization. Those functions which are added to the respective tensions of chords are the most desirable ones for use as axes of the melody. Thus, the axis of the melody above S(5) in four-part harmony is either 7 or 13. Actually such a choice creates *polymodality*, as S(5) d_0 serves as an accompaniment to melody which is d_6 or d_5 respectively.* It is *polymodality* that makes such music expressive.

There follows a table of melodic axes for the respective structures in four and five-part harmony. In some cases there is a choice of more than one. Some of the forms are admitted because there has been practical use of them already—for example, S(5) in five-parts with the melodic axis on d_1 (= 9). It is interesting to note that Σ (13) [XIII] is used most of all, and that it is the most obvious of the master-structures, as it consists of a large S(7) and a major S(5).

Master-Structure: Σ (13) [XIII]

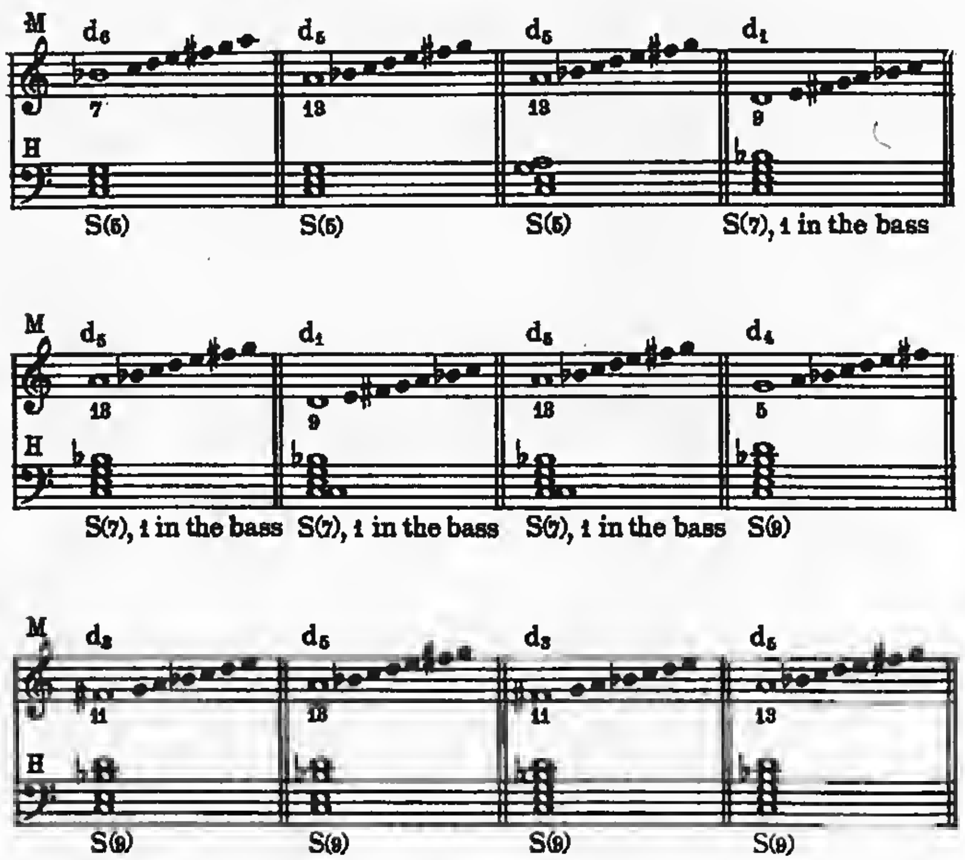


Figure 29. Table of melodic axes in relation to tension of H (continued).

*That is to say: S(5) as a triad in d_0 (that mode which starts on the same tone—the d_0—as that on which the key itself starts) serves to accompany a melody the axis of which is located at the 7 (a third *above* the 5 of the 1, 3, 5) or at the 13 (a third *below* the 1 of the 1, 3, 5), thus putting the melody in d_5 (that mode which starts on the *la* of the key) or in d_6 (that mode which starts on the *ti* of the key). (Ed.)

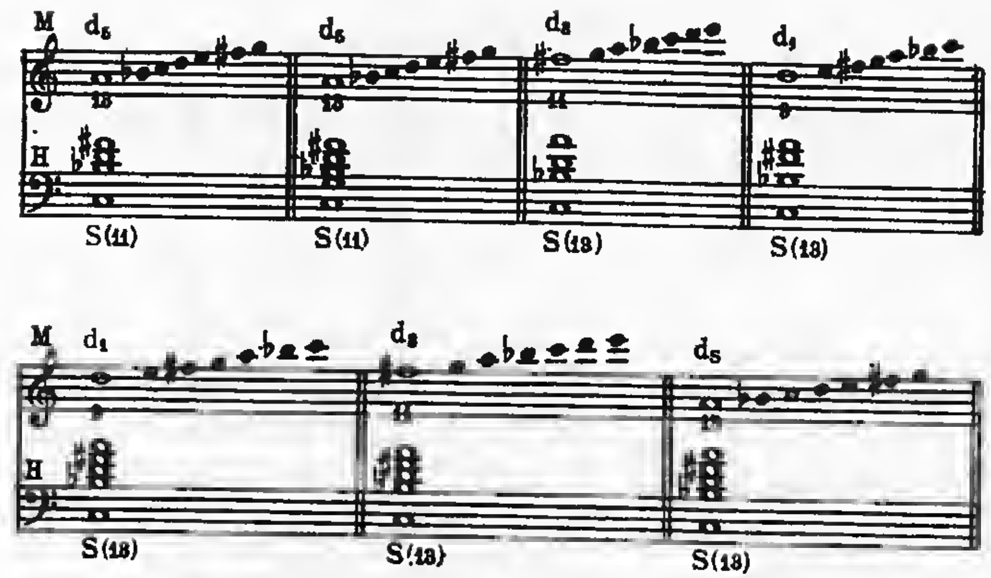


Figure 29. Table of melodic axes in relation to tension of H (concluded).

Using this Σ(13) [XIII] we shall melodize a generalized symmetric progression in four parts in $\frac{M}{H} = a$.

Theme: 2 + 2 + 2 + 1; tension: S(5) + 2S(7) + S(9) + 2S(13)

Σ (13): XIII

Figure 30. Melodizing a generalized symmetric progression in four parts $\frac{M}{H} = a$.

Theme: Type II: $\zeta = 2C_5 + C_{-7} + 2C_3 + C_5$
tension: $S(5) + S(7) + 2S(9) + S(11)$

Σ (13): XIII

$\frac{M}{H} = a$

Figure 31. Theme of Type II.

With ***more than one attack*** of M per H, the quality deriving from the transitions in melody during the chord changes becomes more and more noticeable.

In melodizing each H with more than one attack of M, ***it becomes necessary to perform modulations in melody.*** Such modulations are equivalent to ***polytonal-unimodal*** and ***polytonal-polymodal*** transitions. The technique for this is based on ***common tones***, on ***chromatic alterations***, or on ***identical motifs*** and a full explanation has been provided in the *Theory of Pitch-Scales* (the first group).*

$\frac{M}{H} = 2 + 4 + 3$; $\frac{4}{4}$ series of T.

Figure 32. Symmetric melodization (continued).

*See Book II.

$\frac{M}{H} = 2 + 6 + 3 + 5 + 4$; $\frac{6}{6}$ and $\frac{3}{3}$ series of T.

Figure 32. Symmetric melodization (continued).

Figure 32. Symmetric melodization (concluded).

With this kind of saturated harmonic continuity, the melody often gains in expressiveness by being more stationary than would be desirable in simple diatonic melodization; greater ***stability of tension*** is another desirable characteristic.

When mixing the different master-structures for one harmonic continuity, it is desirable to ***alter either the lower part of the*** Σ *(13)*, i.e., 1, 3, 5, 7, without altering the upper, or the ***upper part*** of it, i.e., 9, 11, 13, without altering the lower.

Let us now produce such a mixed style of master-structures, confining the latter to two—Σ (13) [XIII] and Σ (13) [XVII]. After such a selection has been made, the master-structures may be called simply: Σ_1 and Σ_2. In devising the style, we resort to coefficients of recurrence, for a predominance of one Σ over another is the chief stylistic determinant.

Let us assume the following recurrence-scheme: $2\,\Sigma_1 + \Sigma_2$.

$$\frac{M}{H} = a + 4a;\ \tfrac{4}{4} \text{ series of T.}$$
$$H^{\rightarrow} = 2C_7 + C_5 + C_3 \text{ (type II).}$$
$$S^{\rightarrow} = 2S(9) + S(13).$$

Figure 33. Recurrence scheme: $2\Sigma_1 + \Sigma_2$ *(continued).*

Figure 33. Recurrence scheme: $2\Sigma_1 + \Sigma_2$ *(concluded).*

E. Chromatic Variation of a Symmetric Melodization

Any melody, once it has been evolved by means of symmetric melodization, may be converted into the chromatic type by means of passing and auxiliary chromatic tones. Such chromatic tones do *not* belong to the master-structure. The rhythmic treatment of the durations is to be accomplished by means of split-unit groups.

Theme: figure 33

Figure 34. Chromatic variation of symmetric melodization.

All rhythmic devices—such as composition of attack and duration-groups—are applicable to all forms of symmetric melodization.

F. Chromatic Melodization of Harmony

The chromatic melodization of harmony serves the purpose of melodizing all forms of chromatic continuity. This includes techniques already explained in my discussion of the chromatic system, modulation, enharmonics, altered chords and of hybrid harmonic continuity. Such melodization is applicable to all forms of symmetric progressions; but from this approach we have nothing to gain, for symmetric melodization is itself a more general technique than the technique now being considered.

There are two fundamental forms of chromatic melodization. One of them produces melodies either of the *chromatic* type, or of the *extensively chromatized* type. The other form produces melodies of a purely *diatonic* type from chromatic harmony.

The first technique consists of *anticipating chordal tones and using them as auxiliary tones.* In a sequence, $H_1 + H_2 + H_3 + \ldots$, the chordal tones of H_2 are the auxiliaries and the chordal tones of H_1 are chordal tones while this chord sounds. In the next chord, (H_2), the chordal tones of H_3 are the auxiliaries and the chordal tones of H_2 are chordal tones while this chord sounds. This procedure may be extended *ad infinitum.*

As all of the "disturbing" pitch-units are harmonically justified as soon as the next chord appears, the listener is not aware that nearly every chromatic unit of the whole octave is used against each chromatic group, especially when there are enough attacks of M against H.

The auxiliary tones should be written in the proper manner, i.e., by raising the lower (ascending) auxiliary and by lowering the upper (descending) auxiliary, even if they have a different enharmonic notation when they occur in the following chord.

Figure 35. Chromatic melodization by means of anticipated chordal tones (continued).

Figure 35. Chromatic melodization by means of anticipated chordal tones (concluded).

G. Statistical Melodization of Chromatic Progressions

The second technique derives from the method of constructing a *quantitative scale.* Such a scale may be evolved by a purely *statistical* method. Although it is not obvious even to the most discriminating ear, it is easy to find by plain addition the quantity in which each chromatic pitch-unit appears during the course of a harmonic continuity. To find a quantitative scale, write out a full chromatic scale from *any note* (I do it usually from *c*).

The next procedure is to add up all the *c*-pitches in a given harmonic progression (doubled tones to be counted as one and enharmonics to be included). Then proceed with all of the c♯-pitches, the d-pitches, etc., until we sum up the entire chromatic scale. This produces a quantitative analysis of the full chromatic scale. Now, by eliminating some of those units which have lower marks, we obtain a *quantitative* (diatonic) *scale.*

The unit having the highest total becomes the root-tone of the scale and, possibly, the axis of the future melody. If more than one unit has a high mark, it is up to the composer to select one of them as the axis.

In the chromatic progression of Fig. 35, a quantitative analysis would be:

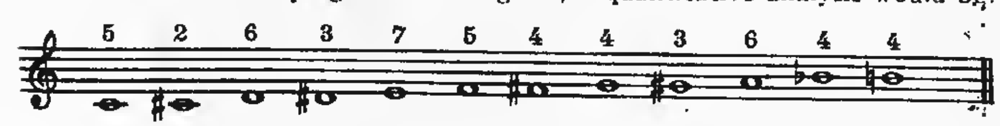


Figure 36. Quantitative analysis of chromatic scale in figure 35.

By excluding all values below 4, we obtain the following nine-unit scale with the root-tone on *e* (maximum value).

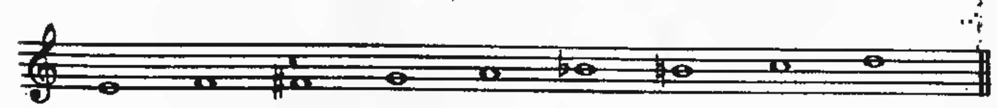


Figure 37 Reduced to nine-unit scale with root-tone on e.

If such a scale seems to be too chromatic, further exclusion of the tones with lower marks will reduce it to a scale of fewer units.

By excluding all the marks below 5 in this case, the scale will be reduced to one of five units and will have a purely diatonic appearance.

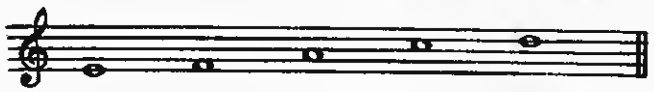


Figure 38. Reduced further by excluding all marks below 5.

The next procedure is the actual melodization, performed according to the diatonic technique. By this method the tones which predominate quantitatively during the course of chromatic continuity (and which affect us as such physiologically, i.e., as excitations) become the units *some of which satisfy every chord.* They attribute great stylistic unity to the entire product of melodization.

The number of attacks of M against H largely depends on the possibilities of melodization.

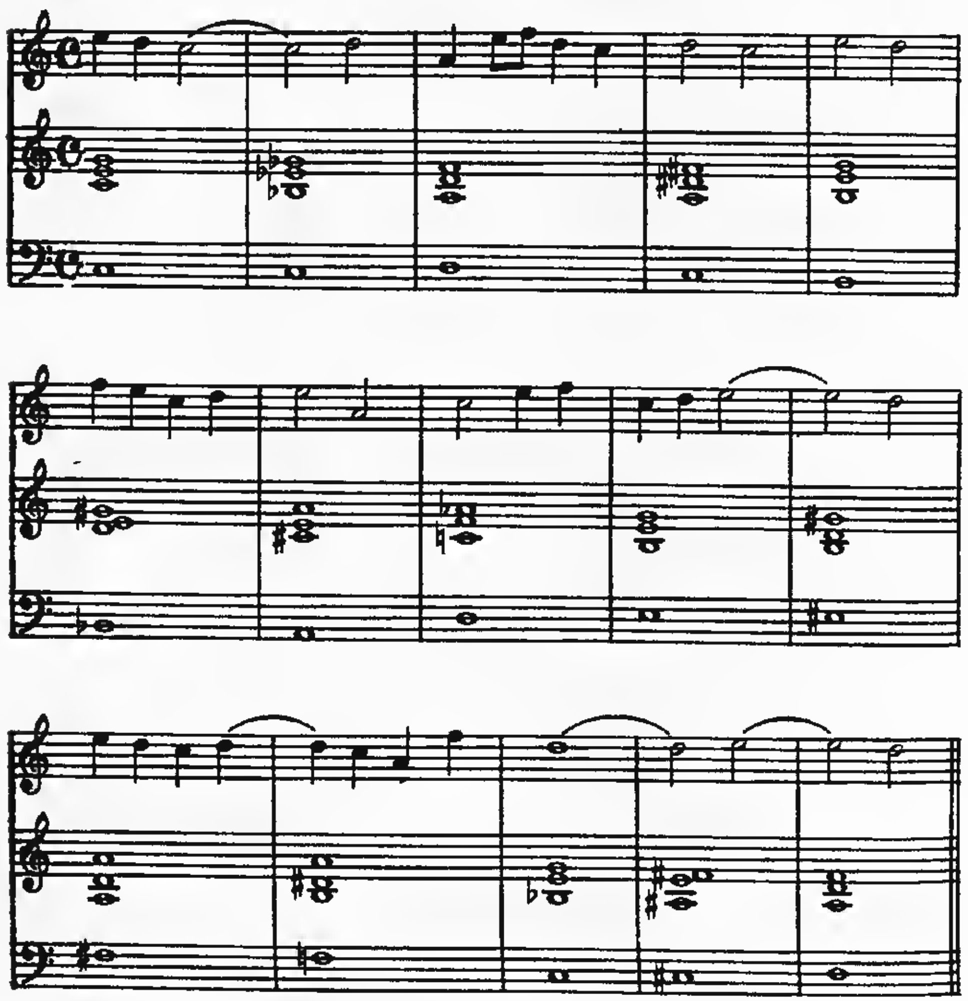


Figure 39. Chromatic melodization by means of quantitative diatonic scale.

These two techniques of chromatic melodization may be combined in sequence. This results in contrasting groups of first a diatonic and then a chromatic nature. The quantity of H covered by one method can be specified by means of the coefficients of recurrence.

For example: 2H di + H ch.

Figure 40. Combining two techniques of chromatic melodization.

CHAPTER 3

THE HARMONIZATION OF MELODY

THE usual approach to the problems of harmonization of melody seems entirely superficial when we consider that the very task of "finding a suitable harmonization" is expected to solve the problem in its entirety. Looking back at music which has already been written, we find a great diversity of styles of harmonization. In some cases the melody has a predominantly diatonic character while the chords seem to form a chromatic progression; in other cases the melody has a predominantly chromatic character while the accompanying harmony is entirely diatonic. Operatic works by Rimsky-Korsakov and Borodin illustrate the first type; music by Chopin, Schumann and Liszt supply examples of the second type. This raises the whole question of an accurate and systematic classification of the styles of harmonization.

By the pure method of combinations, we arrive at the following forms of harmonization:

(1) Diatonic harmonization of a diatonic melody.
(2) Chromatic harmonization of a diatonic melody.
(3) Symmetric harmonization of a diatonic melody.
(4) Symmetric harmonization of a symmetric melody.
(5) Chromatic harmonization of a symmetric melody.
(6) Diatonic harmonization of a symmetric melody.
(7) Chromatic harmonization of a chromatic melody.
(8) Diatonic harmonization of a chromatic melody.
(9) Symmetric harmonization of a chromatic melody.

In addition to these styles, various hybrids may be formed intentionally—and such hybrids do exist in music written on an intuitive basis. The necessity of handling these hybrid forms of harmonic continuity—which is inevitable not only in popular dance music, but frequently in music of composers who are considered "great" and "classical"—in special arrangements or transcriptions requires a thorough knowledge of all pure, as well as hybrid, forms of harmonization.

A. Diatonic Harmonization of a Diatonic Melody

There are two fundamental procedures required for this method of harmonization:

(a) The distribution of the number of attacks in melody and harmony, i.e., the number of attacks of melody to be harmonized by one chord, or the number of chords harmonizing one attack in melody.
(b) Selection of the range of tension.

Let us take a melody consisting of 12 attacks. Such a melody may be harmonized by 12 different chords, each attack in the melody acquiring its individual chord. But it may offer, as well, *two* attacks of a melody harmonized with each chord; in this case, 6 different chords will constitute the harmonic progression. Further, each 3 attacks of a melody may acquire a chord, making 4 chords necessary for the entire melody. Proceeding in similar fashion, one may ultimately arrive at one chord harmonizing the entire melody—this is quite possible, because no pitch-unit in a diatonic scale may exceed the function of 13th and will merely require an 11th chord for its harmonization, in order to support the 13th as an extreme function in a melody in which all the remaining units of the scale may be present as well.

Let us take, for example, the following melody:

Figure 41. Melody.

In order to harmonize this melody with 12 different chords it is necessary to assign each pitch-unit of the melody to one chord. Such an assignment is based on a selection of the range of tension.

Let us suppose that we decide to make our range of tension from the 5th to the 13th. Having a considerable choiçe in the assignment of pitch-units as chordal functions, we will give *preference* to those forming a *positive* cycle of roots for the chords.

Examples of assignment of the above melody:

$\frac{M}{H} = 1$ Range of tension: 5 − 13

Figure 42. $\frac{M}{H} = 1$ *Range of tension: 5 − 13 (continued).*

Figure 42. $\frac{M}{H} = 1$. *Range of tension: 5 – 13 (concluded).*

But if we now decide to assign two attacks in the melody against 1 chord, it is necessary to conceive of the two adjacent melodic pitches as being both in a scheme of chordal functions—thirds in this case. Thus, the first 2 units, a + b, have to be translated into $\frac{a}{b}$, which may, of course, assume any one of the following assignments:

a	9	11	13
b	3	5	7

Likewise, the pair, c + d, transforms itself into:

c	9	11	13
d	3	5	7

The next two units produce:

e	5	7	9	11	13
c	3	5	7	9	11

The next two units produce:

d	5	7	9	11	13
b	3	5	7	9	11

The next two units produce:

e	9	11	13
f	3	5	7

The next two units produce:

g	9	11	13
a	3	5	7

This group of assignments offers a considerable variety of harmonization, even if we preserve only the *positive* system of progressions.

$\frac{M}{H} = 2$ Range of tension: 3 – 13

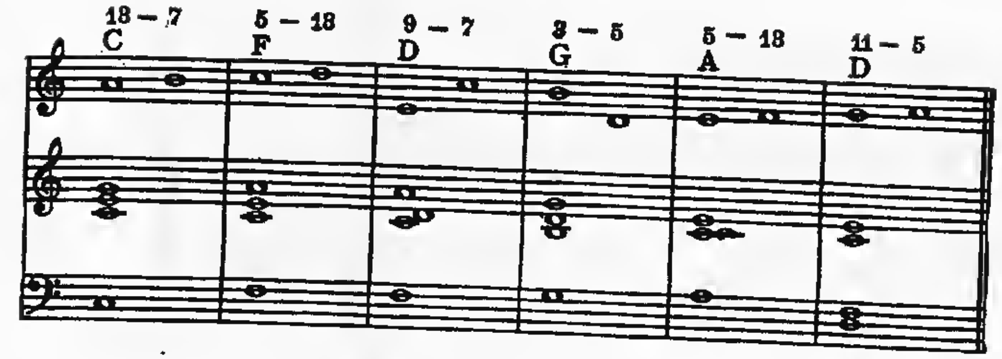


Figure 43. $\frac{M}{H} = 2$ *Range of tension: 3 – 13.*

When we come to assign every three pitch-units of the melody to one chord and to distribute them through the scheme of chordal functions we acquire the following table:

$\frac{M}{H} = 3$ Range of tension: 1 – 13

a.

b.

Figure 44. $\frac{M}{H} = 3$ *Range of tension: 1 – 13.*

And here are examples of the same procedure as applied to harmonization of the 12 melodic tones by three and by two chords respectively:

c.

$\frac{M}{H} = 4$ Range of tension: 1 — 13

Figure 45. $\frac{M}{H} = 4$ *Range of tension: 1 — 13.*

$\frac{M}{H} = 6$ Range of tension: 1 — 13

Figure 46. $\frac{M}{H} = 6$ *Range of tension: 1 — 13.*

d.

$\frac{M}{H} = 12$ Range of tension: 1 — 13

Figure 47. $\frac{M}{H} = 12$ *Range of tension: 1 — 13 (continued).*

Figure 47. $\frac{M}{H} = 12$. *Range of tension: 1 — 13 (concluded).*

Likewise, a ***non-uniform*** group distribution of the pitch-units of a melody may be devised. Rhythmic resultants, or any other material from the procedures already set forth in my theory of rhythm,* may be used as schemes for such distributions.

$\frac{M}{H} = r_{4 \div 3}$ Range of tension: 1 — 9

Figure 48. $\frac{M}{H} = r_{4 \div 3}$ *Range of tension: 1 — 9.*

B. Chromatic Harmonization of a Diatonic Melody

To harmonize a diatonic melody chromatically, it is necessary to obtain first a diatonic harmonization, then to insert passing and auxiliary chromatic tones. These inserted tones must not conflict with any of the pitch-units in the melody. For example, in a c 7th chord, if a melody has b, auxiliary tones may be devised on any of the ***remaining*** chordal functions, i.e., c, e, or g. Such a harmonization will acquire a particularly chromatic appearance if the tones of the figuration are written out together with the chord, thus forming altered chords. The following chromatic harmonization is merely a variation of Figure 44—b, obtained through insertion of the passing and auxiliary chromatic tones.

*See Book I.

*Figure 49. Chromatic harmonization of figure 44*b.

C. Symmetric Harmonization of a Diatonic Melody

Symmetric harmonization of a diatonic melody may be desirable when a certain type of chord structure is preferred to the casual selection that comes when the shapes of the chords are controlled by the diatonic scale. Such chord structures are derivatives of some Σ (13), which may be selected from the complete table of 13th chords. It is usually sufficient to limit the harmonized group to one Σ. In some unavoidable cases, an additional Σ (13) of the same family may be added. A preselected Σ (13) implies a definite harmonic style and brings the structural chord characteristics into prominence. On the other hand, the procedure helps eliminate undesirable or weak sonorities that are inevitable in the purely diatonic system. Any portion of melody consisting of one or more pitch-units may be assigned to be part of a preselected Σ (13) with a definite placement in such a Σ (13). For example, if we take Σ (13) = c – e – g – b♭ – – d – f♯ – a, a melody, the structure of which is in conformity with an incomplete minor S(7), (with omitted 5th), such as c – e♭ – b♭ may be placed on the above Σ as 3 – 5 – 9 or 13 – 1 – 5. No other location of this melodic form is possible with the above Σ.

Figure 50. Melody placed on Σ 13.

After all the melodic forms of one continuous melody are thoroughly analyzed as to their harmonic structure (as in the above case), and after the quantities of attacks of the melody against individual chords are distributed, the next step is to make sure that all such melodic forms will fit a particular Σ (13) selected to satisfy the entire melodic continuity.

To arrive at a practical decision, it is important to verify *all the individual melodic forms* to be harmonized; if necessary, make a corresponding alteration in Σ (13), so as to find a Σ which will satisfy all the forms. Cases in which more than one Σ are needed are comparatively rare, as most of the Σ (13) forms absorb all the partial forms.

Figure 51. Melodic forms of a given Σ 13.

To convert a *symmetric* harmonization of a *diatonic* melody into *chromatic* harmonization, the principle presented in my discussion of how to convert diatonic harmonization into chromatic may be applied, i.e., auxiliary and passing tones are applied within the symmetric chord progressions.

1. More Than One H Against One M

To harmonize one pitch-unit diatonically with *more than one* chord, use a consistent increase of tension. For example, if a melodic note is c, and the chord originally planned is a, thus making c a third, one may add any quantity of chords in addition to that, so that the third eventually becomes a fifth, a ninth, etc. The cycle progression of such chords does not necessarily have to be C_3. For example, the c note in a melody may be 3 – 5 – 9 which requires the following chords: A – F – B.

Chromatization of the diatonic harmony will not require any changes in the above principle; simply supplement the diatonic harmonization by the auxiliary and passing chromatic tones.

In symmetric harmonization of a diatonic melody, when several chords appear against one pitch-unit of a melody, the identical Σ may satisfy the usual requirements of the symmetric harmonization of a diatonic melody. For example, harmonizing with the Σ (13) used in Figure 51, the note c may be satisfied by the following chords:

C, B♭, A♭, G♭, F, E♭, D

arranged in any desirable sequence.

Figure 52. Harmonizing note c with Σ13 of figure 51.

Here is an example of the symmetric harmonization of a complete song, *My Own** (Lyric by Harold Adamson, music by Jimmy McHugh); the entire harmonization is based on one Σ 13th (the first chord in parenthesis). To transform this harmonization into a chromatic one, insert passing auxiliary chromatic tones.

Symmetric harmonization

Figure 53. Symmetric harmonization of "My Own" (continued).

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Figure 53. Symmetric harmonization of "My Own" (continued).

Figure 53. Symmetric harmonization of "My Own" (concluded).

D. Symmetric Harmonization of a Symmetric Melody

There is small probability that any melodies composed from symmetric scales exist outside this system, for the whole conception of symmetric scales itself has hitherto been unknown to the musical world. The problem of harmonization of melodies composed from symmetric scales first requires, therefore, the existence of such melodies. As has been explained in discussing the third and fourth groups of symmetric pitch-scales, melodies may be composed through permutation of pitch-units in the sectional scales (each starting with a new tonic). After the complete melodic form is achieved, the final step consists of superimposition of the rhythm of durations on the continuity of melodic form.

Let us take a scale based on 12 tonics, each sectional scale having a structure 3 + 4; let us limit the entire scale to the first 3 tonics. As scales of the 12-tonic system have a wide range, it is desirable, in many cases, to reduce the range by means of octave-contraction.

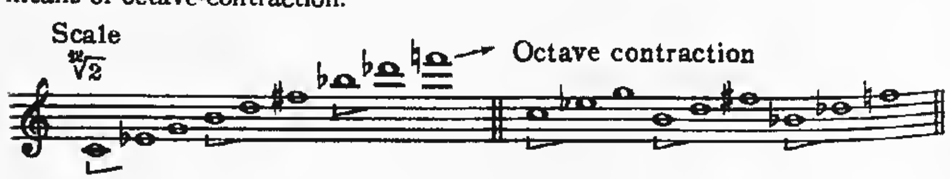


Figure 54. Reducing range of $\sqrt[12]{2}$

The next step is to select a melodic form based on circular permutations of the pitch-units in the above scale, and to select a rhythmic form based on synchronization of $3(2+1)$ and $(2+1)^2$.

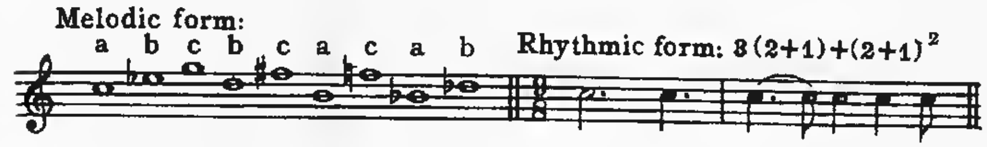


Figure 55. Selecting melodic and rhythmic forms.

By superimposing this rhythm of durations on the melodic form, we obtain an interference between the number of attacks in the melodic form (9) and the number of attacks in the rhythmic form (6). This means that the melodic form will appear twice, and the rhythmic form will appear three times.

Composition of Melodic Continuity

Melodic form consists of 9 attacks

$\frac{9}{6} = \frac{3}{2} \quad \frac{2}{3}\frac{(9)}{(6)}$

Rhythmic form consists of 6 attacks

Melodic Continuity

Figure 56. Interference between number of attacks in melodic and rhythmic forms.

For this melody a sequence of chords will be assigned to each tonic. Thus, the first sectional scale emphasizes 13t: the second, 5t; the third, 13t; the second recurrence of the first, 5t; the second recurrence of the second, 13t; the second recurrence of the third, 5t; and an axis (= 18t) is added to complete the whole.

There are two practical methods of symmetric harmonization of melodies constructed on symmetric pitch scales. The first provides an extraordinary variety of devices—the second is limited to a considerably smaller number of harmonizations.

1. The First Method

The first method assigns the important tones (all pitch-units in this case) of a sectional scale to the three upper functions of a $\Sigma(13)$, adding the remaining functions downward through any desirable selection. The first sectional scale in the sample melody has three pitch-units (c, e♭, g) which we shall originally conceive as 13 — 11 — 9, downwards. The continuation of this chord downwards will produce pitch-units with the following names: a, f, d, b. In the following $\Sigma(13)$ a certain structure is offered as a special case of many other possible Σ's.

Figure 57. $\Sigma 13$.

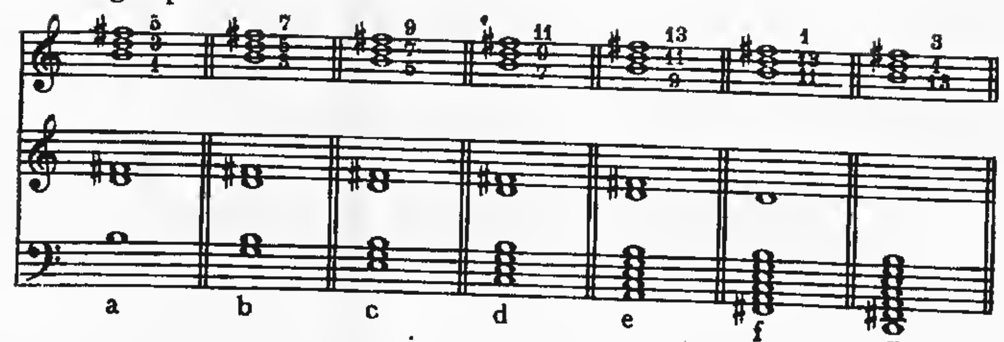
The upper three functions of the chord (denoted in black note-heads in the figure) may produce their own chord in harmony. Thus, the functions 9 — 11 — 13 of the Σ may actually become 1 — 3 — 5. All pitch-units of melody and harmony are identical in this case. (See Figure 58-a). By assigning the same three pitch-units as 3 — 5 — 7 we have to add one function down. (See Figure 58-b).

All further assignments of the three pitch-units, namely 5 — 7 — 9, 7 — 9 — 11, 9 — 11 — 13, 11 — 13 — 1, 13 — 1 — 3 are the c, d, e, f, g, respectively, on Figure 58. This figure offers a complete transposition of all assignments through the three tonics employed in the melody.

C-group

Figure 58. Melodic structures (continued).

B-group



B♭-group

Figure 58. Melodic structures (concluded).

As Figure 58 exhausts all possibilities under the given group of chords, it is possible to exhaust all forms of harmonization for the given melody through various forms of constant and variable assignment of functions. The melody consists of 3 groups; so the sequence of chords with regard to these 3 groups can be read directly from Figure 58. The letters on Figure 59 represent the respective bars of Figure 58 in such a fashion that the first letter refers to the first group of the melody, the second to the second, and the third to the third.

aaa	bbb	ccc	ddd	eee	fff	ggg					
aab	aba	baa	cca	cac	acc	eea	eae	aee	gga	gag	agg
aac	aca	caa	ccb	cbc	bcc	eeb	ebe	bee	ggb	gbg	bgg
aad	ada	daa	ccd	cdc	dcc	eec	ece	cee	ggc	gcg	cgg
aae	aea	eaa	cce	cec	ecc	eed	ede	dee	ggd	gdg	dgg
aaf	afa	faa	ccf	cfc	fcc	eef	efe	fee	gge	geg	egg
aag	aga	gaa	ccg	cgc	gcc	eeg	ege	gee	ggf	gfg	fgg
bba	bab	abb	dda	dad	add	ffa	faf	aff			
bbc	bcb	cbb	ddb	dbd	bdd	ffb	fbf	bff			
bbd	bdb	dbb	ddc	dcd	cdd	ffc	fcf	cff			
bbe	beb	ebb	dde	ded	edd	ffd	fdf	dff			
bbf	bfb	fbb	ddf	dfd	fdd	ffe	fef	eff			
bbg	bgb	gbb	ddg	dgd	gdd	ffg	fgf	gff			

Figure 59. Total number of possible harmonizations (continued).

abc	bcb	cde	def	efg
abd	bce	cdf	deg	
abe	bcf	cdg	dfg	
abf	bcg	cef		
abg	bde	ceg		
acd	bdf	cfg		
ace	bdg			
acf	bef			
acg	beg			
ade	bfg			
adf				
adg				
aef				
aeg				
afg				

Figure 59. Total number of possible harmonizations (concluded).

The total number of possible harmonizations to be derived from Figure 59 is as follows: 7 cases with constant tension: aaa, bbb, etc. $18 \times 7 = 126$ cases on a tension that is constant for 2 of the three groups. $35 \times 6 = 210$ cases with variable tension for all 3 groups. Thus, the total number of harmonizations offered for the melody is $7 + 126 + 210 = 343$.

2. The Second Method

The second method is based on a random selection of a $\Sigma(13)$ based entirely on the composer's preference with regard to sonority. As any Σ (13) has definite substructures, often in limited quantities, the possibilities of harmonization are less varied than through the first method. If one selects $\Sigma(13)$ with b♭ and f♯ on a c scale (see Figure 60) the possibilities of accommodating a sectional scale $3 + 4$ (minor triad) becomes limited to only two assignments, namely, $5 - 7 - 9$ and $13 - 1 - 3$.

Figure 60. $\Sigma(13)$ with b♭ and f♯ on c scale.

Retransposing these functions to the melody assigned for harmonization, we obtain the following results:

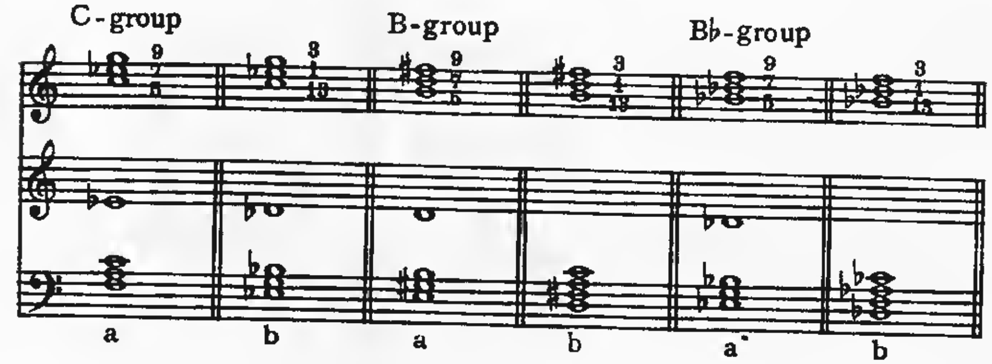


Figure 61. Harmonizing the melody.

It follows from this figure that each sectional scale of melody permits only two versions of chords. By either a constant or variable assignment of the two possible versions, a complete table of possible harmonizations is obtained.

aaa	bbb
aab	bba
aba	bab
baa	abb

Figure 62. Table of possible harmonizations.

The total number of possible harmonizations amounts to 8.

When the sectional scales are too complete, assignment of only certain tones as chordal functions is necessary. For example, in the following scale based on 3 tonics and 5-unit sectional scales, it is sufficient to assign the white notes as chordal functions, then in the melody derived from such a scale, black notes become the auxiliary and passing tones.

Figure 63. Scale based on 3 tonics and 5-unit sectional scales. White notes are chordal. Black are auxiliary.

In some symmetrical scales, the structure of individual sectional scales is such that the sonority of certain pitch-units does not conform to the structures of special harmony (i.e., the harmony of thirds). Some of the units of such sectional scales may be disturbing, and although they may fit as passing tones in some chord structures other than those used in this special harmony, they decidedly do not fit as passing tones in any $\Sigma(13)$. In such a case, each pitch-unit in such sectional scale of a compound symmetric scale must be selected either as a chordal function or as an auxiliary tone with a definite direction. These pairs—i.e., the chordal tone and its auxiliary tone—are then *directional units.*

In composing melodic forms from scales containing such directional units, permute the directional units and not simply the individual pitch-units. After all the units are assigned, the above-described procedure of harmonization (the second method) may be applied.

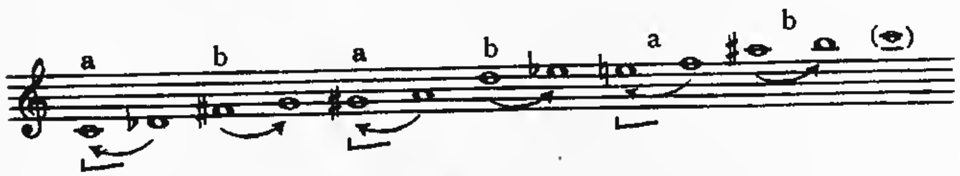


Figure 64. Applying the second method of harmonization.

The arrows on the above figure lead from an auxiliary tone to a chordal function.

E. Chromatic Harmonization of a Symmetric Melody

Chromatic harmonization of a symmetric melody is based on the same principle as chromatic harmonization of a diatonic melody. The procedure consists of inserting passing and auxiliary chromatic tones into symmetric harmonic continuity. As a result of the insertion of passing or auxiliary chromatic tones, altered chords may be formed as independent forms.

This type of harmonization may sound to the listener's ears either as chromatic continuity or as symmetric continuity with passing chromatic tones.

*Deep in a Dream**
by Jimmy Van Heusen and Eddie DeLange.

Figure 65. Four-part hybrid with chromatic harmonization (continued).

Figure 65. Four-part hybrid with chromatic harmonization (continued).

Figure 65. Four-part hybrid with chromatic harmonization (concluded).

If the composer or arranger finds that certain passing or auxiliary tones in the above example sound unsatisfactory, he may eliminate them. The greater the allowance made for altered chords, the greater are the possibilities for giving a chromatic character to a symmetric harmonic continuity.

F. Diatonic Harmonization of a Symmetric Melody

Melodies constructed from symmetric scales cannot be harmonized by a purely diatonic continuity. The style that has the most nearly diatonic characterization is in reality a hybrid of diatonic progressions *symmetrically connected.* This type of harmonization is possible when the melody that has been evolved within the scope of an individual sectional scale is one that can be harmonized by several chords belonging to one key. The relationship of symmetric sectional scales defines the form of symmetric connections between the diatonic portions of harmonic continuity. The diatonic portions of harmonization are brought into conformity with one key.

Symmetrical tonics do not necessarily represent the root chords of a key. For example, a note, c, in a melody scale may be the 1, or the 3, or the 5, etc., of any chord. In most cases, in music of the past, such harmonizations usually pertained to identical motifs in symmetric arrangement—as in the first announcement of a theme by the celli in Wagner's overture to *Tannhäuser*, where identical motifs are arranged through $\sqrt[4]{2}$, and the diatonic portions appear, the first in B minor making a progression IV − I − V − III, the following sections as exact transpositions through the $\sqrt[4]{2}$, i.e., in D minor and F minor respectively.

Figure 66. Identical motifs in symmetric arrangement for "Overture" to Tannhäuser.

In the following example of harmonization, the melody is based on a symmetric scale with three pitch-units (2 + 1) connected through $\sqrt[3]{2}$.

Figure 67. Melody based on symmetric scale with 3 pitch-units connected through $\sqrt[3]{2}$.

Each bar comprises one sectional scale utilizing the melodic form, abcb. As there are many ways of harmonizing such a motif, I shall give here one of them which produces $C_0 + C_7 + C_5$ for each group. All the following groups are identical reproductions of the original group, connected through $\sqrt[3]{2}$.

Figure 68. Harmonizing the motif of figure 67. $C_0 + C_7 + C_5$.

Music by Rimsky-Korsakov, Borodin and Moussorgsky has abundant examples of such forms of harmonization.

In order to transform the above harmonization into a chromatic one, all that is necessary is to insert passing and auxiliary chromatic tones. A diatonic harmonization of those symmetric melodies which have not been composed on the sequence of identical motifs, and in which different portions pertaining to individual sectional scales are connected symmetrically, is possible as well. The latter form is not as obvious and it may seem somewhat incoherent to the ordinary listener.

G. Chromatic Harmonization of a Chromatic Melody

A melody which is to be harmonized chromatically must be a chromatic melody consisting of long durations. Each group of three units of melody must then be assigned to a chromatic operation in a chromatic group of harmony. The usual sequence d — ch — d refers to every three notes where the middle note is a chromatic alteration. In the following melody, the chromatic groups of harmony will be assigned as follows:

Group 1: c — c# — d
Group 2: d — d# — e
Group 3: a — a♭ — g
Group 4: g — g# — a
Group 5: a — a# — b

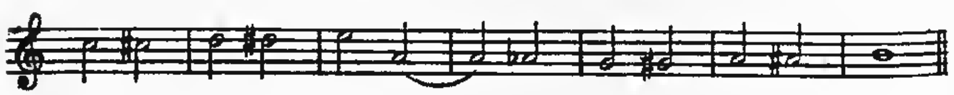


Figure 69. Chromatic melody.

The process of harmonization of a chromatic melody chromatically, consists of two procedures, once the pitch-units have been assigned to some number combinations. As our technique of chromatic harmony deals with 4-part harmony, the melody must become some one of the four parts. Let us assign the chromatic groups to the above melody as follows:

Group 1: 1 — 1 — 1
Group 2: 1 — 1 — 5
Group 3: 5 — 5 — 3
Group 4: 3 — 1 — 1
Group 5: 1 — 1 — 1

In group 3, a♭ is a lowered fifth. In group 5, a# is a raised root tone. The following example represents the above melody in a 4-part setting.

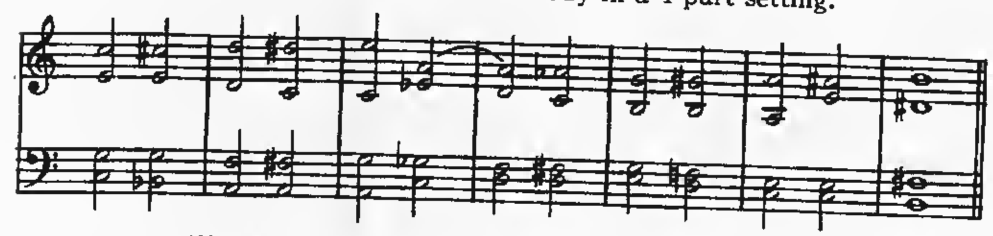


Figure 70. Melody of figure 69 in a four-part setting.

The final procedure in chromatic harmonization of a chromatic melody consists of *isolating* the melody; placing it above the harmony; and melodizing the *remaining 3-part harmony* with an additional voice. This additional voice is devised according to the fundamental forms of melodization, i.e., it may double any of the functions present in the chord, or it may add the function next in rank.

In the following example, the notes in parenthesis represent such an added voice. The functions of this voice are:

g — 5	e — 9
b — 13	c# — 13
a — 5	d — 5
b — 9	e — 5
b — 7	g — 7
c — 7	a — 7

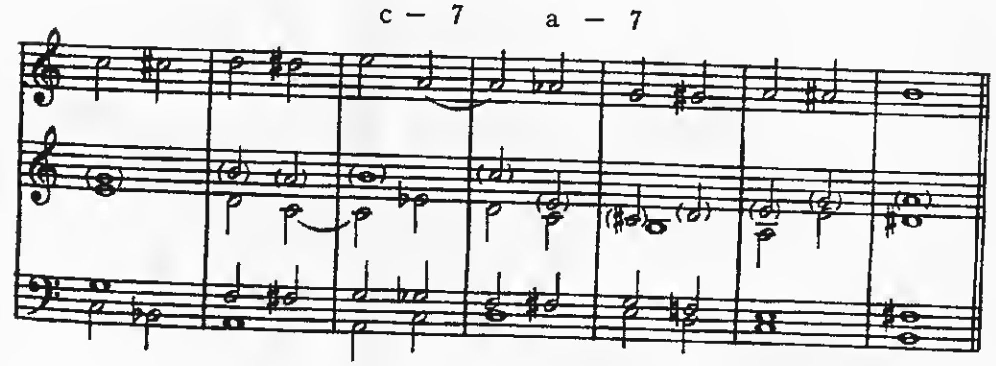


Figure 71. Chromatic harmonization of a chromatic melody.

H. Diatonic Harmonization of a Chromatic Melody

A chromatic melody may be diatonically harmonized when it has a considerable degree of animation (short durations). In such a case, some of the tones are treated as chordal functions and some become auxiliary or passing chromatic tones. The process of determining which functions are to be diatonic then takes place.

The following example is the same melody that was used as an illustration in the preceding section; here it is used in its most animated form.

Figure 72. Chromatic melody in animated form.

By assigning $\begin{bmatrix} c-5 \\ d-13 \end{bmatrix}$ we acquire an F chord. In the next bar, by assigning $\begin{bmatrix} a-5 \\ e-9 \end{bmatrix}$ we obtain a D chord. By assigning $\begin{bmatrix} g-1 \\ a-9 \end{bmatrix}$ we obtain a G chord; and by assigning b 1 — 5, we obtain the B and E chords. In this way the entire melody can be placed in a certain desirable key (C major in this case). The units a♯ and c♯ in the second bar are auxiliary tones to the third and fifth respectively of the G chord. The entire harmonization has a Phrygian character.

Figure 73. Diatonic harmonization.

Another example of harmonization of the same melody will be found on the following page; by assigning the melodic tones to operate as the following functions, we obtain another harmonization:

c — 7	e — 13	g — 5	
d — 1	a — 9	a — 13	b — 3

Figure 74. Another harmonization.

I. Symmetric Harmonization of a Chromatic Melody

Symmetric harmonization of a chromatic melody is used for melodies of long durations. In most cases each pitch-unit of a melody has to be harmonized by a different chord. The advantage of the symmetric method of harmonization is that, if a melody is partly diatonic, there is an opportunity to use one chord against more than one pitch-unit of a melody. Any symmetric harmonization, as in the cases above, must be based on a preselected $\Sigma(13)$.

Let us assign the following $\Sigma(13)$ and use it for harmonization of the same melody as that used in the previous examples. The important considerations in the following procedure are (1) the variation of tension, and (2) the utilization of enharmonics as participants of $\Sigma(13)$; a♭ supplies an equivalent of g♯ for the 13th of a B chord.

Figure 75. Symmetric harmonization is based on a preselected $\Sigma(13)$.

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK VII

THEORY OF COUNTERPOINT

The Technology of Correlated Melodies

BOOK VII

THEORY OF COUNTERPOINT

CHAPTER 1

THE THEORY OF HARMONIC INTERVALS

ANY sequence of two pitch-units produces a ***melodic interval***. A simultaneous combination of two pitch-units produces a ***harmonic interval***. The technique of correlation of simultaneous melodies depends entirely on the composition of harmonic intervals. Any number of simultaneous parts (voices) in counterpoint is formed by the pairs. These pairs may be conceived as voices immediately adjacent in pitch, or in any other form of vertical arrangement (i.e., over 1, over 2, etc.).

The degree of harmonic versatility achieved in counterpoint depends on the manifold of harmonic intervals used in a certain style. A limited number of harmonic intervals results in limited forms of harmonic versatility in counterpoint. The study of harmonic intervals is an important prerequisite to the study of counterpoint.

Harmonic intervals have a *dual* origin:

1. physical
2. musical.

The *physical* origin of harmonic intervals goes back to the simplest ratios. The *musical* origin of intervals is based on selective and combinatory processes. All semitones—i.e., the units of the equal temperament of twelve—are the structural units of all other harmonic intervals available in such equal temperament. As they occur in our hearing, they take the following forms:

$$i = 1, i = 2, i = 3, i = 4,$$
$$i = 5, i = 6, i = 7, i = 8$$
$$i = 9, i = 10, i = 11, i = 12$$

The above, of course, includes the entire selection available within one octave range. The addition of an interval to an octave produces a musically identical interval over one octave, for the similarity of different pitch-units within the ratio of 2 to 1 is so great that they even have identical musical names. The present system of musical notation involves—among other forms of confusion—a dual system of interval nomenclature. An interval containing three semitones, for example, may be called either a minor third or an augmented second.

A. Some Acoustical Fallacies

The simple ratios of acoustical intervals are merely *approximate* equivalents of harmonic intervals in equal temperament. It is not scientifically correct to think—as the majority of acousticians do—that a 5 to 4 ratio is the equivalent of a major third; or a 6 to 5, of a minor third; or a 7 to 4, of a minor seventh, etc. These intervals *deviate* considerably from their equivalents in equal temperament.

It is utterly impossible to follow some acousticians in the comparative relations they establish between the type and quality of intervals in the equal temperament of twelve and the equivalents of these intervals in simple acoustical ratios.* So-called "consonance" is a *totally different* type of interval relationship depending on whether it is considered musically or acoustically. If music actually had to use acoustical consonances only, while being confined to the equal temperament of twelve, the only real consonance available would be the octave; no two pitch-units bearing different names would ever be used, and we would have neither harmony nor counterpoint; for no intervals other than an octave (or a perfect fifth, with a certain allowance) are consonances within equal temperament. All other intervals are quite complicated ratios. The art of music in fact, however, has its own possibilities based on the limitations within the given manifold constituted by our tuning system.

Now, the acoustical consonances produce the so-called "natural harmonic scale," which consists of a fundamental tone with all its partials appearing in the same sequence as a natural harmonic series—1, 2, 3, 4, 5, 6, 7, 8, 9, etc. The ratios of acoustical consonances are equivalent to the ratios of vibrations producing pitches. For example, a $\frac{3}{2}$ ratio means that if the actual quantities representing both the numerator and the denominator were multiplied by a considerable number value, they would actually sound as pitches. While $\frac{3}{2}$, as such, sounds to our ear as the resultant of an interference of 3 to 2, $\frac{300}{200}$ cycles per second sounds to our ear as a perfect fifth.

*Figure 1. Acoustical scale of natural harmonics.***

Our ears accept pitch-units and their ratios in the form in which they reach our ears and our auditory consciousness—and not as they are asked to do according to the traditional musical schooling. For example, a melody played simultaneously in the key of c and in the key of b next to it, or a seventh above, sounds decidedly disturbing to musicians of our time. Yet an interval that is

*Indeed, despite the specific warning of the great acoustician, Helmholz, against careless application of his discoveries to music, the "acoustical fallacy" has vitiated endless quantities of musical theorizing. So we find Sir Donald Francis Tovey—by no means an undistinguished writer on music—lamenting that no "true" harmonic ideas are based on equal temperament, a statement which he can make directly in the face of the best that Western music has produced for more than 400 years. (Ed.)

**This scale is necessarily given in the notation used for equal temperament; the intervals in the acoustical scale—save for the octaves—are, of course, *not* identical with the same intervals in equal temperament. (Ed.)

musically identical is acoustically so different that, being placed three octaves apart, it produces a musically *consonant* impression.* The reason for this is that such an absolute interval as the seventh three octaves apart approximates the 15 to 1 ratio, i.e., the sound of a 15th harmonic in relation to its fundamental—and when the pitches are so far apart, the deviation from equal temperament becomes less obvious in our discrimination of pitch. The following tables offer a group of examples illustrating musically consonant intervals which are usually classified as dissonances, together with their correspondence to the proper location of harmonics. In all these cases, no octave substitution can be made without affecting the actual state of consonance.

Figure 2. Musically consonant intervals usually classified as dissonant.

Likewise, when musical consonances are placed in a wrong pitch register—such as low register—they produce upon our ears the effect of musical dissonances. The reason for this is that, being an approximation of simple ratios, they require the placement of their fundamentals at such low frequencies that they are below the range of audibility. For example, a major third—being associated with $\frac{5}{4}$ ratio—would require that its fundamental be located two octaves below the fourth harmonic; when music is played in major thirds in the contra-octave, the physical existence of such a fundamental is impossible.

*When Schillinger played the fourth example here (a melody coupled to its 7th at a 7 plus 3 octave interval), any number of capable musicians thought it was a 4-octave coupling. (Ed.)

**These numbers correspond to the numbers appearing on the acoustical scale of natural harmonics (Figure 1); they refer to the pitch of the first note. (Ed.)

The following tables offer three examples of the low setting of intervals.

Figure 3. The low setting of intervals.

With these thoughts in mind, we can see that no serious theory of the resolution of dissonant intervals may be devised without specifications as to the *exact octave location* of the intervals. In studying my theory of resolution of intervals, bear in mind that I offer it for the purpose of giving the composer a versatile treatment of progressions of harmonic intervals—not for the purpose of eliminating dissonances. Esthetically as well as physiologically, all of us desire sequences of tension and release. And, as different harmonic intervals produce different degrees of tension, the versatility of the sequence of intervals will satisfy such requirements.

It has often been the case that music written according to the "rules and regulations" of dogmatic counterpoint does not sound esthetically as convincing as real counterpoint in the 16th or 17th centuries. This inferior quality is due to the limited number of harmonic intervals and the forms of treatment of the former.

B. Classification of Harmonic Intervals within the Equal Temperament of Twelve

Any harmonic interval may be classified in one of two ways:

1. With regard to its *density*, i.e., the fullness of sonority;
2. With regard to its *tension*, i.e., the degree of dissonance.

Classification of *density* evolves from the intervals producing the "emptiest" effect upon our ears up to the intervals producing the "fullest" effect. The table on the following page is only a general one; nevertheless, it serves the purpose with a certain degree of approximation—the first few intervals sound decidedly empty; the last few, decidedly full; in the middle, there are some intermediate intervals.

*See the footnote on the preceding page with regard to these numbers. (Ed.)

Figure 4. Classification of intervals according to density.

Classification of intervals according to *tension* is based on a separation of consonances from dissonances—and upon a separation of intervals which are consonances or dissonances *by name* from those which are consonances or dissonances *by sonority*. Every case in which a consonance and a dissonance correspond both in name and sonority is a case implying *diatonic* intervals; all cases in which the consonances and dissonances do *not* correspond with their original names produce *chromatic* intervals. The group of diatonic consonances includes perfect unisons, perfect octaves, perfect fifths, perfect fourths, major thirds, minor thirds, major sixths, minor sixths. The group of diatonic dissonances includes major and minor seconds, major and minor sevenths, major and minor ninths. All the chromatic intervals are classified into *augmented* and *diminished*.

The Augmented Intervals:
Unison, 2nd, 3rd, 4th, 5th, 6th.

The Diminished Intervals:
Octave, 7th, 6th, 5th, 4th, 3rd.

Tension

Figure 5. Diatonic and chromatic dissonances and consonances.

The augmented	unison	is	equivalent	to	minor 2nd	by sonority.
"	" 2nd	"	"	"	minor 3rd	" "
"	" 3rd	"	"	"	perfect 4th	" "
"	" 4th	"	"	"	no diatonic interval.	
"	" 5th	"	"	"	minor 6th	by sonority.
"	" 6th	"	"	"	minor 7th	" "
The diminished	octave	"	"	"	major 7th	" "
"	" 7th	"	"	"	major 6th	" "
"	" 6th	"	"	"	perfect 5th	" "
"	" 5th	"	"	"	no diatonic interval.	
"	" 4th	"	"	"	major 3rd	by sonority.
"	" 3rd	"	"	"	major 2nd	" "

The following "dissonant" intervals are actually consonances by sonority: the augmented 2nd, 3rd, 5th; the diminished 7th, 6th, 4th. All other chromatic intervals will be treated as dissonances, with resolutions corresponding to those of either diatonic or chromatic dissonances.

C. Resolution of Harmonic Intervals

The need for varying the tension results in the procedure known as the resolution of intervals. It is important to realize that the variation of tension may be gradual quite as well as sudden; the transition from a more dissonant harmonic interval to a less dissonant one, and finally into a fully consonant one, is just as desirable as a direct transition from extreme tension to full consonance.

In the following tables, intervals such as the perfect 4th and 5th are included along with the dissonances—not for the purpose of relieving them of tension, but for the purpose of devising different useful manipulations for contrapuntal sequences. The actual number of resolutions known to any composer has a definite effect on the harmonic versatility of his counterpoint. For example, if a composer knows only *four* resolutions of a major 2nd (which is the usual case) as compared to the *twelve* possible resolutions, the number of musical possibilities open to him is considerably restricted. Thinking in terms of variations one can see that the number of permutations available from four elements differs so much from those afforded by twelve elements that they cannot be compared, the first giving twenty-four variations and the second giving 479,001,600 variations. It is easy to see that when a composer suffers such losses as to the quantity of resolutions for each harmonic interval, the loss in the total versatility of his counterpoint is incalculable.

There is no need to memorize all the details for the resolution of intervals, as there are general underlying principles evolved over the centuries:

1. All diatonic intervals resolve through outward, inward, or oblique motion. Each moving voice moves by a semitone or whole tone.*

*An i = 3 is also correct when such an interval represents two adjacent musical names (c — d#, for example). (J.S.)

2. A resolution obtained through oblique motion may be replaced by one in which the formerly sustained voice leaps by a melodic interval of a perfect 4th, either up or down.

3. All intervals known as *2nds* have a tendency to expand. All intervals known as *7ths* have a tendency to contract. All 7ths are the exact equivalent of 2nds in the octave inversion (i.e., pitch-units are identical with those of the 2nds). All the *9ths* have a tendency to *contract*. All the *4ths* and *5ths* are "neutral," i.e., they either *expand* or *contract*.

Thus, the entire range of permutations of semitones and whole tones, with their respective directions, constitutes the entire manifold of resolutions.

The reader may refer to the "Chart of Resolution of Diatonic Intervals" below.

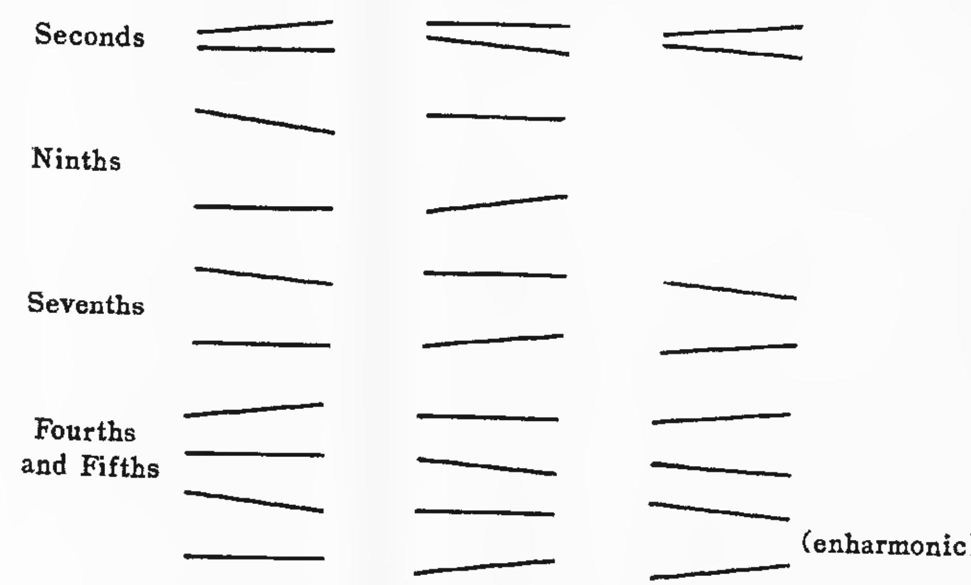


Figure 6. Resolution of diatonic intervals.

The following is a complete table of resolutions of diatonic intervals. The intervals in parentheses are the secondary resolutions, used in all cases in which the first resolution produces a dissonance:

Seconds and Sevenths

Figure 7. Resolution of seconds and sevenths (continued).

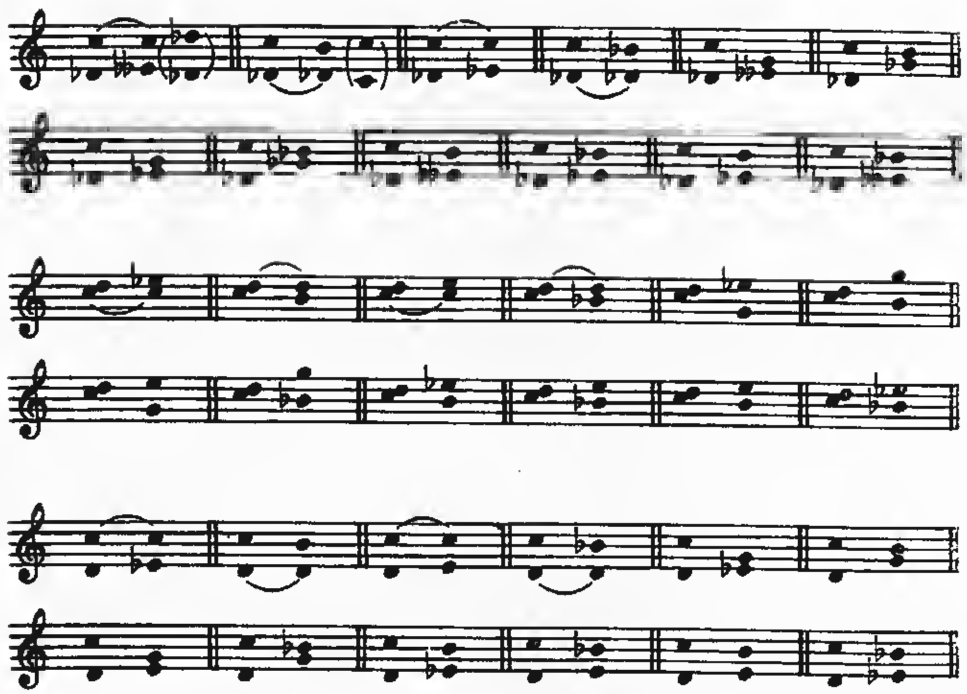


Figure 7. Resolution of seconds and sevenths (concluded).

Fourths and Fifths

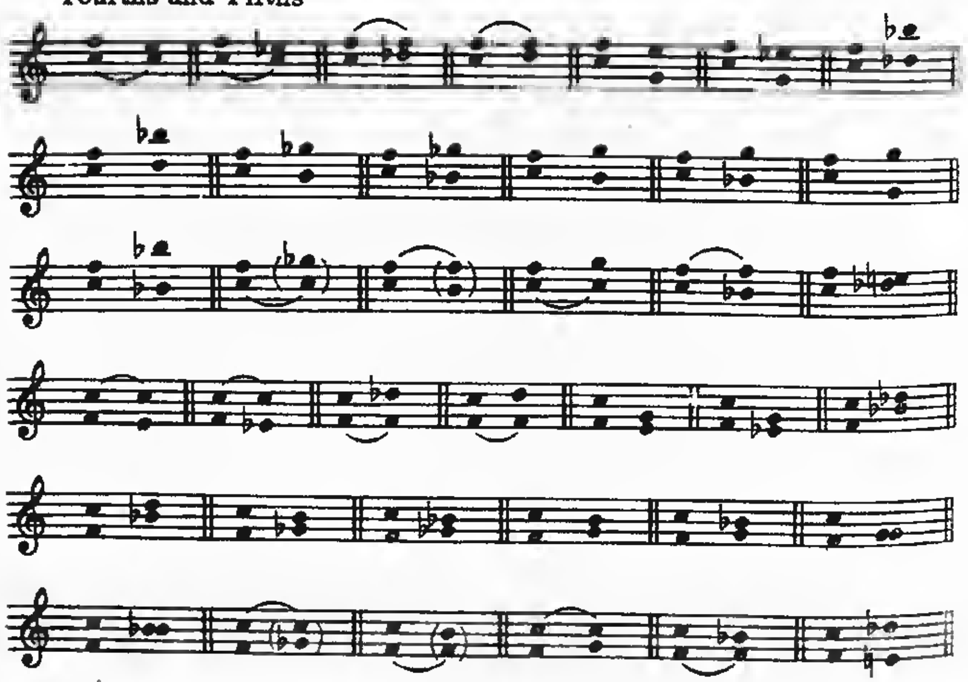


Figure 8. Resolution of fourths and fifths.

Ninths

Figure 9. Resolution of ninths.

D. Resolution of Chromatic Intervals

All chromatic intervals which are augmented have a tendency toward expansion, and all chromatic intervals which are diminished have a tendency toward contraction. The logic of the resolution of augmented or diminished intervals is as follows: $\genfrac{}{}{0pt}{}{d\sharp}{c}$ is a 2nd derived through augmentation of a major second, through either altering the d to d♯, or the c♯ to c♮. Originally it could only have been a 2nd $\genfrac{}{}{0pt}{}{d}{c}$ or $\genfrac{}{}{0pt}{}{d\sharp}{c\sharp}$. Considering the dual origin of such interval, we find the respective resolutions: if d♯ is the alteration of d, its inertia makes it move further in the same direction, to e; or if c♮ is the alteration of c♯, it moves by inertia* to b♮. These two steps taken individually or simultaneously constitute the fundamental resolutions. An analogous procedure must be applied to diminished intervals; the diminutions are produced through inward alteration.

The following is a complete table of resolutions of chromatic intervals. When a chromatic interval resolves into a consonance by sonority, the sign "enh." is placed above it meaning "enharmonic." When the interval of resolution is surrounded by parentheses, the interval of resolution is a dissonance:

Unison

2nd

Figure 10. Resolution of chromatic intervals (augmented).
[*Continued on following page*].

*Note that inertia scientifically refers to the tendency of moving bodies to keep on moving in the same direction, as well as the tendency of motionless bodies to remain motionless. (Ed.)

Figure 10. Resolution of chromatic intervals (augmented) [concluded].

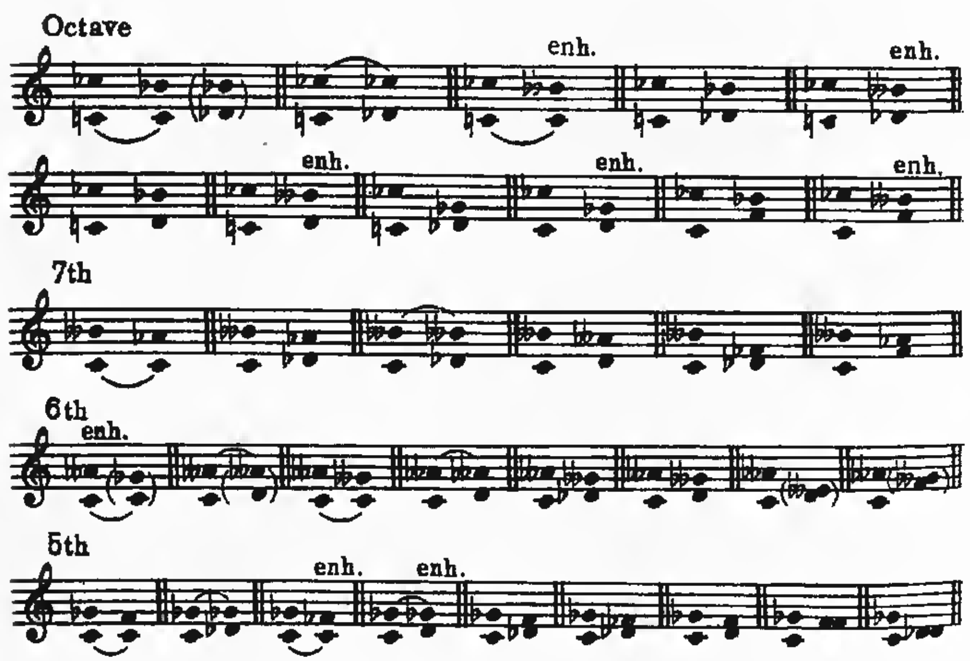


Figure 11. Resolution of chromatic intervals (diminished).
[*Continued on following page*].

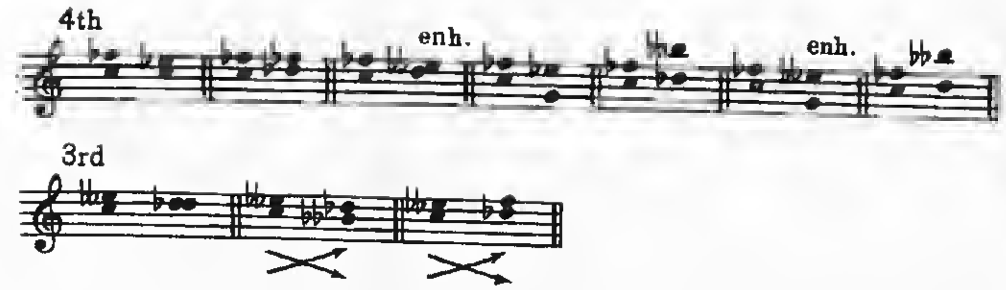


Figure 11. Resolution of chromatic intervals (diminished) [concluded].

In the old counterpoint we often find a type of resolution different from those described above. They were known as *cambiata** resolutions and were conceived of as a melodic step of a 3rd instead of a 2nd. No good explanation has ever been given of the use of such resolutions; I offer an hypothesis to explain these resolutions, which I believe is the correct one.

As the tradition of old counterpoint was developed while the pentatonic (5-unit) scales were in use, some of the pitch-units of full diatonic (heptatonic, 7-unit) scales were absent. If we find that in resolving an interval $\genfrac{}{}{0pt}{}{d}{c}$, d moves to e, while c moves to a (instead of to b), a *cambiata* takes place simply because the scale is a pentatonic scale and the unit, b, does not exist.

This approach offers us a definite principle for resolution of intervals in scales which have not been in use in classical traditional music confining all resolutions merely to the step with the succeeding musical name. For example, in harmonic *a*-minor, the interval $\genfrac{}{}{0pt}{}{a}{g\sharp}$ may be resolved through movement of the lower voice only to f♮, as no other pitch-unit with the name f exists in the scale.

* *Nota cambiata*, i.e., a class of "changing" note. In addition to the requirements of resolution, the classical *cambiata* also observed certain temporal considerations with respect to the accent. (Ed.)

CHAPTER 2

THE CORRELATION OF TWO MELODIES

As counterpoint represents a system of correlation of melodies in simultaneity and continuity, it is absolutely essential that the composer be thoroughly familiar with the constitution of melody. Only through complete familiarity with the material discussed in my exposition of the *Theory of Melody** is the successful accomplishment of such a task possible. The correlation of melodies is usually considered to be one of the most difficult of procedures; this is because the structural constitution of even one melody is unknown in ordinary theory; hence the combination of two unknown quantities is an entirely fantastic task to undertake.

The problem is not only that of putting two voices together, but one of either combining two melodies already made, or making a composition of two melodies with distinct individual characteristics. As *each* melody consists of several components—such as the rhythm of durations, attacks, melodic forms, the forms of trajectorial motion, etc.,—the correlation of *two* melodies adds one more component to those just mentioned: harmonic correlation. Counterpoint can be defined briefly as *a system of correlation of rhythmic, melodic, and harmonic forms in two or more conjugated melodies.*

I shall assume that the forms applying to one individual melody are known through the previous material; we will now cover that field of harmonic correlation which is based on the theory of harmonic intervals in Chapter 1 of this section. After covering this particular subject, I shall then discuss other forms of correlation—so that the composer may be capable of using the complete resources offered by contrapuntal technique.

A. Two-Part Counterpoint

The fundamental technique in writing two-part counterpoint is based on the writing of a new melody to a given melody. A given melody is usually *abstracted* from its rhythm of durations, thus producing a melodic form which may be taken from a choral, as well as from a popular song. The usual way of presenting such an abstracted melodic form is in whole notes, and this is usually called the *cantus firmus* ("firm chant," canonic, or established, chant). The abbreviation we shall use for cantus firmus will be "C.F."; for the melody written to it, counterpoint or "C.P." The first forms of counterpoint will be classified according to the number of attacks in C.P. occurring against one attack in C.F. All of these fundamental forms of counterpoint are devised as follows:

$$\frac{CP}{CF} = 1, 2, 3 \ldots\ldots n$$

*See Book IV.

B. $\boxed{\frac{CP}{CF} = a}$

This form of counterpoint—through international agreement for a number of centuries—implies the usage of consonances only. As we shall have four fundamental forms of harmonic correlation, and as two of these forms will be polytonal (i.e., there will be two different keys used simultaneously), we will have to use consonances by *name* and by *sonority*.

The positive requirements for harmonic correlation in 2-part counterpoint are:

a. A variety of types of interval (i.e., intervals as expressed by different numbers).
b. A variety of density.
c. Well-defined cadences, expressed through the use of leading tones moving into their axes.
d. Crossing of C.P. and C.F. is permissible *when necessary*.

The negative requirements are:

a. Elimination of consecutive intervals which are perfect unisons, octaves, 4ths and 5ths. Dissonances may not be used consecutively; the only intervals to be used in parallel motion are thirds and sixths.
b. There may be motion toward such intervals as unison, octave, 4th, or 5th—only through contrary (outward or inward) directions.
c. There may not be repetition of the same pitch-unit in CP unless it is in a different octave.

The forms of harmonic relations previously used in time continuity (see my earlier discussion of the theory of pitch scales)* will be used in counterpoint as the forms of simultaneous harmonic correlation.

C. Forms of Harmonic Correlation

1. U. — U. Unitonal — Unimodal: (identical scale structure and key signature).
2. U. — P. Unitonal — Polymodal: (a family scale with identical key signature).
3. P. — U. Polytonal — Unimodal: (identical scale structure, different key signature).
4. P. — P. Polytonal — Polymodal: (different scale structure, different key signature).

In the 14th century, in the music of Guillaume de Machault,** we find a fully developed type 2, and, in some cases, an undeveloped type 3. Only the ignorance and vanity of some contemporary composers make them believe that

*See Book II.

**Phonograph records of a Mass written by this composer for the coronation of Charles V are available. (*Les Paraphonistes de St. Jean des Matines* and Brass Ensemble, conducted by Van). A reconstruction of Machault's 2- and 3- part chansons in modern musical notation was published by the Deutsche Musikgesellschaft in 1926, in the edition of Friedrich Ludwig.

they are the discoverers of polytonal counterpoint; the joke being especially good on those modern French composers who make claim to priority, being unaware that it is their own direct musical ancestors who were the originators of this style centuries ago.

It is also unfortunate that the idea of polytonality is commonly associated with so-called "dissonant counterpoint", i.e., the counterpoint of continuous tension without release. Music based on polytonality *with* resolutions is a very fruitful, highly promising, and almost undiscovered field.

The usual length of a C. F. is about 5, 7, 9, or more bars, preferably in odd numbers—this requirement being traditional. The selection of different key signatures for types 3 and 4 is entirely a matter of choice. Any two scales—the root tones of which produce a consonance—may be used for this type of counterpoint. The best way to construct these exercises is to place the C.F. on a central staff, with two staves below and two staves above, assigning a different type of counterpoint to each staff.

In the following group of exercises, each part must be played *individually* with C.F. Each example produces four types of counterpoint with a historical perspective of eight centuries, for the first and second types were considerably developed during the Middle Ages, and the third and the fourth types are mostly used—when at all—in the music of today.

It is important to realize that all forms of traditional contrapuntal writing were based on the conception of *each melody being in a different mode.* One can even trace polytonal forms (although in their embryonic form) as far back as the 13th century.

$\frac{CP}{CF} = a$

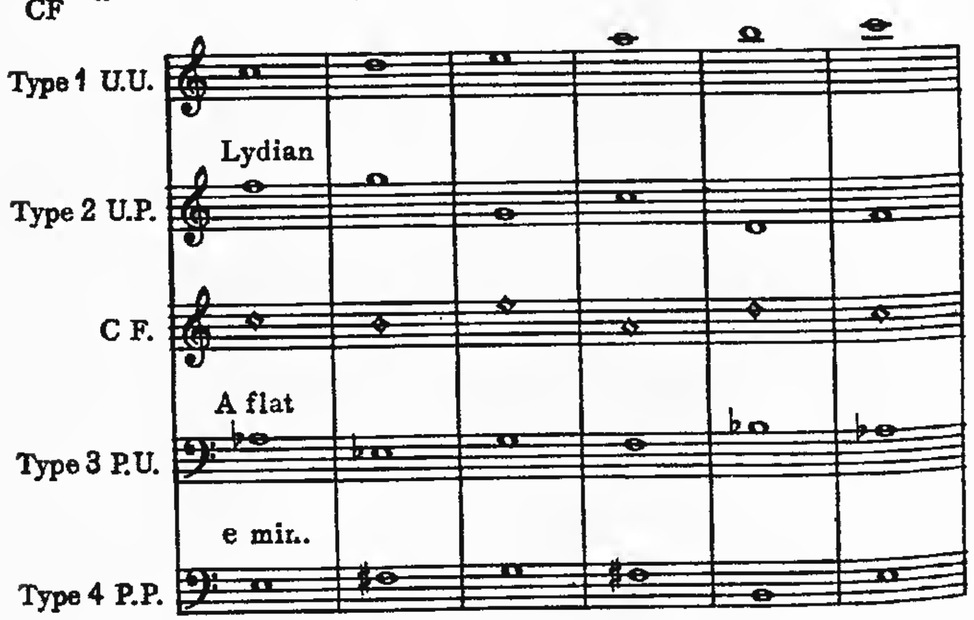


Figure 12. Two-part counterpoint.

As a temporary device for harmonic accompaniment, a double pedal point may be used in addition to the 2-part counterpoint. The root tones of both contrapuntal parts become the axes which must be assigned as chordal functions of a double pedal point. For example, in counterpoint of type 1 (giving the same pitch-units for both voices) the single root tone may be assigned as the root, or 3rd or 5th, etc., of a simple chord structure. Then, inasmuch as c is the axis for both contrapuntal parts in the example, the pedal point will become $\frac{g}{c}$ or $\frac{e}{a}$, $\frac{c}{f}$, etc.

This device is applicable to all four types of counterpoint. For example, in type 2, if one contrapuntal part were in Ionian c and the other in Aeolian a, the two might represent a root and a 3rd, or a 3rd and a 5th, etc., respectively. The pedal point in such a case would be $\frac{e}{a}$ or $\frac{c}{f}$, etc. In types 3 and 4, with any two such axes as c and a♭, we may use $\frac{e\flat}{a\flat}$ or $\frac{c}{f}$, etc., as pedal points. Each double pedal point must last through the entire contrapuntal continuity.

More flexible forms of harmonization of the 2-part counterpoint will be offered later.

D. $\boxed{\frac{CP}{CF} = 2a}$

In devising two attacks of the counterpoint against one attack of the C.F., the following combinations of harmonic intervals are possible:

(c = consonance; d = dissonance)

c — c

c — d

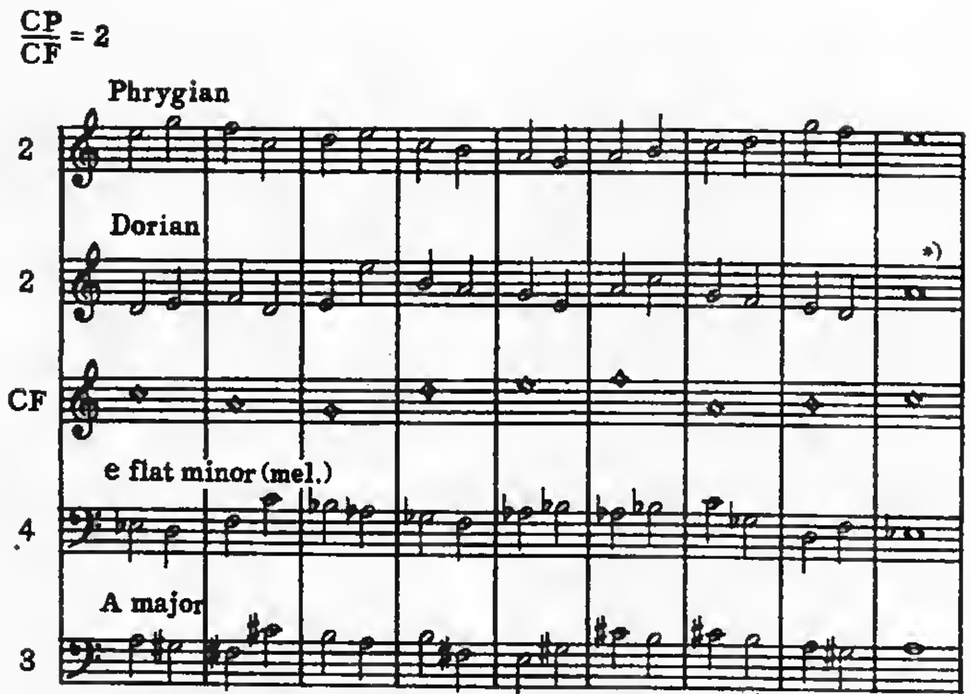
d — c

d — d*

In old counterpoint, all these cases were used in both *strict* and *free* style, with the exception that a dissonance was not supposed to occur on the first of the two attacks.

Each bar may start with either a consonance or a dissonance, and, in the case of $\frac{CP}{CF} = 2$, all dissonances require immediate resolution. The following pages contain a few examples of such contrapuntal exercises.

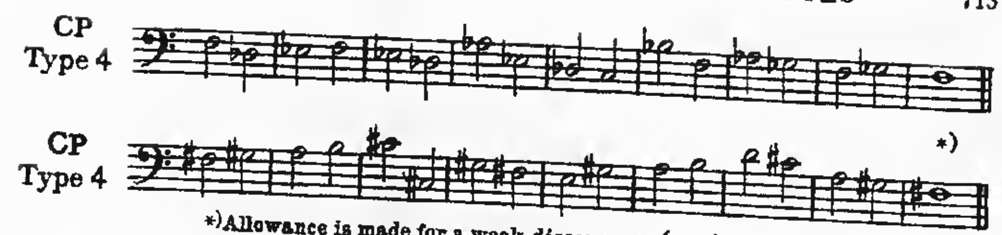
*In scalewise contrary motion only. (J.S.)



*) Aeolian axis caused by necessity of having a consonance for the ending. (J.S.)

Figure 13. Two attacks of C.P. to one of C.F.

Figure 14. Two attacks of C.P. to one of C.F. (continued).



*) Allowance is made for a weak dissonance. (J.S.)

Figure 14. Two attacks of C.P. to one of C.F. (concluded).

Figure 15. Two attacks of C.P. to one of C.F. (continued).

Figure 15. Two attacks of C.P. to one of C.F. (concluded).

E. $\boxed{\frac{CP}{CF} = 3a}$

Three attacks of CP against one attack of CF offer us the following combinations of harmonic intervals:

c — c — c
c — d — c
d — c — c
c — c — d resolution
d — c — d resolution
d — d*— c
c — d — d*

The d — c — c combination offers a new device which only becomes possible with three or more attacks; we shall call it a *delayed* (or indirect) *resolution*. Instead of resolving a tense interval at once, we move it to another consonance, after which we resolve the dissonance.

*In scalewise contrary motion only. (J.S.)

This procedure accomplishes two things:

(1) it produces psychological suspense, thus making the music more interesting;

(2) it produces *ipso facto* a more expressive melodic form.

Figure 16. Examples of delayed resolutions.

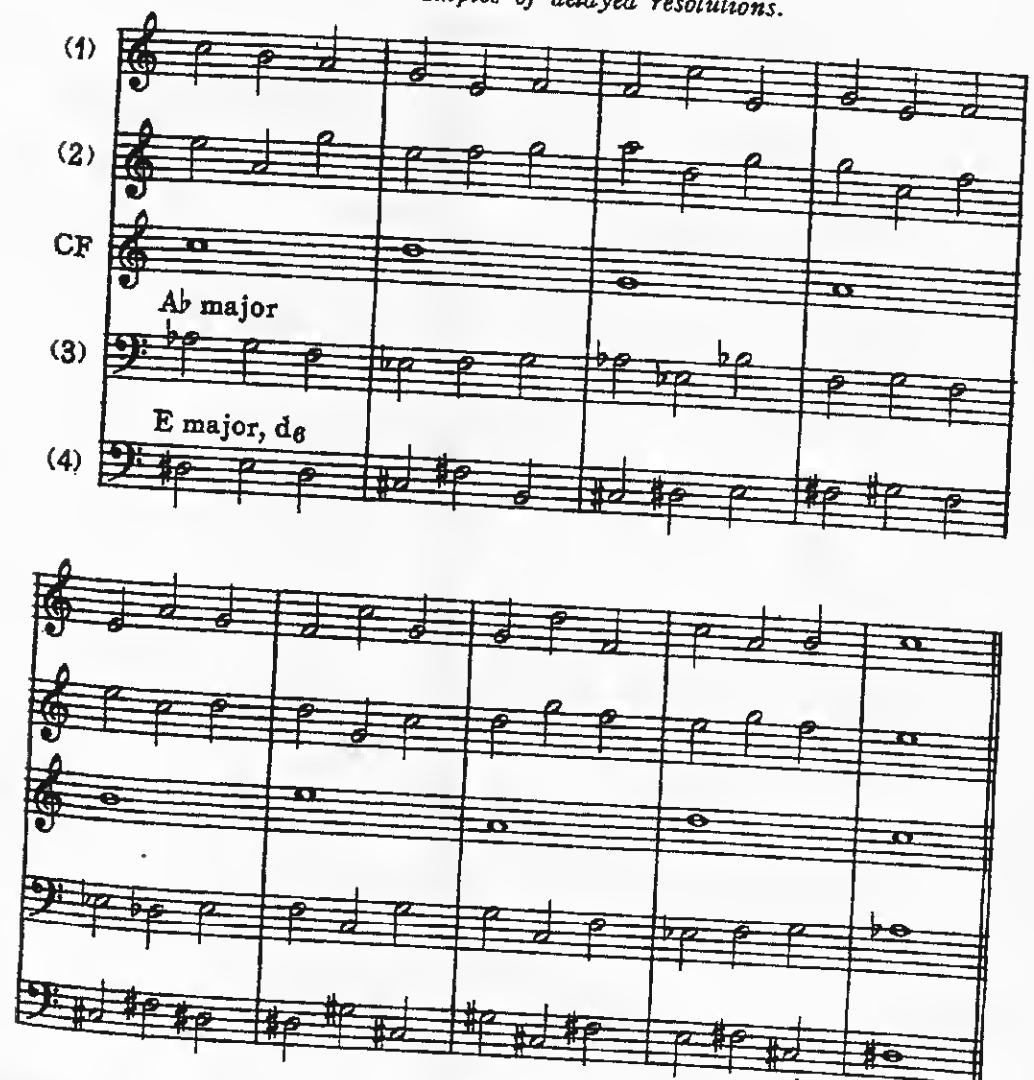


Figure 17. $\frac{CP}{CF} = 3a$.

F. $\boxed{\frac{CP}{CF} = 4a}$

Four attacks of CP against one attack of CF offer still more combinations of harmonic intervals:

c — c — c — c
c — c — c — d resolution
c — c — d — c
c — d — c — c
d — c — c — c
c — c — d — d*
c — d — d* — c
d — d ** c — c
d — c — c — d resolution
c — d — c — d resolution
d — c — d — c

There are wider possibilities in the field of delayed resolution for $\frac{CP}{CF} = 4$.

Parallel axes, centrifugal and centripetal forms now become more prominent among the devices by which the composer may construct the second melody.

Figure 18. Examples of delayed resolutions.

It is also useful to know all the advantageous starting points for those scalewise passages which end with a consonance:

Figure 19. Examples of passages ending with a consonance.

*In scalewise contrary motion only. (J.S.)

**Either the same as in *, or two independent dissonances, both of which are resolved by the following c — c in any order. (J.S.)

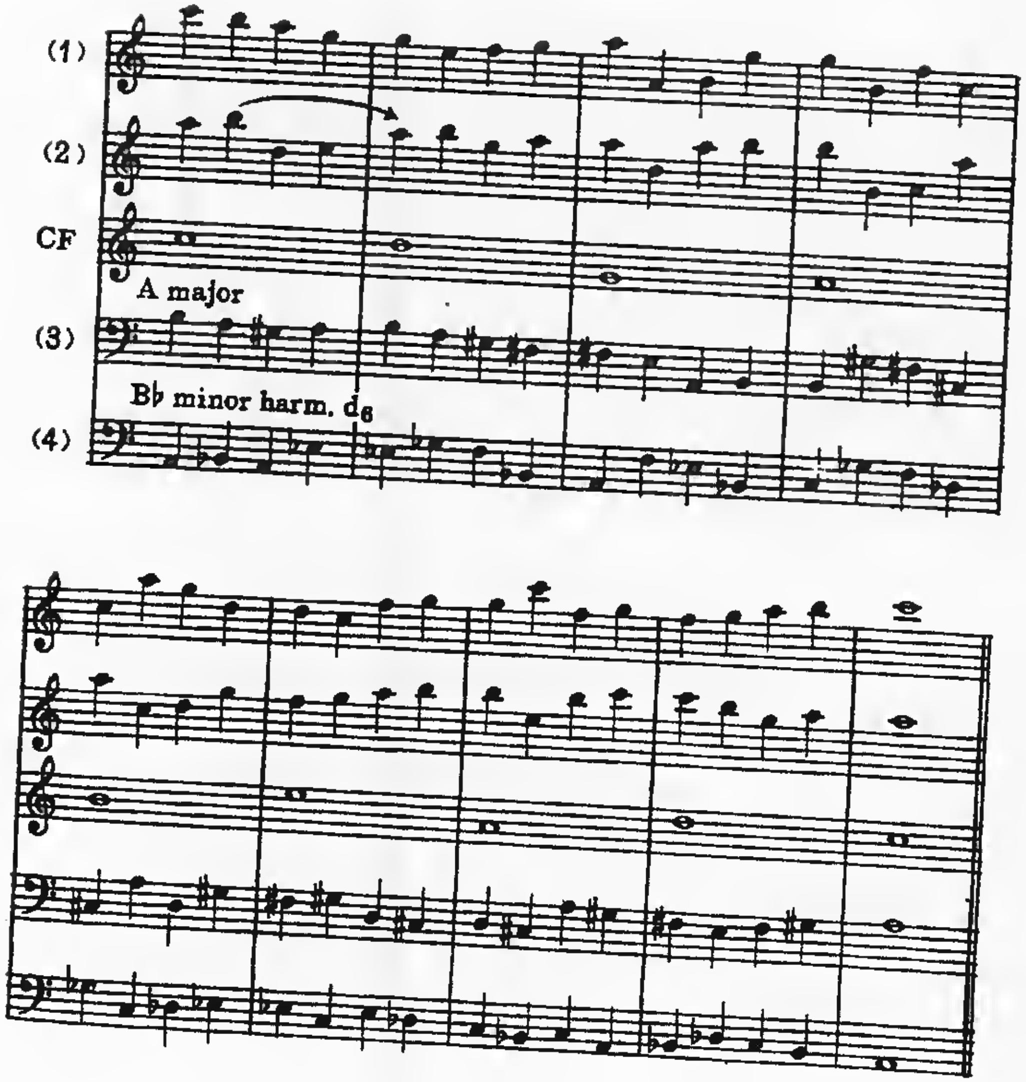


Figure 20. $\frac{CP}{CF} = 4a$.

G. $\boxed{\frac{CP}{CF} = 5a}$

It is no longer necessary to tabulate all the possible combinations of c and d.

The best melodic quality in the CP will result from extensive use of delayed resolutions. Combined with a variety of interval and with scalewise passage, delayed resolutions make available the most versatile forms of melody.

The devices for delayed resolution, impossible for fewer attacks than five, are as follows:

$d_1\ d_2\, c\ \ d_1\, c$—the first dissonance is followed by a second dissonance with its resolution, then by the repetition of the first dissonance with its resolution.

$d_1\ d_2\, c\ \ d_2\, c$—the first dissonance is followed by the second dissonance without resolution, followed by the resolution of the first dissonance, then by the second dissonance and its resolution.

Figure 21. Examples of delayed resolutions.

Figure 22. Scalewise passages ending with a consonance.

Figure 23. $\frac{CP}{CF}$ = *5a (continued).*

Figure 23. $\frac{CP}{CF}$ = *5a (concluded).*

H. $\boxed{\frac{CP}{CF} = 6a}$

Still other devices for delayed resolutions become possible with six attacks:

$d_1\ d_2\ d_1\, c\ \ d_2\, c$—the first dissonance, the second dissonance, repetition of the first dissonance with its resolution, repetition of the second dissonance with its resolution;

$d_1\ d_2\, c\ \ d_2\, c\, c$—the first dissonance, the second dissonance, resolution of the first dissonance, repetition of the second dissonance, the delay, and the resolution of the second dissonance;

$d_1\ d_2\, c\ \ d_1\, c\, c$—the first dissonance, the second dissonance with its resolution, repetition of the first dissonance, delay, and resolution of the first dissonance;

$d_1\, c\, c\ \ d_2\, c\, c$—a combination of two groups of three, each consisting of dissonance, delay, and resolution.

Other combinations may be devised in a similar way, for example, $d_1\, c\ \ d_2\, c\, d_2\, c$—which is the combination, 2 + 4.

In using six attacks against CF, it is easy to devise a great variety of melodic forms and interference patterns, as discussed in the section on melodization of harmony.*

*See Book VI, pages 619-625.

Figure 24. Examples of delayed resolutions

Figure 25. Scalewise passages ending with a consonance.

Figure 26. $\frac{CP}{CF}$ = *6a (continued).*

Figure 26. $\frac{CP}{CF}$ = *6a (concluded).*

I. $\boxed{\frac{CP}{CF} = 7a}$

Seven attacks of CP against one of CF offer new forms of delayed resolutions. The number of new combinations grows, and it becomes quite easy to develop various melodic forms built on parallel, converging, and diverging axes.

Figure 27. Examples of delayed resolutions.

Figure 28. Scalewise passages ending with a consonance.

Figure 29. $\frac{CP}{CF}$ *= 7a (continued).*

Figure 29. $\frac{CP}{CF}$ *= 7a (concluded).*

J. $\boxed{\frac{CP}{CF} = 8a}$

Eight attacks of a CP against one of CF offer a still greater variety of melodic forms. The latter may be obtained through the technique of delayed resolutions. It is equally fruitful to devise melodic forms by means of attack-groups, for example, by thinking of 8 as $\frac{8}{8}$ series represented through its binomials and trinomials. Interference groups may be carried out in counterpoint in the same way as in the melodization of harmony*, in which technique such groups were used against the attacks of H.

Figure 30. Examples of delayed resolutions.

*See Book VI.

All 8-against-1 scalewise passages ending with a consonance must start and end with the same pitch unit, as this is a property of our seven-name musical system.

Figure 31. Scalewise passages ending with a consonance.

Figure 32. $\frac{CP}{CF} = 8a$

The $\frac{CP}{CF} = 8a$ gives the composer sufficient technical equipment for an unlimited number of attacks. It would be desirable for the student now to devise such cases as $\frac{CP}{CF} = 12$ a, and $\frac{CP}{CF} = 16$ a, as they provide very useful material for animated forms of passage-like obligatos. Under the usual or traditional treatment, such groups with many attacks of CP against CF remain uniform or nearly uniform in durations.

The most important conditions for obtaining an expressive counterpoint are:

(1) an abundance of dissonances;
(2) delayed resolutions; and
(3) interference attack-groups.

CHAPTER 3

ATTACK-GROUPS IN TWO-PART COUNTERPOINT

IN all the forms of counterpoint discussed so far, the attack-group of CP against each attack of CF was constant: $\frac{CP}{CF} = A$ const. The monomial attack group consisted of any desirable number of attacks: $A = a, 2a, 3a, \ldots ma$.

Now, however, we arrive at ***binomial*** attack-groups for CP. This situation may be expressed as $\frac{CP}{CF} = A_1 + A_2$, i.e., the counterpoint to be written to two successive attacks of the cantus firmus is to consist of two different attack-groups.

For instance:

(1) $\frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{2a}{a} + \frac{a}{a}$; (2) $\frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{3a}{a} + \frac{2a}{a}$;

(3) $\frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{5a}{a} + \frac{3a}{a}$; (4) $\frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{a}{a} + \frac{8a}{a}$; . . .

The selection of number values for the attacks of CP against the attacks of CF depends on the amount of contrast desired in the two successive attack-groups of CP.

All further details pertaining to this problem are to be found in my earlier discussion of the theory of melodization.*

Binomial attack-groups are subject to permutations. For example, if $\frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{4a}{a} + \frac{2a}{a}$, this binomial attack-group may be varied further through permutations of a higher order. Suppose CF has 8a; then the whole contrapuntal continuity will acquire the following distribution of attack-groups:

$$\frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} + \frac{CP_3}{CF_3} + \frac{CP_4}{CF_4} + \frac{CP_5}{CF_5} + \frac{CP_6}{CF_6} + \frac{CP_7}{CF_7} + \frac{CP_8}{CF_8}, \text{ or}$$

$$\frac{CP_{1-8}}{CF_{1-8}} = \frac{4a}{a} + \frac{2a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{4a}{a} + \frac{2a}{a}.$$

Polynomial attack-groups of CP against CF may be devised in a similar fashion.

The resultants of interference, their variations, involution groups, and series of variable velocities may all be used as material for this purpose.

Examples of polynomial attack-groups of $\frac{CP}{CF}$:

(1) $\frac{CP_{1-6}}{CF_{1-6}} = \frac{3a}{a} + \frac{a}{a} + \frac{2a}{a} + \frac{2a}{a} + \frac{a}{a} + \frac{3a}{a}$;

(2) $\frac{CP_{1-9}}{CF_{1-9}} = \frac{2a}{a} + \frac{a}{a} + \frac{a}{a} + \frac{a}{a} + \frac{2a}{a} + \frac{a}{a} + \frac{a}{a} + \frac{a}{a} + \frac{2a}{a}$;

(3) $\frac{CP_{1-5}}{CF_{1-5}} = \frac{a}{a} + \frac{2a}{a} + \frac{3a}{a} + \frac{5a}{a} + \frac{8a}{a}$;

(4) $\frac{CP_{1-4}}{CF_{1-4}} = \frac{9a}{a} + \frac{6a}{a} + \frac{6a}{a} + \frac{4a}{a}$.

*See Book IV.

The simplest duration-equivalents of attacks will be used in the following examples.

$$\frac{CP}{CF} = \frac{a}{a} + \frac{8a}{a} + \frac{8a}{a}$$

Figure 33. $\frac{CP}{CF} = A$ var.

$$\frac{CP}{CF} = \frac{a}{a} + \frac{5a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{8a}{a} + \frac{8a}{a} + \frac{4a}{a} + \frac{2a}{a} + \frac{5a}{a} + \frac{a}{a}$$

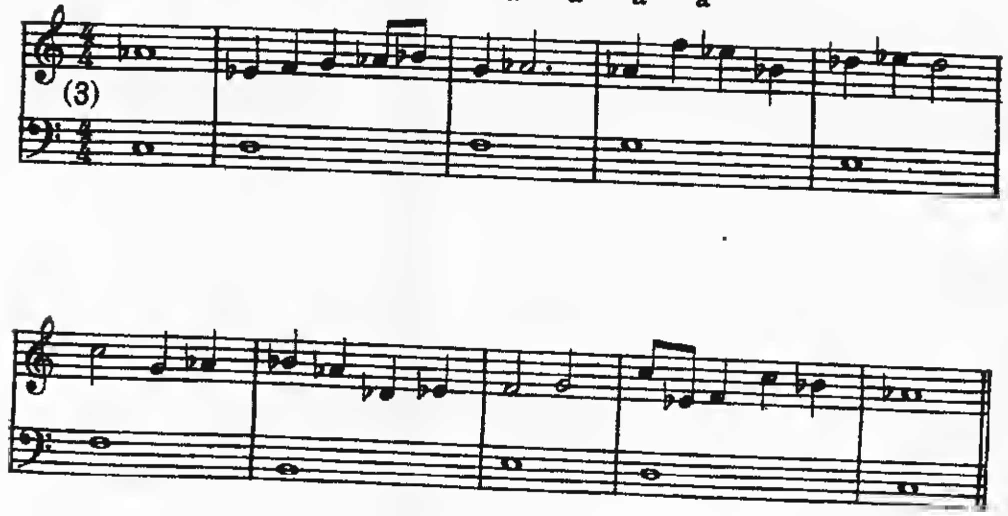


Figure 34. $\frac{CP}{CF} = A$ var.

$$\frac{CP}{CF} = (1+2)^3 = \frac{a}{a} + \frac{2a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{4a}{a} + \frac{8a}{a}$$

Figure 35. $\frac{CP}{CF}$ = A var.

A. More than One Attack of CF to CP

At this stage it should not be difficult for the student to develop the technique of writing *one* attack of CP to a group of attacks of CF. In an exercise, CF must be so constructed as to permit the matching of one attack against a given attack-group. In composing a counterpart to a given melody, it is necessary to compose the attack-groups first. This should be done with a view to the possibilities of resolving the harmonic intervals. Whenever the assumed group does not permit one to carry out the resolution requirements (such as expanding of the second, contracting of the seventh or the ninth, etc.), then the attack-group itself must be reconstructed.

As was mentioned previously, it is entirely practical to re-write the given melody into uniform durations first, then to assign advantageous attack-groups. After the counterpoint has been written, the original scheme of durations may then be reconstructed.

With the equipment which I have so far presented, only such melodies may be used as the cantus firmus which is built on one scale at a time; the scale itself must belong to the first group (see my discussion of the theory of pitch scales).*

The procedure of distributing the attack-groups of a given melody is analogous to that used in the technique of the harmonization of melody,** according to which the attacks of a given melody were distributed in relation to the number of chords accompanying the melodic attacks.

*See Book II. **See Book VI, Chapter 3.

The following example is a melody which has been subjected to different attack treatments in the process of writing a counterpart to it.

$$\frac{CP}{CF} = \frac{a}{8a} + \frac{a}{a} + \frac{a}{2a} + \frac{a}{2a} + \frac{a}{a}$$

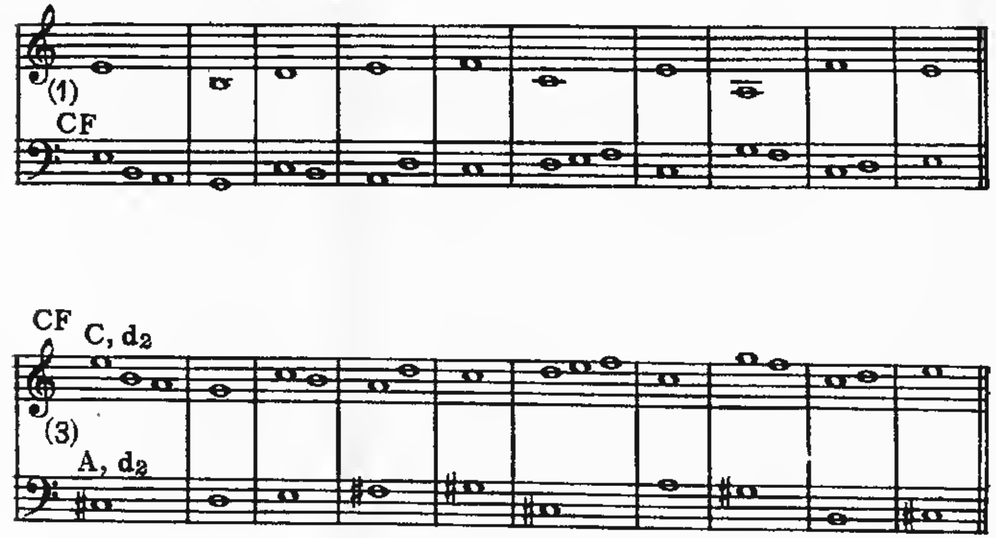


Figure 36. Single attack of CP to many of CF.

$$\frac{CP}{CF} = \frac{a}{6a} + \frac{a}{8a} + \frac{a}{2a} + \frac{a}{4a} + \frac{a}{8a}$$

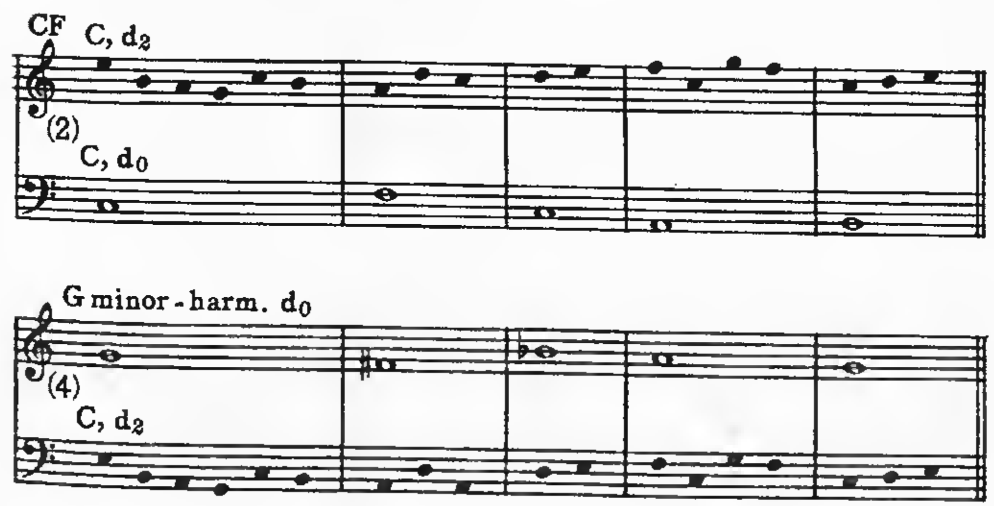


Figure 37. Single attack of CP to many of CF.

In writing a counterpart to a given melody (but *without* consideration of any harmonic accompaniment that may also be given) it is important to consider:

(1) the composition of attacks, and
(2) the composition of durations.

The choice of means for the composition of attacks depends on the degree of animation of the given melody. If a lively melody is to be compensated, then the countermelody should be devised on the basis of reciprocation of attacks and, finally, of durations. All the techniques pertaining to the variation of two elements serve as material for such a two-part compensation (counterbalancing).

If a lively melody is to be contrasted, then the countermelody should be devised by *summing up* groups of attacks together with their durations. The sums of durations of the given melody, with the specified number of attacks against each attack of the countermelody, define the durations of the counterpart.

If a slow melody is to be compensated (counterbalanced) by a slow counterpart, then the technique of reciprocation of attacks and durations should be used. Variations of two elements provide such a technique.

If a slow melody is to be contrasted, then the countermelody should be devised first by defining the number of attacks in the countermelody against each individual attack of the given melody, after which the sum of the attacks of the counterpart will represent the duration, equivalent to the duration of one attack of the given melody.

When one handles melodies which have animated portions alternating with slow ones, or with cadences, it will be found that these are particularly suited for the compensation method. In such a case, when one melody stops, the other moves—and *vice versa.*

Let us analyze the problem, say, of writing a counterpart to a given melody, taking the setting to Ben Jonson's *Drink to Me Only With Thine Eyes.*

The melody is:

Reconstruction of this melody into a CF gives it the following appearance:

Figure 38. C.F. of Drink to Me Only with Thine Eyes.

This is a fairly animated type of melody.

Let us first devise a scheme of durations for CP. One of the simplest solutions for a contrasting CP would be to make each attack of CP correspond to T; we would obtain CP = 4a and a = 6t. For a less moderate contrast, we could assign CP = 8a and a = 3t. To obtain a CP of the counterbalancing type, we would have to assign two contrasting elements, if such can be found in CF. As $T_1 = 2a$ and $T_2 = 6a$, and as $T_3 = 5a$ and $T_4 = a$, this CF provides sufficient material for assigning two elements and for compensating them in CP. There is, of course, no way to counterbalance the original version of this melody. In this way we obtain the following three solutions, each different, but all equally acceptable.

Figure 39. Varying counterpoints to melody of Drink to Me Only with Thine Eyes *(continued).*

Figure 39. Varying counterpoints (continued).

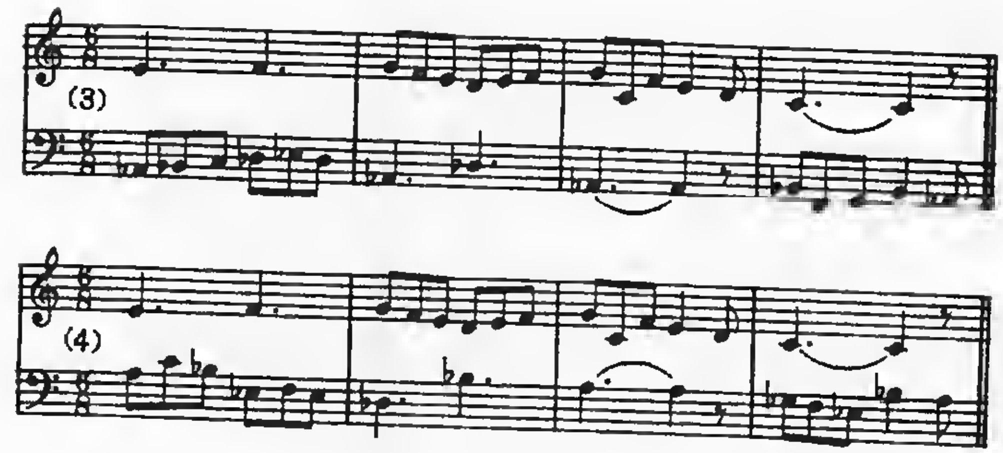


Figure 39. Varying counterpoints (concluded).

B. Direct Composition of Durations in Two-Part Counterpoint

In composing an original two-part counterpoint, it is often desirable first to compose the two counterparts rhythmically. The entire technique of handling binomials and their variations (as set forth in my theory of rhythm)* is applicable in this case.

Counterbalancing (compensation) is achieved through permutation of binomials, and this may follow through the higher orders. For example:

Figure 40. Counterbalancing through permutation of binomials.

It does not matter which part is written first (thus becoming the CF) in such a case. It is essential, however, to write one part *completely*, not section by section. The CP must be written *after* the CF has been completed.

For a more *diversified* rhythmic continuity, resultants with an even number of terms may be used; the binomials constantly reciprocate in such a case. For example, $T = r_{8 \div 7}\ (+\ 8t)$:

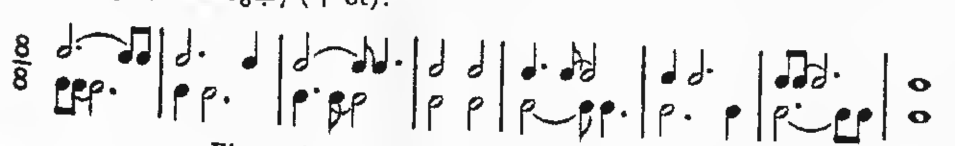


Figure 41. Employing reciprocating binomials.

*Book I.

In all such cases (continuous reciprocation of the variable binomials), the number of attacks of CP against CF remains constant while the durations vary.

Homogeneous effects of rhythm in both counterparts may be achieved through varying the rests or split-unit groups. The groups themselves do not have to be binomials. The two "best" of any polynomial groups are the self-reciprocating members.*

For example: (a) rests

or: (b) tied rests

(c) split-unit groups

Figure 42. Self-reciprocating members.

Any rhythmic group set against its converse provides a satisfactory counterpart. For example: $2\left(r_{\frac{5 \div 4}{2}}\right)$; $T = 4t$.

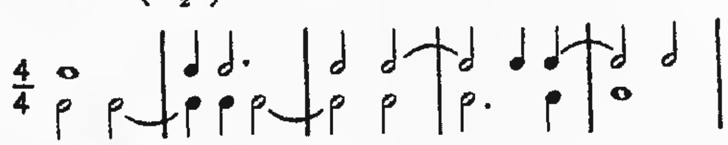


Figure 43. Converse of a rhythmic group provides satisfactory counterpoint.

Any of the series of variable velocities may be used for such a purpose. For example:

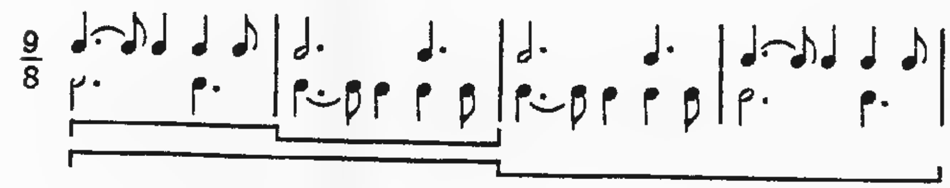
Figure 44. Summation series I.

*In variations or circular permutation of three or more elements, it is selective and desirable to choose only pairs of the resultant groups. The self-reciprocating groups, of which there are rarely more than two, are called the best. They will be found in circular permutations of rests, accents and split-unit groups (also non-uniform durations). See Book I. (Ed.)

Adjacent contrasts for two mutually compensating parts may be achieved by synchronized involution-groups placed in a sequence. The two powers supply the *a* and *b* elements, and thus are treated through the permutations of two elements (any order).

For example: $(2+1)^2 + 3(2+1)$.

$a = (2+1)^2$; $b = 3(2+1)$



Or, for example: $4(2+1+1) + (2+1+1)^2$.

$a = 4(2+1+1)$; $b = (2+1+1)^2$.

Figure 45. Permutation of two elements.

All the above devices permit one to start work with the composition of either part as the CF, and they all refer to counterbalancing (compensation).

The technique of simultaneous harmonic contrasts between CF and CP is based on the distributive involution of the two synchronized parts used simultaneously. Any number of terms may be used as a group. The limitation of *two parts* corresponds to the *two power-groups* (adjacent or non-adjacent powers). In all such cases, the number of attacks of CP against CF is constant, and such a number equals the number of terms in the polynomial. Thus, a binomial squared gives $\frac{CP}{CF} = 2a$; a trinomial squared gives $\frac{CP}{CF} = 3a$, etc.

Still greater contrasts may be achieved either by using larger polynomials, or by synchronizing non-adjacent powers. In the latter case a binomial cubed and used against its synchronized first power gives $\frac{CP}{CF} = 4a$, i.e., 2^2; a trinomial cubed and used against its synchronized first power gives $\frac{CP}{CF} = 9a$, i.e., 3^2, etc.

Nothing prevents the composer from using adjacent higher powers—such as cubes against squares, fourth power groups against cubes, etc.

In all these cases the lower power employed represents the CF, as it is easier to match several attacks against a given single attack, than *vice versa*.

Examples:

(a) CF = 3(2+1); CP = $(2+1)^2$.

(b) CF = 9(2+1); CP = $(2+1)^2$.

(c) CF = 8(2+1+2+1+2); CP = $(2+1+2+1+2)^2$.

(d) CF = 16(2+1+1); CP = $(2+1+1)^2$.

Figure 46. Using larger polynomials for contrast.

In addition to involution-groups, *coefficients of duration* may be used, as in $\frac{CP}{CF} = \frac{2(r_{4\div 3})}{r_{8\div 6}} = \frac{(3+1+2+2+1+3) + (3+1+2+2+1+3)}{6+2+4+4+2+6}$, as well as the resultants of instrumental interference composed for two parts.

In all the following examples, the intonation of CF was composed first.

Figure 47. Two-part counterpoint with pre-composed duration group (continued

Figure 47. Two-part counterpoint with pre-composed duration group (continued).

Coefficients of duration = R 4÷3

Figure 47. Two-part counterpoint with pre-composed duration group (continued).

The resultants of instrumental interference: R 3÷2

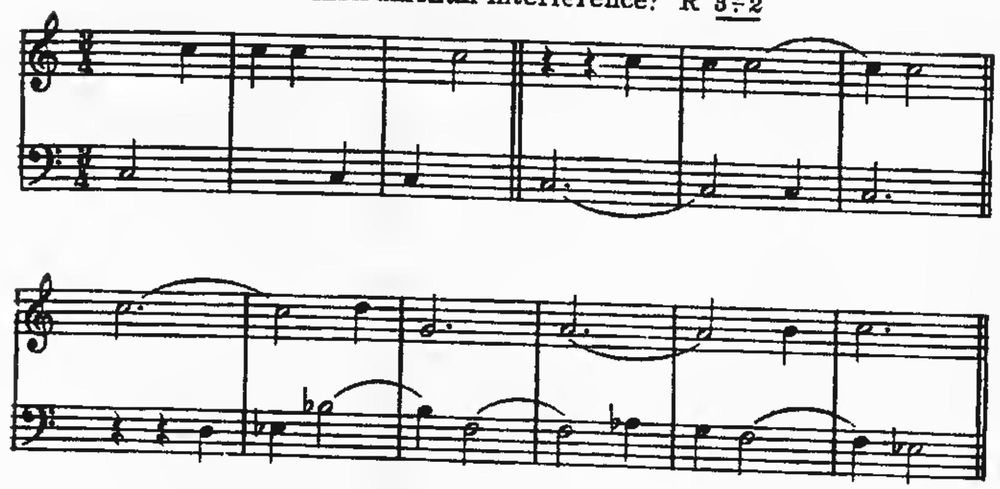


Figure 47. Two-part counterpoint with pre-composed duration group (concluded).

C. Chromatization of Diatonic Counterpoint

It would seem to be easy to write a chromatic counterpart to any diatonic melody, for any suitable pitch-unit may be chosen from anywhere in the entire chromatic scale. But such countermelodies have one general defect, a neutral character which comes with a uniform scale. To the average listener such counterpoint sounds as if any pitch-unit would be just as acceptable as those already set.

This peculiarity of musical perception is due to our inherited and cultivated *diatonic orientation*. The average listener hears chromatic units as an ornamental supplement to a diatonic scale. Chromatic units are commonly used as auxiliary tones moving into the diatonic units of a given scale, forming *directional* units. Now, diatonic units are perceived as *independent* pitches (although in a certain grouping in sequence), but chromatic units are perceived as *dependent* pitches *leading into* diatonic pitches.

Music constructed entirely chromatically, i.e., without diatonic dependence, therefore usually belongs to a category different from diatonic music with directional units; it is known under the name of "atonal", or "twelve-tone" music.

For this reason,* we shall use chromatic counterpoint with diatonic dependence only. This kind of counterpoint may be devised at its best by means of inserting passing or auxiliary chromatic units *after* the diatonic counterpoint has been written.

This technique is applicable to all four types of harmonic relations. It is important to note that the conversion of diatonic into chromatic counterpoint does not affect the established forms of resolutions; remodeling of durations may be accomplished by means of split-unit groups, a device allowing us to preserve the character of the rhythm which was originally set.

*Unless, of course, the composer *wants* to write "atonal" music. (Ed.)

Original

Chromatic variation

Original

Chromatic variation

Original

Figure 48. Chromatic variation of diatonic counterpoint (continued).

Chromatic variation (both parts are chromatized)

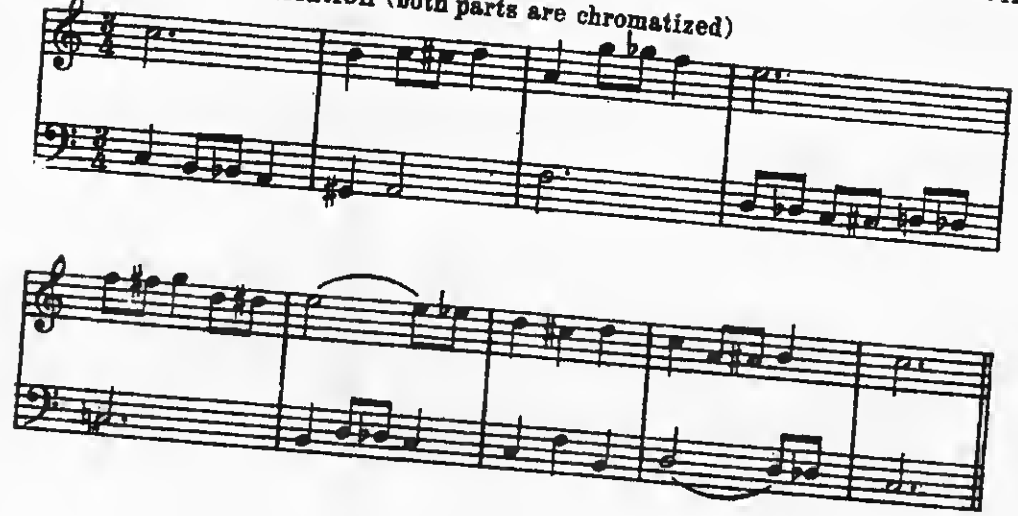


Figure 48. Chromatic variation of diatonic counterpoint (concluded).

CHAPTER 4

THE COMPOSITION OF CONTRAPUNTAL CONTINUITY

THE extension of any given contrapuntal continuity is based on geometrical mutations.

The fundamental technique of these geometrical mutations, in two-part counterpoint, is the interchange of music assigned to CF and CP. Assuming that CF represents the actual melody, and CP represents the actual counterpart, we obtain two variants for each voice: $\frac{CP}{CF}$ and $\frac{CF}{CP}$, where both CF's and both CP's are identical but change their vertical positions.

In the old systems of counterpoint, this device was known as "vertical convertibility in octave." We shall regard it merely as a device formed by two variants of the exposition for any counterpoint; we shall consider such convertibility to be an inherent property of counterpoint as such.

By applying the principle of variation of two elements *ad infinitum*, i.e., through permutations of the higher orders, we can compose an entire piece of music from a single contrapuntal exposition.

Figure 49. Contrapuntal continuity of the third order produced through permutation of parts of the original exposition (continued).

Figure 49. Contrapuntal continuity of the third order produced through permutation of parts of the original exposition (continued).

Figure 49. Contrapuntal continuity of the third order produced through permutation of parts of the original exposition (concluded).

When it is conceived geometrically, any musical exposition becomes subject to ***quadrant rotation*** (as described earlier in my discussion of geometrical projections of music), yielding the four variations of the geometrical position: ⓐ, ⓑ, ⓒ, ⓓ.*

Through vertical permutation of parts, two-part exposition yields two variants. Each variant has four rotational positions; the total number of variants for one two-part contrapuntal exposition is therefore eight:

$$\frac{CF}{CP}ⓐ,\ \frac{CP}{CF}ⓐ,\ \frac{CF}{CP}ⓑ,\ \frac{CP}{CF}ⓑ,\ \frac{CF}{CP}ⓒ,\ \frac{CP}{CF}ⓒ,\ \frac{CF}{CP}ⓓ,\ \frac{CP}{CF}ⓓ.$$

In making a transition from one form to another in the same part, place the respective pitch-unit in its nearest pitch position. This is true of both the octave and the geometrical inversion. ***The axis of inversion*** for ⓒ and ⓓ is the ***axis of CF***, or the part assumed to function as the CF.

*To remind the reader, these geometrical positions are: ⓐ the original; ⓑ the same but backwards; ⓒ the original backwards and upside down; ⓓ the original upside down. See Book III. (Ed.)

Type I

Figure 50. Variants of one exposition. Type I and quadrant rotation (continued).

Figure 50. Variants of one exposition. Type I and quadrant rotation (concluded).

Type II

Figure 51. Type II and quadrant rotation (continued).

Figure 51. Type II and quadrant rotation (concluded).

Type III and/or IV

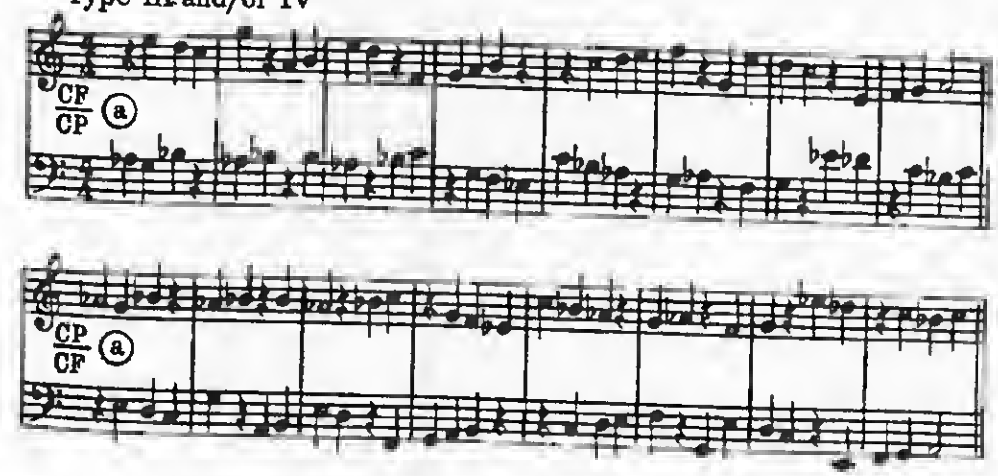


Figure 52. Type III and/or IV. Quadrant rotation (continued).

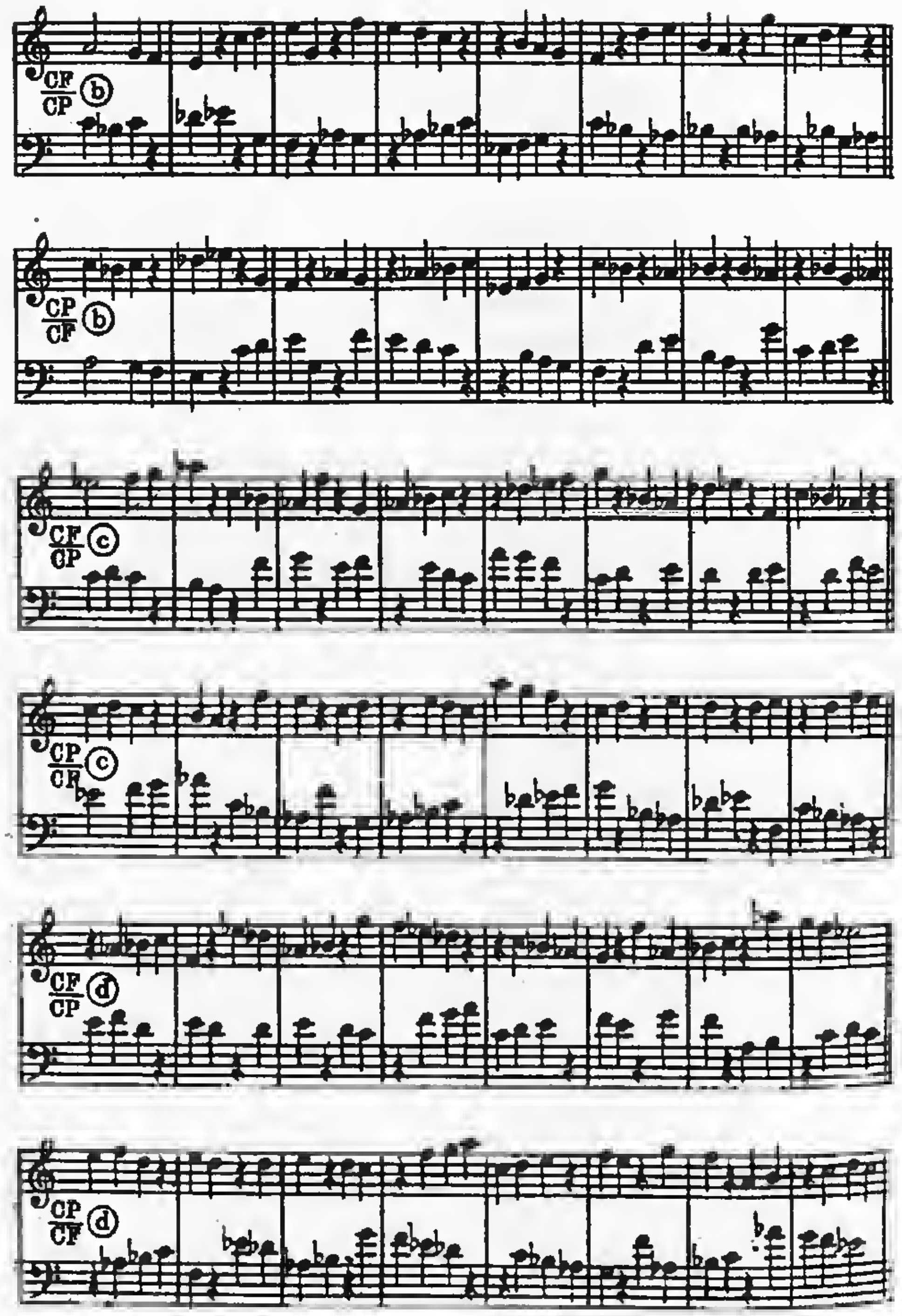


Figure 52. Type III and/or IV. Quadrant rotation (concluded).

These eight variants of contrapuntal exposition may be selected in any desirable combination. Any combination of the selected variants produces a complete form of continuity, i.e., a whole composition.

The selection of various geometrical inversions must be guided by a definite tendency with regard to the number and distribution of contrasts; all considerations pertaining to this matter were discussed in the section on geometrical projections of music. *

The most important principles to remember are:

(1) a and b are identical in intonation and converse in temporal structure;
(2) c and d are identical in intonation and converse in temporal structure;
(3) a and d are converse in intonation and identical in temporal structure;
(4) a and c are converse in intonation and converse in temporal structure;
(5) b and c are converse in intonation and identical in temporal structure;
(6) b and d are converse in intonation and converse in temporal structure.

There is a way of developing identical temporal structures for all geometrical inversions: any symmetrical group is identical with its converse; for instance:

(1) $r_{5 \div 4} = 4 + 1 + 3 + 2 + 2 + 3 + 1 + 4$

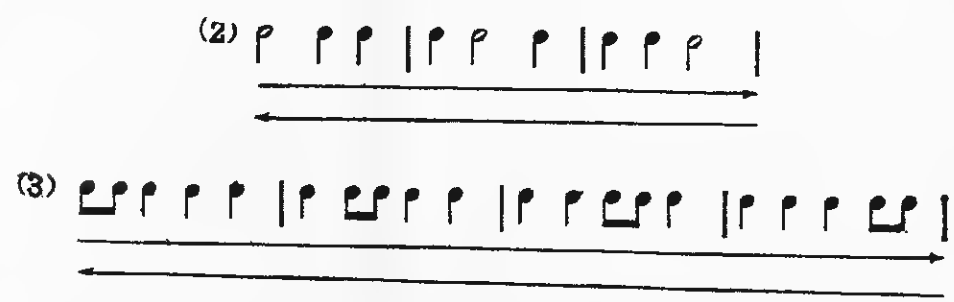


Figure 53. A symmetrical group is identical with its converse.

There is also a way of developing an identical pitch-scale for all geometrical inversions, when such is desirable. The original scale must be symmetrically constructed (which does not necessarily place it into the third or fourth group). In such a case the pitch units in a and d are not identical but the scale structures (that is, the sets of intervals) are identical.

For instance:

a	c — eb — f — g — bb	(3 + 2 + 2 + 3) ↑
b	bb — g — f — eb — c	(3 + 2 + 2 + 3) ↓
c	d — f — g — a — c	(3 + 2 + 2 + 3) ↑
d	c — a — g — f — d	(3 + 2 + 2 + 3) ↓

Figure 54. Symmetric scale is identical for all geometrical inversions.

*See Book III.

Here are some examples of complete forms of contrapuntal continuity based on geometrical inversions:

(1) $\frac{CP}{CF}a + \frac{CF}{CP}a + \frac{CF}{CP}d + \frac{CP}{CF}b + \frac{CF}{CP}a + \frac{CF}{CP}c + \frac{CP}{CF}b$;

(2) $\frac{CF}{CP}a + \frac{CP}{CF}b + \frac{CF}{CP}c + \frac{CP}{CF}d + \frac{CF}{CP}a$;

(3) $\frac{CP}{CF}c + \frac{CF}{CP}b + \frac{CP}{CF}a + \frac{CF}{CP}d + \frac{CF}{CP}a$;

(4) $\frac{CF}{CP}a + \frac{CF}{CP}c + \frac{CP}{CF}b + \frac{CP}{CF}d + \frac{CF}{CP}a$.

Figure 55. Forms of contrapuntal continuity.

We shall apply the first of the above schemes of continuity to the theme based on the exposition in type II, Fig. 51. The theme will be used in its original ST version (i.e., without the added balance).

Figure 56. Scheme 1 applied to exposition in figure 51 (continued).

Figure 56. Scheme 1 applied to exposition in figure 51 (concluded).

As we have seen before, the interchangeability of CF and CP produces two forms for each geometrical position. This property may be utilized for the purpose of producing continuity based on *imitation*. The two reciprocal expositions following one another are planned in such a manner that the first one consists of an unaccompanied CF only, whereas the second has both parts. When CF exchanges its positions, the resulting effect is imitation.

In the following example, Fig. 52, type III, will serve as a theme. The complete continuity will follow this scheme: $CF\,a + \frac{CP}{CF}a + \frac{CF}{CP}d + \frac{CP}{CF}c + \frac{CF}{CP}b$

Figure 57. Exposition of figure 52 developed by geometrical inversions (continued).

Figure 57. Exposition of figure 52 developed by geometrical inversions (concluded).

CHAPTER 5

CORRELATION OF MELODIC FORMS IN TWO-PART COUNTERPOINT

THUS FAR, we have been concerned with the harmonic and the temporal correlation of two melodic parts. The melodic forms we have used have been planned in some general way, but many details were merely the outcome of the harmonic treatment of intervals.

Now, however, it is time to consider a systematic method for correlating *melodic* forms. Melody is expressed, fundamentally, by means of an *axial* combination; the correlation of two melodies, then, becomes essentially a problem of coordination between the two axial groups.*

A. Use of Monomial Axes

We shall begin our analytical survey with a glance at *monomial* axes for *both* CF and CP. The following 25 forms become possible:

$$\frac{CF}{CP} = \frac{0}{0};\ \frac{a}{0};\ \frac{0}{a};\ \frac{b}{0};\ \frac{0}{b};\ \frac{c}{0};\ \frac{0}{c};\ \frac{d}{0};\ \frac{0}{d};$$

$$\frac{a}{a};\ \frac{b}{a};\ \frac{a}{b};\ \frac{c}{a};\ \frac{a}{c};\ \frac{d}{a};\ \frac{a}{d};\ \frac{b}{b};\ \frac{c}{b};\ \frac{b}{c};$$

$$\frac{d}{b};\ \frac{b}{d};\ \frac{c}{c};\ \frac{d}{c};\ \frac{c}{d};\ \frac{d}{d}.$$

It is important to note that the various forms of balancing and unbalancing are *inherent* in the above combinations. Analysis of two parts as being parallel or contrary is *not* sufficient, as, under either condition, one voice may be balancing and the other may be unbalancing, both voices may be balancing, or both may be unbalancing.

For example: $\frac{CF}{CP} = \frac{b}{b};\ \frac{d}{b};\ \frac{b}{c};\ \frac{a}{d}.$

In the first case, both voices are parallel and balancing; in the second case, both voices are parallel, but CF is unbalancing, and CP is balancing; in the third case, both voices are contrary, but both are balancing; in the fourth case, both voices are contrary, but both are unbalancing.

It follows from the above considerations that the correct way to achieve continuous motion in two-part counterpoint is to introduce an unbalancing axis in one of the parts when the other part is moving toward balance, unless a cadence is desired. The music of J. S. Bach contains more parallel motion than is usually realized; but he always managed to avoid unintentional cadencing. On the other hand, many academic theoreticians advocate an abundance of contrary motion as being essentially contrapuntal; contrary motion is in itself of little importance,

*See Book IV.

however; it actually becomes a source of monotony unless it is used along with the proper constitution of balance relations between CF and CP.

The selection of axial combinations for the two counterparts (or for one counterpart to a given part) depends on the form of expression.

Axial relations with regard to their directions are: (1) parallel; (2) contrary; (3) oblique.

Axial relations with regard to their balancing tendencies are:

$$(1)\ \frac{U}{U};\ (2)\ \frac{U}{B};\ (3)\ \frac{B}{U};\ (4)\ \frac{B}{B}.$$

In addition, the zero-axis expresses a continuous state of balance.

All further development of the technique of correlating axial combinations of two melodies follows the ratio development of the quantities of axes in one part in relation to those in another.

Under such conditions, all the above described cases refer to one category only: $\frac{CP}{CF} = ax$, i.e., one secondary axis of counterpoint corresponds to one secondary axis of the cantus firmus, *ax* being used as an abbreviation of the word, "axis."

B. Binomial Axial Groups

Coming now to the binomial relations of axial groups of the counterpoint in relation to the cantus firmus, we see that:

$$\frac{CP}{CF} = \frac{2ax}{ax},\ \text{or}\ \frac{ax}{2ax}$$

Under such conditions, a monomial axis of one part corresponds to a binomial axial combination of another. For instance:

$$\frac{CP}{CF} = \frac{0+a}{0};\ \frac{a+b}{b};\ \frac{c+d}{a};\ \frac{b+0}{c};\ \frac{d+a}{0};\ \ldots,\ \text{etc.}$$

$$\frac{CP}{CF} = \frac{0}{0+a};\ \frac{b}{a+b};\ \frac{a}{c+d};\ \frac{c}{b+0};\ \frac{0}{d+a};\ \ldots,\ \text{etc.}$$

It is easy to see that there are 200 such simultaneous combinations, as there are 10 original binomial axial combinations, each having 2 permutations. Twenty combinations are now combined vertically with 5 monomials (0, a, b, c, d). This produces $20 \cdot 5 = 100$. Finally, 100 must be multiplied by 2, as each simultaneous combination can be inverted.

The period of duration of one axis equals the sum of durations of the two axes constituting the binomial. Thus, in a combination:

$\frac{CP}{CF} = \frac{2ax}{ax} = \frac{axmt + axnt}{axpt} = \frac{T}{T} = 1$, the time period for both parts is the same.

Time ratios for binomial axes must be selected in accordance with the series which the monomial axis represents. If, for instance, the duration of ax of CF is 8T, then CP may be matched as any binomial of $\frac{8}{5}$ series. We might select the 5+3 binomial of this series.

Now we can define the simultaneous temporal relations as follows:

$$\frac{CP}{CF} = \frac{ax5T + ax3T}{ax8T}$$

In a simultaneous combination of a binomial-against-a-monomial axial combination, we find that the following is significant: during the period of the monomial axis (balanced, balancing, or unbalancing) its counterpart has two phases, which may be any of these pairs: U+U; U+B; B+U; or B+B. If we single out a continuous balance (0-axis) as an independent form, we obtain 12 forms of balance relations between CP and CF, when one of them is a binomial and the other a monomial.

$$\frac{CF}{CP} = \frac{ax}{2ax} = \frac{0}{U+U};\ \frac{0}{U+B};\ \frac{0}{B+U};\ \frac{0}{B+B};$$

$$\frac{U}{U+U};\ \frac{U}{U+B};\ \frac{U}{B+U};\ \frac{U}{B+B};$$

$$\frac{B}{U+U};\ \frac{B}{U+B};\ \frac{B}{B+U};\ \frac{B}{B+B}.$$

Just as many are available for $\frac{CP}{CF} = \frac{ax}{2ax}$. If the 0-axis participates in a binomial, there are 15 more combinations: 0+U, 0+B, B+0, 0+0 multiplied by 3.

Let us select *one* of the many possible combinations. Let it be $\frac{CP}{CF} = \frac{2ax}{ax} = \frac{U+U}{B} = \frac{d+a}{c}$. Suppose that CF = 8T, and suppose that we match the previously selected time-ratio for CP. Then the correlation of $\frac{CP}{CF}$ appears as follows: $\frac{CP}{CF} = \frac{d5T + a3T}{c8T}$. In this case CP unbalances for 5T in the direction below its P.A. (primary axis) and unbalances still further in the direction above its P.A. for 3T. While this happens, CF moves steadily toward its own P.A. in the upward direction during the course of 8T. The graph of this would be:

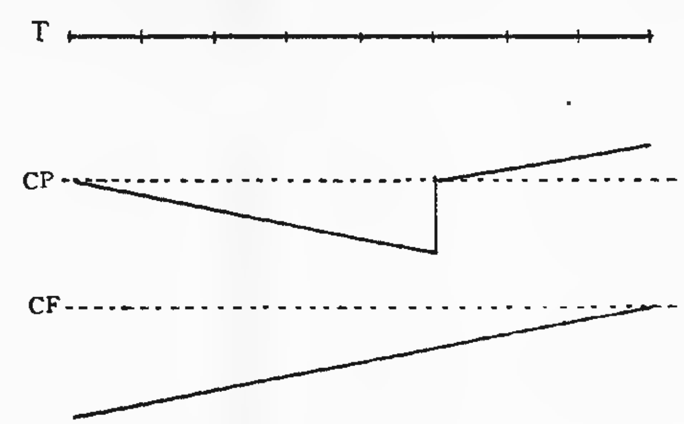


Figure 58. $\frac{CP}{CF} = \frac{d5T + a3T}{c8T}$

C. Trinomial Axial Combinations

In the same fashion, trinomial axial combinations of one part may be correlated with a monomial axis of another. The number of simultaneous combinations equals the number of trinomials times 5.

There are 60 trinomials with two identical terms (as noted in my discussion of the theory of melody)* and 60 trinomials with all terms different. This yields $(120 \cdot 5 =)$ 600 for $\frac{CP}{CF}$, and the same quantity for $\frac{CF}{CP}$.

As the number of axes is three in one part and one in the other part, we may write:

$$\frac{CP}{CF} = \frac{3ax}{ax} \text{ or } \frac{CP}{CF} = \frac{ax}{3ax}.$$

In each case, the trinomial requires three temporal coefficients, the sum of which equals that of the monomial.

$$\frac{CP}{CF} = \frac{3ax}{ax} = \frac{axmt + axnt + axpt}{axT}, \text{ where } mt + nt + pt = T. \text{ Let T equal 5.}$$

Then, by selecting 2+2+1 which is one of the trinomials of $\frac{5}{5}$ series, we obtain:

$$\frac{CP}{CF} = \frac{ax2T + ax2T + axT}{ax5T}$$

The trinomial distribution of the O, U and B yields the following number of the forms of balance.

O+O+U; O+O+B; U+U+O; U+U+B; B+B+O; B+B+U.

Each of the above 6 combinations has 3 permutations, giving a total of $6 \cdot 3 = 18$. When each of these variations is placed against O, U or B in the counterpart, the number of forms becomes tripled: $18 \cdot 3 = 54$. Thus, $\frac{CP}{CF}$ and $\frac{CF}{CP}$ have 54 forms each.

But the above forms contain trinomials with two identical terms. The addition of trinomials without identical terms produces one combination: O+U+B, which has 6 permutations. These 6 forms, placed against the three possible forms of the counterpart, produce $(6 \cdot 3 =)$ 18 combinations.

$\frac{CP}{CF}$ and $\frac{CF}{CP}$ have 18 forms each.

The total of trinomial combinations of balance of $\frac{CP}{CF}$ is $(54 + 18 =)$ 72, and the same number for $\frac{CF}{CP}$.

When secondary axes are substituted for the forms of balance, each case gives more than one solution. For example: if $\frac{CP}{CF} = \frac{U + 0 + B}{U}$, then—

	CP	CF
(1)	U = a; U = d;	U = a; U = d.
(2)	O = O	
(3)	B = b; B = c;	

*See Book IV.

—and the following solutions are available:

$$\frac{CP}{CF} = \frac{a+0+b}{a};\ \frac{a+0+b}{d};\ \frac{a+0+c}{a};\ \frac{a+0+c}{d};$$

$$\frac{d+0+b}{a};\ \frac{d+0+b}{d};\ \frac{d+0+c}{a};\ \frac{d+0+c}{d}.$$

Let us assign the previously discussed $\frac{5}{5}$ series trinomial time ratio. We obtain the following solutions:

$$\frac{CP}{CF} = \frac{a2T + 02T + bT}{a5};\ \frac{a2T + 02T + bT}{d5T};\ \frac{a2T + 02T + cT}{a5T};$$

$$\frac{a2T + 02T + cT}{d5T};\ \frac{d2T + 02T + bT}{a5T};\ \frac{d2T + 02T + bT}{d5T};$$

$$\frac{d2T + 02T + cT}{a5T};\ \frac{d2T + 02T + cT}{d5T};$$

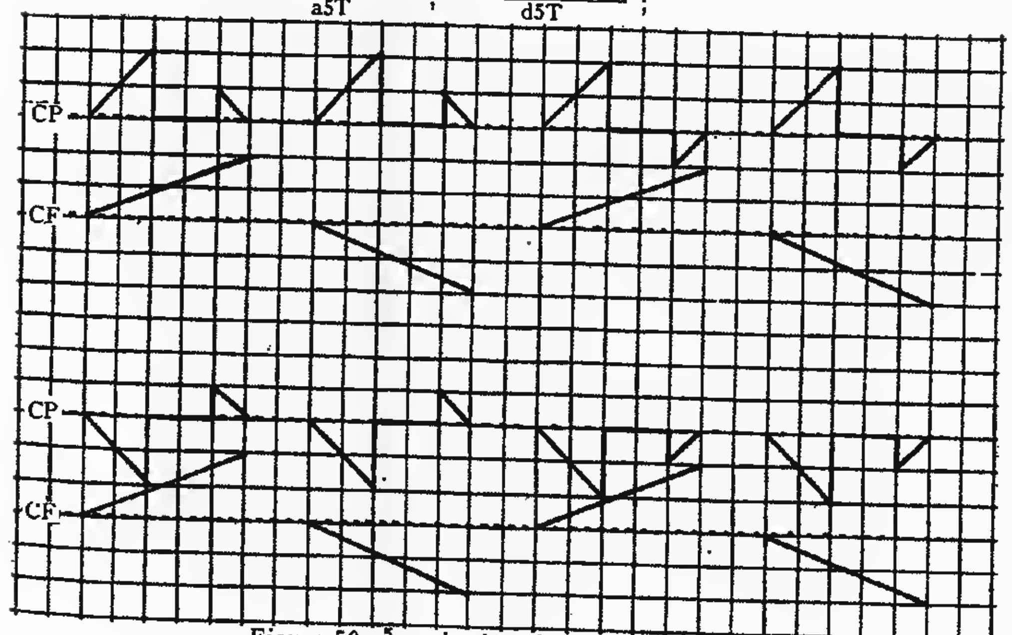


Figure 59. $\frac{5}{5}$ series in trinomial time ratio.

D. Polynomial Axial Combinations

Ultimately, a polynomial axial combination may serve as counterpart to a monomial axis. The effect of such a correlation is instability (polynomial) versus stability (monomial). The selection of forms of O, U and B depends on the effects of balance necessary in each particular case. An abundance of unbalancing axes results in restless, disquieting, unstable melodies—such melodies are often called dramatic, passionate, ecstatic, etc. An abundance of balancing and O-axes produces restful, quiet, stable melodies, usually termed contemplative, epical, or serene.

Examples of compositions of $\frac{CP}{CF} = \frac{max}{ax}$.

Let m = 5; then: $\frac{CP}{CF} = \frac{5ax}{ax}$.

Let us consider our balance-group to be U+B+U+B+U, and assume that the two extreme terms are identical, but different from the middle one. Then the possibilities for the U's are:

(1) a+d+a and (2) d+a+d

In the first combination, let us assume that both B's are identical but on the opposite side of P.A. from the two identical U's. Then we get c+c for the B+B. The entire axial combination for CP appears as follows:

CP = a+c+d+c+a

Let CF be represented by B, and let it be b in order to achieve greater variety of balancing forms of CP in relation to CF.

$$\frac{CP}{CF} = \frac{a + c + d + c + a}{b}$$

Let the duration of the entire group be 16T. Let the temporal coefficients correspond to $\frac{8}{8}$ series on the basis of t = 2T. Then, by selecting a quintinomial (for the five axes of CP), we obtain the following temporal scheme:

$$\frac{CP}{CF} = \frac{a4T + c2T + d4T + c2T + a4T}{b16T}$$

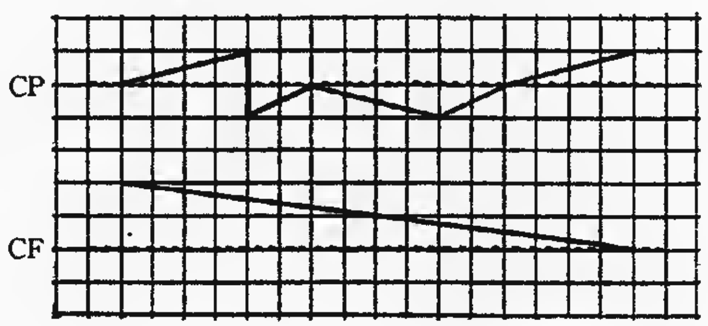


Figure 60. Graph of $\frac{CP}{CF} = \frac{a4T + c2T + d4T + c2T + a4T}{b16T}$

E. Developing Axial Relations Through Attack-Groups

The temporal ratios, discussed so far, referred to the form $\frac{CP}{CF} = 1, 2, 3, \ldots m$.

Such axial relations may be further developed into polynomial groups in both CF and CP:

(1) through the technique previously applied to the composition of attack-groups, as in *Melodization of Harmony;**

(2) by direct application of ratios producing interference.

*See Book VI.

The first technique makes it possible to match any desirable number of axes of the CP against each axis of the CF.

Let us take a CF with 4 axes. We may match 2, 3, or more axes of CP against each axis of CF and in any desirable sequence.

For example: $\frac{CP}{CF} = \frac{2ax}{ax} + \frac{2ax}{ax} + \frac{2ax}{ax} + \frac{2ax}{ax}$.

By assigning temporal coefficients in such a way that the sum of durations in each 2ax of CP corresponds to the duration of ax of CF, we acquire a synchronized $\frac{CP}{CF}$. With the temporal coefficients based on $r_{5 \div 4}$, for instance, we obtain the following correlation:

$$\frac{CP}{CF} = \frac{ax4T + axT}{ax5T} + \frac{ax3T + ax2T}{ax5T} + \frac{ax2T + ax3T}{ax5T} + \frac{axT + ax4T}{ax5T}$$

Let 0+b+c+a be the axial combination of CF, and let (0+a) + (0+b) + + (b+0) + (a+0) be the axial combination of CP. Then $\frac{CP}{CF}$ acquires the following appearance:

$$\frac{CP}{CF} = \frac{04T + aT}{05T} + \frac{03T + b2T}{b5T} + \frac{b2T + 03T}{c5T} + \frac{aT + 04T}{a5T}$$

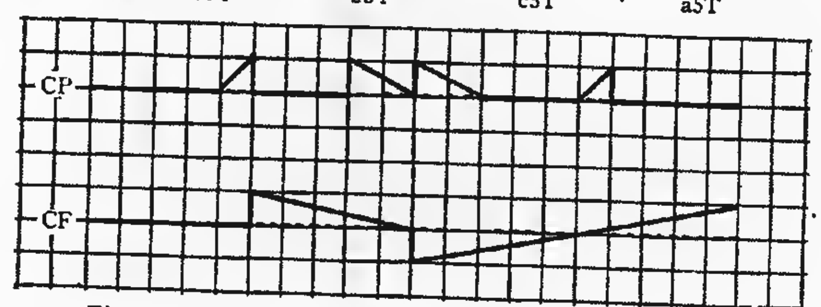


Figure 61. Two axes of CP matched to each one of CF.

When proportionate relations of the temporal coefficients of $\frac{CP}{CF}$ are desirable, and when a constant number of the axes of CP is assigned against each axis of CF, the technique of *distributive involution* solves the problem.

For example: $\frac{CP}{CF} = \frac{9ax}{3ax} = \frac{3ax}{ax} + \frac{3ax}{ax} + \frac{3ax}{ax}$.

To carry out this form of correlation in proportions, we may select the square of 2+1+1 of the $\frac{4}{4}$ series.

$$\frac{CP}{CF} = \frac{ax4T + ax2T + ax2T}{ax8T} + \frac{ax2T + axT + axT}{ax4T} + \frac{ax2T + axT + axT}{ax4T}$$

Let the axial combination for both CP and CF be the trinomial a+b+c. Then:

$$\frac{CP}{CF} = \frac{a4T + b2T + c2T}{a8T} + \frac{a2T + bT + cT}{b4T} + \frac{a2T + bT + cT}{c4T}.$$

See musical illustration on following page.

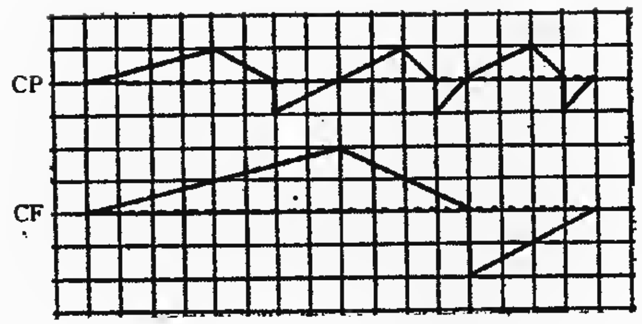


Figure 62. Proportionate relation of temporal coefficients of CP and CF.

F. Interference of Axis-Groups

The most complex temporal relations result when the respective axes in CP and CF produce interference ratios. I shall discuss here only the simplest forms of such interference, those which require uniform temporal coefficients for both CP and CF, and differ only in value. This corresponds to *binary synchronization* as described in my earlier discussion of the *Theory of Rhythm.** In this sense, an $\frac{a}{b}$ ratio represents the number of secondary axes in the two counterparts.

Let us take the $\frac{3}{2}$ ratio. Under such conditions $\frac{CP}{CF} = \frac{3ax}{2ax}$, or $\frac{CP}{CF} = \frac{2ax}{3ax}$. After synchronization, the first expression appears as follows:

$$\frac{CP}{CF} = \frac{ax2T + ax2T + ax2T}{ax3T + ax3T}$$

Let CF consist of 0+d and CP—of a+d+0. Then:

$$\frac{CP}{CF} = \frac{a2T + d2T + 02T}{03T + d3T}$$

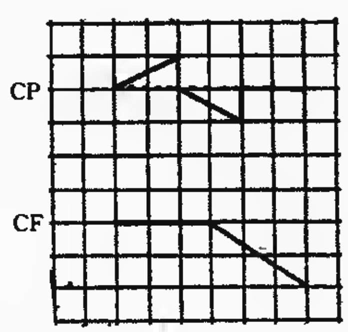


Figure 63. More complex temporal relations of CP and CF.

*See Book I.

Series of accelerations used in their reciprocal directions serve as additional material for the temporal coefficients of $\frac{CP}{CF}$. This technique produces two counterparts in the form of "growth" against "decline."

An example:

$$\frac{CP}{CF} = \frac{axT + ax2T + ax3T + ax5T}{ax5T + ax3T + ax2T + axT}$$

Axial combinations: $\frac{CP}{CF} = \frac{a+b+c+d}{a+b+c+d}$. Hence:

$$\frac{CP}{CF} = \frac{aT + b2T + c3T + d5T}{a5T + b3T + c2T + dT}$$

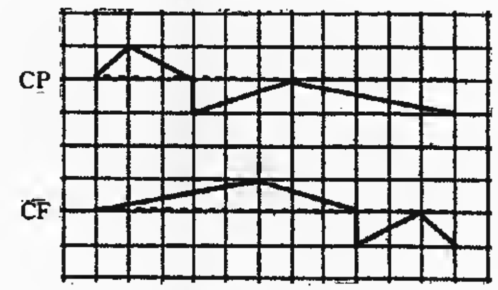


Figure 64. Adding series of accelerations.

This case illustrates the fact that even identical axial combinations in both counterparts may be made contrasting by the reciprocation of temporal coefficients.

An obvious contrast, that of some axial combinations against their own magnified versions, may be achieved by means of coefficients of duration applied to the original group of temporal coefficients.

An example:

$$\frac{CP}{CF} = \frac{2(ax3T + axT + ax2T + ax2T)}{ax6T + ax2T + ax4T + ax4T}$$

Axial combination: $\frac{CP}{CF} = \frac{a+b+c+d}{a+b+c+d}$. Hence:

$$\frac{CP}{CF} = \frac{a3T + bT + c2T + d2T + a3T + bT + c2T + d2T}{a6T + b2T + c4T + d4T}$$

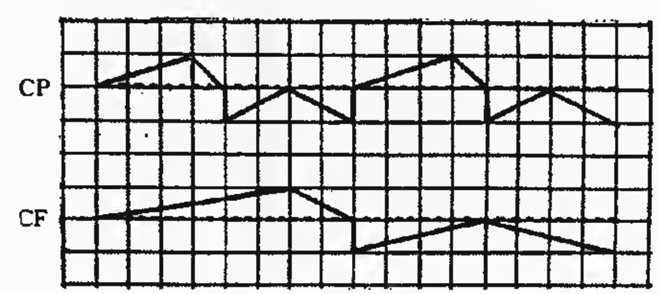


Figure 65. Applying coefficients of duration.

G. Correlation of Pitch-Time Ratios of the Axes

After correlation of temporal coefficients has been established, correlation of pitch ranges of both counterparts is the next step.*

Secondary axes that are otherwise identical may have different rates of speed. In terms of pitch ranges, it means that a greater range in one axis may be covered in the same period of time required by another axis to traverse a smaller range.

Use of identical axes having different pitch-ranges produces a noticeable amount of contrast.

$\frac{CP}{CF} = \frac{axT2P}{axTP}$. Let a be the axis in both parts.

Then: $\frac{CP}{CF} = \frac{aT2P}{aTP}$.

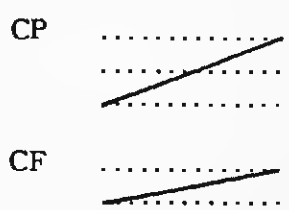


Figure 66. Different pitch ranges for identical axes.

When the two counterparts are represented by axes identical with respect to balance, but non-identical in structure, the contrast becomes still more obvious.

(1) $\frac{CP}{CF} = \frac{B}{B}$.

$\frac{CP}{CF} = \frac{b2P}{cP}; \frac{c2P}{bP}; \frac{b3P}{cP}; \frac{c3P}{bP}; \frac{b3P}{c2P}; \frac{c3P}{b2P}; \ldots$

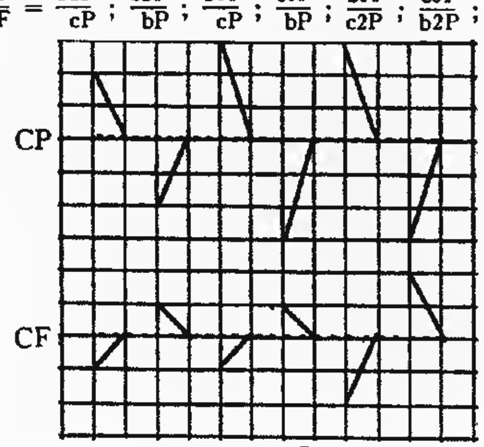


Figure 67. $\frac{CP}{CF} = \frac{B}{B}$

*The student should be cautioned that these—and similar—passages in the text as to the *calculation* of music in advance of writing it are not simply mathematical curiosities, but are the very core of Schillinger's system. Maximum efficiency and fluent coordination of all the factors involved in "good" music cannot be achieved without just such exact planning as is being illustrated in these portions of the text. (Ed.)

(2) $\frac{CP}{CF} = \frac{U}{U}$.

$\frac{CP}{CF} = \frac{a2P}{dP}; \frac{d2P}{aP}; \frac{a3P}{dP}; \frac{d3P}{aP}; \frac{a3P}{d2P}; \frac{d3P}{a2P};$

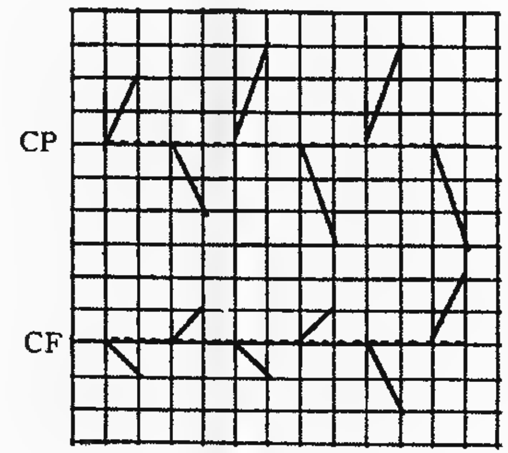


Figure 68. $\frac{CP}{CF} = \frac{U}{U}$

Still greater contrasts result from juxtaposition of pitch ranges of the two counterparts when the axial structures differ with respect to balance.

$\frac{CP}{CF} = \frac{U}{B}$.

$\frac{CP}{CF} = \frac{a2P}{bP}; \frac{a2P}{cP}; \frac{d2P}{bP}; \frac{d2P}{cP}; \ldots$

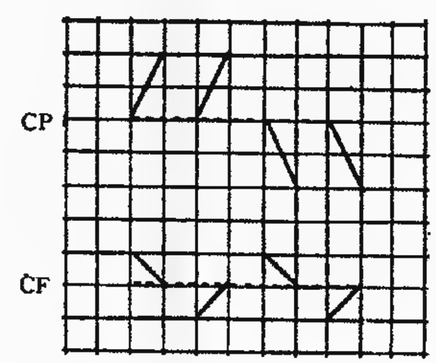


Figure 69. $\frac{CP}{CF} = \frac{U}{B}$

The 0-axis need not detain us in calculations aimed at correlating the pitch-ranges of the two counterparts.

As pitch-ratios may be in direct, oblique or inverse relations with the time-ratios in each part, the correlation of the two counterparts offers the following fundamental possibilities:

$$\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ direct}};\ \frac{T \div P \text{ oblique}}{T \div P \text{ direct}};\ \frac{T \div P \text{ inverse}}{T \div P \text{ direct}}$$

$$\frac{T \div P \text{ oblique}}{T \div P \text{ oblique}};\ \frac{T \div P \text{ inverse}}{T \div P \text{ oblique}};\ \frac{T \div P \text{ inverse}}{T \div P \text{ inverse}}$$

The second, the third, and the fifth forms have another variant, each by inversion. The total number of the above relations is 6+3 = 9.

Examples:

$$\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ direct}}$$

$$(1)\ \frac{CP}{CF} = \frac{bTP + c2T2P + a4T4P}{d4T4P + b3T3P};$$

$$(2)\ \frac{CP}{CF} = \frac{aTP + b2T2P + a3T3P + d4T4P}{04T + a3T3P + c2T2P + bTP}.$$

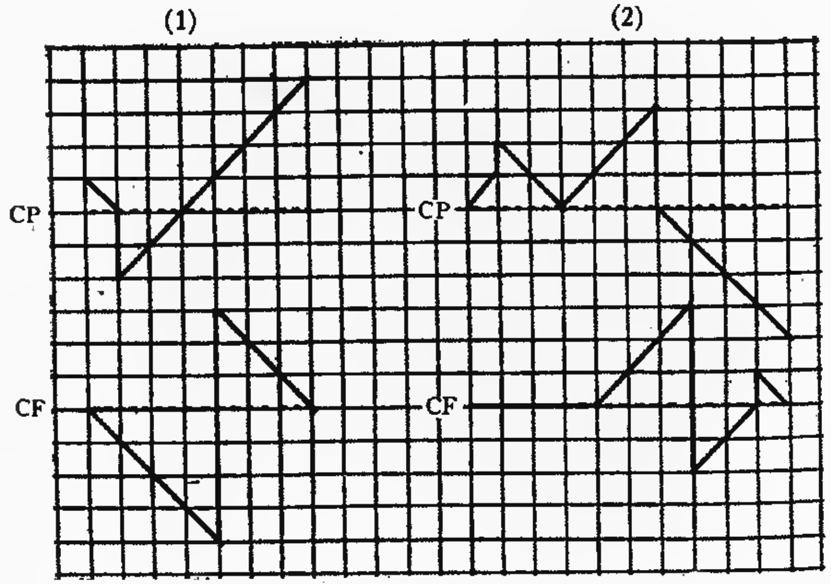


Figure 70. Inverting various forms $\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ direct}}$

$$\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ oblique}}$$

$$(1)\ \frac{CP}{CF} = \frac{a4T4P + c2T2P}{dT3P + c2T2P + d3T1P};$$

$$(2)\ \frac{CP}{CF} = \frac{b3T3P + dTP + c2T2P + a2T2P}{dT4P + b3T3P + c4T1P}$$

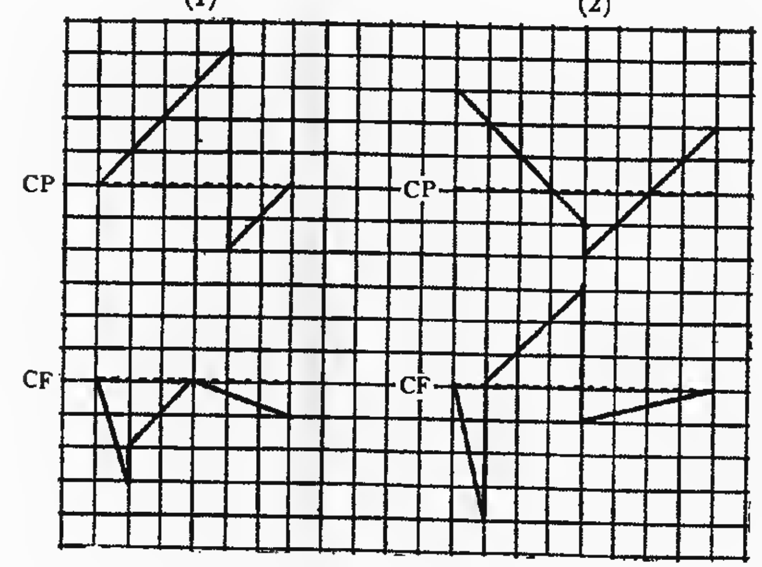


Figure 71. $\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ oblique}}$

$$\frac{CP}{CF} = \frac{T \div P \text{ inverse}}{T \div P \text{ direct}}$$

$$(1)\ \frac{CP}{CF} = \frac{a6T2P + b3T4P}{b4T4P + d2T2P + c2T2P + dTP};$$

$$(2)\ \frac{CP}{CF} = \frac{a2T2P + d2T2P + aTP + dTP + a2T2P + d2T2P}{c4T1P + c3T2P + c2T3P + cT4P}$$

See the corresponding illustrations on the following page.

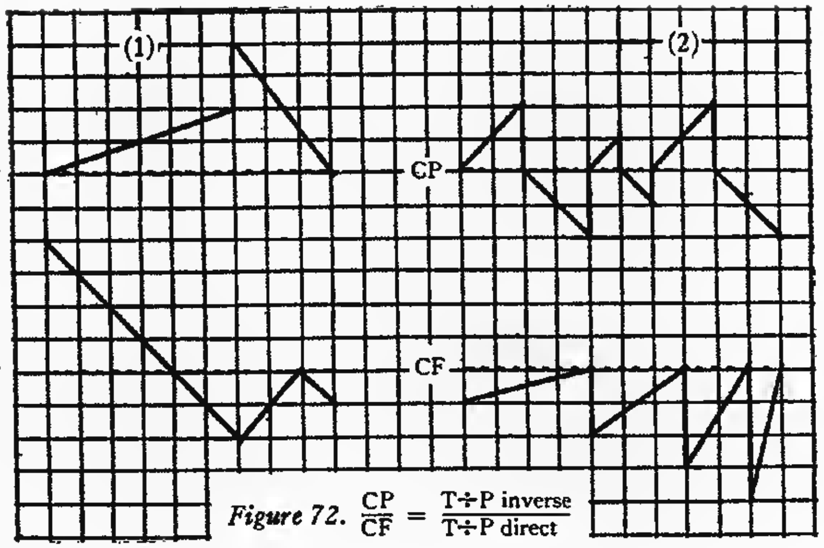


Figure 72. $\frac{CP}{CF} = \frac{T \div P \text{ inverse}}{T \div P \text{ direct}}$

$$\frac{CP}{CF} = \frac{T \div P \text{ oblique}}{T \div P \text{ oblique}}$$

$$(1)\quad \frac{CP}{CF} = \frac{a3T1P + a2T2P + bT3P + b3T1P + b2T2P + aT3P}{c3T5P + d4T4P + c5T3P}$$

$$(2)\quad \frac{CP}{CF} = \frac{bT5P + a2T4P + d3T3P + b4T2P + a5T1P}{a7T3P + b5T5P + c3T7P}$$

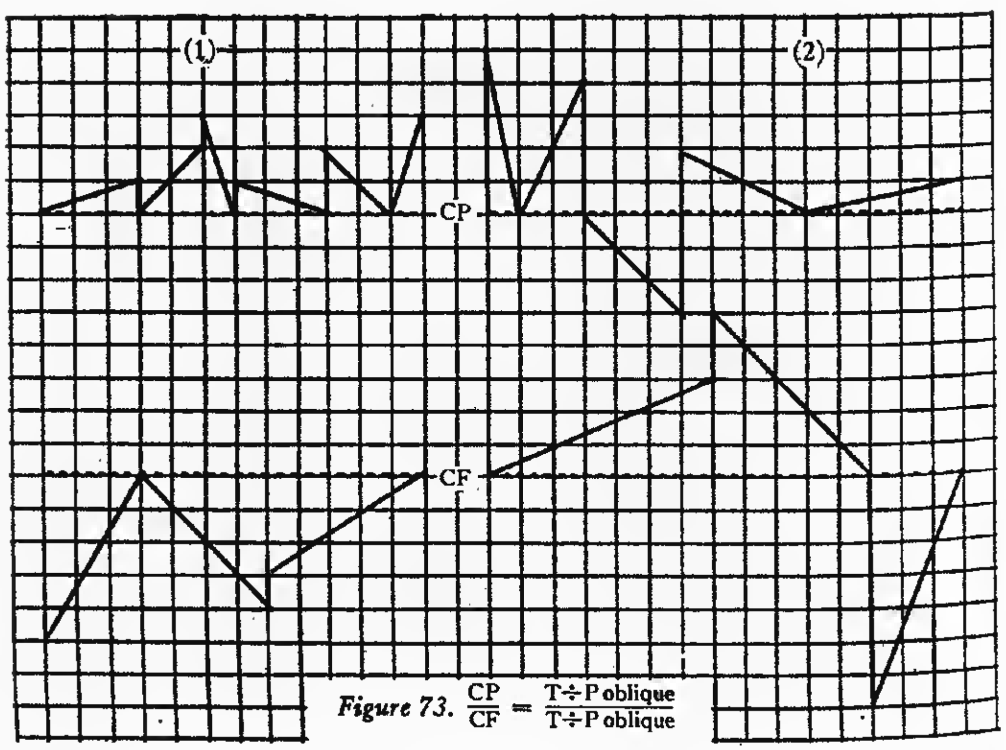


Figure 73. $\frac{CP}{CF} = \frac{T \div P \text{ oblique}}{T \div P \text{ oblique}}$

$$\frac{CP}{CF} = \frac{T \div P \text{ oblique}}{T \div P \text{ inverse}}$$

$$(1)\quad \frac{CP}{CF} = \frac{b3T2P + c3T3P + b2T3P}{aT2P + b2T1P + c2T1P + d3T1P}$$

$$(2)\quad \frac{CP}{CF} = \frac{a4T3P + d3T3P + a3T4P}{cT4P + b2T3P + b3T2P + c4T1P}$$

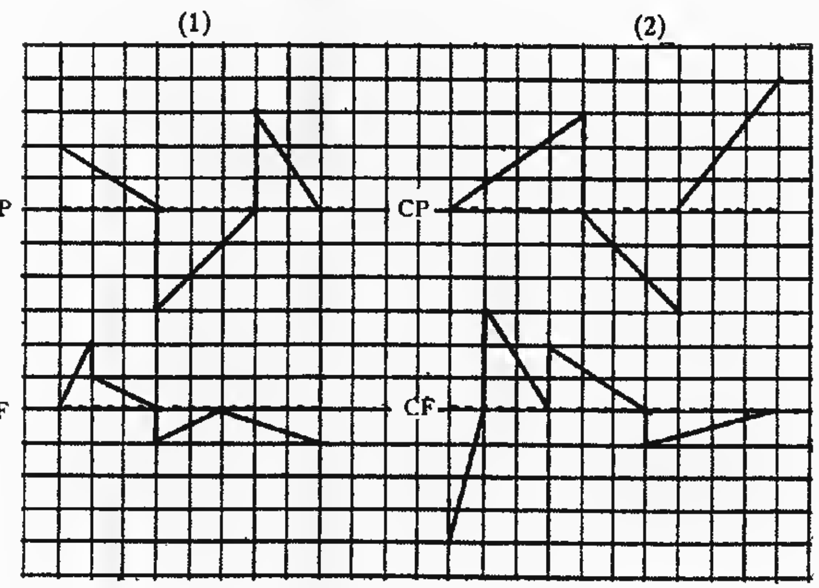


Figure 74. $\frac{CP}{CF} = \frac{T \div P \text{ oblique}}{T \div P \text{ inverse}}$

$$\frac{CP}{CF} = \frac{T \div P \text{ inverse}}{T \div P \text{ inverse}}$$

$$(1)\quad \frac{CP}{CF} = \frac{a3T1P + cT3P + c3T1P + aT3P}{a5T3P + b3T5P}$$

$$(2)\quad \frac{CP}{CF} = \frac{cT2P + c2T1P + b2T1P + b4T2P}{d6T3P + d3T6P}$$

See the corresponding illustrations on the following page.

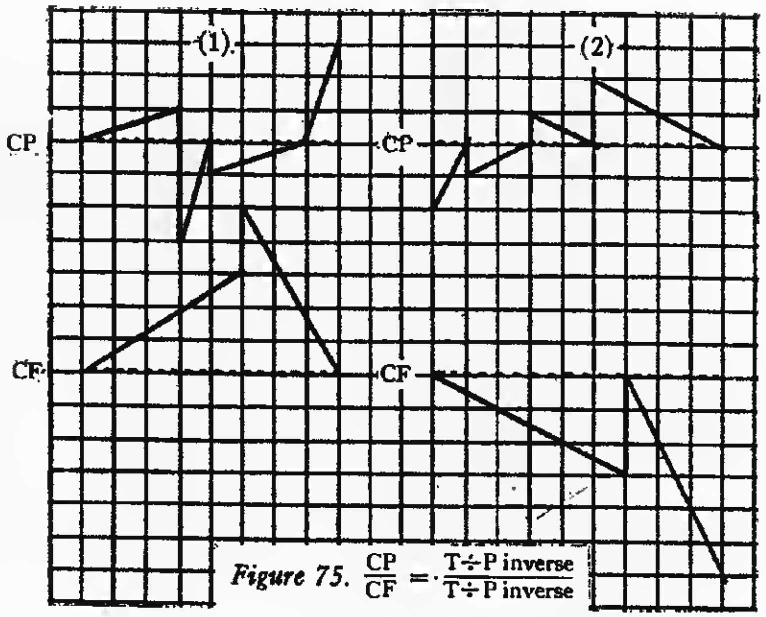


Figure 75. $\frac{CP}{CF} = \frac{T \div P \text{ inverse}}{T \div P \text{ inverse}}$

Example of Application

$$\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ inverse}}$$

$$\frac{CP}{CF} = \frac{a4T4P + b3T3P + a3T3P + b2T2P}{b8T1P + d4T2P}$$

$$T(CF) = (4+3+3+2)^2 = (16+12+12+8) + (12+9+9+6) + \\ + (12+9+9+6) + (8+6+6+4).$$

$$T(CP) = (\boxed{1}+1+1+1+1+1+1+1+1+1+1+1) \circlearrowright$$

Axial combination of $\frac{CP}{CF}$ in its general form:

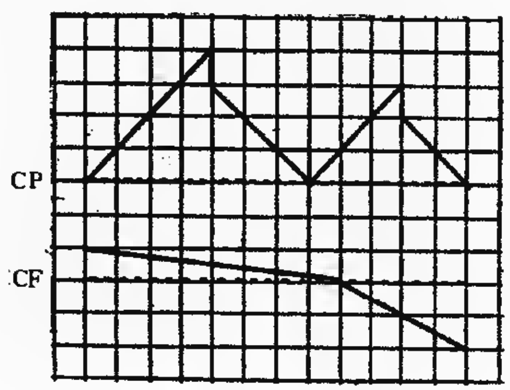


Figure 76. $\frac{T \div P \text{ direct}}{T \div P \text{ inverse}}$

Let CF be constructed from C-maj. nat. d_0 scale and CP—from A♭-maj. nat. d_6 scale.* Let P = 5p with approximation. Under such conditions, the range of CF will be about an octave and a half, and the range of CP—about two octaves.

Figure 77. Melody for preceding figure.

**C-maj. nat. d_0 scale* means—to refer to the material on pitch-scales—"key of C, the natural major scale (the "all white keys" scale), zero displacement (i.e., the tonic in C itself)." *A♭-maj. nat. d_6* means "key of A-flat, natural major, sixth displacement (i.e., the mode starting on G as its tonic). (Ed.)

H. Composition of a Counterpart to a Given Melody by Means of Axial Correlation

In order to correlate counterparts by means of axial correlation, it is necessary just to reconstruct the axial group of the given melody.*

After this analysis of the TP ratios of CF has been accomplished, it is important to detect whether the $T \div P$ is of direct, oblique, or inverse form.** After this, the general planning of the CP axial combination must follow—first, with respect to the $T \div P$ correlation; second, with respect to the axial combination itself and its own $T \div P$ ratios.

The following graph is a transcription of the first four measures of the common musical setting of Ben Jonson's "Drink to Me Only With Thine Eyes."

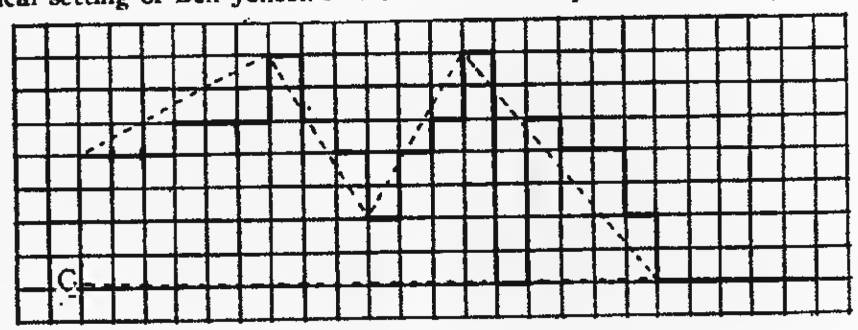


Figure 78. Graph of Drink to Me Only with Thine Eyes.

On analysis, we find that this melody contains a modal modulation, for $P.A._1$ is Phrygian (d_2), and $P.A._2$ is Ionian (d_0). The entire axial group gradually gravitates toward $P.A._2$, where it reaches its absolute balance. If we take into account all the minute crossings, an analysis of the axial group will appear as follows:

$$P.A._1 = a6t + b2t + dt + ct + a2t + b3t + d3t$$

$$P.A._2 = b3t + 05t + \boxed{t}.$$

The modulation here from one mode (d_2) to another (d_0) is performed by establishing a correspondence between d3t ($P.A._1$) and b3t ($P.A._2$). We can say that d3t ($P.A._1$) $\equiv$ b3t ($P.A._2$). As the pitch ranges are approximately equal the TP ratio may be regarded as constant.

*This is a technique indispensable in modern "arranging"—and in virtually all good orchestration of any style. (Ed.)

**The pitch-time ratio ("TP" ratio, or $T \div P$) means just what it says: The duration of the particular axis divided by its "height" or "depth" measured vertically in semitones. Composers seeking to perfect a style based on tastes they have already formed will find it useful to analyze, say, a hundred of their "favorite melodies," noting the axes—0, a, b, c and d—, the sequence of axes in groups, the durations (T) of each axis, the pitch-range (P) of each axis, and the TP ratios involved. (Ed.)

Let us now devise a counterpart in $1 \div 4$ time-ratio, meaning that CP will have only one secondary axis. As the general tendency of the CF is that of gradual gravitation toward balance in the course of two oscillations (which correspond to four directions and eight individual axes), we shall introduce a b-axis for the counterpart.* Then CP will consist of one direction, consistently gravitating toward balance. Under such conditions, $\frac{CP}{CF}$ represents a complete cycle of development.

This counterpart corresponds to case (2) in group (a) of Figure 39, where CP has an Aeolian P.A. (d_6).

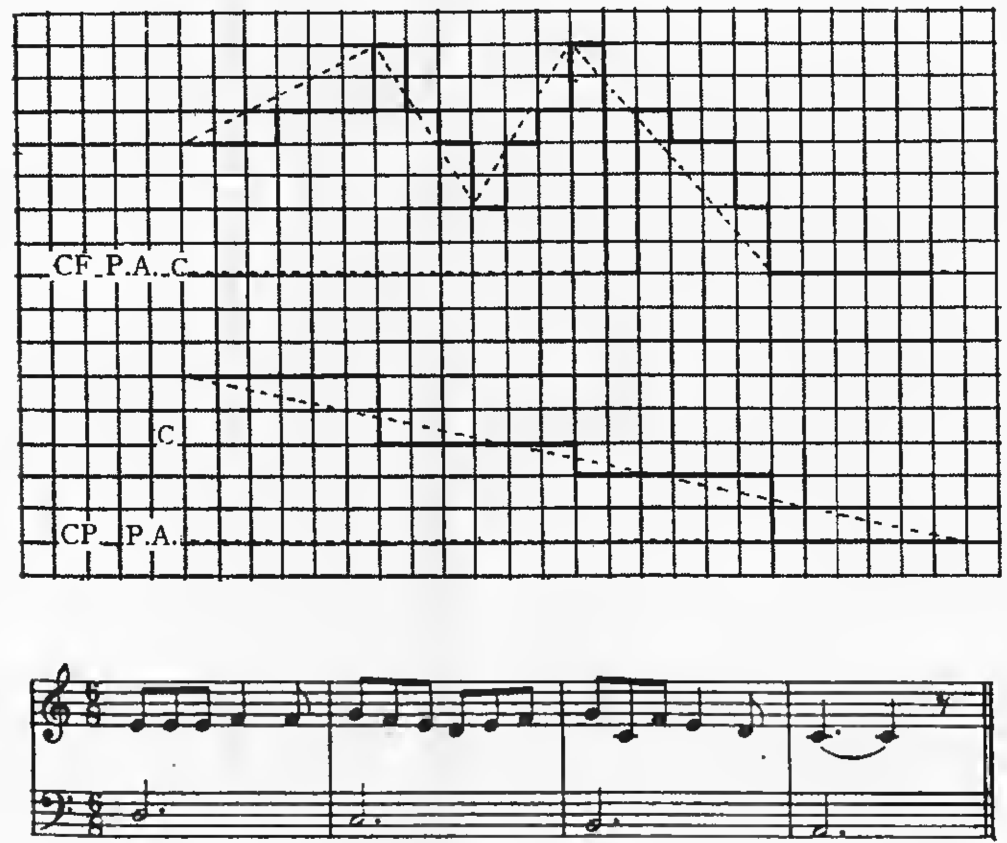


Figure 79. Counterpart in $1 \div 4$ time-ratio.

*That is, in this case, the over-all, general trend of CF, regardless of the oscillations, is in the general form of a b (downwards to the P.A.) axis, which same axis is here chosen as the form for the entire CP. (Ed.)

CHAPTER 6

TWO-PART COUNTERPOINT WITH SYMMETRIC SCALES

UNITY of style requires that both the *cantus firmus* and its counterpart be based on symmetric scales if one of them is.

Scales of the third group and scales of the fourth group, mostly in contracted form, serve as material for counterpoint. It is acceptable to have one counterpart in the third group and another in either the third or the fourth group.* When the two counterparts are in scales which belong to different groups, two cases may be distinguished:

(1) both scales have an *identical set of pitches;*
(2) each scale has a *different set of pitches.*

Example:

(1)
T_1 T_2 T_3 T_1
$\overrightarrow{S_1}$ = c – f – a♭ – d♭ – e – a♮ – c
T_1 T_2 T_3 T_1
$\overrightarrow{S_2}$ = c – d♭ – e – f – g♯ – a – c

(2)
T_1 T_2 T_1
$\overrightarrow{S_1}$ = c – d – e♭ – f – f♯ – g♯ – a – b – c
T_1 T_2 T_3 T_1
$\overrightarrow{S_2}$ = c – d – f – g – a♭ – b♭ – d♭ – e♭ – e♮ – f♯ – a – b – c

Figure 80. Identical and different sets of pitches.

The relations between the harmonic axes of the two counterparts may be carried out in all four of the forms previously used. Their meaning with regard to symmetric scales appears as follows:

Type I (U.U.) [Unitonal-unimodal]: both scales have the same T_1, the same number of tonics, and an identical set of pitch-units.

Type II (U.P.) [Unitonal-polymodal]: both scales have the same number of tonics, their sets of pitch-units are identical, but their harmonic axes are on different tonics.

Type III (P.U.) [Polytonal-unimodal]: both scales have an identical form of symmetry (the quantity of tonics) and an identical set of pitch-units; none of the tonics of one scale has pitches in common with the tonics of the other, i.e., the two sets of tonics belong to the mutually exclusive sets of pitches.

*Third group scales are one octave in range with 2, 3, 4, 6 or 12 symmetrically arranged tonics; fourth group scales are of *more than* one octave in range, and of 2 or more symmetric tonics. (Ed.)

Type IV (P.P.) [Polytonal-polymodal]: the two scales belong to either identical or non-identical forms of symmetry; their sectional scales are of non-identical structure, yet they belong to one family (according to the classification offered in my discussion of the first group of scales*); the two sets of tonics belong to mutually exclusive sets of pitches.

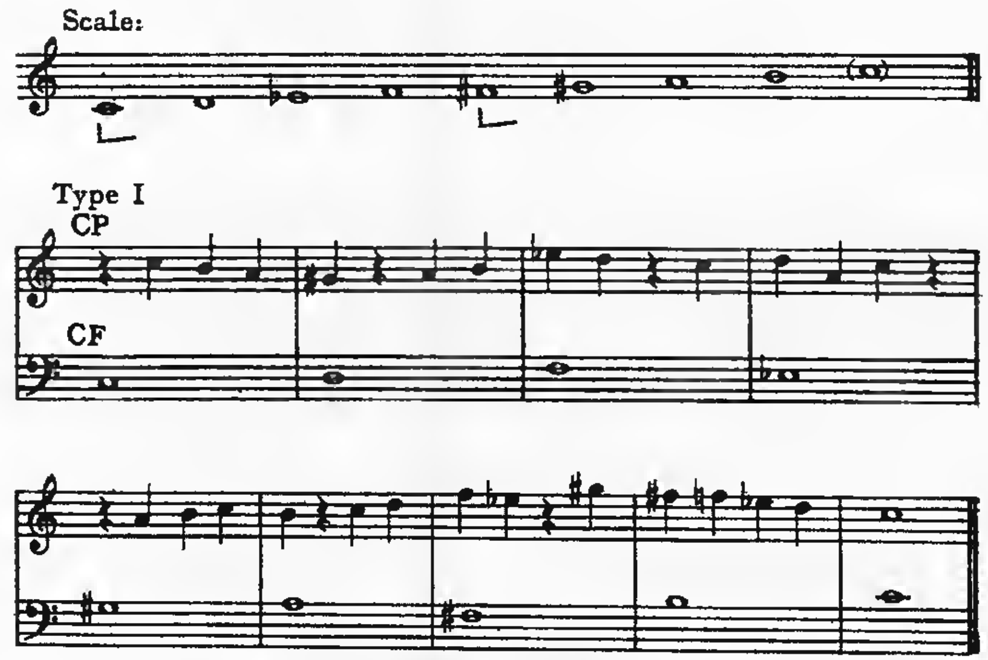


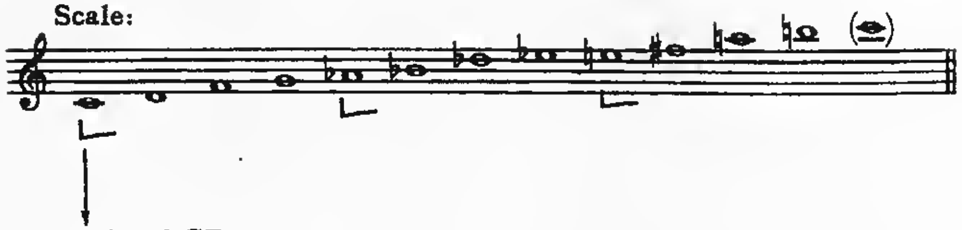
Figure 81. Two-part counterpoint in scale of third group. Type I.

Figure 82. Two part counterpoint in scale of third group. Type II (continued).

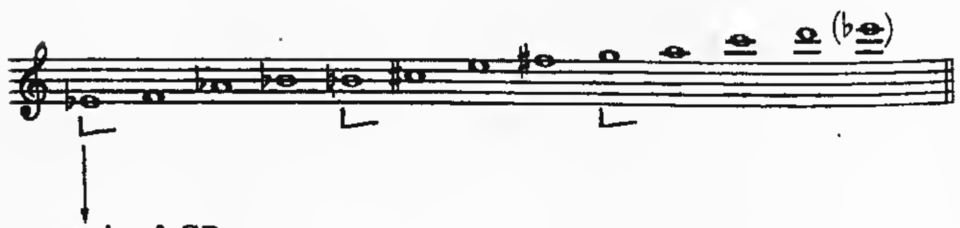
*See Book II, *Theory of Pitch-Scales.*

Figure 82. Two part counterpoint in scale of third group. Type II (concluded).

Scale:



axis of CF



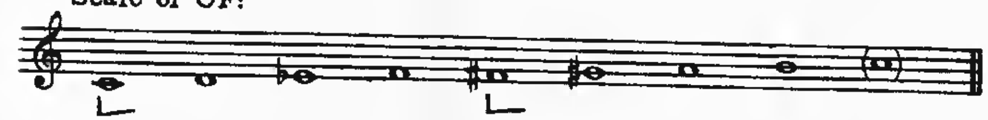
axis of CP

Type III

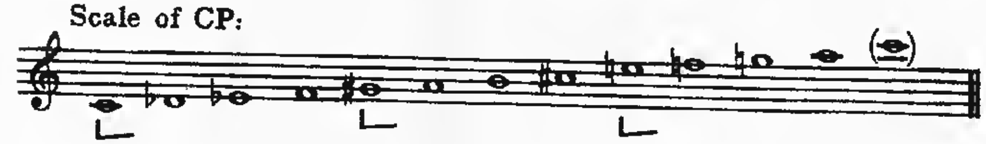
Figure 83. Two-part counterpoint in scale of fourth group. Type III (continued).

Figure 83. Two-part counterpoint in scale of fourth group. Type III (concluded).

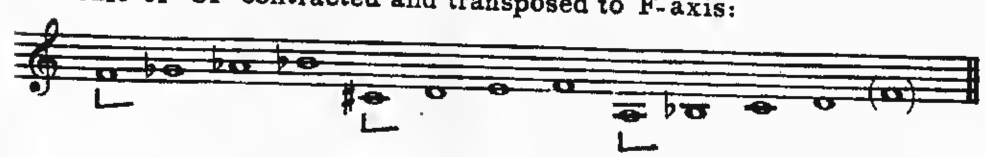
Scale of CF:



Scale of CP:



Scale of CP contracted and transposed to F-axis:



Type IV

Figure 84. Two-part counterpoint in scale of fourth group. Type IV (continued).

Figure 84. Two-part counterpoint in scale of fourth group. Type IV (concluded).

CHAPTER 7

CANONS AND CANONIC IMITATION

THE source of ***continuous imitation***, usually known as ***canonic***, is the well known phenomenon of acoustical resonance which bears the name of the Hellenic nymph, Echo. Before any composer existed on this planet, nature created, by chance, a quintuple echo—the "Lorelei" (which can be justly called a five-part canon) discovered on the Rhine river. The Russian Admiral Wrangel described a place in Siberia where the river Lena enters a canyon about 600 feet high and where a pistol shot rapidly repeats itself more than a hundred times. How would you like that for a canon?

But music theorists, as is typical of the species, think the canon is a purely esthetic development. Whatever they think, canon is actually a natural phenomenon and is the most ancient form of musical continuity.

The common belief is that it requires great skill to write a canon; but the real cause of whatever difficulty is encountered in writing in this form is simply methodological incompetence. Both the music theorists and the composers are guilty, for neither have been able to formulate the principles of continuous imitation. I shall not discuss the case of Sergei Ivanovich Taneiev, as his interpretation of canon requires knowledge of his work, *Convertible Counterpoint in Strict Style*—a highly complicated system which deals only with the strict style and which fails to bring us any solution to melodic and rhythmic forms; it is preoccupied with vertical and horizontal convertibility of intervals in the harmonic sense.

A ***canon*** is a complete composition written in the form of ***continuous imitation***.

The usual academic approach to this form is such that the student is taught first how to write an "ordinary" imitation (scientifically: ***discontinuous imitation***). After not getting anywhere with this form of imitation, the student next begins to struggle with the canon. Inasmuch as, from the very start, the principles of imitation have not been disclosed to him, it does not make any difference whether the imitation is discontinuous or continuous. But once such principles are defined and the technique is specified, it becomes obvious that *discontinuous* imitation is merely a ***special*** case of ***continuous imitation***.

With this in mind, let us now establish the actual principles of continuous imitation. ***Continuous imitation consists of one melody coexisting in two or more different parts in its different phases and at a velocity that remains constant in any given part.*** This melody, being of identical structure in both parts, may vary in intonation; such variance occurs only when the scale-structure itself varies.

The temporal organization of continuous imitation has no direct influence on the duration of a canon. Longer rhythmic groups are preferable, however, as continuous recurrence of the same rhythmic structure eventually becomes monotonous.

The main source of continuous self-stimulation in a canon is its *melodic form*, i.e., the axial group. With the devices offered in my theory of melody discussed earlier,* it is possible to evolve an axial group of great extension and, if necessary, *without* repetitions. In this way the continuance of the melodic flow may be completely insured.

The correlation of harmonic types and the treatment of harmonic intervals remain the same as for all other forms of contrapuntal technique. This permits us to compose canons in unitonal as well as in polytonal types.

A. Temporal Structure of Continuous Imitation

A complete composition based on continuous imitation is known as a *canon*.

The duration of continuous imitation—or of a canon—is some multiple of its *temporal structure*. The temporal structure of a two-part canon is related to the theme of the canon as $3 \div 1$. The first third of the whole is the announcement; the second third is the imitation of the announcement in the first voice and the counterpoint in the second voice; and the last third is the imitation of the first portion of counterpoint in the second voice and the second portion of counterpoint in the first voice. After the temporal scheme is exhausted, it begins to repeat itself with new intonations.

We shall designate the first entering voice as $\overrightarrow{P_I}$ (whether upper or lower), the second entering voice as $\overrightarrow{P_{II}}$, the first announcement as CP_1, the first portion of counterpoint as CP_2, and the second portion of counterpoint as CP_3, etc. The temporal structure of a canon then appears as follows:

$\dfrac{\overrightarrow{P_I}}{\overrightarrow{P_{II}}} = \dfrac{CP_1 + CP_2 + CP_3}{CP_1 + CP_2}$. The continuation of the temporal structure does not alter the process; it merely increases the subnumerals of CP in the original relation:

$$\frac{\overrightarrow{P_I}}{\overrightarrow{P_{II}}} = \frac{CP_1 + CP_2 + CP_3}{CP_1 + CP_2} + \frac{CP_4 + CP_5}{CP_3 + CP_4} + \frac{CP_6 + CP_7}{CP_5 + CP_6} + \dots$$

The temporal structure of any two-part canon is based on two elements, which appear as reciprocal permutations. All forms of variation of two elements are applicable therefore to two-part canons (see my earlier discussion of the *Theory of Rhythm*).** Let a and b be two elements each representing any kind of duration-group. Then,

$\dfrac{\overrightarrow{P_I}}{\overrightarrow{P_{II}}} = \dfrac{a+b+a}{a+b}$, and the continuation of the temporal structure assumes the following appearance:

$$\frac{\overrightarrow{P_I}}{\overrightarrow{P_{II}}} = \frac{a+b+a}{a+b} + \frac{b+a}{a+b} + \frac{b+a}{a+b} + \dots$$

*See Book IV. **See Book I.

The duration of some temporal structure is the factor really controlling the flow of the canon. The longer the structure (not as to speed, but as to the number of attacks), the greater the fluidity of the canon. Duration-groups of all kinds are acceptable as temporal structures for continuous imitation and for the canon.

1. Temporal structures composed from the parts of resultants.

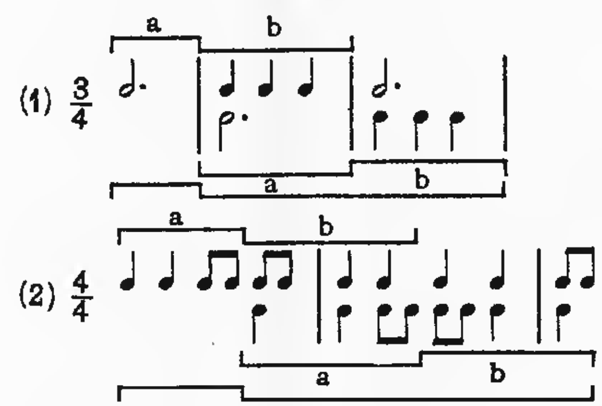


Figure 85. Temporal structures based on resultants.

2. Temporal structures composed from complete resultants.

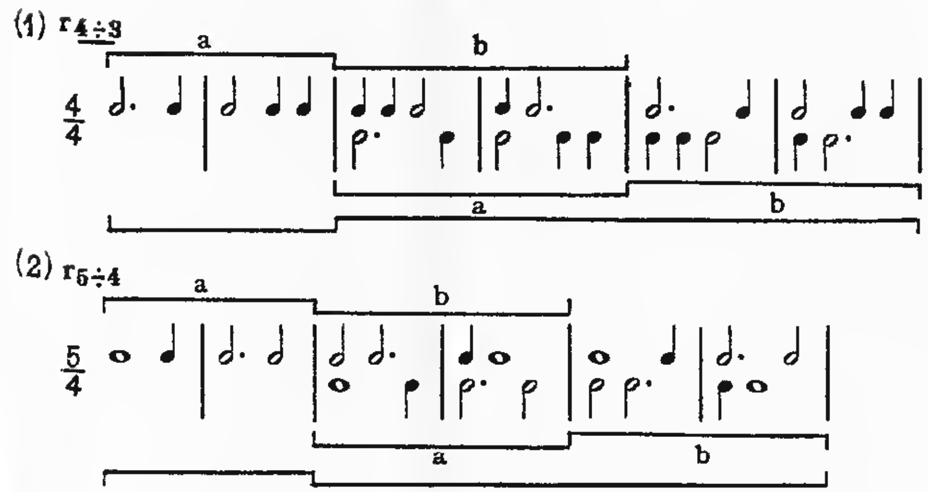


Figure 86. Temporal structures based on complete resultants (continued).

Figure 86. Temporal structures based on complete resultants (concluded).

3. Temporal structures evolved by means of permutations.

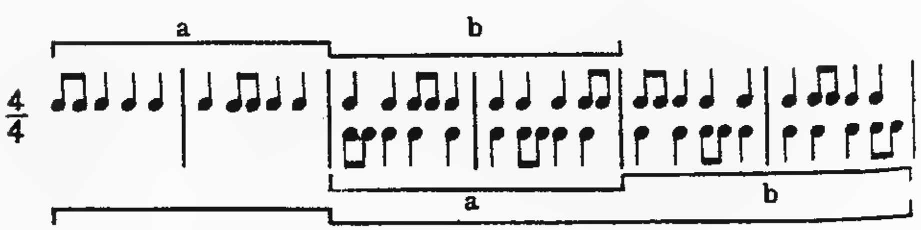


Figure 87. Temporal structures based on permutations.

4. Temporal structures composed from synchronized involution-groups.

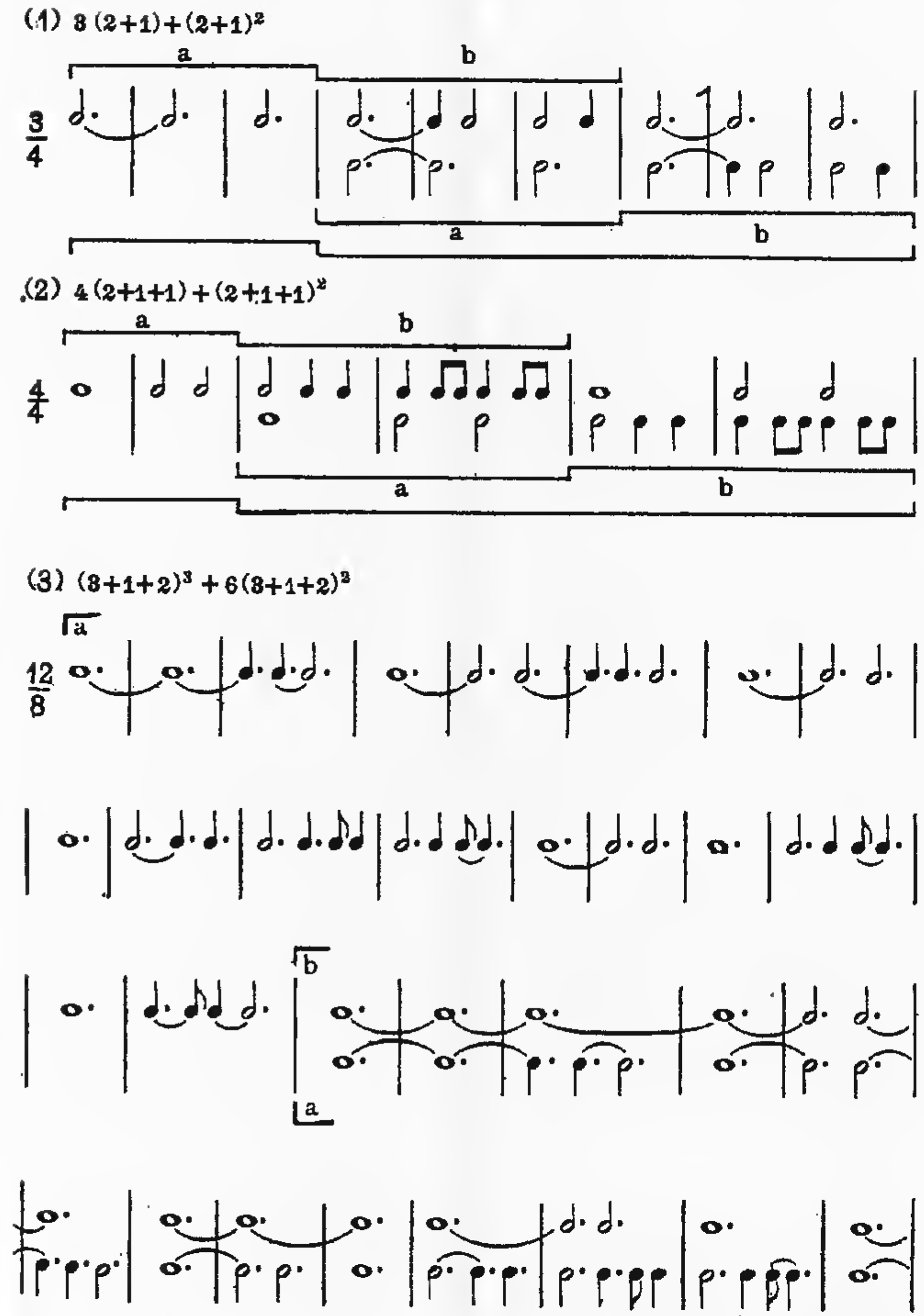


Figure 88. Temporal structures based on involution groups (continued).

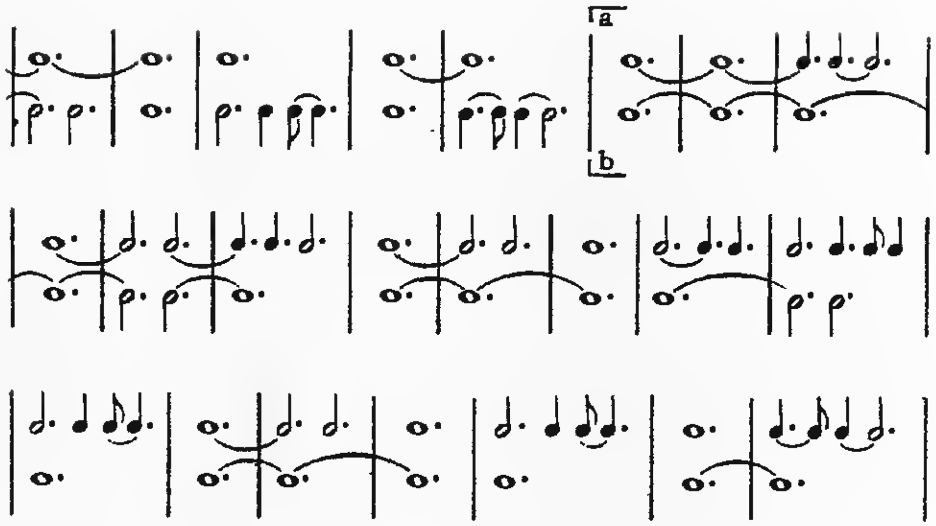


Figure 88. Temporal structures based on involution groups (concluded).

5. Temporal structures composed from acceleration-groups and their inversions.

Figure 89. Temporal structures based on acceleration-groups and their inversions.

B. Canons in All Four Types of Harmonic Correlation

As a canon is a duplication of melody at a certain time interval, differences of intonation in the two counterparts are due to scale-structures. Counterpoint of Type I (U.U.) produces identical intonations; type II (U.P.), non-identical intonations; type III (P.U.), identical intonations; and type IV (P.P.), non-identical intonations. The choice of axes for all four forms of correlation remains based on the original principle, consonance between the two axes of the two counterparts. In Types II and IV, the starting P.A. may be in a dissonant relation with the P.A. of the first voice, but it must end on a consonance.

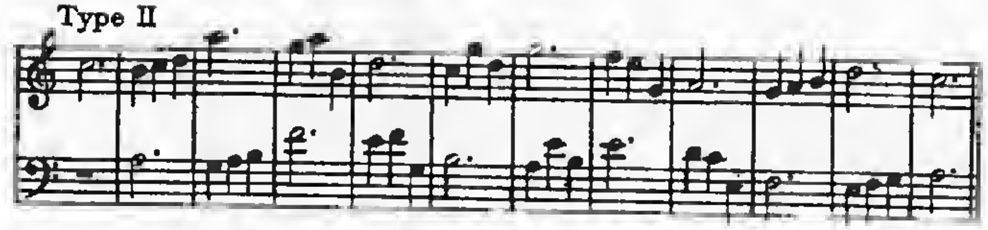
As continuous imitation can go on indefinitely, we need to know the exact technique of bringing such an imitation to a close. Cadences are produced by leading tones moving into the primary axis. Since in canon, what happens in the first moving voice defines what happens in the second voice, all that is necessary, if we wish to cadence, is to produce a leading tone in the first moving voice. When this portion of melody is transferred to the second voice, the first voice produces its own leading tone, after which both voices close on their respective primary axes.

Symmetric pitch-scales may be used in canons.

Examples of two-part canons in all four types of harmonic correlation

Type I

Type II



Type III

Figure 90. Two-part canon in four types (continued).

Type IV
A, d8

Figure 90. Two-part canon in four types (concluded).

Type II

Type III

Figure 91. Two-part canon in three types (continued).

Type IV

Figure 91. Two-part canon in three types (continued)

Figure 91. Two-part canon in three types (concluded).

Type I

Type I symmetric

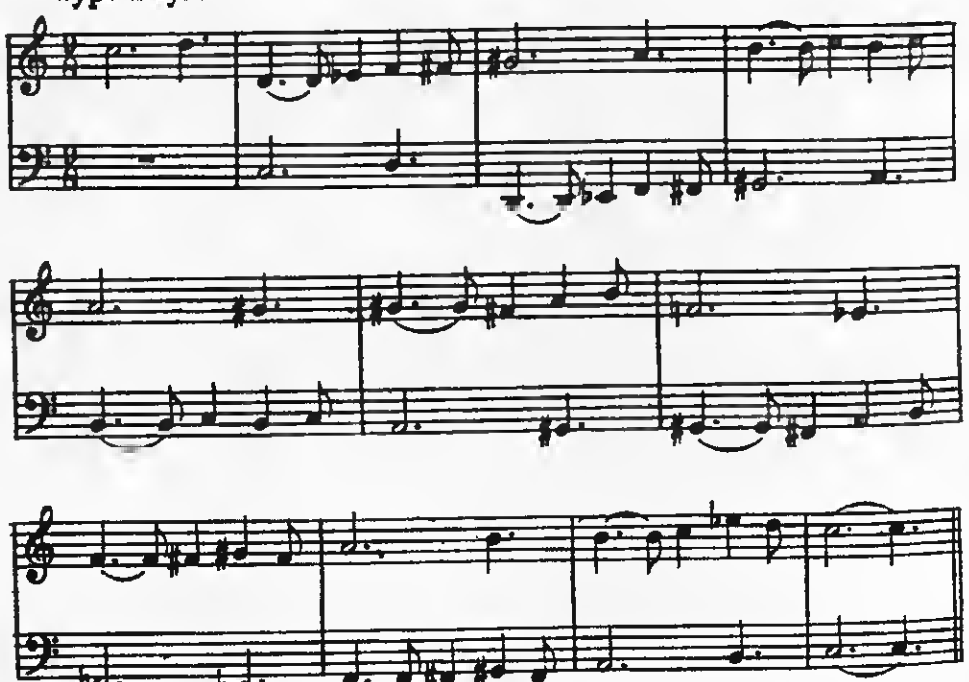


Figure 92. Two-part canon in Type I and Type I Symmetric.

C. Composition of Canonic Continuity by Means of Geometrical Inversions

The original version of a canon may be considerably extended by means of geometrical inversions.

The voice entering first produces an axis of inversion for the positions ⓒ and ⓓ. The final balance of the last cadence must not participate in the sequence of inversions, as this would disrupt the continuous flow of the canon. It must be used only at the very end of the composition if the canon ends in position ⓐ or ⓓ. Otherwise a new balance must be added.

Under such conditions, the canon consists of several contrasting and independent sections of continuous imitation.

Example of a canon developed through the use of geometrical inversions.

Figure 93. A canon developed by geometrical inversion (continued).

Figure 93. A canon developed by geometrical inversions (continued).

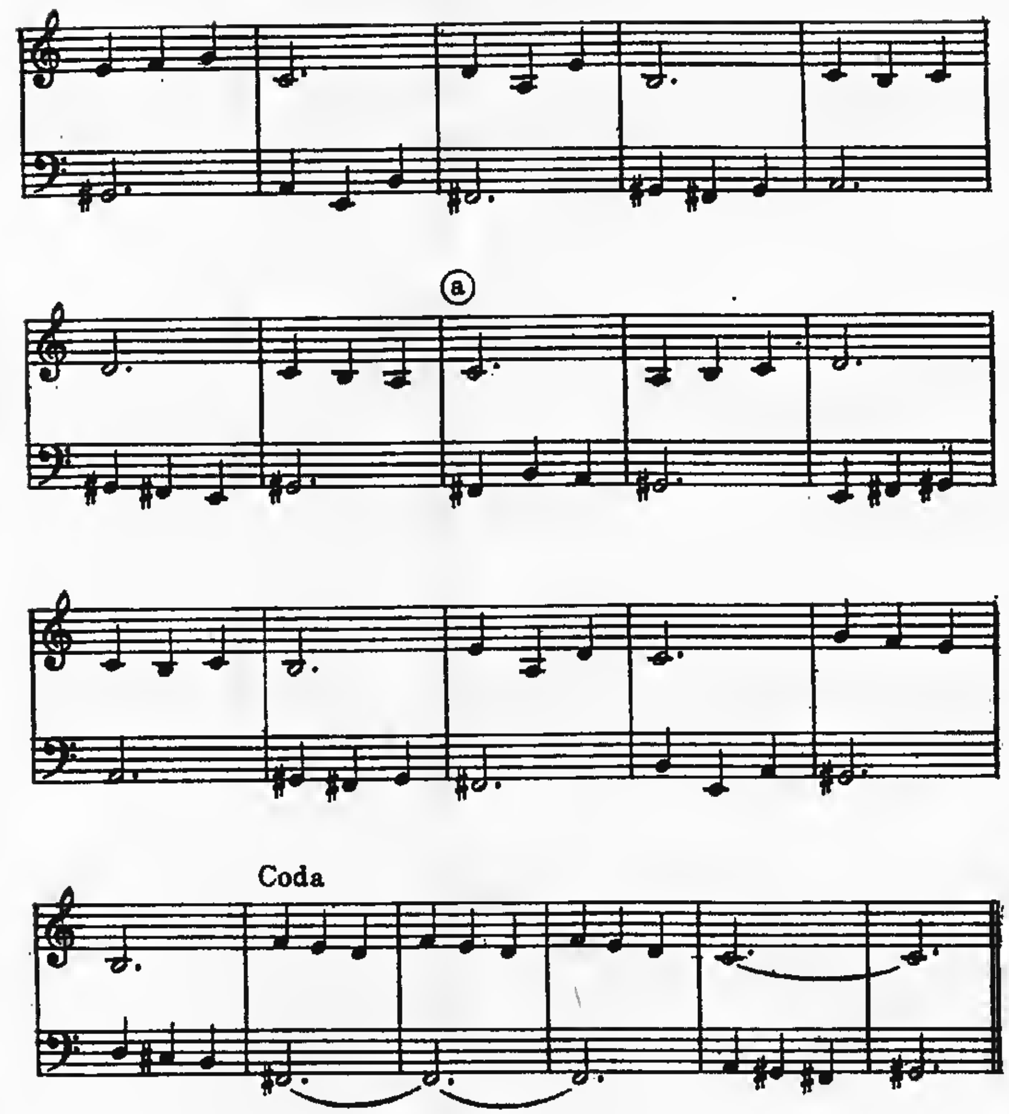


Figure 93. A canon developed by geometrical inversions (concluded).

Each geometrical inversion allows the use of two vertical permutations of the counterparts. Octave readjustment of the parts becomes a necessity under such conditions.

CHAPTER 8

THE ART OF THE FUGUE

IN the generalized sense a ***fugue*** may be defined as a complete composition based on ***discontinuous imitation.***

A ***fragmentary*** (incomplete) composition based on ***discontinuous imitation*** constitutes a ***fugato.*** A fugato usually appears as a polyphonic episode in an otherwise homphonic composition.

All other names established in the past—such as sinfonia, invention, praeludium (sometimes), fughetta—refer to the same fundamental form, the fugue. The difference lies mainly in the magnitude of the composition or in the type of harmonic correlation of the counterparts. A fugue which is unitonal-unimodal is called an invention, a praeludium, or a sinfonia—praeludium being the loosest term of all, as in many cases it has nothing in common with the fugue. A fugue (in this general sense) which is unitonal but polymodal (and of a *specified* polymodality) is called a fugue (in the specific sense).

It is my opinion that the presence or absence of polymodality or of polytonality is a matter of harmonic specifications which vary with time and place. Therefore, I feel that any complete composition based on discontinuous imitation may rightly be called a ***fugue.***

A. The Form of a Fugue

The temporal structure of a fugue depends on the number of themes ("subjects"). It is customary to call a fugue with one theme a "single fugue"; the fugue with two themes, a "double fugue". Triple fugues are very rare; indeed, a real triple fugue requires many parts (voices) if the idea that each part is a theme is not to become nonsensical.

For this reason it is expedient to confine fugues in two-part counterpoint to fugues with but one theme.

The general characteristic of all fugues is the appearance of the theme in all parts in sequence. This complete thematic cycle is known as an ***exposition.*** In two-part counterpoint, the first voice entering announces the theme (we shall call it CF, for the sake of uniformity in terminology), after which the second voice enters with the imitation—the imitation is usually called "reply" and might as well be called "echo". In fact, the imitation is the same theme, sometimes with differences caused by the form of harmonic correlation. The ***reply*** in types I and III is identical with the ***theme,*** whereas in types II and IV it is non-identical because the scale-structure is modified.

At the time the second voice entering makes its announcement (CF), the first entering voice evolves a counterpart (CP) to it. The form of ***the first exposition*** (E_1) is—

$$E_1 = \frac{P_I}{P_{II}} = \frac{CF+CP}{CF}$$

—and the form of ***any other exposition*** (E_n) is—

$$E_n = \frac{P_I}{P_{II}} = \frac{CF+CP}{CP+CF}.$$

In both cases the voice entering first (P_I) and the second voice entering (P_{II}) may be inverted.

In a fugue in which CF and CP represent the only thematic material and no interludes are used, the entire composition acquires the following form:

$$F = E_1 + E_2 + E_3 + \ldots + E_n.$$

In homophonic music this would correspond to a ***theme with variations.*** In the fugue the variation technique consists of geometrical inversions of the original exposition.

The counterpoint to the theme may be either ***constant*** (i.e., the CP is carried out through the entire fugue), or ***variable*** (i.e., a new CP is composed for each exposition). Statistically, the use of constant or variable CP is about 50 percent. In the 17th and 18th centuries a constant CP was something of a luxury, for counterpoint which we may now consider to be general technique was at that time known as "vertically convertible counterpoint," which was believed to be more difficult to execute. On the other hand, the older composers did not know the technique of geometrical inversions; they used tonal inversions instead and merely as a trick on some special occasions.

With the systematic use of geometrical inversions, the fugue becomes greatly diversified, with the result that constant CP becomes merely a practical convenience. Once the theme and the counterpoint are composed (which we will call the preparation of one exposition), one may develop the entire fugue by means of quadrant rotation arranged in any desirable sequence. If rotations refer to the entire E, the fugue assumes the following appearance:

$F = E_1 m + E_2 n + E_3 p + \ldots$, where m, n and p are any of the geometrical inversions.

For example:

(1) $F = E_{1a} + E_{2d} + E_{3c} + E_{4b} + E_{5a}$

(2) $F = E_{1a} + E_{2b} + E_{3a} + E_{4d} + E_{5a} + E_{6c} + E_{7a}$

(3) $F = E_{1a} + E_{2a} + E_{3c} + E_{4b} + E_{5b} + E_{6d}$

Such schemes are subject to variation according to the composers' inventiveness.

Quadrant rotation may affect each appearance of the theme; in that case, the theme and reply appear in different geometrical positions.

For example:

$$(1)\quad E = \frac{P_I}{P_{II}} = \frac{CF\textcircled{a} + CP}{CF\textcircled{d}}$$

$$(2)\quad E = \frac{P_I}{P_{II}} = \frac{CF\textcircled{b} + CP}{CP \quad + CF\textcircled{a}}$$

$$(3)\quad E = \frac{P_{II}}{P_I} = \frac{CF\textcircled{b}}{CF\textcircled{c} + CP}$$

It is important to note that the position is always identical for two simultaneous parts; $CF\textcircled{d}$ means that CP set against it is also in position $\textcircled{d}$.

Quadrant rotation applied to theme and reply produces altogether 16 geometrical forms of exposition.

B. Forms of Imitation Evolved Through Four Quadrants

$\frac{\textcircled{a}}{\textcircled{a}}$	$\frac{\textcircled{b}}{\textcircled{a}}$	$\frac{\textcircled{c}}{\textcircled{a}}$	$\frac{\textcircled{d}}{\textcircled{a}}$
$\frac{\textcircled{a}}{\textcircled{b}}$	$\frac{\textcircled{b}}{\textcircled{b}}$	$\frac{\textcircled{c}}{\textcircled{b}}$	$\frac{\textcircled{d}}{\textcircled{b}}$
$\frac{\textcircled{a}}{\textcircled{c}}$	$\frac{\textcircled{b}}{\textcircled{c}}$	$\frac{\textcircled{c}}{\textcircled{c}}$	$\frac{\textcircled{d}}{\textcircled{c}}$
$\frac{\textcircled{a}}{\textcircled{d}}$	$\frac{\textcircled{b}}{\textcircled{d}}$	$\frac{\textcircled{c}}{\textcircled{d}}$	$\frac{\textcircled{d}}{\textcircled{d}}$

Figure 94. Imitation evolved through four quadrants.

All those cases which involve one geometrical position for the entire E form the diagonal arrangement (heavily outlined) on the above table; they are special cases of the general rotary system.

It is easy to see that with this technique a fugue of any length may be composed with little effort.

An example of fugal scheme employing quadrant rotation:

$$F = \left(\frac{CF\textcircled{a} + CP\textcircled{a}}{CF\textcircled{a}}\right)E_1 + \left(\frac{CF\textcircled{d} + CP\textcircled{d}}{CP\textcircled{d} + CF\textcircled{d}}\right)E_2 +$$
$$+\left(\frac{CF\textcircled{a} + CP\textcircled{d}}{CP\textcircled{a} + CF\textcircled{d}}\right)E_3 + \left(\frac{CF\textcircled{d} + CP\textcircled{a}}{CP\textcircled{d} + CF\textcircled{a}}\right)E_4 +$$
$$+\left(\frac{CF\textcircled{c} + CP\textcircled{c}}{CP\textcircled{c} + CF\textcircled{c}}\right)E_5 + \left(\frac{CF\textcircled{a} + CP\textcircled{c}}{CP\textcircled{a} + CF\textcircled{c}}\right)E_6 +$$
$$+\left(\frac{CP\textcircled{b} + CF\textcircled{d}}{CF\textcircled{b} + CP\textcircled{d}}\right)E_7 + \left(\frac{CP\textcircled{b} + CF\textcircled{c}}{CF\textcircled{b} + CP\textcircled{c}}\right)E_8 +$$
$$+\left(\frac{CF\textcircled{d} + CP\textcircled{b}}{CP\textcircled{d} + CF\textcircled{b}}\right)E_9$$

Figure 95. Quadrant rotation in a fugal scheme.

As this example shows, the CF may appear in the same voice successively when its geometrical position *alters*.

The form of a fugue in which the counterpoint is varied with some, or with each, of the expositions may also be subjected to quadrant rotation.

The general scheme of such a fugue is:

$$F = \left(\frac{CF \;+ CP_1}{CF}\right)E_1 + \left(\frac{CF \;+ CP_1}{CP_1 + CF}\right)E_2 + \left(\frac{CF \;+ CP_2}{CP_2 + CF}\right)E_3 +$$
$$+\left(\frac{CF \;+ CP_3}{CP_3 + CF_3}\right)E_4 + \;. \;. \;.$$

An example with application of the quadrant rotation

$$F = \left(\frac{CF \;+ CP_1}{CF}\right)\textcircled{a}\; E_1 + \left(\frac{CF \;+ CP_2}{CP_1 + CF}\right)\textcircled{d}\; E_2 +$$
$$+\left(\frac{CF\;\textcircled{b} + CP_3\textcircled{a}}{CP_2\textcircled{b} + CF\;\textcircled{a}}\right)E_3 + \left(\frac{CF \;+ CP_2}{CP_1 + CF}\right)\textcircled{b}\; E_4 +$$
$$+\left(\frac{CF \;+ CP_3}{CP_2 + CF}\right)\textcircled{c}\; E_5 + \left(\frac{CP_2\textcircled{b} + CF\;\textcircled{c}}{CF\;\textcircled{b} + CP_1\textcircled{c}}\right)\dot{E}_6$$

Figure 96. Quadrant rotation.

In the old fugue form, the elimination of monotony was usually achieved by means of *interludes*. An interlude (we shall term it: I) is a contrapuntal sequence of either the imitation or the general type. Statistical analysis of actual fugues shows that about 50 of every 100 interludes are thematic (i.e., based on elements of CF or CP); the others are neutral, i.e., they use thematic elements of their own.

As in the case of counterpoint itself, an I may be composed once and rotated afterwards. In other cases, a new I may be composed each time. In the old classical fugues, the interludes served mainly as bridges between the E's, each I leading into a new key.

In our fugues of types I and II, the I's may serve the same purpose, but in types III and IV the interludes are hardly necessary, for key variety is already inherent in the group of different symmetric tonics. As we shall see later, the fact that we have two parts does not limit the number of tonics.

The general scheme of a fugue with interludes appears as follows:

$$F = E_1 + I_1 + E_2 + I_2 + E_3 + I_3 + \ldots + E_n + I_n.$$

This form is equivalent to the *first rondo* form of homophonic music.

$I_1, I_2, I_3, \ldots$ may be either identical (although in different geometrical positions) or totally different. I_n, i.e., the last interlude, is a rather common feature in the old fugues and has the function of a *conclusion* (coda). By rotating the same interlude, we acquire *new modulatory directions.*

C. Steps in the Composition of a Fugue

The method of composing a fugue by this system consists of the following stages:

(1) Composition of the theme;
(2) Composition of the counterpoint (one or more) to the theme; this is equivalent to the "preparation" of an exposition;
(3) Preparation of the exposition (or of all expositions if there is more than one counterpoint) in four geometrical positions:

$$\frac{CF}{CP}\ ⓐ\ ;\ \frac{CF}{CP}\ ⓑ\ ;\ \frac{CF}{CP}\ ⓒ\ ;\ \frac{CF}{CP}\ ⓓ\ ;$$

(4) Composition of the interlude(s), if there are to be any;
(5) Preparation of the four geometrical positions of the interlude(s), if any;
(6) Composition of the scheme of the fugue; and
(7) Assembly of the fugue.

D. Composition of the Theme of a Fugue

In a fugue the theme (or "subject") is of the utmost importance; it constitutes at least one half of the entire composition. It is therefore odd that no one has hitherto defined clearly the requirements for a fugal theme.

A good fugal theme is usually ascribed to the composer's "genius," and this neither helps nor consoles a student of the subject, for what we want to know, precisely, is: what makes the melody a suitable fugal theme? Experience shows that not every "good" or expressive melody makes a suitable fugal theme, and that not every suitable fugal theme is a good melody for any *other* purpose. Many composers who were outstanding melodists nevertheless failed to show any important achievements as contrapuntalists—e.g., Chopin, Schumann, Liszt, Chaikovsky, and others.

The first requirement of a fugal theme is that it be an *incomplete* melodic form. In the best and most typical fugues by J. S. Bach, we find that such *incomplete melodic forms used as themes are succeeded by their completions during the counterpoint* which evolves in the course of the announcement of the theme in the second voice.

An incomplete melodic form in this case means that *at the moment the second voice starts the theme, the first voice has not arrived at its own primary axis.*

For an illustration, take Fugue II, Vol. I, *The Well Tempered Clavichord* (later to be referred to by the abbreviation, "W.T.C.") by J. S. Bach.

Figure 97. Fugue II of W.T.C.

This particular theme ends on the first sixteenth note of the third measure, while the melodic form completes itself on the third quarter of the same bar. It is interesting to note that the theme (and the whole melodic form) is constructed on the binary axis: $\frac{0}{d}$.

In order to present his announcement clearly, Bach used $\frac{1}{t}$ (= 𝅘𝅥𝅯) at the very point where the theme might otherwise have stopped; he reserves the use of the eighth note until the reply is well on its way of development. In this way, Bach eliminates the danger of stopping—which, indeed, had it occurred, would have spoiled the entire fugue. Another important detail is the juxtaposition of the db-axis in CP versus the 0-axis in CF.

All the other requirements of a fugal theme actually derive from the first one: *all such resources of temporal rhythm and axial forms may be used as will demonstrate an unfinished melodic structure in the very process of formation.*

The presence of any one of the following structural characteristics, as well as of any combinations of the latter, will increase its suitability as a fugal theme.

(1) The presence of rests; particularly a decreasing series of rests, combined with an increasing number of attacks; "stop-and-go" effects; the effect of "gaining momentum."

(2) A sequence of decreasing duration-values: rhythmic acceleration in the broadest sense.

(3) "Dialogue" effects achieved by means of *binary* axes, and by means of attack-groups contrasting in form, such as a legato-staccato contrast.

(4) Effects of growth, achieved by means of binary and ternary *diverging* axes.

(5) The presence of resistance forms, including repetition, phasic and periodic rotation, particularly those forms that lead to climaxes.

Combinations of the above techniques applied to one theme make the latter more saturated and tense, which increases its fugal characteristics.

Fugal themes by J. S. Bach--and by "just J.S."

(Numbers in musical examples refer to the preceding classifications).

Figure 98. Fugal themes of W.T.C.

Figure 99. Fugal themes by J. S.

Figure 100. Fugal themes of W.T.C.

by J.S.

Figure 101. Fugal themes by J.S.

W. T. C. by J. S. Bach Vol. I, No. III

Figure 102. Fugal themes of W.T.C.

by J.S. also Vol. I, No. XI
also Vol. I, No. XXI
also Vol. II, No. XII

Figure 103. Fugal themes by J. S.

W. T. C. by J. S. Bach Vol. I, No. II

Figure 104. Fugal themes of W.T.C.

by J.S.

Figure 105. Fugal themes by J. S.

W.T.C. by J.S.Bach Vol. II, No. V

Figure 106. Fugal themes of W.T.C. (continued).

Vol. I, No. VI

Vol. I, No. XV

Vol. II, No. X

Figure 106. Fugal themes of W.T.C. (concluded).

by J.S.
(Symmetric: $\sqrt[3]{2}$)

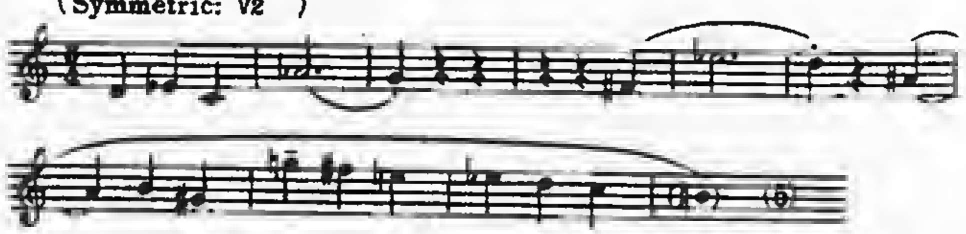


Figure 107. Fugal themes by J. S.

As indicated by the above examples, the total duration of a theme (in terms of the number of attacks, or in terms of measures) largely depends on the composer's own decision. However, the following generalization is true for most classical fugues: the duration of the fugal theme in inverse proportion to the number of parts. Indeed, the first theme of Fugue IV, Vol. I, W.T.C. has but five attacks; the theme in Fugue XXII, Vol. I, W.T.C. has six attacks. Both of these fugues are written in five parts. On the other hand, Fugue X of the same volume, written in two parts, has a theme of twenty-six attacks.

It is not important that the reply should enter on the same time-unit of the measure as the theme; on the contrary, a difference in the starting moments (in relation to the measure divisions) adds interest to the whole composition as it produces an element of surprise.

Themes which are unsuitable for fugues may be subjected to alterations which will make them suitable.

It can be demonstrated, by reversing the procedure, that the simple addition of a 0-axis to any melodic form will render it suitable as a fugal theme. J. S. Bach's theme from his *Toccata and Fugue in D-minor* for organ, if it is deprived of its 0-axis, loses all its fugal quality. When the 0-axis is taken out, the axial combination becomes b+a+c+a, and the theme seems to have nothing but rotation in relatively narrow range. But the inclusion of the 0-axis produces an effect of growing resistance, and the axial combination becomes:

$$\frac{0}{d+c+c}$$

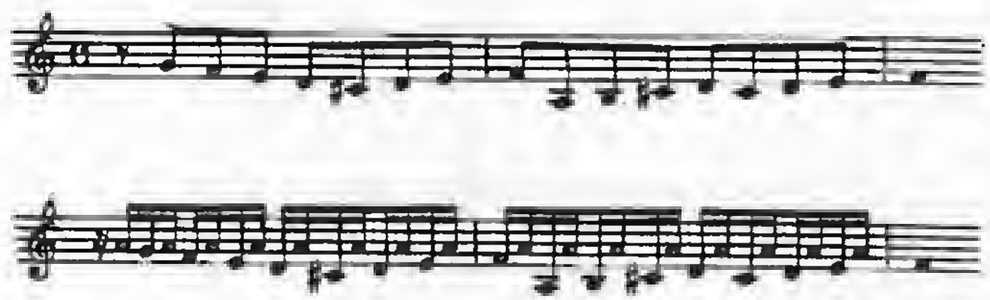


Figure 108. $\frac{0}{d+c+c}$

The number of measures in a fugal theme is optional; it may be even or odd; it may be integral or fractional. Both odd and fractional are preferable to even and integral, because the latter two suggest a cadence at the end of the theme. I believe that one of the factors that influenced Buxtehude and all the Bachs is their awareness of *cantus firmus* (in a strict sense) as a theme—*cantus firmi* usually had an odd number of attacks, as noted earlier.

E. Preparation of the Exposition

After selecting the theme, the composer must prepare the fugal exposition.

As it is easy, with this method, to write four types of fugues on one theme, so it becomes desirable to prepare four expositions for future fugues. In a two-part fugue, the entire preparation of E consists simply of writing a CP to the CF. It is advisable that the exposition prepared for each type should be written out in all four geometrical positions; this saves time during the process of assembling the fugue. Fugues of type IV often require preparation of two expositions, for when the axes exchange in $\frac{CP}{CF}$, CP may not fit, and a new counterpoint must be written (CP_{II}).

To make the demonstration of all techniques pertaining to fugue concise, I shall use a very brief theme.

CF (Theme)

Type I: $\frac{CF}{CP}$ (a)

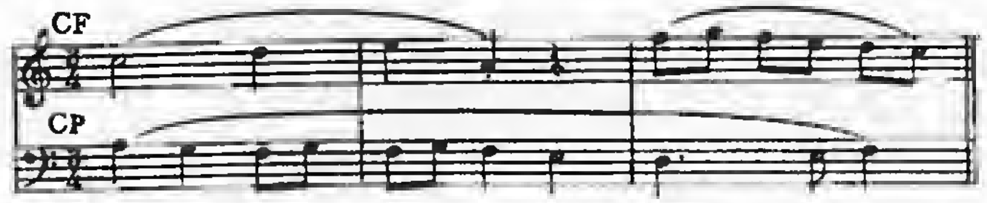
Type I: $\frac{CF}{CP}$ (b)

Type I: $\frac{CF}{CP}$ (d)

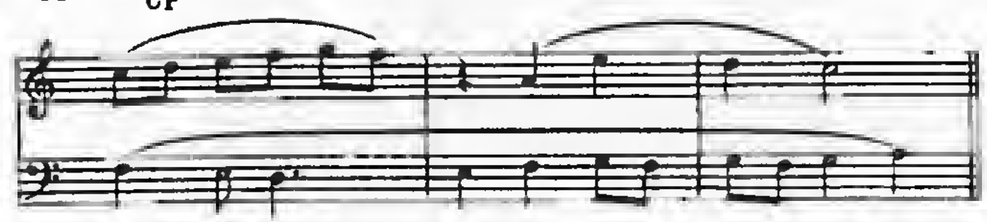
Figure 109. A brief theme and the various fugal techniques (continued).

Type I: $\frac{CF}{CP}$ (c)

Type II: $\frac{CF}{CP}$ (a)



Type II: $\frac{CF}{CP}$ (b)



Type II: $\frac{CF}{CP}$ (d)

Type II: $\frac{CF}{CP}$ (c)

Figure 109. Technique of the fugue (continued).

Type III: $\frac{CF}{CP}$ (a)

Type III: $\frac{CF}{CP}$ (b)

Type III: $\frac{CF}{CP}$ (d)

Type III: $\frac{CF}{CP}$ (c)

Type IV: $\frac{CF}{CP_I}$ (a)

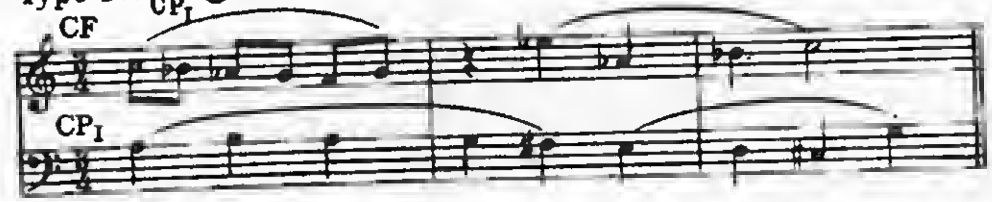
Type IV: $\frac{CP_{II}}{CF}$ (a)

Figure 109. Technique of the fugue (continued).

Type IV: $\frac{CF}{CP_I}$ (b)

Type IV: $\frac{CP_{II}}{CF}$ (b)

Type IV: $\frac{CF}{CP_I}$ (c)



Type IV: $\frac{CP_{II}}{CF}$ (c)

Type IV: $\frac{CF}{CP_I}$ (d)

Type IV: $\frac{CP_{II}}{CF}$ (d)

Figure 109. Technique of the fugue (concluded).

F. Composition of the Expositions

Composition of the expositions in type I involves no special considerations, for both parts have an identical P.A.

In type II, the modal modulations of CF and its respectively related CP must be in one system of modal sequence. For example, if P.A. of CF_1 is c and P.A. of CP_1 is e, the axis of CF_2 ("reply") must be a and CP_2 (counterpoint to "reply") must have P.A. on c in order to retain axial unity in the first part for the course of one exposition, and in order to preserve the vertical relation of $\frac{CP}{CF}$ as it was originally conceived.

The entire structure of the fugue (from the above relations) appears as follows:

$$F = \left[\frac{(CF_1 + CP_1)\,c}{CF_2\;\;a}\right]E_1 + \left[\frac{(CF_3 + CP_3)a}{CP_2c + CF_4\;f}\right]E_2 + \left[\frac{(CF_5 + CP_5)f}{CP_4a + CF_6\;d}\right]E_3 + \;.\;.\;.$$

where c, a, f, d, are the primary axes of the respective parts.

Likewise, if $\frac{CF_1}{CP_1} = \frac{c}{f}$, then the sequence of P.A.'s becomes: $\frac{c}{g} + \frac{g}{c+d} + \frac{d}{g+a} + \dots$

In type III, the tonal (key) modulations of CF and of its respectively related CP, must be in one system of symmetric sequence. This sequence preserves its constant $\frac{CP}{CF}$ relation only when CP_2 (the reply) forms its P.A. in symmetric inversion to the original setting.

Let us take the symmetry of $\sqrt[4]{2}$; for example: $\frac{CP}{CF} = \frac{e\flat}{c}$. In order to preserve the axial relation where CP is 3 semitones above CF, the reply must appear from the opposite equidistant point, i.e., from a. This permits a relative stability of both parts, as CP_1—being three semitones above CF—requires the c-axis.

The structure of such a fugue, evolved on four points of symmetry (tonics), appears as follows:

$$F = \left[\frac{(CF_1 + CP_1)\,c}{CF_2\;\;a}\right]E_1 + \left[\frac{(CF_3 + CP_3)a}{CP_2c + CF_4\,f\sharp}\right]E_2 + \left[\frac{(CF_5 + CP_5)\,f\sharp}{CP_4a + CF_6\;\;e\flat}\right]E_3 +$$

$$+\left[\frac{(CF_7 \;\; + CP_7)\,e\flat}{CP_6 f\sharp + CF_8\;\;c}\right]$$

A similar case evolved from three points of symmetry ($\sqrt[3]{2}$), where $\frac{CP}{CF} = \frac{e}{c}$, gives the following sequence of P.A.'s:

$$\frac{c}{a} + \frac{a\flat}{c+e} + \frac{e}{a\flat + c}$$

In type IV, in order to carry out the sequence of P.A.'s in symmetric inversion of the original setting, it often becomes necessary to prepare two independent expositions—

$$E = \frac{CP_I}{CF} \text{ and } E^1 = \frac{CP_{II}}{CF^1}$$

—as CP may be in an intervallic relation to CF_2 different from the relation to CF_1. The difference usually appears in variations of a semitone or whole tone, which results in the most disturbing relations—such as a second instead of a third. For this reason, the example in Figure 109 offers two expositions.

It is easy to see that CP_I is unfit to be a counterpoint to the reply, by exchanging it with P.A. or CF.

The sequence of symmetric P.A.'s in type IV of Fig. 109 would develop on the basis of its pre-set expositions:

$$E = \frac{CP_I}{CF} = \frac{e\sharp}{c} \text{ and } E^1 = \frac{CP_{II}}{CF^1} = \frac{c}{e\sharp}.$$

Considering the enharmonic equality of e♯ and f, a♯ and b♭, etc., and the fact that CF is evolved in natural major d_0 and CF^1 in natural major d_6, we obtain the following structure for the fugue:

$$F = \left[\frac{(CF + CP_{II})c}{CF^1\;\;e\sharp}\right]E_1 + \left[\frac{(CF + CP_{II})\,f}{(CP_I + CF^1)\,a\sharp}\right]E_2 + \left[\frac{(CF + CP_{II})b\flat}{(CP_I + CF^1)\;d\sharp}\right]E_3 + \;.\;.\;.$$

In the old classical fugues the reply appears on the dominant (i.e., seven semitones above or five semitones below the theme). If there was a sequence of expositions before the interlude took place, the theme would usually return to the tonic. According to our type II, if $CF_1 = c$ and $CF_2 = g$, CF_3 should have been d, CF_4 should have been a, etc. However, this was not the case in the fugues of the classical period, and there was a good reason for it: the tuning of *mean* temperament (the two-coordinate system: $\frac{3}{2}$ and $\frac{5}{4}$) developed an aberration in pitches deviating from the tuning center (= 1), and so it was not possible to get satisfactory intonation in the course of a sequence traveling through C_5 or C_{-5} P.A.'s. Although *equal* temperament has since overcome this defect, the habit remained with composers until the end of the 19th century.

G. Preparation of the Interludes

Interludes (I_1, I_2, . . . I_m) serve as bridges between the expositions. The last interlude, if the fugue ends with one, would be called a *postlude* or *coda*.

Interludes serve one or both of two purposes:

(1) to divert the listener's attention away from the persistence of theme;

(2) to produce a modulatory transition from one key-axis to another.

Interludes of the first form are confined to one key but may have any number of successive P.A.'s, thus producing modal modulations (U.-P.) between the two adjacent expositions having the same key-axis (U.-U. and U.-P.). The second form contains different key-axes (P.-U. and P.-P.) and connects the two adjacent expositions having different key-axes (P.-U. and P.-P.).

Both forms of interludes may be either *neutral* or *thematic*. *Neutral* interludes are based on material of rhythm, or intonation, or both, which does not appear in any of the exposition. *Thematic* interludes borrow their material of rhythm, or intonation, or both, from either the CF or the CP of the exposition. Any of the above described types of interludes may be executed either in *general* or in *imitative* counterpoint.

The duration of an interlude depends on the duration of the exposition and the number of interludes. The form of an interlude itself has an influence upon its duration. In order to construct a perfect fugue, the duration of interludes must be put into some definite relationship with the duration of expositions. Assuming that one exposition is the temporal unit (T), we arrive at the following fundamental schemes for the temporal organization of interludes:

(1) T (E) = T (I), i.e., the duration of an interlude equals that of an exposition. This presupposes an equal duration for each of the interludes.

(2) T (E) > T (I), i.e., the duration of an exposition is longer than that of an interlude. An exact ratio must be established in each case.

(3) T (E) < T (I), i.e., the duration of an interlude is longer than that of an exposition. An exact ratio must be established in each case.

(4) $I^{\rightarrow} = I_1T + I_2 2T + I_3 3T + \ldots$, i.e., each successive interlude becomes longer. The durations of consecutive interludes may evolve in any desirable type of progression (natural, arithmetic, geometric, involution, summation, etc.). The resulting effect of such fugue-structures is that the interludes in the course of time begin to dominate the theme. Thus, the persistence of the theme diminishes.

(5) $I^{\rightarrow} = I_1 nT + I_2(n-1)T + I_3(n-2)T + \ldots$, i.e., each successive interlude becomes shorter. The resulting effect is opposite to that of (4); the domination of theme over interludes grows in the course of time.

(6) $I^{\rightarrow}$, i.e., the sequence of interludes develops according to some form of rhythmic grouping.

As convertibility and quadrant rotation are general properties, the same interlude may be used several times during the course of a fugue. This, in combination with key-transpositions, offers an enormous variety of resources—and at the same time conserves the composer's energy.

H. Non-Modulating Interludes

(Types I and II)

Non-modulating interludes may be either neutral or thematic and they can be evolved in either general or imitative counterpoint.

(1) An example of Interlude type II executed in general counterpoint. Non-thematic (Neutral).

(2) An example of Interlude type II executed in imitative counterpoint. This one is thematic with reference to CF of Fig. 109.

See the corresponding musical illustrations on the opposite page.

Figure 110. Interlude type II.

I. Modulating Interludes

1. Modulating counterpoint evolved through harmonic technique.

Contrary to the general notion, J. S. Bach's counterpoint is less "contrapuntal" than it is generally believed to be. This is especially true of his tonal (key-to-key) modulations. It is obvious that Bach, as well as many other important contrapuntalists, thought of key-to-key transitions in terms of *modulating chords;* see, for example, J. S. Bach's *W.T.C.*, Vol. I, fugue No. X (a two-part fugue) in E-minor—the harmonic background of this fugue is very distinct, and this fugue is typical rather than exceptional.

It is easy to convert any modulating chord-progression written in four-part harmony into two-part harmony.

The chord structures in two-part harmony have the following functions:

(1) S(3) = 1, 3; used instead of the S(5) of three-part structure;
(2) S(5) = 1, 5; used instead of the S(5) of three-part structure;
(3) S(7) = 1, 7; used instead of the S(7) of four-part structure.*

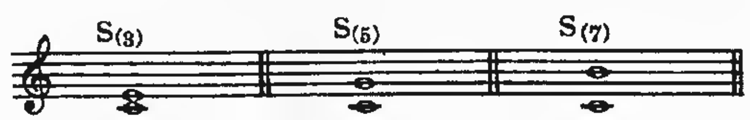


Figure 111. Chord structures in two-part harmony.

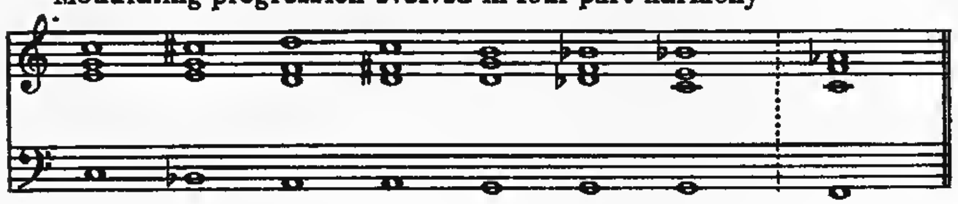
*Or a two-part incomplete S(7) = 3, 7 may be used instead of the S(7) in four-part harmony—as in the fourth chord in the example of translation that follows. (Ed.)

In order to obtain an interlude from a four-part chord-progression, it is necessary to select those corresponding chordal functions which will *translate* the four-part structures into two-part structures. The voice-leading pertaining to two-part harmony will not be discussed here, as all positions of the two functions are equally as acceptable for the present purpose. Both parts are more or less in the vicinity of the four-part harmony range. The final step consists of developing melodic figuration in both parts, with somewhat contrasting rhythms of durations and attacks.

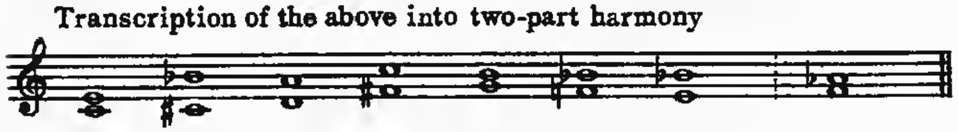
Modulating interludes may be either neutral (general counterpoint) or thematic (imitative counterpoint). In the latter case, thematic material is either borrowed from the CF or the CP of the expositions—or it is entirely independent.

(1) Neutral and (2) Thematic.

Modulating progression evolved in four-part harmony



Transcription of the above into two-part harmony



Interlude (1)

Figure 112. Modulating interludes (continued).

Interlude (2)

Figure 112. Modulating interludes (concluded).

An interlude may be used in the same fugue more than once, appearing in the different geometrical positions. It may also be transposed to any desirable key-axis in any of the four quadrants.

2. Modulating counterpoint evolved through melodic technique.

I offer this new technique in order to enable the composer to compose in pure contrapuntal style even when a key-to-key transition is desirable.

Modulating counterpoint consists of two independently modulating melodies (see my earlier discussion of *modulation* in the book dealing with the *Theory of Pitch-Scales*)* whose primary axes are in a constant, simultaneous relationship at any given key-point of the sequence. After vertical dependence has been established (the harmonic interval between CP and CF), it becomes necessary *to assign to the primary axis of CP the meaning of that tonic which is nearest to CF through the scale of key-signatures.*

Let the exposition end in the key of C, and let CF end on *c* and CP end on *a*. Then *a* becomes a-minor (as the key nearest to the key of C through the scale of key signatures; A- major would be far more remote). Thus, we have established a constant dependence where CP is the minor key three semitones below CF.

The next step consists of planning the modulation of P_I (originally: CF) Let the modulation be to the key of f- minor.

Then:

$$\overrightarrow{P_I} = C + d + G + f$$

Now we assume that in order to retain the original vertical dependence between P_I and P_{II}, each axis of a major key must be reciprocated by a minor

*See Book II.

key, and vice versa. Then:

$\frac{\overrightarrow{P_I}}{\overrightarrow{P_{II}}} = \frac{C+d+G+f}{a+F+e+A\flat}$, i.e., while P_I modulates from C to d, P_{II} modulates from a to F, and when P_I modulates from d to G, P_{II} modulates from F to e; finally both parts arrive at CF having an A♭-axis and CP having an f-axis.

The period of modulation from key to key in both parts is approximately the same.

(1) Neutral and (2) Thematic

Figure 113. Modulating interludes.

The easiest way to compose modulating interludes by contrapuntal technique is through a sequence of procedures:

(1) P_I modulates to the first intermediate key;
(2) P_{II} " " " " " "
(3) P_I " " " second " "
(4) P_{II} " " " " " "

and so on, until the entire modulation is completed.

J. Assembly of the Fugue

The process of assembling a fugue consists of planning the general sequence of expositions, interludes, their geometrical positions and their primary axes (key-axes).

In the following group of fugues only such materials were used as had been prepared in advance (see Fig. 109, 110, 112, and 113).

The first three fugues have interludes (of both harmonic and melodic type), while the fourth has none, as the key-variety is sufficiently great without interludes. The last fugue has independent counterpoints for the theme and the reply. The latter are interchanged in E_5.

The form of Fugue I (Fig. 114)

$$E_1 a + I_1 + E_2 a + E_3 d + I_2 c + E_4 d c + I_3 b + E_5 b$$

The form of Fugue II (Fig. 115):

$$\overset{C}{\overline{E_1 a + E_2}}\; a + I_1 + \overset{F}{\overline{E_3 a}} + \overset{C}{\overline{E_4 d + E_5 d c}} + I_2 + E_6 a b$$

The form of Fugue III (Fig. 116)

$$\overset{C}{\overline{(E_1}} + E_2)\; a + (E_3 + E_4)\; d + I_1 + \overset{A\flat}{\overline{(E_5 + E_6)}}\; a + E_7 \overset{A\flat E\flat C}{\overline{b\, a\, a}}$$

The form of Fugue IV (Fig. 117):

$$(E_1 + E_2 + E_3 + E_4 + E_5)\; a + E_6 c + E_7 d + E_8 b$$

(1) Fugue Type I

Figure 114. Fugue of type I (continued).

Figure 114. Fugue of type I (continued).

Figure 114. Fugue of type I (concluded).

(2) Fugue Type II

Figure 115. Fugue of type II (continued).

Figure 115. Fugue of type II (continued).

Figure 115. Fugue of type II (concluded).

(3) Fugue Type III

Figure 116. Fugue of type III (continued).

Figure 116. Fugue of type III (continued).

Figure 116. Fugue of type III (continued).

Figure 116. Fugue of type III (concluded).

(4) **Fugue Type IV**

Figure 117. Fugue of type IV (continued).

Figure 117. Fugue of type IV (continued).

Figure 117. Fugue of type IV (concluded).

(Editor's note. The original manuscript included material on the generalization of 2-part counterpoint into 3 or more part counterpoint. This is omitted because it was largely in outline form. Various students who studied privately with Schillinger have presented their notes to the editors, and have made it possible to complete Schillinger's outlines. This material will be published at a later date.)

CHAPTER 9

TWO-PART CONTRAPUNTAL MELODIZATION OF A GIVEN HARMONIC CONTINUUM

WE are now to concern ourselves with the technique of writing two correlated melodies (two-part counterpoint) to a given chnrd-progression. The counterpoint itself must satisfy all the requirements applying to harmonic intervals. Each of the melodic parts (to be designated as M_I and M_{II}, or as CP_I and CP_{II}) must also satisfy the requirements pertaining to melodization of harmony.

The sequence in which such a two-part melodization should be accomplished is:

(1) the writing of $H^{\rightarrow}$;
(2) the writing of the M with a fewer number of attacks per H;
(3) the writing of the M with the greater number of attacks per H.

It does not matter which of the two melodies is selected to be M_I and which is to be M_{II}.

In view of the fact that the natural physical scale of frequencies increases in the upward direction of musical pitch, we shall first produce that melody which has the fewer number of attacks in a position immediately above harmony, and the melody with the greater number of attacks we shall develop *above* the first melody. Such an arrangement will be considered to be fundamental; it may later be rearranged.

We arrive at the two possible settings:

$$(1)\ \frac{\dfrac{M_I}{M_{II}}}{H^{\rightarrow}} \quad \text{and} \quad (2)\ \frac{\dfrac{M_{II}}{M_I}}{H^{\rightarrow}}$$

Octave-convertibility (exchange of the positions of M_I and M_{II}) is possible only when the harmonic intervals of both melodic parts are chosen with an eye to such convertibility—and this is mainly a matter of supporting certain higher functions (such as 11) by the immediately preceding function (such as 9).

All forms of quadrant rotation (ⓐ, ⓑ, ⓒ and ⓓ) are acceptable on the condition that M_I and M_{II} always remain *above* the chord progression, $H^{\rightarrow}$.

Just as melodization of harmony by means of *one* part produced different types of melody in relation to the different types of harmonic progressions, the same possibilities still exist for *two*-part melodization.

It is to be remembered that some types of melody in one-part melodization were the outcome of new techniques. For instance, the technique of a modulating symmetric melody above all forms of symmetric harmony, or the technique of a diat ic melody evolved from a quantitative scale above all forms of chromatic harr -both are forms not known in music prior to my development of the pri r these procedures. All such new techniques may be applied now tc elodization. This, naturally, will result in new types of counter-

The distribution of attacks of M_I, M_{II} and $H^{\rightarrow}$ is a matter of considerable complexity and will be discussed more fully later. For the present, we shall distribute the attacks for all three parts (M_I, M_{II} and $H^{\rightarrow}$) uniformly and by means of multiples.

Some elementary forms of the distribution of attacks.

M_I	a	2a	a	3a	a	4a	a	4a	2a	6a	2a	8a	2a	6a	3a	8a	4a
M_{II}	a	a	2a	a	3a	a	4a	2a	4a	2a	6a	2a	8a	3a	6a	4a	8a
$H^{\rightarrow}$	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H

M_I	9a	3a	12a	3a	12a	4a	15a	3a	16a	4a
M_{II}	3a	9a	3a	12a	4a	12a	3a	15a	4a	16a
$H^{\rightarrow}$	H	H	H	H	H	H	H	H	H	H

Figure 118. Distribution of attacks.

Here the quantities of attacks in $\frac{M_I}{M_{II}}$ are designated as the attacks per chord.

Each original setting of two simultaneous melodies accompanied by a chord-progression offers seven forms of exposition:

$$(1)\ M_I;\ (2)\ M_{II};\ (3)\ H^{\rightarrow};\ (4)\ \frac{M_I}{M_{II}};\ (5)\ \frac{M_I}{H^{\rightarrow}};\ (6)\ \frac{M_{II}}{H^{\rightarrow}};\ (7)\ \begin{matrix}M_I\\ \overline{M_{II}}\\ \overline{H^{\rightarrow}}\end{matrix}$$

A. Melodization of Diatonic Harmony by Means of Two-Part Diatonic Counterpoint (*Type I and II*)

The melody with the lower number of attacks and which appears immediately above the harmony must conform to the principles of diatonic melodization. It is desirable not to use higher functions (9, 11) in this melody (we shall call it M_{II}), for the latter should be reserved for use in the melody with the larger number of attacks (we shall call it M_I), so that the higher functions of M_I may be supported by M_{II}. Scales of both melodies must have a common source of derivation; this common source is the diatonic scale of the harmony.* Any derivative scales of the original d_0 may be employed.

The harmony itself may be devised in four or in five parts; four-part harmony is preferable, for the texture of a duet accompanied by five parts is somewhat heavy.

None of the melodies should produce consecutive octaves with any of the harmonic parts.

*That is, the diatonic scale that results from converting the Σ (E_I) of the given chord into its zero expansion (E_0). For example, the chord (reading upwards in thirds) C – E – G – B♭ – D – F♯ – A (Σ XIII) becomes, when contracted to E_0, the third displacement or mode (d_3) of the natural G minor scale, C – D – E – F♯ – G – A – B♭. (Ed.)

M_I should be written as a counterpoint to M_{II} and as a melodization of the given chord-progression.

Identical as well as non-identical scales (which derive through permutation of the pitch-units of d_0) may be used in M_I, M_{II} and $H^{\rightarrow}$. Under such conditions, any d_0 produces 35 possibilities of modal relations between the above-mentioned three components.

As we are employing seven-unit scales, and as—

$${}_7C_3 = \frac{7!}{3!\,(7-3)\,!} = \frac{5040}{6\cdot 24} = \frac{5040}{144} = 35$$

—the number of possible two-part melodizations to one chord-progression (written in one definite d) is:

$${}_7C_2 = \frac{7!}{2!\,(7-2)\,!} = \frac{5040}{2\cdot 120} = \frac{5040}{240} = 21$$

(1)

(2)

Figure 119. Diatonic two-part melodization. (continued).

(3)

(4)

(5)

Figure 119. Diatonic two-part melodization (continued).

(6)

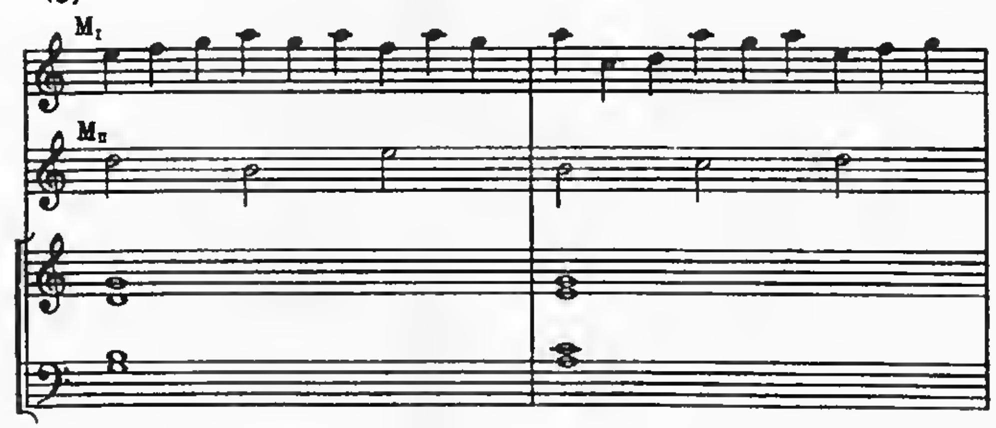


Figure 119. Diatonic two-part melodization (concluded).

B. Chromatization of Diatonic Two-Part Melodization

In order to produce a greater contrast between M_I and M_{II} either one can be subjected to chromatic variation. If desirable, both melodies can be used in their chromatic version.

Chromatic variation is achieved by means of passing or auxiliary chromatic tones.

By means of combining the two variations of Fig. 120, we obtain a new version in which chromatic sections alternate with the diatonic ones.

Theme: Figure 119 (3)

Var. I

Var. II

Figure 120. Chromatic variations. Var. I, II and III (continued).

Var. III

Figure 120. Chromatic variations. Var. I, II and III (concluded).

C. Melodization of Symmetric Harmony

(Type II, III and Generalized) by means of Two-Part Symmetric Counterpoint

Symmetric melodization is based on the pitch-scale which is a contraction of the particular Σ 13 that corresponds to each individual H. Theoretically, each chord requires a new scale. The quality of the melody, however, depends on the number of tones there are in common among the successive Σ 13's upon which the $\overrightarrow{S}$'s are based; this is true of both M_I and M_{II} of two-part melodization.

The requirements for two-part symmetric melodization may be stated as follows:

(1) Adherence of one M to a particular set of pitch-units, producing a scale.

(2) Graduality of melodic modulation, which is executed by means of common tones, chromatic alterations and identical motifs.

(3) Strict adherence to contrapuntal treatment of the harmonic intervals between M_I and M_{II}.

Figure 121. Symmetric two-part melodization (continued).

Figure 121. Symmetric two-part melodization (concluded).

D. Chromatization of a Symmetric Two-Part Melodization

This technique is identical with chromatization of diatonic counterpoint. The passing and auxiliary chromatic tones are not part of the Σ 13. Either of the two contrapuntal parts may be chromatized. Alternation of chromatic and symmetric sections in both melodies is fully satisfactory.

Theme: Figure 121 (2)

Var. I

Var. II

Figure 122. Chromatization of a symmetric two-part melodization (continued).

Var. III

Figure 122. Chromatization of a symmetric two-part melodization (concluded).

E. Melodization of Chromatic Harmony by Means of Two-Part Counterpoint

As we know, one-part melodization of chromatic harmony is possible by two distinctly different procedures:

(1) that based on directional units, and
(2) that based on quantitative scale.

Chromatic melodization in two parts is therefore possible in the following combinations of the above techniques:

M_I	di	ch	di	ch
M_{II}	di	di	ch	ch
$H^{\rightarrow}$	ch	ch	ch	ch

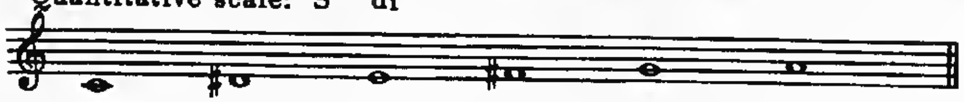
where *di* (diatonic) represents the quantitative scale; *ch* of M represents the directional-units method, and *ch* of $H^{\rightarrow}$ stands for chromatic harmonic continuity.

If a contrast is to be achieved between M_I and M_{II}, one of them becomes di; the other, ch.

If a similarity is preferable (contrast may still be achieved by juxtaposition of the quantities of attacks of $\frac{M_I}{M_{II}}$), both melodies are either di or ch. The first has a diatonic character (due to adherence to one particular pitch-scale) and the second has a modulating character which abounds in semitonal directional units.

Progression: H → ch

Quantitative scale: S → di



(1)

(2)

Figure 123. Melodization of chromatic harmony (continued).

(3)

(4)

Figure 123. Melodization of chromatic harmony (concluded).

CHAPTER 10

ATTACK-GROUPS FOR TWO-PART MELODIZATION

THE number of attacks as among $\begin{array}{c}M_I\\ \hline M_{II}\\ \hline H\end{array}$ may be either constant or variable.

We say it is a ***constant*** form of the attack-group when each individual H has a definite corresponding number of attacks in M_I and M_{II}, which number remains the same for every consecutive H.

$$\begin{array}{c}M_I\\ \hline M_{II}\\ \hline H\end{array} = A \text{ const.}$$

Constant-A does not necessarily mean an even distribution in $\frac{a\,(M_I)}{a\,(M_{II})}$. An even distribution may be considered as merely a special case of this relationship.

Examples of an even distribution of A:

M_I	4a	6a	6a	8a	8a	9a	12a	12a
M_{II}	2a	2a	3a	2a	4a	3a	3a	4a
H	a	a	a	a	a	a	a	a

Examples of uneven distribution of A:

M_I	2a+3a	4a+2a	4a+2a	4a+6a
M_{II}	a+a	a+a	2a+a	2a+2a
H	a	a	a	a

M_I	4a+2a+3a+6a	6a+3a+6a+4a+2a+9a
M_{II}	2a+a+a+2a	3a+a+2a+2a+a+3a
H	a	a

We have a variable form of the attack-group when A emphasizes a group of chords, and when each consecutive H has a specified number of attacks for a definite number of chords.

For example: $A^{\rightarrow} = A_1 + A_2 + A_3$

Let $A_1 = \begin{array}{c}M_I\\ \hline M_{II}\\ \hline H\end{array} = \begin{array}{c}2a+a\\ \hline a+a\\ \hline a\end{array}$ and let $A_2 = \begin{array}{c}M_I\\ \hline M_{II}\\ \hline H\end{array} = \begin{array}{c}4a+3a\\ \hline 2a+a\\ \hline a\end{array}$

and let $A_3 = \begin{array}{c}M_I\\ \hline M_{II}\\ \hline H\end{array} = \begin{array}{c}4a+6a+3a\\ \hline 2a+2a+a\\ \hline a\end{array}$ then:

$$\begin{array}{c}M_I\\ \hline M_{II}\\ \hline H^{\rightarrow}\end{array} = \left(\begin{array}{c}2a+a\\ \hline a+a\\ \hline a\end{array}\right) H_1 + \left(\begin{array}{c}4a+3a\\ \hline 2a+a\\ \hline a\end{array}\right) H_2 + \left(\begin{array}{c}4a+6a+3a\\ \hline 2a+2a+a\\ \hline a\end{array}\right) H_3$$

All other considerations concerning the distribution and the number of attacks are identical with those I have discussed as part of one-part melodization.*

Example of Correlated Attack-Groups
in Two-Part Melodization

$$\begin{array}{c}M_I\\ \hline M_{II}\\ \hline H^{\rightarrow}\end{array} = \left(\begin{array}{c}2a+3a\\ \hline a+a\\ \hline a\end{array}\right) H_1 + \left(\begin{array}{c}3a+4a\\ \hline a+a\\ \hline a\end{array}\right) H_2 + \left(\begin{array}{c}4a+3a+2a\\ \hline a+a+a\\ \hline a\end{array}\right) H_3$$

$H^{\rightarrow} = 6\sqrt[6]{2}$, S(9) const.; $\Sigma 13$ XIII; $S = \frac{3p}{p}$; transformation: ↻

$T'' = 12t$ in $\frac{3}{4}$ time.

Figure 124. Correlated attack-groups in two-part melodization (continued).

*See Book V.

Figure 124. Correlated attack-groups in two-part melodization (concluded).

A. Composition of Durations

Durations and duration-groups which will satisfy the attack-groups composed for two-part melodization may either be selected from the various series of the *evolution-of-rhythm* families (in which case there is no interference between the attacks of the attack-group and the attacks of the duration-group. They may also be based on a *direct composition of duration-groups* (which may, or may not, produce an interference between the attacks of the attack-group and the attack of the duration-group) superimposed upon the attack-groups.

When the respective attack-groups are represented by durations selected from style-series, and the number of individual attacks in the attack sub-groups does *not* correspond to the number of attacks in the duration-groups, it is necessary to split the respective duration-units. This consideration concerns only the first technique, that is, the matching of attack-groups by a series of durations.

The musical example of Figure 125 is a translation of its corresponding attack-group into $\frac{3}{8}$ series, where three types of split-unit groups were used: $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. One exception to the series was made at the cadence, where a musical quarter was split into $\frac{4}{4}$ series binomial, i.e., 3+1.

The numerical representation of this example of melodization appears as follows:

$$\begin{array}{l} M_I = \\ M_{II} = \\ H^{\rightarrow} \end{array} \left(\begin{array}{c} 1/2t+1/2t+1/2t+1/2t+t \\ \hline t \quad + \quad 2t \\ \hline 3t \end{array} \right) H_1 \begin{array}{c} + \\ + \end{array}$$

$$\begin{array}{c} + \\ + \end{array} \left(\begin{array}{c} 1/3t+1/3t+1/3t+1/2t+1/2t+1/2t+1/2t \\ \hline t \quad + \quad 2t \\ \hline 3t \end{array} \right) H_2 \begin{array}{c} + \\ + \end{array}$$

$$\begin{array}{c} + \\ + \end{array} \left(\begin{array}{c} 1/4t+1/4t+1/4t+1/4t+1/3t+1/3t+1/3t+1/2t+1/2t \\ \hline t \quad + \quad t \quad + \quad t \\ \hline 3t \end{array} \right) H_3$$

Figure 125. Numerical representation of Figure 124.

The abundance of split units and split-unit groups in this instance is due to the abundance of attacks over each H and to a relatively low value of the series. With a series of higher value, the splitting of units would be greatly reduced.

We shall next translate the same example into $\frac{9}{8}$ series:

$$\begin{array}{l} M_I = \\ M_{II} = \\ H^{\rightarrow} \end{array} \left(\begin{array}{c} t+3t+t+3t+t \\ \hline 4t \quad +5t \\ \hline 9t \end{array} \right) H_1 \begin{array}{c} + \\ + \end{array} \left(\begin{array}{c} t+2t+t+t+2t+t+t \\ \hline 4t \quad +5t \\ \hline 9t \end{array} \right) H_2 \begin{array}{c} + \\ + \end{array}$$

$$\begin{array}{c} + \\ + \end{array} \left(\begin{array}{c} t+t+t+t+t+t+t+t+t \\ \hline 4t \quad +3t \quad +2t \\ \hline 9t \end{array} \right) H_3$$

Figure 126. The attack-pattern of figure 125 translated into $\frac{9}{8}$ series.

We shall next take a case in which the attack-groups and the duration-groups are composed independently. Let $r_{5\div4}$ represent the number of attacks of M_I to each attack of M_{II}, and let every 2 attacks of M_{II} correspond to one attack of $H^{\rightarrow}$. The distribution of attacks for all three parts will be as follows:

$$\begin{array}{l} a(M_I) \\ a(M_{II}) \\ a(H^{\rightarrow}) \end{array} \begin{array}{c} = \\ = \end{array} \begin{pmatrix} 4a+a \\ a+a \\ a \end{pmatrix} H_1 \begin{array}{c} + \\ + \end{array} \begin{pmatrix} 3a+2a \\ a+\ a \\ a \end{pmatrix} H_2 \begin{array}{c} + \\ + \end{array} \begin{pmatrix} 2a+3a \\ a+\ a \\ a \end{pmatrix} H_3 \begin{array}{c} + \\ + \end{array} \begin{pmatrix} a+4a \\ a+\ a \\ a \end{pmatrix} H_4$$

Superimpose the following duration-group:

$$T = r_{4\div3} = 16t;\ 10a$$

Then: $\dfrac{a(A)}{a(T)} = \dfrac{20}{10} = \dfrac{2}{1};\ \begin{array}{l} 1(20) \\ 2(10) \end{array}$

Hence, $T' = 16t \cdot 2 = 32t$

Let $T'' = 8t$, then: $N_{T''} = \frac{32}{8} = 4$

Each $a(M_I)$ corresponds to an individual term of T; each $a(M_{II})$ corresponds to the sum of the respective durations of M_I; each $a(H^{\rightarrow})$ corresponds to the sum of 2 durations of M_{II}.

The final temporal scheme of this two-part melodization takes the following form:

$$\begin{array}{l} M_I \\ M_{II} \\ H^{\rightarrow} \end{array} \begin{array}{c} = \\ = \end{array} \begin{pmatrix} 3t+t+2t+t+t \\ 7t\qquad +t \\ 8t \end{pmatrix} H_1 \begin{array}{c} + \\ + \end{array} \begin{pmatrix} t+t+2t+t+3t \\ 4t\qquad +4t \\ 8t \end{pmatrix} H_2 \begin{array}{c} + \\ + \end{array}$$

$$\begin{array}{c} + \\ + \end{array} \begin{pmatrix} 3t+t+2t+t+t \\ 4t\quad +4t \\ 8t \end{pmatrix} H_3 \begin{array}{c} + \\ + \end{array} \begin{pmatrix} t+t+2t+t+3t \\ t+7t \\ 8t \end{pmatrix} H_4$$

Figure 127. Two-part melodization. Attack-groups and duration-groups composed independently (continued).

Figure 127. Two part melodization (concluded).

B. Direct Composition of Durations

Direct composition of durations becomes particularly valuable when one desires a *proportionate distribution of durations for a constant number of attacks* among the component parts (M_I, M_{II} and $H^{\rightarrow}$). Distributive involution of three synchronized powers solves this problem.

It follows from my theory of rhythm* that the cube of a binomial produces an eight-term polynomial, the square of a binomial produces a quadrinomial and the first-power group remains a binomial. Thus, the ratio of attacks in any pair of adjacent part $\frac{M_I}{M_{II}}$ and $\frac{M_{II}}{H^{\rightarrow}}$ is two. Cubing of a trinomial gives a twenty-seven-term polynomial, the synchronized square producing nine terms, and the first-power group producing three terms. The ratio of attacks between pairs of adjacent parts remains three. The number of terms of the original polynomial thus equals the number of attacks between each pair of adjacent parts.

We shall devise now a correlated proportionate system of duration-groups. The distributive cube will serve as T for M_I, the synchronized distributive square as T for M_{II} and the synchronized first-power group as T for $H^{\rightarrow}$.

We shall operate from the trinomial of the $\frac{4}{3}$ series. This yields the following attack-group correlation:

$$\begin{array}{l} a(M_I) \\ a(M_{II}) \\ a(H^{\rightarrow}) \end{array} \begin{array}{c} = \\ = \end{array} \begin{array}{c} 9a \\ 3a \\ a \end{array}$$

The entire temporal scheme assumes the form shown on the following page:

*See Book I.

$$\begin{array}{c} T(M_I) \\ \hline T(M_{II}) \\ \hline T(H^{\rightarrow}) \end{array} = \begin{array}{c} [(8t+4t+4t)+(4t+2t+2t)+(4t+2t+2t)] \\ \hline (16t \qquad + 8t \qquad + 8t \qquad) \\ \hline 32tH_1 \end{array} + $$

$$+ \begin{array}{c} [(4t+2t+2t)+(2t+t+t)+(2t+t+t)] \\ \hline (8t \qquad + 4t \qquad + 4t \qquad) \\ \hline 16tH_2 \end{array} + $$

$$+ \begin{array}{c} [(4t+2t+2t)+(2t+t+t)+(2t+t+t)] \\ \hline (8t \qquad + 4t \qquad + 4t \qquad) \\ \hline 16tH_3 \end{array}$$

In addition to this technique, ***coefficients of duration*** may be used for correlation of durations in two-part melodization.

Example:

$$\begin{array}{c} M_I \\ \hline M_{II} \\ \hline H^{\rightarrow} \end{array} = \begin{array}{c} (3t+t+2t+2t)+(3t+t+2t+2t)+(3t+t+2t+2t)+(3t+t+2t+2t) \\ \hline (6t+2t+4t+4t)+(6t+2t+4t+4t) \\ \hline 12tH_1+4tH_2+8tH_3+8tH_4 \end{array}$$

Figure 128. Direct composition of durations through distributive involution of three synchronized powers (continued).

Figure 128. Direct composition of durations (concluded).

C. Composition of Continuity

The seven forms of exposition previously classified may be now incorporated into a continuity of two-part melodization. The meaning of these seven forms as applied to composition may be expressed as follows:

(1) M_I — Solo melody: theme A;

(2) M_{II} — Solo melody: theme B;

(3) $H^{\rightarrow}$ — Solo harmony: theme C;

(4) $\dfrac{M_I}{H^{\rightarrow}}$ — Solo melody with harmonic accompaniment (theme A accompanied);

(5) $\dfrac{M_{II}}{H^{\rightarrow}}$ — Solo melody with harmonic accompaniment (theme B accompanied);

(6) $\dfrac{M_I}{M_{II}}$ — Duet of two melodies $\left(\dfrac{\text{Theme A}}{\text{Theme B}}\right)$

(7) $\begin{array}{c} M_I \\ \hline M_{II} \\ \hline H^{\rightarrow} \end{array}$ — Duet of two melodies with harmonic accompaniment $\left(\begin{array}{c}\text{Theme A} \\ \hline \text{Theme B} \\ \hline \text{Theme C}\end{array}\right)$

The above seven forms serve as thematic elements of a composition in which they appear in an organized sequence producing a complete musical whole.

Themes A, B and C must be considered as component parts of the whole in which they express their particular characteristics. The particular characteristics which distinguish A from B and from C are:

(1) High mobility of A (maximum quantity of attacks);
(2) Medium mobility of B (medium quantity of attacks);
(3) Low mobility of C (minimum quantity of attacks)—combined with maximum density (four or five parts).

The planning of the continuity must be based on a definite pattern for variation of the density, combined with variation in the quantity of attacks.

The scale of density may be arranged from low to high density, as follows:

$$(1)\quad A,\ \frac{A}{B},\ C,\ \frac{A}{C},\ \begin{matrix}A\\ \hline B\\ \hline C\end{matrix}\ ;\ \text{or as}$$

$$(2)\quad B,\ \frac{A}{B},\ C,\ \frac{B}{C},\ \begin{matrix}A\\ \hline B\\ \hline C\end{matrix}\ .$$

The relatively extreme points of any such scale produce contrasts; for instance:

$$(1)\ \begin{matrix}A\\ \hline B\\ \hline C\end{matrix} + B + \begin{matrix}A\\ \hline B\\ \hline C\end{matrix} + A + \begin{matrix}A\\ \hline B\\ \hline C\end{matrix} + A + C + B + C + \begin{matrix}A\\ \hline B\\ \hline C\end{matrix}\ ;$$

$$(2)\ A + C + B + C + A + \begin{matrix}A\\ \hline B\\ \hline C\end{matrix} + B + \begin{matrix}A\\ \hline B\\ \hline C\end{matrix} + A + B$$

Durations corresponding to one individual attack of the component of lowest mobility (mostly $H^{\rightarrow}$) become time-units of the continuity. Such units (we shall call them T) may be arranged in any form of rhythmic distribution.

Correlation of the thematic duration-groups (T's, with their coefficients) with the different forms of density constitutes the composition.

Assuming that there are three forms of density and three forms of mobility, we obtain the following ***combined thematic forms*** (low, medium, high): $3^2 = 9$.

Density	Low	Low	Medium	Medium	Low	High
Mobility	Low	Medium	Low	Medium	High	Low

Medium	High	High
High	Medium	High

For instance: $\dfrac{\text{Density}}{\text{Mobility}} = \dfrac{\text{Low}}{\text{Low}} \equiv M_{II}\ ;\ \dfrac{\text{Density}}{\text{Mobility}} = \dfrac{\text{High}}{\text{Low}} \equiv H^{\rightarrow}\ ;$

$$\frac{\text{Density}}{\text{Mobility}} = \frac{\text{High}}{\text{Medium}} \equiv \frac{M_{II}}{H^{\rightarrow}},\ \text{etc.}$$

Let us now devise a composition in which gradual and sudden variations of both mobility and density will be combined.

It is desirable to use a scheme of two-part melodization which will be *cyclic* and *recapitulating*, i.e., one permitting a correct transition from the end to the beginning for all three components.

For the present, we shall not resort to any additional techniques (such as inversions, expansions, etc.); the complete synthesis of all these and other procedures will be accomplished in my later discussion of *composition** as such.

Let Figure 127 serve as the fundamental scheme for two-part melodization, as this material is both cyclic and recapitulating.

Let us adopt the following scheme of density and mobility:

$$\frac{\text{Density}}{\text{Mobility}} = \frac{\text{Low}}{\text{Low}} + \frac{\text{Low}}{\text{High}} + \frac{\text{Medium}}{\text{High}} + \frac{\text{High}}{\text{Medium}} + \frac{\text{High}}{\text{Low}} + \frac{\text{Medium}}{\text{High}} + \frac{\text{High}}{\text{High}}$$

A sequence of thematic elements and their combinations, corresponding to the seven *forms of expositions* and satisfying the above scheme of *thematic forms*, may be selected as follows:

$$E^{\rightarrow} = M_{II}E_1 + M_IE_2 + \frac{M_I}{H^{\rightarrow}}E_3 + \frac{M_{II}}{H^{\rightarrow}}E_4 + H^{\rightarrow}E_5 + \frac{M_I}{M_{II}}E_6 + \left(\begin{matrix}M_I\\ \hline M_{II}\\ \hline H^{\rightarrow}\end{matrix}\right)E_7.$$

Let us make T correspond to H, and establish the following sequence for the T's: $T = r_{5\div 3}$.

$$T^{\rightarrow} = T_1 3H + T_2 2H + T_3 H + T_4 3H + T_5 H + T_6 2H + T_7 3H$$
$$T^{\rightarrow} = 7T\ 15H.$$

The 7T of $T^{\rightarrow}$ produce no interference in relation to the 7E of $E^{\rightarrow}$. But there is an interference between $T^{\rightarrow}E^{\rightarrow}$ and $H^{\rightarrow}$, however, for $H^{\rightarrow} = 8H$.

$$\frac{T^{\rightarrow}E^{\rightarrow}}{H^{\rightarrow}} = \frac{7}{8};\quad \begin{matrix}8(7)\\ 7(8)\end{matrix}\qquad E^{\rightarrow\prime} = 7\cdot 8 = 56\ TE.$$

As 7 TE corresponds to 15 H, there will be 7 TE·8 = 56 TE and 15 H·8 = 120 H.

The complete composition after synchronization evolves into the following form:

$$T^{\rightarrow\prime}E^{\rightarrow\prime} = 56\ TE\ 120\ H;\ T'' = H;\ N_{T''} = 120.$$

As, in Figure 127, $T'' = TH$, the entire composition consumes 120 measures —which is 15 times the duration of the original scheme of melodization.**

*See Book XI.

**Observe that the original source material (MS), whereas the same composition in score plus the formula requires about 2½ pages requires 8 pages. (Ed.)

Here is the final layout of the composition:

$$T^{\rightarrow}E^{\rightarrow} = [M_{II}\ (H_1 + H_2 + H_3)\ T_1E_1 + M_I\ (H_4 + H_5)\ T_2E_2 + \frac{M_I}{H^{\rightarrow}}(H_6)T_3E_3 +$$

$$+ \frac{M_{II}}{H^{\rightarrow}}\ (H_7 + H_8 + H_1)\ T_4E_4 + H^{\rightarrow}(H_2)\ T_5E_5 + \frac{M_I}{M_{II}}(H_3 + H_4)\ T_6E_6 +$$

$$+ \frac{\frac{M_I}{M_{II}}}{H^{\rightarrow}}\ (H_5 + H_6 + H_7)\ T_7E_7] + [M_{II}\ (H_8 + H_1 + H_2)T_8E_8 +$$

$$+ M_I\ (H_3 + H_4)\ T_9E_9 + \frac{M_I}{H^{\rightarrow}}(H_5)\ T_{10}E_{10} + \frac{M_{II}}{H^{\rightarrow}}\ (H_6 + H_7 + H_8)$$

$$T_{11}E_{11} + H^{\rightarrow}\ (H_1)\ T_{12}E_{12} + \frac{M_I}{M_{II}}(H_2 + H_3)\ T_{13}E_{13} +$$

$$+ \frac{\frac{M_I}{M_{II}}}{H^{\rightarrow}}\ (H_4 + H_5 + H_6)\ T_{14}E_{14}] + [M_{II}\ (H_7 + H_8 + H_1)\ T_{15}E_{15} +$$

$$+ M_I\ (H_2 + H_3)\ T_{16}E_{16} + \frac{M_I}{H^{\rightarrow}}\ (H_4)\ T_{17}E_{17} +$$

$$+ \frac{M_{II}}{H^{\rightarrow}}\ (H_5 + H_6 + H_7)\ T_{18}E_{18} + H^{\rightarrow}(H_8)\ T_{19}E_{19} + \frac{M_I}{M_{II}}(H_1 + H_2)T_{20}E_{20} +$$

$$+ \frac{\frac{M_I}{M_{II}}}{H^{\rightarrow}}\ (H_3 + H_4 + H_5)\ T_{21}E_{21}] + [M_{II}\ (H_6 + H_7 + H_8)\ T_{22}E_{22} +$$

$$+ M_I\ (H_1 + H_2)\ T_{23}E_{23} + \frac{M_I}{H^{\rightarrow}}\ (H_3)\ T_{24}E_{24} +$$

$$+ \frac{M_{II}}{H^{\rightarrow}}\ (H_4 + H_5 + H_6)\ T_{25}E_{25} + H^{\rightarrow}\ (H_7)\ T_{26}E_{26} +$$

$$+ \frac{M_I}{M_{II}}\ (H_8 + H_1)\ T_{27}E_{27} + \frac{\frac{M_I}{M_{II}}}{H^{\rightarrow}}\ (H_2 + H_3 + H_4)\ T_{28}E_{28}] +$$

$$+ [M_{II}\ (H_5 + H_6 + H_7)\ T_{29}E_{29} + M_I\ (H_8 + H_1)\ T_{30}E_{30} +$$

$$+ \frac{M_I}{H^{\rightarrow}}\ (H_2)\ T_{31}E_{31} + \frac{M_{II}}{H^{\rightarrow}}(H_3 + H_4 + H_5)\ T_{32}E_{32} + H^{\rightarrow}\ (H_6)\ T_{33}E_{33} +$$

$$+ \frac{M_I}{M_{II}}\ (H_7 + H_8)\ T_{34}E_{34} + \frac{\frac{M_I}{M_{II}}}{H^{\rightarrow}}\ (H_1 + H_2 + H_3)\ T_{35}E_{35}] +$$

$$+ [M_{II}\ (H_4 + H_5 + H_6)\ T_{36}E_{36} + M_I\ (H_7 + H_8)\ T_{37}E_{37} +$$

(*Continued on opposite page*).

$$+ \frac{M_I}{H^{\rightarrow}}\ (H_1)\ T_{38}E_{38} + \frac{M_{II}}{H^{\rightarrow}}\ (H_2 + H_3 + H_4)\ T_{39}E_{39} + H^{\rightarrow}\ (H_5)\ T_{40}E_{40} +$$

$$+ \frac{M_I}{M_{II}}\ (H_6 + H_7)\ T_{41}E_{41} + \frac{\frac{M_I}{M_{II}}}{H^{\rightarrow}}\ (H_8 + H_1 + H_2)\ T_{42}E_{42}] +$$

$$+ [M_{II}\ (H_3 + H_4 + H_5)\ T_{43}E_{43} + M_I\ (H_6 + H_7)\ T_{44}E_{44} +$$

$$+ \frac{M_I}{H^{\rightarrow}}\ (H_8)\ T_{45}E_{45} + \frac{M_{II}}{H^{\rightarrow}}(H_1 + H_2 + H_3)\ T_{46}E_{46} +$$

$$+ H^{\rightarrow}(H_4)\ T_{47}E_{47} + \frac{M_I}{M_{II}}\ (H_5 + H_6)\ T_{48}E_{48} + \frac{\frac{M_I}{M_{II}}}{H^{\rightarrow}}\ (H_7 + H_8 + H_1)\ T_{49}E_{49}] +$$

$$+ [M_{II}\ (H_2 + H_3 + H_4)\ T_{50}E_{50} + M_I\ (H_5 + H_6)\ T_{51}E_{51} +$$

$$+ \frac{M_I}{H^{\rightarrow}}\ (H_7)\ T_{52}E_{52} + \frac{M_{II}}{H^{\rightarrow}}(H_8 + H_1 + H_2)\ T_{53}E_{53} + H^{\rightarrow}(H_3)\ T_{54}E_{54} +$$

$$+ \frac{M_I}{M_{II}}\ (H_4 + H_5)\ T_{55}E_{55} + \frac{\frac{M_I}{M_{II}}}{H^{\rightarrow}}\ (H_6 + H_7 + H_8)\ T_{56}E_{56}].$$

Figure 129. Numerical layout of a complete two-part melodization.

Below you will find the complete composition based on musical representation of the numerical layout just given:

Figure 130. Musical representation of figure 129 (continued).

Figure 130. Musical representation of figure 129 (continued).

Figure 130. Musical representation og figure 129 (continued).

Figure 130. Musical representation of figure 129 (*continued*).

Figure 130. Musical representation of figure 129 (*continued*).

Figure 130. Musi al representation of figure 129 (continued).

Figure 130. Musical representation of figure 129 (continued).

Figure 130. Musical representation of figure 129 (*continued*).

Figure 130. Musical representation of figure 129 (*concluded*).

CHAPTER 11

HARMONIZATION OF TWO-PART COUNTERPOINT

THE main procedure in writing a harmonic accompaniment to the duet of two contrapuntal parts consists of ***assigning harmonic consonances to be chordal functions.***

Each combination of two pitch-units producing a simultaneous consonance becomes a pair of chordal functions—this premise concerns all types of counterpoint and all types of harmonization.

Those pitch-units which produce dissonances are perceived by us, through auditory association, as auxiliary and passing tones. When what we might call "justification" of the consonance as a pair of chordal functions takes place, the harmonic accompaniment acquires a proper meaning.

A. Diatonic Harmonization

Under the conditions imposed by Special Harmony, the kind of two-part counterpoint which can be harmonized by Special Harmony must be constructed from seven-unit scales of the first group, not containing identical intonations.

All three components—M_1, M_2 and H—must belong to one key. According to the definition of diatonic, the only types of counterpoint which can be diatonically harmonized are types I and II.

It is important for the composer to realize the versatility of relations which may exist among the modes of the three components. M_I may be written in any of the seven modes (d_0, d_1, d_2, d_3, d_4, d_5, d_6) of one scale; so may M_{II}, and so may the $\overrightarrow{H}$. The total number of these modal variations for one scale is: $7^3 = 343$. This, of course, includes all the identical as well as all the non-identical combinations; practically, however, this quantity must be somewhat diminished, if we want to preserve a consonant relation between the P.A.'s of M_I and M_{II}.

The number of seven-unit scales not containing identical units is 36; therefore, the total manifold of relations of $M_I : M_{II} : \overrightarrow{H}$ in diatonic counterpoint of types I and II is:

$$343 \cdot 36 = 12{,}348.$$

Any given combination may be modified into a new system of intonations, i.e., into a new scale, by simply readjusting the accidentals. All the above quantities, naturally, do not include the attack-relations which have to be established for the harmonization.

As the attacks of $\frac{M_I}{M_{II}}$ are fixed groups, the only relation that must be established concerns $\overrightarrow{H}$. The most refined form of harmonization results from assigning ***each*** harmonic consonance to one H. If counterpoint contains many delayed resolutions of one dissonance, then the number of attacks of M_I is quite great and the changes of H are not as frequent. On the other hand, *direct* resolutions produce *frequent* chord changes.

The assignment of two successive harmonic consonances to one H, amplifies the number of chords satisfying such a set, but at the same time neutralizes somewhat the character of the $\overrightarrow{H}$. This technique, however, permits a greater variety of attack-relations among the three components.

Let us see how we would harmonize counterpoint of type II, when $\frac{M_I}{M_{II}} = a$. In such a case, all the harmonic intervals are consonances. Therefore, we can have the following matching of attacks:

$$\begin{array}{lll} M_I = a & M_I = 2a & M_I = 3a \\ M_{II} = a & M_{II} = 2a & M_{II} = 3a, \text{ etc.} \\ \overrightarrow{H} = a & \overrightarrow{H} = a & \overrightarrow{H} = a \end{array}$$

Examples of Diatonic Harmonization of Two-Part Counterpoint when $\frac{M_I}{M_{II}} = a$.

Theme:

M_I d_0

Harmonization (1)

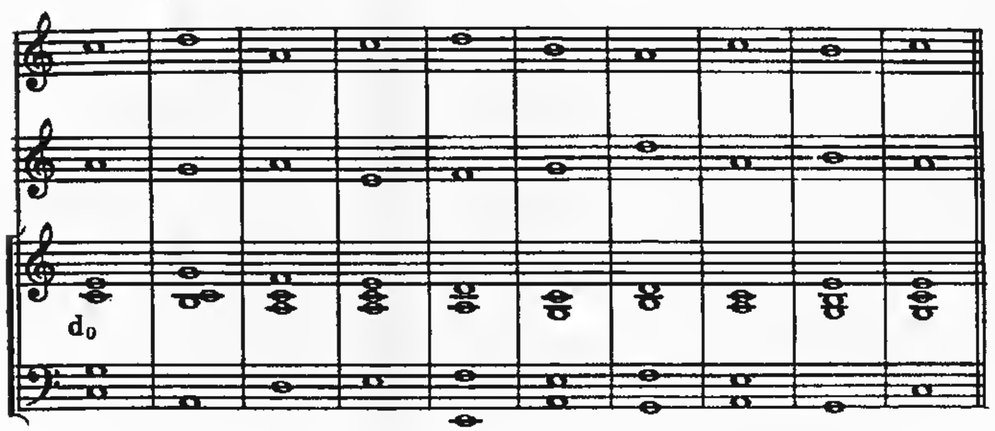
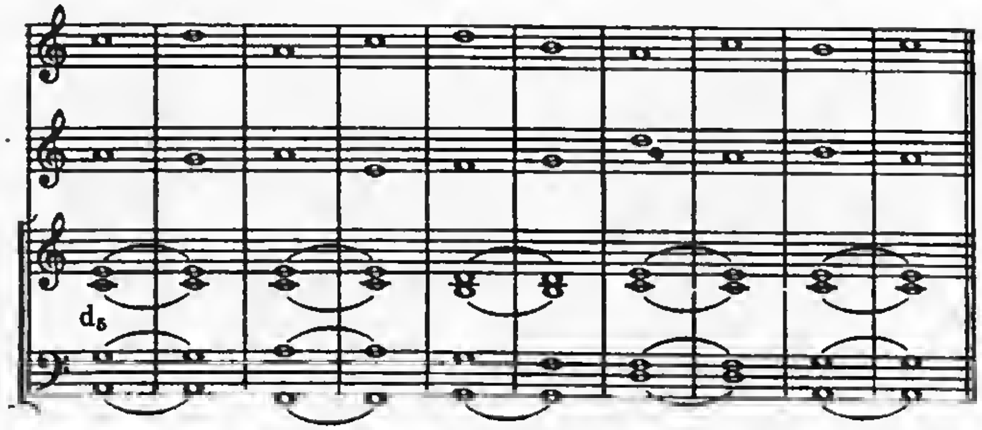


Figure 131. Diatonic harmonization of two-part counterpoint; $\frac{M_I}{M_{II}} = a$ *(continued).*

Harmonization (2)



Harmonization (3)

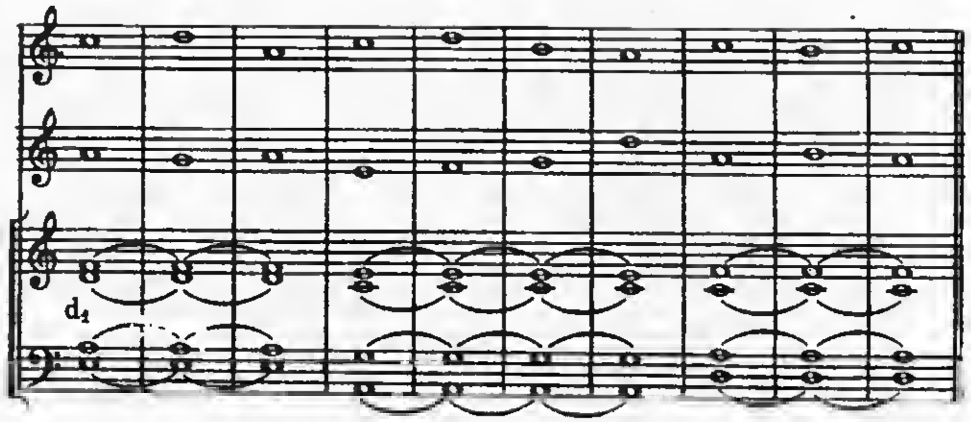


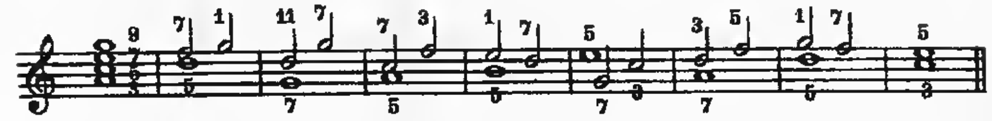
Figure 131. Diatonic harmonization of two-part counterpoint; $\frac{M_I}{M_{II}} = a$ *(concluded).*

Examples of Diatonic Harmonization of Two-Part Counterpoint when $\frac{M_I}{M_{II}} = \frac{3a}{a}$

Theme:

Figure 132. Diatonic harmonization of two-part counterpoint; $\frac{M_I}{M_{II}} = \frac{3a}{a}$ *(continued).*

Harmonization (1)

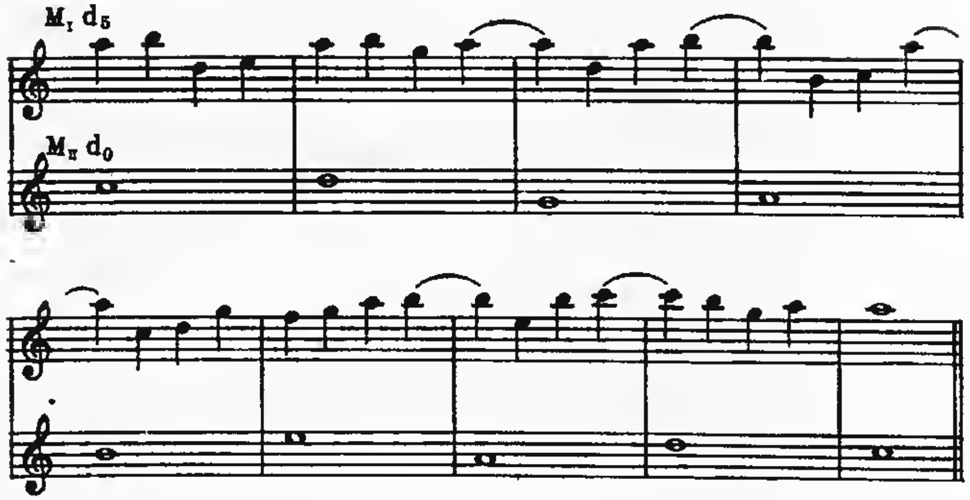


Harmonization (2)

Figure 132. Diatonic harmonization of two-part counterpoint: $\frac{M_I}{M_{II}} = \frac{3a}{a}$ *(concluded).*

Examples of Diatonic Harmonization of Two-Part Counterpoint when $\frac{M_I}{M_{II}} = \frac{4a}{a}$ and $\frac{M_I}{M_{II}} = \frac{6a}{a}$

Theme: (1)



Harmonization:

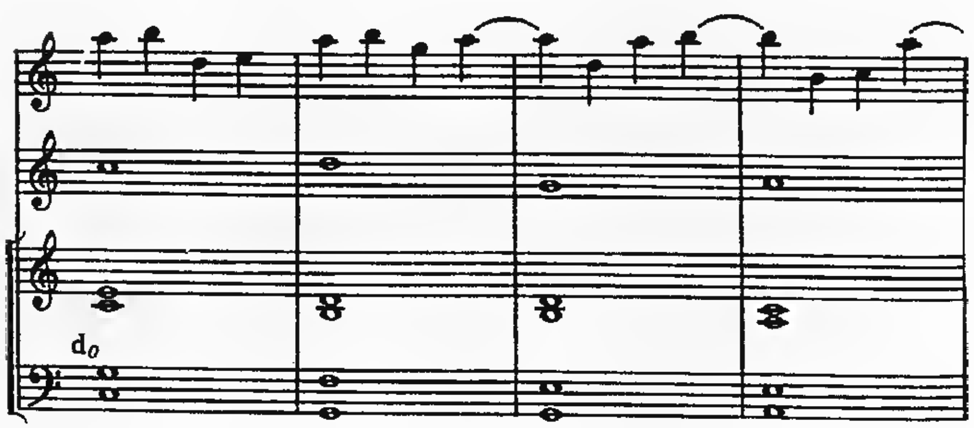


Figure 133. Diatonic harmonization of two-part counterpoint: $\frac{M_I}{M_{II}} = \frac{4a}{a}$ *(continued).*

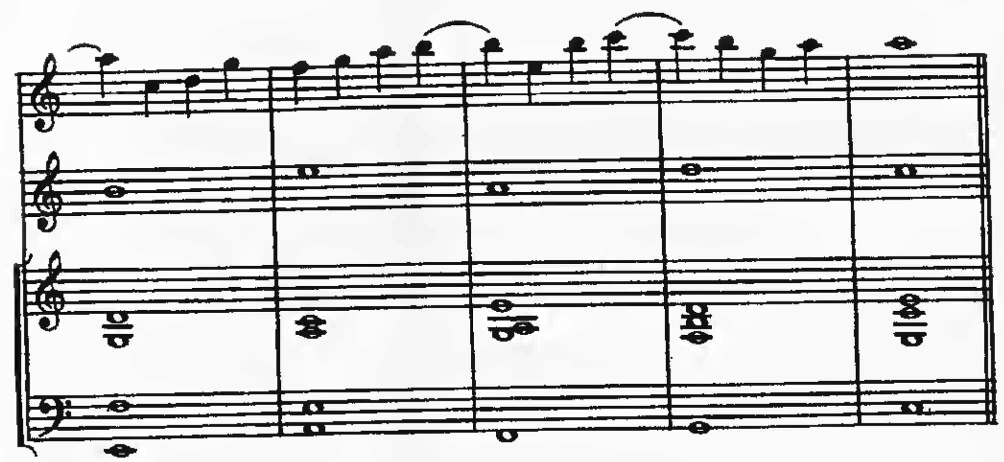


Figure 133. $\frac{M_I}{M_{II}} = \frac{4a}{a}$ *(concluded).*

Theme: (2)

Harmonization:

Figure 134. $\frac{M_I}{M_{II}} = \frac{6a}{a}$ *(continued).*

Figure 134. $\frac{M_I}{M_{II}} = \frac{6a}{a}$ *(concluded).*

B. Chromatization of Harmony Accompanying Two-Part Diatonic Counterpoint (Types I and II)

Chromatic variation of diatonic harmony accompanying two-part counterpoint may be obtained by means of auxiliary and passing chromatic tones. Of course, such altered tones must not conflict in any way with the two melodies.

For our example, we shall take the two-part counterpoint diatonically harmonized from Figure 133 (2).

Figure 135. Chromatization of harmonic counterpoint (continued).

Figure 135. Chromatization of harmonic counterpoint (concluded).

C. Diatonic Harmonization of Chromatic Counterpoint Whose Origin is Diatonic (Types I and II)

The principle of this form of harmonization is that of assigning the *diatonic* consonances as chordal functions; chromatic consonances, as well as all other forms of harmonic interval, are to be ignored so far as the $H^{\rightarrow}$ goes.

The number of successive consonances which should correspond to one H is optional; it is practical to make T, or 2T, or 3T correspond to one H.

When harmonizing a chromatic counterpoint whose diatonic original is known, one can assign chordal functions directly from the diatonic original; doing this obviously eliminates any possible confusion of the diatonic and the chromatic consonances.

We shall now harmonize a duet in which both parts are chromatic. The theme is taken from Figure 50 of Chapter 4. For clarity's sake, we shall write out both the original and the chromatized version. We shall choose the following relationship between $H^{\rightarrow}$ and $T^{\rightarrow}$:

$$H^{\rightarrow}T^{\rightarrow} = HT + H2T + HT + HT + HT + H2T + HT$$

which is a modified version of the $r_{3\div2}$, and which permits us to demonstrate diversified forms of attacks groups of M_I and M_{II} in relation to $H^{\rightarrow}$

Example of Diatonic Harmonization of Chromatic Counterpoint

Original

Chromatic variation

Figure 136. Chromatic counterpoint (continued).

Figure 136. Chromatic counterpoint (concluded).

When the diatonic origin of the chromatic counterpoint is unknown, an analysis of diatonic consonances must precede the planning of the harmonization.

D. Symmetric Harmonization of Diatonic Two-Part Counterpoint (Types I, II, III and IV)

The principle of *symmetric* harmonization of two-part counterpoint is that of assigning *all* harmonic intervals to be chordal functions.

The fewer the attacks of M_I and M_{II} that correspond to one H, the easier it is to perform such harmonization by means of one $\Sigma 13$. But when a considerable number of attacks (even in only one of the two melodies) corresponds to one H, it becomes necessary to introduce two, and sometimes even three, $\Sigma 13$'s. The forms of the latter should vary only slightly, making sure that any change is for the purpose only of rectifying the particular non-corresponding pitch-unit. For instance, in using a $\Sigma 13$ XIII as Σ_1, a correction of the eleventh to f♮ gives a satisfactory solution for most cases; Σ_2 in this instance will differ from Σ_1 only with respect to the 11.

The selection of the original Σ 13 is a matter of harmonic character. For example, the use of Σ 13 XIII attributes to music a definitely "Ravelian" quality. However, harmonic quality still remains virgin territory awaiting the composer's exploration; most of the 36 forms of the Σ 13 have not been utilized at all.

The fact that counterpoint belongs to types I and II, or to types III and IV does not help us select any particular Σ 13. Whereas symmetric harmonization of counterpoint of types I and II is a luxury, it is an actual necessity for types III and IV, as the latter correlate two different *key*-axes.

The fact that *two different keys*, with identical or with non-identical scales, may be united by *one* chord is of particular importance. This is so because the quality of a selected Σ 13 can influence the two melodies. In our musical civilization, our ears are so much conditioned by harmony that most of our listeners have lost any ability to enjoy melodic line as such. If the ear of an average music-lover can relate one diatonic melody to only one chord progression, the harmonic association of two melodies belonging to two different keys becomes impossible; the role of a harmonic master-structure (Σ 13 in this case) is that of *synthesizer*.

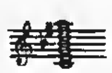
The simplest way to assign harmonic functions is to relate the latter first to consonances. The master-structure used in the following harmonization is Σ 13 XIII.*

Symmetric Harmonization of Diatonic Two-Part Counterpoint of Types I and II

(1)

(2)

Figure 137. Symmetric harmonization of diatonic two-part counterpoint of types I and II (continued).

*Σ 13 (XIII) has the following form  The complete list of Σ 13's is presented in Book VI, Chapter 2. (Ed.)

(3) **Chromatic variation of harmony** (2)

Figure 137. Symmetric harmonization of diatonic two-part counterpoint of types I and II (concluded).

The chromatic variation of $H^{\rightarrow}$ in the foregoing example was obtained through the usual technique: the insertion of passing and auxiliary chromatic units.

Symmetric Harmonization of Diatonic Two-Part Counterpoint of Types III and IV

(1)

(2) Chromatic variation of harmony (1)

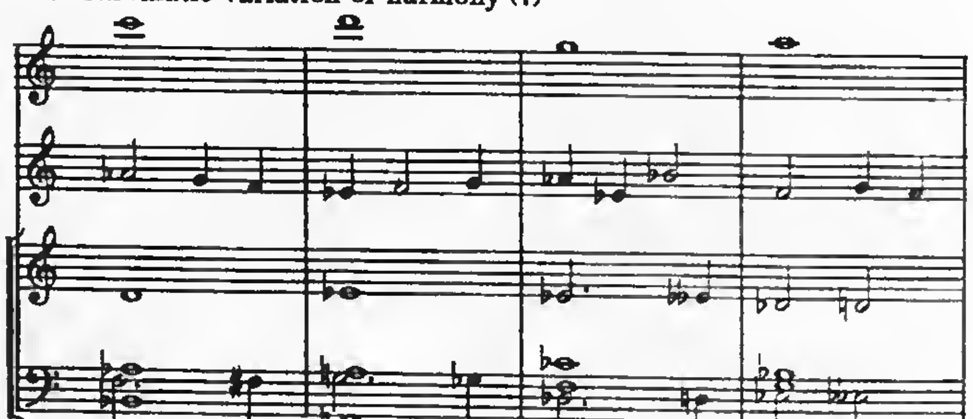


Figure 138. Symmetric harmonization of diatonic two-part counterpoint of types III and IV (continued).

Figure 138. Symmetric harmonization of diatonic two-part counterpoint of types III and IV (concluded).

E. Symmetric Harmonization of Chromatic Two-Part Counterpoint Whose Origin Is Diatonic (Types I, II, III and IV)

The principle of symmetric harmonization of chromatic two-part counterpoint is that of assigning *all the diatonic pitch-units of both melodies* to be chordal functions of the master-structure (Σ 13)—but neglecting all the chromatic pitch-units as not belonging to the scale; it does not matter whether the chromatic units belong to the master-structure or not. When the diatonic original of the two-part counterpoint is unknown, then the diatonic units of both melodies should be isolated before proceeding.

Original (1)

Figure 139. Symmetric harmonization of chromatic two-part counterpoint (continued).

Chromatic variation harmonized (1)

Original (2)

Figure 139. Symmetric harmonization of chromatic two-part counterpoint (continued).

Chromatic variation harmonized (2)

Figure 139. Symmetric harmonization of chromatic two-part counterpoint (concluded).

Counterpoint executed in symmetric scales of the third and fourth groups may be harmonized by means of a symmetric master-structure. This master-structure is independent of the system of symmetry of the pitch-scales involved. As in previous cases, all units corresponding to one H must belong to one Σ 13.

After the harmonization is performed, it may be subjected, if desired, to chromatic variation.

F. Symmetric Harmonization of Symmetric Two-Part Counterpoint

Theme:

Harmonization

Figure 140. Symmetric harmonization of symmetric two-part counterpoint (cont.).

Chromatic variation of harmony

Figure 140. Symmetric harmonization of symmetric two-part counterpoint (concluded).

All forms of contrapuntal continuity and complete compositions in the form of canon and fugue may be harmonized by this technique. Any of the correspondences described above between counterpoint and harmony may be established by the composer. One should remember that *overloading* harmonic accompaniments is more a sin than a virtue; for this reason, the technique of *variable density* should receive the utmost consideration.

CHAPTER 12

MELODIC, HARMONIC, AND CONTRAPUNTAL OSTINATO

FORMS of ***ostinato*** or ***ground motion*** have been known since time immemorial. They appear in different folk and traditional music as a fundamental form of improvisation around a given theme. The characteristic of *ostinato* (literally: obstinate) is the continuous repetition of a certain thematic group—which may be either rhythm, melody, or harmony. As one example, the dance beat of 4/4 in a fox-trot is one of just such fundamental forms of ***ostinato***. And, as a matter of fact, a ***rhythmic ostinato*** is ever-present in ***all*** the developments in classical symphonies! Take, for example, the first motif of Beethoven's *Fifth Symphony*, consisting of 4 notes, and follow it through the development (middle section of the first movement); the motif, rhythmically the same, changes its forms of intonation either melodically or in the form of accompanying harmony.

Repetitions of groups of chords, or repetitions of melodic fragments accompanied by continuously changing chords, are both forms of *ostinato*. Ostinato is one of the traditional forms of thematic growth and, as such, is very well known in the forms called chaconne (ciaconna) and passacaglia. In many Irish jigs, ***ostinato*** appears in the form of pedal point, as well as in repetitious melodic fragments. When portions of the same melody appear in succession, being harmonized every time anew, (which may be found even in such works as Chopin's mazurkas), we have still another case of ostinato.

A. Melodic Ostinato (Basso Ostinato)

Melodic *ostinato*, better known under the name of "ground bass," is a ***harmonization*** of an ever-repeating melody by continuously changing chords. *Ostinato* groups produce one uninterrupted continuity in which the recurrence of the bass form produces the unity and the accompanying harmony produces the variety. All forms of harmonization may be applied to the continuously repeating melody, regardless of whether it appears in the bass or in any of the middle voices, or in the upper voice (above the harmony).

As every harmonic setting of chords is subject to vertical permutations, a ***basso ostinato*** may be transformed into tenor, or alto, or soprano ***ostinato***, i.e., it may appear in any desirable voice and in any desirable sequence after the harmonization has been completed.

In the following example, the *ostinato* of the theme is a melody in whole notes in the bass (the first four bars); later it repeats itself two more times. The form of harmonization is symmetric in this case, although it could have been diatonic or in any of the chromatic forms. This device may be used as a form of thematic ***development***, —and in arranging it may be used with effect for the purpose of constructing introductions or transitions. Any characteristic melodic pattern may be converted into ***basso ostinato*** either with the preservation of its original rhythm or in an entirely new setting.*

*See Arensky's *Basso Ostinato* for piano. (J.S.)

Melodic Ostinato
Basso Ostinato (Ground Bass)

Symmetric Harmonization of the Bass.

Figure 141. Melodic ostinato.

B. Harmonic Ostinato

Harmonic *ostinato* might also be called, by analogy, "ground harmony." It consists of the repetition of a group of chords, in relation to which a continuously changing melody is evolved. This form of ostinato is the one which J. S. Bach employed in his D-minor chaconne for violin; it is also used in numerous other compositions—by Bach and other composers, too. Among my own students, George Gershwin used this device successfully in an exercise which later, at my suggestion, he put into the musical comedy, *Let 'Em Eat Cake*, as the song hit, *Mine*.

This form of *ostinato* may be applied to any type of harmonic progression. The technical procedure is exactly the opposite of the first one. In this case we deal with ***melodization*** of harmony. As in the previous case, the melody evolved against chords may be transferred to a different position in relation to the chord by means of vertical permutation. Naturally, not every melody will be as good in the bass as in the soprano, for the chordal functions represented by melody are more advantageous for an upper part than for the lower, or *vice versa*.

In the following example, the harmonic theme of *ostinato* emphasizes four different chords (the first two bars), and is based on a Σ 13 [XIII]. The melody evolves through the principle of symmetric melodization forming its axis points in relation to the chord structure itself. The main resource by which variety is obtained against the uniformity of the *ostinato* is the manifold of melodic forms.

Harmonic Ostinato (Ground Harmony)
Symmetric Melodization of Harmony

Figure 142. Harmonic ostinato.

C. Contrapuntal Ostinato

The form, contrapuntal *ostinato*, is well known in the works of old masters. It was usually evolved against a melody, a *cantus firmus*. If a C.F. repeats itself continuously a number of times while the contrapuntal part or parts evolve in relation to it producing different relations with every appearance of the C.F., the result is a contrapuntal ostinato.

In the following example, the theme of the *ostinato* is taken from Figure 141 and the accompanying counterpoint is evolved through type II, adhering to a rhythmic *ostinato*, as well, except for a few intentional permutations. Naturally, both voices may be exchanged, or may be subjected to any of the variations through geometrical positions ⓐ, ⓑ, ⓒ, and ⓓ.

Contrapuntal Ostinato
Basso Ostinato (Ground Bass)

CP Type II

Figure 143. Contrapuntal ostinato.

Likewise, a counterpoint may be evolved to the soprano voice through the use of the same principle. In Figure 144, the same theme is employed, altered rhythmically; the counterpoint, in its rhythmic setting, produces a constant interference against the C.F., as it consists of a 3-bar group. The harmonic setting of this example is in type III: the C.F. is in natural C major, and the counterpoint is in natural A♭ major.

Soprano Ostinato (Ground Melody)

Figure 144. Soprano ostinato.

The latter two forms of *ostinato*—harmonic and contrapuntal—are extremely adaptable in all cases in which it is desirable to repeat one motif and yet introduce variety into the obligato. These characteristics make the devices extremely useful for introductions, transitions, and codas in arranging.

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THE SCHILLINGER SYSTEM OF MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

IN TWO VOLUMES

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BOOK VIII

INSTRUMENTAL FORMS

BOOK EIGHT

INSTRUMENTAL FORMS

CHAPTER 1

MULTIPLICATION OF ATTACKS

INSTRUMENTAL form will mean, so far as this discussion is concerned, a modification of the original melody and/or harmony which renders them fit for execution on an instrument. ***Instrumental*** can thus be regarded as an ***applied*** form of ***pure*** music. Depending on the degree of virtuosity which can be expected from singers, instrumental forms may be applied to vocal music as well as orchestral.

The main technical characteristic of the instrumental (i.e., of applied, as against pure) form is that it emphasizes the ***development of quantities (multiplication) and the forms of attacks*** from the original attack. We shall here be concerned only with the former—i.e., with quantities and their uses in composition—and leave the latter, the forms of attack (such as durable, abrupt, bouncing, oscillating, etc.), to that branch of this theory called ***orchestration.***

Multiplication of attacks may be applied directly to single pitch-units as well as to pitch-assemblages. The number of instrumental forms available is dependent upon the number of pitch-units in an assemblage. When the number of pitch-units (parts) in an assemblage is few, the number of instrumental forms is low. When the pitch-units (parts) in an assemblage are abundant, the number of instrumental forms is high, permitting greater variety in a composition insofar as its instrumental aspect is concerned.

The paucity of instrumental forms derivable from but one pitch-unit (part) often compels us to resort to ***couplings.*** By the addition of one coupling to one part, we achieve a two-part setting with all its instrumental implications. Likewise, the addition of two couplings to one part transforms the latter into a three-part assemblage, etc.

What we are to discuss here is all forms of arpeggio and their applications in the field of melody, harmony, and correlated melodies.

A. NOMENCLATURE:

Σ—score (group of instrumental strata)
S—stratum (instrumental stratum)
p—part (function, coupling)
a—attack

Preliminary Data:

(1) $p = a\ ;\ p = 2a\ ;\ .\ .\ .\ p = na$
(2) $S = p\ ;\ S = 2p\ ;\ .\ .\ .\ S = np$
(3) $\Sigma = S\ ;\ \Sigma = 2S\ ;\ .\ .\ .\ \Sigma = nS$

B. SOURCES OF INSTRUMENTAL FORMS

(a) Multiplication of S is achieved by 1 : 2 : 4 : 8 : . . . ratio (i.e., by the octaves).

(b) Multiplication of p in S is achieved by coupling or by harmonization. It is applicable to melody (p), to correlated melodies (2p, . . . np), and to harmony (2p . . . 4p). The material for p is to be found in my previous exposition of the *Theory of Pitch-Scales** and the *Theory of Melody.*** The material for 2p, . . . np acting as melodies was discussed in the theory of correlated melodies (*Counterpoint*).*** The material for 2p, . . . np acting as parts of harmony is presented in the previous *Special Theory of Harmony,*** * and in the discussion that is to come on the *General Theory of Harmony.*****

(c) Multiplication of a is achieved by repetition and sequence of p's (arpeggio).

(d) Different S's and different p's, as correlated melodies of Σ, may have independent instrumental forms.

C. DEFINITION OF INSTRUMENTAL FORMS:

I. (a) *Instrumental Forms of Melody*: I (M = p):

Repetition of pitch-units represented by the duration-group and expressed through its common denominator. The number of a equals the number of t.

$$\text{If } \frac{1}{nt} = nt, \text{ then } nt = na$$

Rhythmic composition of durations assigned to each attack.

(b) *Instrumental Forms of Melody*: I (M = np):

Repetition of pitch-units (p_I) and their couplings (p_{II}, p_{III}, . . . p_N) and transition (sequence) from one p to another, represented by the duration group and expressed through its common denominator. Instrumental groups of p's consisting of repetitions and sequences are subject to permutations.

(α) *Instrumental Forms of the Simultaneous Groups of Melody*:

$$M = \frac{p_{II}}{p_I};\ \frac{p_I}{p_{II}};\ \frac{\frac{p_{III}}{p_I}}{p_{II}};\ \frac{\frac{p_{II}}{p_I}}{p_{III}};\ \frac{\frac{p_I}{p_{II}}}{p_{III}};\ \frac{\frac{p_I}{p_{III}}}{p_{II}};\ \frac{\frac{p_{III}}{p_{II}}}{p_I};\ \frac{\frac{p_{II}}{p_{III}}}{p_I};\ \ldots$$

(β) *Instrumental Forms of the Sequent Groups of Melody:*

$$M = p_I + p_{II};\ p_{II} + p_I;\ p_I + p_{II} + p_{III};\ p_I + p_{III} + p_{II};\ p_{III} + p_I + p_{II};\ p_{II} + p_I + p_{III};\ p_{II} + p_{III} + p_I;\ p_{III} + p_{II} + p_I.$$

$$M = p_I + p_{II} + p_I;\ p_I + p_{II} + p_{III} + p_I;\ p_I + p_{II} + p_{III} + p_{II};\ p_I + p_{II} + p_{III} + p_{III},\ \ldots .$$

*See Vol. I, p. 101 ff. **See Vol. I, p. 227 ff. ***See Vol. I, p. 708 ff. ****See Vol. I, p. 359 ff. *****See p. 1063 ff.

(γ) *Instrumental Forms of the Combined Groups of Melody*:

$$M = \frac{p_{II}}{p_I} + \frac{p_{III}}{p_I} + \frac{p_{III}}{p_{II}};\ \frac{p_{II}}{p_I} + \frac{p_{III}}{p_I} + \frac{p_{IV}}{p_I} + \frac{p_{III}}{p_{II}} + \frac{p_{IV}}{p_{II}} + \frac{p_{IV}}{p_{III}};\ \ldots$$

$$M = \frac{\frac{p_{III}}{p_{II}}}{p_I} + \frac{\frac{p_{IV}}{p_{II}}}{p_I} + \frac{\frac{p_{IV}}{p_{III}}}{p_I} + \frac{\frac{p_{IV}}{p_{III}}}{p_{II}};\ \ldots$$

II. *Instrumental Forms of Correlated Melodies*:

(a) $I\left(\frac{M_{II} = p}{M_I = p}\right)$: correlation of instrumental forms of the two uncoupled melodies (M_I and M_{II}) by means of correlating their a's.

M_I (nt = na); M_{II} (nt = 2na; 3na; . . . mna)

$$\frac{M_{II}\ (t = a)}{M_I\ (t = 2a)};\ \frac{M_{II}\ (t = 2a)}{M_I\ (t = a)};\ \frac{M_{II}\ (t = a)}{M_I\ (t = 3a)};\ \frac{M_{II}\ (t = 3a)}{M_I\ (t = a)}$$

$$\frac{M_{II}\ (t = 2a)}{M_I\ (t = 3a)};\ \frac{M_{II}\ (t = 3a)}{M_I\ (t = 2a)};\ \frac{M_{II}\ (t = a)}{M_I\ (t = 4a)};\ \frac{M_{II}\ (t = 4a)}{M_I\ (t = a)};$$

$$\frac{M_{II}\ (t = 2a)}{M_I\ (t = 4a)};\ \frac{M_{II}\ (t = 4a)}{M_I\ (t = 2a)};\ \frac{M_{II}\ (t = 3a)}{M_I\ (t = 4a)};\ \frac{M_{II}\ (t = 4a)}{M_I\ (t = 3a)};\ \ldots$$

$$\ldots\ \frac{M_{II}\ (t = na)}{M_I\ (t = ma)}$$

(b) $I\left(\frac{M_{II} = np}{M_I = mp}\right)$: this form corresponds to combinations of (α), (β) and (γ) of I (b).

$$\frac{M_{II}\ (\alpha)}{M_I\ (\alpha)};\ \frac{M_{II}\ (\alpha)}{M_I\ (\beta)};\ \frac{M_{II}\ (\beta)}{M_I\ (\alpha)};\ \frac{M_{II}\ (\beta)}{M_I\ (\beta)};$$

$$\frac{M_{II}\ (\alpha)}{M_I\ (\gamma)};\ \frac{M_{II}\ (\gamma)}{M_I\ (\alpha)};\ \frac{M_{II}\ (\beta)}{M_I\ (\gamma)};\ \frac{M_{II}\ (\gamma)}{M_I\ (\beta)};\ \frac{M_{II}\ (\gamma)}{M_I\ (\gamma)}.$$

III. *Instrumental Forms of Harmony*:

I (S = p, 2p, 3p, 4p): this corresponds to one-part harmony, which is the equivalent of M; two-part harmony, which is the equivalent of two correlated uncoupled melodies; three-part harmony, which is the equivalent of three correlated uncoupled melodies; four-part harmony, which is the equivalent of four correlated uncoupled melodies.

The source of the harmony may be the *Theory of Pitch-Scales*, the *Special Theory of Harmony*, and the *General Theory of Harmony.** Parts (p's) in their simultaneous and sequent groupings correspond to a. b, c, d.

$$p_I = a;\ p_{II} = b;\ p_{III} = c;\ p_{IV} = d.$$

*See Vol. I, p. 101 ff.; p. 359 ff. and Vol. II, p. 1063 ff.

CHAPTER 2

STRATA OF ONE PART

THERE being, by definition, but one part to strata of this type, we need not classify the attack forms in any general way, but may proceed at once to discuss the instrumental forms for $S = p$. The material for these forms is:

(a) melody;
(b) any one of the correlated melodies;
(c) one part harmony;
(d) harmonic form of one unit scale;
(e) one part of any harmony.

$I = a; 2a; 3a; ma; A$ var.
$nt = na$

(a) Theme

Var. I: $nt = na$

Var. I: $A = a2t + at$

Figure 1. Melody.

(b) Theme

Var. I (M_I): $a = 2t$

Var. I (M_{II}): $a = t$

Figure 2. Correlated melody (continued).

Var. I $\left(\frac{M_2}{M_1}\right)$: $\frac{a = t}{a = 2t}$

Figure 2. Correlated melodies (concluded).

(c) Theme

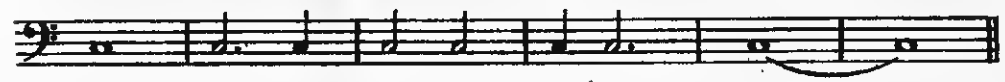
Var. I (B): a = t

Figure 3. One-part harmony (continued).

Var. I (B): A = 3a + a + 2a + 2a + a + 3a; T″: $\frac{4}{4}$ series

Figure 3. One-part harmony (concluded).

(d) Ped. Point. Theme



Var. I (p): a = t

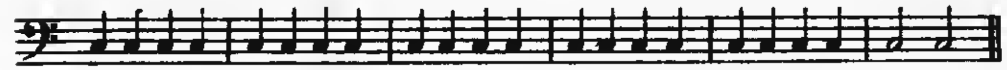


Figure 4. Harmonic form of one-unit scale.

(e) Theme

Var. I (S): a = t + 2t + t; T″: $\frac{8}{8}$ series

Var. I (ABTS): a = t; T″ $\frac{8}{8}$ series

CHAPTER 3

STRATA OF TWO PARTS

A. General Classification of I (S = 2p)

(*A table of the combinations of attacks for a and b.*)

A = a; 2a; 3a; 4a; 5a; 6a; 7a; 8a; 12a.*

I give here a complete table of all forms of I (S = 2p). Included are ***all the combinations and permutations*** for 2, 3, 4, 5, 6, 7, 8 and 12 attacks.

A = 2a; $a + b$.

$P_2 = 2! = 2$

Total of general permutations: 2

Total of circular permutations: 2

A = 3a; $2a + b; a + 2b$.

$$P_3 = \frac{3!}{2!} = \frac{6}{2} = 3$$

Each of the above 2 permutations of the coefficients has 3 general permutations.

Total: $3 \cdot 2 = 6$

The total number of cases: A = 3a

General permutations: 6

Circular permutations: 6

A = 4a

Forms of the distribution of coefficients:

4 = 1+3; 2+2 ; 3+1

$A = a + 3b; 3a + b$.

$$P_4 = \frac{4!}{3!} = \frac{24}{6} = 4$$

Each of the above 2 permutations of the first form of distribution of the coefficients of recurrence has 4 general permutations.

Total: $4 \cdot 2 = 8$

$A = 2a + 2b$

$$P_4 = \frac{4!}{2!\,2!} = \frac{24}{2 \cdot 2} = 6$$

The above invariant form of distribution has 6 general permutations.

The total number of cases: A = 4a

General permutations: 8 + 6 = 14

Circular permutations: $4 \cdot 3 = 12$

*In this chapter and several ensuing chapters we are to be concerned with tables of combinations; it should be said that the tables are included not merely as items of interest, but as actual sources on which the composer or arranger may draw—above all, if he is insufficiently familiar with the techniques of making permutations, combinations, and related groups. (Ed.)

A = 5a

Forms of the distribution of coefficients:

5 = 1+4; 2+3.

$A = a + 4b; 4a + b$

$$P_5 = \frac{5!}{4!} = \frac{120}{24} = 5$$

Each of the above 2 permutations of the first form of distribution has 5 general permutations.

Total: $5 \cdot 2 = 10$

$A = 2a + 3b; 3a + 2b$

$$P_5 = \frac{5!}{2!\,3!} = \frac{120}{2 \cdot 6} = 10$$

Each of the above 2 permutations of the second form of distribution has 10 general permutations.

Total: $10 \cdot 2 = 20$

The total number of cases: A = 5a

General permutations: 10 + 20 = 30

Circular permutations: $5 \cdot 4 = 20$

A = 6a

Forms of the distribution of coefficients:

6 = 1+5; 2+4; 3+3.

$A = a + 5b; 5a + b$.

$$P_6 = \frac{6!}{5!} = \frac{720}{120} = 6$$

Each of the above 2 permutations of the first form of distribution has 6 general permutations.

Total: $6 \cdot 2 = 12$

$A = 2a + 4b; 4a + 2b$.

$$P_6 = \frac{6!}{2!\,4!} = \frac{720}{2 \cdot 24} = 15$$

Each of the above 2 permutations of the second form of distribution has 15 general permutations.

Total: $15 \cdot 2 = 30$

$A = 3a + 3b$

$$P_6 = \frac{6!}{3!\ 3!} = \frac{720}{6 \cdot 6} = 20$$

The above invariant (third) form of distribution has 20 general permutations.

The total number of cases: A = 6a
General permutations: 12 + 30 + 20 = 62
Circular permutations: 6·5 = 30

A = 7a

Forms of the distribution of coefficients:

7 = 1+6; 2+5; 3+4.

$A = a + 6b;\ 6a + b.$

$$P_7 = \frac{7!}{6!} = \frac{5040}{720} = 7$$

Each of the above 2 permutations of the first form of distribution has 7 general permutations.

Total: 7·2 = 14

$A = 2a + 5b;\ 5a + 2b.$

$$P_7 = \frac{7!}{2!\ 5!} = \frac{5040}{2 \cdot 120} = 21$$

Each of the above 2 permutations of the second form of distribution has 21 general permutations.

Total: 21·2 = 42

$A = 3a + 4b;\ 4a + 3b.$

$$P_7 = \frac{7}{3!\ 4!} = \frac{5040}{6 \cdot 24} = 35$$

Each of the above 2 permutations of the third form of distribution has 35 general permutations.

Total: 35·2 = 70
The total number of cases: A = 7a
General permutations: 14 + 42 + 70 = 126
Circular permutations: 7·6 = 42

A = 8a

Forms of the distribution of coefficients:

8 = 1+7; 2+6; 3+5; 4+4.

$A = a + 7b;\ 7a + b.$

$$P_8 = \frac{8!}{7!} = \frac{40{,}320}{5{,}040} = 8$$

Each of the above 2 permutations of the first form of distribution has 8 general permutations.

Total: 8·2 = 16

$A = 2a + 6b;\ 6a + 2b.$

$$P_8 = \frac{8!}{2!\ 6!} = \frac{40{,}320}{2 \cdot 720} = 28$$

Each of the above 2 permutations of the second form of distribution has 28 general permutations.

Total: 28·2 = 56

$A = 3a + 5b;\ 5a + 3b.$

$$P_8 = \frac{8!}{3!\ 5!} = \frac{40{,}320}{6 \cdot 120} = 56$$

Each of the above 2 permutations of the third form of distribution has 56 general permutations.

Total: 56·2 = 112

$A = 4a + 4b$

$$P_8 = \frac{8!}{4!\ 4!} = \frac{40{,}320}{24 \cdot 24} = 70$$

The above invariant (fourth) form of distribution has 70 general permutations.

The total number of cases: A = 8a
General permutations: 16 + 56 + 112 + 70 = 254
Circular permutations: 8·7 = 56

A = 12a

Forms of the distribution of coefficients:

12 = 1+11; 2+10; 3+9; 4+8; 5+7; 6+6

$A = a + 11b;\ 11a + b.$

$$P_{12} = \frac{12!}{11!} = \frac{479{,}001{,}600}{39{,}916{,}800} = 12$$

Each of the above 2 permutations of the first form of distribution has 12 general permutations.

Total: 12·2 = 24

$A = 2a + 10b;\ 10a + 2b.$

$$P_{12} = \frac{12!}{2!\ 10!} = \frac{479{,}001{,}600}{2\cdot 3{,}628{,}800} = 66$$

Each of the above 2 permutations of the second form of distribution has 66 general permutations.

Total: $66\cdot 2 = 132$

$A = 3a + 9b;\ 9a + 3b.$

$$P_{12} = \frac{12!}{3!\ 9!} = \frac{479{,}001{,}600}{6\cdot 362{,}880} = 220$$

Each of the above 2 permutations of the third form of distribution has 220 general permutations.

Total: $220\cdot 2 = 440$

$A = 4a + 8b;\ 8a + 4b.$

$$P_{12} = \frac{12!}{4!\ 8!} = \frac{479{,}001{,}600}{24\cdot 40{,}320} = 495$$

Each of the above 2 permutations of the fourth form of distribution has 495 general permutations.

Total: $495\cdot 2 = 990$

$A = 5a + 7b;\ 7a + 5b.$

$$P_{12} = \frac{12!}{5!\ 7!} = \frac{479{,}001{,}600}{120\cdot 5{,}040} = 792$$

Each of the above 2 permutations of the fifth form of distribution has 792 general permutations.

Total: $792\cdot 2 = 1584$

$A = 6a + 6b$

$$P_{12} = \frac{12!}{6!\ 6!} = \frac{479{,}001{,}600}{720\cdot 720} = 924$$

The above invariant (sixth) form of distribution has 924 general permutations.

The total number of cases: $A = 12a$

General permutations: $24 + 132 + 440 + 990 + 1584 + 924 = 4094.$

Circular permutations: $12\cdot 11 = 132$

The interval of an octave may be changed to any other interval. For the groups with more than 6 attacks, only circular permutations are included. See figures 6-12 inclusive.

A = a A = 2a; 2 forms A = 3a: 2a+b; a+2b. 2 combinations.

3 permutations each. Total 2·3 = 6

Figure 6. A = *2a.*

A = 4a: 3a+b: 2a+2b: a+3b

4 forms

6 forms

4 forms

Total: 4+6+4 = 14

Figure 7. A = *4a.*

A = 5a: 4a+b: 3a+2b· 2a+3b; a+4b

5 forms

10 forms

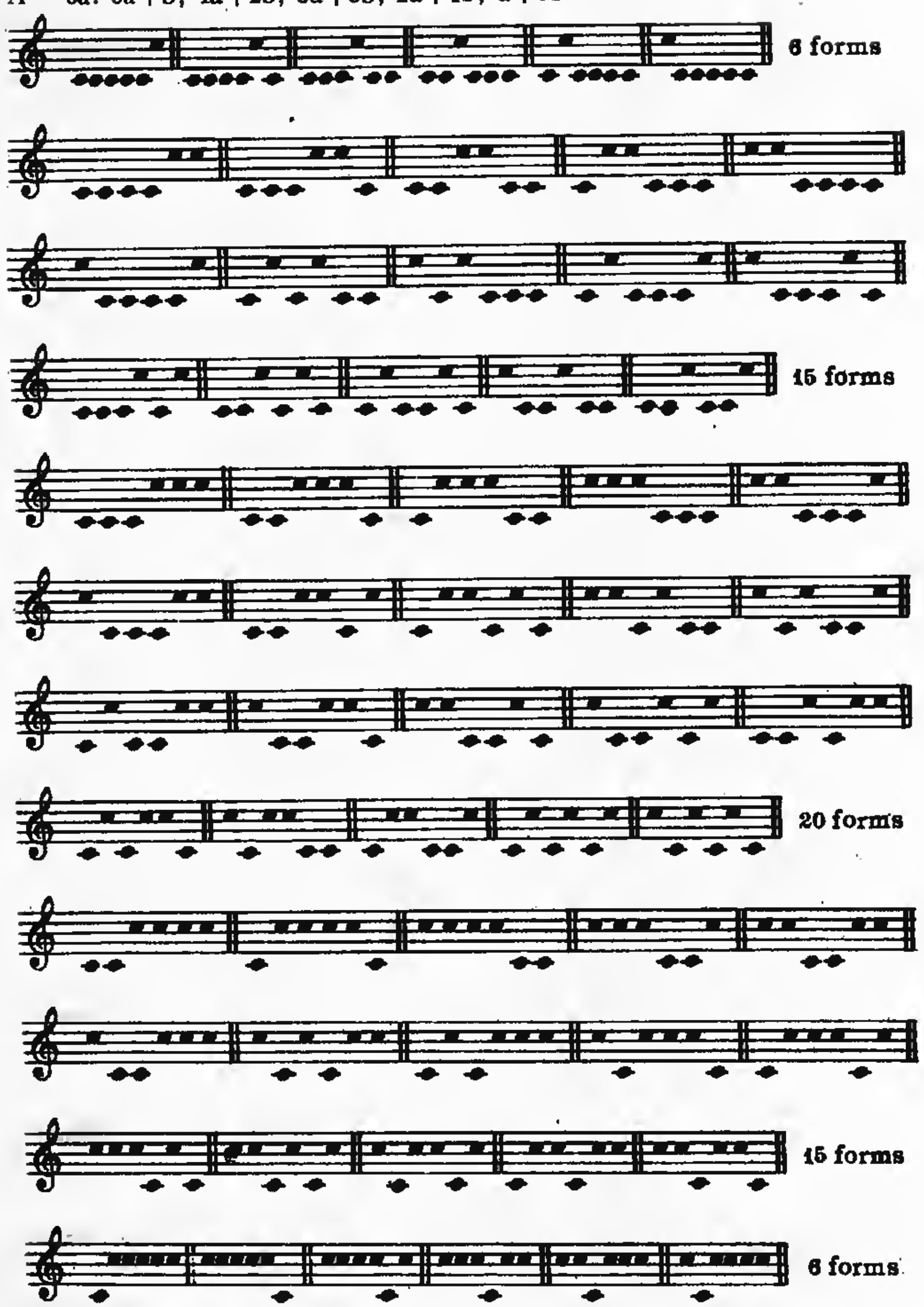
10 forms

5 forms

Total: 5+10+10+5 = 30

Figure 8. A = *5a.*

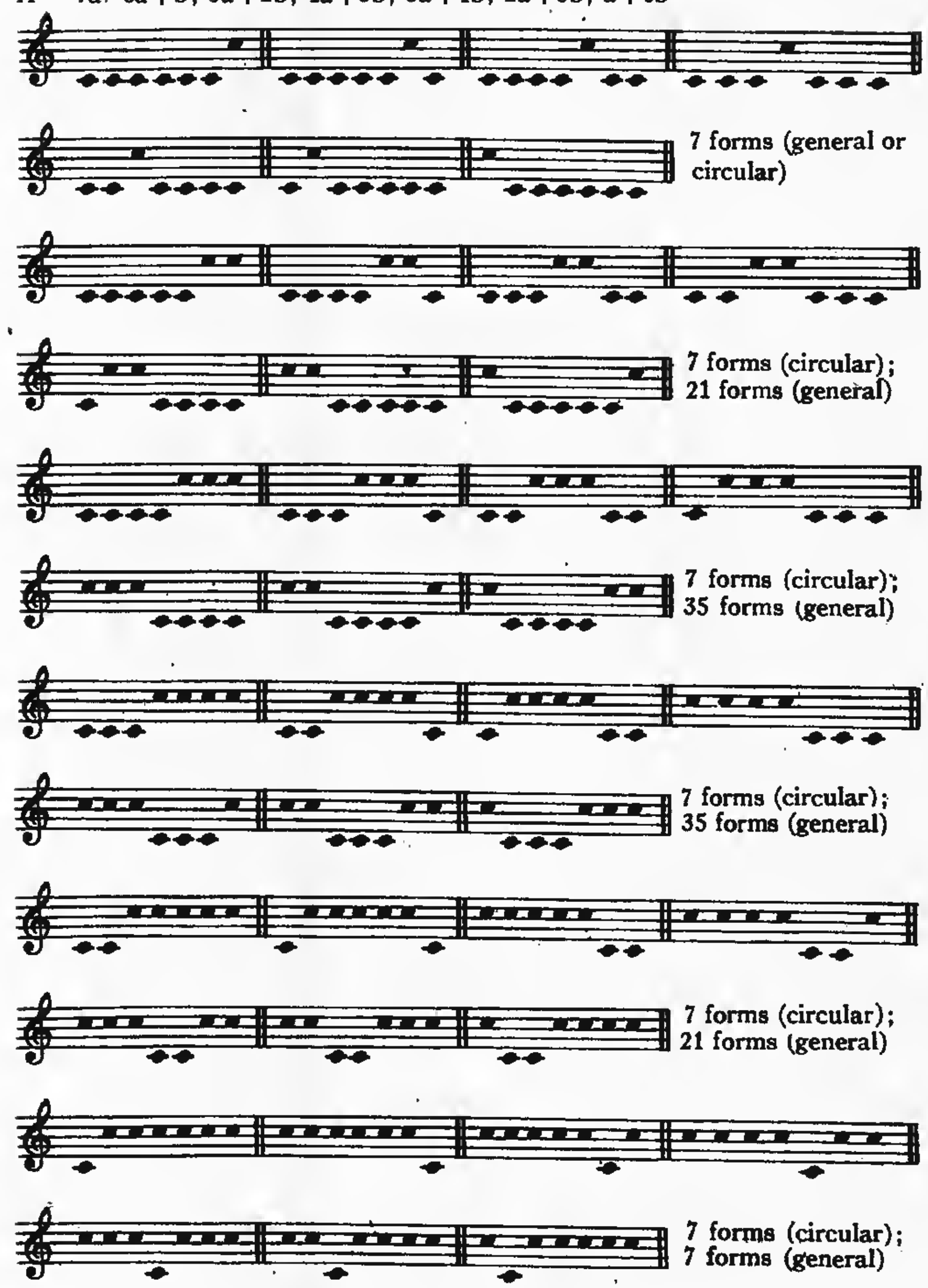
A = 6a: 5a+b; 4a+2b; 3a+3b; 2a+4b; a+5b



Total: 6+15+20+15+6 = 62

Figure 9. A = *6a.*

A = 7a: 6a+b; 5a+2b; 4a+3b; 3a+4b; 2a+5b; a+6b



Total: 7+21+35+35+21+7 = 126

Figure 10. A = 7a.

A = 8a: 7a+b; 6a+2b; 5a+3b; 4a+4b; 3a+5b; 2a+6b; a+7b

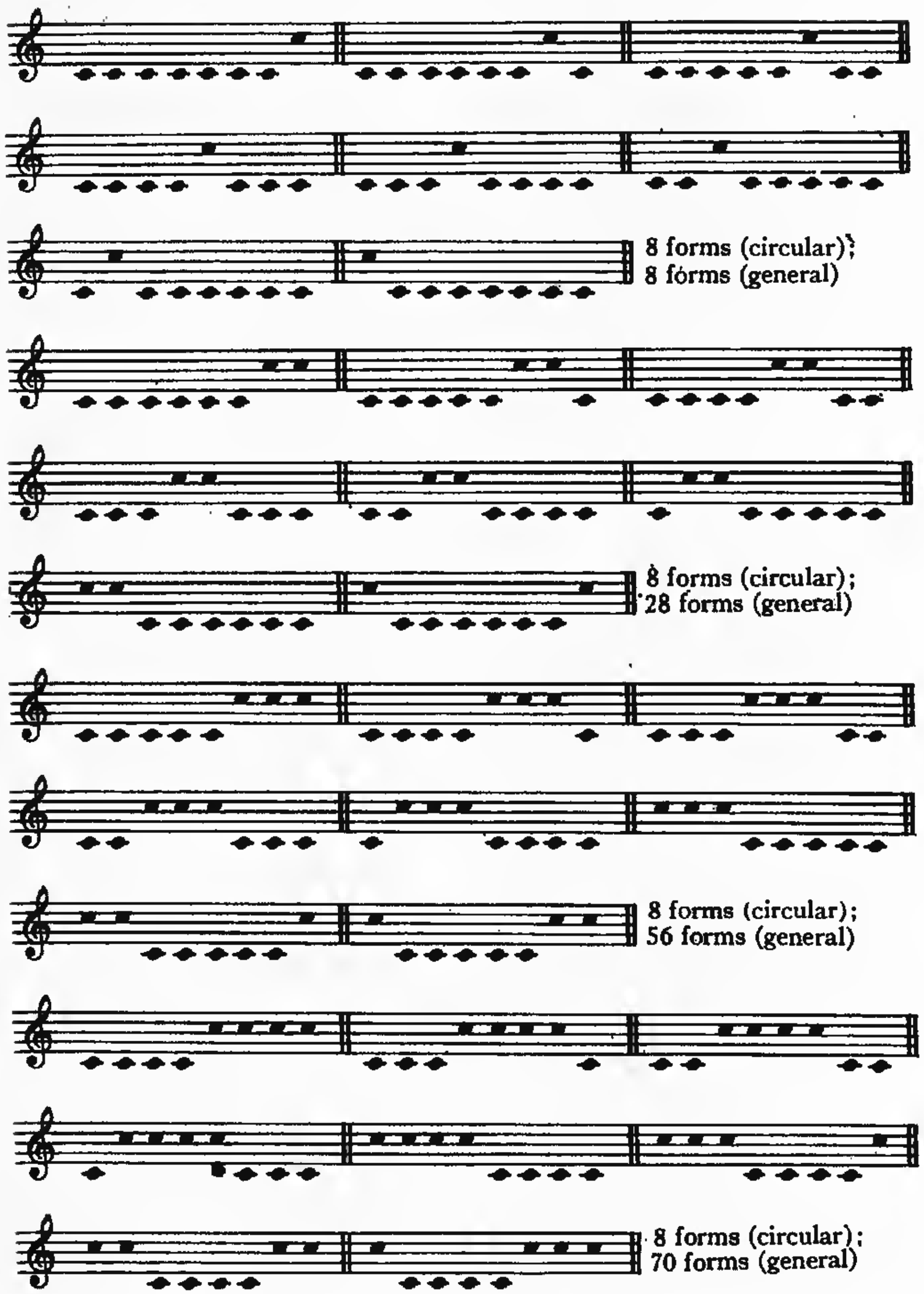
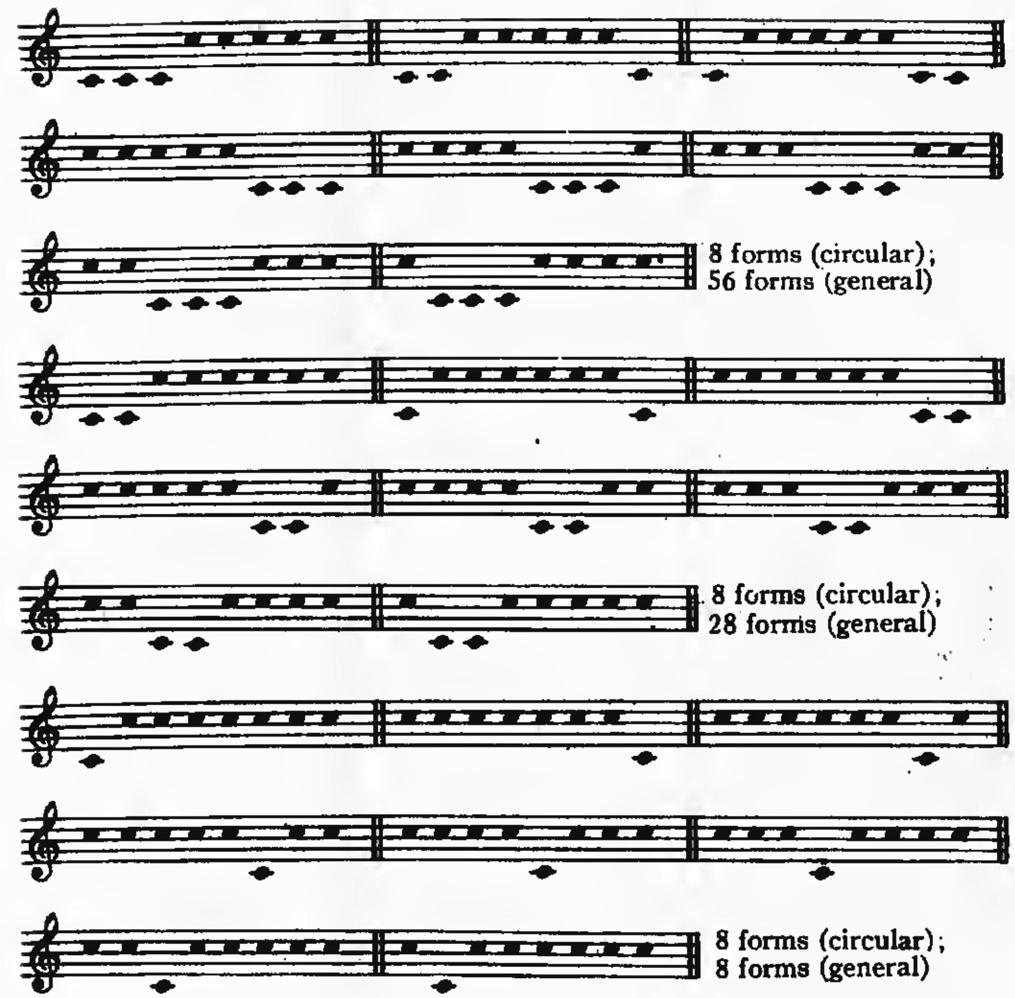


Figure 11. A = *8a.* (*continued*).



Total: 8+28+56+70+56+28+8 = 254

Figure 11. A = *8a.* (*concluded*).

A = 12a: 11a+b; 10a+2b; 9a+3b; 8a+4b; 7a+5b; 6a+6b; 5a+7b; 4a+8b; 3a+9b; 2a+10b; a+11b

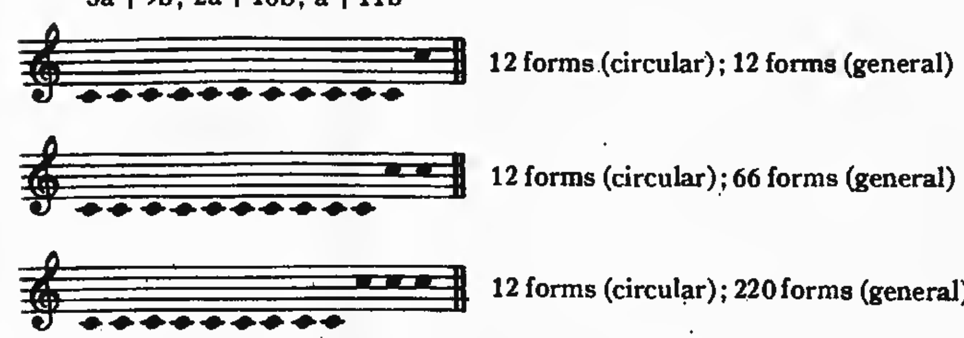
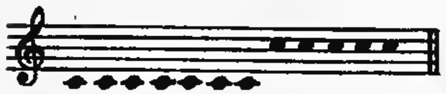
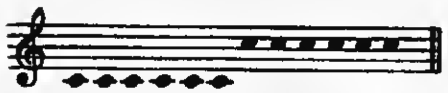


Figure 12. A = *12a.* (*continued*).


12 forms (circular); 495 forms (general)


12 forms (circular); 792 forms (general)

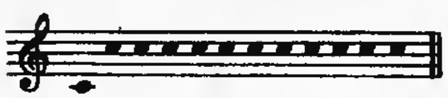

12 forms (circular); 924 forms (general)


12 forms (circular); 792 forms (general)


12 forms (circular); 495 forms (general)


12 forms (circular); 220 forms (general)


12 forms (circular); 66 forms (general)

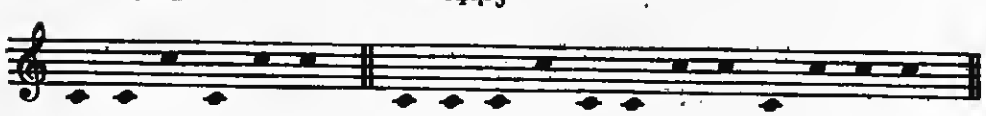

12 forms (circular); 12 forms (general)

Total: $12+66+220+495+792+924+792+495+220+66+12 = 4094$

Figure 12. A = 12a. (concluded).

Examples of the polynomial attack-groups (coefficients of recurrence).

$A = r_{3\div2}$ $A = r_{4+3}$



$A = r_{5\div4}$

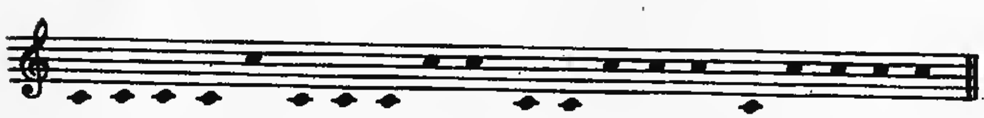
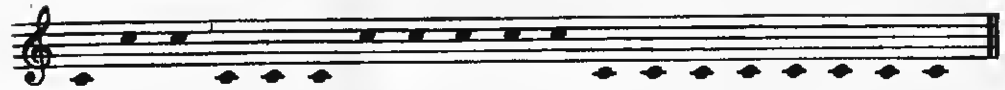


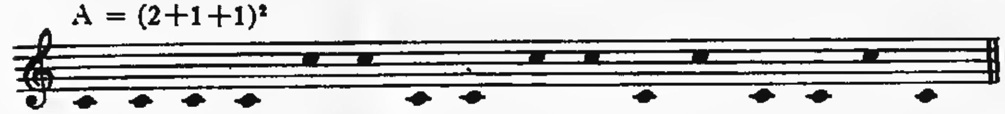
Figure 13. Polynomial attack-groups (continued).

A = Summation Series I

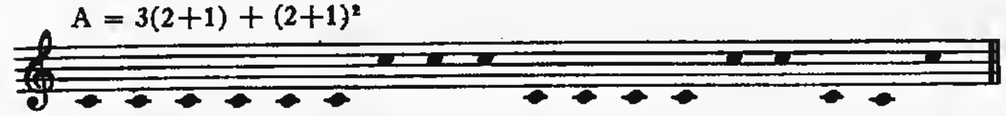


A = Summation Series II

$A = (2+1+1)^2$



$A = 3(2+1) + (2+1)^2$



$A = (3+1+1) + (1+3+1) + (1+1+3)$

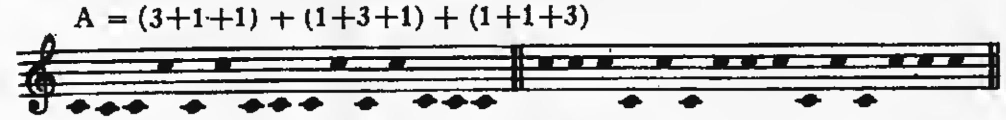


Figure 13. Polynomial attack-groups (concluded).

B. Instrumental Forms of $S = 2p$

The material for these forms is:

(a) coupled melody: $M\left(\frac{p_I}{p_{II}}\right)$;
(b) harmonic forms of two-unit scales;
(c) two-part harmony;
(d) two-parts of any harmony.

$$I = a: \frac{p_{II}}{p_I}, \frac{p_I}{p_{II}}; \frac{a_2}{b_2} + \frac{b_2}{a_2}; \frac{ma_2}{mb_2} + \frac{nb_2}{na_2}$$

I = ab, ba: permutations of the higher orders.

Coefficients of recurrence: $2a+b$; $a+2b$; . . . . . . $ma + nb$.

(a) Var.

Figure 14. Coupled melody (continued).

Var.

Var.

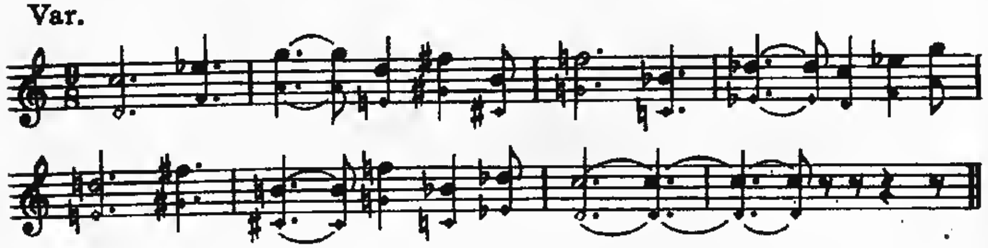
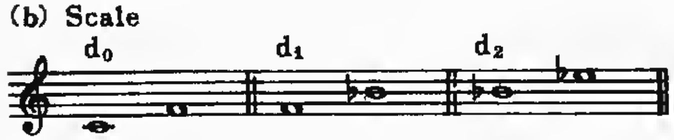


Figure 14. Coupled melody (concluded).

(b) Scale



$M = 2a_2 + 2b_2 + a_2 + b_2 + 2a_2 + 2b_2; d_0 + d_2 + d_1 \quad T = r_{5\div2}; \ T'' = 8t$

Theme

Figure 14A. Harmonic forms of two-unit scales (continued).

Figure 14A. Harmonic forms of two-unit scales (concluded).

Var. I: $\dfrac{2a_2+2b_2+a_2+b_2+2a_2+2b_2}{2b_2+2a_2+b_2+a_2+2b_2+2a_2}$

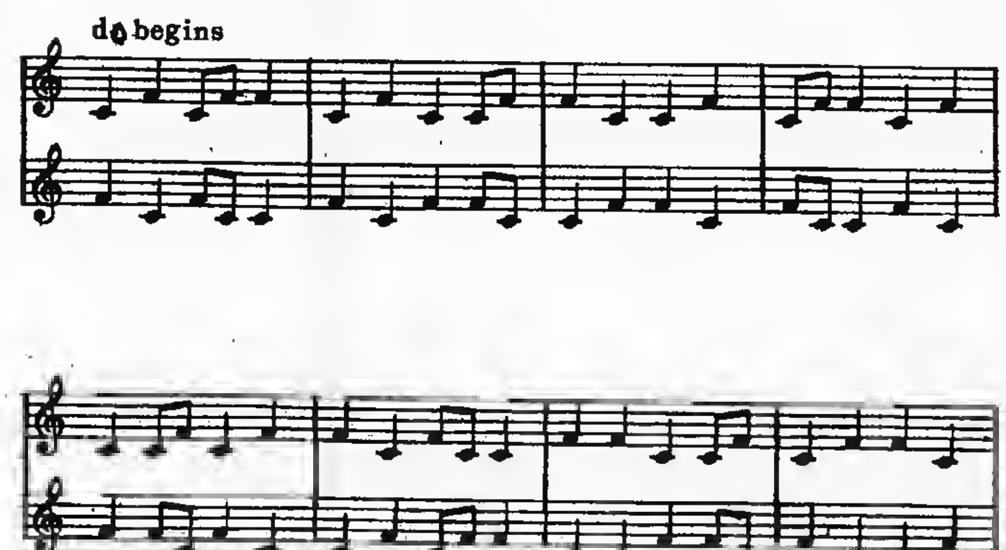
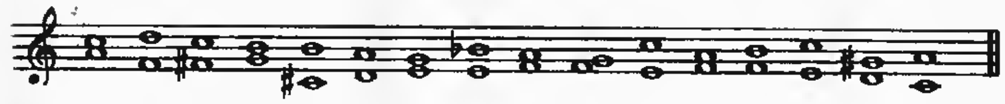


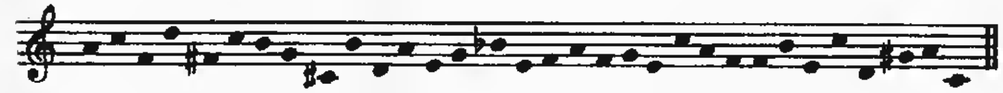
Figure 14B. Harmonic forms of two-unit scales (continued).

Figure 14B. Harmonic forms of two-unit scales (concluded).

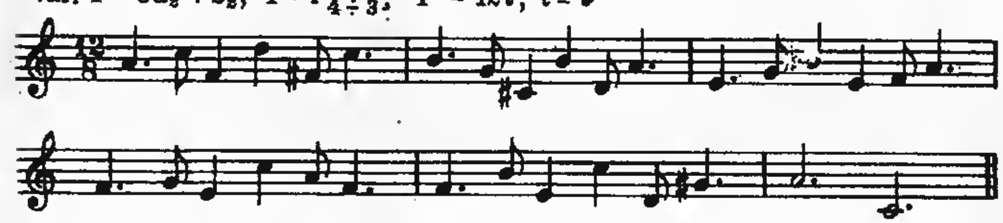
(c) Theme: $S = 2p$



Var. $I = 3a_2 + b_2$



Var. $I = 3a_2 + b_2$; $T = r_{4 \div 3}$; $T'' = 12t$; $t = ♪$



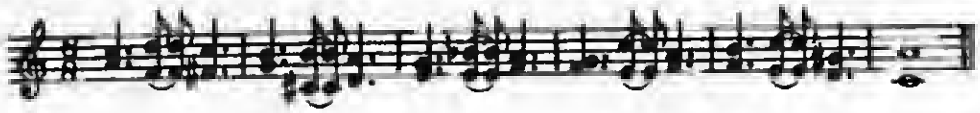
$T'' = 16t$; $t = 𝅘𝅥𝅯$

Var. I $\dfrac{2a_2 + 2b_2}{2b_2 + 2a_2} + \dfrac{2b_2 + 2a_2}{2a_2 + 2b_2}$

Figure 15. Two-part harmony (continued).

When the progression of chords ($H^{\rightarrow}$) has an assigned duration group, instrumental form (I) can be carried out through t.

Theme



Var. I = $a_4 + b_4$; t = ♪

Figure 15. Two-part harmony (concluded).

(d) Theme: $S = 4p$

Var. I $\left(\frac{pIV}{pIII}\right) = (b_2 + a_2 + b_2)\ H_1 + (a_2 + b_2 + a_2)\ H_2$

Figure 16. Two parts of any harmony. (continued).

Var. I $\left(\frac{pII}{pI}\right) = (a_2 + b_2 + a_2)\ H_1 + (b_2 + a_2 + b_2)\ H_2$

Var.: the two preceding variations combined

Theme: $T = r_{5 \div 4}$

Figure 16. Two parts of any harmony (continued).

Var. I $\left(\frac{p\text{III},\ p\text{IV}}{p\text{I},\ p\text{II}}\right) = \frac{2a_2 + b_2 + a_2 + 2b_2}{2b_2 + a_2 + b_2 + 2a_2}$; t = ♪

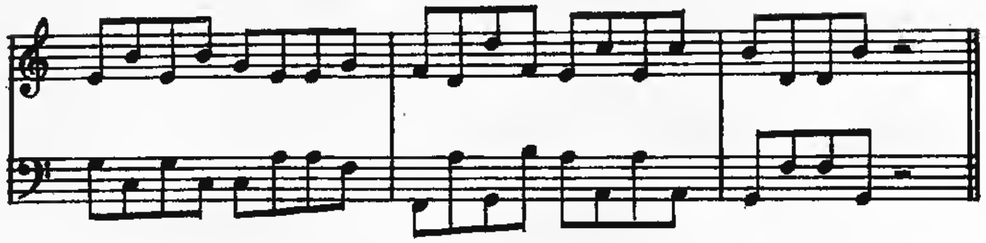


Figure 16. Two parts of any harmony (concluded).

Individual attacks emphasizing one or two parts can be combined into one attack-group of any desirable form.

Example:

b bb bb bbb bbbb b
I (S = 2p): aa ; aaaa ; aaa aa ; aa a aa ; . . .

Theme

b b b
Var.: a a a

Var.: (a a a $\overset{\text{b b b b}}{}$ a) H.

Figure 17. I (S = 2p.) (continued).

b b b b
Var.: a a a a; H = 4a = 4t.

Figure 17. I (S = 2p) (concluded).

CHAPTER 4

STRATA OF THREE PARTS

A. General Classification of I (S = 3p)

(A table of the combinations of attacks for a, b, and c)

A = a; 2a; 3a; 4a; 5a; 6a; 7a; 8a; 12a.

The following is a complete table of all forms of I(S = 3p). It includes *all the combinations and permutations* for 2, 3, 4, 5, 6, 7, 8 and 12 sequent attacks.*

(1) I = ap (one part, one attack).

Three invariant forms: a or b or c.
A = ap, 2ap, . . . map.
This is equivalent to I(S = p).

(2) I = a2p→ (one attack to a part, two sequent parts)

Three invariant forms: ab, ac, bc.
Each invariant form produces 2 attacks and has 2 permutations.
This is equivalent to I(S = 2p).
Further combinations of ab, ac, bc are not necessary as it corresponds to the forms of (3).

(3) I = a3p→ (one attack to a part, three sequent parts).

One invariant form: abc.
The invariant form produces 3 attacks and has 6 permutations:
abc, acb, cab, bac, bca, cba.
All other attack groups (A = 3 + n) develop from this source by means of the coefficients of recurrence.

*Here, as on other occasions, Schillinger uses convenient and brief rather than the full mathematical expressions to indicate relationships. For example, in an expression like S = p the coefficients are understood to be 1, i.e., 1 S = 1 p. It does *not* mean, as it would in strict mathematical form, that the number—any number—of strata equals the number of parts. Nor does the juxtaposition of, say, *a* and *p* as in *ap* imply multiplication; on the contrary, it means, as the text makes clear, "one attack to one part"—which would be expressed in rigid mathematical form as $\frac{A_1 p}{1 p} = 1$. (Ed.)

I(S = 3p): attack-groups for one simultaneous p.

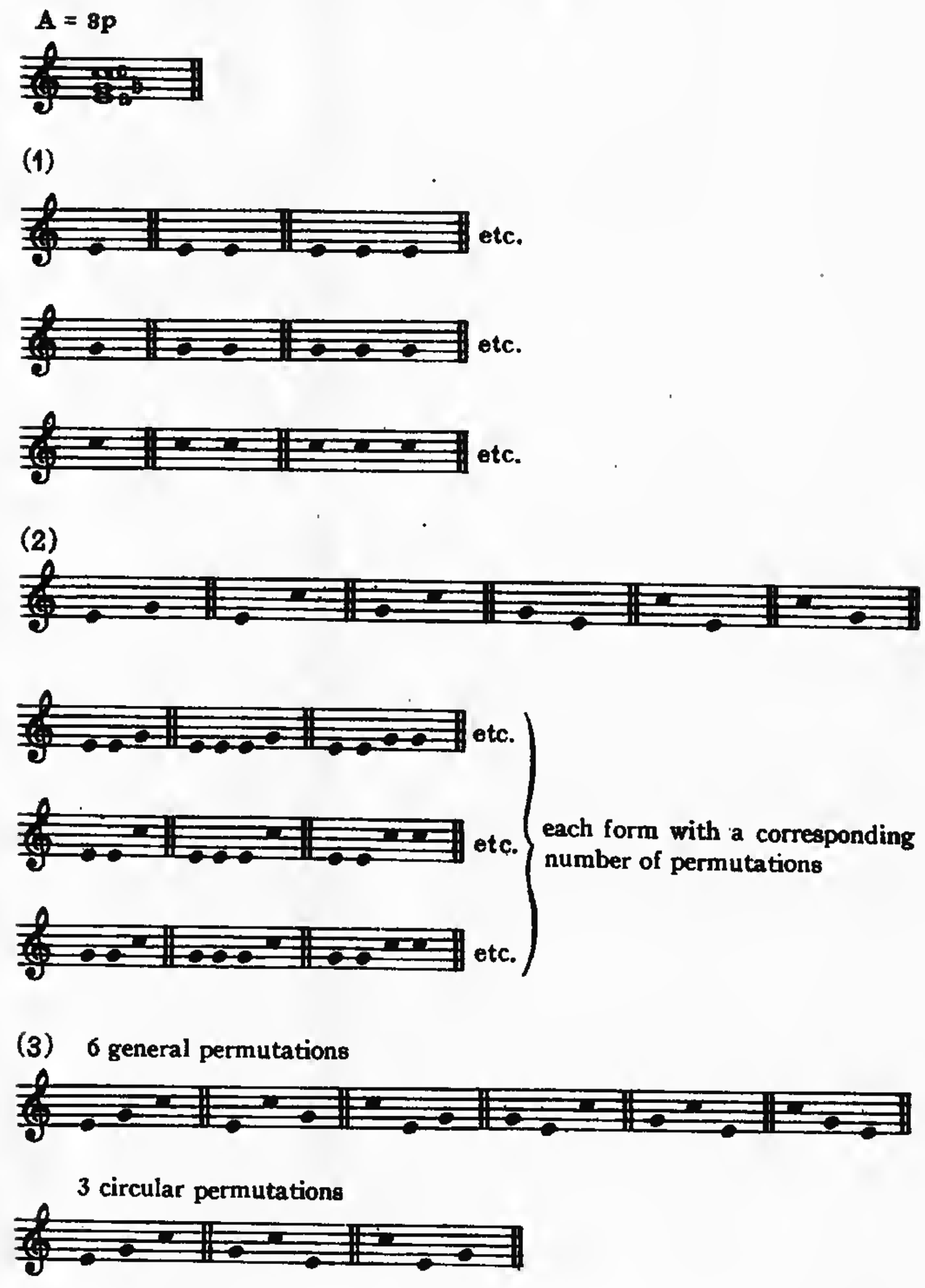


Figure 18. I (S = 3p)

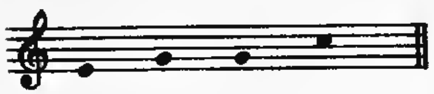
B. Development of Attack-Groups by Means of the Coefficients of Recurrence

A = **4a**; 2a+b+c; a+2b+c; a+b+2c.

$$P_4 = \frac{4!}{2!} = \frac{24}{2} = 12$$

Each of the above 3 permutations of the coefficients has 12 general permutations.


12 general or 4 circular permutations


12 general or 4 circular permutations

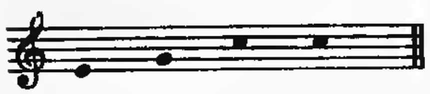

12 general or 4 circular permutations

Figure 19. A = 4a; 2a+b+c; a+2b+c; a+b+2c

Total in general permutations: 12+12+12 = 36
Total in circular permutations: 4+4+4 = 12

A = 5a.

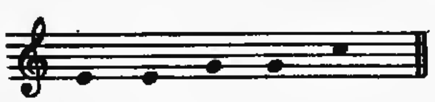
Forms of the distribution of coèfficients:

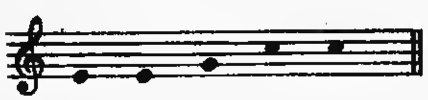
5 = 2+2+1 and 5 = 1+1+3

$A = 2a+2b+c;\ 2a+b+2c;\ a+2b+2c$

$$P_5 = \frac{5!}{2!\ 2!} = \frac{120}{2\cdot 2} = 30$$

Each of the 3 permutations of the first form of distribution has 30 general permutations. Total: 30·3 = 90.


30 general or 5 circular permutations


30 general or 5 circular permutations


30 general or 5 circular permutations

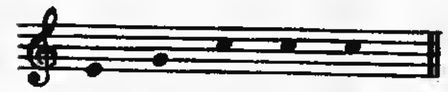
Figure 20. A = 5a; 2a+2b+c; 2a+b+2c; a+2b+2c

Total in general permutations: 30+30+30 = 90
Total in circular permutations: 5+5+5 = 15

$A = a+b+3c;\ a+3b+c;\ 3a+b+c.$

$$P_5 = \frac{5!}{3!} = \frac{120}{6} = 20$$

Each of the above 3 permutations of the second form of distribution has 20 general permutations. Total: 20·3 = 60.


20 general or 5 circular permutations


20 general or 5 circular permutations

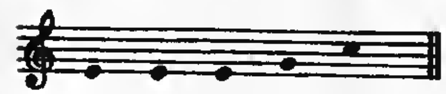

20 general or 5 circular permutations

Figure 21. A = 5a; a+b+3c; a+3b+c; 3a+b+c.

Total in general permutations: 20+20+20 = 60
Total in circular permutations: 5+5+5 = 15
The entire total for 5 attacks: in general permutations: 150
in circular permutations: 30

A = 6a.

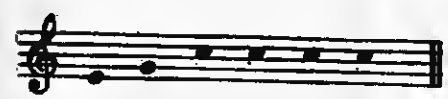
Forms of the distribution of coefficients:

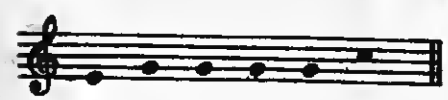
6 = 1+1+4; 1+2+3; 2+2+2.

$A = a+b+4c;\ a+4b+c;\ 4a+b+c.$

$$P_6 = \frac{6!}{4!} = \frac{720}{24} = 30$$

Each of the above 3 permutations of the first form of distribution has 30 general permutations.


30 general or 6 circular permutations


30 general or 6 circular permutations

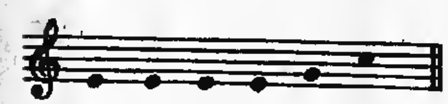

30 general or 6 circular permutations

Figure 22. A = 6a; a+b+4c; a+4b+c; a+b+4c.

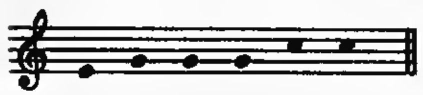
Total in general permutations: 30·3 = 90
Total in circular permutations: 6·3 = 18

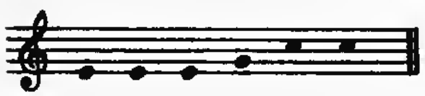
$A = a+2b+3c;\ a+3b+2c;\ 3a+b+2c;\ 2a+b+3c;\ 2a+3b+c;\ 3a+2b+c.$

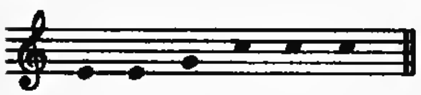
$$P_6 = \frac{6!}{2!\,3!} = \frac{720}{2\cdot 6} = 60$$

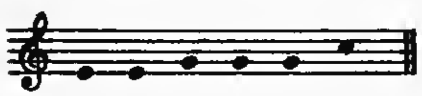
Each of the above 6 permutations of the second form of distribution has 60 general permutations.


60 general or 6 circular permutations


60 general or 6 circular permutations


60 general or 6 circular permutations


60 general or 6 circular permutations


60 general or 6 circular permutations

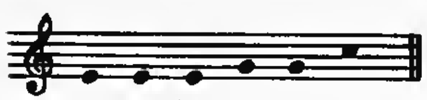

60 general or 6 circular permutations

Figure 23. $A = 6a;\ a+2b+3c;\ a+3b+2c;\ 3a+b+2c;\ 2a+b+3c;\ 2a+3b+c;\ 2a+2b+c$

Total in general permutations: $60\cdot 6 = 360$
Total in circular permutations: $6\cdot 6 = 36$

$A = 2a+2b+2c.$

$$P_6 = \frac{6!}{2!\,2!\,2!} = \frac{720}{2\cdot 2\cdot 2} = 90$$

The third form of distribution (invariant) has 90 general permutations

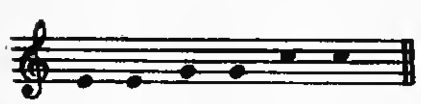

90 general or 6 circular permutations

Figure 24. $A = 6a$: $2a+2b+2c$.

The entire total for 6 attacks: in general permutations: 540
in circular permutations: 60

A = 7a.

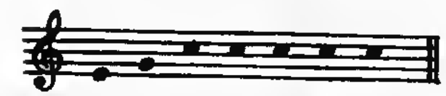
Forms of the distribution of coefficients:

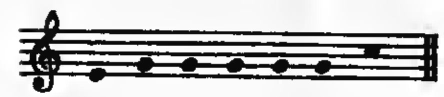
$7 = 1+1+5;\ 1+2+4;\ 2+2+3;\ 3+3+1$

$A = a+b+5c;\ a+5b+c;\ 5a+b+c,.$

$$P_7 = \frac{7!}{5!} = \frac{5040}{120} = 42$$

Each of the above 3 permutations of the first form of distribution has 42 general permutations.


42 general or 7 circular permutations


42 general or 7 circular permutations

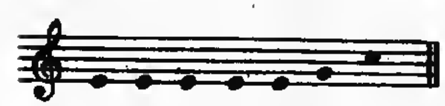

42 general or 7 circular permutations

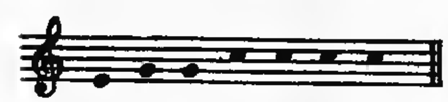
Figure 25. $A = 7a;\ a+b+5c;\ a+5b+c;\ 5a+b+c.$

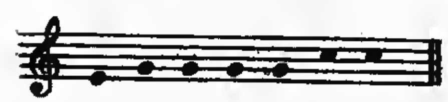
Total in general permutations: $42\cdot 3 = 126$
Total in circular permutations: $7\cdot 3 = 21$

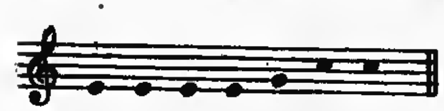
$A = a+2b+4c;\ a+4b+2c;\ 4a+b+2c;\ 2a+b+4c;\ 2a+4b+c;\ 4a+2b+c.$

$$P_7 = \frac{7!}{2!\,4!} = \frac{5040}{2\cdot 24} = 105$$

Each of the above 6 permutations of the second form of distribution has 105 general permutations.


105 general or 7 circular permutations


105 general or 7 circular permutations


105 general or 7 circular permutations

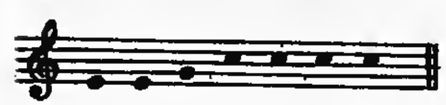

105 general or 7 circular permutations

Figure 26. $A = 7a;\ a+2b+4c;\ a+4b+2c;\ 4a+b+2c;\ 2a+b+4c;\ 2a+4b+c;\ 4a+2b+c$ (*continued*).


105 general or 7 circular permutations

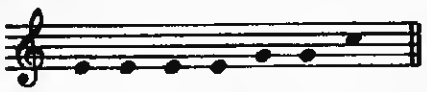

105 general or 7 circular permutations

Figure 26. $A = 7a;\ a+2b+4c;\ a+4b+2c;\ 4c+b+2c;\ 2a+b+4c;\ 2a+4b+c;\ 4a+2b+c$ (*concluded*).

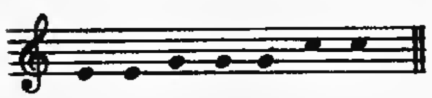
Total in general permutations: $105 \cdot 6 = 630$
Total in circular permutations: $7 \cdot 6 = 42$

$A = 2a+2b+3c;\ 2a+3b+2c;\ 3a+2b+2c.$

$$P_7 = \frac{7!}{2!\ 3!\ 2!} = \frac{5040}{2 \cdot 6 \cdot 2} = 210$$

Each of the above 3 permutations of the third form of distribution has 210 general permutations.


210 general or 7 circular permutations


210 general or 7 circular permutations

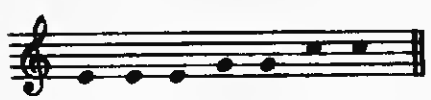

210 general or 7 circular permutations

Figure 27. $A = 7a;\ 2a+2b+3c;\ 2a+3b+2c;\ 3a+2b+2c.$

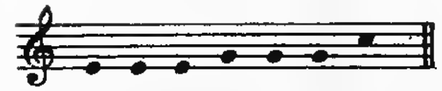
Total in general permutations: $210 \cdot 3 = 630$
Total in circular permutations: $7 \cdot 3 = 21$

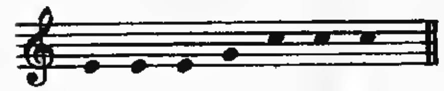
$A = 3a+3b+c;\ 3a+b+3c;\ a+3b+3c.$

$$P_7 = \frac{7!}{3!\ 3!} = \frac{5040}{6 \cdot 6} = 140$$

Each of the above 3 permutations of the fourth form of distribution has 140 general permutations.

See Figure 28 on the following page.


140 general or 7 circular permutations


140 general or 7 circular permutations


140 general or 7 circular permutations

Figure 28. $A = 7a;\ 3a+3b+c;\ 3a+b+3c;\ a+3b+3c.$

Total in general permutations: $140 \cdot 3 = 420$
Total in circular permutations: $7 \cdot 3 = 21$

The entire total for 7 attacks: in general permutations: 1806
in circular permutations: 105

A = 8a.

Forms of the distribution of coefficients:

$8 = 1+1+6;\ 1+2+5;\ 1+3+4;\ 2+2+4;\ 2+3+3$

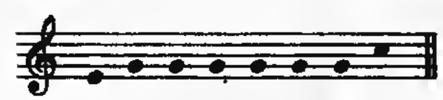
$A = a+b+6c;\ a+6b+c;\ 6a+b+c.$

$$P_8 = \frac{8!}{6!} = \frac{40{,}320}{720} = 56$$

Each of the two above 3 permutations of the first form of distribution has 56 general permutations.

Total: $56 \cdot 3 = 168$


56 general or 8 circular permutations


56 general or 8 circular permutations

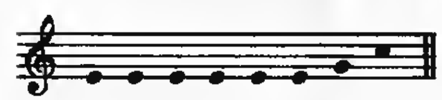

56 general or 8 circular permutations

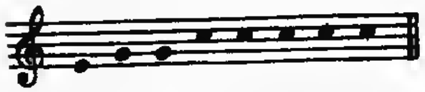
Figure 29. $A = 8a;\ a+b+6c;\ a+6b+c;\ 6a+b+c.$

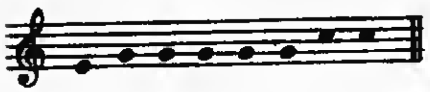
Total in general permutations: $56 \cdot 3 = 168$
Total in circular permutations: $8 \cdot 3 = 24$

$A = a+2b+5c; a+5b+2c; 5a+b+2c; 2a+b+5c; 2a+5b+c; 5a+2b+c.$

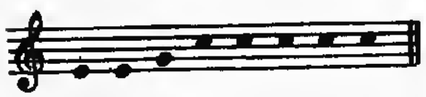
$$P_8 = \frac{8!}{2!\ 5!} = \frac{40{,}320}{2 \cdot 120} = 168$$

Each of the above 6 permutations of the second form of distribution has 168 general permutations.


168 general or 8 circular permutations


168 general or 8 circular permutations


168 general or 8 circular permutations


168 general or 8 circular permutations


168 general or 8 circular permutations

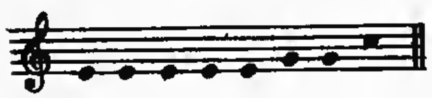

168 general or 8 circular permutations

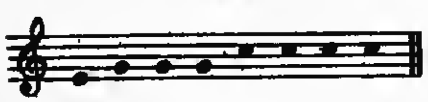
Figure 30. $A = 8a;\ a+2b+5c;\ a+5b+2c;\ 5a+b+2c;\ 2a+b+5c;\ 2a+5b+c;\ 5a+2b+c.$

Total in general permutations: $168 \cdot 6 = 1008$
Total in circular permutations: $8 \cdot 6 = 48$

$A = a+3b+4c; a+4b+3c; 4a+b+3c; 3a+b+4c; 3a+4b+c; 4a+3b+c.$

$$P_8 = \frac{8!}{3!\ 4!} = \frac{40{,}320}{6 \cdot 24} = 280$$

Each of the above 6 permutations of the third form of distribution has 280 general permutations.


280 general or 8 circular permutations

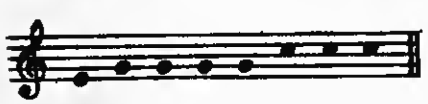
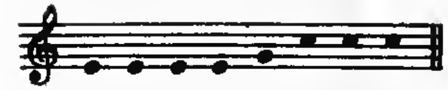
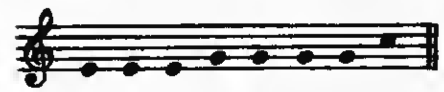

280 general or 8 circular permutations

Figure 31. $A = 8a;\ a+3b+4c;\ a+4b+3c;\ 4a+b+3c;\ 3a+b+4c;\ 3a+4b+c;\ 4a+3b+c.$ *(continued)*


280 general or 8 circular permutations


280 general or 8 circular permutations


280 general or 8 circular permutations

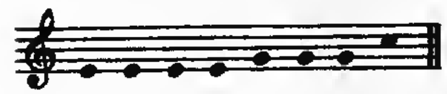

280 general or 8 circular permutations

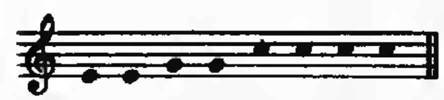
Figure 31. $A = 8a;\ a+3b+4c;\ a+4b+3c;\ 4a+b+3c;\ 3a+b+4c;\ 3a+4b+c;\ 4a+3b+c$ *(concluded).*

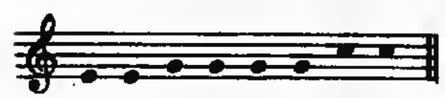
Total in general permutations: $280 \cdot 6 = 1680$
Total in circular permutations: $8 \cdot 6 = 48$

$A = 2a+2b+4c; 2a+4b+2c; 4a+2b+2c$

$$P_8 = \frac{8!}{2!\ 2!\ 4!} = \frac{40{,}320}{2 \cdot 2 \cdot 24} = 420$$

Each of the above 3 permutations of the fourth form of distribution has 420 general permutations.


420 general or 8 circular permutations


420 general or 8 circular permutations

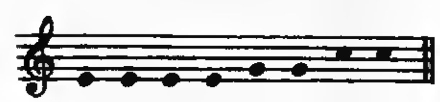

420 general or 8 circular permutations

Figure 32. $A = 8a;\ 2a+2b+4c;\ 2a+4b+2c;\ 4a+2b+2c.$

Total in general permutations: $420 \cdot 3 = 1260$
Total in circular permutations: $8 \cdot 3 = 24$

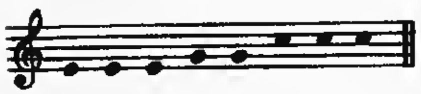
$A = 2a+3b+3c; 3a+2b+3c; 3a+3b+2c$

$$P_8 = \frac{8!}{2!\ 3!\ 3!} = \frac{40{,}320}{2 \cdot 6 \cdot 6} = 560$$

Each of the above 3 permutations of the fifth form of distribution has 560 general permutations.

See Figure 33 on the following page.

 560 general or 8 circular permutations

 560 general or 8 circular permutations

 560 general or 8 circular permutations

Figure 33. $A = 8a;\ 2a+3b+3c;\ 3a+2b+3c;\ 3a+3b+2c.$

Total in general permutations: $560\cdot3 = 1680$
Total in circular permutations: $8\cdot3 = 24$

The total number of cases: A = 8a
General permutations: 168 + 1008 + 1680 + 1260 + 1680 = 5796
Circular permutations: 24 + 48 + 48 + 24 + 24 = 168

A = 12a.

Forms of the distribution of coefficients:

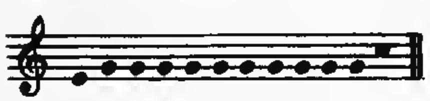
$8 = 1+1+10;\ 1+2+9;\ 1+3+8;\ 1+4+7;\ 1+5+6;\ 2+2+8;\ 2+3+7;\ 2+4+6;\ 2+5+5;\ 3+3+6;\ 3+4+5;\ 4+4+4.$

$A = a+b+10c;\ a+10b+c;\ 10a+b+c$

$$P_{12} = \frac{12!}{10!} = \frac{479{,}001{,}600}{3{,}628{,}800} = 132$$

Each of the above 3 permutations of the first form of distribution has 132 general permutations.

 132 general or 12 circular permutations

 132 general or 12 circular permutations

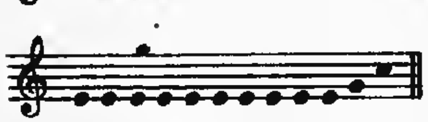
 132 general or 12 circular permutations

Figure 34. $A = 12a;\ a+b+10c;\ a+10b+c;\ 10a+b+c.$

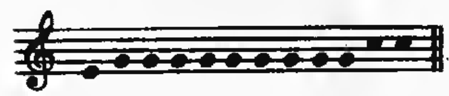
Total in general permutations: $132\cdot3 = 396$
Total in circular permutations: $12\cdot3 = 36$

$A = a+2b+9c;\ a+9b+2c;\ 9a+b+2c;\ 2a+b+9c;\ 2a+9b+c;\ 9a+2b+c.$

$$P_{12} = \frac{12!}{2!\ 9!} = \frac{479{,}001{,}600}{2\cdot362{,}880} = 660$$

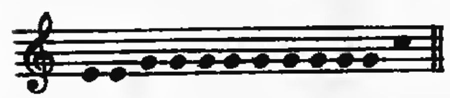
Each of the above 6 permutations of the second form of distribution has 660 general permutations.

 660 general or 12 circular permutations

 660 general or 12 circular permutations

 660 general or 12 circular permutations

 660 general or 12 circular permutations

 660 general or 12 circular permutations

 660 general or 12 circular permutations

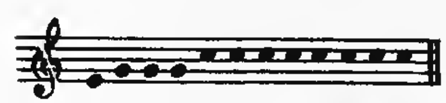
Figure 35. $A = 12a;\ a+2b+9c;\ a+9b+2c;\ 9a+b+2c;\ 2a+b+9c;\ 2a+9b+c;\ 9a+2b+c.$

Total in general permutations: $660\cdot6 = 3960$
Total in circular permutations: $12\cdot6 = 72$

$A = a+3b+8c;\ a+8b+3c;\ 8a+b+3c;\ 3a+b+8c;\ 3a+8b+c;\ 8a+3b+c.$

$$P_{12} = \frac{12!}{3!\ 8!} = \frac{479{,}001{,}600}{6\cdot40{,}320} = 1980$$

Each of the above 6 permutations of the third form of distribution has 1980 general permutations.

 1980 general or 12 circular permutations

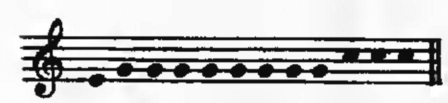
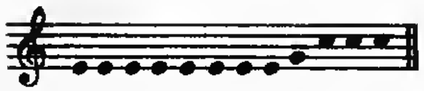
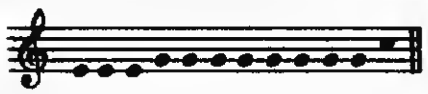
 1980 general or 12 circular permutations

Figure 36. $A = 12a;\ a+3b+8c;\ a+8b+3c;\ 8a+b+3c;\ 3a+b+8c;\ 3a+8b+c;\ 8a+3b+c$ (*continued*).


1980 general or 12 circular permutations


1980 general or 12 circular permutations


1980 general or 12 circular permutations


1980 general or 12 circular permutations

Figure 36. $A = 12a;\ a+3b+8c;\ a+8b+3c;\ 8a+b+3c;\ 3a+b+8c;\ 3a+8b+c;\ 8a+3b+c.$

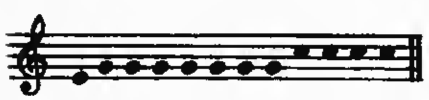
Total in general permutations: $1980 \cdot 6 = 11{,}880$
Total in circular permutations: $12 \cdot 6 = 72$

$A = a+4b+7c;\ a+7b+4c;\ 7a+b+4c;\ 4a+b+7c;\ 4a+7b+c;\ 7a+4b+c.$

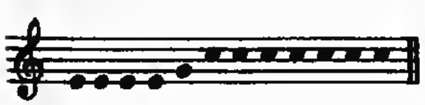
$$P_{12} = \frac{12!}{4!\,7!} = \frac{479{,}001{,}600}{24 \cdot 5{,}040} = 3960$$

Each of the above 6 permutations of the fourth form of distribution has 3960 general permutations.


3960 general or 12 circular permutations


3960 general or 12 circular permutations


3960 general or 12 circular permutations


3960 general or 12 circular permutations

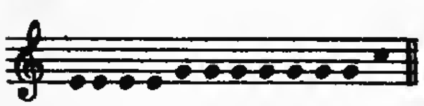

3960 general or 12 circular permutations

Figure 37. $A = 12a;\ a+4b+7c;\ a+7b+4c;\ 7a+b+4c;\ 4a+b+7c;\ 4a+7b+c;\ 7a+4b+c$ (*continued*).

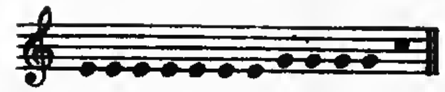

3960 general or 12 circular permutations

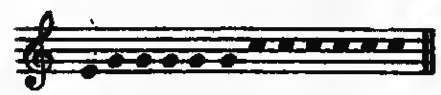
Figure 37. $A = 12a;\ a+4b+7c;\ a+7b+4c;\ 7a+b+4c;\ 4a+b+7c;\ 4a+7b+c;\ 7a+4b+c.$ (*concluded*.)

Total in general permutations: $3960 \cdot 6 = 23{,}760$
Total in circular permutations: $12 \cdot 6 = 72$

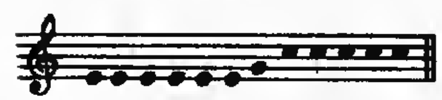
$A = a+5b+6c;\ a+6b+5a;\ 6a+b+5c;\ 5a+b+6c;\ 5a+6b+c;\ 6a+5b+c.$

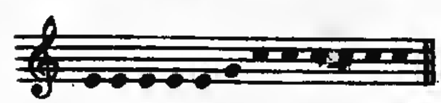
$$P_{12} = \frac{12!}{5!\,6!} = \frac{479{,}001{,}600}{120 \cdot 720} = 5544$$

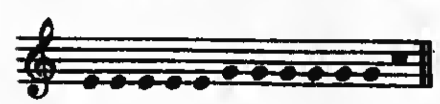
Each of the above 6 permutations of the fifth form of distribution has 5544 general permutations.


5544 general or 12 circular permutations


5544 general or 12 circular permutations


5544 general or 12 circular permutations


5544 general or 12 circular permutations


5544 general or 12 circular permutations

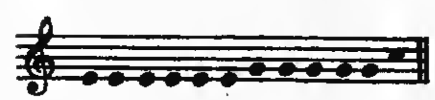

5544 general or 12 circular permutations

Figure 38. $A = 12a;\ a+5b+6c;\ a+6b+5c;\ 6a+b+5c;\ 5a+b+6c;\ 5a+6b+c;\ 6a+5b+c.$

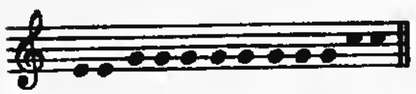
Total in general permutations: $5544 \cdot 6 = 32{,}264$
Total in circular permutations: $12 \cdot 6 = 72$

$A = 2a+2b+8c;\ 2a+8b+2c;\ 8a+2b+2c.$

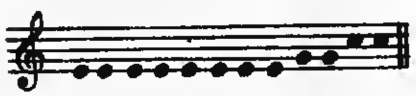
$$P_{12} = \frac{12!}{2!\ 2!\ 8!} = \frac{479{,}001{,}600}{2\cdot 2\cdot 40{,}320} = 2970$$

Each of the above 3 permutations of the sixth form of distribution has 2970 general permutations.

2970 general or 12 circular permutations



2970 general or 12 circular permutations



2970 general or 12 circular permutations

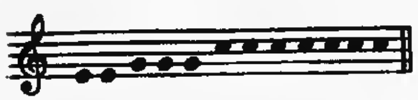
Figure 39. $A = 12a;\ 2a+2b+8c;\ 2a+8b+2c;\ 8a+2b+2c$

Total in general permutations: $2970\cdot 3 = 8910$
Total in circular permutations: $12\cdot 3 = 36$

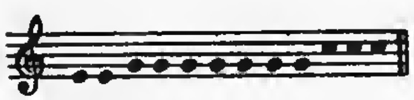
$A = 2a+3b+7c;\ 2a+7b+3c;\ 7a+2b+3c;\ 3a+2b+7c;\ 3a+7b+2c;\ 7a+3b+2c.$

$$P_{12} = \frac{12!}{2!\ 3!\ 7!} = \frac{479{,}001{,}600}{2\cdot 6\cdot 5{,}040} = 7920$$

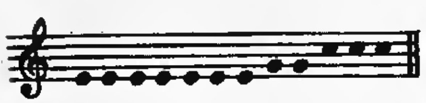
Each of the above 6 permutations of the seventh form of distribution has 7920 general permutations.



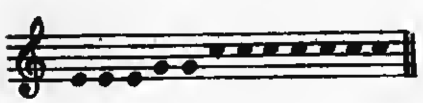
7920 general or 12 circular permutations



7920 general or 12 circular permutations

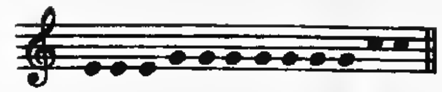


7920 general or 12 circular permutations

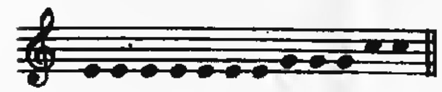


7920 general or 12 circular permutations

Figure 40. $A = 12a;\ 2a+3b+7c;\ 2c+7b+3c;\ 7a+2b+3c;\ 3a+2b+7c;\ 3c+7b+2c;\ 7a+3b+2c.$ (*continued*).



7920 general or 12 circular permutations



7920 general or 12 circular permutations

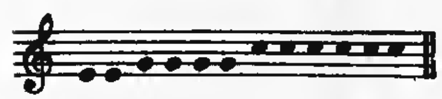
Figure 40. $A = 12a;\ 2a+3b+7c;\ 2a+7b+3c;\ 7a+2b+3c;\ 3a+2b+7c;\ 3a+7b+2c;\ 7a+3b+2c.$ (*concluded*).

Total in general permutations: $7920\cdot 6 = 47{,}520$
Total in circular permutations: $12\cdot 6 = 72$

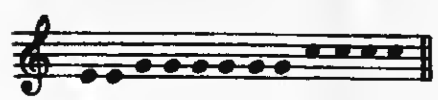
$A = 2a+4b+6c;\ 2a+6b+4c;\ 6a+2b+4c;\ 4a+2b+6c;\ 4a+6b+2c;\ 6a+4b+2c.$

$$P_{12} = \frac{12!}{2!\ 4!\ 6!} = \frac{479{,}001{,}600}{2\cdot 24\cdot 720} = 1386$$

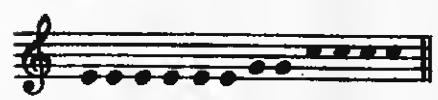
Each of the above 6 permutations of the eighth form of distribution has 1386 general permutations.



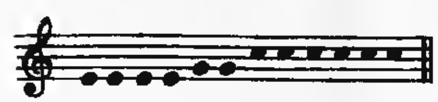
1386 general or 12 circular permutations



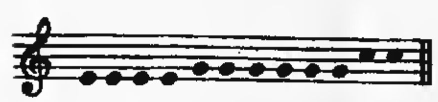
1386 general or 12 circular permutations



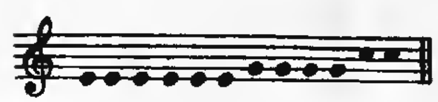
1386 general or 12 circular permutations



1386 general or 12 circular permutations



1386 general or 12 circular permutations



1386 general or 12 circular permutations

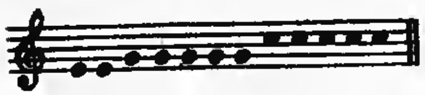
Figure 41. $A = 12a;\ 2a+4b+6c;\ 2a+6b+4c;\ 6a+2b+4c;\ 4a+2b+6c;\ 4a+6b+2c;\ 6a+4b+2c.$

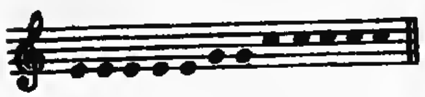
Total in general permutations: $1386\cdot 6 = 8316$
Total in circular permutations: $12\cdot 6 = 72$

$A = 2a+5b+5c;\ 5a+2b+5c;\ 5a+5b+2c.$

$$P_{12} = \frac{12!}{2!\ 5!\ 5!} = \frac{479{,}001{,}600}{2 \cdot 120 \cdot 120} = 16{,}632$$

Each of the above 3 permutations of the ninth form of distribution has 16,632 general permutations.


16,632 general or 12 circular permutations


16,632 general or 12 circular permutations


16,632 general or 12 circular permutations

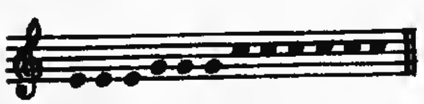
Figure 42. $A = 12a;\ 2a+5b+5c;\ 5a+2b+5c;\ 5a+5b+2c.$

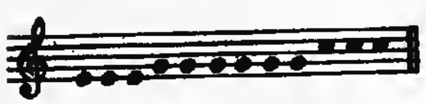
Total in general permutations: $16{,}632 \cdot 3 = 49{,}896$
Total in circular permutations: $12 \cdot 3 = 36$

$A = 3a+3b+6c;\ 3a+6b+3c;\ 6a+3b+3c.$

$$P_{12} = \frac{12!}{3!\ 3!\ 6!} = \frac{479{,}001{,}600}{6 \cdot 6 \cdot 720} = 18{,}480$$

Each of the above 3 permutations of the tenth form of distribution has 18,480 general permutations.


18,480 general or 12 circular permutations


18,480 general or 12 circular permutations

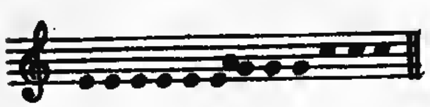

18,480 general or 12 circular permutations

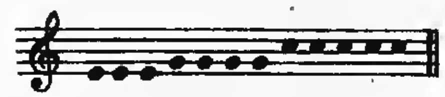
Figure 43. $A = 12a:\ 3a+3b+6c:\ 3a+6b+3c:\ 6a+3b+3c$

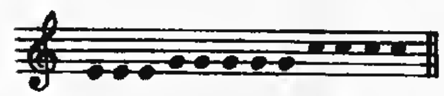
Total in general permutations: $18{,}480 \cdot 3 = 55{,}440$
Total in circular permutations: $12 \cdot 3 = 36$

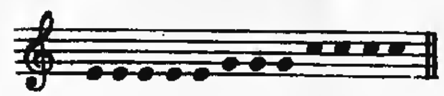
$A = 3a+4b+5c;\ 3a+5b+4c;\ 5a+3b+4c;$
$4a+3b+5c;\ 4a+5b+3c;\ 5a+4b+3c.$

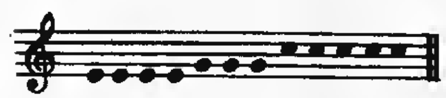
$$P_{12} = \frac{12!}{3!\ 4!\ 5!} = \frac{479{,}001{,}600}{6 \cdot 24 \cdot 120} = 27{,}720$$

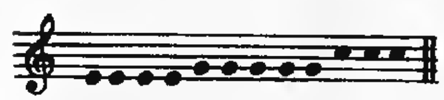
Each of the above 6 permutations of the eleventh form of distribution has 27,720 general permutations.


27,720 general or 12 circular permutations


27,720 general or 12 circular permutations


27,720 general or 12 circular permutations


27,720 general or 12 circular permutations


27,720 general or 12 circular permutations

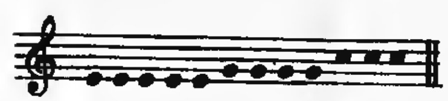

27,720 general or 12 circular permutations

Figure 44. $A = 12a;\ 3a+4b+5c;\ 3a+5b+4c;\ 5a+3b+4c;$
$4a+3b+5c;\ 4a+5b+3c;\ 5a+4b+3c.$

Total in general permutations: $27{,}720 \cdot 6 = 166{,}320$
Total in circular permutations: $12 \cdot 6 = 72$

$A = 4a+4b+4c$

$$P_{12} = \frac{12!}{4!\ 4!\ 4!} = \frac{479{,}001{,}600}{24 \cdot 24 \cdot 24} = 34{,}650$$


34,650 general or 12 circular permutations

Figure 45. $A = 12a;\ 4a+4b+4c.$

The total number of cases: $A = 12a$.

General permutations: $396 + 3960 + 11{,}880 + 23{,}760 + 32{,}264 + 8910 + 47{,}520 + 8316 + 49{,}896 + 55{,}440 + 166{,}320 + 34{,}650 = 443{,}312$.

Circular permutations: $36 + 72 + 72 + 72 + 72 + 36 + 72 + 72 + 36 + 36 + 72 + 12 = 660$

(4) **I = a2p (one attack to a combination or two simultaneous parts).***

The three invariant forms of (2) become elements of the second order:

$$\frac{b}{a} = a_2\,;\quad \frac{c}{a} = b_2\,;\quad \frac{c}{b} = c_2$$

Further combinations in sequence necessitate the inclusion of all three parts.

Sequent combinations by two:

$$a_2 + b_2\,;\ a_2 + c_2\,;\ b_2 + c_2$$

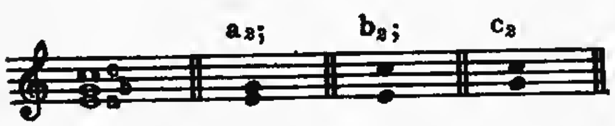
This corresponds to two consecutive attacks. The growth of attack-groups is achieved by means of the coefficients of recurrence:

$$2a_2 + b_2\,;\ 3b_2 + c_2\,;\ 2a_2 + 3b_2\,;\ 2a_2 + c_2 + a_2 + 2c_2$$

The latter, in turn, become subject to permutations (general or circular), as well as to permutations of the higher orders.

Sequent combination by three (there is only one such combination): $a_2 + b_2 + c_2$. The latter with its permutations becomes an element of the third order: $a_2 + b_2 + c_2 = a_3$. The development of attack groups by means of the coefficients of recurrence corresponds to figures 19-45 inclusive in classification and quantity.

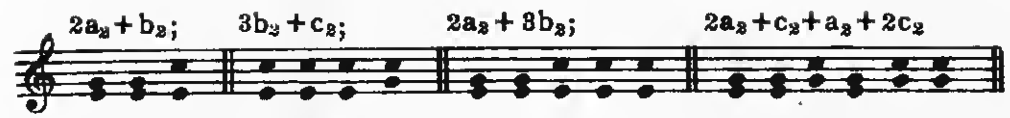
Table of $I(S = 3p) = a2p$.

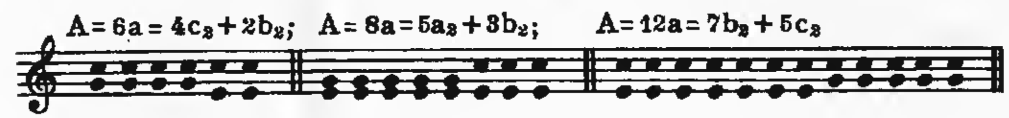


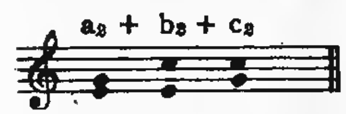
2 permutations to each combination

Figure 46. I (S = 3p) = a2p (continued).

*We are here concerned no longer with I = a2p→, in which the 2p→ denotes two sequent parts, but with I = a2p, in which the absence of the → denotes two simultaneous parts. (Ed.)

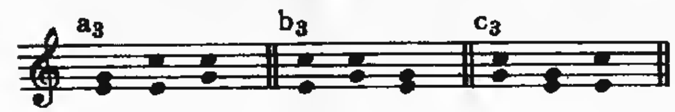






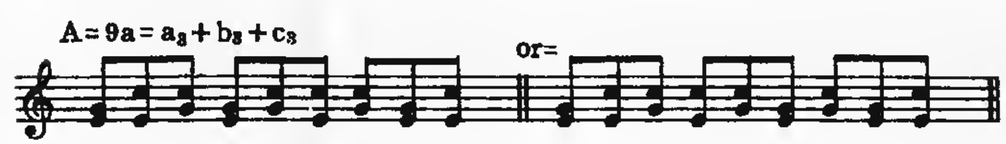
6 general or 3 circular permutations

(general)



(circular)

Combinations of the higher orders:



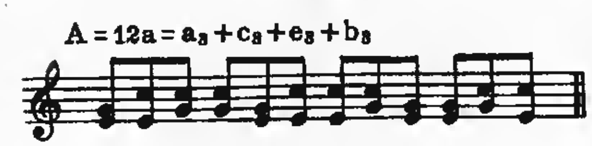


Figure 46. I (S = 3p) = a2p (concluded).

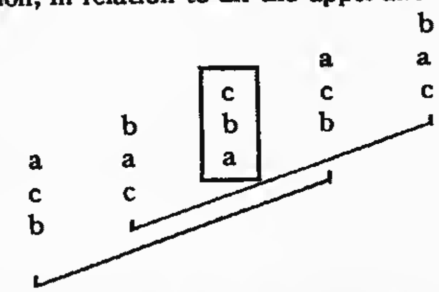
(5) **I = a3p (one attack to a combination of three simultaneous parts)**

One invariant form: $\frac{\frac{c}{b}}{a} = a_3$

Multiplication of attacks is achieved by direct repetition: A = a_3; $2a_3$; $3a_3$; . . . ma_3.

Further variations may be obtained by means of permutations of the vertical (simultaneous) arrangement of parts. The extreme p→ of a given position must serve as a limit, that is, for a position above the original, *c* is the limit for the lower function, and for a position below the original, *a* is the limit for the upper function.

The original position, in relation to all the upper and all the lower positions, is:*



The positions indicated by the brackets are ***identical but in different octaves***. It is desirable to use the adjacent positions in a sequence. From the above variations of the original position, any number of attacks can be devised.

Table of I(S = 3p) = a3p

Figure 47. I (S = 3p) = a3p (continued).

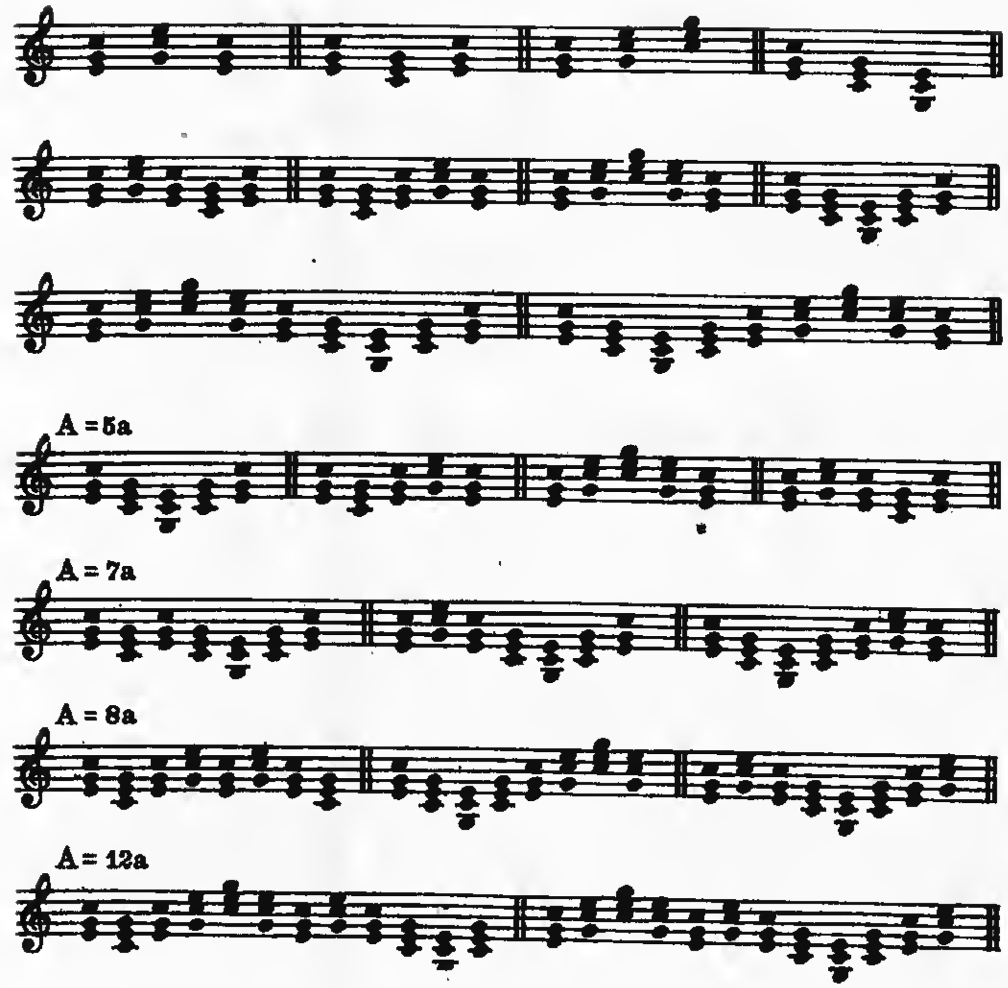


Figure 47. I (S = 3p) = a3p (concluded).

C. Instrumental Forms of S = 3p

Material:

(1). melody with two couplings: M $\begin{pmatrix} \frac{p_{III}}{\frac{p_{II}}{p_I}} \end{pmatrix}$;
(2). harmonic forms of three-unit scales;
(3). three-part harmony;
(4). three parts of any harmony.

$$1 = a: \begin{array}{c} p_{III} \\ \hline p_{II} \\ \hline p_{I} \end{array} \quad \text{(6 general or 3 circular permutations);}$$

$$\begin{array}{c} c_2 \\ \hline b_2 \\ \hline a_2 \end{array} \quad \text{(6 general or 3 circular permutations);}$$

$$\begin{array}{c} mc_2 \\ \hline mb_2 \\ \hline ma_2 \end{array} + \begin{array}{c} nc_2 \\ \hline nb_2 \\ \hline na_2 \end{array} + \begin{array}{c} pc_2 \\ \hline pb_2 \\ \hline pa_2 \end{array} \quad \text{(6 general or 3 circular permutations of the coefficients m, n, p)}$$

1. Melody with two couplings: $M \begin{pmatrix} p_{III} \\ \hline p_{II} \\ \hline p_{I} \end{pmatrix}$. Illustrated by a theme and six variations. See figures 48 to 54 inclusive.

Examples of application of the 1(S = 3p)

Figure 48. Theme.

Figure 49. Variation: abc constant.

Figure 50. Variation: abc + bca + cab.

Figure 51. Variation: 2a + b + a + 2c

Figure 52. Variation: $a_2 + b_2 + c_2$ (continued).

Figure 52. Variation: $a_2 + b_2 + c_2$ *(concluded).*

Figure 53. Variation: $(2a_2+b_2+c_2) + (a_2+2b_2+c_2) + (a_2+b_2+2c_2)$

Figure 54. Variations: $a_2 \nearrow\searrow \searrow\nearrow$ *(continued).*

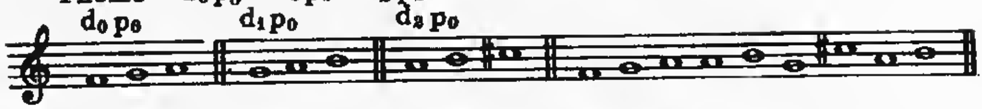
Figure 54. Variations: $a_2 \nearrow\searrow \searrow\nearrow$ *(concluded).*

2. Harmonic forms of three-unit scales.

Illustrated by a series of themes and variations. See figures 55, 56 and 57.

Figure 55. Theme and two variations.

Theme $= d_0p_0 + d_1p_1 + d_2p_2$



Var.: $T=(2+1+1)$; $t=$ ♪

Figure 56. Theme and variation.

Theme:

Var.: simultaneity of six general permutations

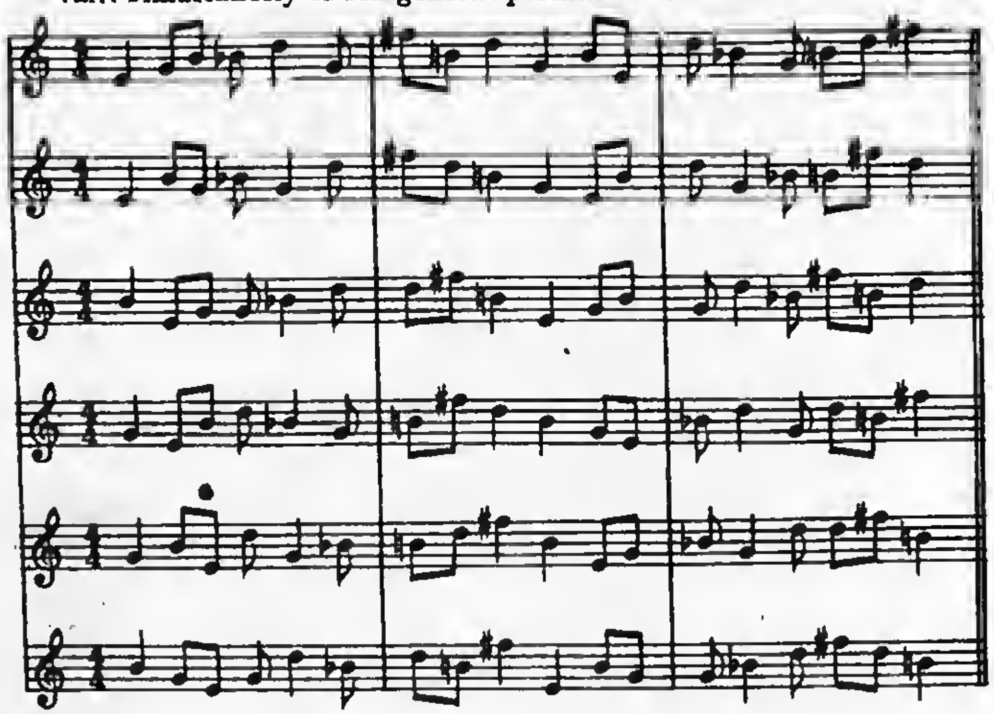
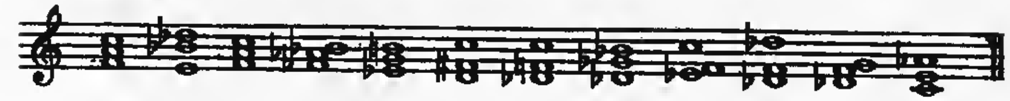


Figure 57. Theme and variation.

3. Three-part harmony. Illustrated by themes and variations. See figures 58 to 67 inclusive.

Theme:



Var.: $I = a_2 + b_2 + c_2$

Var.: $I\ (abc + abc + ab) + (bca + bc + bca) + (ca + cab + cab)$

 etc.

Var.: $I = 3\overrightarrow{p} = \begin{array}{c}\underline{3c_2}\\ \underline{2b_2}\\ \underline{2a_2}\end{array} + \begin{array}{c}\underline{a_2}\\ \underline{c_2}\\ \underline{b_2}\end{array} + \begin{array}{c}\underline{3b_2}\\ \underline{3a_2}\\ \underline{3c_2}\end{array}$

Figure 58. Theme and variations (continued).

Figure 58. Theme and Variations $I = 3p^{\rightarrow}$ *(concluded).*

Var.: $I = 6p^{\rightarrow}$ (general permutations); $A = 5a = a+2b+2c$.

Sequent circular permutations of the coefficients:

$$(a+2b+2c) + (2a+c+2b) + (2c+2a+b) +$$
$$+ (b+2a+2c) + (2b+c+2a) + (2c+2b+a).$$

$$T = 6t = (2+1+1+1+1) + (1+1+2+1+1) + (1+1+1+1+2)$$

Figure 59. Variation. $I = 6p^{\rightarrow}$ *(continued).*

Figure 59. Variation. $T = 6p^{\rightarrow}$ *(concluded).*

Var.: I = a2p: $(2a_2+b_2+c_2)+(a_2+2b_2+c_2)+(a_2+b_2+2c_2)$

Figure 60. Variation. I = a2p.

Var.: I = a2p: $3a_2+2b_2+c_2$. Three simultaneous parts in circular permutations: $(3a_2+2b_2+c_2)+(2b_2+c_2+3a_2)+(c_2+3a_2+2b_2)$.

Figure 61. Variation. I = a2p (continued).

Figure 61. Variation. I = a2p (concluded).

Var.: I = a2p: $a_2+b_2+c_2+d_2+e_2+f_2$ in simultaneous general permutations. $T = 2+1+1$. $T'' = 8t$.

Figure 62. Variation. I = a2p (continued).

Figure 62. Variation. I = a2p (concluded).

Var.: I= a8p; A = 6a:

Figure 63. Variation. I = a3p.

FAST MOTION *without final recurrence of the original position* must follow the ORIGINAL SCHEME of voice-leading.

Var.: I= a8p; A = 8a:

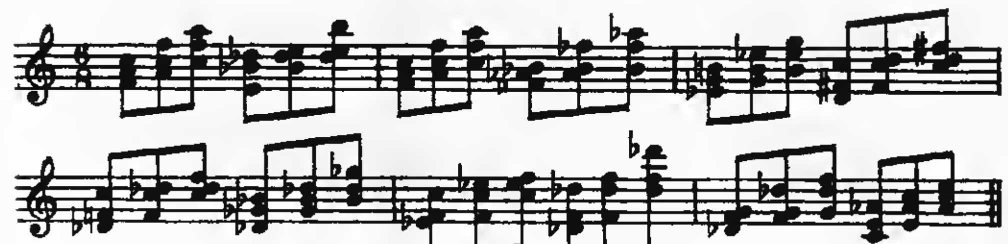


Figure 64. Variation. I = a3p.

SLOW MOTION *without final recurrence of the original position* must follow the voice-leading of the ADJACENT POSITIONS.

Var.: I= a8p; A = 8a: +

Figure 65. Variation. I = a3p.

Theme:

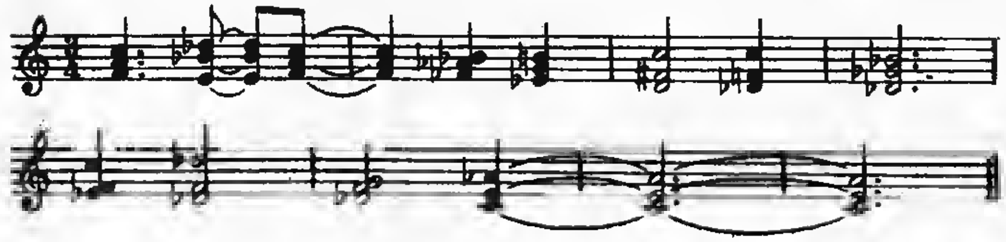
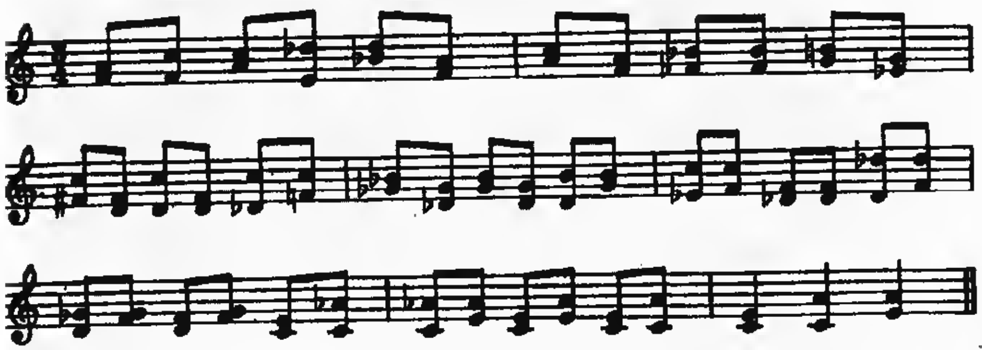


Figure 66. Theme.

Var.: I = ap: $a_2 + b_2 + c_2$ in circular permutations

Var.: I = a2p: conditions as above

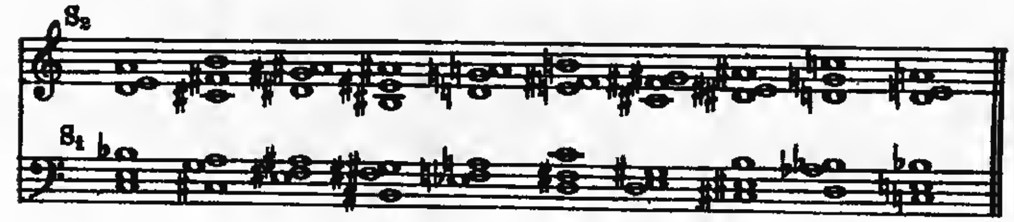


Var.: I = a3p:

Figure 67. Variations. I = a3p.

4. Three parts of any harmony. Illustrated with a theme and variations. See figure 68.

Theme: $\Sigma = 2S$



Var.: $\left[(a+b+2c)+(a+2b+c)+(2a+b+c)\right] S_1$

Var.: $\left[\left(\begin{smallmatrix} c & \\ b & b \\ & a \end{smallmatrix}\right) H_1 + \left(\begin{smallmatrix} & c \\ b & \\ a & a \end{smallmatrix}\right) H_2\right] S_2$

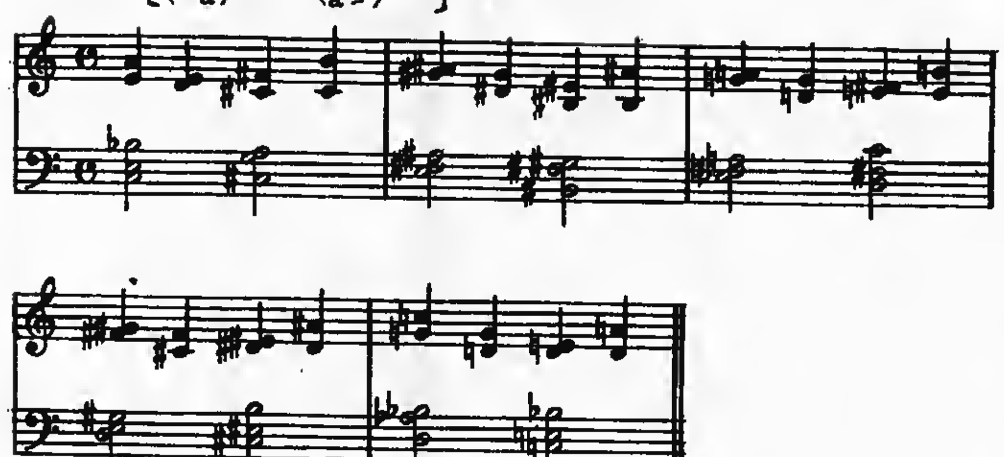


Figure 68. Theme Σ = 2S (continued).

Var.: The two preceding variations combined

Figure 68. Theme Σ = 2S (concluded).

Individual attacks emphasizing one, two, or three parts may be combined into one attack-group of any desirable form.

Examples

c cc cc c c cc c
b b b b b b bb bb
I(S = 3p): a aa aa ; a a aa a ;

c cc c c c ccccc c ccc
b bbbb bbbbb b bbb b
aaaa a ; aaa a aa ; aa a a ;

c ccccccc
bbbbbb bbb
aaaaa aaa a ; . . .

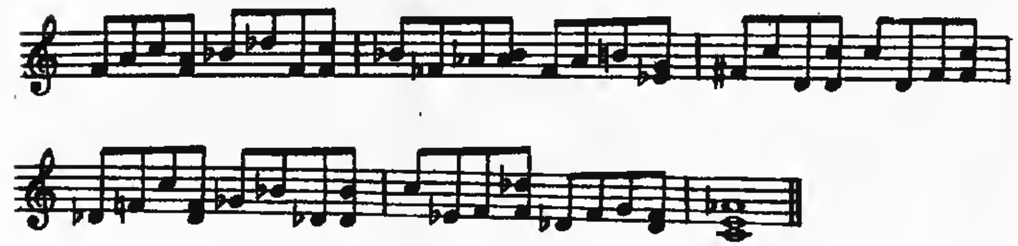
b
aaa
ccccc c c c
bb b b bb bb
a aaaaa ; . . .
c

Theme:

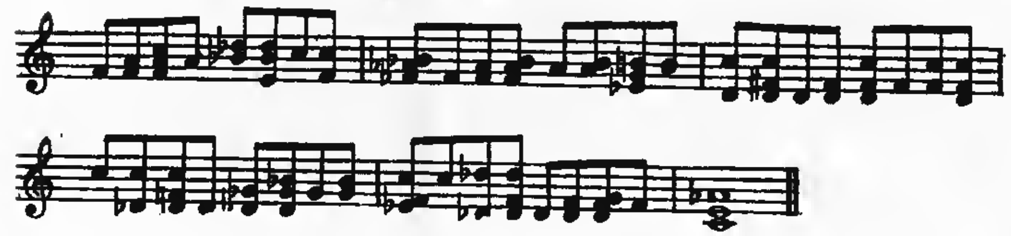
Figure 69. Theme and variations (continued).

c c c c c
Var.: I (S=3p) = b b b b b b ; a=t
a aa a a

c c cc c
Var.: I (S=3p) = b bb bb ; a=t
a a aa a



c ccccc
Var.: I (S=3p) = bbbbb b ; a=t
aaa a ac



b
aaa
ccccc c c c
Var.: I (S=3p) = bb b b bb bb ; a=t
a aaaaa
c

Figure 69. Theme and Variations.

CHAPTER 5

STRATA OF FOUR PARTS

General Classification of I(S = 4p)
(*Table of the combinations of attacks for a, b, c and d*).

A = a; 2a; 3a; 4a; 5a; 6a; 7a; 12a.

The following is a complete table of all forms of I(S = 4p). It includes *the combinations and permutations* for 2, 3, 4, 5, 6, 7, 8 and 12 attacks.

(1) I = ap (one part, one attack).

Four invariant forms: a, b, c, d. [*See figure 70* (*1*)]
A = ap, 2ap, . . . map.
This is equivalent to I(S = p).

(2) I = a2p→ (one attack to a part, two sequent parts).

Six invariant forms: ab, ac, ad, bc, bd, cd. [*See figure 70* (*2*)]
Each invariant form produces 2 attacks and has 2 permutations.
This is equivalent to I(S = 2p).

(3) I = a3p→ (one attack to a part, three sequent parts).

Four invariant forms: abc, abd, acd, bcd. [*See figure 70* (*3*)]
Each invariant form produces 3 attacks and has 6 general or 3 circular permutations.

(4) I = a4p→ (one attack to a part, four sequent parts).

One invariant form: abcd.
The invariant form produces 4 attacks and has 24 general or 4 circular permutations. [*See figure 70* (*4*)]
All other attack-groups (A = 4+n) develop from this source by means of the coefficients of recurrence.

I(S = 4p): attack-groups for one simultaneous p.

A = 4p

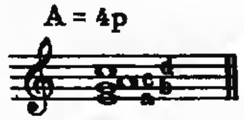
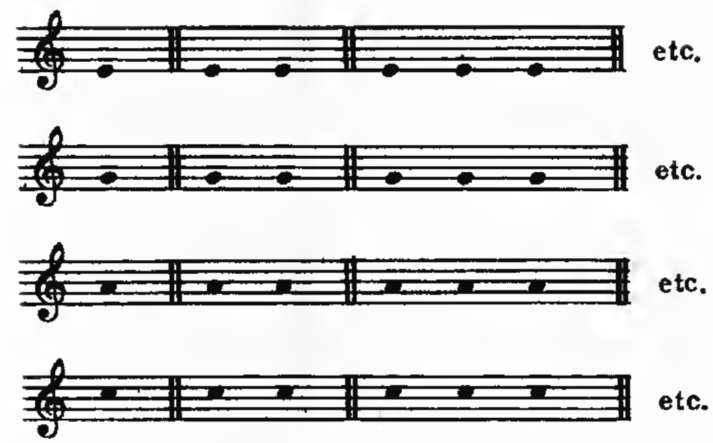


Figure 70. I (S = 4p) (continued).

(1)

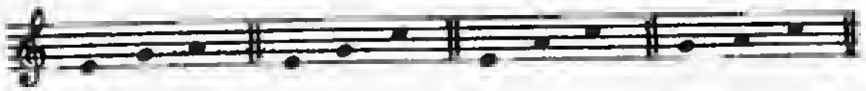


(2)

Each form with a corresponding number of permutations.

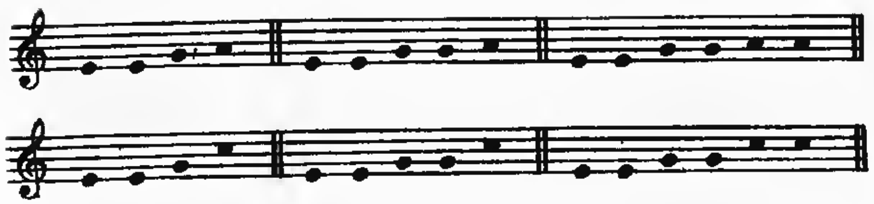
Figure 70. I (S = 4p) (continued).

(3)



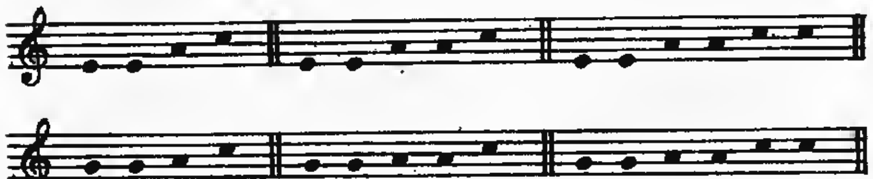
etc.

Each of the above forms has 6 general or 3 circular permutations



etc.

etc.



etc.

etc.

Each form with a corresponding number of permutations

(4) 24 general permutations

4 circular permutations

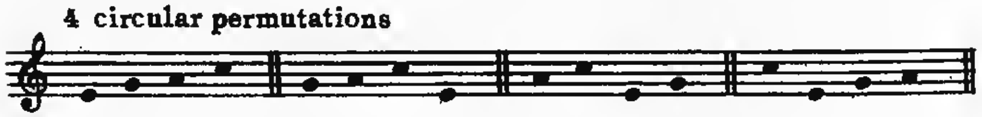


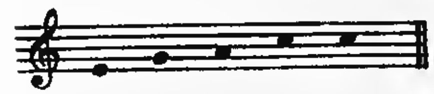
Figure 70. I (S = 4p) (concluded).

B. Development of Attack-Groups by Means of Coefficients of Recurrence

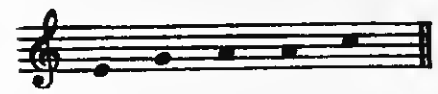
A = 5a; a+b+c+2d; a+b+2c+d; a+2b+c+d; 2a+b+c+d.

$$P_5 = \frac{5!}{2!} = \frac{120}{2} = 60$$

Each of the above 4 permutations of the coefficients has 60 general permutations.

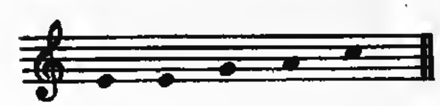


60 general or 5 circular permutations



60 general or 5 circular permutations

60 general or 5 circular permutations



60 general or 5 circular permutations

Figure 71. A = 5a.

Total in general permutations: 60·4 = 240
Total in circular permutations: 5·4 = 20

A = 6a

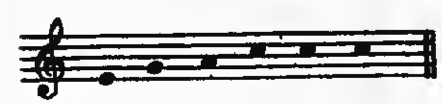
Forms of the distribution of coefficients:

6 = 1+1+1+3; 1+1+2+2

A = a+b+c+3d; a+b+3c+d; a+3b+c+d; 3a+b+c+d

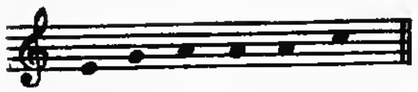
$$P_6 = \frac{6!}{3!} = \frac{720}{6} = 120$$

Each of the above 4 permutations of the first form of distribution has 120 general permutations.



120 general or 6 circular permutations

Figure 72. A = 6a. Permutations of 1+1+1+3 (continued).



120 general or 6 circular permutations

120 general or 6 circular permutations

120 general or 6 circular permutations

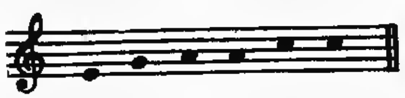
Figure 72. A = 6a. Permutations of 1+1+1+3. (concluded).

Total in general permutations: $120 \cdot 4 = 480$
Total in circular permutations: $6 \cdot 4 = 24$

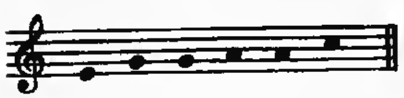
$A = a+b+2c+2d;\ a+2b+2c+d;\ 2a+2b+c+d;\ 2a+b+c+2d;$
$a+2b+c+2d;\ 2a+b+2c+d.$

$$P_6 = \frac{6!}{2!\,2!} = \frac{720}{4} = 180$$

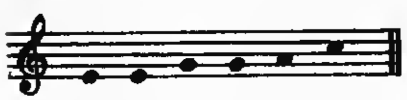
Each of the above 6 permutations of the second form of distribution has 180 general permutations.



180 general or 6 circular permutations

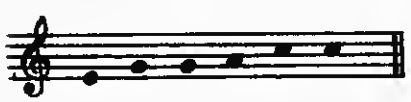


180 general or 6 circular permutations

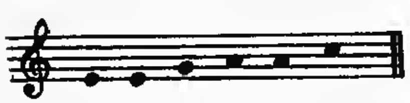


180 general or 6 circular permutations

180 general or 6 circular permutations



180 general or 6 circular permutations



180 general or 6 circular permutations

Figure 73. A = 6a. Permutations of 1+1+2+2.

Total in general permutations: $180 \cdot 6 = 1080$
Total in circular permutations: $6 \cdot 6 = 36$

The total number of cases: A = 6a
General permutations: 480 + 1080 = 1560
Circular permutations: 24 + 36 = 60

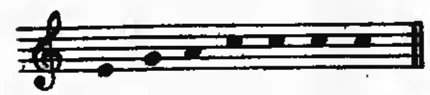
A = 7a

Forms of the distribution of coefficients:

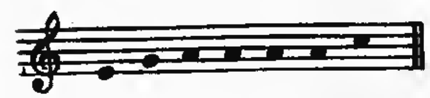
$7 = 1+1+1+4;\ 1+1+2+3;\ 1+2+2+2$
$A = a+b+c+4d;\ a+b+4c+d;\ a+4b+c+d;\ 4a+b+c+d$

$$P_7 = \frac{7!}{4!} = \frac{5040}{24} = 210$$

Each of the above 4 permutations of the first form of distribution has 210 general permutations.

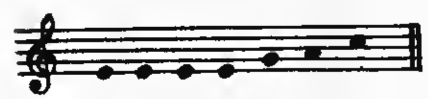


210 general or 7 circular permutations



210 general or 7 circular permutations

210 general or 7 circular permutations



210 general or 7 circular permutations

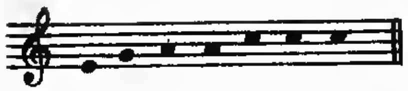
Figure 74. A = 7a. Permutations of 1+1+1+4.

Total in general permutations: $210 \cdot 4 = 840$
Total in circular permutations: $7 \cdot 4 = 28$

$A = a+b+2c+3d;\ a+2b+3c+d;\ 2a+3b+c+d;\ 3a+b+c+2d;$
$a+b+3c+2d;\ a+3b+2c+d;\ 3a+2b+c+d;\ 2a+b+c+3d;$
$a+2b+c+3d;\ 2a+b+3c+d;\ a+3b+c+2d;\ 3a+b+2c+d$

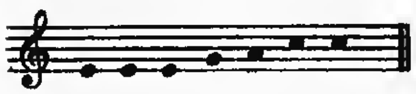
$$P_7 = \frac{7!}{2!\,3!} = \frac{5040}{2 \cdot 6} = 420$$

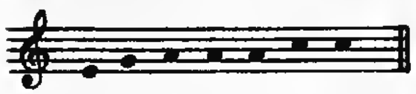
Each of the above 12 permutations of the second form of distribution has 410 general permutations. See Figure 75 on the following page.

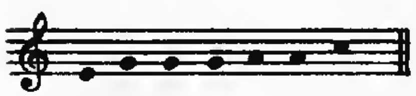
 420 general or 7 circular permutations

 420 general or 7 circular permutations

 420 general or 7 circular permutations

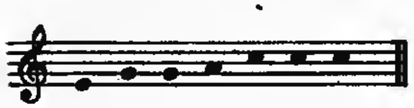
 420 general or 7 circular permutations

 420 general or 7 circular permutations

 420 general or 7 circular permutations

 420 general or 7 circular permutations

 420 general or 7 circular permutations

 420 general or 7 circular permutations

 420 general or 7 circular permutations

 420 general or 7 circular permutations

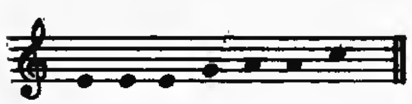
 420 general or 7 circular permutations

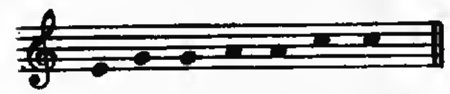
Figure 75. A = 7a. Permutations of 1+1+2+3.

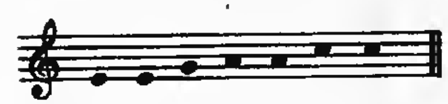
Total in general permutations: $420 \cdot 12 = 5040$
Total in circular permutations: $7 \cdot 12 = 84$

A = a+2b+2c+2d; 2a+b+2c+2d; 2a+2b+c+2d; 2a+2b+2c+d

$$P_7 = \frac{7!}{2!\,2!\,2!} = \frac{5040}{8} = 630$$

Each of the above 4 permutations of the third form of distribution has 630 general permutations.

 630 general or 7 circular permutations

 630 general or 7 circular permutations

 630 general or 7 circular permutations

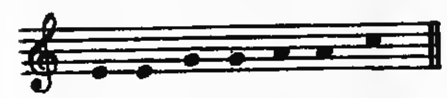
 630 general or 7 circular permutations

Figure 76. A = 7a. Permutations of 1+2+2+2.

Total in general permutations: $630 \cdot 4 = 2520$
Total in circular permutations: $7 \cdot 4 = 28$

The total number of cases: A = 7a
General permutations: 840 + 5040 + 2520 = 8400
Circular permutations: 28 + 84 + 28 = 140

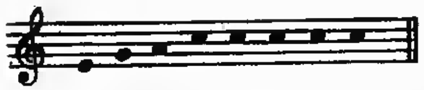
A = 8a

Forms of the distribution of coefficients:

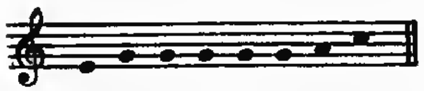
8 = 1+1+1+5; 1+1+2+4; 1+1+3+3; 1+2+2+3; 2+2+2+2
A = a+b+c+5d; a+b+5c+d; a+5b+c+d; 5a+b+c+d

$$P_8 = \frac{8!}{5!} = \frac{40{,}320}{120} = 336$$

Each of the above 4 permutations of the first form of distribution has 336 general permutations. See Figure 77 on the following page.


336 general or 8 circular permutations


336 general or 8 circular permutations


336 general or 8 circular permutations


336 general or 8 circular permutations

Figure 77. A = 8a. Permutations of 1+1+1+5.

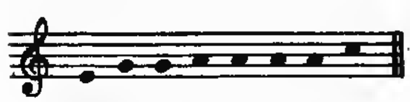
Total in general permutations: $336 \cdot 4 = 1344$
Total in circular permutations: $8 \cdot 4 = 32$

$A = a+b+2c+4d$; $a+2b+4c+d$; $2a+4b+c+d$; $4a+b+c+2d$;
$a+b+4c+2d$; $a+4b+2c+d$; $4a+2b+c+d$; $2a+b+c+4d$;
$a+2b+c+4d$; $2a+b+4c+d$; $a+4b+c+2d$; $4a+b+2c+d$.

$$P_8 = \frac{8!}{2!\,4!} = \frac{40{,}320}{2 \cdot 24} = 840$$

Each of the above 12 permutations of the second form of distribution has 840 general permutations.


840 general or 8 circular permutations


840 general or 8 circular permutations


840 general or 8 circular permutations

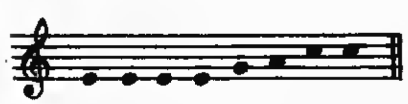

840 general or 8 circular permutations

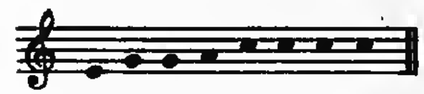
Figure 78. A = 8a. Permutations of 1+1+2+4 (continued).

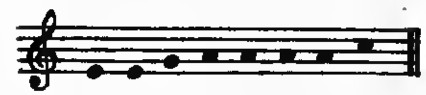

840 general or 8 circular permutations


840 general or 8 circular permutations


840 general or 8 circular permutations


840 general or 8 circular permutations


840 general or 8 circular permutations


840 general or 8 circular permutations


840 general or 8 circular permutations

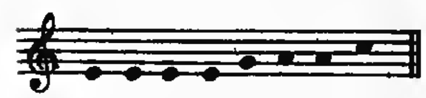

840 general or 8 circular permutations

Figure 78. A = 8a. Permutations of 1+1+2+4 (concluded).

Total in general permutations: $840 \cdot 12 = 10{,}080$
Total in circular permutations: $8 \cdot 12 = 96$

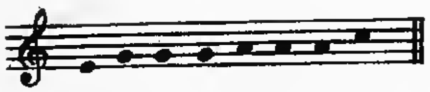
$A = a+b+3c+3d$; $a+3b+3c+d$; $3a+3b+c+d$; $3a+b+c+3d$;
$a+3b+c+3d$; $3a+b+3c+d$.

$$P_8 = \frac{8!}{3!\,3!} = \frac{40{,}320}{6 \cdot 6} = 1120$$

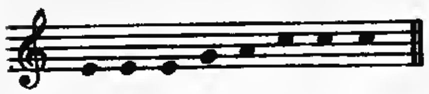
Each of the above 6 permutations of the third form of distribution has 1120 general permutations.

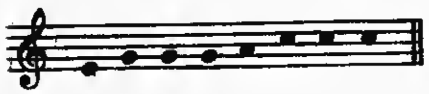

1120 general or 8 circular permutations

Figure 79. A = 8a. Permutations of 1+1+3+3 (continued).


1120 general or 8 circular permutations


1120 general or 8 circular permutations


1120 general or 8 circular permutations


1120 general or 8 circular permutations

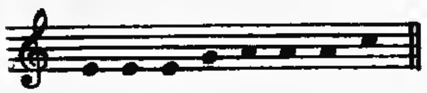

1120 general or 8 circular permutations

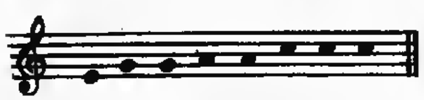
Figure 79. A = 8a. Permutations of 1+1+3+3 (concluded).

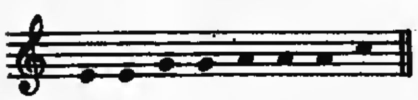
Total in general permutations: $1120 \cdot 6 = 6720$
Total in circular permutations: $8 \cdot 6 = 48$

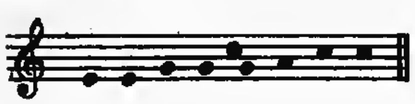
A = a+2b+2c+3d; 2a+2b+3c+d; 2a+3b+c+2d; 3a+b+2c+2d;
a+2b+3c+2d; 2a+3b+2c+d; 3a+2b+c+2d; 2a+b+2c+3d;
a+3b+2c+2d; 3a+2b+2c+d; 2a+2b+c+3d; 2a+b+3c+2d.

$$P_8 = \frac{8!}{2!\ 2!\ 3!} = \frac{40{,}320}{2 \cdot 2 \cdot 6} = 1680$$

Each of the above 12 permutations of the fourth form of distribution has 1680 general permutations.


1680 general or 8 circular permutations


1680 general or 8 circular permutations


1680 general or 8 circular permutations

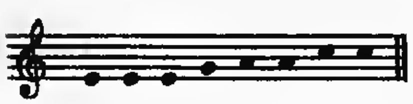
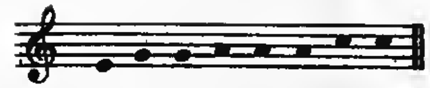
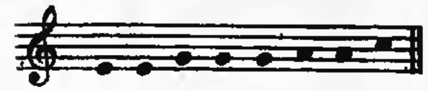

1680 general or 8 circular permutations

Figure 80. A = 8a. Permutations of 1+2+2+3 (continued).

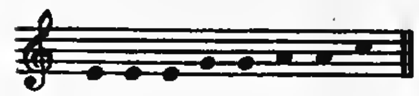

1680 general or 8 circular permutations

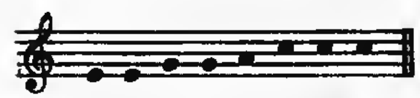

1680 general or 8 circular permutations


1680 general or 8 circular permutations


1680 general or 8 circular permutations


1680 general or 8 circular permutations


1680 general or 8 circular permutations


1680 general or 8 circular permutations

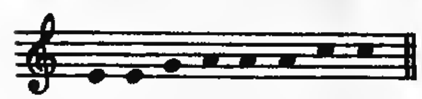

1680 general or 8 circular permutations

Figure 80. A = 8a. Permutations of 1+2+2+3 (concluded).

Total in general permutations: $1680 \cdot 12 = 20{,}160$
Total in circular permutations: $8 \cdot 12 = 96$

A = 2a+2b+2c+2d

$$P_8 = \frac{8!}{2!\ 2!\ 2!\ 2!} = \frac{40{,}320}{2 \cdot 2 \cdot 2 \cdot 2} = 2520$$

The above invariant (fifth) form of distribution has 2520 general permutations.

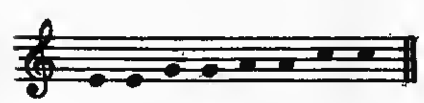

2520 general or 8 circular permutations

Figure 81. A = 8a. 2a+2b+2c+2d.

The total number of cases: A = 8a

General permutations: 1344 + 10,080 + 6720 + 20,160 + + 2520 = 40,824

Circular permutations: 32 + 96 + 48 + 96 + 8 = 280

A = 12a

Forms of the distribution of coefficients:

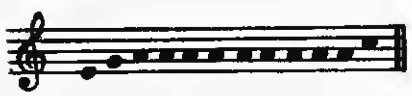
1+1+1+9; 1+1+2+8; 1+1+3+7; 1+1+4+6; 1+1+5+5; 1+2+2+7; 1+2+3+6; 1+2+4+5; 1+3+3+5; 1+3+4+4; 2+2+2+6; 2+2+3+5; 2+2+4+4; 2+3+3+4; 3+3+3+3.

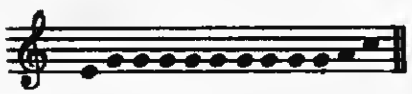
A = a+b+c+9d; a+b+9c+d; a+9b+c+d; 9a+b+c+d

$$P_{12} = \frac{12!}{9!} = \frac{479{,}001{,}600}{362{,}880} = 1320$$

Each of the above 4 permutations of the first form of distribution has 1320 general permutations.


1320 general or 12 circular permutations


1320 general or 12 circular permutations


1320 general or 12 circular permutations

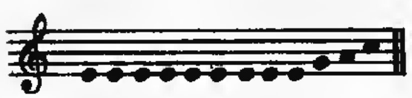

1320 general or 12 circular permutations

Figure 82. A = 12A. Permutations of 1+1+1+9.

Total in general permutations: 1320·4 = 5280
Total in circular permutations: 12·4 = 48

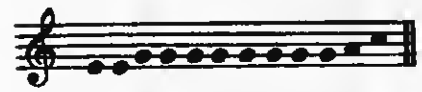
A = a+b+2c+8d; a+2b+8c+d; 2a+8b+c+d; 8a+b+c+2d; a+b+8c+2d; a+8b+2c+d; 8a+2b+c+d; 2a+b+c+8d; a+8b+c+2d; 8a+b+2c+d; a+2b+c+8d; 2a+b+8c+d.

$$P_{12} = \frac{12!}{2!\,8!} = \frac{479{,}001{,}600}{2 \cdot 40{,}320} = 5940$$

Each of the above 12 permutations of the second form of distribution has 5940 general permutations. See Figure 83 on the following page.

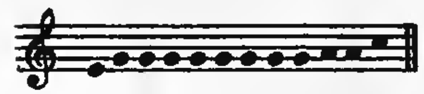

5940 general or 12 circular permutations

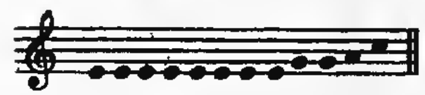

5940 general or 12 circular permutations

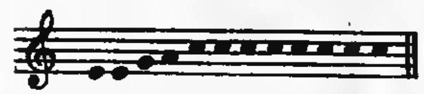

5940 general or 12 circular permutations

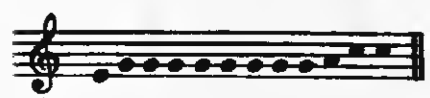

5940 general or 12 circular permutations

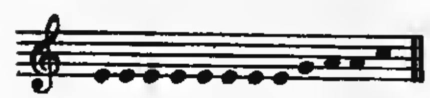

5940 general or 12 circular permutations

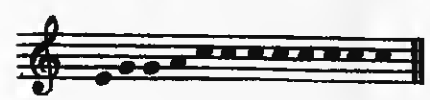

5940 general or 12 circular permutations


5940 general or 12 circular permutations


5940 general or 12 circular permutations


5940 general or 12 circular permutations


5940 general or 12 circular permutations


5940 general or 12 circular permutations

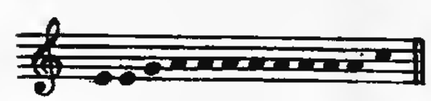

5940 general or 12 circular permutations

Figure 83. A = 12a. Permutations of 1+1+2+8.

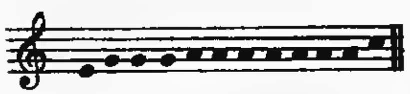
Total in general permutations: 5940·12 = 71,280
Total in circular permutations: 12·12 = 144

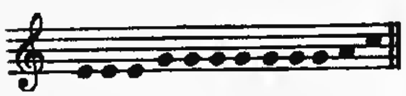
A = a+b+3c+7d; a+3b+7c+d; 3a+7b+c+d; 7a+b+c+3d;
a+b+7c+3d; a+7b+3c+d; 7a+3b+c+d; 3a+b+c+7d;
a+3b+c+7d; 3a+b+7c+d; a+7b+c+3d; 7a+b+3c+d.

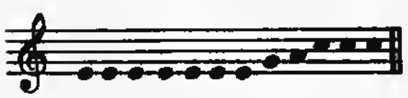
$$P_{12} = \frac{12!}{2!\ 7!} = \frac{479{,}001{,}600}{6 \cdot 5040} = 15{,}840$$

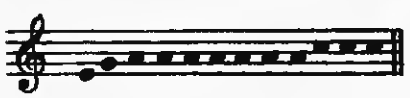
Each of the above 12 permutations of the third form of distribution has 15,840 general permutations.


15,840 general or 12 circular permutations

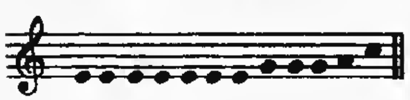

15,840 general or 12 circular permutations

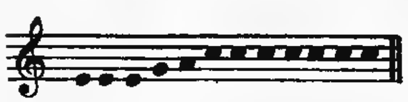

15,840 general or 12 circular permutations

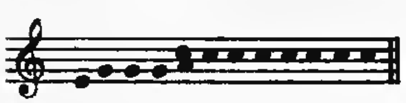

15,840 general or 12 circular permutations


15,840 general or 12 circular permutations


15,840 general or 12 circular permutations


15,840 general or 12 circular permutations


15,840 general or 12 circular permutations


15,840 general or 12 circular permutations

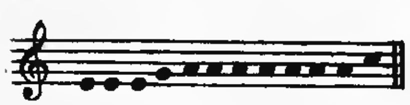

15,840 general or 12 circular permutations

Figure 84. A = 12a. Permutations of 1+1+3+7 (continued).


15,840 general or 12 circular permutations

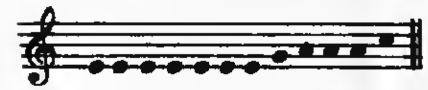

15,840 general or 12 circular permutations

Figure 84. A = 12a. Permutations of 1+1+3+7 (concluded).

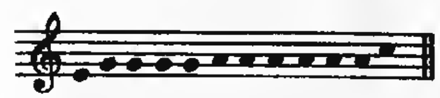
Total in general permutations: 15,840·12 = 190,080
Total in circular permutations: 12·12 = 144

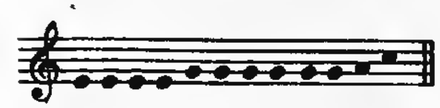
A = a+b+4c+6d; a+4b+6c+d; 4a+6b+c+d; 6a+b+c+4d;
a+b+6c+4d; a+6b+4c+d; 6a+4b+c+d; 4a+b+c+6d;
a+4b+c+6d; 4a+b+6c+d; a+6b+c+4d; 6a+b+4c+d.

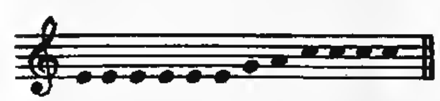
$$P_{12} = \frac{12!}{4!\ 6!} = \frac{479{,}001{,}600}{24 \cdot 720} = 27{,}720$$

Each of the above 12 permutations of the fourth form of distribution has 27,720 general permutations.

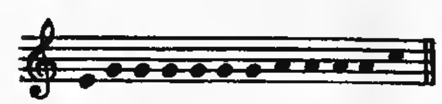

27,720 general or 12 circular permutations


27,720 general or 12 circular permutations


27,720 general or 12 circular permutations


27,720 general or 12 circular permutations


27,720 general or 12 circular permutations


27,720 general or 12 circular permutations


27,720 general or 12 circular permutations

Figure 85. A = 12a. Permutations of 1+1+4+6 (continued).

 27,720 general or 12 circular permutations

 27,720 general or 12 circular permutations

 27,720 general or 12 circular permutations

 27,720 general or 12 circular permutations

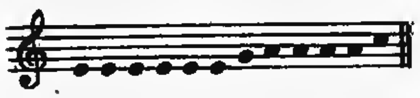
 27,720 general or 12 circular permutations

Figure 85. A = 12a. Permutations of 1+1+4+6 (concluded).

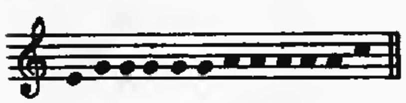
Total in general permutations: 27,720·12 = 332,640
Total in circular permutations: 12·12 = 144

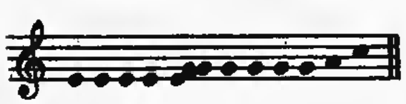
A = a+b+5c+5d; a+5b+5c+d; 5a+5b+c+d; 5a+b+c+5d;
a+5b+c+5d; 5a+b+5c+d.

$$P_{12} = \frac{12!}{5!\,5!} = \frac{479{,}001{,}600}{120\cdot 120} = 33{,}264$$

Each of the above 6 permutations of the fifth form of distribution has 33,264 general permutations.

 33,264 general or 12 circular permutations

 33,264 general or 12 circular permutations

 33,264 general or 12 circular permutations

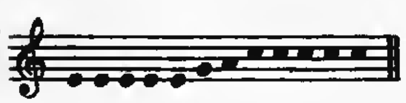
 33,264 general or 12 circular permutations

Figure 86. A = 12a. Permutations of 1+1+5+5 (continued).

 33,264 general or 12 circular permutations

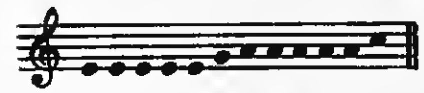
 33,264 general or 12 circular permutations

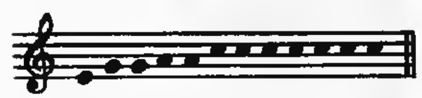
Figure 86. A = 12a. Permutations of 1+1+5+5 (concluded).

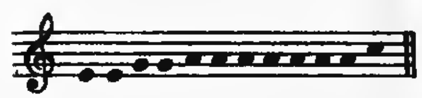
Total in general permutations: 33,264·6 = 199,584
Total in circular permutations: 12·6 = 72

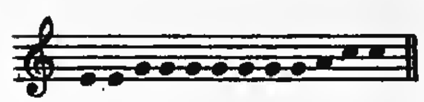
A = a+2b+2c+7d; 2a+2b+7c+d; 2a+7b+c+2d; 7a+b+2c+2d;
a+2b+7c+2d; 2a+7b+2c+d; 7a+2b+c+2d; 2a+b+2c+7d;
7a+2b+2c+d; 2a+2b+c+7d; 2a+b+7c+2d; a+7b+2c+2d.

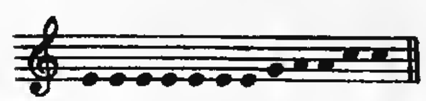
$$P_{12} = \frac{12!}{2!\,2!\,7!} = \frac{479{,}001{,}600}{2\cdot 2\cdot 5040} = 23{,}760$$

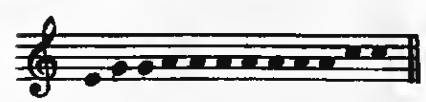
Each of the above 12 permutations of the sixth form of distribution has 23,760 general permutations.

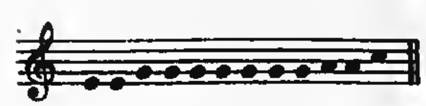
 23,760 general or 12 circular permutations

 23,760 general or 12 circular permutations

 23,760 general or 12 circular permutations

 23,760 general or 12 circular permutations

 23,760 general or 12 circular permutations

 23,760 general or 12 circular permutations

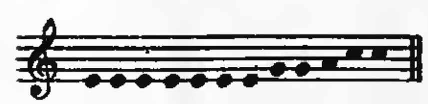
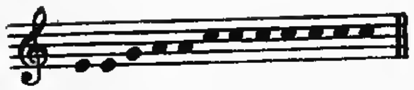
 23,760 general or 12 circular permutations

Figure 87. A = 12a. Permutations of 1+2+2+7 (continued).


23,760 general or 12 circular permutations


23,760 general or 12 circular permutations


23,760 general or 12 circular permutations


23,760 general or 12 circular permutations


23,760 general or 12 circular permutations

Figure 87. A = 12a. Permutations of 1+2+2+7 (concluded).

Total in general permutations: $23{,}760 \cdot 12 = 285{,}120$
Total in circular permutations: $12 \cdot 12 = 144$

A = a+2b+3c+6d; a+2b+6c+3d; a+6b+2c+3d; 6a+b+2c+3d;
a+3b+2c+6d; a+3b+6c+2d; a+6b+3c+2d; 6a+b+3c+2d;
3a+b+2c+6d; 3a+b+6c+2d; 3a+6b+c+2d; 6a+3b+c+2d;
2a+b+3c+6d; 2a+b+6c+3d; 2a+6b+c+3d; 6a+2b+c+3d;
2a+3b+c+6d; 2a+3b+6c+d; 2a+6b+3c+d; 6a+2b+3c+d;
3a+2b+c+6d; 3a+2b+6c+d; 3a+6b+2c+d; 6a+3b+2c+d.

$$P_{12} = \frac{12!}{2!\,3!\,6!} = \frac{479{,}001{,}600}{2 \cdot 6 \cdot 720} = 55{,}440$$

Each of the above 24 permutations of the seventh form of distribution has 55,440 general permutations.


55,440 general or 12 circular permutations

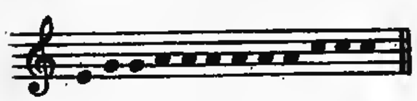
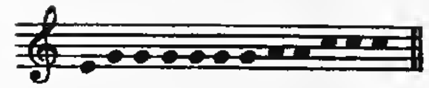
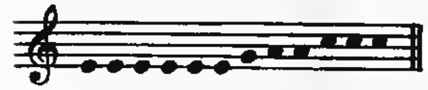

55,440 general or 12 circular permutations

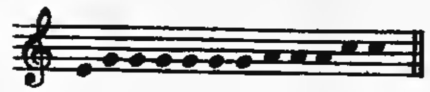
Figure 88. A = 12a. Permutations of 1+2+3+6 (continued).

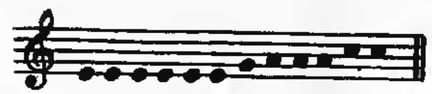

55,440 general or 12 circular permutations

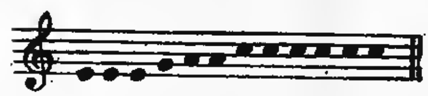

55,440 general or 12 circular permutations

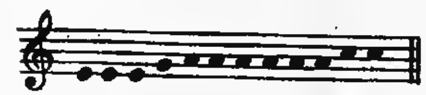

55,440 general or 12 circular permutations

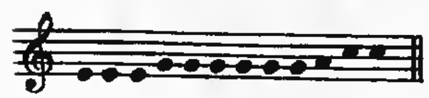

55,440 general or 12 circular permutations

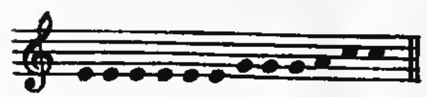

55,440 general or 12 circular permutations


55,440 general or 12 circular permutations

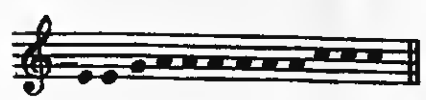

55,440 general or 12 circular permutations


55,440 general or 12 circular permutations


55,440 general or 12 circular permutations


55,440 general or 12 circular permutations


55,440 general or 12 circular permutations


55,440 general or 12 circular permutations

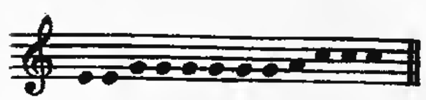
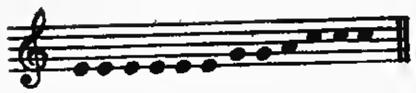
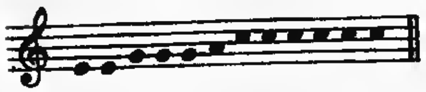
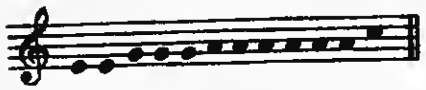

55,440 general or 12 circular permutations

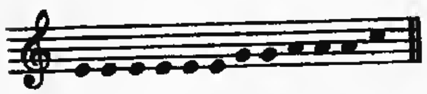
Figure 88. A = 12a. Permutations of 1+2+3+6 (continued).


55,440 general or 12 circular permutations


55,440 general or 12 circular permutations

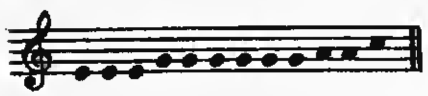

55,440 general or 12 circular permutations


55,440 general or 12 circular permutations


55,440 general or 12 circular permutations


55,440 general or 12 circular permutations


55,440 general or 12 circular permutations


55,440 general or 12 circular permutations

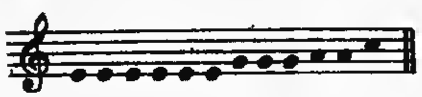

55,440 general or 12 circular permutations

Figure 88. A = 12a. Permutations of 1+2+3+6 (concluded).

Total in general permutations: $55{,}440 \cdot 24 = 1{,}330{,}560$
Total in circular permutations: $12 \cdot 24 = 288$

A = a+2b+4c+5d; a+2b+5c+4d; a+5b+2c+4d; 5a+b+2c+4d;
a+4b+2c+5d; a+4b+5c+2d; a+5b+4c+2d; 5a+b+4c+2d;
4a+b+2c+5d; 4a+b+5c+2d; 4a+5b+c+2d; 5a+4b+c+2d;
2a+b+4c+5d; 2a+b+5c+4d; 2a+5b+c+4d; 5a+2b+c+4d;
2a+4b+c+5d; 2a+4b+5c+d; 2a+5b+4c+d; 5a+2b+4c+d;
4a+2b+c+5d; 4a+2b+5c+d; 4a+5b+2c+d; 5a+4b+2c+d.

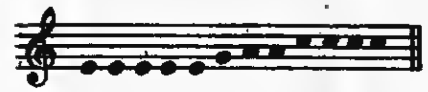
$$P_{12} = \frac{12!}{2!\,4!\,5!} = \frac{479{,}001{,}600}{2 \cdot 24 \cdot 120} = 83{,}160$$

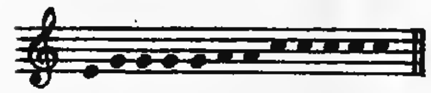
Each of the above 24 permutations of the eighth form of distribution has 83,160 general permutations. See Figure 89 on next page.

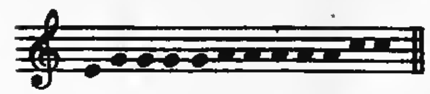

83,160 general or 12 circular permutations


83,160 general or 12 circular permutations

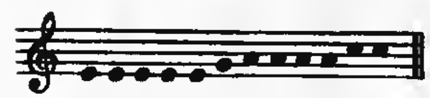

83,160 general or 12 circular permutations

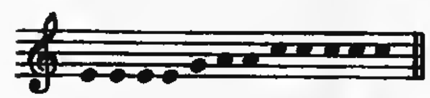

83,160 general or 12 circular permutations

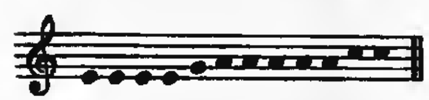

83,160 general or 12 circular permutations

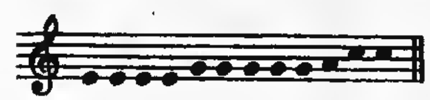

83,160 general or 12 circular permutations

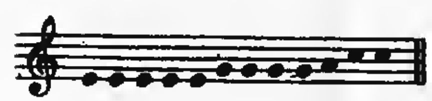

83,160 general or 12 circular permutations


83,160 general or 12 circular permutations


83,160 general or 12 circular permutations


83,160 general or 12 circular permutations


83,160 general or 12 circular permutations


83,160 general or 12 circular permutations

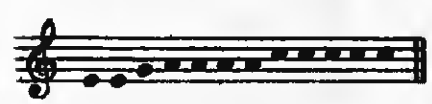
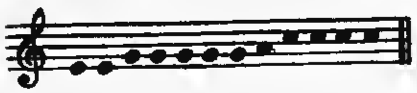
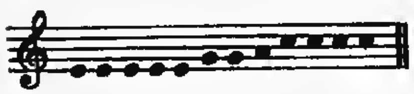
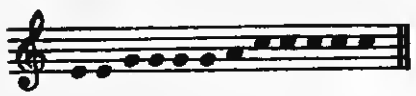

83,160 general or 12 circular permutations

Figure 89. A = 12a. Permutations of 1+2+4+5 (continued).

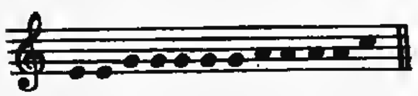

83,160 general or 12 circular permutations

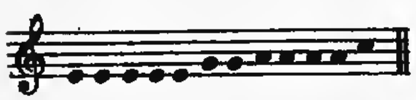

83,160 general or 12 circular permutations

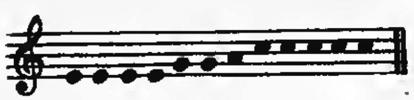

83,160 general or 12 circular permutations

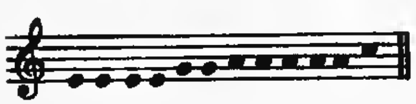

83,160 general or 12 circular permutations

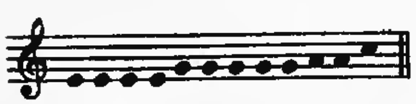

83,160 general or 12 circular permutations


83,160 general or 12 circular permutations


83,160 general or 12 circular permutations


83,160 general or 12 circular permutations


83,160 general or 12 circular permutations


83,160 general or 12 circular permutations

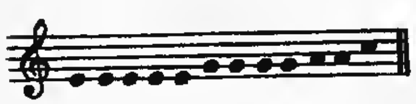

83,160 general or 12 circular permutations

Figure 89. A = 12a. Permutations of 1+2+4+5 (concluded).

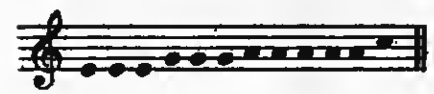
Total in general permutations: $83{,}160 \cdot 24 = 1{,}995{,}840$
Total in circular permutations: $12 \cdot 24 = 288$

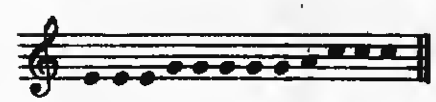
A = a+3b+3c+5d; 3a+3b+5c+d; 3a+5b+c+3d; 5a+b+3c+3d;
a+3b+5c+3d; 3a+5b+3c+d; 5a+3b+c+3d; 3a+b+3c+5d;
5a+3b+3c+d; 3a+3b+c+5d; 3a+b+5c+3d; a+5b+3c+3d.

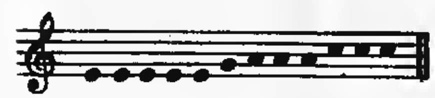
$$P_{12} = \frac{12!}{3!\ 3!\ 5!} = \frac{479{,}001{,}600}{6 \cdot 6 \cdot 120} = 110{,}880$$

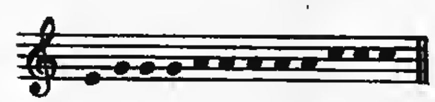
Each of the above 12 permutations of the ninth form of distribution has 110,880 general permutations.

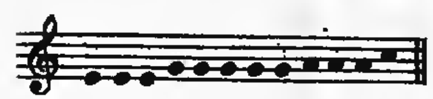

110,880 general or 12 circular permutations

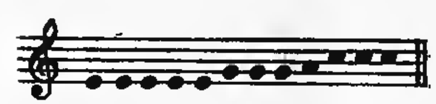

110,880 general or 12 circular permutations

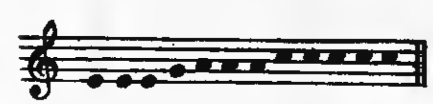

110,880 general or 12 circular permutations


110,880 general or 12 circular permutations


110,880 general or 12 circular permutations


110,880 general or 12 circular permutations


110,880 general or 12 circular permutations


110,880 general or 12 circular permutations

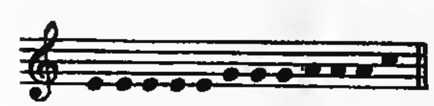

110,880 general or 12 circular permutations

Figure 90. A = 12a. Permutations of 1+3+3+5 (continued).

 110,880 general or 12 circular permutations

 110,880 general or 12 circular permutations

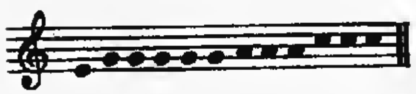
 110,880 general or 12 circular permutations

Figure 90. A = 12a. Permutations of 1+3+3+5 (concluded).

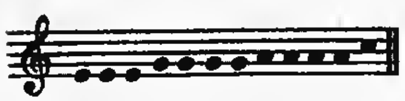
Total in general permutations: 110,880·12 = 1,330,560
Total in circular permutations: 12·12 = 144

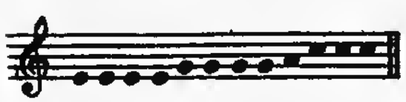
A = a+3b+4c+4d; 3a+4b+4c+d; 4a+4b+c+3d; 4a+b+3c+4d;
3a+b+4c+4d; a+4b+4c+3d; 4a+4b+3c+d; 4a+3b+c+4d;
a+4b+3c+4d; 4a+3b+4c+d; 3a+4b+c+4d; 4a+b+4c+3d.

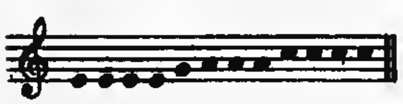
$$P_{12} = \frac{12!}{3!\ 4!\ 4!} = \frac{479{,}001{,}600}{6\cdot 24\cdot 24} = 138{,}600$$

Each of the above 12 permutations of the tenth form of distribution has 138,600 general permutations.

 138,600 general or 12 circular permutations

 138,600 general or 12 circular permutations

 138,600 general or 12 circular permutations

 138,600 general or 12 circular permutations

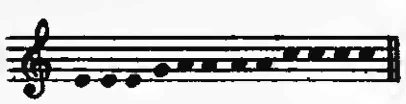
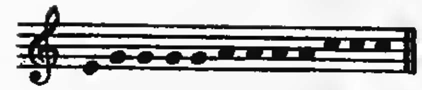
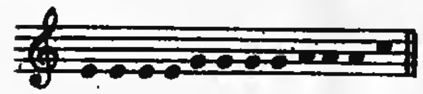
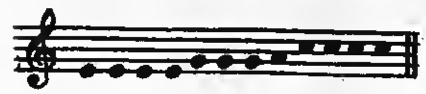
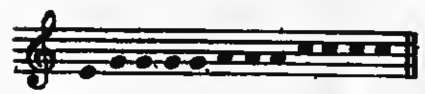
 138,600 general or 12 circular permutations

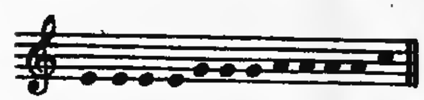
Figure 91. A = 12a. Permutations of 1+3+4+4 (continued).

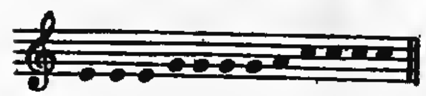
 138,600 general or 12 circular permutations

 138,600 general or 12 circular permutations

 138,600 general or 12 circular permutations

 138,600 general or 12 circular permutations

 138,600 general or 12 circular permutations

 138,600 general or 12 circular permutations

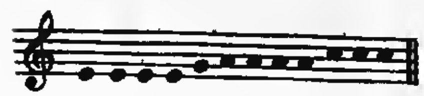
 138,600 general or 12 circular permutations

Figure 91. A = 12a. Permutations of 1+3+4+4. (concluded).

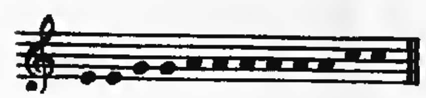
Total in general permutations: 138,600·12 = 1,663,200
Total in circular permutations: 12·12 = 144

A = 2a+2b+2c+6d; 2a+2b+6c+2d; 2a+6b+2c+2d; 6a+2b+2c+2d

$$P_{12} = \frac{12!}{2!\ 2!\ 2!\ 6!} = \frac{479{,}001{,}600}{2\cdot 2\cdot 2\cdot 720} = 83{,}160$$

Each of the above 4 permutations of the eleventh form of distribution has 83,160 general permutations.

 83,160 general or 12 circular permutations

 83,160 general or 12 circular permutations

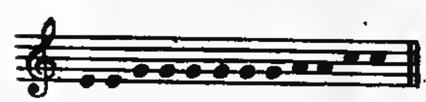
 83,160 general or 12 circular permutations

Figure 92. A = 12a. Permutations of 2+2+2+6 (continued).

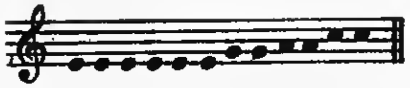

83,160 general or 12 circular permutations

Figure 92. A = 12a. Permutations of 2+2+2+6 (concluded).

Total in general permutations: $83,160\cdot4 = 332,640$
Total in circular permutations: $12\cdot4 = 48$

A = 2a+2b+3c+5d; 2a+3b+5c+2d; 3a+5b+2c+2d; 5a+2b+2c+3d;
2a+2b+5c+3d; 2a+5b+3c+2d; 5a+3b+2c+2d; 3a+2b+2c+5d;
2a+3b+2c+5d; 3a+2b+5c+2d; 2a+5b+2c+3d; 5a+2b+3c+2d.

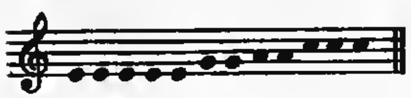
$$P_{12} = \frac{12!}{2!\ 2!\ 3!\ 5!} = \frac{479,001.600}{2\cdot2\cdot6\cdot120} = 166,320$$

Each of the above 12 permutations of the twelfth form of distribution has 166,320 general permutations.

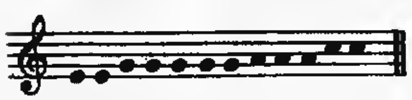

166,320 general or 12 circular permutations

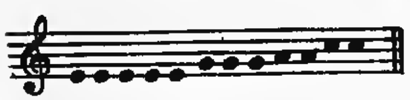

166,320 general or 12 circular permutations


166,320 general or 12 circular permutations


166,320 general or 12 circular permutations


166,320 general or 12 circular permutations


166,320 general or 12 circular permutations


166,320 general or 12 circular permutations

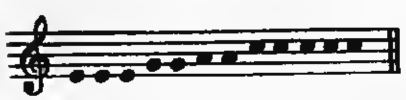
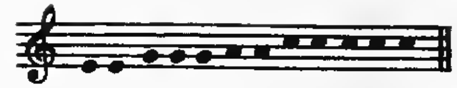
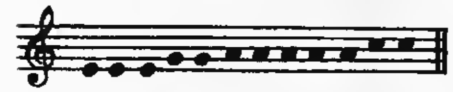

166,320 general or 12 circular permutations

Figure 93. A = 12a. Permutations of 2+2+3+5 (continued).


166,320 general or 12 circular permutations


166,320 general or 12 circular permutations


166,320 general or 12 circular permutations

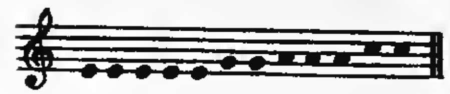

166,320 general or 12 circular permutations

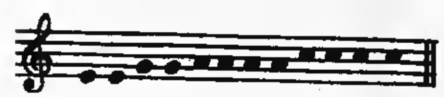
Figure 93. A = 12a. Permutations of 2+2+3+5 (concluded).

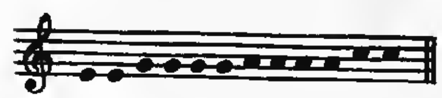
Total in general permutations: $166,320\cdot12 = 1,995,840$
Total in circular permutations: $12\cdot12 = 144$

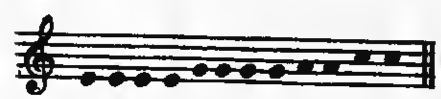
A = 2a+2b+4c+4d; 2a+4b+4c+2d; 4a+4b+2c+2d; 4a+2b+2c+4d;
2a+4b+2c+4d; 4a+2b+4c+2d.

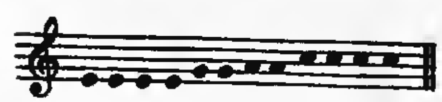
$$P_{12} = \frac{12!}{2!\ 2!\ 4!\ 4!} = \frac{479,001,600}{2\cdot2\cdot24\cdot24} = 207,900$$

Each of the above 6 permutations of the thirteenth form of distribution has 207,900 general permutations.


207,900 general or 12 circular permutations


207,900 general or 12 circular permutations


207,900 general or 12 circular permutations


207,900 general or 12 circular permutations

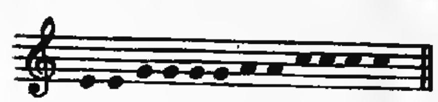
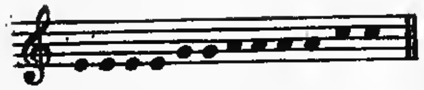

207,900 general or 12 circular permutations

Figure 94. A = 12a. Permutations of 2+2+4+4 (continued).



207,900 general or 12 circular permutations

Figure 94. A = 12a. Permutations of 2+2+4+4 (concluded).

Total in general permutations: 207,900·6 = 1,247,400
Total in circular permutations: 12·6 = 72

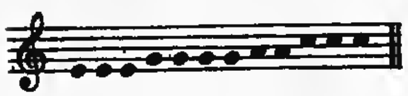
A = 2a+3b+3c+4d; 3a+3b+4c+2d; 3a+4b+2c+3d; 4a+2b+3c+3d;
2a+3b+4c+3d; 3a+4b+3c+2d; 4a+3b+2c+3d; 3a+2b+3c+4d;
4a+3b+3c+2d; 3a+3b+2c+4d; 3a+2b+4c+3d; 2a+4b+3c+3d.

$$P_{12} = \frac{12!}{2!\ 3!\ 3!\ 4!} = \frac{479{,}001{,}600}{2\cdot6\cdot6\cdot24} = 277{,}200$$

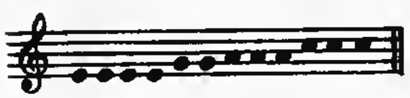
Each of the above 12 permutations of the fourteenth form of distribution has 277,200 general permutations.

277,200 general or 12 circular permutations

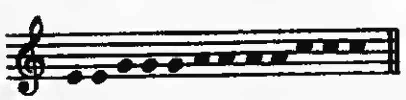
277,200 general or 12 circular permutations



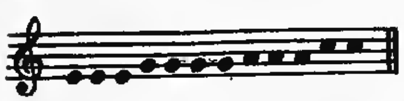
277,200 general or 12 circular permutations



277,200 general or 12 circular permutations

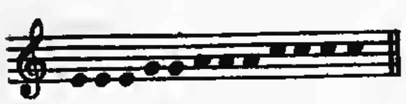


277,200 general or 12 circular permutations



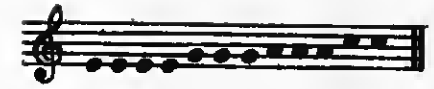
277,200 general or 12 circular permutations

277,200 general or 12 circular permutations

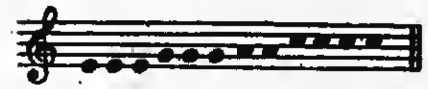


277,200 general or 12 circular permutations

Figure 95. A = 12a. Permutations of 2+3+3+4 (continued).

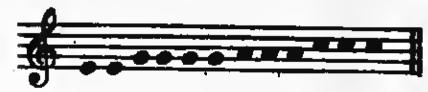


277,200 general or 12 circular permutations



277,200 general or 12 circular permutations

277,200 general or 12 circular permutations



277,200 general or 12 circular permutations

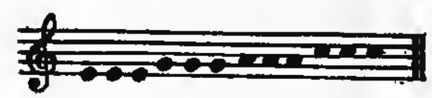
Figure 95. A = 12a. Permutations of 2+3+3+4 (concluded).

Total in general permutations: 277,200·12 = 3,326,400
Total in circular permutation: 12·12 = 144

A = 3a+3b+3c+3d

$$P_{12} = \frac{12!}{3!\ 3!\ 3!\ 3!} = \frac{479{,}001{,}600}{6\cdot6\cdot6\cdot6} = 369{,}600$$

The above invariant (fifteenth) form of distribution has 369,600 general permutations.



369,600 general or 12 circular permutations

Figure 96. A = 12a. 3a+3b+3c+3d.

The total number of cases: A = 12a

General permutations: 5280 + 71,280 + 190,080 + 332,640 + 199,584 + + 285,120 + 1,330,560 + 1,995,840 + 1,330,560 + 1,663,200 + 332,640 + + 1,995,840 + 1,247,400 + 3,326,400 + 369,600 = 14,646,024

Circular permutations: 48 + 144 + 144 + 144 + 72 + 144 + 288 + 288 + + 144 + 144 + 48 + 144 + 72 + 144 + 12 = 1960

(5) I = a2p (one attack to a combination of two simultaneous parts).

The six invariant forms of (2) become elements of the second order:

$$\frac{b}{a} = a_2;\ \frac{c}{a} = b_2;\ \frac{d}{a} = c_2;\ \frac{c}{b} = d_2;\ \frac{d}{b} = e_2;\ \frac{d}{c} = f_2$$

Table of $I(S = 4p) = a2p$

Figure 97. Forms of $I = a2p$.

Combinations of these forms in sequence, within the limits of a to c or b to d, require the inclusion of the three lower or the three upper parts.

Combinations of these forms in sequence, within the limit of a to d, require the inclusion of all four parts.

Sequent combinations by two:

$a_2 + b_2;\ a_2 + c_2;\ a_2 + d_2;\ a_2 + e_2;\ a_2 + f_2;$
$b_2 + c_2;\ b_2 + d_2;\ b_2 + e_2;\ b_2 + f_2;$
$c_2 + d_2;\ c_2 + e_2;\ c_2 + f_2;$
$d_2 + e_2;\ d_2 + f_2;$
$e_2 + f_2.$

Figure 98. Sequent combinations by 2.

The table above corresponds to two consecutive attacks. Each of the above combinations has 2 permutations.

Further development of attacks is achieved by means of the coefficients of recurrence:

$2a_2 + b_2;\ 3a_2 + 2c_2;\ \ldots$
$2b_2 + d_2 + b_2 + 2d_2;\ 3c_2 + f_2 + 2c_2 + 2f_2 + c_2 + 3f_2;\ \ldots$

Figure 99. Coefficients of recurrence.

The latter, in turn, become subject to permutations (general or circular), as well as to permutations of the higher orders:

$a_2 + b_2;\ b_2 + a_2;\ a_2 + c_2;\ c_2 + a_2;\ \ldots$

$a_2 + b_2 = a_3;\ b_2 + a_2 = b_3;\ \ldots$

Sequent combinations by three:

$a_2 + b_2 + c_2;\ a_2 + b_2 + d_2;\ a_2 + b_2 + e_2;\ a_2 + b_2 + f_2;$
$a_2 + c_2 + d_2;\ a_2 + c_2 + e_2;\ a_2 + c_2 + f_2;$
$a_2 + d_2 + e_2;\ a_2 + d_2 + f_2;$
$a_2 + e_2 + f_2;$
$b_2 + c_2 + d_2;\ b_2 + c_2 + e_2;\ b_2 + c_2 + f_2;$
$b_2 + d_2 + e_2;\ b_2 + d_2 + f_2;$
$b_2 + e_2 + f_2;$
$c_2 + d_2 + e_2;\ c_2 + d_2 + f_2;$
$c_2 + e_2 + f_2;$
$d_2 + e_2 + f_2.$

Figure 100. Sequent combinations by 3.

The above corresponds to three consecutive attacks. Each of the above combinations has 6 general or 3 circular permutations. The latter may develop still further through permutations of the higher orders:

$a_2 + b_2 + c_2 = a_3;\ a_2 + c_2 + b_2 = b_3;\ \ldots\ c_2 + b_2 + a_2 = f_3;$
or:
$a_2 + b_2 + c_2 = a_3;\ b_2 + c_2 + a_2 = b_3;\ c_2 + a_2 + b_2 = c_3.$

Further development of attacks is achieved by means of the coefficient-groups, which may assume any form, i.e., trinomials, polynomials whose terms are divisible by 2, or interference groups:

$3a_2 + b_2 + 2c_2;\ 3a_2 + c_2 + 2e_2 + 2a_2 + c_2 + 3e_2;$

$2b_2 + d_2 + 2f_2 + b_2 + 2d_2 + f_2;\ \ldots$

See Figure 101 on the following page.

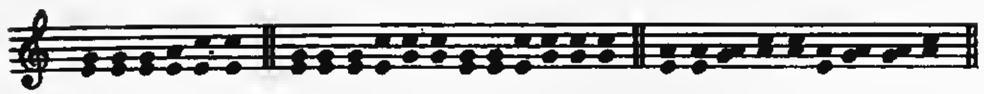


Figure 101. Coefficients of recurrence.

The latter, in turn, become subject to permutations (general or circular) as well as to permutations of the higher orders.

Sequent combinations by four:

$a_2 + b_2 + c_2 + d_2$; $a_2 + b_2 + c_2 + e_2$; $a_2 + b_2 + c_2 + f_2$;
$a_2 + b_2 + d_2 + e_2$; $a_2 + b_2 + d_2 + f_2$;
$a_2 + b_2 + e_2 + f_2$;
$a_2 + c_2 + d_2 + e_2$; $a_2 + c_2 + d_2 + f_2$;
$a_2 + c_2 + e_2 + f_2$;
$a_2 + d_2 + e_2 + f_2$;
$b_2 + c_2 + d_2 + e_2$; $b_2 + c_2 + d_2 + f_2$;
$b_2 + c_2 + e_2 + f_2$;
$b_2 + d_2 + e_2 + f_2$;
$c_2 + d_2 + e_2 + f_2$.

Figure 102. Sequent combinations by 4.

The above corresponds to four consecutive attacks. Each of the above combinations has 24 general or 4 circular permutations. The latter may develop still further through permutations of the higher orders:

$a_2 + b_2 + c_2 + d_2 = a_3$; $a_2 + b_2 + d_2 + c_2 = b_3$; . . .
or:
$a_2 + b_2 + c_2 + d_2 = a_3$; $b_2 + c_2 + d_2 + a_2 = b_3$; . . .

Further development of attacks is achieved by means of the coefficient-groups, which may assume any form, i.e., quadrinomials, polynomials divisible by 4, or interference groups:

$4a_2 + b_2 + 3c_2 + 2d_2$; $2a_2 + b_2 + c_2 + 2d_2$; $3a_2 + b_2 + 3c_2 + d_2$;
$4a_2 + b_2 + 3c_2 + 2d_2 + 2a_2 + 3b_2 + c_2 + 4d_2$;
$3a_2 + b_2 + 2c_2 + 3d_2 + a_2 + 2b_2 + 3c_2 + d_2 + 2a_2 + 3b_2 + c_2 + 2d_2$; . . .

See Figure 103 on the following page

Figure 103. Coefficients of recurrence.

The latter, in turn, become subject to permutations (general or circular), as well as to permutations of the higher orders.

Sequent combinations by five:

$a_2 + b_2 + c_2 + d_2 + e_2$; $a_2 + b_2 + c_2 + d_2 + f_2$; $a_2 + b_2 + c_2 + e_2 + f_2$;
$a_2 + b_2 + d_2 + e_2 + f_2$;
$a_2 + c_2 + d_2 + e_2 + f_2$;
$b_2 + c_2 + d_2 + e_2 + f_2$.

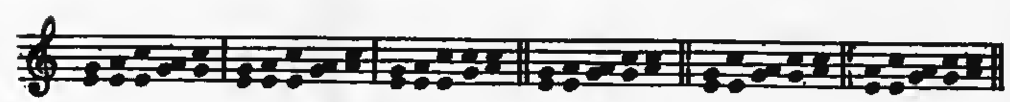


Figure 104. Sequent combinations by 5.

The above corresponds to five consecutive attacks. Each of the above combinations has 120 general or 5 circular permutations. The latter may develop still further through permutations of the higher orders:

$a_2 + b_2 + c_2 + d_2 + e_2 = a_3$; $a_2 + b_2 + c_2 + e_2 + d_2 = b_3$; . . .
or:
$a_2 + b_2 + c_2 + d_2 + e_2 = a_3$; $b_2 + c_2 + d_2 + e_2 + a_2 = b_3$; . . .

Further development of attacks is achieved by means of coefficient-groups, which may assume any form, i.e., quintinomials, polynomials divisible by 5, or interference groups:

$2a_2 + b_2 + 2c_2 + d_2 + 2e_2$;
$5a_2 + b_2 + 4c_2 + 2d_2 + 3e_2 + 3a_2 + 2b_2 + 4c_2 + d_2 + 5e_2$;
$3a_2 + b_2 + 3c_2 + d_2 + 3e_2 + a_2 + 3b_2 + c_2 + 3d_2 + e_2$; . . .

See Figure 105 on the following page

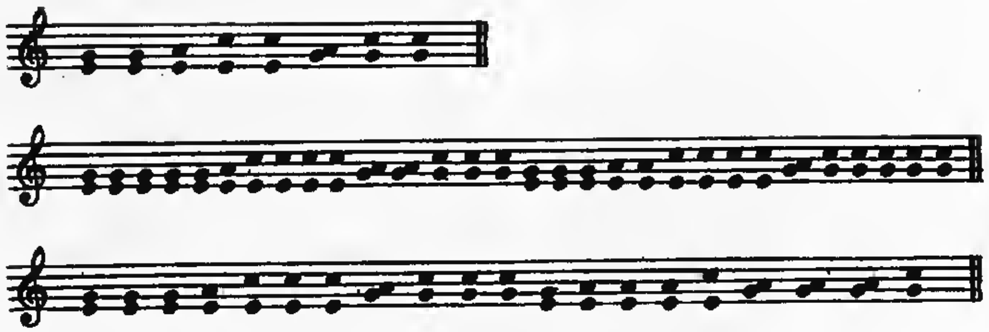


Figure 105. Coefficients of recurrence.

The latter, in turn, become subject to permutations (general or circular), as well as to permutations of the higher orders.

The sequent combination by six $(a_2 + b_2 + c_2 + d_2 + e_2 + f_2)$ has 720 general or 6 circular permutations.

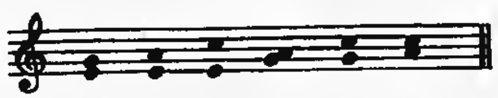


Figure 106. Sequent combinations by 6.

The latter may develop still further through permutations of the higher orders:

$a_2 + b_2 + c_2 + d_2 + e_2 + f_2 = a_3$; $a_2 + b_2 + c_2 + d_2 + f_2 + e_2 = b_3$; . . . .
or:
$a_2 + b_2 + c_2 + d_2 + e_2 + f_2 = a_3$; $b_2 + c_2 + d_2 + e_2 + f_2 + a_2 = b_3$; . . .

Further development of attacks is achieved by means of the coefficient-groups, which may assume any form, i.e., sextinomials, polynomials divisible by 6, or interference groups:

$3a_2 + b_2 + 2c_2 + 2d_2 + e_2 + 3f_2$;
$3a_2 + b_2 + 2c_2 + 2d_2 + 3e_2 + f_2 + 2a_2 + 2b_2 + 3c_2 + d_2 + 2e_2 + 2f_2$;
$5a_2 + 4b_2 + 3c_2 + 2d_2 + e_2 + 5f_2 + 4a_2 + 3b_2 + 2c_2 + d_2 + 5e_2 + 4f_2 +$
$+ 3a_2 + 2b_2 + c_2 + 5d_2 + 4e_2 + 3f_2 + 2a_2 + b_2 + 5c_2 + 4d_2 + 3e_2 +$
$+ 2f_2 + a_2 + 5b_2 + 4c_2 + 3d_2 + 2e_2 + f_2$; . . .

Figure 107. Coefficients of recurrence (continued).

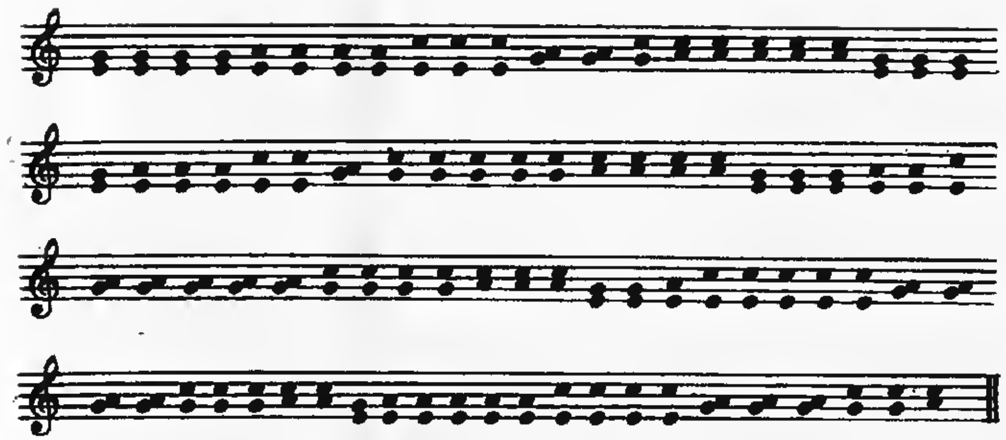


Figure 107. Cofficients of recurrence (concluded).

The latter, in turn, become subject to permutations (general or circular), as well as to permutations of the higher orders.

(6) I = a3p (one attack to a combination of three simultaneous parts).

The four invariant forms of (3) become elements of the second order:

$$\begin{array}{c} c \\ \hline b \\ \hline a \end{array} = a_2; \quad \begin{array}{c} d \\ \hline b \\ \hline a \end{array} = b_2; \quad \begin{array}{c} d \\ \hline c \\ \hline a \end{array} = c_2; \quad \begin{array}{c} d \\ \hline c \\ \hline b \end{array} = d_2.$$

Figure 108. Forms of I = a3p.

Any combination of these forms in sequence requires the inclusion of all four parts.

Sequent combinations by two:
$a_2 + b_2$; $a_2 + c_2$; $a_2 + d_2$;
$b_2 + c_2$; $b_2 + d_2$;
$c_2 + d_2$.

Figure 109. Sequent combinations by 2.

The above corresponds to two consecutive attacks. Further development of attacks is achieved by means of the coefficients of recurrence.

Each of the preceding combinations has 2 permutations. The latter may develop further through permutations of the higher orders:

$a_2 + b_2 = a_3$; $b_2 + a_2 = b_3$.

Further development of attacks is achieved by means of the coefficient-groups, which may assume any form, i.e., binomials, polynomials divisible by 2, or interference groups:

$2a_2 + b_2$; $3a_2 + 2b_2$; . . .
$2a_2 + b_2 + a_2 + 2b_2$; . . .
$3a_2 + 2b_2 + a_2 + 3b_2 + 2a_2 + b_2$; . . .

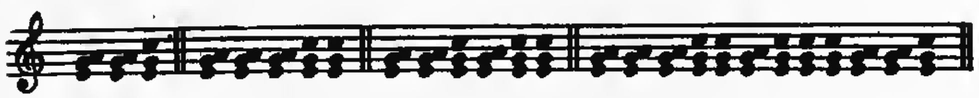


Figure 110. Coefficients of recurrence.

The latter in turn become subject to permutations (general or circular), as well as to permutations of the higher orders.

Sequent combinations by three:

$a_2 + b_2 + c_2$; $a_2 + b_2 + d_2$; $a_2 + c_2 + d_2$; $b_2 + c_2 + d_2$.

Figure 111. Sequent combinations by 3.

These correspond to three consecutive attacks. Further development of attacks is achieved by means of the coefficients of recurrence. Each of the above combinations has 6 general or 3 circular permutations. The latter may develop further through permutations of the higher orders:

$a_2 + b_2 + c_2 = a_3$; $a_2 + c_2 + b_2 = b_3$; . . .
or:
$a_2 + b_2 + c_2 = a_3$; $b_2 + c_2 + a_2 = b_3$; . .

Further development of attacks is achieved by means of the coefficient-groups, which may assume any form, i.e., trinomials, polynomials divisible by 3, or interference groups:

$3a_2 + b_2 + 2c_2$; . . .
$3a_2 + b_2 + 2c_2 + 2a_2 + b_2 + 3c_2$; . . .
$2a_2 + b_2 + c_2 + a_2 + 2b_2 + c_2 + a_2 + b_2 + 2c_2$; . . .

See Figure 112 on the following page.

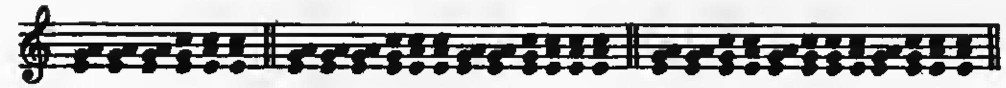


Figure 112. Coefficients of recurrence.

The latter, in turn, become subject to permutations (general or circular), as well as to permutations of the higher orders.

The sequent combination by four ($a_2 + b_2 + c_2 + d_2$) has 24 general or 4 circular permutations.

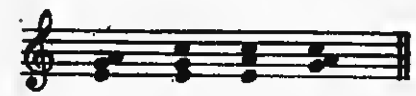


Figure 113. Sequent combination by 4.

The latter may develop further through permutations of the higher orders:

$a_2 + b_2 + c_2 + d_2 = a_3$; $a_2 + b_2 + d_2 + c_2 = b_3$; . . .
or:
$a_2 + b_2 + c_2 + d_2 = a_3$; $b_2 + c_2 + d_2 + a_2 = b_3$; . . .

Further development of attacks is achieved by means of the coefficient-groups, which may assume any form, i.e., quadrinomials, polynomials divisible by 4, or interference groups:

$4a_2 + b_2 + 3c_2 + 2d_2$;
$5a_2 + b_2 + 4c_2 + 2d_2 + 2a_2 + 4b_2 + c_2 + 5d_2$; . . .
$2a_2 + b_2 + c_2 + 2d_2 + a_2 + b_2 + 2c_2 + d_2 + a_2 + 2b_2 + c_2 + d_2$; . . .

Figure 114. Coefficients of recurrence.

(7) I = a4p (one attack to a combination of four simultaneous parts).

One invariant form: $\frac{d}{\frac{c}{\frac{b}{a}}} = a_2$.

Multiplication of attacks is achieved by direct repetition: $A = a_2;\ 2a_2;\ 3a_2;\ .\ .\ .\ ma_2$.

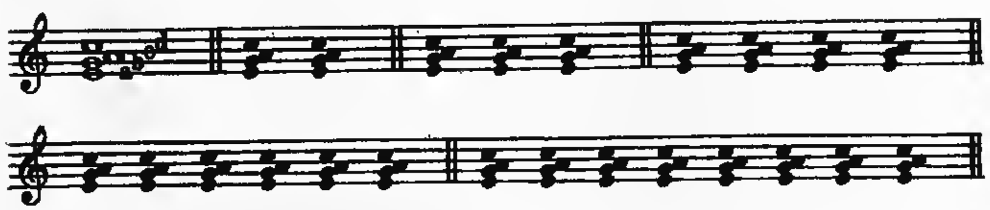


Figure 115. Multiplication of attacks.

Further variations may be obtained by means of permutations of the vertical (simultaneous) arrangement of parts. The extreme $p^{\rightarrow}$ of a given position must serve as a limit, that is, for a position above the original, the function, d, is the limit for the lower function. For a position below the original, the function, a, is the limit for the upper function.

The original position in relation to all the upper and all the lower positions is as follows:

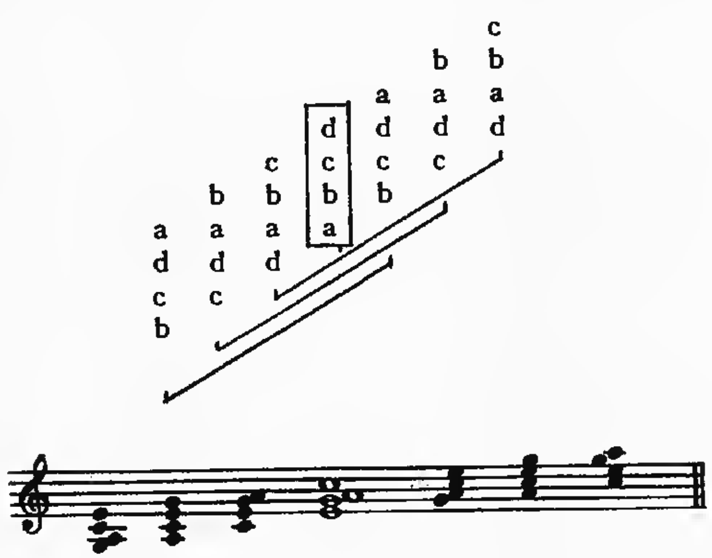


Figure 116. Relation to original position.

Positions indicated by the brackets are identical in the different octaves.

It is desirable to use the adjacent positions in sequence.

From the above variations of the original position, any number of attacks may be devised.

Voice-leading from the adjacent positions
(long durations)

Voice-leading from the original positions
(short durations)

Figure 117. Voice-leading.

From the above variations of the original position any number of attacks may be devised.

Figure 118. Multiplication of attacks (continued).

Figure 118. Multiplication of attacks (concluded).

C. Instrumental Forms of S = 4p

Material:

(1) melody with three couplings:

$$M\begin{bmatrix} \dfrac{p_{IV}}{\dfrac{p_{III}}{\dfrac{p_{II}}{p_I}}} \end{bmatrix}$$

(2) harmonic forms of four-unit scales;
(3) four-part harmony;
(4) four part stratum (S) of any compound harmony (Σ)

$$I = a: \ \dfrac{p_{IV}}{\dfrac{p_{III}}{\dfrac{p_{II}}{p_I}}} \text{ (24 general or 4 circular permutations)}$$

$$\dfrac{d_2}{\dfrac{c_2}{\dfrac{b_2}{a_2}}} \text{ (24 general or 4 circular permutations)}$$

$$\dfrac{md_2}{\dfrac{mc_2}{\dfrac{mb_2}{ma_2}}} + \dfrac{nd_2}{\dfrac{nc_2}{\dfrac{nb_2}{na_2}}} + \dfrac{pd_2}{\dfrac{pc_2}{\dfrac{pb_2}{pa_2}}} + \dfrac{qd_2}{\dfrac{qc_2}{\dfrac{qb_2}{qa_2}}}$$

(24 general or 4 circular permutations of the coefficients m, n, p, q).

1. Melody with three couplings. Illustrated by theme and variations. See figures 119 to 125 inclusive.

Figure 119. Theme.

Figure 120. Theme with couplings.

Figure 121. Variation I.

Figure 122. Variation II.

Figure 123. Variation III.

Figure 124. Variation IV.

Figure 125. Variation V.

2. Harmonic forms of four-unit scales. Illustrated by theme and ten variations. See figures 126 to 136 inclusive.

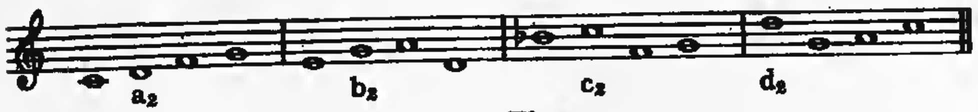


Figure 126. Theme.

Figure 127. Variation I.

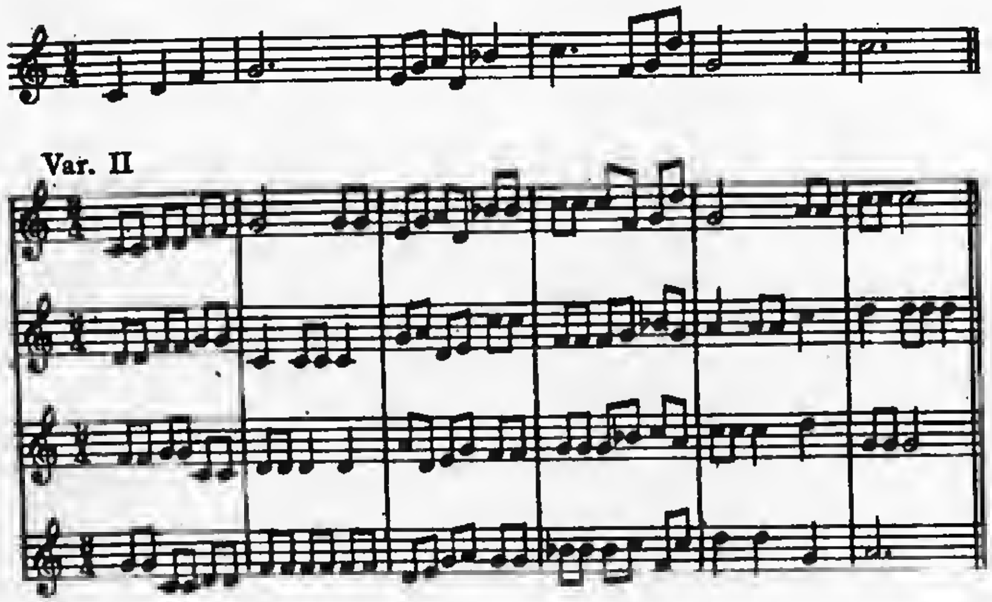


Figure 128. Rhythmic variation of the theme.

Figure 129. Variation III. (The two best reciprocals)

Figure 130. Variation IV. (Eight p→: 4 and 4 reciprocals)

Figure 131. Variation V.

Figure 132. Variation VI.

Figure 133. Variation VII.

Figure 134. Variation VIII.

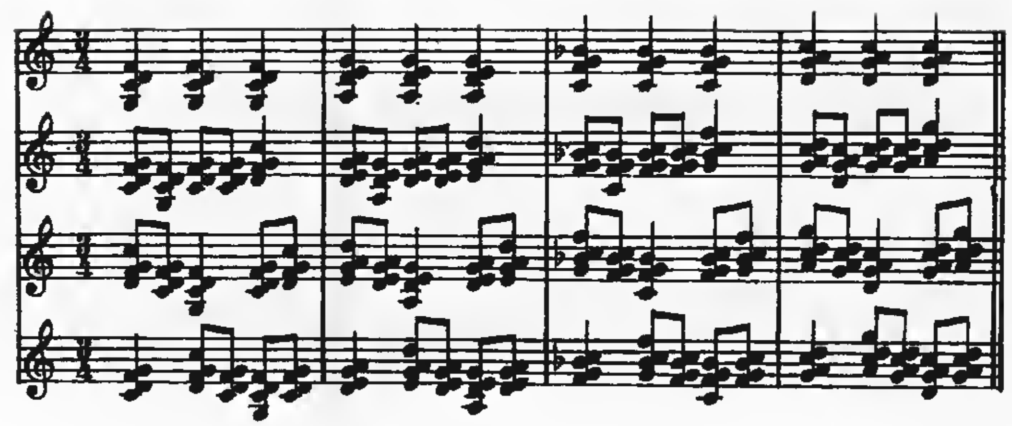


Figure 135. Variation IX.

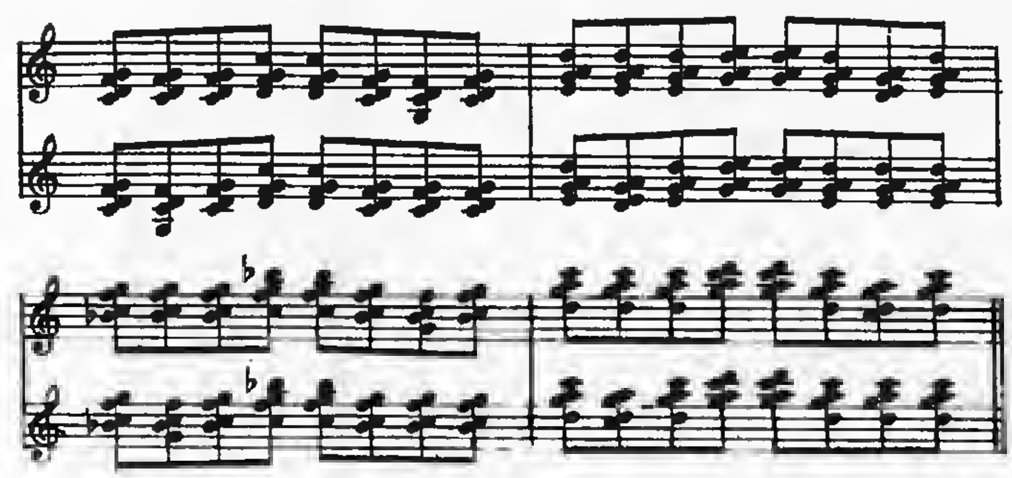


Figure 136. Variation X.

3. Four-part harmony. Illustrated by a theme and six variations. See figures 137 to 143 inclusive.

Figure 137. Theme and rhythmic variation.

Figure 138. Variation I. $I = 2a_2 + b_2 + c_2 + 2d_2$.

Figure 139. Variation II. Theme in rhythmic variation.
$I = 2a_2 + b_2 + c_2 + 2d_2$

Figure 140. Variation III. $I = a_2 + b_2 + c_2 + d_2 + e_2 + f_2$.

Figure 141. Variation IV. Theme in rhythmic variation.
$I = a_2 + b_2 + c_2 + d_2$

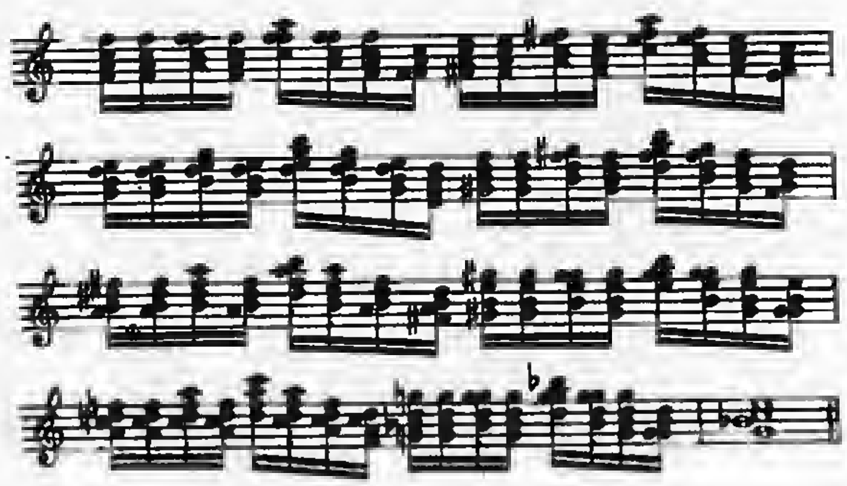


Figure 142. Variation V.

Figure 143. Variation VI.

(4) Four-part stratum (S) of any compound harmony (Σ). Illustrated by a theme and three variations. See figures 144 to 147 inclusive.

Figure 144. Theme.

Figure 145. Variation I.

Figure 146. Variation II.

Figure 147. Variation III.

Individual attacks emphasizing one, two, three or four parts can be combined into one attack-group of any desirable form.

Examples:

d d d dddd ddd
c c ccc c c c cccc
b b b bbb b b b b bbbb
I(S = 4p): aaaaaa ; aa a a ; aaa aa ; aa a ;

d ddd ddddd ddddddd d d ddd d d d d d ;
ccc ccc ccc cc ccccc c ccc c c cc c c c
bb b bbbbb b bbbb bbb bbb b b bbb b bb bb b
aaaaaa a a a ; aaaaaa a ; aa a a a ; aa a a ; aa aa a ;
d c
c d

a
dddddd dddd d d
ccccccc ccc ccc cc
bbbbbbb bb b bbb b
aaaaaa ; a aaaaaa ; . . .
d

Figure 148. Combining individual attacks.

Figure 149. Theme and variations.

CHAPTER 6

THE COMPOSITION OF INSTRUMENTAL STRATA

A. Identical Octave-Positions

In order to employ various instrumental groups as strata (S) in a simultaneous coordinated performance, it is necessary to arrange these instrumental strata into ***identical octave-positions***—a requirement which must be carried out with utmost rigidity, as any deviation from it will result in a loss of acoustical quality, particularly when one is dealing with orchestration.

When simultaneous pitch assemblages are in identical positions, their harmonics and their combination-tones (tones of the difference)* are similar. When such assemblages are in non-identical positions, their harmonics and their combination-tones do not appear in acoustical balance, the latter being achieved only when the ratio between all audible tones bearing identical names equals 2 or 4 or 8, etc.

This principle refers to all cases when the strata constitute a multiplication of ***one harmonic stratum.*** However, when different ***harmonic strata*** are used in superimposition (as I shall shortly show when I discuss my general theory of harmony),** their positions are independent; but if any of the superimposed harmonic strata of a harmonic Σ (compound harmonic structure) are duplicated in adjacent octaves as ***instrumental*** strata, the principle of ***identical positions*** for one harmonic stratum holds true.

To achieve acoustical balance between clockwise ("open") and counterclockwise ("close") positions of the assemblages, it is necessary to align both instrumental strata in such a way that their ***upper instrumental functions will be identical.***

If we designate the lower instrumental stratum as S_1 and the upper adjacent stratum as S_2, then the instrumental score (Σ) takes on the following form:

$$S = 2p; \quad \Sigma = 2S$$

$$\Sigma \left[\begin{array}{l} S_2 \left.\right] \frac{b}{a} \\ \\ S_1 \left[\right. \frac{b}{a} \end{array}\right] \text{ identical positions}$$

*Tones of the difference—or differential tooes—are tones produced by pairs of other tnoes. The frequency of a differential tooe is equal to the frequency of the higher tooe minus that of the lower tqoe. The differential tone is a real tone and may be heard clearly on instruments producing nearly "pure" frequencies. —(Ed.)

**See pp. 1074 ff., 1110 ff., 1139 ff.

Superimposition of two *non-identical* positions for S = 2p is obviously impossible; there is, however, another variant foı the identical positions:

$$\Sigma\begin{bmatrix} S_2 \begin{bmatrix} \frac{a}{b} \end{bmatrix} \\ S_1 \begin{bmatrix} \frac{a}{b} \end{bmatrix} \end{bmatrix} \text{ identical positions}$$

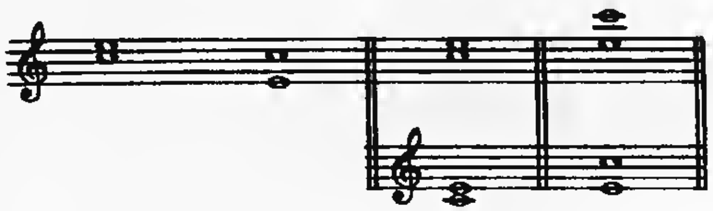


Figure 150. S = 2p.

Theme:

Instrumental octave-coupling

Instrumental Variation I.

Figure 151. Theme and instrumental variations (continued).

Instrumental Variation II.

Figure 151. Theme and instrumental variations. (concluded)

In three-part assemblages both identical and non-identical positions may be used in the octave-couplings.

S = 3p; Σ = 2S

$$\Sigma\begin{bmatrix} S_2 \\ \\ S_1 \end{bmatrix}\begin{array}{c|c|c|c|c|c} (1) & (2) & (3) & (4) & (5) & (6) \\ \frac{c}{\frac{b}{a}} & \frac{b}{\frac{c}{a}} & \frac{b}{\frac{a}{c}} & \frac{c}{\frac{a}{b}} & \frac{a}{\frac{c}{b}} & \frac{a}{\frac{b}{c}} \\ \frac{c}{\frac{b}{a}} & \frac{b}{\frac{c}{a}} & \frac{b}{\frac{a}{c}} & \frac{c}{\frac{a}{b}} & \frac{a}{\frac{c}{b}} & \frac{a}{\frac{b}{c}} \end{array} \text{ identical positions}$$

Figure 152. S = 3p. Identical positions.

$$\Sigma\begin{bmatrix} S_2 \\ \\ S_1 \end{bmatrix}\begin{array}{c|c|c} (1) & (2) & (3) \\ \frac{a}{\frac{c}{b}} & \frac{b}{\frac{a}{c}} & \frac{c}{\frac{b}{a}} \\ \frac{a}{\frac{b}{c}} & \frac{b}{\frac{c}{a}} & \frac{c}{\frac{a}{b}} \end{array} \text{ non-identical positions}$$

See Figure 153 on the following page.

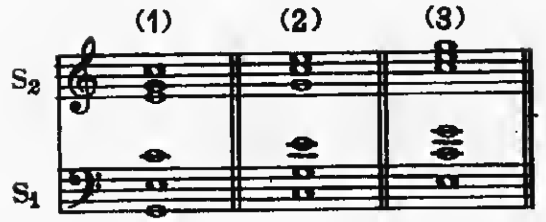


Figure 153. Non-identical positions.

The principles on which close and open positions of the same assemblage, S, can be brought into octave coordination, may be expressed as follows:

(1) both instrumental strata are in close position;
(2) both instrumental strata are in open position;
(3) the lower instrumental stratum (S_1) is in open position, and the upper instrumental stratum (S_2) is in close position.

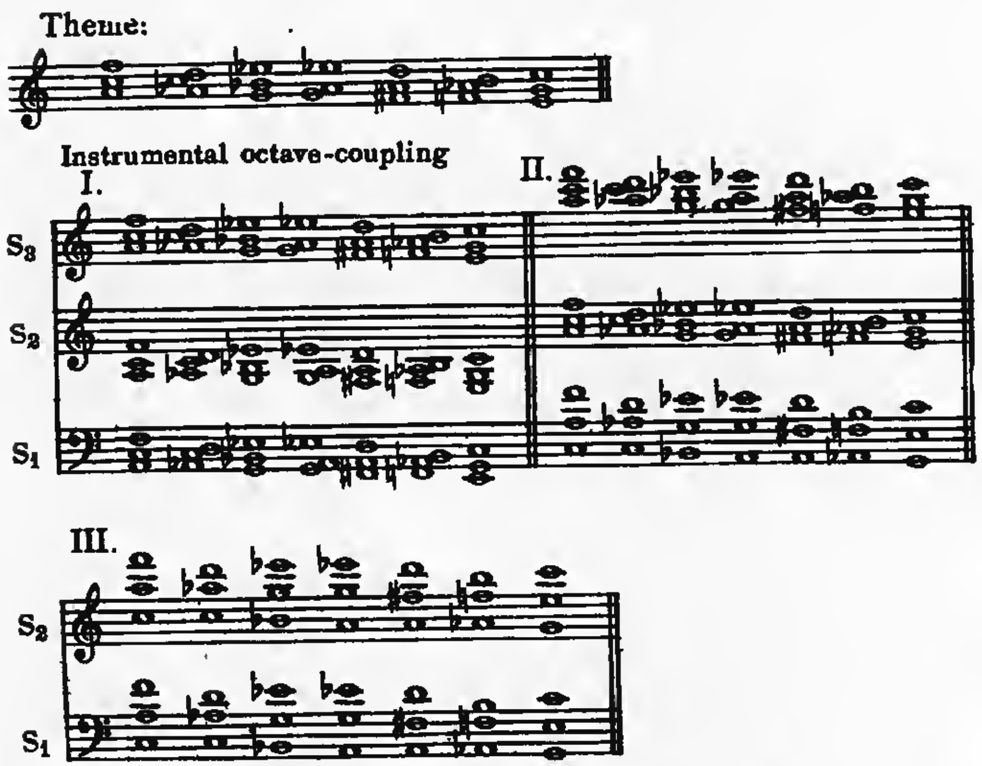


Figure 154. Theme and instrumental octave-coupling.

The reversal of (3) conflicts with the normal distribution of harmonics, which will deprive the Σ of its acoustical clarity. This means: whatever the number of instrumental strata aligned in octave coordination, ***there must never be an open position above a close.***

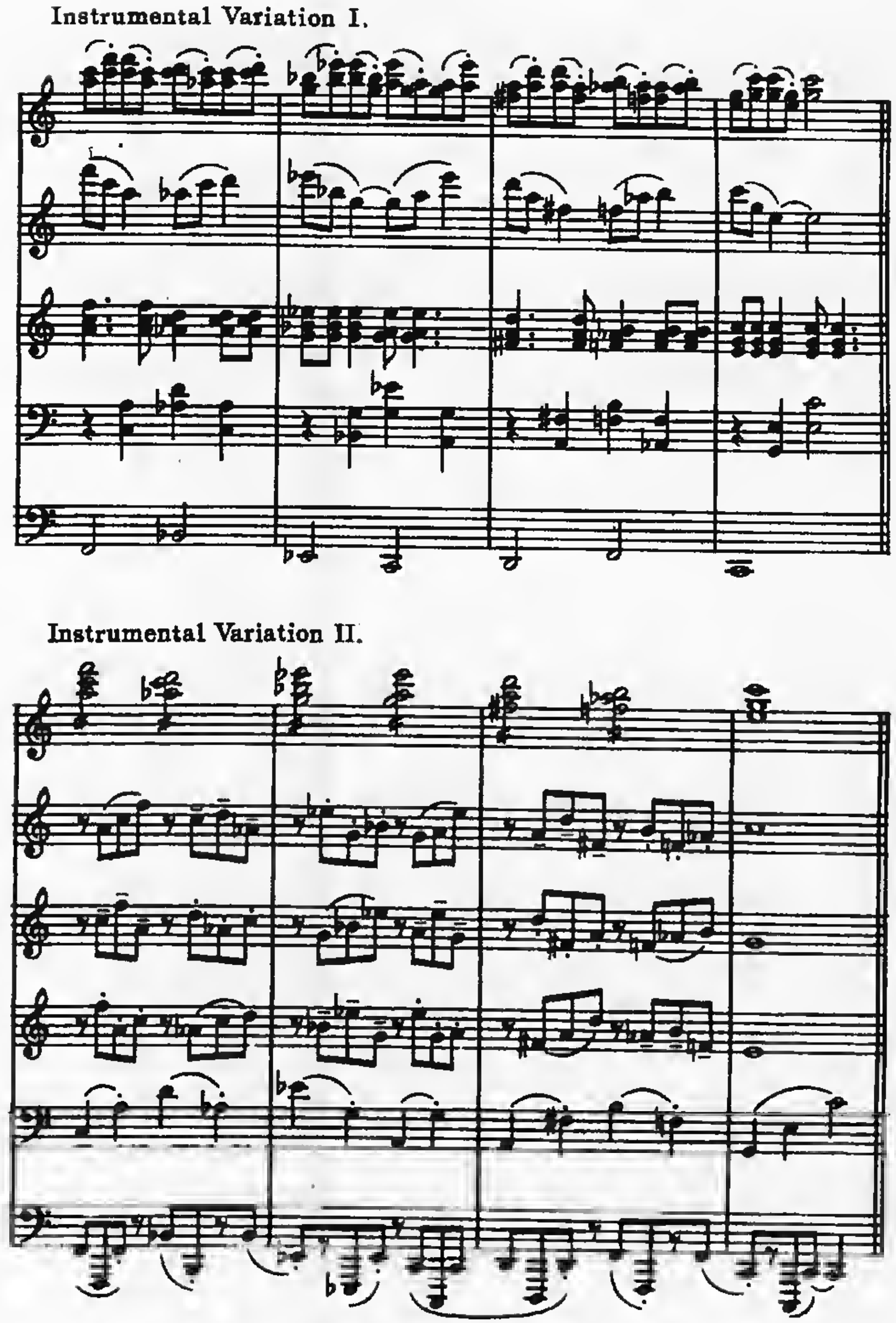


Figure 155. Two instrumental variations.

All the above described principles and regulations hold true for the four-part assemblages as well.

$S = 4p;\ \Sigma = 2S$

Σ												
S_2	d c b a	c d b a	c b d a	c b a d	d b c a	b d c a	b c d a	b c a d	d b a c	b d a c	b a d o	b a c d
S_1	d c b a	c d b a	c b d a	c b a d	d b c a	b d o a	b c d a	b c a d	d b a c	b d a c	b a d c	b a c d
	*)			*)							*)	

Figure 156. S = 4p. Identical positions.

Σ												
S_2	d c a b	c d a b	c a d b	c b a d	d a c b	a d c b	a c d b	a c b d	d a b c	a d b c	a b d c	a b o d
S_1	d o a b	o d a b	o a d b	c b a d	d a c b	a d c b	a o d b	a c b d	d a b c	a d b c	a b d c	a b c d
						*)						

See Figure 157 on the following page.

Figure 157. Identical Positions.

The above table shows all cases of identical positions. The forms marked by the asterisk are the practical ones for general use, as the distribution of all four functions is confined to a one-octave range. This permits more than one octave-duplication when necessary. All other positions of this table are practical mostly for one stratum instrumental forms, particularly for fingerboard and key-board instruments.

Non-identical positions with identical upper functions are most practical when constructed from the preceding forms marked by the asterisk (the latter are clockwise circular permutations when read upward):

Σ								
S_2	d c b a	a d c b	b a d c	c b a d	d o b a	a d c b	b a d c	c b a d
S_1	d a b c	a b c d	b c d a	c d a b	d b c a	a c d b	b d o a	c d b a

Σ												
S_2	d c b a	a d c b	b a d c	c b a d	d c b a	a d o b	b a d c	c b a d	d c b a	a d c b	b a d c	c b a d
S_1	d b a c	a c b d	b c a d	c b d a	d c a b	a d b c	b d a c	c a d b	d a o b	a b d c	b a c d	c a b d

See Figure 158 on the following page.

Figure 158. Most practical forms of the non-identical positions.

The above table represents matched pairs of S, the upper (S_2) being in close—and the lower (S_1) in open position. The choice of one or another form depends on its suitability to the type of orchestration—considerations of range, register, and adaptability to instrumental execution.

It is desirable that, in the case of octave duplication of an open position, all instrumental strata (in the open position) be *identical*.

If extra parts are added to a three-part or a four-part assemblage, such individual consecutive parts form their own instrumental strata and may be subjected to couplings for such a purpose. Whether the added part appears *below* or *above* the assemblage, its couplings must be always constructed *in the outward direction*. Thus, melody appearing above harmony must have couplings above its original functions:

$M = \frac{p_{II}}{p_I};\ \frac{p_{III}}{\frac{p_{II}}{p_I}};\ \ldots$, where p_I is the original function of melody and p_{II}, p_{III}, . . . are its couplings.

The *bass*, on the contrary, must have couplings *below* its original functions:

$B = \frac{p_I}{p_{II}};\ \frac{\frac{p_I}{p_{II}}}{p_{III}};\ \ldots$, where p_I is the original function of the bass and p_{II}, p_{III}, . . . are its couplings.

Forms of instrumental strata appearing in simultaneous coordination may assume *different degrees of density*. For instance:

(1)

$$\Sigma \left[\begin{array}{l} I(S_3) = a3p \\ I(S_2) = a2p \\ I(S_1) = ap \end{array}\right.$$

(2)

$$\Sigma \left[\begin{array}{l} I(S_4) = a4p \\ I(S_3) = a3p \\ I(S_2) = ap \\ I(S_1) = a2p \end{array}\right.$$

(3)

$$\Sigma \left[\begin{array}{l} I(S_3) = ap \\ I(S_2) = a2p \\ I(S_1) = a3p \end{array}\right.$$

(4)

$$\Sigma \left[\begin{array}{l} I(S_4) = ap \\ I(S_3) = a2p \\ I(S_2) = a3p \\ I(S_1) = a4p \end{array}\right.$$

Different instrumental strata may have different arrangements of time elements—including durations, rests, etc.

B. Acoustical Conditions for Setting the Bass

The form, $S = 3p$, either appears independently or in octave duplications. To such three-part harmony, a fourth part may be added and it is usually the harmonic bass, which is actually *an added part* and must be treated as an independent $S = p$ when it has no couplings. This fourth part may also be subjected to outward couplings. Neither the bass nor any of its couplings should ever cross any of the functions of the adjacent upper assemblage.

$S = 4p$ appears independently or in octave duplications. In hybrid five-part harmony, the bass is an added part and must be treated as an independent $S = p$; as in the preceding case, it may acquire outward couplings, but neither its original functions nor the couplings should ever cross any of the functions of the adjacent upper assemblage.

When four-part harmony appears independently, that is, without a bass as such, the entire S must be subjected to octave-coupling, but never any individual functions nor any combinations thereof. This principle applies to close positions. Harmony appearing in open position and in the lower instrumental stratum may have an octave doubling of its lower function (i.e., the bass voice of four-part harmony), whichever meaning this function may assume harmonically. This does not prevent us from doubling the entire stratum in the adjacent upper stratum either in open or in close position.

Theme:

Figure 159. Theme and instrumental octave-coupling (continued).

Instrumental octave-coupling

Figure 159. Instrumental octave coupling (continued).

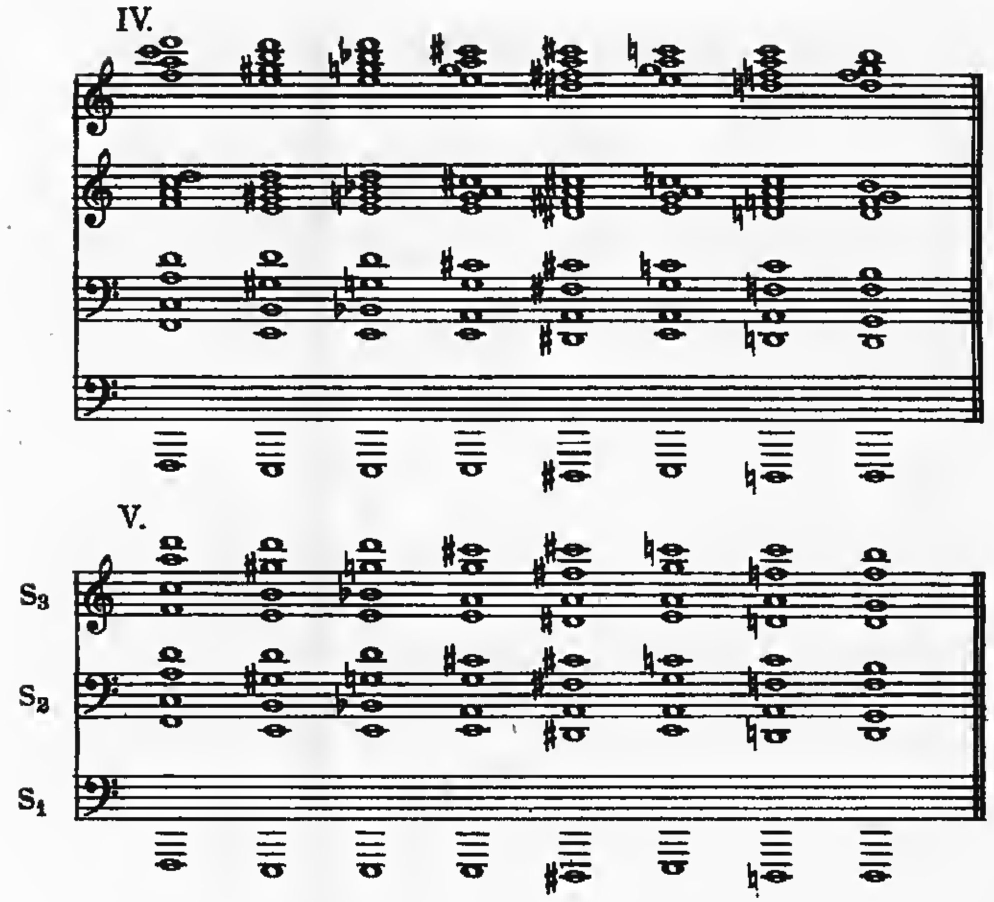


Figure 159. Instrumental octave coupling (concluded).

Instrumental Variation I.

I = 8abcbcd + bcdabc.

Figure 160. Five instrumental variations (continued).

Figure 160. Instrumental variation I = 3abcbcd + bcdabc

Instrumental Variation II.

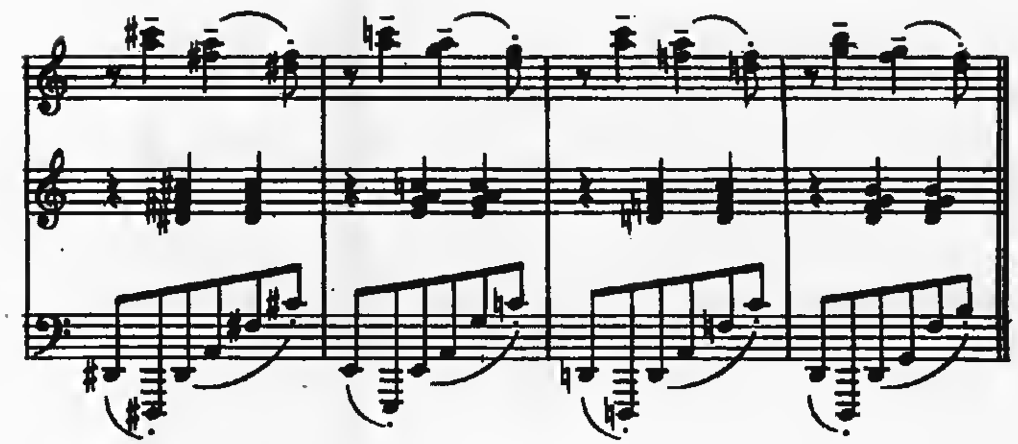
Figure 160. Instrumental variations (continued).

Instrumental Variation III.

Figure 160. Instrumental variations (continued).

Instrumental Variation IV.

Figure 160. Instrumental variations (continued).



Instrumental Variation V.

Figure 160. Instrumental variations (concluded).

It is obvious that instrumental variations of Figure 160 are complete and self-sufficient scores of harmonic accompaniments. They may be subjected to orchestration without their forms being changed.

CHAPTER 7

SOME INSTRUMENTAL FORMS OF ACCOMPANIED MELODY

NOW that a systematic classification of all instrumental forms—1(S = p, 2p, 3p and 4p)—and their applications to individual fields of melody and harmony has been completed, we shall evolve some of the most typical forms of combined applications. The most universal of the latter is, undoubtedly, ***melody with harmonic accompaniment***, and this involves both harmonization and melodization.

A. Melody with Harmonic Accompaniment

The following considerations specifically pertain to this problem:

(1) The melody should not cross any of the harmonic parts; it may be placed above, between, or below any of the harmonic strata—the various styles of melodization and harmonization being each subject to limitations. When the melody is below the lower instrumental stratum of harmony, any harmonic bass must be completely eliminated. The number of instrumental strata depends on the range of melody (or of melody with its couplings). None of the couplings of melody should ever cross any of the parts of the adjacent harmonies, whether above or below.

(2) Couplings added to the original melody may be placed above it, or below it, or they may surround it. The number of couplings is optional. The most common form of coupling is the octave. Other intervals—as well as the filling in of the octave with other intervals—may also be used. Consonant as well as dissonant harmonic intervals may be used, the selection of one or the other being a matter of style. The 19th century favored thirds and sixths; the 20th century, on the contrary, features fourths, fifths, sevenths, and seconds as couplings. All couplings of melody accompanied by harmony must be *diatonic*, i.e., they must conform to the pitches of accompanying harmonic structure (auxiliary tones being neglected). Thus, if a third is selected as the coupling, it may be major against some chords and minor against some other chords.

Instrumental Variation of Accompanied Melody

Figure 161. Theme. Melodization of harmony of figure 108 (continued).

Figure 161. Melodization of harmony of figure 108 (concluded).

Figure 162. Instrumental variation I.

Figure 163. Instrumental variation II (continued).

Figure 163. Instrumental variation II (concluded).

Figure 164. Instrumental variation III (continued).

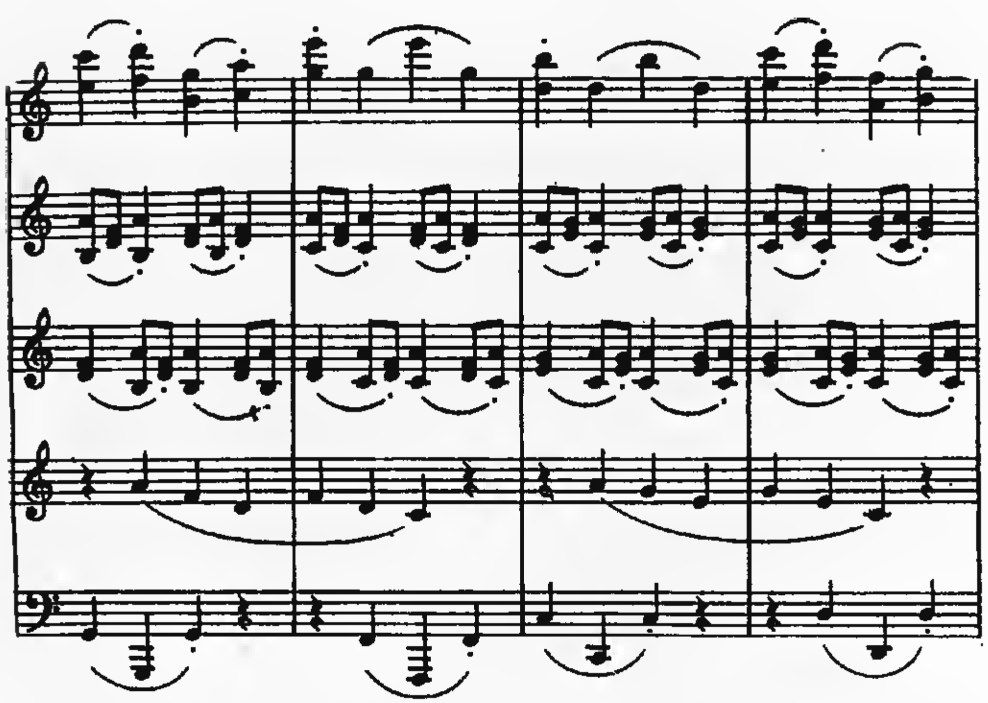


Figure 164. Instrumental variation III (continued).

Figure 164. Instrumental variation III (concluded).

B. Instrumental Forms of Duet with Harmonic Accompaniment

The principles on which instrumental variations of an accompanied duet may be devised are:

(1) If M_I and M_{II} do not cross each other at any point, then *diatonic* couplings may be used in either or in both parts. If both parts are coupled, their respective couplings may be either identical or non-identical. Neither of the two melodies nor any of their couplings may cross any of the parts of accompanying harmony. Crossing melodies should have no couplings.

(2) The harmonic bass may be used only if both melodies are placed above the harmony; in all other cases, such a bass must be eliminated. All the following positions are acceptable ($H^{\rightarrow}$ referring to harmony without bass):

(a) $\begin{array}{c} M_I \\ \hline H^{\rightarrow} \\ \hline M_{II} \end{array}$ (b) $\begin{array}{c} M_I \\ \hline M_{II} \\ \hline H^{\rightarrow} \\ \hline \text{Bass} \end{array}$ (c) $\begin{array}{c} H^{\rightarrow} \\ \hline M_I \\ \hline M_{II} \end{array}$ (d) $\begin{array}{c} H^{\rightarrow} \\ \hline M_I \\ \hline H^{\rightarrow} \\ \hline M_{II} \end{array}$ (e) $\begin{array}{c} M_I \\ \hline H^{\rightarrow} \\ \hline M_{II} \\ \hline H^{\rightarrow} \end{array}$

Instrumental Variation of Accompanied Duet.

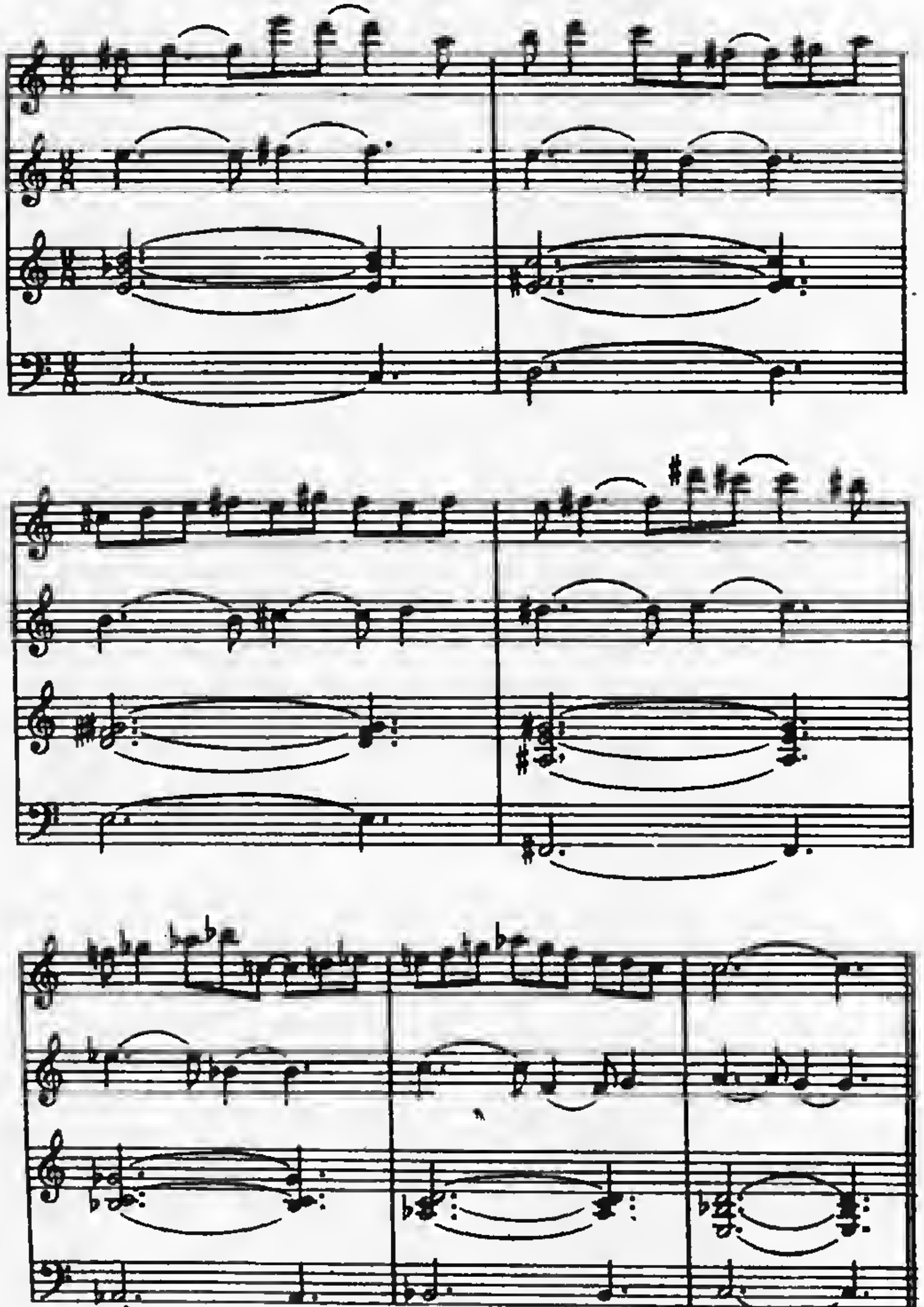


Figure 165. Theme. Two part melodization of figure 3.

Instrumental Variation

Figure 166. Instrumental variation (continued).

Figure 166. Instrumental variation (concluded).

CHAPTER 8

THE USE OF DIRECTIONAL UNITS IN INSTRUMENTAL FORMS OF HARMONY

ONCE the auxiliary tones to be used have been pre-set, they may be used as a part of the general technique of instrumental forms. There are no limitations to the ***sequent*** use of auxiliary tones in instrumental strata. Any instrumental stratum may or may not have directional units. In the case of one instrumental stratum, this proposition will always hold true; in the case of several instrumental strata broken into various forms of arpeggio in sequence (single, double, triple and quadruple attacks), it is preferable to adhere to the acoustical set, i.e., to use directional units in the uppermost stratum.

In simultaneous groups of strata, directional units may be used in strata of identical octave-duplication of simultaneous assemblages ***only*** when such strata belong to ***different*** tone-qualities; otherwise the subsequent orchestration will lack clarity. In some instances, a compromise may be affected by juxtaposition of contrasting attacks or by extremely contrasting speeds in the two respective instrumental strata. For example, a part with directional units may be played ***legato***, and a part with neutral units only may be played ***staccato***; or one part may move by sustained half-notes while the other produces instrumental figuration in eighth-notes, with the latter using the directional units.

All other forms of melodic figuration—such as suspensions, anticipations, and passing tones—must either not be used at all, or else be treated as chordal functions, which would mean they should be in the instrumental strata evolved through octave-duplication.

Examples of the Instrumental Forms of Harmony Containing Melodic Figuration

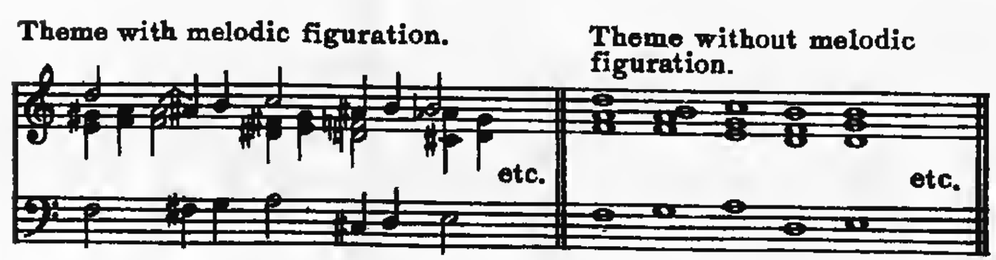


Figure 167. Theme and instrumental variations (continued).

Instrumental Variation I.

Instrumental Variation II.

etc.

Instrumental Variation III.

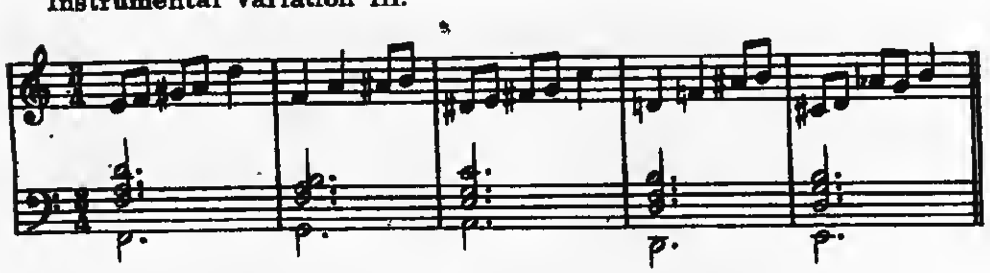


Figure 167. Instrumental variations (continued).

Instrumental Variation IV.

Figure 167. Instrumental variations (concluded).

Theme: suspended, passing and anticipated tones applied to a given chord-progression.

etc.

Figure 168. Theme and instrumental variations (continued).

Instrumental Variation

Figure 168 Instrumental variations(continued)

Figure 168. Instrumental variations (concluded).

CHAPTER 9

INSTRUMENTAL FORMS OF TWO-PART COUNTERPOINT

THE principles we have already established for instrumental forms of accompanied duet, apply to an unaccompanied duet as well. Thus, a canon or a fugue may be subjected to instrumental variation. However, as polyphonic duets have a considerable degree of mobility, the main aspect of this technique lies in the utilization of *couplings* as such.

When correlated melodies are unaccompanied their couplings become *automatic*, i.e., once the coupling has been selected, its form—or their forms—does not vary throughout the entire composition. Couplings of M_I and M_{II} may have independent forms. Selection of the automatic couplings is left to the composer's discretion. Such couplings attribute to the counterpoint a certain persistent harmonic flavor. It is to be expected that the two contrapuntal parts supplied with couplings will frequently clash with each other; but without this, the music would lack harmonic contrasts.

The number of couplings added to each part is also optional. The two contrapuntal parts may each have a different number of couplings. For ordinary purposes, the addition of one or two couplings to each part suffices, and doing this attributes to the polyphonic texture a definite and individual harmonic quality.

With a considerable number of couplings added to each contrapuntal part, composition of continuity based on variable density (low, medium, high) becomes possible. Schemes of density variation may be worked out in a fashion similar to that used in the treatment of density as described in my earlier discussion of two-part melodization.* All the more detailed and elaborate forms of continuity based on coupled polyphony will be discussed when I come later to the general theory of composition. For the time being, many valuable results may be obtained through the use of initiative in combining factorial continuity with couplings and instrumental forms.

Below is a table which suggests in detail the system by which more forms of couplings may be obtained. As in most cases there is a definite predominance of a certain harmonic interval occurring as between M_I and M_{II}, it is advisable to select a specific coupling which satisfies some particular occasion in relation to this predominant interval; then the chances of producing this particular harmonic sonority will increase.

Couplings, as in all earlier cases, may be distributed below or above the original pitch unit. Pitch units as well as their couplings are subject to octave-couplings.

*See Vol. I, p. 700.

Figure 169. Exemplary table of automatic couplings.

The couplings are marked by the black notes. Similar tables may be developed with regard to other harmonic intervals. We shall now refer to examples of application of this technique.

Fugue with Automatic Couplings
(Two Parts)

Two Part Fugue*

Type III: Superimposed Coupling

$5T\frac{0}{CF} + 5T\frac{CF}{CP} + 5T\frac{CF+cpl}{CP} + 5T\frac{CF}{CP+cpl}$ etc.

by Richard Benda

Form: cpl

Figure 170. Fugue by student Richard Benda (continued).

Figure 170. Fugue by student Richard Benda (continued).

Figure 170. Fugue by student Richard Benda (continued).

Figure 170. Fugue by student Richard Benda (continued).

Figure 170. Fugue by student Richard Benda (concluded).

Homophonic Compositions Developed from Two-Part Counterpoint

(1) Original; (2) Couplings; (3) Instrumental forms: Var. I and II.

(1)

(2)

Figure 171. Two-part counterpoint.

(3) Var. I.

Figure 172. Variation (continued).

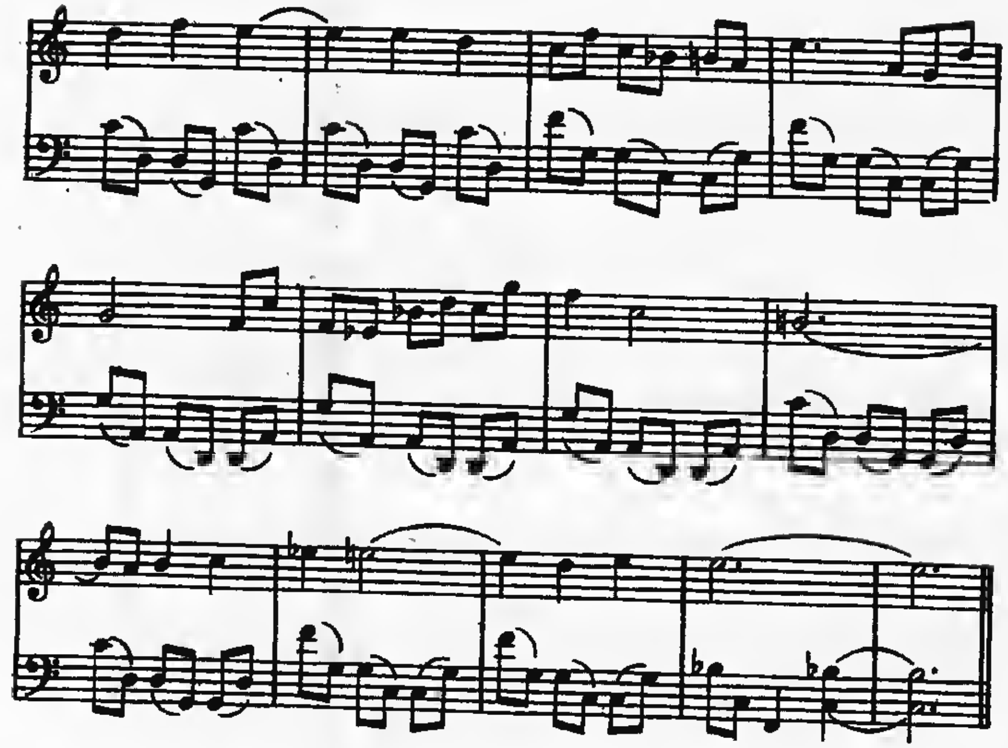


Figure 172. Variation (concluded).

(3) Var. II.

Figure 173. Variation (continued).

Figure 173. Variation (concluded).

CHAPTER 10

INSTRUMENTAL FORMS FOR PIANO COMPOSITIONS

WRITING for the pianoforte requires a highly specialized technique because of peculiarities in execution of music for this instrument. Human beings are bi-fold; they have right and left arms and hands, and they have two sets of fingers arranged in bi-fold symmetry. Because of the strength of the thumb and the relative weakness of some of the other fingers, an extensive exercise system has been developed for the purpose of equalizing the striking power of the various fingers. But this equalization has never been completely achieved. The better pianists, however, have a fair approximation to uniformity in this respect—close enough for practical purposes.

Nevertheless, certain characteristics remain invariant owing to the bi-foldness of the finger arrangement. One of these characteristics is the excessive striking power of the thumb; it leads to an adaptation of some patterns of instrumental forms to piano writing. For example, it is easy and natural in a consecutive group of arpeggio figures to single out the *lower* instrumental function (producing the effect of self-accompanied melody) when such figures are played by the *right hand*, or to single out the *upper* function, when played by the left hand.

This fact and existing piano literature—to which techniques of execution are more or less adjusted (e.g., the convention that instrumental forms of harmony are to played mostly with the left hand)—have created a whole system of digital habits which are so crystallized by now that very few composers—particularly if they are pianists themselves—can develop any really independent style of piano writing.

The purpose of this discussion is to demonstrate the inexhaustible resources of instrumental forms and possibilities, so as to enable the composer to develop any number of his own individual styles.

The principles of natural acoustical arrangement, i.e., the contraction of harmonic intervals in the direction of increasing frequencies (upward direction of pitch) and the octave duplication of identical positions of assemblages combined with the principle of outward coupling, hold true in piano writing as well. The use of directional units remains the same as in all other instrumental forms previously described.

The only peculiarity which is typical of piano writing is the execution of melody in octave coupling filled out by other functions of the same assemblage. In some cases, not all of the functions of an assemblage are used, although certain fingers will remain unengaged. The most customary forms are the thirds from the upper or the lower instrumental function, or the third from one function and the fourth from another. However, these conventional forms of duplication are influenced by their common origin, which is *harmonic*, i.e., the use of S(5) and its inversions.

This viewpoint is well confirmed by present-day American dance music (it has many trade names: jazz,* swing, blues, boogie, etc.), in which it is customary to fill any octave coupling with the remaining functions of S(7), or S(5) with added 13th. This method of coupling melody has become so universal that its use is a permanent feature of many arrangements and orchestrations under the trade name of "block-harmony." This leads me to the belief that the first arrangers and orchestrators of such music were *pianists*, for the orchestral conception of these instrumental strata couplings is acoustically much more sound; the latter correspond to the forms described in this branch.

Many pianist-composers of the past, such as Chopin and Schumann, had very chaotic styles of piano writing, from both the acoustical and the harmonic standpoints. This is due to the fact that their compositions emerged from piano-improvising—and the latter was based, in their cases, on comfortable positions of hands, which in many instances conflicted with the standards of voice-leading. And although the piano acoustically can stand almost anything because it is primarily a percussive instrument (i.e., an instrument whose sounds fade out very rapidly), the orchestral works of these pianist-composers show how they had to pay the penalty. Chopin's own scores of his piano concertos, for example, are not played in the composer's own orchestration!

As the piano is a frequent participant in ensembles and orchestra, being used both as a solo and as an accompanying instrument, it is very desirable indeed to apply only such instrumental couplings as are used in orchestral writing. It would be of great advantage, both harmonically and acoustically, if the amateurish "block-harmony" were eliminated and piano writing were restricted to the general forms of instrumental couplings.

This requirement may be met by following either of these two procedures:

(1) an octave coupling of melody may be used only in the absence of other couplings of the same melody;

(2) any assemblage may be coupled in identical positions in the adjacent octave; *all* units of the assemblage must be included in the instrumental octave coupling if the latter takes place.

Octave coupling of melody or bass is a comfortable interval for most hands. It can be struck without much danger of being missed—hence the popularity of octave coupling on the keyboard.

*Schillinger has suggested that, inasmuch as pre-swing jazz is performed in rhythms deriving from $\frac{8}{8}$ series, and swing—although written in $\frac{4}{4}$—is actually performed in $\frac{12}{8}$ series rhythms, the term "jazz" be reserved for $\frac{8}{8}$ style and "Swing" be used to denote $\frac{12}{8}$ style. See his article in *Metronome*, July, 1942. The outstanding feature of boogie-woogie is the *basso ostinato*; it must also always be in $\frac{12}{8}$ series—i.e., the characteristic group is (as written in $\frac{4}{4}$) the triplet, , instead of the eighth, (Ed.)

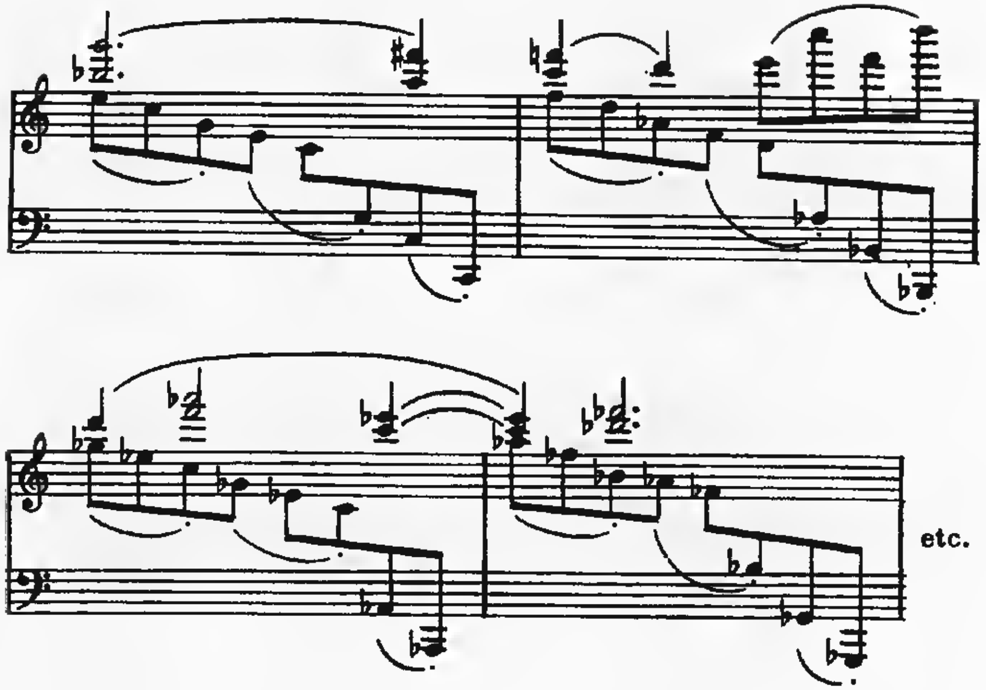
Examples of Conventional Instrumental Piano Forms.

Figure 174. Theme. Melodization of harmony.

Instrumental Variation I.

Figure 175. Instrumental variation I.

Instrumental Variation II.



Instrumental Variation III.

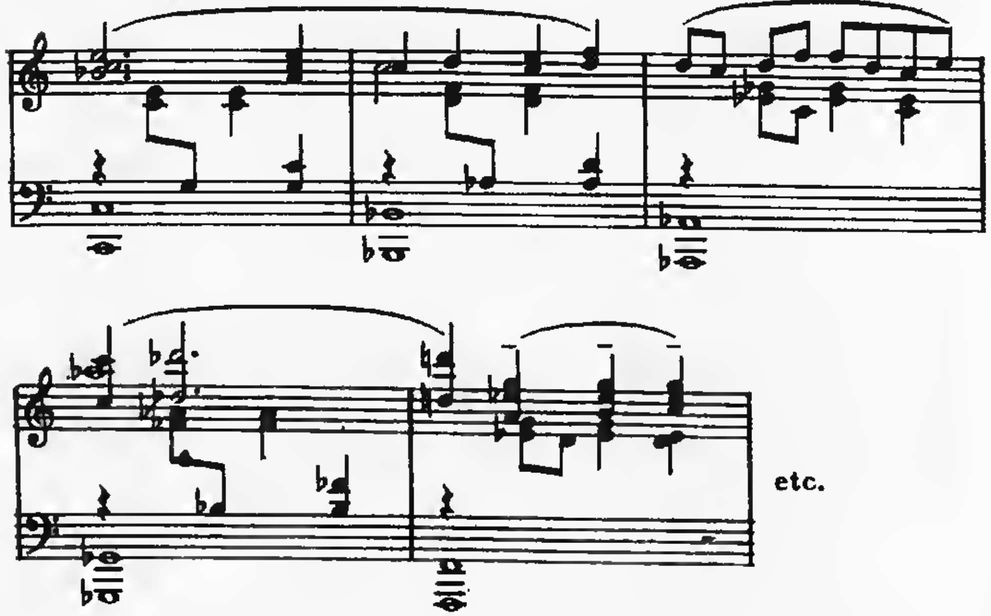


Figure 176. Instrumental variations II, III and IV (continued).

Instrumental Variation IV.

Figure 176. Variation (concluded).

The reader may use his own researches to verify how the problem of the instrumental form for piano writing was solved by Chopin, Mendelssohn, Schumann, Liszt, Rachmaninov, Scriabine, Debussy, and Ravel. Observe the evolution of piano styles toward normal acoustical forms* from Scarlatti, Clementi, etc., down to Liszt and Rachmaninov. Particular attention should be paid to the piano compositions of Nicholas Medtner.**

All problems pertaining to the piano's possibilities as to tone qualities, forms of attacks, and dynamics will be discussed when we come to a discussion of orchestration.

The main subject of the present study is the systematization of piano forms in their relation both to hands and the keyboard.

*See p. 1043.

**Nicolas Medtner is a contemporary Russian composer who was born in Moscow in 1880 and is now living in London. He served as professor of pianoforte at the Moscow Conservatory for a period of years and has toured England, France, and the United States as concert pianist. His best-known compositions are for the piano. (Ed.)

A. The Positions of Hands (R and L) with Respect to the Keyboard

Designating the right hand as R and the left hand as L, we shall evolve and demonstrate the inexhaustible possibilities and diversity of piano styles.

Fundamental principles:

(1) L is located below R; or
(2) L is located above R, crossing over it; or
(3) R is located below L, crossing over it;
(4) there are different *registral* positions for both L and R, and *each* such position emphasizes and corresponds to *one* instrumental stratum.
(5) The reasons for crossing R and L are:
  (a) excessive mobility of the instrumental form;
  (b) more comfortable control over a certain instrumental stratum (often the melody);
(6) avoidance of overloading each hand with too many scalewise passages.

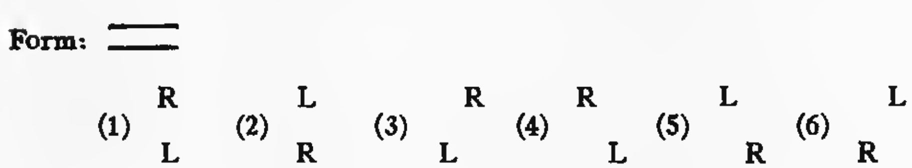
The latter principle was strictly followed by Debussy, but was neglected by his predecessors. The utilization of five fingers (and therefore five points) in one passage is a very sound and economical principle, quite in contrast to the old-fashioned, conventional finger-twisting. To be sure, not too much can be done toward revising the fingering in old compositions, but we are here concerned with the writing of *new* works rather than with the execution of old ones.

The positions of R and L in their different distributions through the strata may refer either to melody, or to harmony, or to a combination of both, as well as to two or more correlated melodies; the following examples of positions, in other words, may be applied in more than one way.

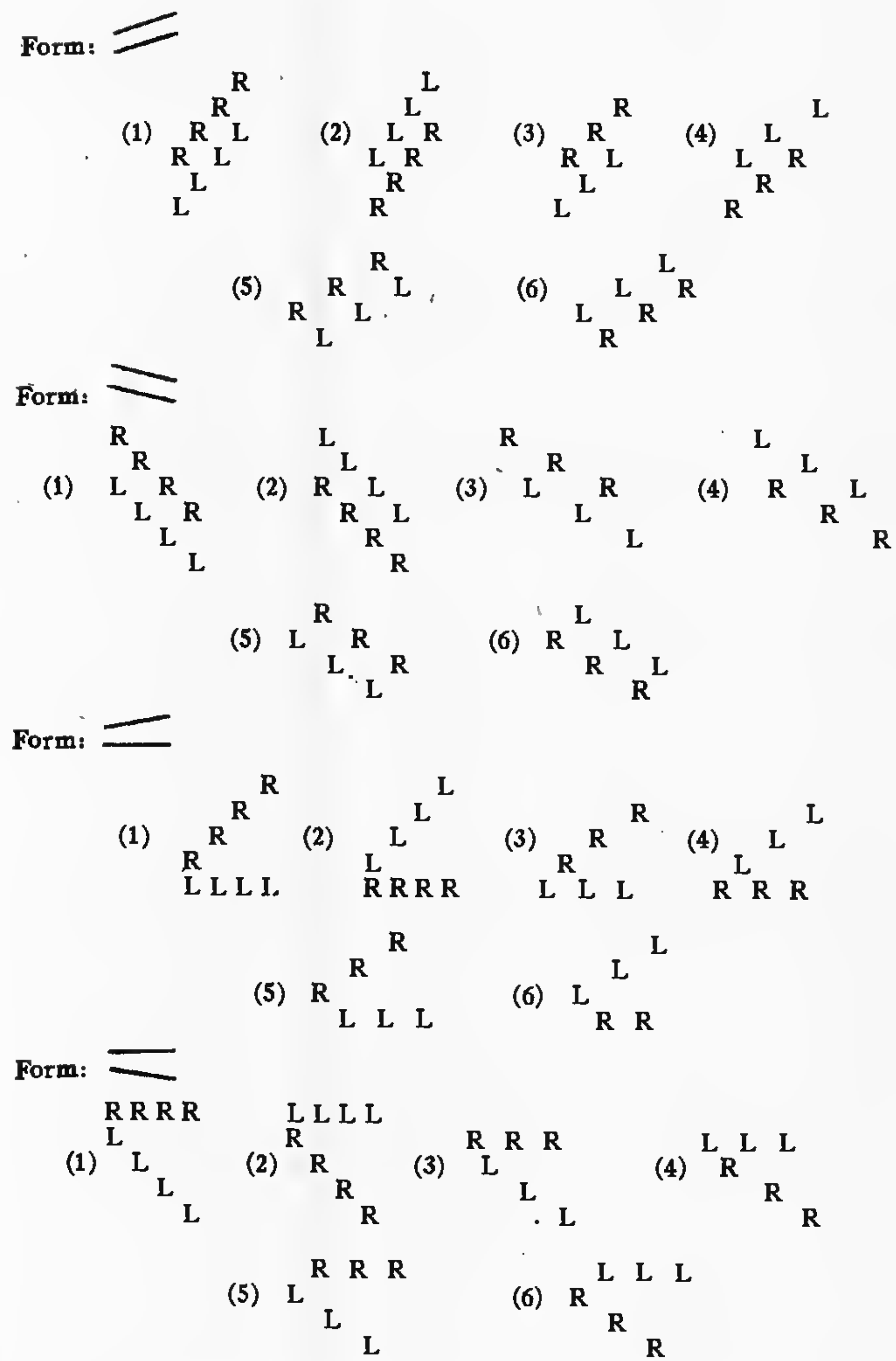
The different levels in the table represent the different instrumental strata The time sequence of the different positions is represented in the usual manner i.e., from left to right. Time periods for the different sequent positions are not specified. The entire scheme is evolved geometrically and is based on level, ascending, and descending directions—and on the number of instrumental strata involved.

Classification of R and L with Respect to Keyboard, Time Sequence and the Number of Instrumental Strata.

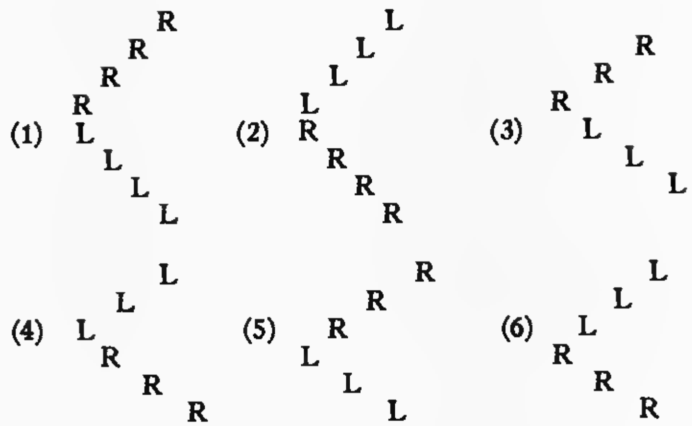
$\Sigma = 2S$; Two Staves



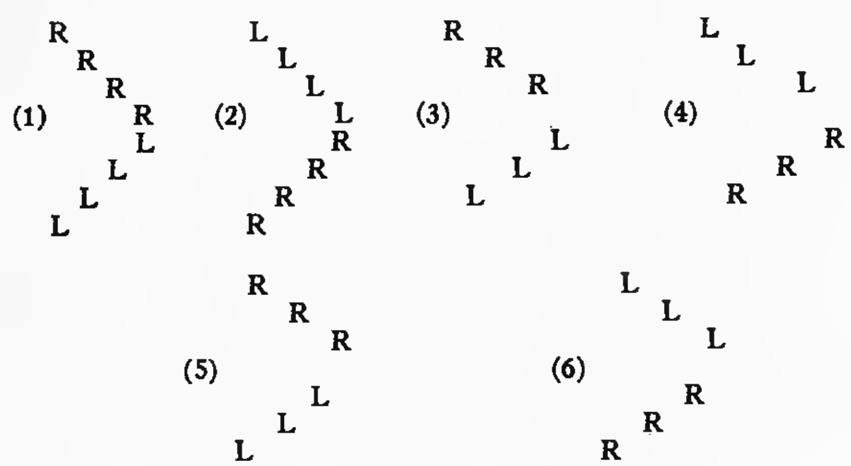
$\Sigma = NS$; Two Staves



Form:

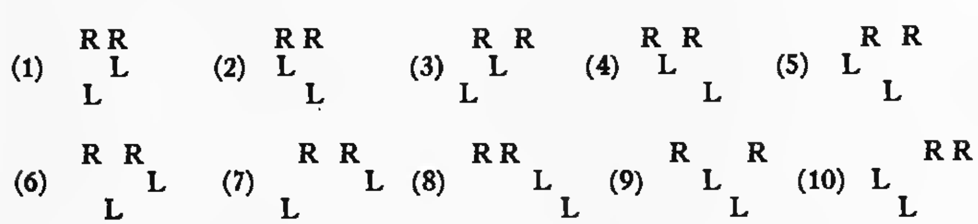


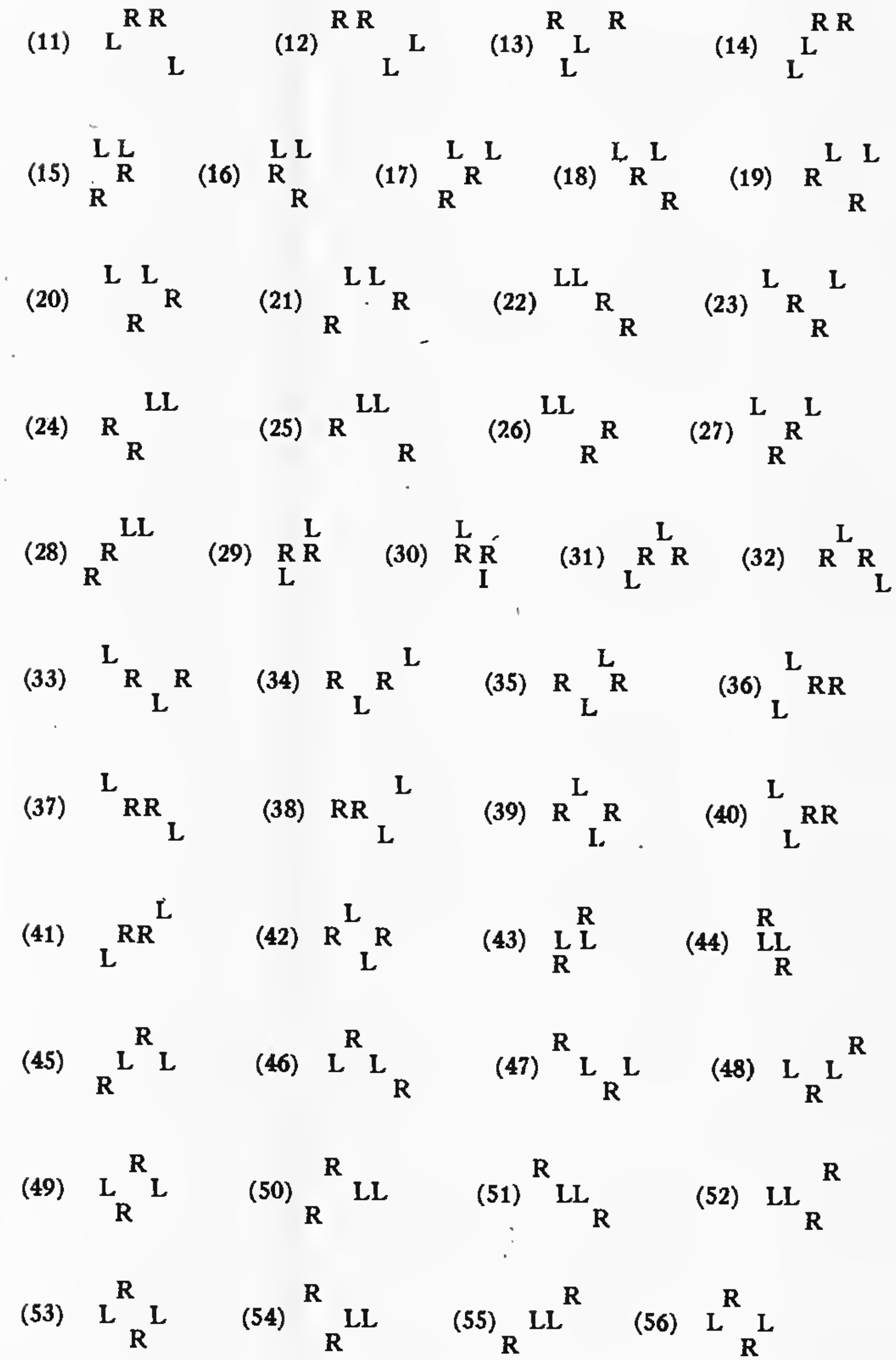
Form:



$\Sigma = 3S$; Three Staves

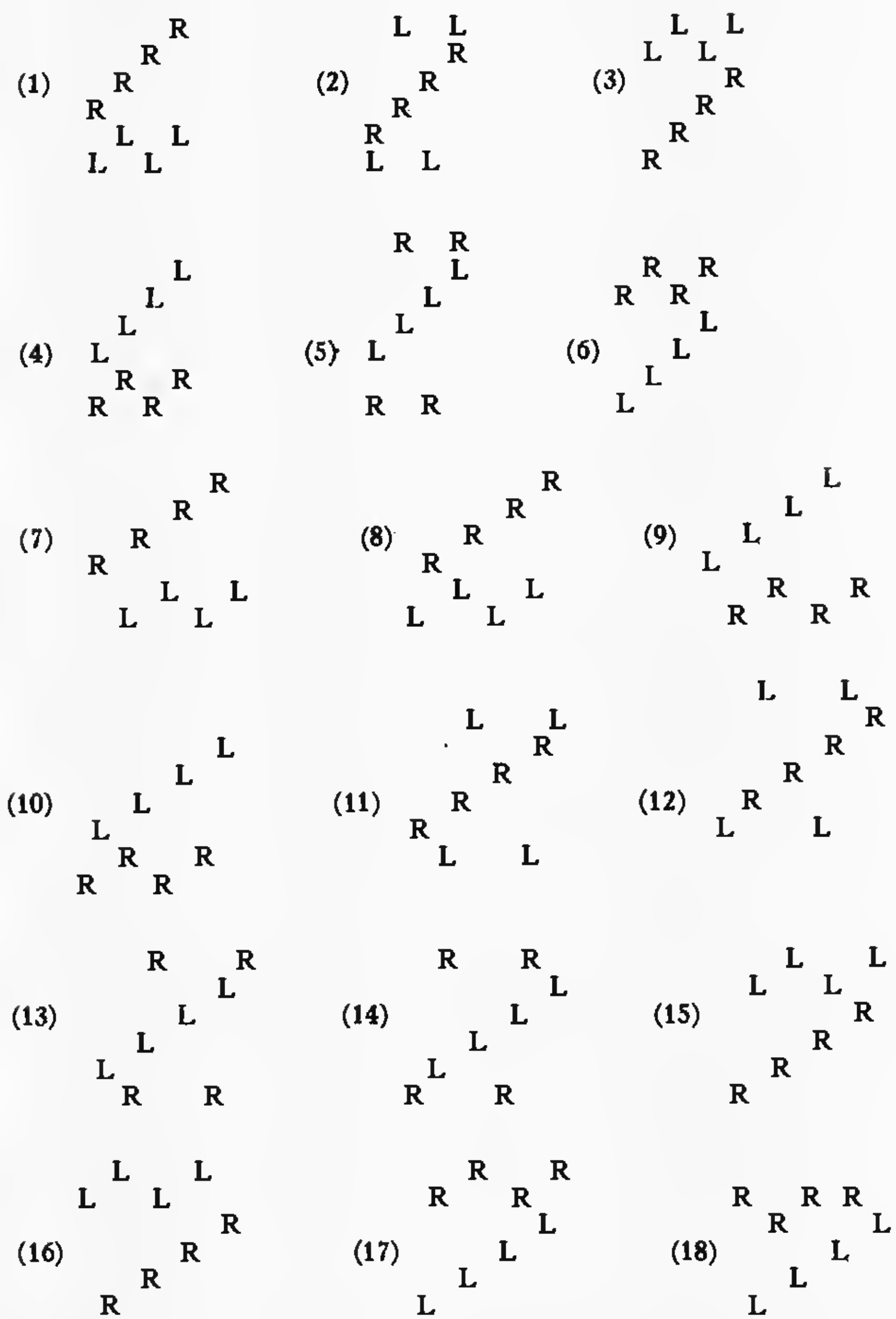
Form:





Σ = NS; 3 Staves

Form: 



In a similar way, simultaneous and sequent groups of R and L may be developed from the following forms:

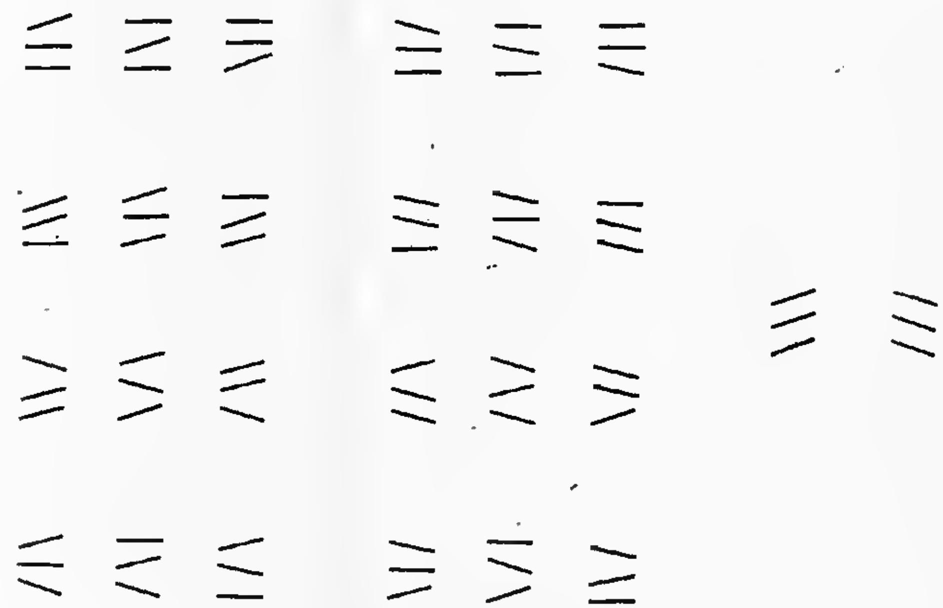


Figure 176. Positions of R and L.

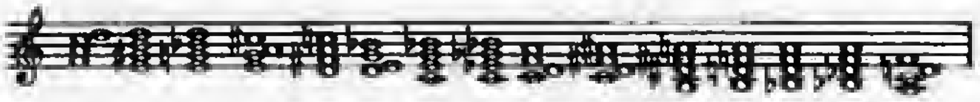
A still greater degree of complexity may be achieved by means of four-staff positions for R and L. It is not necessary to tabulate such forms; they are not likely to be used frequently and may be selected for each particular use, if and when desirable.

Many of the cases, which contain several instrumental strata, become sufficiently complex to be represented on more than the two customary piano staves. Depending on the position of hands which predominates in each particular case, different combinations of staves with regard to R and L may be considered practical. For instance, a harmonic accompaniment, emphasizing two or three instrumental strata played by the L, with melody above it played by the R requires three staves, the lower two (bass and treble clefs) being executed by L; the upper, by R. The case in which L plays the lower and the upper strata while R plays the middle stratum requires three staves also, the two extreme staves should refer to L; the middle staff, to R.

A number of composers have utilized the three-staff arrangement. We find, moreover, a four-staff arrangement in Rachmaninov's *Prelude* in C#-minor, and a five-staff arrangement in N. Cherepnin's First Piano Concerto. In the latter case, in my opinion, three staves would have been entirely adequate.

Examples of Positions of R and L

Theme:



Instrumental Variation: $R_L R_L R^L$ 8 staves

$H = T = 8t$; $T = 2t + t + 2t + t + 2t$.

etc.

Figure 177. Theme and instrumental variations (continued).

Instrumental Variation: $R\overset{L}{R}\overset{L}{R}\underset{L}{R}\underset{L}{R}$ 8 staves

$H_1 = T = 15t$; $H_2 + H_3 = 9t + 6t$; ---

etc.

Figure 177. Instrumental variation (concluded).

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK IX

GENERAL THEORY OF HARMONY

(STRATA HARMONY)

BOOK NINE

GENERAL THEORY OF HARMONY: STRATA HARMONY

APPLICATIONS OF GENERAL HARMONY

INTRODUCTION TO STRATA HARMONY

My general theory of harmony denotes the whole manifold of techniques, which enable the composer to write *directly* for groups of instruments or voices. Every score (chamber music, symphonic, choral or operatic) consists of parts for such individual instruments as piano, harp, or organ, and for those instruments which generally appear in groups, such as clarinets, violins, or trombones.

To evolve the required techniques for composing these scores, it is necessary to discover, first, the principles which control the behavior of individual parts and groups; and, second, the principles by which these individual parts and groups may be coordinated.

We also know that the field to which the theory of the behavior of groups or assemblages of pitch-units belongs is the field we call harmony. Therefore, the solution to this whole problem lies in the *generalization of harmonic principles*. This generalization must emphasize structures, their coordination in simultaneity and continuity, progressions, and directional units; it must generalize *structure* to such an extent that *sequent* structures will be *convertible* into *simultaneous* structures and vice versa. This means the introduction of scientific system in place of the old musical dualism of melody and harmony. Our theory must also enable us to coordinate any number of melodies, the derivation of which is harmonic. Thus we see that the manifold of harmonic techniques, although it is immense in its scope *per se*, becomes merely a subsidiary propedeutics to the *art of composing for groups*.

My general theory of harmony, I may say, satisfies all of the requirements just stated; it is the first scientific system crossing the threshold of the *sanctum sanctorum* of musical creation.

Contrary to what was the case in my special theory of harmony, this system has not been based on observation and analysis of existing musical facts only; it is entirely *inductive*. General harmony does not conflict with any of the principles of special harmony, but it gives them a broader interpretation instead. As a system, then, special harmony is but one case of general harmony.

The *General Theory of Harmony* discloses the real principles of harmonic creation.* It is particularly gratifying to me that, being an inductive system, this theory gives us direct interpretations of musical facts found in such remote regions of musical creation as the polyphony of Palestrina, the symphonic style of Mozart, the "bizarre" harmonies of Ravel, or the tone-clusters of some of our

*"The main purpose of the *General Theory of Harmony*: *Strata Harmony*," Schillinger writes in Chapter 7 of the present book, "is to satisfy demands for the scoring of all possible combinations of instruments, voices, or both . . ." In other words, the present book lays the groundwork and presents some of the basic principles of Schillinger's Theory of Orchestration. One aspect of orchestration not included in this book is the assigning of instrumental combinations to harmony "where the mobility of the instrumental form of a part defines the quantity of harmonic parts": i.e., where one instrument may perform an instrumental form of 2, or 3, or 4 part harmony. Nevertheless, the arrangements made by students who had completed this book were so rich and arresting that other students, who had not yet reached this book, assumed that such arrangements had been made on the basis of the *Theory of Orchestration*. (Ed.)

contemporaries. Besides covering all known styles of music, both in folklore and in the creations of individual composers, it shows that an inexhaustible number of *new* individual styles is available, and that the possibilities of the twelve-unit equal temperament scale can outlive the life-span of music itself.

The nomenclature I shall use is:

p, 2p, 3p, and 4p = simultaneous pitch-units (parts).
S = simultaneous structure, stratum.
Σ (sigma) = compound structure of strata.
Σ (Σ) (the sigma of a sigma) = complex compound structure (compound structure of sigmae).
p_I, p_{II}, p_{III}, etc. = parts of simultaneity = a, b, c, . . . etc.
S_I, S_{II}, S_{III}, etc. = structures of simultaneity.
Σ_I, Σ_{II}, Σ_{III}, etc. = structures of simultaneity.
S_1, S_2, S_3, etc. = structures of continuity.
Σ_1, Σ_2, Σ_3, etc. = structures of continuity.
i = pitch-interval unit (semitone).
I = pitch-interval group (of semitones).
$p_{\rightarrow}$ = ascending directional unit ($a_{\rightarrow}$, $b_{\rightarrow}$, $c_{\rightarrow}$, $d_{\rightarrow}$).
$p^{\rightarrow}$ = descending directional unit ($a^{\rightarrow}$, $b^{\rightarrow}$, $c^{\rightarrow}$, $d^{\rightarrow}$).
$p\rightarrow$ = sequent part (sequent pitch unit).
$S^{\rightarrow}$ = sequent structure (pitch-scale, directional pitch-scale).
$\Sigma^{\rightarrow}$ = sequent compound structure (pitch-scale derived from all strata of the Σ).
H_1, H_2, H_3, etc. = chords in successive enumeration.
$H^{\rightarrow}$ = progression of chords.

CHAPTER 1

ONE-PART HARMONY

A. One Stratum of One-Part Harmony (S = p)

THE Sp represents a constant or a variable function of a potential assemblage, Σ. It may also have an independent existence, in which case it represents a constant *function a*, since it is a root-tone. In both cases, it becomes a melody harmonically defined.

Progressions of Sp may be evolved through any desirable scale selected from any of the four groups. Either tonal cycles or simply permutations of the pitch-units may control such progressions. It follows from the foregoing statement that progressions of Sp may be either diatonic or symmetric. (It is correct to think of all the diatonic scales as *special cases* of symmetry—where symmetric roots are 2, 4, 8, . . . n, i.e., where they are arranged in an octave or in a multiple-octave recurrence).

A one-unit scale of the first (and *ipso facto* of the second) group constitutes the progression known as pedal point.* A one-unit scale of the third or the fourth group constitutes a progression consisting of a group of successive pedal points, each pedal point representing a root of symmetry.**

All other forms of Sp progressions, in most known instances, represent a *basso continuo* (so-called general bass, figured bass, or thorough bass). Under the conditions of general harmony, Sp may appear in any vertical relationship to any other S of the Σ, which means that when Sp assumes the role of a bass, it is simply one special case among the possible cases. In special harmony, Sp progressions appeared as a constant root-tone in harmony composed of S(5) in the classical system, and as a variable chordal function of S(5) in harmony of S(6) and $S(^{6}_{4})$, as the third or the fifth of the chord.

As one chordal function cannot reciprocate with itself, it has no transformations. Its variability depends on a potential Σ, as in the cases described above. Yet, as we learned from the theory of melodization, a constant function, a, may become a constant function b, or c, or d . . . etc., which is dependent on the potential Σ.

The meaning of these constant chordal functions in the light of special harmony is confined to function *a*'s being the root, function *b*'s being the third, function *c*'s being the fifth, etc. But one must now bear in mind that the root, the third, the fifth, etc., are nothing but the degrees of certain seven-unit scales in their E_1. Therefore, the constancy of a chordal function may refer to any degree of any scale in any of the four groups.

We shall make extensive use of Sp progressions in this study as a desirable —and often necessary—supplement to other strata of the Σ. No illustrations of independent progressions of Sp are necessary.

*That is to say, when the scale is a one-note scale, the progressions available are: *one*. A single-note progression, so far as one-part harmony goes, is a pedal point. (Ed.)

**The reference here is to the four groups of scales described in Vol. I, pp. 103, 133, 148 and 155. (Ed.)

CHAPTER 2

TWO-PART HARMONY

A. One Stratum of Two-Part Harmony ($S = 2p$)

ASSEMBLAGES which serve as two-part harmonic structures are the two pitches of two-unit scales brought into simultaneity. As the number of two-unit pitch-scales is eleven,* there are that many two-part harmonic structures.

Illustrations of $S = 2p$

All structures of S2p

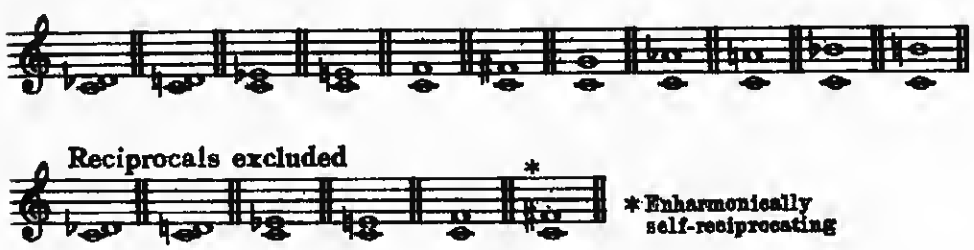


Figure 1. All structures of S2p.

Each scale, as we know, may be expressed through the quantities of interval-units; if we enumerate the possible structures as S_1, S_2, S_3, . . . we obtain their equivalents in the forms of I.

$$I(S_1) = i;\ I(S_2) = 2i;\ I(S_3) = 3i;\ \ldots\ I(S_{11} = 11i)$$

Progressions of S2p for any one of the eleven forms of S may be evolved through any desirable scale from all four groups. Either tonal cycles or permutation of pitch-units may control such progressions, which may be executed in any form of symmetry, including generalized symmetry as well. It is expedient, for this reason, to develop progressions of S2p under different diatonic and symmetric conditions. However, only the seven-unit scales with non-identical pitches permit the use of all types of structure in diatonic progressions —and even then "all structures" means ***all structures within the diatonic scale.***

In this system we shall regard all the possible diatonic structures as consisting of adjacent pitch-units in a given scale under a certain form of tonal expansion. Therefore, the structures of E_0 of a natural major are all seconds: $\frac{d}{c}, \frac{e}{d}, \frac{f}{e}, \ldots$, etc. Likewise, all structures of E_1, in the same scale are all *thirds;* $\frac{e}{c}, \frac{f}{d}, \frac{g}{e}, \ldots$, etc.

*That is, 11 within the limits of a 12-semitone octave. (Ed.)

Structures, diatonic with respect to natural major.

S(E_3), S(E_4), S(E_5) constitute reciprocity

Structures, diatonic with respect to Chinese Pentatonic.

S(E_3) reciprocates with S(E_0)

Figure 2. Diatonic structures.

As the number of diatonic structures (i.e., structures corresponding to combinations of musical names and not to the exact quantities of i) corresponds to the number of tonal expansions (including E_0), the number of such structures in any of the above defined seven-unit scales is six:

$$S(E_0) \equiv \text{second};\ S(E_1) \equiv \text{third};\ S(E_2) \equiv \text{fourth};$$
$$S(E_3) \equiv \text{fifth};\ \ S(E_4) \equiv \text{sixth};\ S(E_5) \equiv \text{seventh}.$$

This number has to be reduced practically to three, for the six forms include three mutually reciprocating pairs in octave-inversion.

Whether S(E_1) be assumed to be $\frac{c}{a}$—causing S(E_4) to be merely an inversion of S(E_1), or whether it is the opposite, makes a purely theoretical difference. Once the transformations take place, the forms begin to reciprocate, and the question as to whether the sixth is an inversion of the third, or the third is an inversion of the sixth, is a metaphysical rather than real one.

It is easy to see from the above discussion that scales with fewer than seven units (providing they are diatonic and not symmetric) do not provide diatonically constant structures under any desirable tonal expansion.

For this reason, whenever the composer wishes to use a constant diatonic structure in a diatonic progression, he should evolve his harmonic progressions from the seven-unit scales with non-identical pitches.

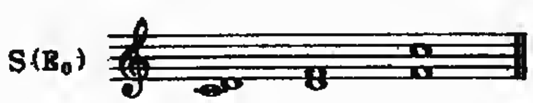
In all other types of harmonic progression we shall use any of the eleven forms of S2p, whatever the stylistic authenticity may be with regard to the progression itself.

Transformations

Transformations* of two-part assemblages of ***diatonically identical*** structures are reduced to one possible form: a⥀b, i.e., a transforms into b, while b transforms into a. This concerns both the positions and voice-leading. A two-part assemblage of any form may be called a *diad.*

Transformations of two-part assemblages of ***diatonically non-identical*** structures have an additional const. ab transformation: $a \rightarrow a^1$ and $b \rightarrow b^1$—i.e., the a-function of the first structure transforms into the a-prime function of the following structure, and the b-function of the first structure transforms into the b-prime function of the following structure. Once the transformation is performed, H_1 is assumed to be the original structure (i.e., $\frac{b}{a}$ and not $\frac{b^1}{a^1}$)—so that H_2, the subsequent structure, in turn may be $\frac{b}{a}$ or $\frac{b^1}{a^1}$, depending on the diatonic identity with the preceding structure.

Structures, diatonic with respect to I = 2i + 3i.



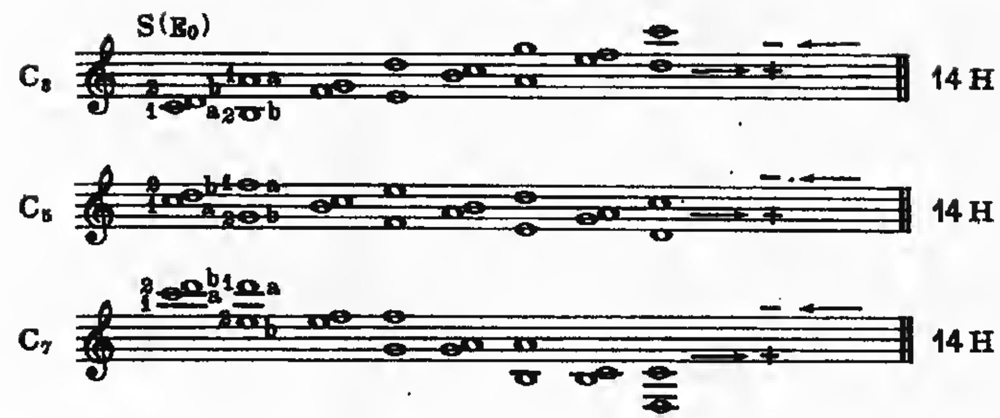
S(E_1) reciprocates with S(E_0)

Figure 3. S(E_0), S(E_1), and S(E_2) (continued).

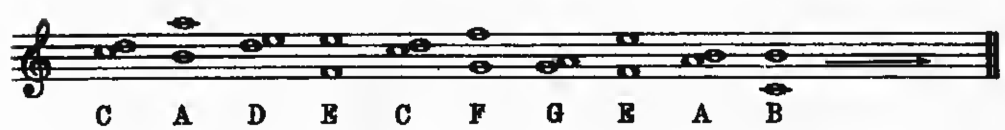
*If the reader happens to have forgotten it, Schillinger uses the term *transformations* to mean *voice-leading*, so far as general (and special) harmony is concerned. A 2-part structure, $\frac{a}{b}$, transforms–i.e., its voices lead–into a structure, $\frac{b}{a}$. For example, if a = 1, b = 3, and two successive roots are C & F, then $\frac{a}{b}$ $(\frac{1}{3})$ on C is $\frac{C}{E}$, which transforms to $\frac{b}{a}$ $(\frac{3}{1})$ on F, or $\frac{A}{F}$. In other words the upper voice leads from C to A, while the lower voice leads from F to C, the cycle (C, F) being C_5. Transformations are the general form of *all* voice-leading, of no matter what kind—and the student who grasps this single principle will never have any trouble with even the most complex problems of this sort. (Ed.)

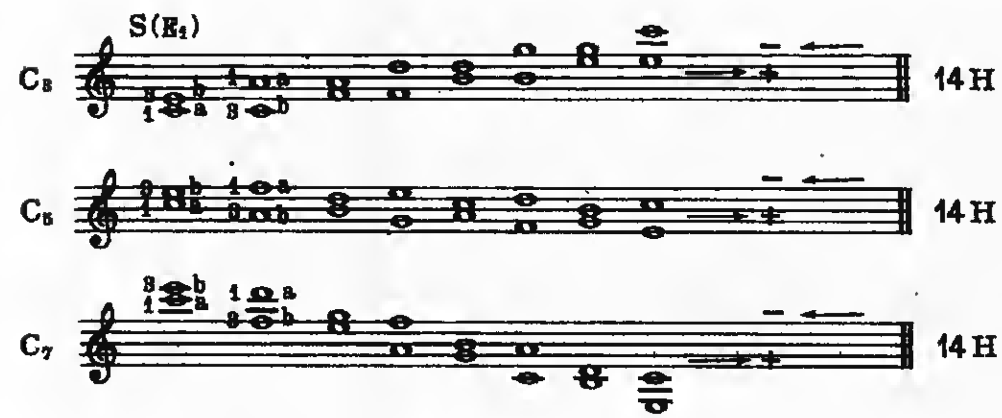
All the following examples are reversible.

Examples of Diatonic Progressions in Natural Major



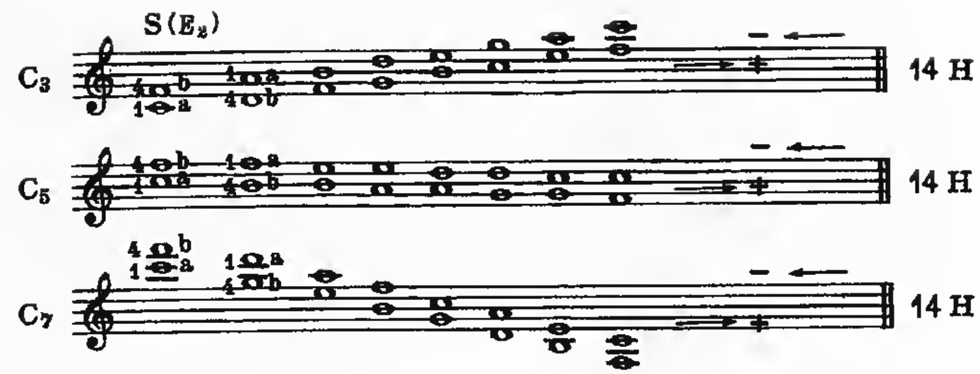
$C^{\rightarrow} = C_3 + C_5 + C_7$





$C^{\rightarrow} = C_3 + C_5 + C_7$

Figure 3. S(E_0), S(E_1), and S(E_2) (continued).



$C^{\rightarrow} = C_3 + C_5 + C_7$

Figure 3. $S(E_0)$, $S(E_1)$ *and* $S(E_2)$ *(concluded).*

Diatonic-Symmetric Progressions.

Structure: I = 2i

Structure: I = 3i

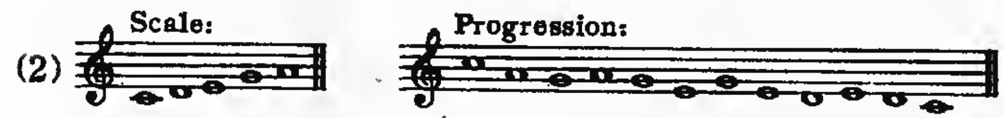
Structure: I = 4i

Figure 4. Diatonic-symmetric progression. Structures I = 2i to I = 6i (continued).

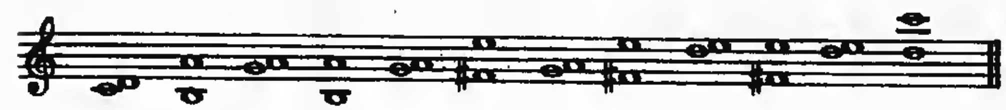
Structure: I = 5i

Structure: I = 6i

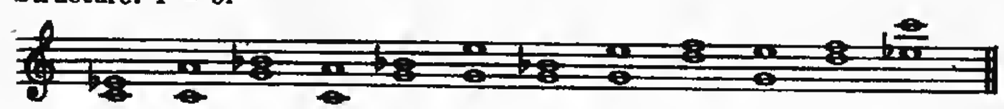
Figure 4. Diatonic-symmetric progression. Structures I = 2i to I = 6i (concluded).



Structure: I = 2i

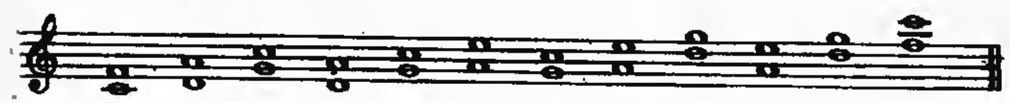


Structure: I = 3i



Structure: I = 4i

Structure: I = 5i



Structure: I = 6i

Figure 5. Diatonic-symmetric progressions. Structures I = 2i to I = 6i.

Figure 6. Diatonic-symmetric progression. Structure I = 2i to I = 6i.

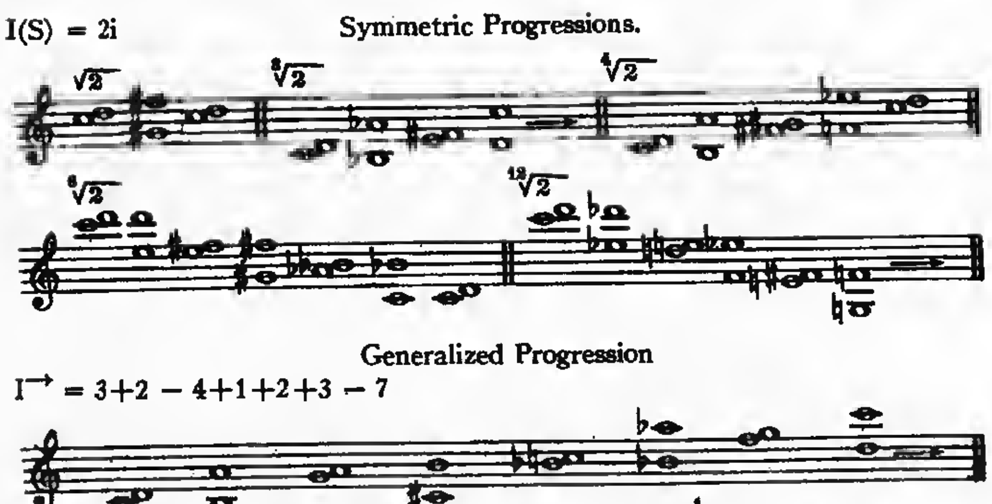


Figure 7. Symmetric and generalized progression. I(S) = 2i.

Figure 8. Generalized progression. I→(S) = 3i

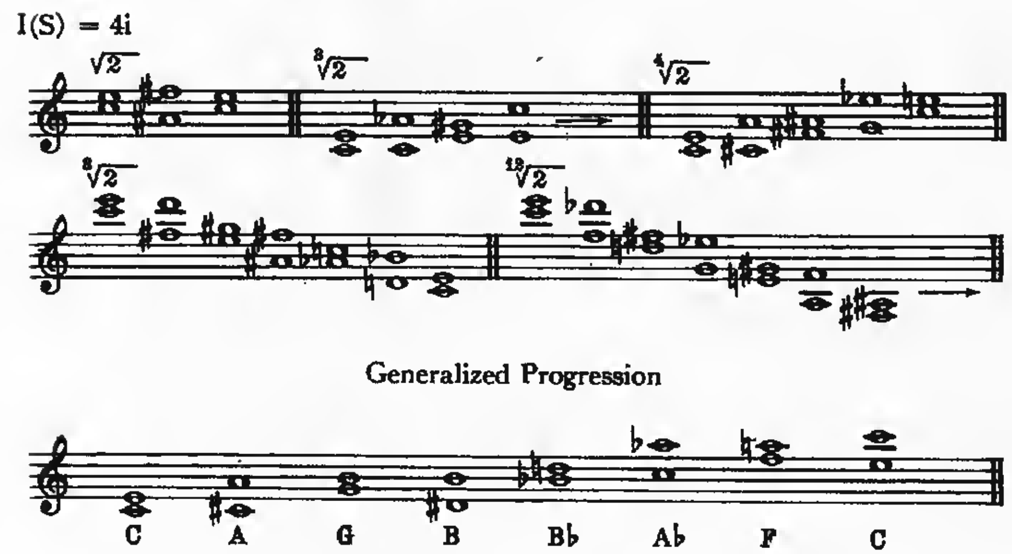


Figure 9. I(S) = 4i.

Figure 10. I(S) = 5i (continued).

Generalized Progression

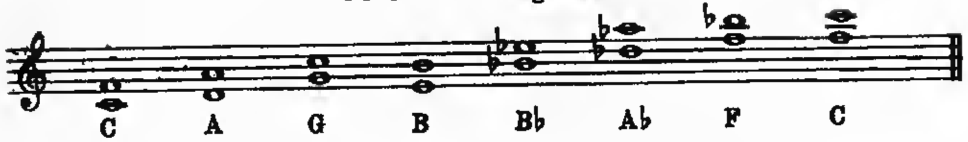


Figure 10. $I(S) = 5i$ *(concluded).*

$I(S) = 6i$

Figure 11. $I(S) = 6i$.

B. Sequence of Variable Structures in One Two-Part Stratum

Variable structures may appear in any type of harmonic progression.

Diatonic variable structures may be referred to the different forms of tonal expansion: $S(E_0)$, $S(E_1)$, $S(E_2)$, . . ., etc., which may be selected in any desirable quantities and forms of distribution. However, in view of our auditory habits, it is advisable to use low coefficients of recurrence.

To simplify the notation, we shall represent the correspondence between structures and forms of expansion as follows:

$$\Sigma_1 = S(E_0);\ \Sigma_2 = S(E_1);\ \Sigma_3 = S(E_2);\ \ldots$$

In composing the continuity of structures, we may select a coefficient-group from any source discussed in the *Theory of Rhythm*.*

Examples of composition of the structure-groups:

(1) $\overrightarrow{\Sigma} = \Sigma_2 + \Sigma_1$; (2) $\overrightarrow{\Sigma} = \Sigma_2 + \Sigma_1 + \Sigma_3$; (3) $\overrightarrow{\Sigma} = 2\Sigma_1 + \Sigma_3$;
(4) $\overrightarrow{\Sigma} = 3\Sigma_1 + \Sigma_2 + 2\Sigma_3$; (5) $\overrightarrow{\Sigma} = 2\Sigma_3 + \Sigma_2 + \Sigma_2 + 2\Sigma_3$;
(6) $\overrightarrow{\Sigma} = 3\Sigma_1 + \Sigma_2 + 2\Sigma_3 + 2\Sigma_1 + \Sigma_2 + 3\Sigma_3$;
(7) $\overrightarrow{\Sigma} = 4\Sigma_1 + 2\Sigma_3 + 2\Sigma_1 + \Sigma_3$;

*See Vol. I, p. 12 ff.

(8) $\overrightarrow{\Sigma} = 9\Sigma_3 + 3\Sigma_1 + 3\Sigma_2 + \Sigma_1$;
(9) $\overrightarrow{\Sigma} = 4\Sigma_1 + 2\Sigma_2 + 2\Sigma_3 + 2\Sigma_1 + \Sigma_3 + \Sigma_2 + 2\Sigma_1 + \Sigma_3 + \Sigma_2$;
(10) $\overrightarrow{\Sigma} = \Sigma_3 + 2\Sigma_1 + 3\Sigma_3 + 5\Sigma_1$;
(11) $\overrightarrow{\Sigma} = \Sigma_3 + 2\Sigma_1 + 4\Sigma_3 + 8\Sigma_1$.

Structures of seven-unit scales with non-identical pitches produce *diatonically*-identical structures, i.e., Σ_1 are seconds, Σ_2 are thirds, Σ_3 are fourths. In other scales, structures of one expansion are diatonically non-identical. Yet it is better, and a more general method, to select diatonic structures with respect to their expansions.

The choice of general structures (out of the manifold of eleven) may be made freely, and any combination of structures in any form of distribution is acceptable. Such a use of eleven structures in any combinations and arrangement is applicable to progressions of type II, III, and the generalized forms of consecutive symmetry.

Examples of Progressions with Variable Structures

Diatonic

$\overrightarrow{\Sigma} = 3\Sigma_3 + \Sigma_1 + 2\Sigma_2 + 2\Sigma_3 + \Sigma_1 + 3\Sigma_2 + \Sigma_1$.
$\overrightarrow{C} = 2C_3 + C_5 + C_3 + 2C_7$; Scale = Nat. Major.

Diatonic Symmetric (The same scheme).

Generalized Symmetric ($\overrightarrow{\Sigma}$ identical with the above)

Figure 12. Progression with variable structures.

C. Two Hybrid Strata

$$\Sigma = 2S; S_I = p; S_{II} = 2p$$

The addition of an Sp to any form of S2p progressions produces a *hybrid* three-part harmony.

Actual selection of a function for Sp is a matter of the style of the harmonic structures. Depending on the structure of S2p, the addition of a function of Sp may produce either greater tension or less tension.

It is easy to compute the actual quantity of all possible forms of the three-part hybrid structures. The total quantity of Sp structures is eleven. The latter are built from the twelve symmetric pitch units of equal temperament and represent all combinations by two from the original unit. Assuming that each of the 11 S2p structures may be accompanied by any of the 12 functions of the full tuning scale, we acquire the total of $11 \cdot 12 = 132$ structures.

Out of this total number, all the diatonic structures (with respect to seven musical names) may be classified as well. There are six diatonic structures, corresponding to six expansions of the complete diatonic scale, and seven diatonic units which can be added to any one of them. The total of the diatonic hybrid three-part structures amounts to: $6 \cdot 7 = 42$.

D. Table of Hybrid Three-Part Structures

(a) General and (b) Diatonic

(a)

Figure 13. General hybrid three-part structures (continued).

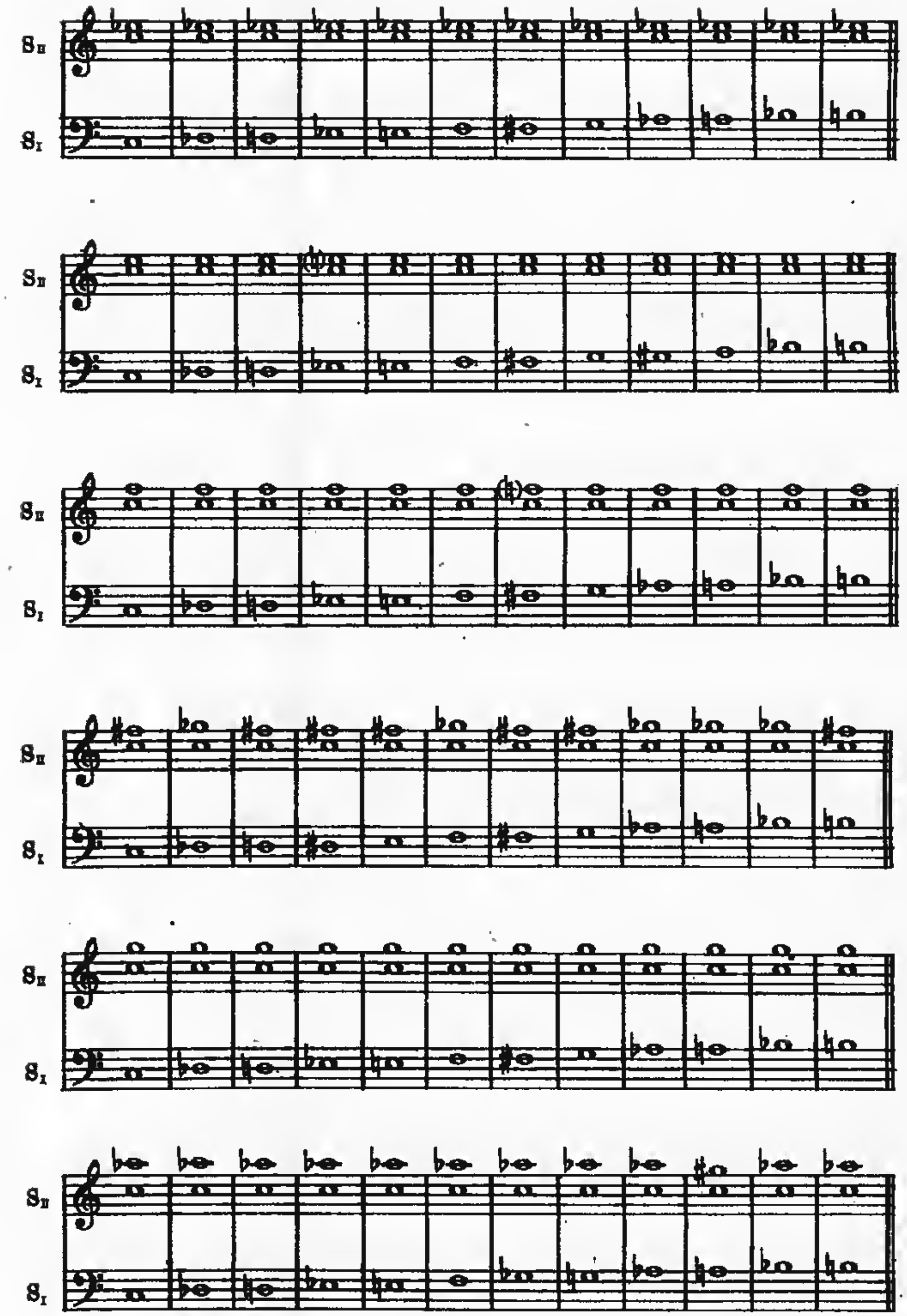


Figure 13. General hybrid three-part structures (continued).

Figures 13. General hybrid three-part structures (concluded).

(b)

Figure 14. Diatonic hybrid three-part structures (continued).

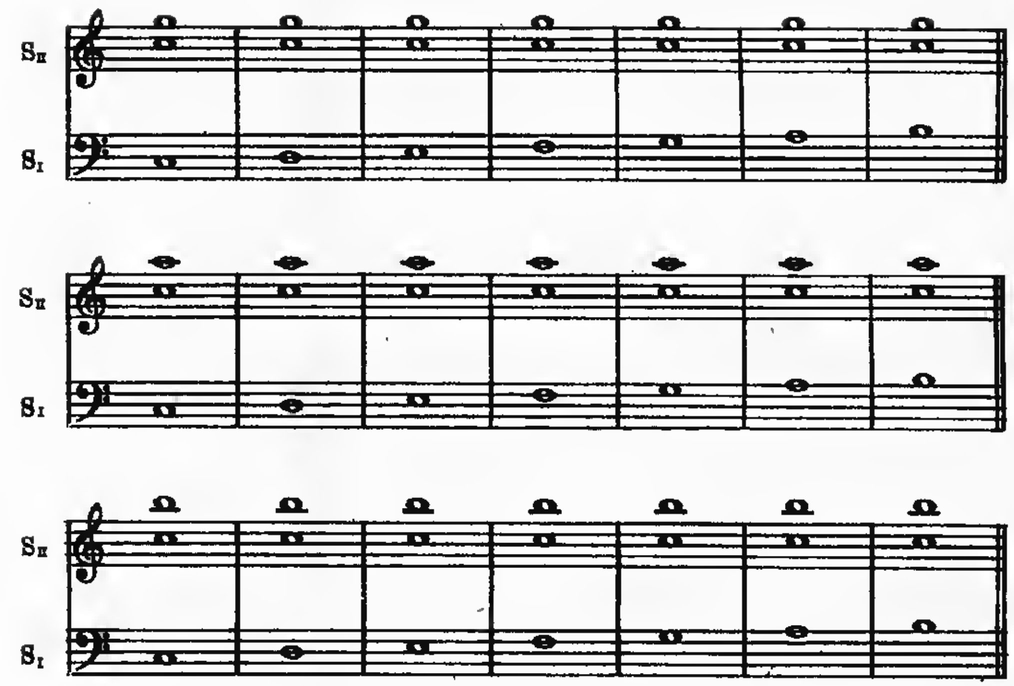


Figure 14. Diatonic hybrid three-part structures (concluded).

All diatonic hybrid three-part structures may acquire *any one system* of accidentals at a time.

Sp may be placed either below or above S2p.

As the sequence of S2p structures may be varied in a progression, the addition of an Sp is a matter of the individual selection of a function for *each* structure of S2p.

In the following notation, we shall use this scheme:

C-chord

(1) Diatonic nomenclature:

c	d	e	f	g	a	b
1	2	3	4	5	6	7

(2) Symmetric nomenclature:

c	c♯	d	d♯	e	f	f♯	g	g♯	a	b♭	b♮
1	2	3	4	5	6	7	8	9	10	11	12

Let us see how such numerical notation can be applied to either system of structures.

We shall take, for example, $S_{II} = 5i$. This represents: (reading from c) $\frac{f}{c}$, or in numerical notation: $\frac{4}{1}$. If we decided to add d as S_I, the latter becomes const. 2. Therefore the entire Σ may be read as follows:

$$\Sigma = \frac{S_{II} = 1,\ 4}{S_I\ = 2 \text{ const.}}$$

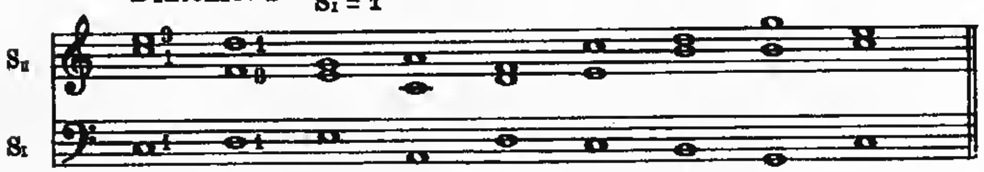
which is the *diatonic* form of numerical notation, where $a(S_{II}) = 1$ and $b(S_{II}) = 4$, and where $a\ (S_I) = 2$. The same case, when represented in the *symmetric* form of numerical notation, assumes the following form:

$$\Sigma = \frac{S_{II} = 1,\ 6}{S_I\ = 3 \text{ const.}}, \text{ where } a(S_{II}) = 1 \text{ and } b(S_{II}) = 6, \text{ and where } a(S_I) = 3.$$

In case of coincidence in pitch of the function of S_I with either of the functions of S_{II}, only the fundamental form of transformation ($a \circlearrowright b$) may be used—otherwise consecutive octaves are unavoidable.

E. Examples of Hybrid Three-Part Harmony

Diatonic: $\Sigma = \frac{S_{II} = 1,8}{S_I = 1}$



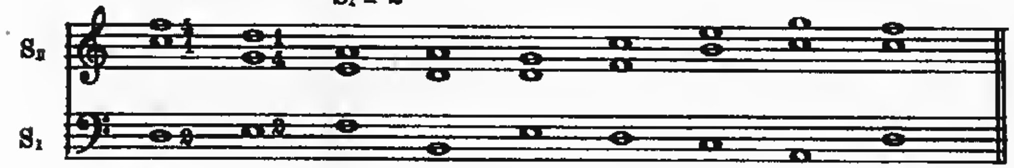
Diatonic: $\Sigma = \frac{S_{II} = 1,8}{S_I = 6}$

Diatonic: $\Sigma = \frac{S_{II} = 1,8}{S_I = 2}$

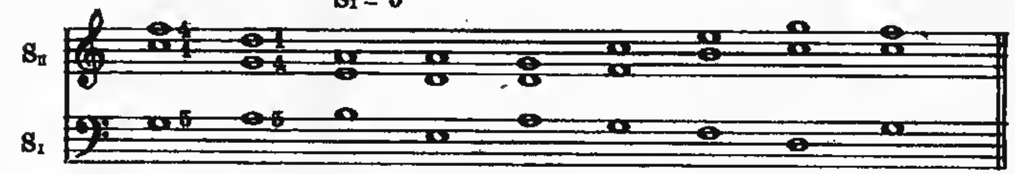
Figure 15. Hybrid three-part harmony (continued).

Diatonic: $\Sigma = \frac{S_{II} = 1,4}{S_I = 1}$

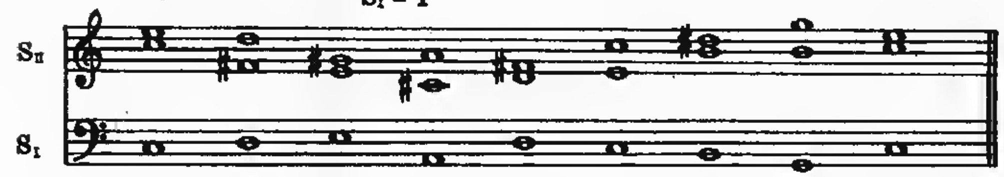
Diatonic: $\Sigma = \frac{S_{II} = 1,4}{S_I = 2}$



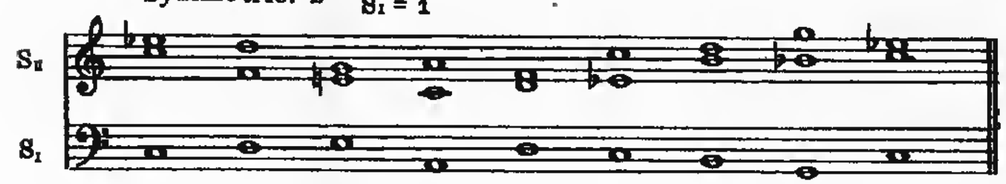
Diatonic: $\Sigma = \frac{S_{II} = 1,4}{S_I = 5}$



Symmetric: $\Sigma = \frac{S_{II} = 1,5}{S_I = 1}$



Symmetric: $\Sigma = \frac{S_{II} = 1,4}{S_I = 1}$

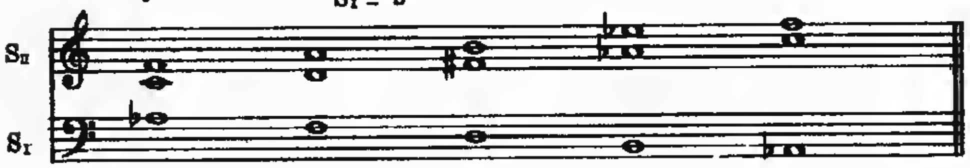


Symmetric: $\Sigma = \frac{S_{II} = 1,6}{S_I = 11}$

Figure 15. Hybrid three-part harmony (continued).

Symmetric: $\Sigma = \frac{S_{II} = 1,7}{S_I = 8}$

Symmetric: $\Sigma = \frac{S_{II} = 1,6}{S_I = 9}$; $C^{\rightarrow} = \sqrt[4]{2}$



Symmetric: $\Sigma = \frac{S_{II} = 1,4}{S_I = 9}$; $C^{\rightarrow} = \sqrt[6]{2}$

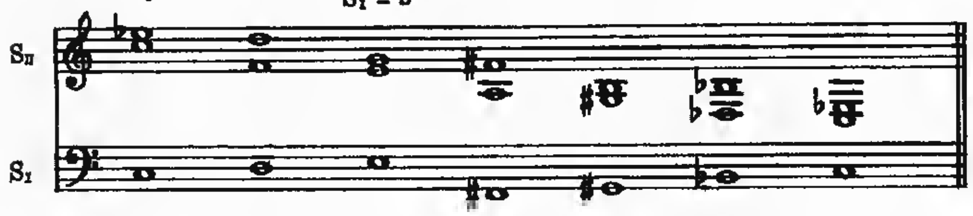


Figure 15. Hybrid three-part harmony (concluded).

Progressions of Mixed Structures.

$$\Sigma^{\rightarrow} = 2\Sigma_1 + \Sigma_2 + \Sigma_1 + 2\Sigma_3$$

Diatonic Progression: $C^{\rightarrow} = 2C_3 + C_7 + C_5 + C_{-7}$
Scale of Roots: Nat. Major. d_4.

Figure 16. Mixed structures (continued).

Symmetric Progression on the same Scale of Roots.

Figure 16. Mixed structures (concluded).

F. Two Strata of Two-Part Harmonies

$$\Sigma = 2S; \ S = 2p$$

Two two-part harmonic structures may be coordinated into a simultaneous Σ and subjected to independent transformations in each stratum. The latter result in four-part progressions in which the two component strata act independently. This technique solves many problems in composing for two heterogeneous pairs of instruments. For example, two clarinets may play a stratum not only against another stratum of two French horns, but even against two violin parts. The quality of orchestration can be well affected by different forms of distribution of the same four-part harmony developed into Σ = 2S.

The number of *general* structures of Σ = 2S2p equals the quantity of S2p, times the number of combinations of S2p in the two strata, times the number of possible positions between S_I and S_{II}: $11^3 = 1,331$.

The number of *diatonic* structures (which represents a portion of the total quantity) equals: $6^3 = 216$.

It means that there are 1,331 general and 216 diatonic chord structures from any pitch-unit designated as a starting point (root-tone).

As tabulation of all forms (since the quantity increases so rapidly) becomes impractical, we shall give some samples only of such tables.

Examples of Structures: Σ = 2S2p

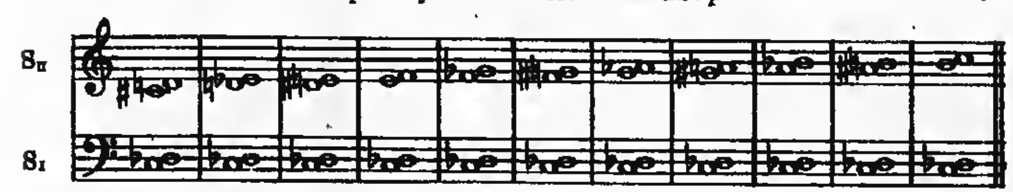


Figure 17. General structures of Σ = 2S2p (continued).

Figure 17. General structures of Σ = 2S2p (concluded).

Figure 18. Diatonic structures of Σ = 2S2p (continued).

Figure 18. Diatonic structures of Σ = 2S2p (concluded).

In order to eliminate consecutive octaves between any pair of parts in strata, assign identical pitches to non-identical functions. If, for example, pitch d appears in both strata, one of them should become function ⓐ and the other should become function ⓑ.

G. Examples of Progressions in Two Strata:

(a) through the different forms of distribution of a given four-part harmony

(a) Theme

Var. I

Figure 19. Theme and variations (continued).

Var. II

Var. III

Figure 19. Theme and variations (concluded).

(b) through independent structures of $\Sigma = 2S2p$

(b) $\Sigma = \frac{S_{II} = 5i}{S_I = 5i}$ I = 10i; Type II; Scale = Nat. major

$\Sigma = \frac{S_{II} = 10i}{S_I = 8i}$ I = 9i; Type II; Scale = Nat. major

$\Sigma = \frac{S_{II} = 5i}{S_I = 10i}$ I = 4i; Type III: $\sqrt[6]{2^5}$

Figure 20. Independent structures of $\Sigma = 2S2p$ (continued).

$\Sigma = \frac{S_{II} = 8i}{S_I = 5i}$ I = 8i; Generalized progression: 8 + 2 − 7

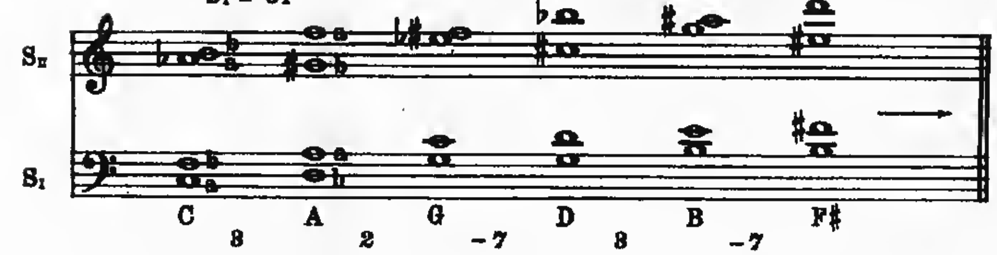


Figure 20. Independent structures of $\Sigma = 2S2p$ (concluded).

In the above examples, the structures are defined by i and I $\left(\frac{S_{II}}{S_I}\right)$.

For the time being, we shall use one const. Σ for the entire progression, unless such a progression is the given progression, which can be traced to the sources of special harmony, and which is subject to strata transcription (variation).

H. Three Hybrid Strata

$$\Sigma = 3S;\ S_I = p;\ S_{II} = 2p;\ S_{III} = 2p$$

An Sp may be added to $\Sigma = 2S2p$. This additional stratum may be either below both S2p, or it may be surrounded by the latter, or it may be above it. This permits three arrangements:

$$(1)\ \Sigma = \frac{\frac{S2p}{S2p}}{Sp};\quad (2)\ \Sigma = \frac{\frac{S2p}{Sp}}{S2p};\quad (3)\ \Sigma = \frac{\frac{Sp}{S2p}}{S2p}$$

An interchange of the positions in a written continuity is acceptable only when the total sonority of the Σ does not suffer from such an interchange. Often a high chordal function, originally placed in the upper stratum, sounds unsatisfactory when moved to the bass; such a rearrangement of parts often changes the very meaning of the Σ itself.

In many instances Sp may acquire a constant coupling or two. Such couplings are particularly practical for the extreme positions of Sp, i.e., either below all or above all other strata. Couplings may be constructed either upward or downward from a given function, provided that such coupling does not cross the functions of adjacent stratum. An octave coupling may be considered universal, i.e., applicable to any function. Couplings by perfect fifths for the lower stratum, and couplings by fifths, fourths, and practically all other intervals for the upper stratum are acceptable. The particular choice of couplings should follow to some extent the natural distribution of pitches (upward contraction of intervals). The coupling of a root-tone with the fifth is the commonest after the coupling of a root-tone by its octave.

Some structures, seemingly meaningless by themselves, become powerful tools of harmonic expression when supplemented by an Sp and a coupling.

Examples of Addition to Sp and Coupled Sp to Σ 2S2p

(Originals are taken from Figure 19. Type 1 progressions may be obtained by cancelling the accidentals, or by superimposing another constant group of accidentals).

Figure 21. Σ2S2p + Sp + coupled Sp (continued).

Figure 21. Σ2S2p + Sp + coupled Sp (concluded).

I. Three, Four and More Strata of Two-Part Harmonies: Hybrid Strata

Now that the principle of composing strata and of forming couplings for them has been established, we may proceed with the evolution of more complex forms of Σ.

Since the number of structures grows beyond the practical possibility of exhausting them, we shall refrain from tabulating them any further. We shall confine each case of Σ to a few samples of structures, and we shall choose the latter according to the principle established before, i.e., the structures and the intervals separating the strata will be conceived as forms of tonal expansions, or both will be evolved on the basis of interval symmetry.

In some instances a certain degree of variety may be achieved by alternating the original positions in the adjacent strata. For instance:

$$S_I = \frac{b}{a};\ S_{II} = \frac{a}{b};\ S_{III} = \frac{b}{a};\ S_{IV} = \frac{a}{b};\ \ldots$$

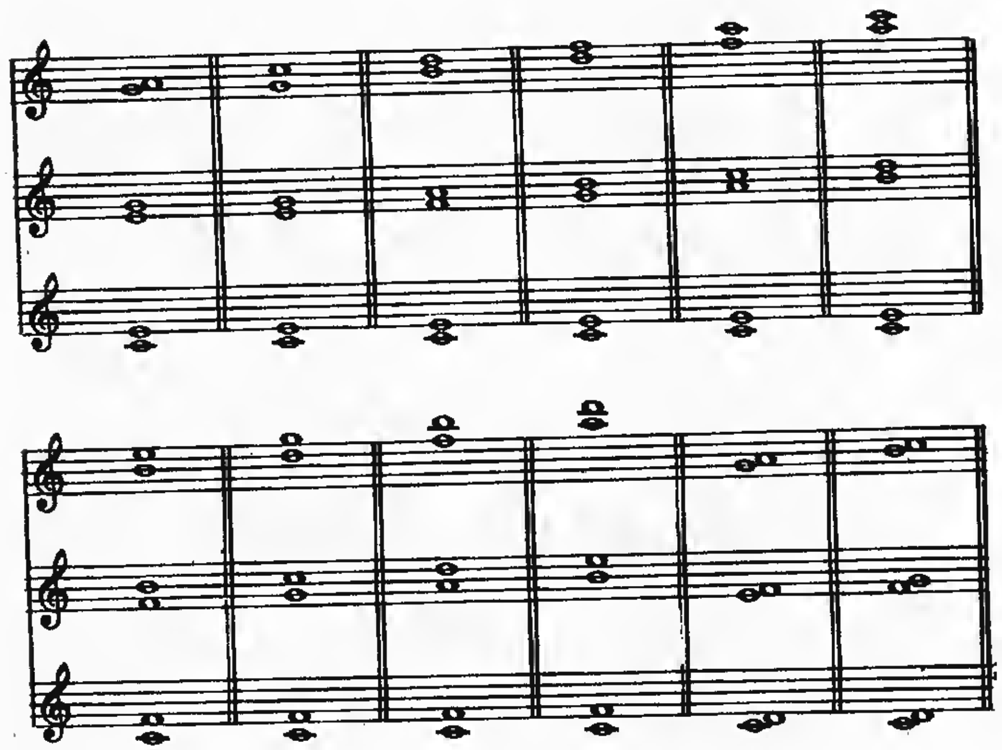
The first example of progressions (Fig. 22) compares favorably with six-part counterpoint of the type: $\frac{CP}{CF} = a$.

Examples of Harmonic Structures and Progressions in Three and more Strata

$\Sigma = 3S2p$

Structures

Diatonic Forms Depend on the Number of Pitch-Units in the Scale.



Symmetric Forms have Three or more Roots;

All Symmetric Two-Unit Scales belong to this group.

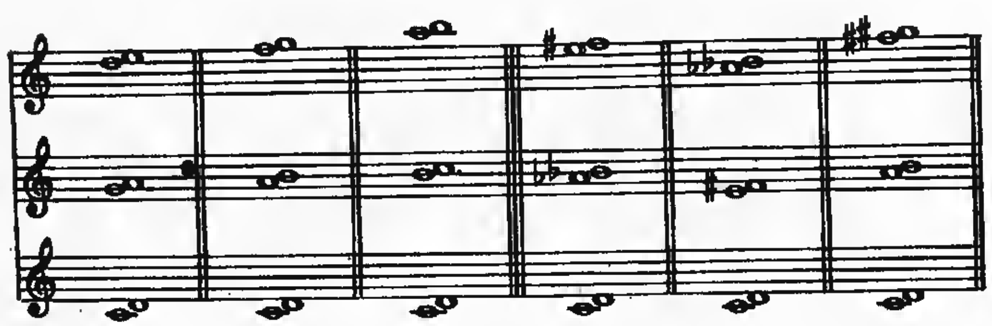


Figure 22. Structures and progressions of $\Sigma = 3S2p$ (continued).

Progressions

Figure 22. Structures and progressions of $\Sigma = 3S2p$ (concluded).

$\Sigma = 3S2pSp$

Figure 23. $\Sigma = 3S2p\,Sp$.

$\Sigma = 4S2p$

Diatonic Structures

Symmetric structures: 4 or more roots

Figure 24. $\Sigma = 4S2p$ (*continued*).

Progressions

Figure 24. $\Sigma = 4S2p$ *(concluded).*

$\Sigma = 4S2pSp$

$I(\Sigma) = \sqrt[4]{2^5}$

Figure 25. $\Sigma = 4S2pSp$.

J. Diatonic and Symmetric Limits and the Compound Σ of Two-Part Strata

The diatonic limit of a sigma composed from two-part strata may be expressed as $\Sigma = NS2p$, where N represents the number of pitch-units of a given scale.

A three-unit scale produces a maximum of three strata, or six parts (even seven parts if one includes a possible added root-tone). A five-unit scale produces $\Sigma = 5S2p$, or 10 parts in 5 strata (11 with the added root-tone). A complete seven-unit scale produces $\Sigma = 7S2p$, or 14 parts in 7 strata (15 with the added root-tone). Such limit-sigmae may be arranged according to one or another tonal expansion with regard to structures and the intervals between the strata. Selection of one or another tonal expansion controls the range of the Σ.

In practical application such limit-sigmae often require the overlapping of adjacent strata. In orchestration the strata which overlap are assigned to different orchestral groups, a method of tone-quality selection which prevents the score from losing its clarity in actual sound.

Only non-overlapping strata may belong to one orchestral group. For example, assuming that all adjacent strata are overlapping, but that no stratum overlaps the stratum next-but-one, we acquire the following possibilities for orchestration:

Σ	S_{IV}	Strings	Wind	Woodwind	Brass	Strings	Brass
	S_{III}	Wind	Strings	Brass	Woodwind	Brass	Strings
	S_{II}	Strings	Wind	Woodwind	Brass	Strings	Brass
	S_{I}	Wind	Strings	Brass.	Woodwind	Brass	Strings

More complex forms of sigmae with overlapping adjacent strata are developed in the form of tutti, i.e., with participation of all orchestral groups and often with the addition of soloists and choirs. This device is also practical when one orchestral group is broken into two or more heterogeneous groups by means of variation of instrumental forms—such as a legato against a pizzicato and against a muted tremolo.

In calculating the symmetric limit of a sigma, N represents the number of symmetric roots. Two tonics produce $\Sigma = 2S2p$, or two strata in 4 parts (or five parts with the addition of the root-tone). Twelve tonics, being the ultimate symmetric limit produce ($\Sigma = 12S2p$), 12 strata in 24 parts, in which case the overlapping of adjacent strata becomes unavoidable (25 parts with the addition of root-tone).

K. Compound Sigmae

I introduce now the concept of a *compound sigma*, or *the sigma of a sigma:* $\Sigma(\Sigma)$.

A compound sigma consists of more than one sigma. Each of the sigmae (i.e., Σ_I, Σ_{II}, Σ_{III}, ... Σ_N) consists of diatonic or of symmetric strata and is combined with another sigma, also consisting of several strata and connected to the first sigma by some form of interval-symmetry. In most cases of $\Sigma(\Sigma)$, overlapping becomes unavoidable. The lower stratum of Σ_I and the lower stratum of Σ_{II} produce a definite interval, which, as a consequence, controls the degree of overlapping.

In $\Sigma(\Sigma)$, diatonic sigmae are connected by a symmetric interval; symmetric sigmae are connected by an interval which is in a mutually excluding form of symmetry* with the structures of strata and the intervals connecting the latter.

The number of strata and of parts in the compound sigma equals the number of strata and of parts in each component sigma multiplied by the number of sigmae. For example, a compound sigma obtained from a five unit scale and three roots of symmetry for each component sigma produces a compound limit of 15 strata in 30 parts (or 31 with an addition of the root of Σ_I):

$$\Sigma(\Sigma)\left[\begin{array}{ll}
\Sigma_{III} = \begin{array}{c} S_V \\ \hline S_{IV} \\ \hline S_{III} \\ \hline S_{II} \\ \hline S_I \end{array} & \\
& I = \sqrt[3]{4} \\
\Sigma_{II} = \begin{array}{c} S_V \\ \hline S_{IV} \\ \hline S_{III} \\ \hline S_{II} \\ \hline S_I \end{array} & \\
& I = \sqrt[3]{4} \\
\Sigma_{I} = \begin{array}{c} S_V \\ \hline S_{IV} \\ \hline S_{III} \\ \hline S_{II} \\ \hline S_I \end{array} & \text{Only } \Sigma_I \text{ may have an added root-tone.}
\end{array}\right.$$

*That is, the interval is such as to ensure that the pitch-units composing one Σ do not coincide with those composing another Σ (Ed.)

It follows, from the above, that the limit for a seven-unit scale evolved into Σ (Σ) through 12 symmetric points, becomes $\Sigma(\Sigma) = 7S2p \cdot 12 = 84S2p = 168p$.

The ultimate compound sigma composed from two-part strata is $(12S2p \cdot 12 = 144S2p =)\ 288p$. This is the ultimate limit for a score composed in twelve-unit equal temperament out of two-part harmonies. Such a score of 289 parts (with an addition of the root-tone) may be used in practice for some group of combined orchestras, or choirs, or both. The place and time for such an occasion would be some such event as a World's Fair, an Eucharistic Congress, a world peace celebration, or an event of similar character calling for resources of such magnitude. With the knowledge of these possibilities, it is pitiful to recollect the experience of New York World's Fair of 1940—with the dozen or so pianos playing the *Second Rhapsody* of Liszt—a la "Roxy" in unison!

Examples of the Limit-Structures and the Compound Structures; Σ (Σ)

Diatonic Limit

Figure 26. Limit-structures of Σ (Σ).

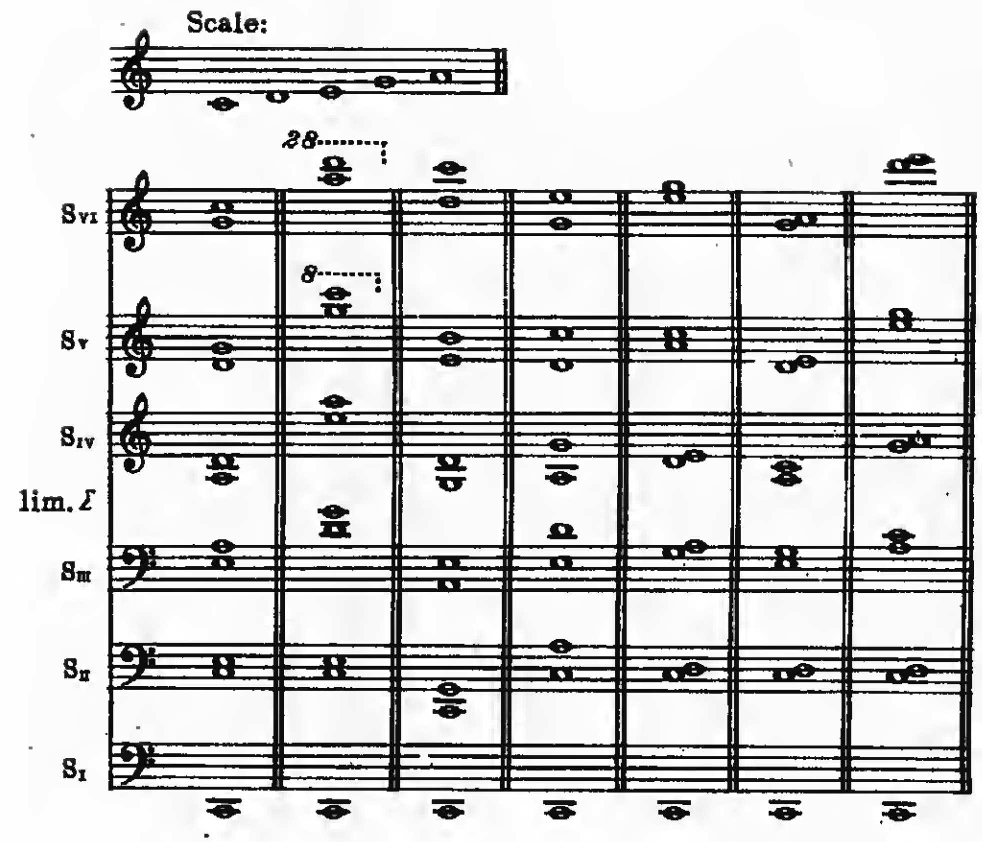


Figure 27. Limit-structures of Σ (Σ).

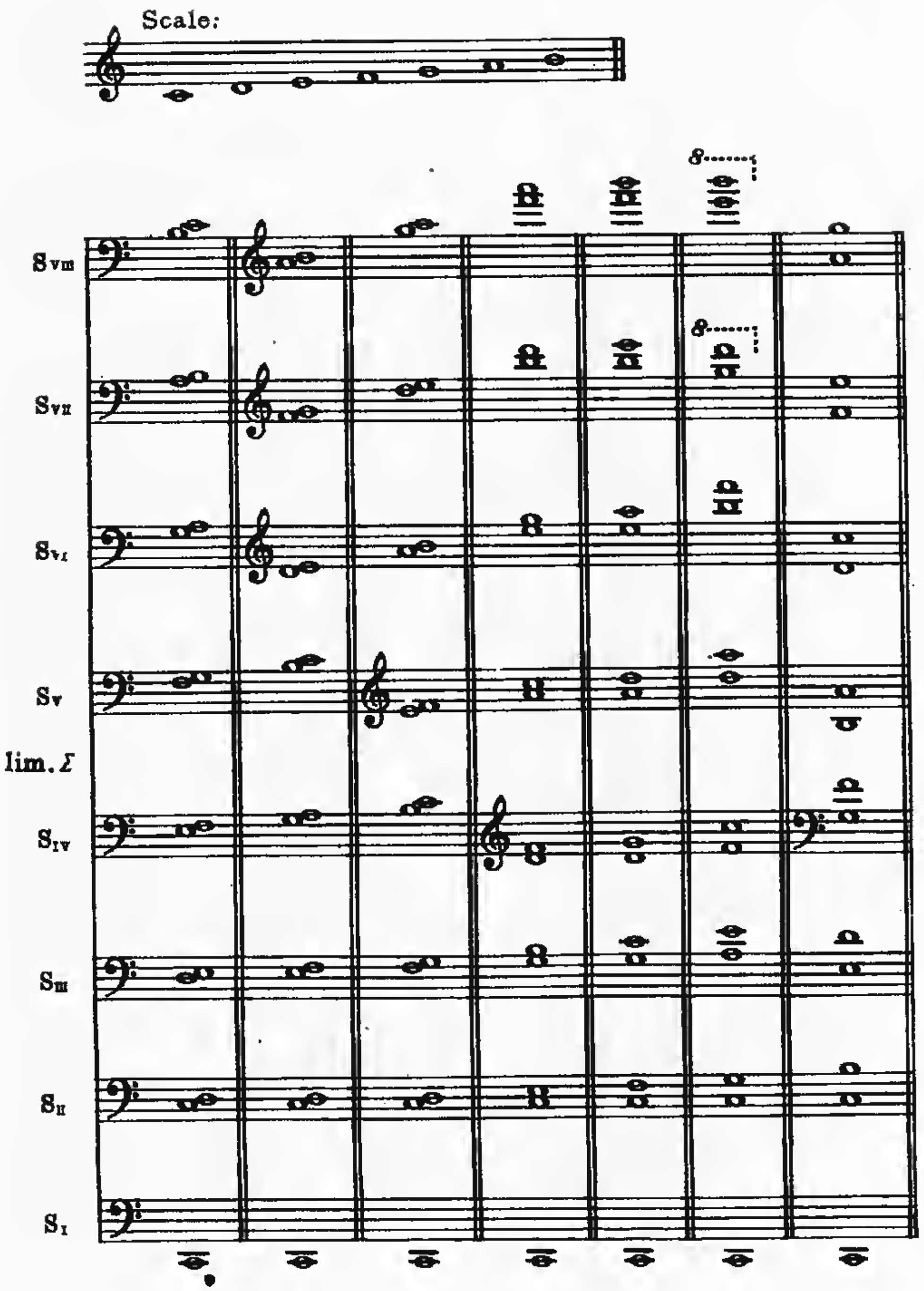


Figure 28. Limit-structures of Σ (Σ).

* **Hypothetical case.**
** **Can be executed with the aid of Hammond Organ.**

Figure 29. Symmetrik limit of Σ (Σ).

Compound Symmetric Limit

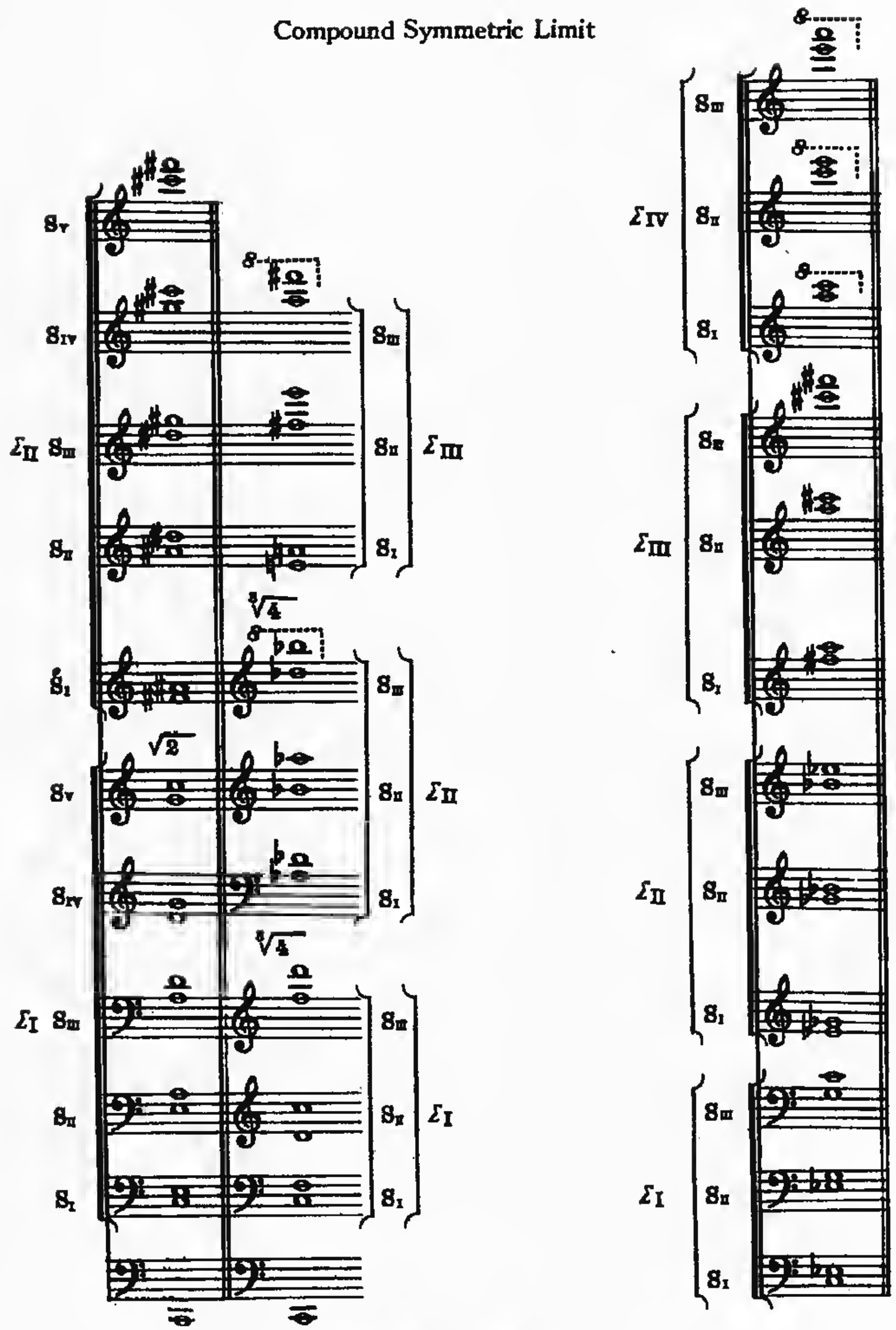


Figure 30. Compound symmetric limit of Σ (Σ).

CHAPTER 3

THREE-PART HARMONY

A. One Stratum of Three-Part Harmony (S = 3p)

ASSEMBLAGES which serve as three-part harmonic structures are the pitches of three-unit scales brought into simultaneity. Since the number of three-unit pitch-scales is 55, there are that many harmonic three-part structures. Each structure may be used in its original or in an expanded form, (E_0 and E_1).

All other conditions remain the same as for S2p.

In one stratum, with or without addition of a constant Sp or Sp with a coupling, we shall use either one constant structure or a group of structures belonging to one family. In the latter case, the added Sp must be assigned to each structure individually.

Table of Structures; Σ = S3p

Figure 31. Σ = S3p (continued).

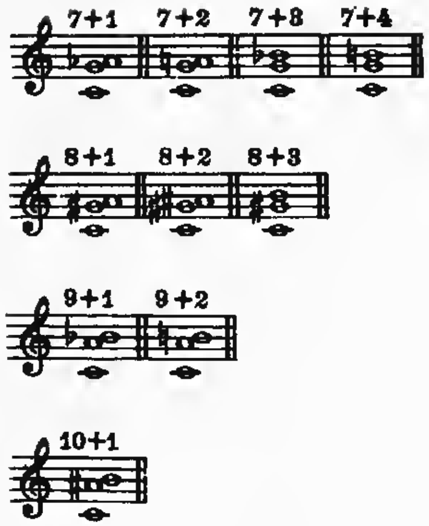


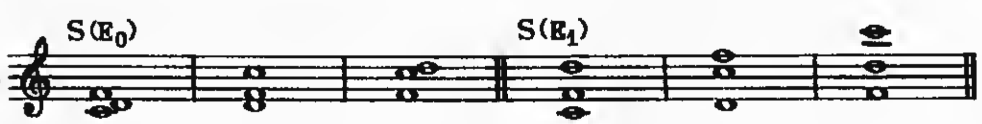
Figure 31. $\Sigma = \backslash S3p$ *(concluded).*

It is particularly important to approach the study of structures of S3p from the viewpoint of tonal expansions of the complete seven-unit scales. Such an approach makes it possible to acquire six diatonic forms of structure per stratum.

Three-part assemblages of any form may be called *triads*. For this reason it is correct to state that there are 6 forms of diatonic triads which derive from the complete seven-unit scales. Each form of a triad corresponds to the respective expansion.

(1) $S3p \equiv E_0$; (2) $S3p \equiv E_1$; (3) $S3p \equiv E_2$;
(4) $S3p \equiv E_3$; (5) $S3p \equiv E_4$; (6) $S3p \equiv E_5$.

Structures, diatonic with respect to: $I(S) = 2i + 3i$



Structures, diatonic with respect to Chinese Pentatonic.

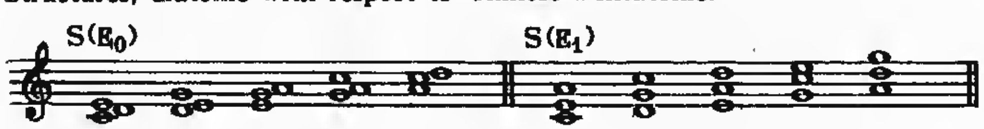
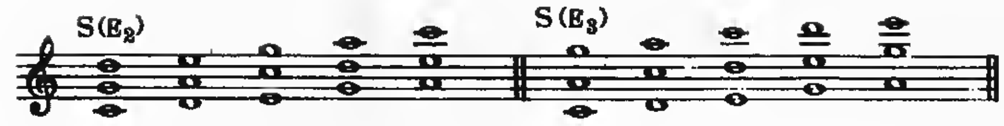


Figure 32. Diatonic structures (continued).



Structures, diatonic with respect to natural major.

Figure 32. Diatonic structures (concluded).

Of these, the following pairs contain identical pitch-units, in their respective triads, in a different form of distribution:

(1) $S(E_0)$ and $S(E_5)$; (2) $S(E_1)$ and $S(E_4)$; (3) $S(E_2)$ and $S(E_3)$.

B. Transformations of S = 3p

Transformations which control three-part assemblages are identical with those described in my discussion of hybrid four-part harmony in the *Theory of Special Harmony*.* They control the positions, (the first two transformations) i.e., the distribution of pitch-units, now serving as chordal functions, and they control voice-leading, (all six) i.e., the transformation of chordal functions in time continuity. The following forms may be used with discretion, depending on the cycles and the possibilities of instrumental execution.

Transformations of S3p

↻	↺	Const. a	Const. b	Const. c	Const. abc
a → b′	a → c′	a → a′	a → c′	a → b′	a → a′
b → c′	b → a′	b → c′	b → b′	b → a′	b → b′
c → a′	c → b′	c → b′	c → a′	c → c′	c → c′

Const. a, Const. b, and Const. c permit the isolation of a heterogeneous instrument from the remaining two, as an independent function, and this solves many important problems in orchestration.

When the structure is constant, a′ = a, b′ = b, c′ = c.

Progressions of Σ = S3p are evolved through the previous means: type I, II, III and the generalized symmetric.

Examples of Transformations

(a) Positions (b) Voice-leading

(a)

Figure 33. Positions (continued).

*See Vol. I. p. 478.

Figure 33. Positions (concluded).

(b)

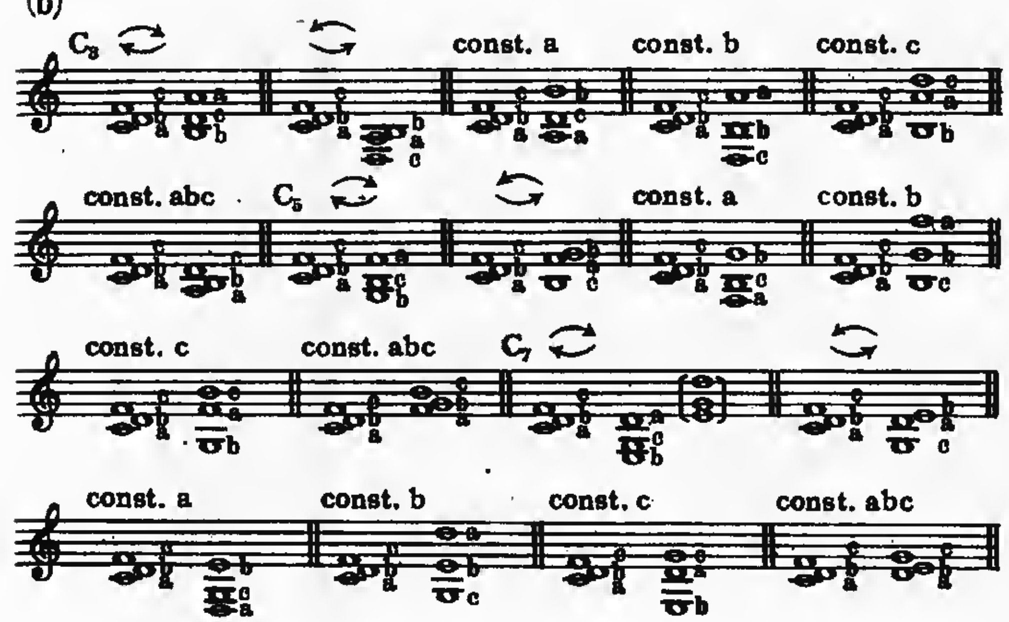


Figure 34. Voice-leading.

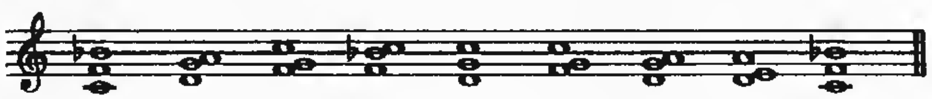
It follows from the example—(b) above—that the second chord of the connection appears in all six possible positions developed from any one position of the first chord when all six transformations are applied. For this reason, progressions may be written by selecting any position for the second chord which is adjacent to the given position of the first chord. However, a thorough knowledge of the patterns of motion through all cycles and through all transformations remains very desirable.

Examples of Progressions of $S = 3p$

Constant and Variable Structures.
Hybrid Four-Part Structures (added Sp and coupled Sp).

$S(E_2)$, nat. major

$I(S) = 5i + 5i$; Type II; Scale of root tones: d_4 of nat. major



The same with addition of coupled Sp.

$I(S) = 6i + 5i$; the same progression and scale of root-tones; new coupled Sp

The second structure combined with the first Sp;
Sp is symmetrically superimposed through $\sqrt[3]{2^2}$

Figure 35. Added Sp and coupled Sp (continued).

The first structure combined with the second Sp.

Figure 35. Added Sp and coupled Sp (concluded).

Progression of Structures $= 2S_2 + S_3 + S_2 + 2S_1$
Scale of root-tones: d_4 of nat. major

The same with Sp.

The same with coupled Sp.

Figure 36. Added Sp and Coupled Sp.

C. Two Strata of Three-Part Harmonies

$$\Sigma = 2S;\ S = 3p$$

Three-part harmonic structures may be coordinated into a simultaneous Σ and subjected to independent transformations in each stratum. As the number of transformations for S3p is 6, the number of transformations for Σ 2S3p is: $6^2 = 36$.

It is practical to study the fundamental forms first:

$$(1)\ \Sigma\left[\frac{S_{II}\circlearrowleft}{S_{I}\circlearrowleft}\right.;\quad (2)\ \Sigma\left[\frac{S_{II}\circlearrowleft}{S_{I}\circlearrowright}\right.;\quad (3)\ \Sigma\left[\frac{S_{II}\circlearrowright}{S_{I}\circlearrowleft}\right.;\quad (4)\ \Sigma\left[\frac{S_{II}\circlearrowright}{S_{I}\circlearrowright}\right.;$$

Σ 2S3p offers solutions to all problems in orchestration in which two groups of three identical instruments are used.

Positions of S_I and S_{II} may be either identical or non-identical. The variety of forms of transformation even permits the use of partly-identical pitch-assemblages in the two strata without producing consecutive octaves as between some of the parts of the two strata.

The number of possible *general* structures of Σ 2S3p equals the quantity of S3p, times the number of combinations of S3p in the two strata, times the number of positions between S_I and S_{II}: $55^2 \cdot 11 = 33{,}275$ (identical positions of S_I and S_{II} are excluded).

Of these, 216 are *diatonic;* $6^3 = 216$.

Examples of Structures of Σ 2S3p.

(1) Diatonic (all scales of three and more units);
(2) Symmetric (all three-unit scales in all forms of symmetry): (General)

(1)

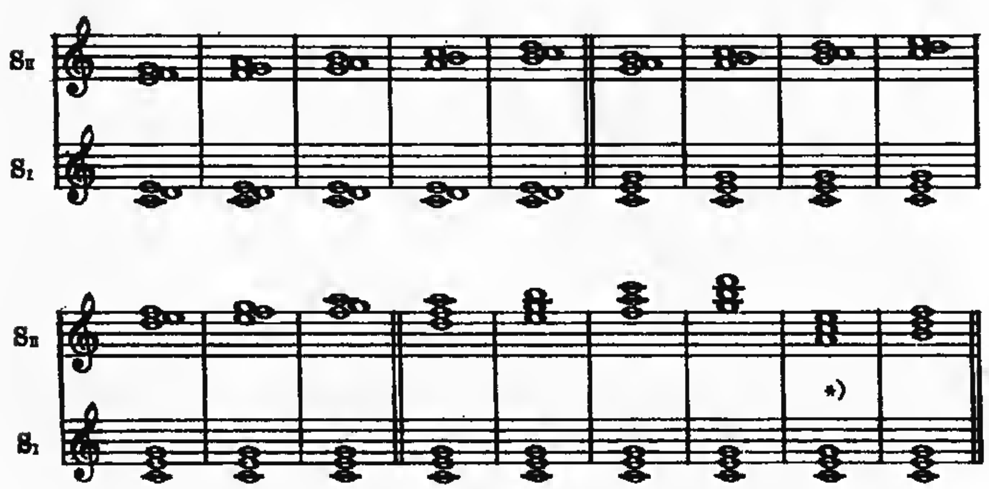


Figure 37. Structures of Σ 2S3p (continued).

*) A useful form of S(5) with added 13, for a combination of two groups by three, like 3 Trumpets and 3 Trombones.

(2)

Figure 37. Structures of Σ 2S3p (continued).

Figure 37. Structures of Σ2S3p (concluded).

For the time being, use one constant Σ with Σ>S.

Examples of Progressions of Σ 2S3p and Hybrid Forms Resulting from the Addition of Sp (uncoupled or coupled).

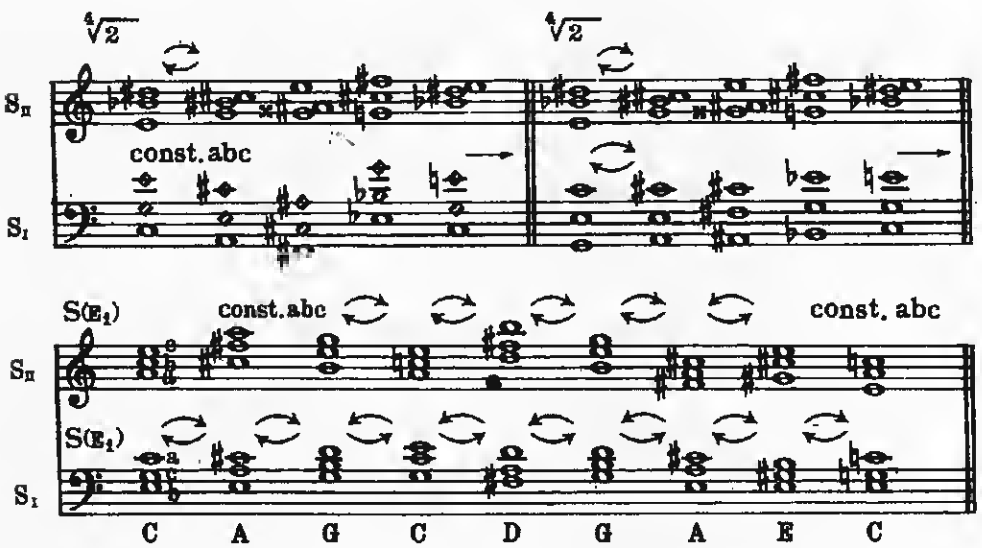


Figure 38. Progressions of Σ 2S3p and hybrid forms (continued).

The same with an added coupled Sp. and identical transformations in both strata

Figure 38. Progression of Σ2S3p and hybrid forms (concluded).

Figure 39. Identical transformations in both strata (continued).

Figure 39. Identical transformations in both strata (concluded).

D. Three Strata of Three-Part Harmonies

$$\Sigma = 3S; \; S = 3p$$

Structures of Σ 3S3p may be evolved from diatonic scales with three or more units, and from three-unit symmetric scales having three or more symmetric roots.

Since all principles remain the same as in harmony of the 2S3p type, we shall proceed with the illustrations.

Examples of Structures of Σ 3S3p.

(1) Diatonic;
(2) Symmetric:

(General)

(1)

(2)

Figure 40. Structures of Σ3S3p (continued).

Figure 40. Structures of Σ3S3p (concluded).

Examples of Progressions of Σ 3S3p and Hybrid Forms Resulting from the Addition of Sp (uncoupled or coupled).

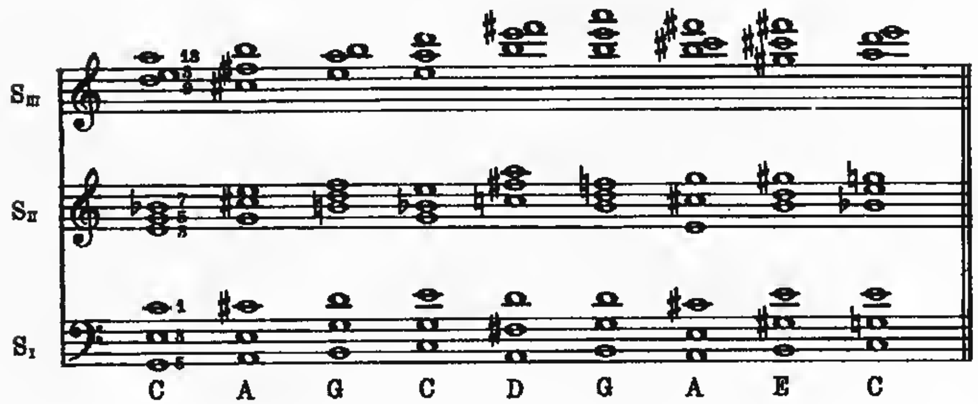


Figure 41. Progressions of Σ3S3p and hybrid forms (continued).

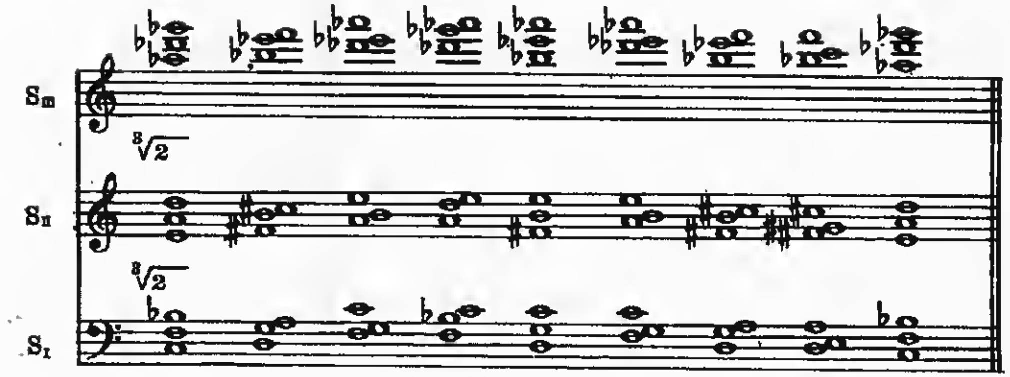


Figure 41. Progressions of Σ3S3p and hybrid forms (concluded).

E. Four and More Strata of Three-Part Harmonies

Structures of Σ = 4S3p are available from all diatonic scales having three units and more. They are desirable when it is advantageous to distribute the latter in groups of 3.

In the example presented below, structures of thirds and fourths are offered as characteristic structures and typical forms of arrangement. Structures derived from symmetric scales lend themselves particularly well to distribution in four strata when there are 4 tonics with 3 unit sectional scales. When the number of tonics exceeds 4, the 6 tonic system, also with 3 unit sectional scales, is practical when 4 out of 6 tonics are used. The same concerns scales constructed on 12 tonics with 3 unit scales from each tonic.

$$\Sigma = 4S;\ S = 3p$$

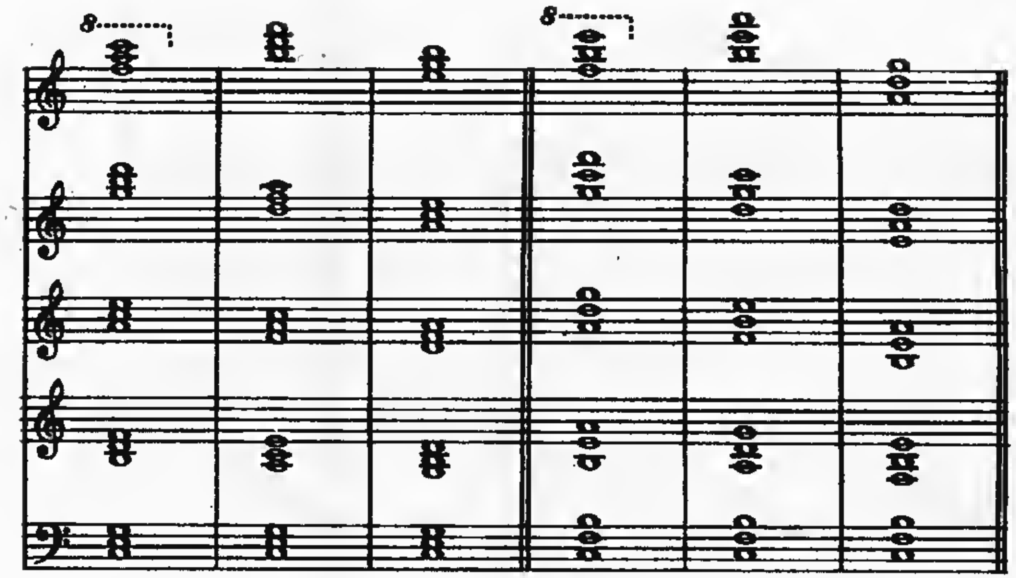
Figure 42. Structures of Σ = 4S3p (continued).

Figure 42. Structures of $\Sigma = 4S3p$ (concluded).

When $\Sigma = 5S$ and $S = 3p$, the diatonic arrangement of the groups of 3 usually adheres to the 3rds or the 4ths.

In order to build symmetric strata in five groups, it is necessary to consider the $\sqrt[6]{2}$, and the $\sqrt[12]{2}$ as the practical forms of symmetry without duplication of strata. The following table illustrates the general procedure of building $\Sigma = 5S$.

$\Sigma = 5S$;
Diatonic Structures



$S = 3p$
Symmetric Structures

Figure 43. Structures of $\Sigma = 5S3p$.

F. The Limits of Three-Part Harmonies

1. Diatonic Limit

By increasing the number of strata in any diatonic scale, we eventually reach the limit. In any diatonic limit the number of strata equals the number of pitch units in a given scale. When a scale has 3 units, the Σ limit = 3S. When the scale consists of 5 units, the Σ limit = 5S. The commonly used 7 unit diatonic scales have their limit in 7 strata, or 21 parts. The chord structures developed from any diatonic scale for each stratum are derived through tonal expansion. The following table illustrates:

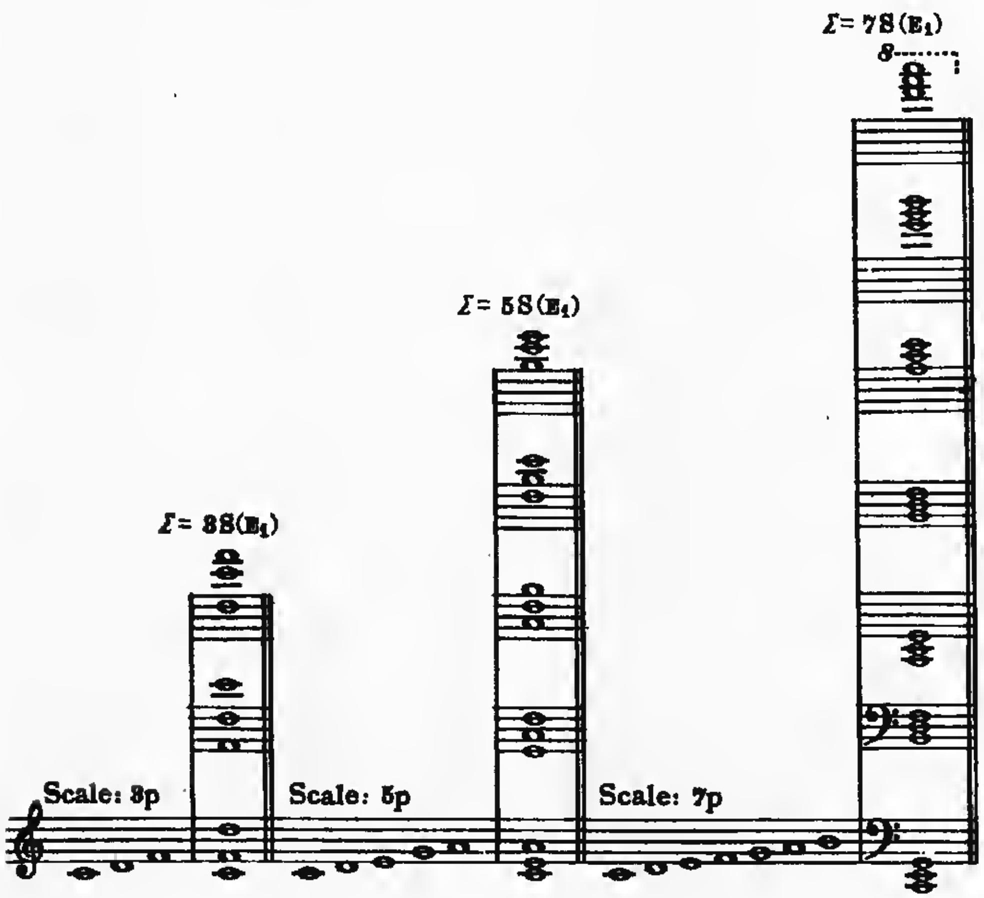


Figure 44. Diatonic limit of S3p.

2. Symmetric Limit

Symmetric limit depends on the number of symmetric roots from which the individual strata are constructed. The two-tonic system produces two strata in 6 parts; the 3-tonic system produces 3 strata in 9 parts; the 4-tonic system produces 4 strata in 12 parts; the 6-tonic system produces 6 strata in 18 parts; and the final symmetric limit on 12 tonics produces 12 strata in 36 parts. The latter is the ultimate symmetric limit.

In the following chart the last bar is a form of re-distribution of strata from the preceding bar. This variation is meant to show a more practical form of vertical arrangement. As a general requirement for satisfactory sonority, the lower stratum must be used in its expanded form (open position) unless it is placed in the middle register or higher.

Figure 45. Symmetric limit of S3p.

3. Compound Symmetric Limit: The Σ (Σ S 3p)

A compound symmetric limit depends on the number of symmetric points from which each individual Σ is constructed. In the following example, Σ consists of 3 strata and is developed from a 3-unit scale, thus representing the diatonic limit for such a scale. The second bar of the example represents a simultaneous vertical arrangement of the original Σ taken thrice through the symmetrical points of the two octave range, ($\sqrt[3]{4}$); thus the first Σ evolves its strata from c, the second evolves its strata from a♭, and the third Σ evolves its strata from e.

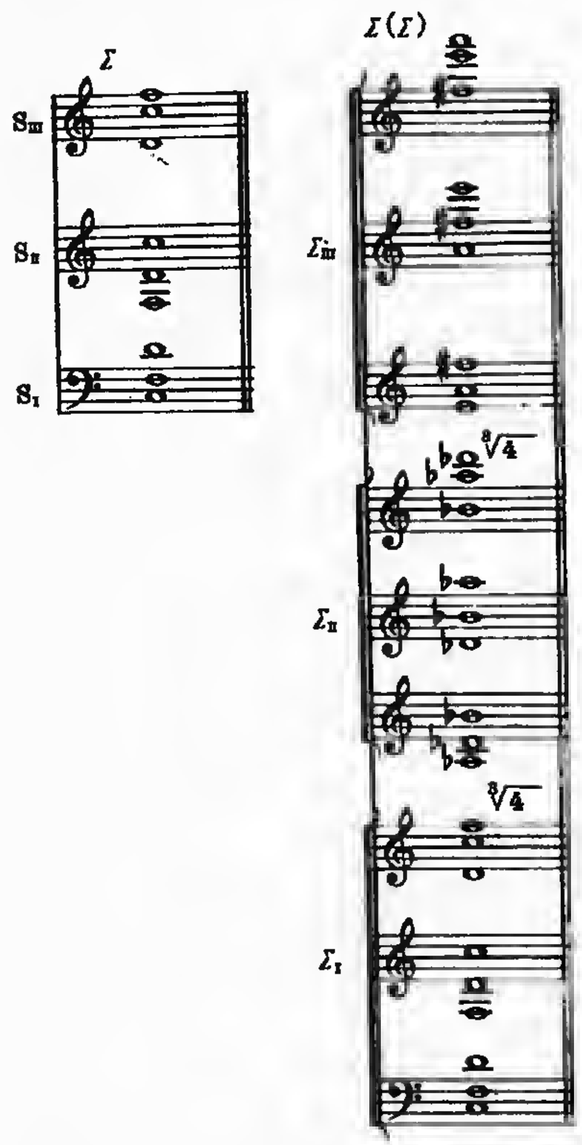


Figure 46. Compound symmetric limit Σ(ΣS3p).

The limits of Σ (Σ)'s go beyond the practical possibilities of today. It is possible to construct a Σ limit consisting of 12 strata in 36 parts and to arrange 12 of such structures in simultaneity; the Σ (Σ) for such a case, being the absolute compound limit for the groups of 3 parts, equals 12 × 36 = 432 parts. The addition of Sp would make it 433 p.

The practical significance of this kind of strata technique is mainly in its application to choral or to orchestral scoring, which is concerned with the individual development of groups and parts, and with a better acoustical quality for the whole sonority of the score.

CHAPTER 4

FOUR-PART HARMONY

A. One Stratum of Four-Part Harmony

(S = 4p)

Assemblages which serve as four-part harmonic structures are the pitches of four-unit scales brought into simultaneity.

There are 165 general S4p structures, which correspond to the 165 four-unit scales. The distribution of functions in any one S4p structure corresponds to E_0, E_1 and E_2.

Four-part assemblages of any form may be called *tetrads*. There are 6 forms of tetrads evolved from the complete seven-unit scales. Each form of a tetrad corresponds to the respective expansion.

(1) $S4p \equiv E_0$; (2) $S4p \equiv E_1$; (3) $S4p \equiv E_2$;
(4) $S4p \equiv E_3$; (5) $S4p \equiv E_4$; (6) $S4p \equiv E_5$.

Table of General Structures of S4p

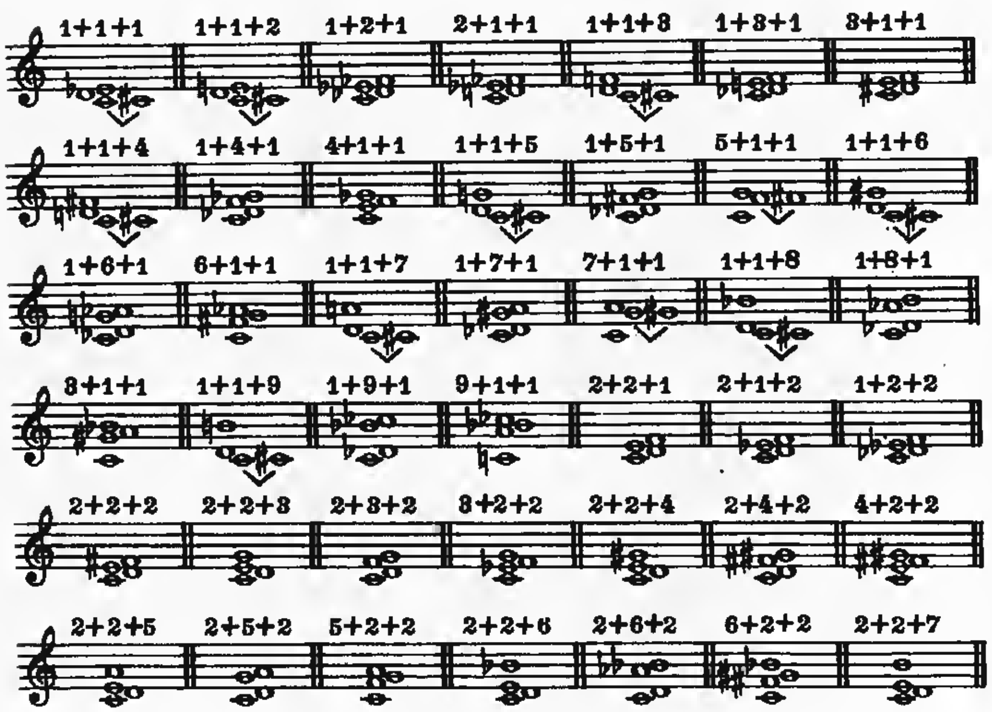


Figure 47. General structures of 4Sp (continued).

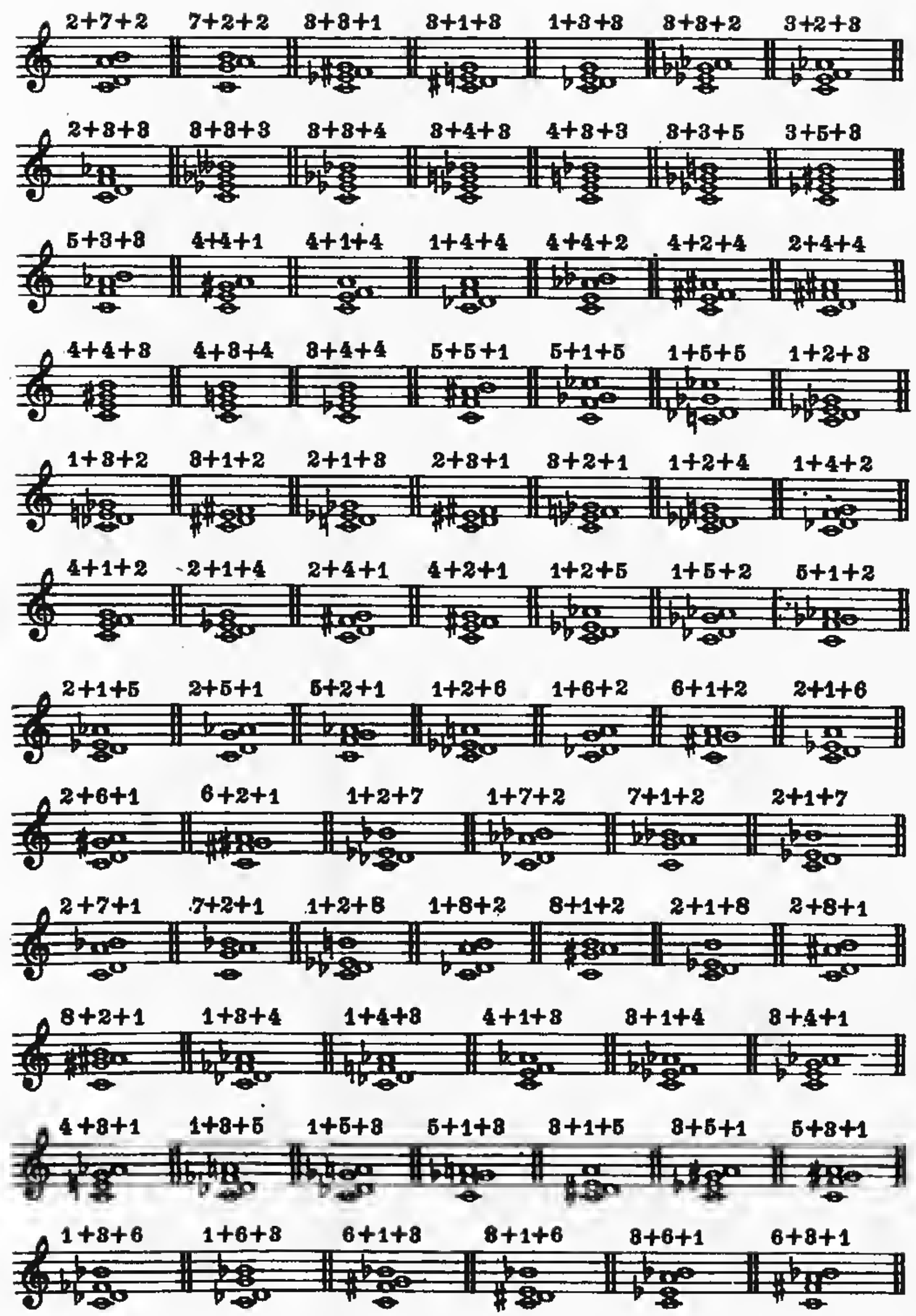


Figure 47. General structures of S4p (continued).

Figure 47. General structures of S4p (concluded).

Table of Diatonic Structures of S4p

Structures, diatonic with respect to: I = 2i + 3i + 2i.

Structures, diatonic with respect to Chinese Pentatonic

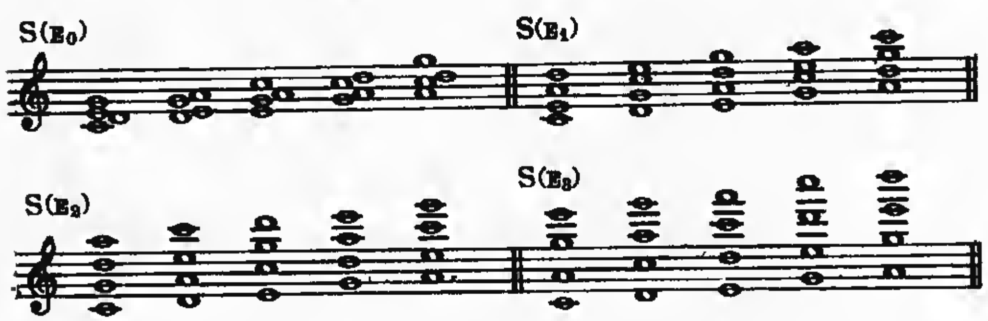


Figure 48. Diatonic structures of S4p (continued).

Structures, diatonic with respect to Natural Major

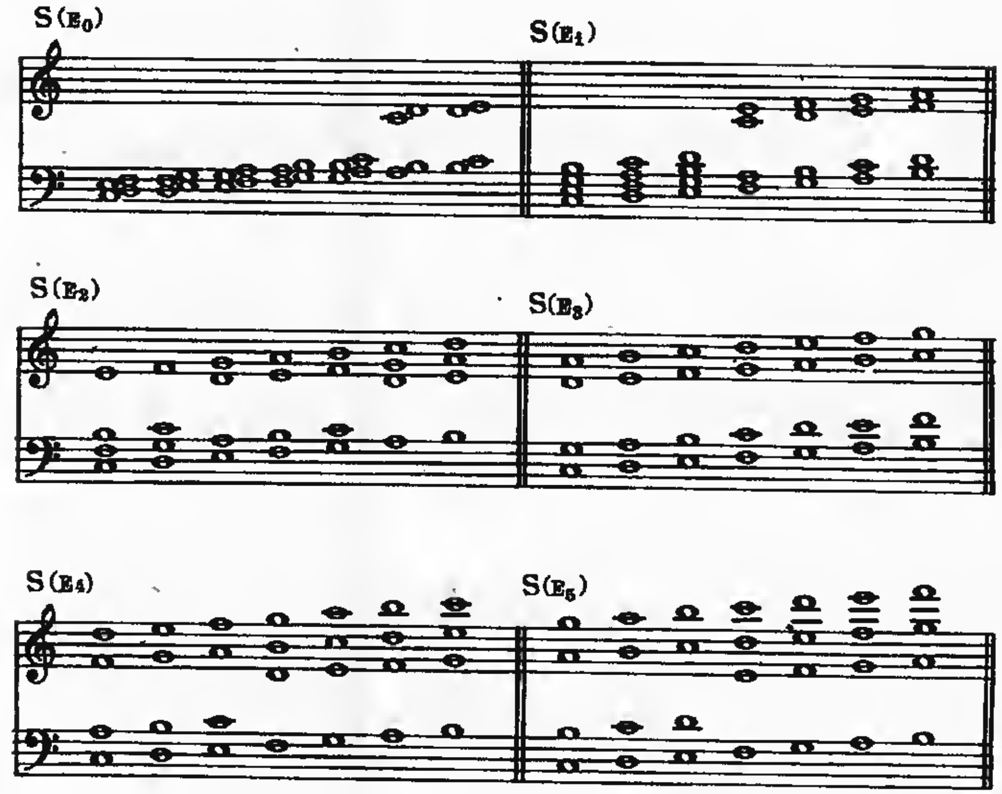


Figure 48. Diatonic structures of S4p (concluded).

Of these, the following pairs contain identical pitch-units, in their respective tetrads, in a different form of distribution:
(1) S(E_0) and S(E_5); (2) S(E_1) and S(E_4); (3) S(E_2) and S(E_3).

B. Transformations of S = 4p.

The classical system of harmony, based on the postulate of resolving 7th, emphasizes only one form of transformation with each tonal cycle. For example, C_3 requires a transformation; C_5, ; and C_7, . But in general forms of transformation not bound to the classical system—i.e., discarding the resolution of the 7th—all forms of transformation may be used with each cycle, giving us three forms for each cycle. In addition to this, one function (either one) of an assemblage may become a constant, producing hybrid 4-part harmony, where the remaining functions are subject to transformations of 3 elements. This produces 4 additional transformations with the direction for the three functions and one constant, and 4 transformations for the three functions and one constant.

In addition to this, two functions may become constant, permitting the other two to produce their only possible transformation.

There are six combinations with two constant functions. When the structures are variable, a constant transformation of all 4 functions may become practical. Summing up all forms, we get altogether 18 forms of transformations for each cycle. The following table includes all forms.

Transformations of S4p

			Const. abc
a → b′ b → c′ c → d′ d → a′	a → c′ c → a′ b → d′ d → b′	a → d′ d → c′ c → b′ b → a′	a → a′ b → b′ c → c′ d → d′

Const. a	Const. b	Const. c	Const. d
a → a′ b → c′ c → d′ d → b′	b → b′ a → c′ c → d′ d → a′	c → c′ a → b′ b → d′ d → a′	d → d′ a → b′ b → c′ c → a′

Const. a	Const. b	Const. c	Const. d
a → a′ b → d′ d → c′ c → b′	b → b′ a → d′ d → c′ c → a′	c → c′ a → d′ d → b′ b → a′	d → d′ a → c′ c → b′ b → a′

Const. ab	Const. ac	Const. ad	Const. bc	Const. bd	Const. cd
a → a′ b → b′ c → d′ d → c′	a → a′ c → c′ b → d′ d → b′	a → a′ d → d′ b → c′ c → b′	b → b′ c → c′ a → d′ d → a′	b → b′ d → d′ a → c′ c → a′	c → c′ d → d′ a → b′ b → a′

The above transformations are applicable to all structures of S4p.

The following table represents all transformations in application to S(7). When there is a crossing of voices, a respective crossing pair may be transposed into a different octave as shown on the table.

Certain cases that are undesirable from the viewpoint of orchestral selection may be eliminated. When one has so many cases, it is easy to select the most desirable ones, as well as to cope with all situations of 4-part orchestration of a melody.

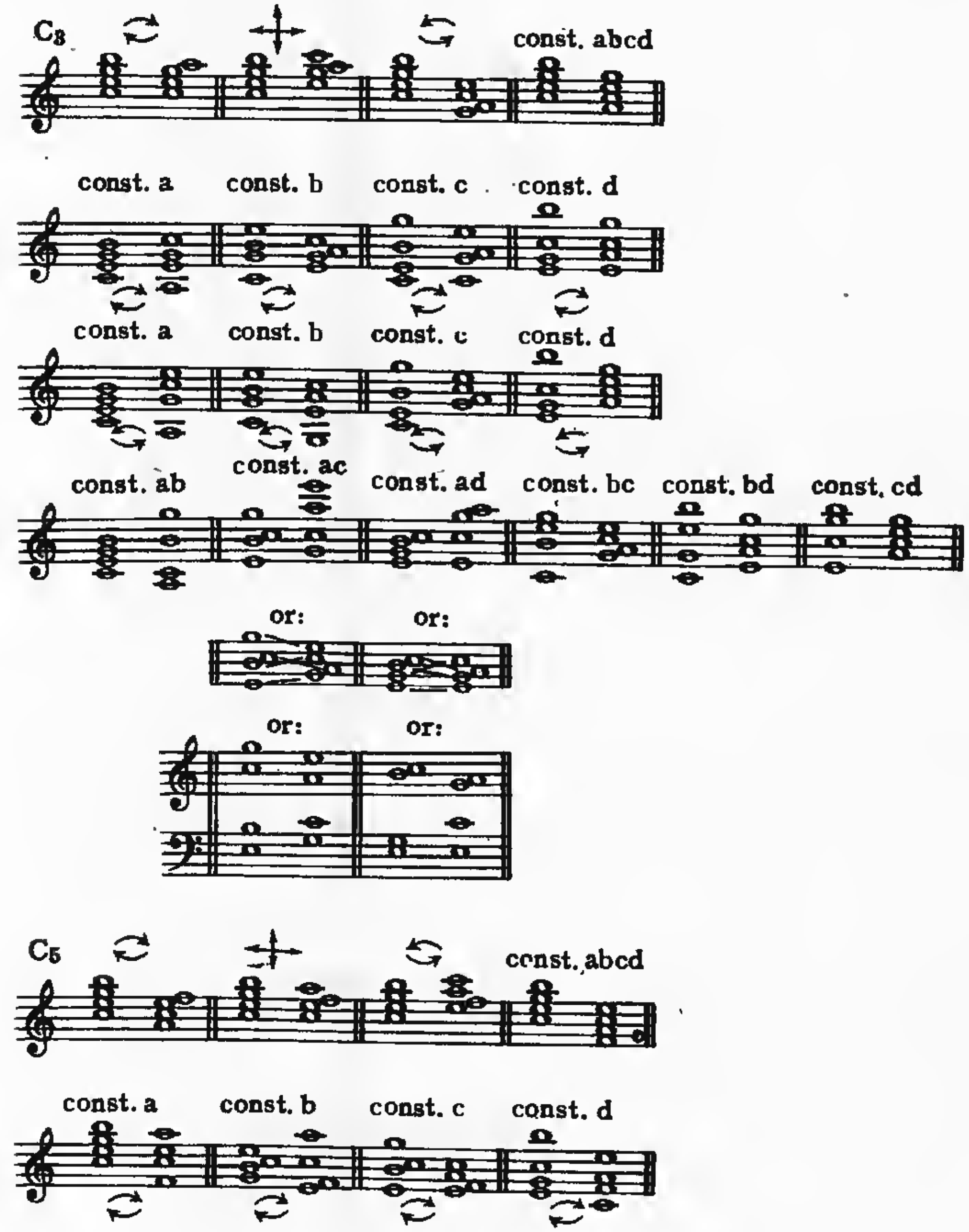


Figure 49. All transformations of S(7) (continued).

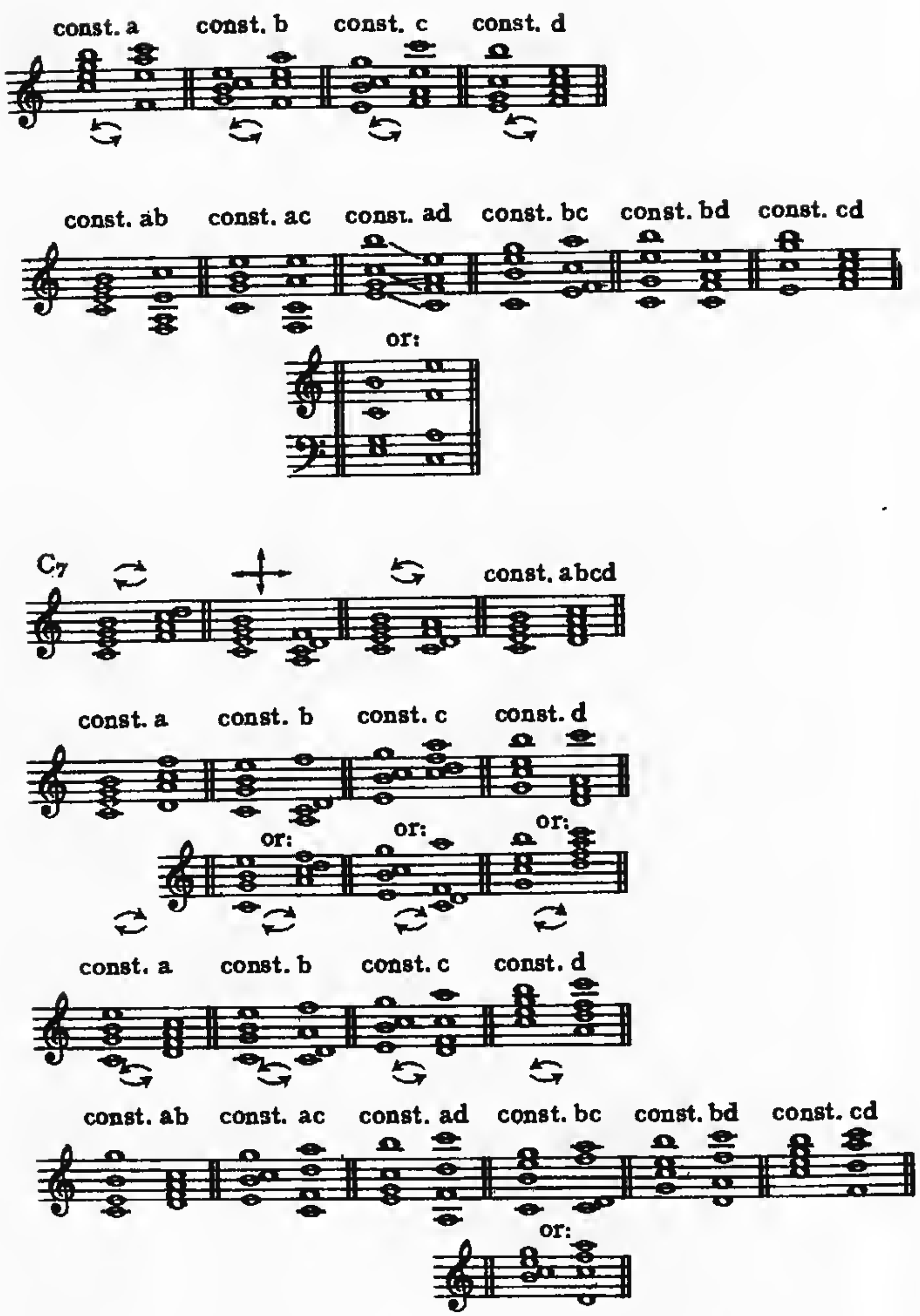
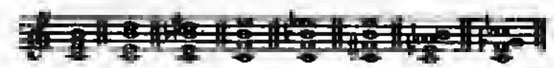


Figure 49. All transformations of S(7) (concluded).

It will be desirable to evolve similar tables for S4p structures in one stratum in all cycles for the following chord structures. An extra Sp may be added to any of these structures in order to obtain a hybrid 5-part harmony.

*After completing the tables, compose continuity selecting any of the following forms as a constant Σ. (—J.S.)



*This is one of the few "homework" directions included in the MS, which it has been thought wise to append here as a footnote. (Ed.)

C. Examples of Progressions of S = 4p

Constant and Variable Structures.
Hybrid Five-Part Structures (added Sp and coupled Sp)

At this point, we may observe that the number of transformations can be increased by a *new positional arrangement of the four functions* (a, b, c, d). This is practical for S(E_2) and the wider expansions, where crossing of parts is admissible, since in many instances it becomes unavoidable. The main advantage of having these new transformations in addition to the 18 already offered lies in the fact that in some cases these additional forms give the smoothest voice-leading, i.e., voice-leading with a maximum of common tones and nearest positions. The additional transformations are of the clockwise, the crosswise, and the counterclockwise forms.

For the sake of drawing comparisons between the three fundamental transformations (↻, ✛, ↺) in the original positional arrangement of the four functions (abcd) and the two new forms (acdb and acbd), we offer a complete table of 9 transformations for the three positional arrangements.

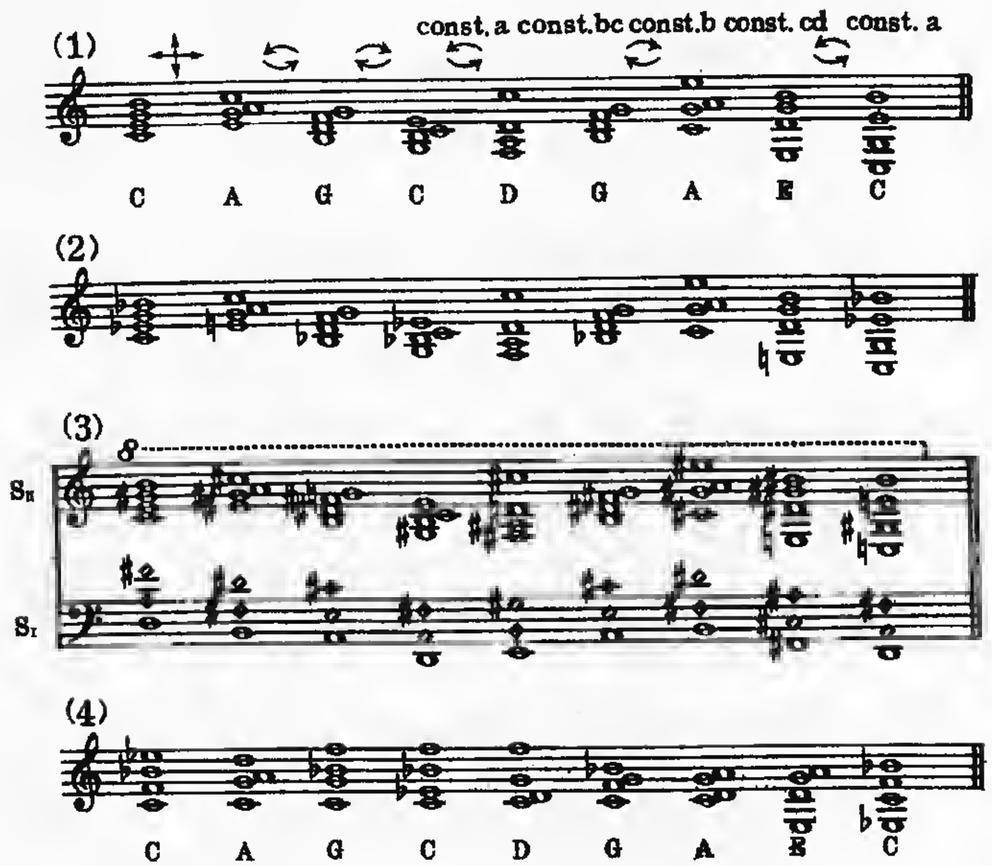


Figure 50. Progressions of S = 4p (continued).

Figure 50. Progressions of S = 4p (concluded).

Fundamental Transformations of S4p in the Three Positional Arrangements of Functions.

	a d b c	a b c d	a d c b
↻	a → b b → c c → d d → a	a → c c → d d → b b → a	a → c c → b b → d d → a
✛	a → c b → d c → a d → b	a → d c → b d → a b → c	a → b c → d b → a d → c
↺	a → d b → a c → b d → c	a → b c → a d → c b → d	a → d c → a b → c d → b

Some of these forms are used in Fig. 50, (4), (5) and (6).

CHAPTER 5

THE HARMONY OF FOURTHS

HARMONIES built on the intervals of a fourth—which is equivalent to the second tonal expansion of the complete seven-unit scales—still remain practically an unexplored region of musical harmony.

Some composers (Scriabine, Ravel, Hindemith) have used such chord-structures, but they have never subjected the latter to any systematic treatment. Neither have they discovered the principle upon which the progressions and the voice-leading are based.

We have seen, in three-part harmony, that 6 forms of transformation control all the possibilities of voice-leading. Existing music offers more evidence as to the correct way of handling progressions of S3p than the way of handling S4p when they both evolve in E_2; for this reason it is highly practical to present the real foundation for composing four-part harmonies built on fourths.

According to the definitions given in the *Special Theory of Harmony*,* the positive form of structures is the respective tonal expansion of the original scale (E_2 in this case), whereas the scale of chord progressions corresponds to the same set in position ⓑ, i.e., in backward motion. For this reason the functions of the assemblage become *1, 4, 7, 10* and the tonal cycles of the positive form become *cycle of the fourth* (C_4), *cycle of the seventh* (C_7) and *cycle of the tenth* (C_{10}). Cadences evolve as the first and the last steps of the cycles and their combinations.

We shall present now a comparative table of S3p and S4p as they evolve in E_2 of the complete seven-unit scales. Only the fundamental transformations will be used in order to present the matter with the utmost clarity.

HARMONY OF FOURTHS

Tonal Cycles (Positive Form)

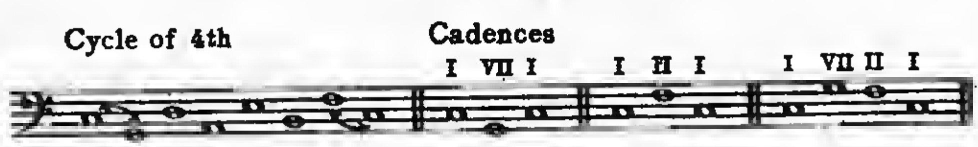
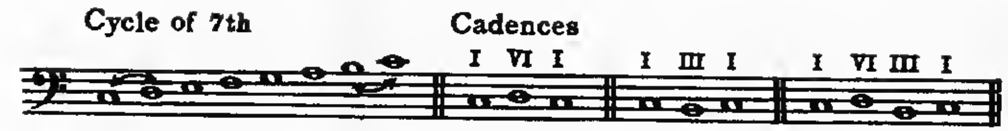


Figure 51. Harmony of fourths (continued).

*See Vol. I, p. 361 ff.



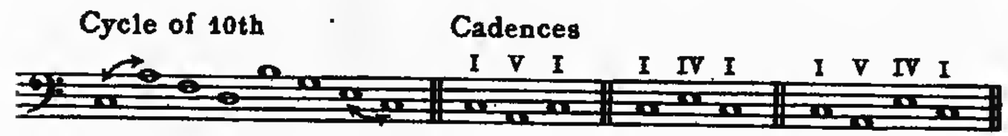


Figure 51. Harmony of fourths (concluded).

Cycle of the 4th

Cycle of the 7th

Figure 52. Voice-leading, cycle of the 4th, 7th, and 10th (continued).

Cycle of the 10th

Figure 52. Voice-leading, cycle of the 4th, 7th, and 10th (concluded).

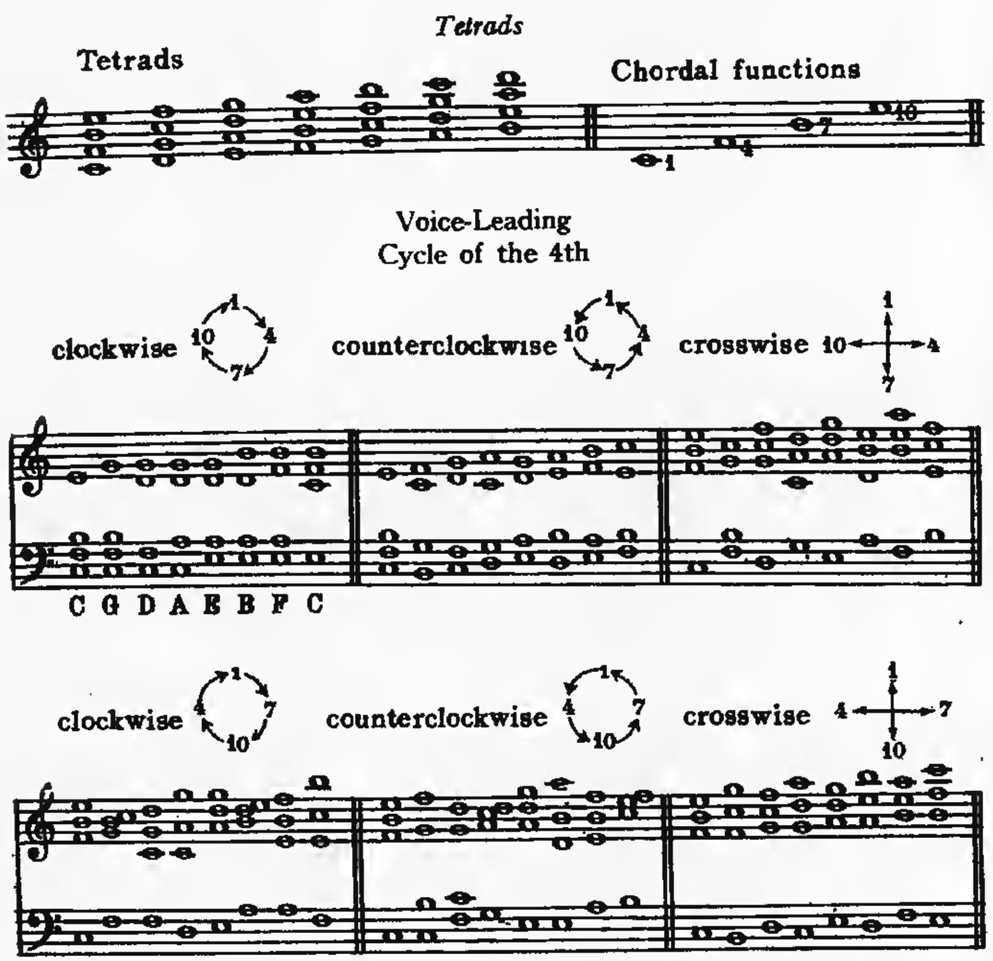


Figure 53. Tetrads. Voice-leading, cycle of the 4ths (continued).

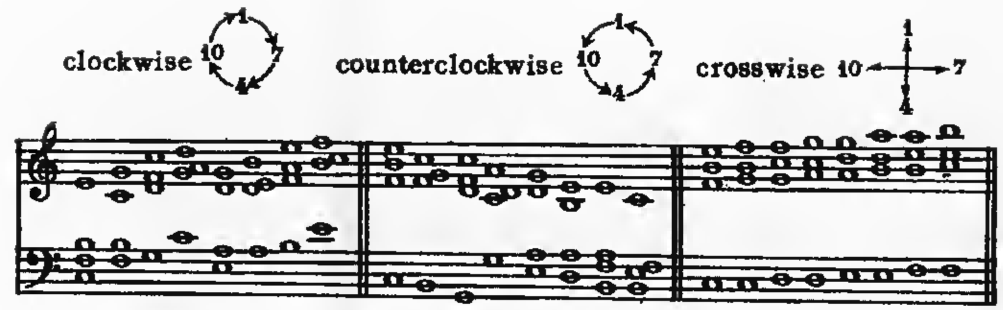


Figure 53. Tetrads. Voice-leading, cycle of the 4ths (concluded).

Cycle of the 7th

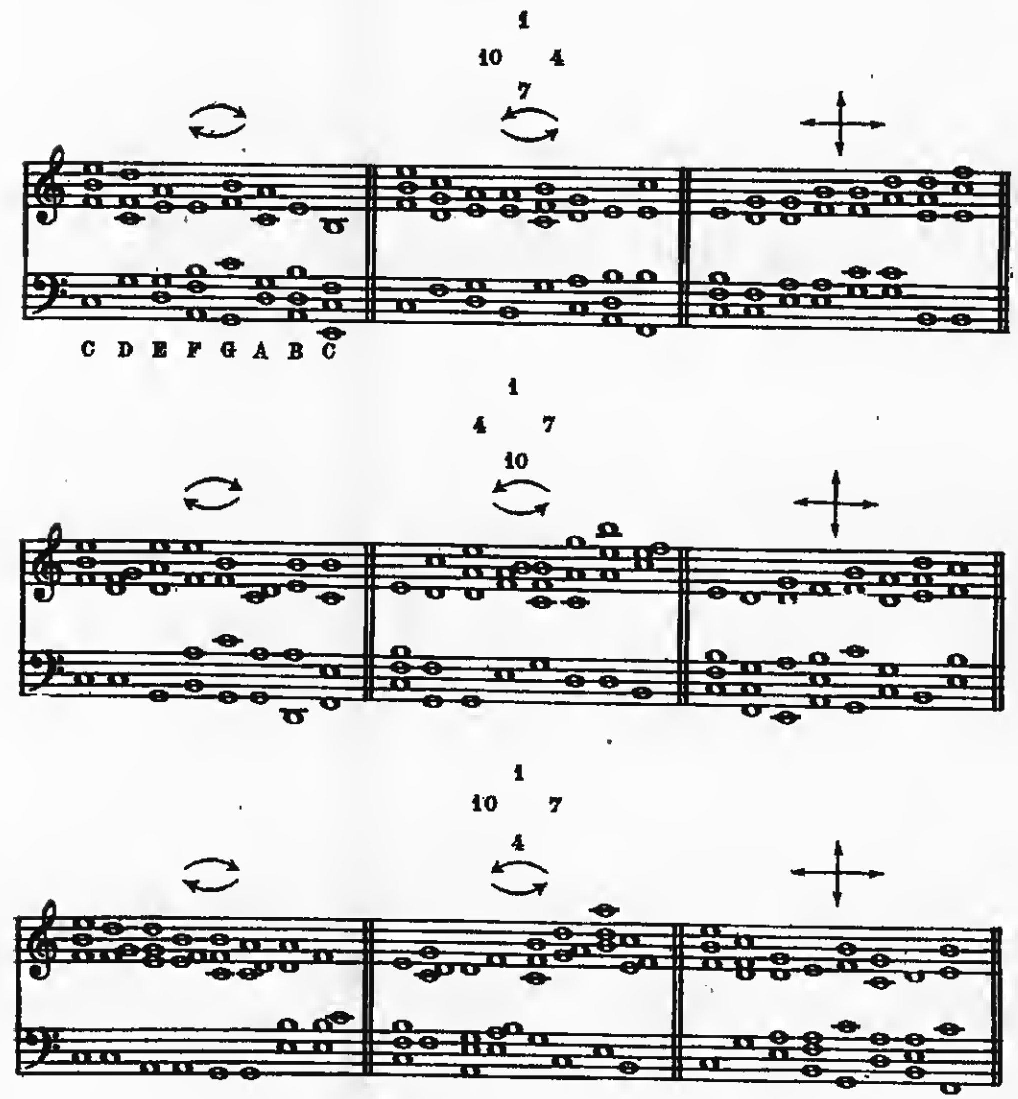


Figure 54. Voice-leading, cycle of the 7th.

Cycle of the 10th

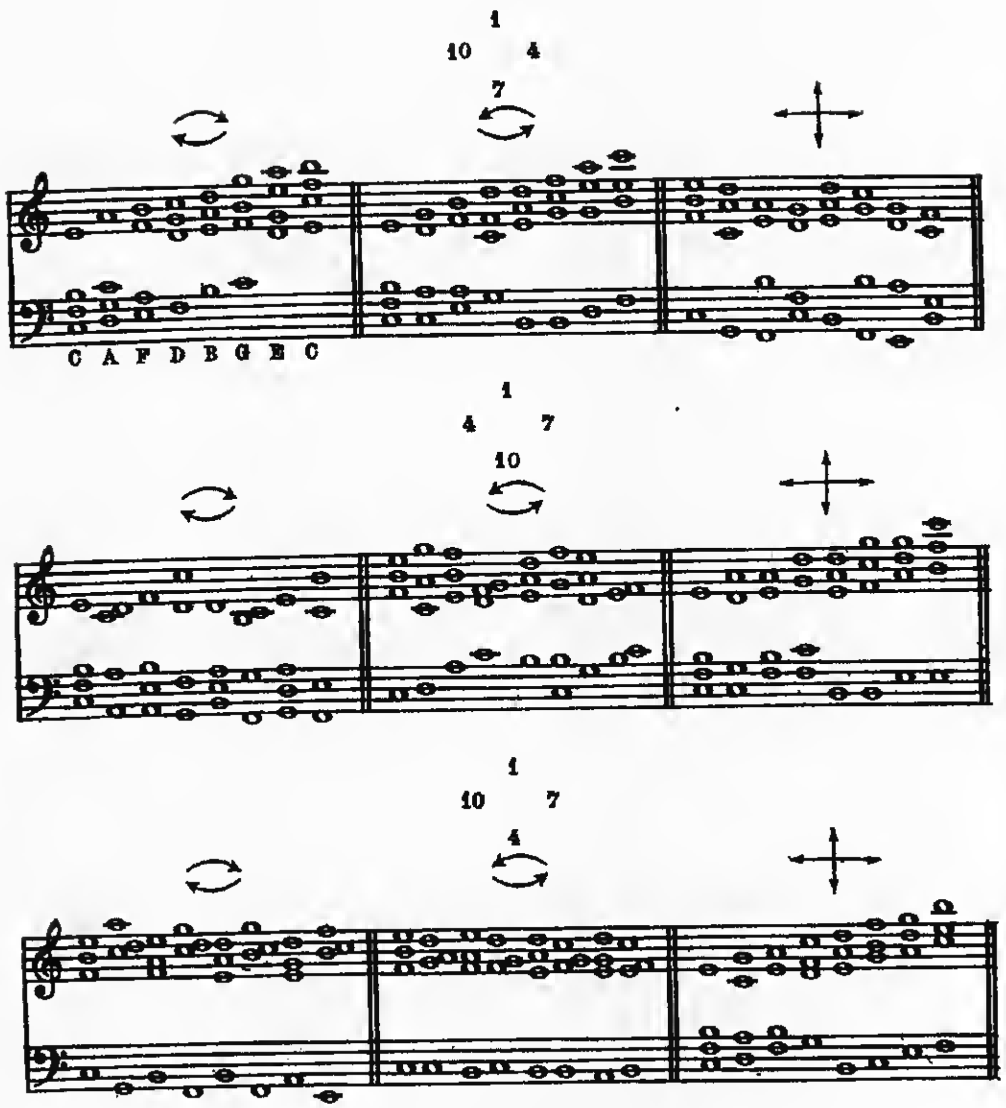


Figure 55. Voice-leading, cycle of the 10th.

CHAPTER 6

ADDITIONAL DATA ON 4-PART HARMONY

A. Special Cases of Four-Part Harmonies in Two Strata

1. Reciprocating Strata

When the number of parts in a harmonic stratum reaches four, it becomes practical to evolve S_{II} to a given S_I by means of inversion of the original stratum. Either the upper or the lower stratum may be considered original.

Tonal inversion (tonal ⓘ) is appropriate for the diatonic progressions; geometrical inversion ⓘ is appropriate for all other types of progressions. However, type II, III and the generalized may be assigned to a common Σ for both strata (as in figure 56).

The axis of inversion is common for both strata. The cycles are common for both strata but have opposite signs. If one of the strata has a positive progression, the other has a negative progression. However, this does not circumscribe the form of the structure. The structure of a certain stratum may be positive, while its progression may be either positive or negative. The reverse is also true.

If S_{II} has the form of S(7), read 1, 3, 5, 7 from the c-axis (i.e., $c - e - g - b$), S_I, being in the tonal inversion ⓘ, acquires the form of a negative S(7), read 1, −3, −5, −7 from the same axis (i.e., $c-a-f-d$, downward).

It is interesting to note that both strata, when moving through any one cycle, coincide in structure on the dominant (the *G*-chord in the key of *c*), which is always an S(7).

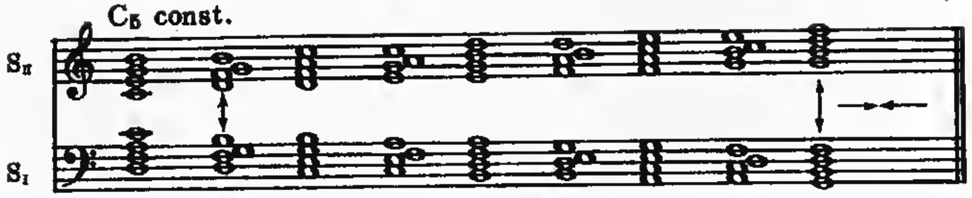
In using this technique for progressions type II, III and the generalized, assign one const. Σ, whichever you choose.

The technique of inversions for evolving the second stratum can be extended to all 165 structures.

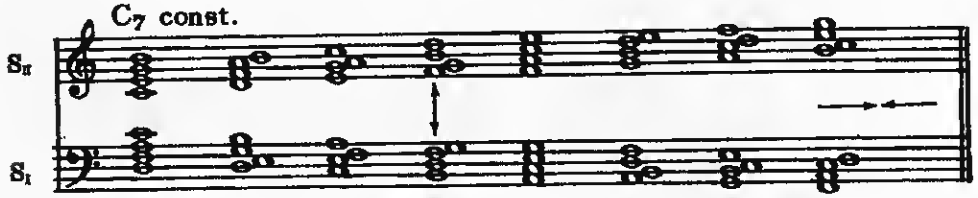
Examples of Two Mutually Reciprocating Strata.
$\Sigma = 2S4p$.

Figure 56. Two mutually reciprocating strata. $\Sigma = 2S4p$ (continued).

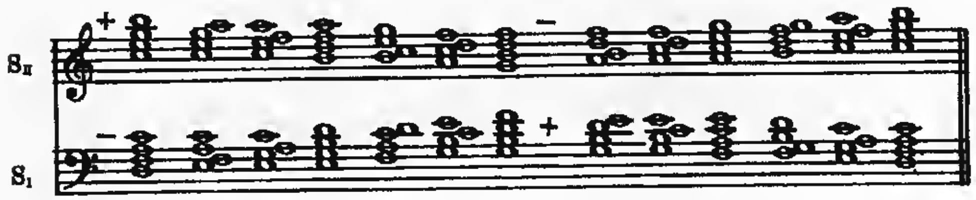
C_5 const.



C_7 const.



Diatonic progression $= 2C_3 + C_5 + 2C_7 + C_5$



Diatonic-Symmetric progression: The same cycles.
Scale of roots = Nat. major. $\Sigma = \Sigma$ (13) XIII. Roots in the lower stratum.

The same progression in $S(E_2)$ structures.

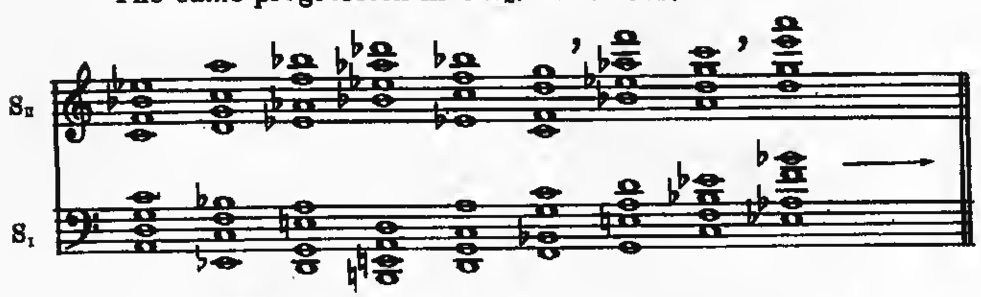


Figure 56. Two mutually reciprocating strata. $\Sigma = 2S4p$ *(concluded).*

2. Hybrid Symmetric Strata

There is a special case of two strata which deserves particular attention. It offers a technical interpretation of many not quite satisfactory attempts made by Ravel (particularly in the *Daphnis et Chloé* suite) and by Stravinsky (in *Petrouchka*, *Le Sacre du Printemps* and *Les Noces*) in their urge for *harmonic polytonality*. The latter, in fact, is a *superimposition of two symmetric strata*—to use the terminology of this system. I mention these two composers because they are the only originators of such a harmonic style and because, in the above mentioned works, this tendency of theirs is the more apparent. Ravel is more consistent than Stravinsky in this respect; but neither of the two composers has succeeded in achieving real consistency and clarity in this style—qualities which become possible now with the development of this theory.

Theirs is a special case of adding *Sp constant* as an upper stratum, mostly in $1 = \sqrt[4]{2}$, and often with symmetric couplings. The main characteristic of this style, which is to be expected, is the *large S(7)* as a permanent fixture of S_I (the lower stratum).

We shall use this style merely as a basis for building S_{II} as a symmetric superimposition upon S_I of Sp, S2p, S3p and, finally, S4p, when we accumulate the full Σ 2S4p. We shall also adhere to the large S(7) as the structure for S_I. At the same time our S_{II} will be developed in two basic ways:

(1) the structure of S_{II} is a part of $\Sigma(13)$ XIII, applied from one root-tone in the various possible roots of symmetry;

(2) the structure of S_{II} is a part of $\Sigma(13)$ XIII, *transposed* to the respective *root of symmetry.*

Examples of the Special Case of Harmonic Polytonality.

Figure 57. Special case of harmonic polytonality (continued).

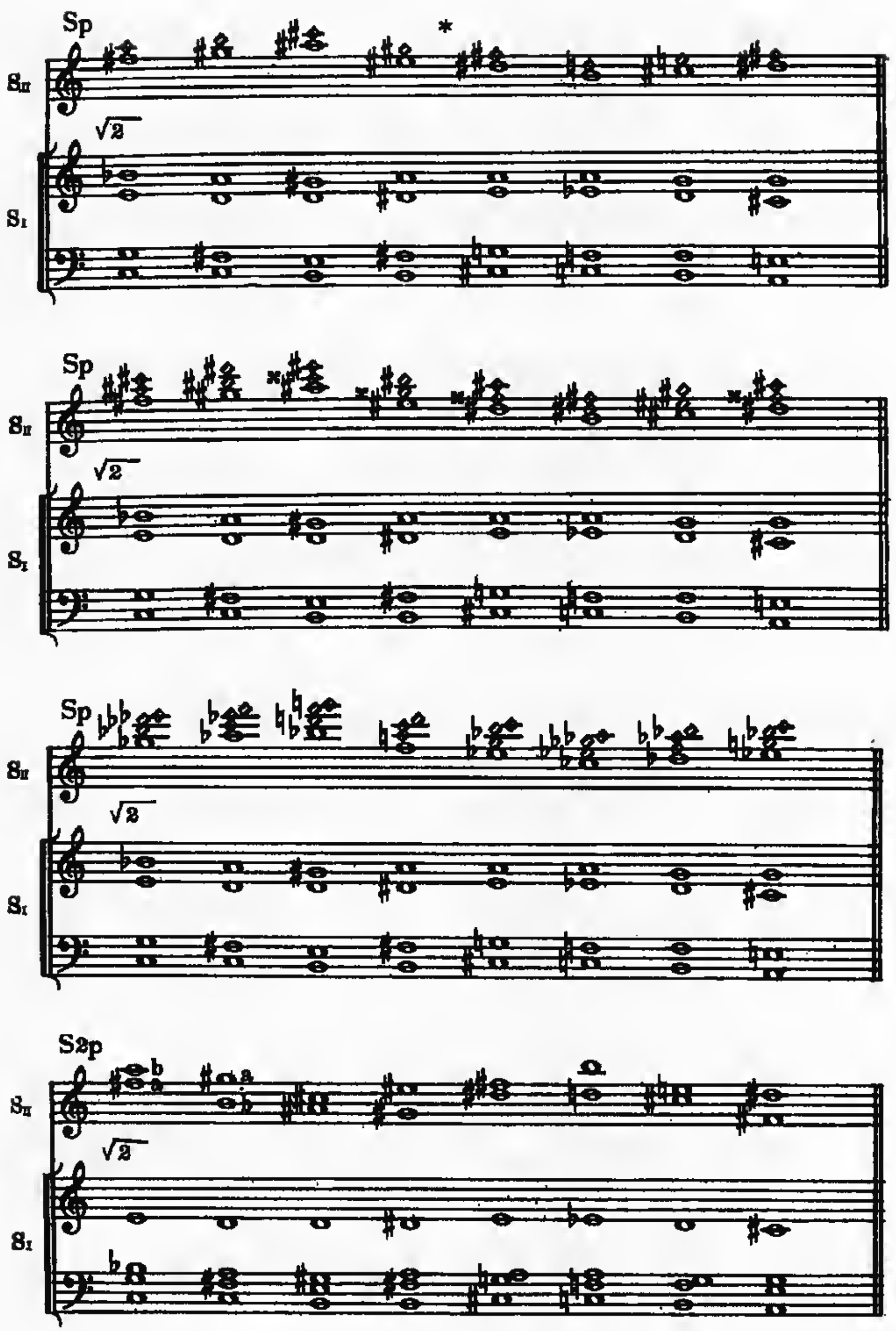


Figure 57. Special case of harmonic polytonality (continued).

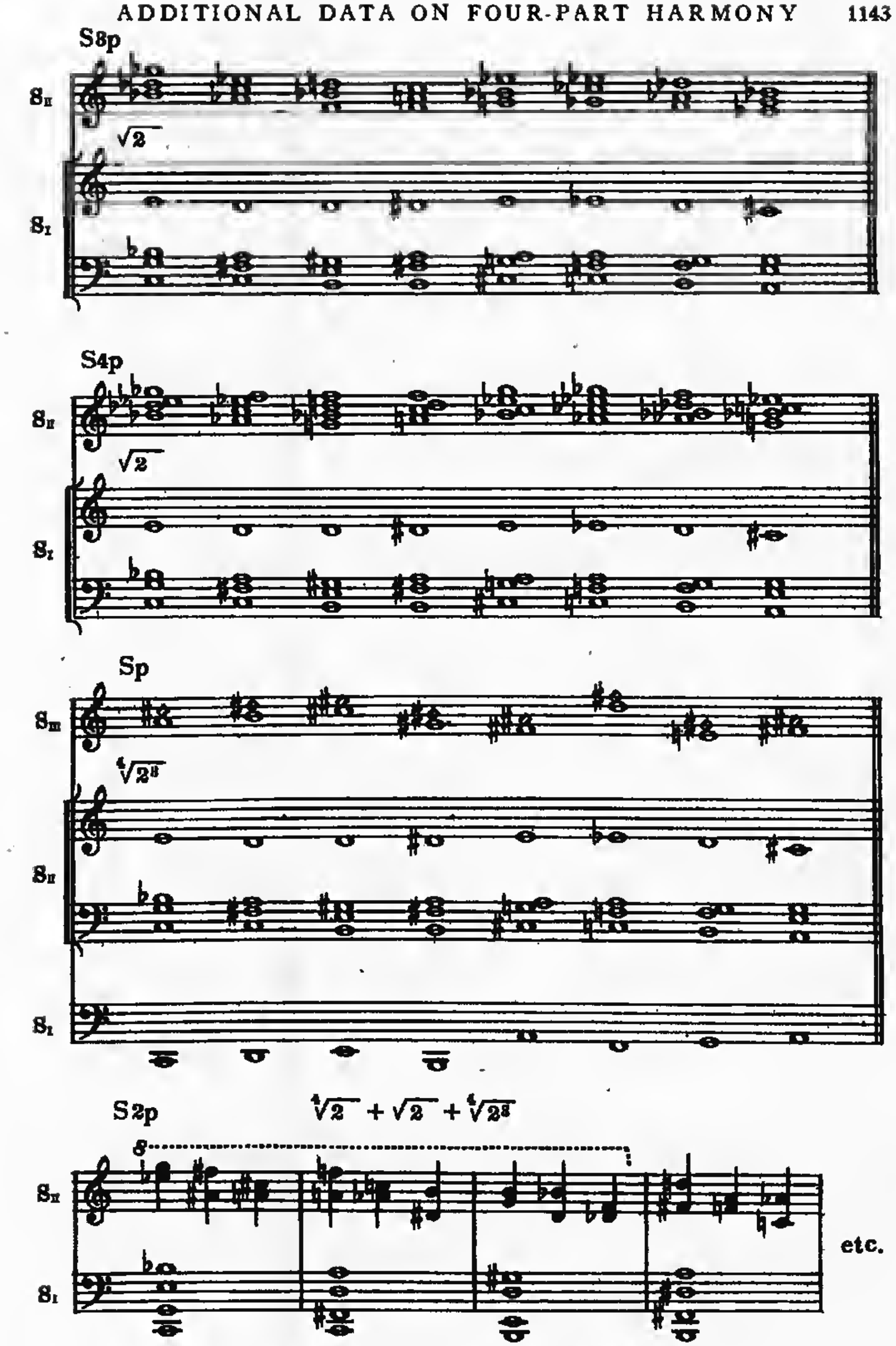


Figure 57. Special case of harmonic polytonality (continued).

S8p $\sqrt[4]{2}+\sqrt{2}+\sqrt[4]{2^3}$

S4p $\sqrt[4]{2^3}+\sqrt{2}+\sqrt[4]{2}$

S8p $\sqrt[3]{2}+\sqrt[3]{2^2}$

Figure 57. Special case of harmonic polytonality (continued).

Sp $\sqrt[12]{2}+\sqrt[6]{2}$

Figure 57. Special case of harmonic polytonality (concluded).

All other cases of two strata belong to the next chapter.

B. Two Strata of Four-Part Harmonies

Generalization of the $\Sigma = 2S$; $S = 4p$

Four-part harmonic structures may be coordinated into a simultaneous Σ and subjected to independent transformation in each stratum. As the number of all transformations for S4p is 24 (18, and the six additional ones), the number of transformations for Σ 2S4p is $24^2 = 576$. Combinations of the 9 fundamental forms alone are sufficient for general use since their quantity amounts to:

$$ {}_9C_2 = \frac{9!}{2!\,(9-2)\,!} = \frac{362{,}880}{2\cdot 5040} = 36 $$

The latter represent the combinations of $\circlearrowright$, $\leftrightarrow$ and $\circlearrowleft$ distributed through two strata and having three forms of the positional arrangement of functions: abcd, acdb and acbd.

All other transformations serve the purpose of isolating one or two parts from the stratum of 4p.

Positions of S_I and S_{II} may be either identical or non-identical. The variety of the forms of transformation permits the use even of completely identical pitch-assemblages, without causing consecutive octaves between any pair of parts of the two strata.

The number of possible *general structures* of Σ 2S4p equals the quantity of S4p, multiplied by the number of combinations of S4p in the two strata, multiplied by the number of positions between S_I and S_{II}: $165^2\cdot 11 = 299{,}475$ (excluding identical positions between S_I and S_{II}). Of these 216 are *diatonic;* $6^3 = 216$.

Examples of Structures of $\Sigma\,2S4p$

(1) Diatonic (all scales with four and more units);

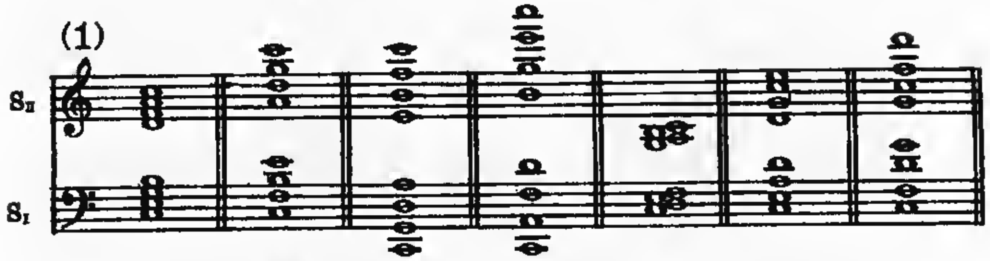


Figure 58. Diatonic structures of $\Sigma\,2S4p$.

(2) Symmetric (all four-unit scales in all forms of symmetry): (General).

Figure 59. Symmetric structures of $\Sigma\,2S4p$.

For the time being, use one constant Σ, when $\Sigma > S$.

Examples of Progressions of $\Sigma\,2S4p$ and Hybrid Forms Resulting from the Addition of Sp.
(uncoupled or coupled)

Figure 60. Progressions of $\Sigma 2S4p$ (continued).

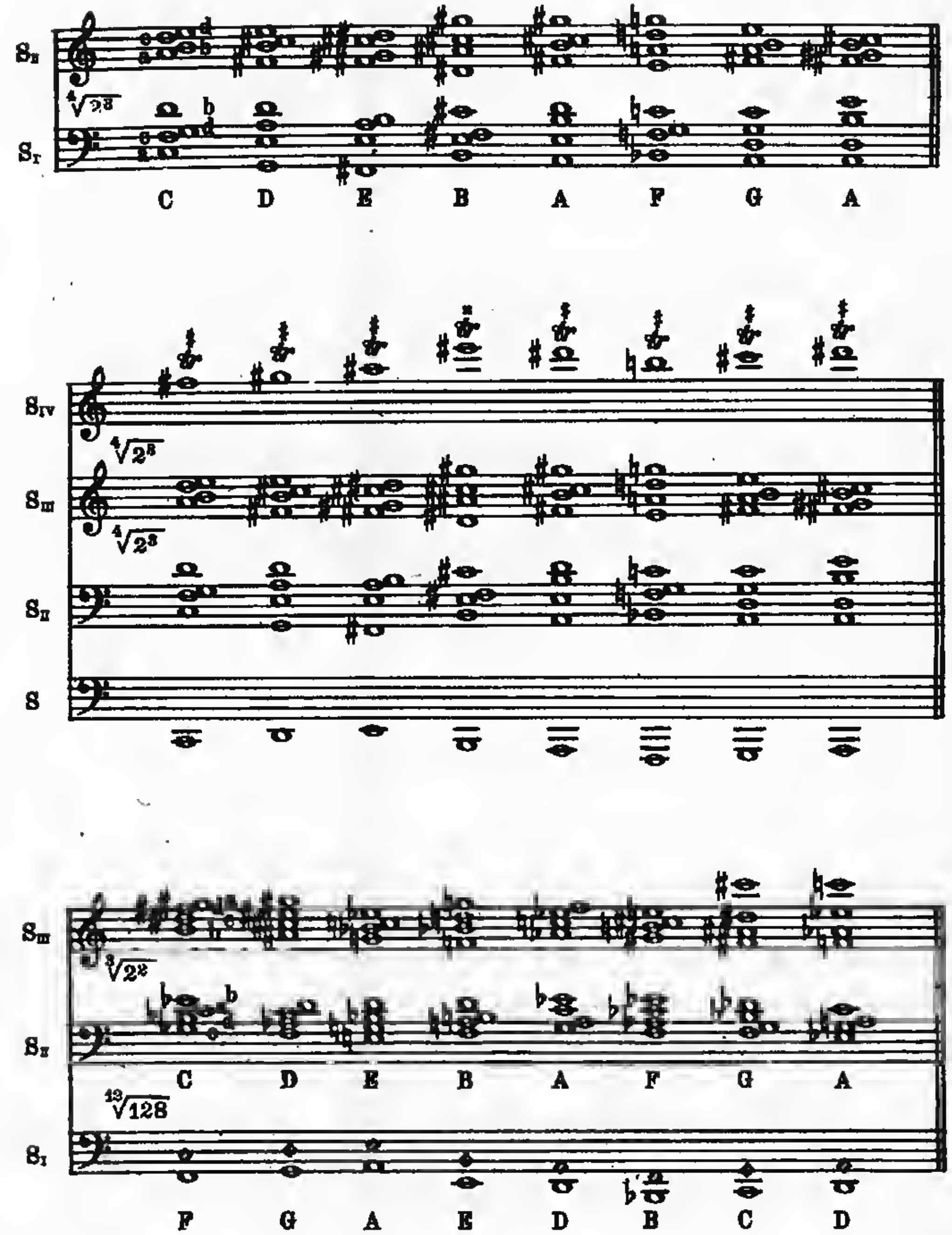


Figure 60. Progressions of $\Sigma 2S4p$ (concluded).

C. Three Strata of Four-Part Harmonies

$$\Sigma = 3S;\ S = 4p$$

Structures of Σ 3S4p can be evolved from the diatonic scales with four or more units and from symmetric scales having three or more symmetric roots. All principles remain the same as in the Σ 2S4p.

Examples of Structures of Σ 3S4p

(1) Diatonic;

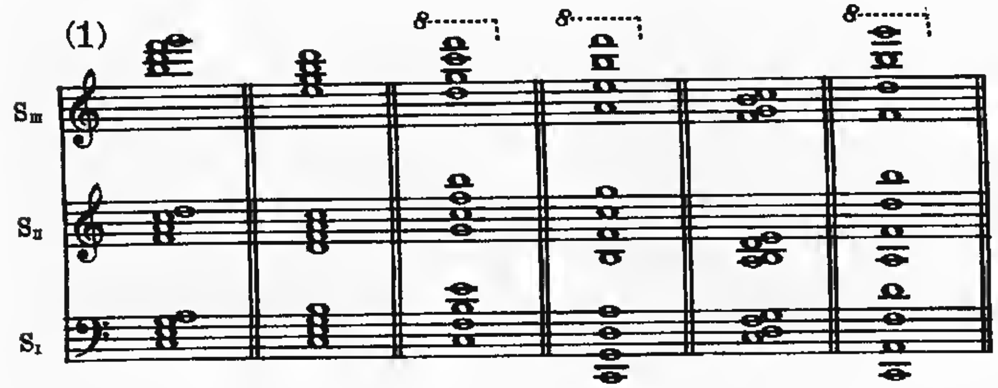


Figure 61. Diatonic structure of Σ 3S4p.

(2) Symmetric:

(General)

Figure 62. Symmetric structure of Σ 3S4p.

Examples of Progressions of Σ 3S4p and Hybrid Forms Resulting from the addition of Sp
(uncoupled or coupled)

Figure 63. Progressions of Σ 3S4p.

D. Four and More Strata of Four-Part Harmonies

Structures of $\Sigma = 4S4p$ are available from all diatonic scales having four units or more. They are desirable when it is advantageous to distribute the latter in groups of 4. Four-unit sectional scales in four and more tonics serve as material for symmetric structures. To this group belongs one of the forms gaining considerable popularity today. It is the large S(7) distributed through the $\sqrt[4]{2}$.

We shall now offer a few examples of multi-strata structures.

Examples of Structures of Σ 4S4p

(1) Diatonic;

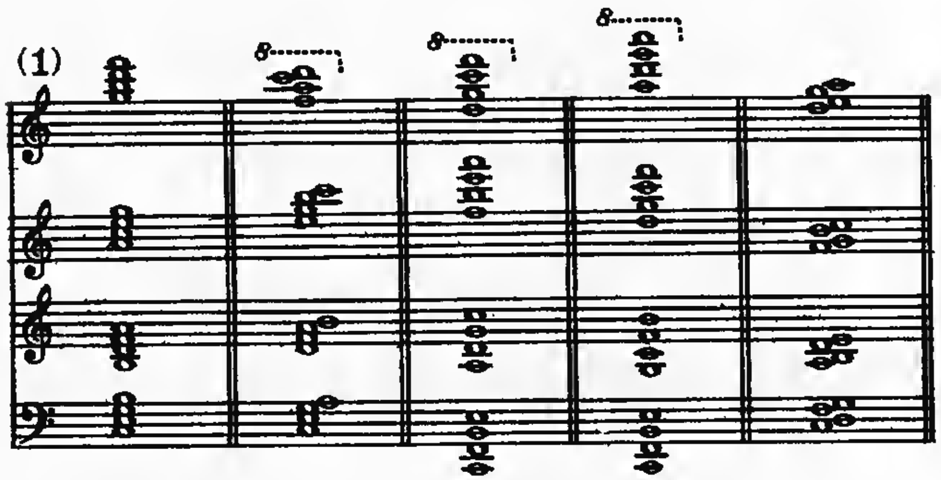
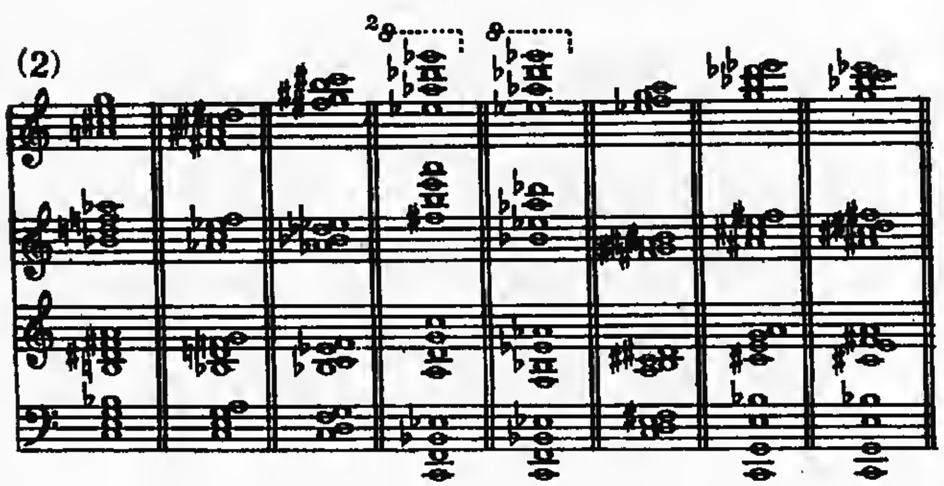


Figure 64. Diatonic structure of Σ 4S4p.

(2) Symmetric:
(General)



E. The Limits of Four-Part Harmonies

Diatonic limit for S4p is defined by the number of pitch-units, which in this case correspond to the number of strata. The minimum number of units is 4.

The limit for a four-unit scale is Σ 4S4p, i.e., 4 strata in 16 parts (or, with the addition of Sp, 17 parts). The diatonic limit for a seven-unit scale is $\Sigma = 7S4p$, i.e., 7 strata in 28 parts (or 29 with the addition of Sp).

Overlapping in most cases is unavoidable.

Example of the Diatonic Limits of S4p

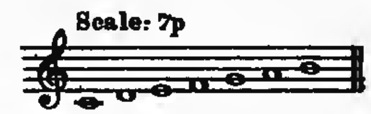


Figure 66. Diatonic limits of S4p.

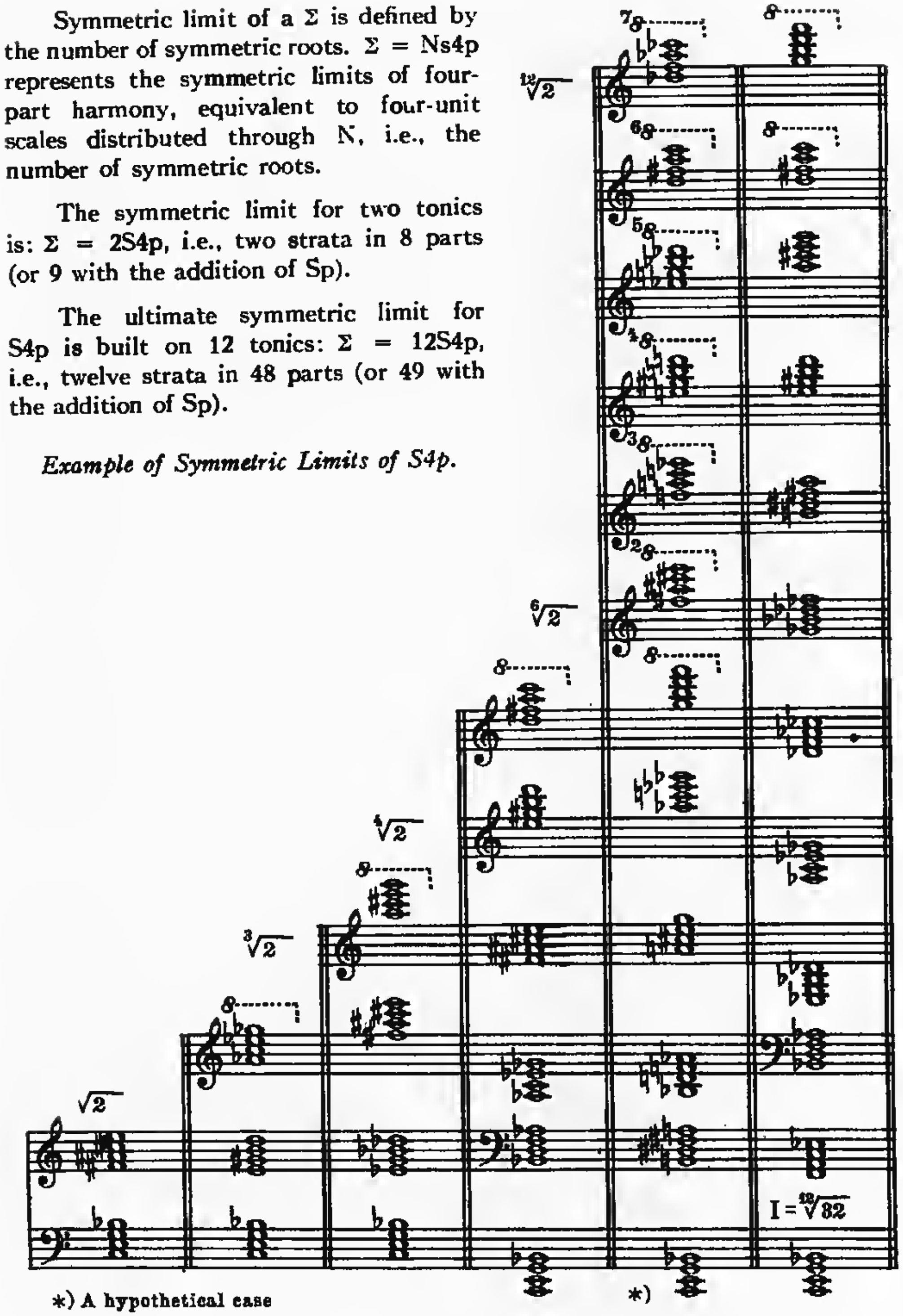
2. Symmetric Limit

Symmetric limit of a Σ is defined by the number of symmetric roots. Σ = Ns4p represents the symmetric limits of four-part harmony, equivalent to four-unit scales distributed through N, i.e., the number of symmetric roots.

The symmetric limit for two tonics is: Σ = 2S4p, i.e., two strata in 8 parts (or 9 with the addition of Sp).

The ultimate symmetric limit for S4p is built on 12 tonics: Σ = 12S4p, i.e., twelve strata in 48 parts (or 49 with the addition of Sp).

Example of Symmetric Limits of S4p.



*) A hypothetical case

Figure 67. Symmetric limits of S4p.

3. Compound Symmetric Limit: Σ (Σ S4p)

A compound symmetric limit depends upon the number of symmetric points from which each individual Σ is constructed. Thus, for example, the diatonic limit of a four-unit scale, being used as a Σ structure and being coordinated with another identical Σ from the $\sqrt{2}$, would produce a compound Σ(Σ) = 2Σ4S4p, i.e., two sigmae of four strata each, 16 parts to each sigma, making a total of 32 parts. The same structure, being coordinated through the $\sqrt[12]{2}$, would produce: Σ(Σ) = 12Σ4S4p, i.e., twelve sigmae, four strata each, 16 parts each: 12·4 = 48 strata; 16·12 = 192 parts. The same procedure being applied to a complete seven-unit scale would produce: Σ(Σ) = 12Σ7S4p, i.e., 12·7·4 = 336 parts.

The ultimate compound limit of S4p can be obtained from a twelve unit scale set through twelve points of symmetry: Σ (Σ) = 12Σ12S4p, i.e., 12·12·4 = = 576 parts in 144 four-part strata of the twelve sigmae (or 577 parts with the addition of Sp).

Such is the incredible number of parts possible within the twelve-unit equal temperament scale.

The practical uses of compound symmetric limits require overlapping and serve the purpose of alternate arrangement (distribution) of the superimposed orchestral or vocal groups, in the same way as was described for the compound limits of S2p.

Overlapping is unavoidable because of the limitations of auditory response to a certain frequency range. For this reason it would not be sensible to construct musical instruments (possible through electronics) *for musical purposes,* which exceed the range of audible pitch.

We shall limit the table of structures of the Σ(Σ) to a few practical illustrations. See Figure 68 on the following page.

Examples of $\Sigma\,(\Sigma)\,S4p$

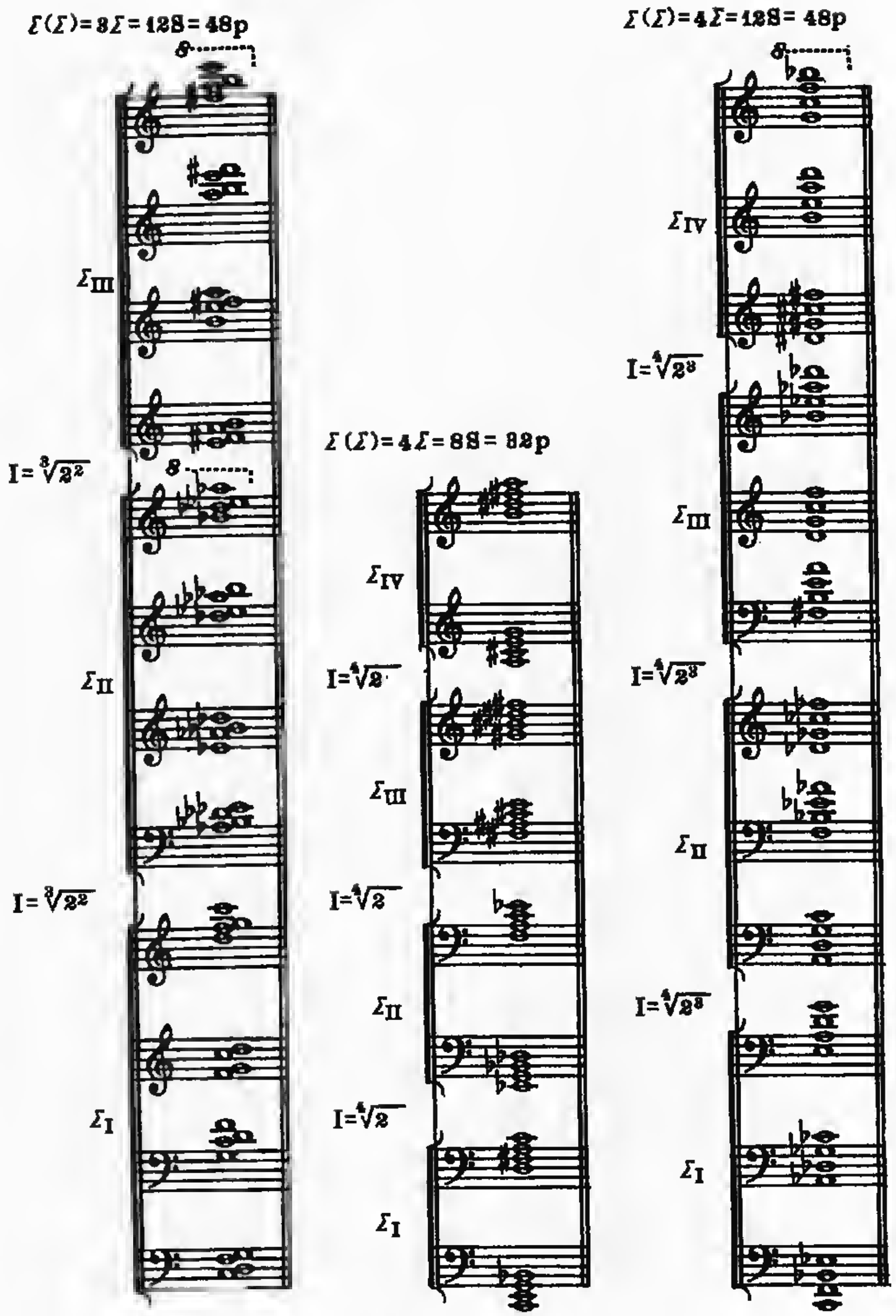


Figure 68. $\Sigma\,(\Sigma)S4p$.

CHAPTER 7

VARIABLE NUMBER OF PARTS IN THE DIFFERENT STRATA OF A SIGMA

As the main purpose of the *General Theory of Harmony* is to satisfy demands for the scoring of all possible combinations of instruments, or voices, or both, it should be flexible enough to make any instrumental combination practical.

If the score must, for some reason, consist of several orchestral groups represented by a different number of instruments in each group, harmony must be evolved for the corresponding number of strata and parts.

A score of 4 violins, 3 clarinets and 2 trombones, fundamentally, requires a Σ, where $S_I = 2S$, $S_{II} = 3S$ and $S_{III} = 4S$.

There are two ways of assigning instrumental combinations to harmony: (1) the fundamental way, where each instrument corresponds to one part, and (2) where the mobility of the instrumental form of a part defines the quantity of harmonic parts; in the latter conception one instrument produces: Sp, S2p, S3p or S4p.

The first form is illustrated by the above case of violins, clarinets and trombones. In the second form *one violin* may perform an instrumental form of 2, or 3, or 4 part harmony, depending on the degree of mobility required. For this reason, in cases where chords change at a low rate of speed and the instrumental form implies high mobility, it is desirable to evolve more than one harmonic part for one individual orchestral part (which may be an individual instrument like the clarinet, or a group-unison like that of the violas).

Later on these considerations will be developed into basic principles of the *Theory of Orchestration.* At present, we shall look upon this problem as a purely harmonic one: *correlation of strata into sigma in simultaneity and continuity.*

There is no specific order *per se* in which the number of parts in the various strata may be distributed. This means that the lower S may have only one or all four parts. The same is true for any other S. There may be denser harmonic assemblages in the upper register and more rarified in the middle or lower register, but the opposite is equally true.

So far as *types of structures* are concerned, there are several considerations which dictate the means of evolving sigmae:

(1) const. or var. E's as components of the strata structures;
(2) const. or var. E's as intervals between the strata structures;
(3) symmetric arrangement of the strata roots in the vertical monomial or group symmetry;
(4) identical or non-identical structures in the different symmetric arrangements of roots;
(5) different strata having a different number of parts;
(6) the mirror Σ (inversion of structures by means of an axis of symmetry).

Examples of the Types of Sigmae
(evolved through the above six classificatione)

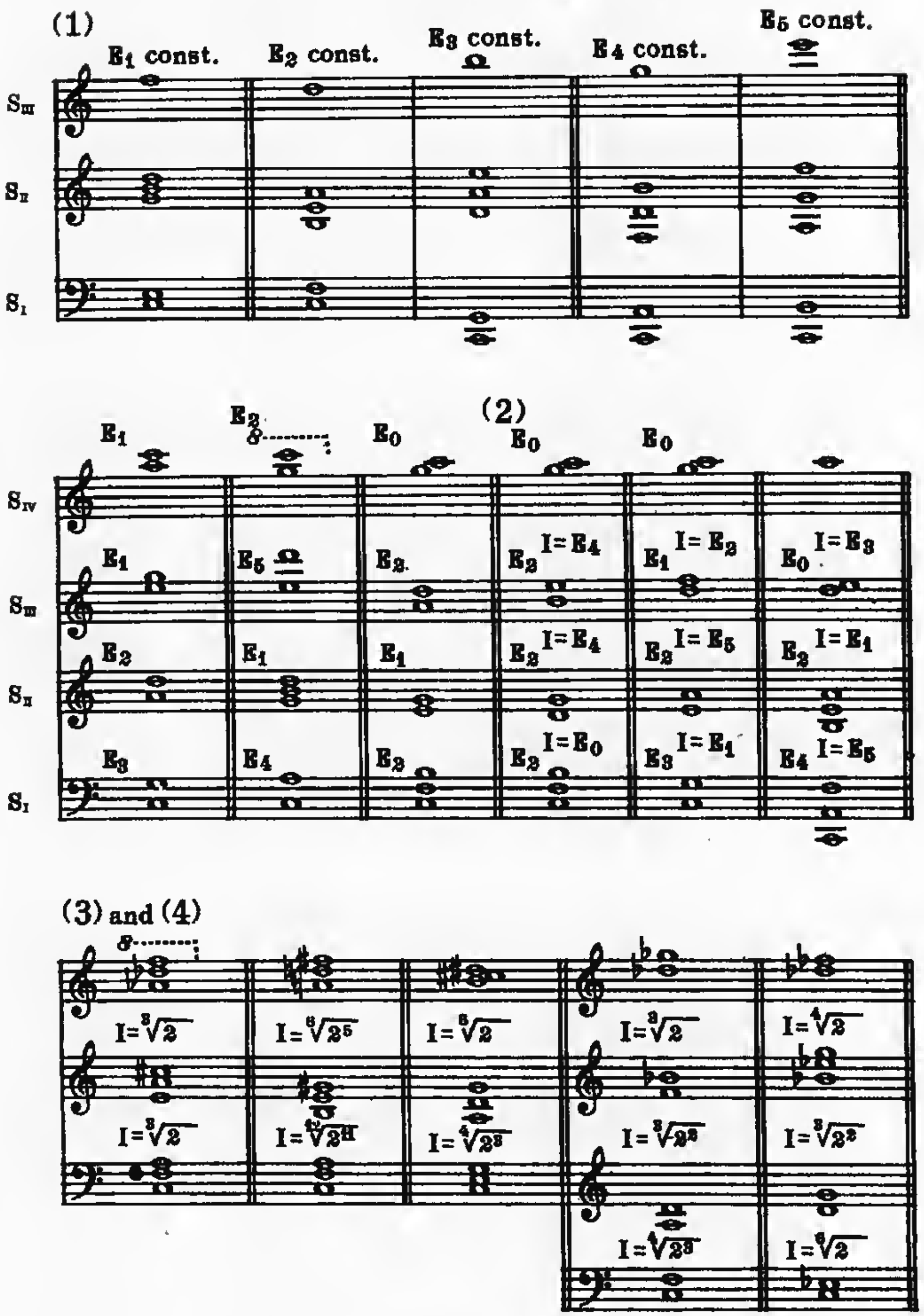


Figure 69. Types of sigmae (continued).

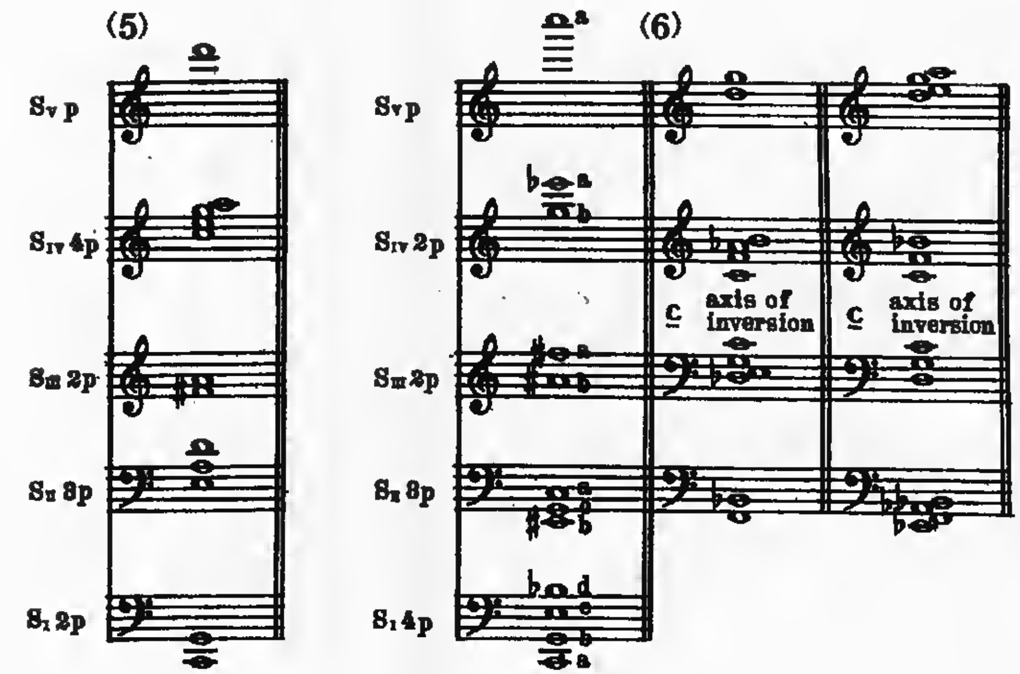


Figure 69. Types of sigmae (concluded).

Examples of Progressions of Σ with a Different Number of Parts in the Different Strata.

Figure 70. Progressions of Σ (continued).

Figure 70. Progressions of Σ (concluded).

A. Construction of Sigmae Belonging to One Family (Style)

1. $\Sigma = S$.

We shall consider $\Sigma = S$ as a special case of Σ. The structure of an assemblage representing a chord is defined by the interval-units (i) constituting such a structure. In the case of $\Sigma = S$ all structures belonging to one family are obtained by means of permutations of interval-groups of the original Σ. Thus a group of sigmae belonging to one family derive from the original Σ as permutation-groups. Therefore: $\Sigma_1 \equiv \Sigma p_0$, $\Sigma_2 \equiv \Sigma p_1$, $\Sigma_n \equiv \Sigma p_{n-1}$.

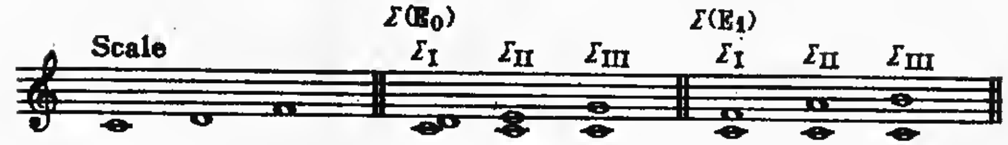
We have used this method already in evolving pitch-scales of one family through the permutation of intervals (see *Theory of Pitch-Scales*)* and have applied the same procedure to structures of E_1 (*Special Theory of Harmony*).** For this reason, there is really nothing essentially new in extending the same technique to all Sp, S2p, S3p and S4p structures.

In *diatonic classification*, all structures of one particular expansion *ipso facto* belong to one family, regardless of the number of parts. Thus Sp(E_0), S2p(E_0), S3p(E_0) and S4p(E_0) belong to one family even if their corresponding interval-groups are not identical. Likewise Sp(E_1), S2p(E_1). S3p(E_1) and S4p(E_1) belong to one family. The same is true of all other expansions.

*See Vol. I, p. 117 ff. **See Vol. I, p. 361.

Examples of the Diatonic Families of $\Sigma = S$

Sigmae of one Family



Sigmae of one Family

$\left.\begin{array}{l}\Sigma(E_0) \text{ and } \Sigma(E_3)\\ \Sigma(E_1) \text{ and } \Sigma(E_2)\end{array}\right\}$ are mutually reciprocating

One Σ to a Family

Mutually Reciprocating Pairs: $\Sigma(E_0)$ and $\Sigma(E_5)$; $\Sigma(E_1)$ and $\Sigma(E_4)$; $\Sigma(E_2)$ and $\Sigma(E_3)$.

Figure 81. Diatonic families of $\Sigma = S$.

In the *general classification* of $\Sigma = S$, Sp and S2p do not evolve any structural families: Sp has no interval to go by and S2p has one I(S), thus being invariant in each case. Families of triads (S3p) are based on permutations of two interval-groups in each Σ. Families of tetrads (S4p) are based on permutations of three interval-groups.

The number of families of triads equals the number of combinations of the interval-groups by two and not exceeding eleven semitones as a sum: $I \not> mi + ni$. There are 29 such families, 5 of which contain only one member (Σ).

The number of families of tetrads equals the number of combinations of the interval-groups by three and not exceeding eleven semitones as a sum: $I \not> mi + ni + pi$. There are 40 such families, 3 of which contain only one member (Σ).

Examples of Triads Belonging to One Family

(1) $I(\Sigma_1) = 2i + 3i$; $I(\Sigma_2) = 3i + 2i$.
(2) $I(\Sigma_1) = 5i + 3i$; $I(\Sigma_2) = 3i + 5i$.
(3) $I(\Sigma_1) = 4i + 3i$; $I(\Sigma_2) = 3i + 4i$.

Examples of Tetrads Belonging to One Family

(1) $I(\Sigma_1) = 3i + 3i + 2i$; $I(\Sigma_2) = 3i + 2i + 3i$;
$I(\Sigma_3) = 2i + 3i + 3i$.

(2) $I(\Sigma_1) = 2i + 3i + 5i$; $I(\Sigma_2) = 2i + 5i + 3i$;
$I(\Sigma_3) = 5i + 2i + 3i$; $I(\Sigma_4) = 3i + 2i + 5i$;
$I(\Sigma_5) = 3i + 5i + 2i$; $I(\Sigma_6) = 5i + 3i + 2i$.

The number of members of one family depends on the number of possible permutations of the interval-groups. If the number of interval-groups is one, there is but one member to a family. If the number of interval-groups is two, and both interval-groups are identical, there is but one member to a family. If both interval groups are non-identical, there are two members to a family. If the number of interval-groups is three, and they are all alike, there is but one member to a family. If the number of interval-groups is three, two of which are identical, there are three members to a family. If the number of interval-groups is three and all three are different, there are six members to a family. Full information on this matter is to be found in my book "Kaleidophone."*

Continuity of variable Σ can be composed from combinations of the members of one particular family, arranged in any desirable order and accompanied by coefficients of recurrence.

For instance: $\overrightarrow{\Sigma} = 2\Sigma_2 + \Sigma_1 + 3\Sigma_3$.

2. Σ = NS.

In a compound structure (Σ), ***all the substructures*** (S) ***and the intervals between the latter belong to one family.*** The different members of one family of compound structures have interval-groups in the substructures, identical with the corresponding original substructures, and interval-groups between the substructures, identical with that of the original compound structure. The difference between the various members of one particular family of the compound structures ***lies in the arrangement*** of the original interval-groups; this refers to both the substructures and the intervals between the latter.

It is assumed that the interval between the adjacent strata is either one of the interval-groups of the Σ or 0i (zero i). This 0i refers to the interval between the upper function of S placed immediately below the adjacent upper S and the lower function of the latter, or between the lower function of S placed immediately above the adjacent lower S and the upper function of the latter.

Examples:

$$(1)\quad \begin{array}{ll} S_{III}: & a \\ & \quad \Big\} \; I = 0i \\ & \underline{c} \\ S_{II}: & \underline{b} \\ & a \\ & \quad \Big\} \; I = 0i \\ S_{I}: & \underline{b} \\ & a \end{array} \qquad\qquad (2)\quad \begin{array}{ll} & \underline{c} \\ S_{II}: & \underline{b} \\ & a \\ & \quad \Big\} \; I = 0i \\ & \underline{d} \\ S_{I}: & \underline{c} \\ & \underline{b} \\ & a \end{array}$$

*Published by M. Witmark and Sons, 1940.

We shall evolve now a family of compound structures in which the *Master-Structure* (the original structure) is represented by $I(S) = 5i + 3i + 3i$.

The compound interval-group, which, in this case consists of the variants of three permutations, offers $\Sigma = 3S$ as the most natural solution. From the original Master-Structure we evolve the Compound Master-Structure in three substructures, in which the adjacent functions of adjacent strata are connected by $I = 0i$.

The following is a complete table of the members of this family: $I(S) = 5i + 3i + 3i$.

All numbers express interval-groups.

Σ_1 (Master)	Σ_2	Σ_3	Σ_4	Σ_5	Σ_6	Σ_7	Σ_8	Σ_9
3	3	5	3	3	3	5	3	3
3	5	3	5	3	3	3	3	3
5	3	3	3	5	5	3	5	5
— 0	— 0	— 0	— 0	— 0	— 0	— 0	— 0	— 0
3	3	5	3	3	3	3	5	3
3	5	3	3	5	3	3	3	3
5	3	3	5	3	5	5	3	5
— 0	— 0	— 0	— 0	— 0	— 0	— 0	— 0	— 0
3	3	5	3	3	3	3	3	5
3	5	3	3	3	5	3	3	3
5	3	3	5	5	3	5	5	3
Σ_{10}	Σ_{11}	Σ_{12}	Σ_{13}	Σ_{14}	Σ_{15}	Σ_{16}	Σ_{17}	Σ_{18}
5	3	3	3	3	3	5	3	3
3	5	5	3	5	5	3	3	3
3	3	3	5	3	3	3	5	5
— 0	— 0	— 0	— 0	— 0	— 0	— 0	— 0	— 0
3	5	3	3	3	3	3	5	3
5	3	5	5	3	5	3	3	3
3	3	3	3	5	3	5	3	5
— 0	— 0	— 0	— 0	— 0	— 0	— 0	— 0	— 0
3	3	5	3	3	3	3	3	5
5	5	3	5	5	3	3	3	3
3	3	3	3	3	5	5	5	3

This table is continued on the following page.

Σ_{19}	Σ_{20}	Σ_{21}	Σ_{22}	Σ_{23}	Σ_{24}	Σ_{25}	Σ_{26}	Σ_{27}
3	3	3	5	3	3	5	3	3
5	3	3	3	5	5	3	3	3
3	5	5	3	3	3	3	5	5
— 0	— 0	— 0	— 0	— 0	— 0	— 0	— 0	— 0
3	3	3	3	5	3	3	5	3
3	5	3	5	3	3	3	3	5
5	3	5	3	3	5	5	3	3
— 0	— 0	— 0	— 0	— 0	— 0	— 0	— 0	— 0
3	3	3	3	3	5	3	3	5
3	3	5	3	3	3	5	5	3
5	5	3	5	5	3	3	3	3

This table has four more variants as the interval-groups between the adjacent strata offer the following possibilities:

$\frac{0}{0}$	$\frac{3}{3}$	$\frac{5}{3}$	$\frac{3}{5}$	$\frac{5}{5}$

The total number of Σ — members of this family is: $N(S = 5+3+3) = 27 \cdot 5 = 135$.

It is hardly necessary, or desirable, to use such an enormous number of structures in one musical composition. Two or three members are perfectly sufficient for such a purpose. It is equally true, however, that one such family may constitute ***the life-time harmonic manifold of one composer***, expressing himself in one harmonic style. The harmonic vocabulary of such a composer would positively dwarf that of Bach, Beethoven and Wagner put together.

(The Master Σ) Some examples from the table

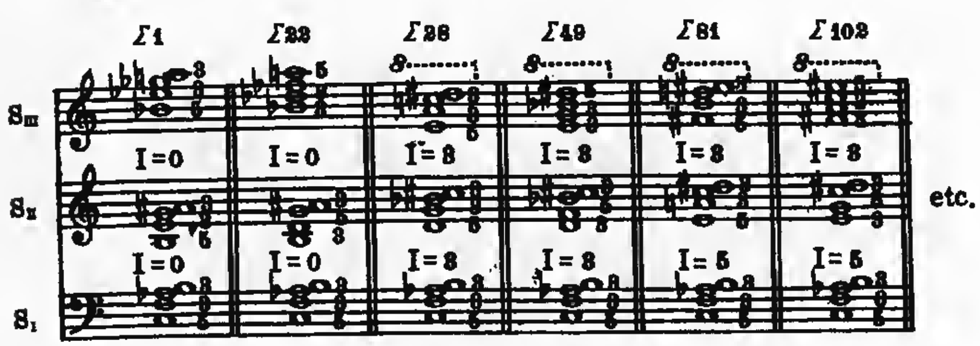


Figure 82. Some examples from table of Master structure.

B. Progressions with Variable Sigma

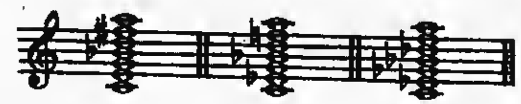
Different sigmae belonging to one family, which can be used in one harmonic continuity, may have a different number of parts in each stratum, but the number does not vary for each individual stratum. For instance:

$$\Sigma_1 = 4S;\ S_I = p,\ S_{II} = 4p,\ S_{III} = 3p,\ S_{IV} = 2p;$$
$$\Sigma_2 = 4S;\ S_I = p,\ S_{II} = 4p,\ S_{III} = 3p,\ S_{IV} = 2p;$$
$$\Sigma_3 = 4S;\ S_I = p;\ S_{II} = 4p,\ S_{III} = 3p,\ S_{IV} = 2p;\ \ldots$$

We shall now present this case in harmonic progression.

Example of a Progression with Variable Sigma and a Different Number of Parts in the Different Strata.

Selective Group of One Family:



Strata Distribution

Progression: $2\Sigma_1+\Sigma_3+\Sigma_1+2\Sigma_2$

Figure 83. Progression with variable sigma.

C. Distribution of a Given Harmonic Continuity Through Strata

Strata arrangement of a given harmonic continuity serves as an auxiliary technique for the orchestration of music which has been already written. The common notion of assigning parts existing in the sketch of a composition directly to the instruments and groups of the orchestra is rather primitive. Since the average sketch contains 2, 3 or 4 and seldom 5 parts, and an average orchestral score contains between 20 and 30 parts, it is no wonder that there is so much duplication of parts. Many instruments are compelled to play identical notes because of the composer's lack of mathematical judgment. Such scores lack acoustical clarity and consume an enormous amount of rehearsing time in order to be made to sound acceptable.

Many prominent composers of the past and present have felt the necessity of individualizing the orchestral parts in a score. Not all of them solved this problem with success. In Mozart's scores we witness a tendency toward rhythmic independence of the duplicated parts, attained by the variation of instrumental forms. In Wagner, the struggle for the individualization of orchestral parts is often achieved by the technique which I call "contrapuntal variation of harmony". But since the advent of the so-called "French Impressionists" (Debussy, Roger-Ducasse, Delage, Ravel), the individualization of orchestral parts by means of *segregation of the harmonic groups* has become a prominent tendency of orchestral writing.

A student of this system can compose his scores directly in harmonic strata. However, whether he plans to use it for the purpose of composing or not, it is necessary for him to know, for the purpose of orchestrating, how to convert a given part-continuity into strata, or how to convert his own sketch of an arrangement into strata, prior to scoring it for the instrumental or vocal combination of parts.

The greater the number of parts in the original continuity, the greater the number of strata which can be developed therefrom. For this reason, if a given harmonic continuity contains too few parts to permit development into the number of parts required by the selection of a larger orchestral combination, it becomes necessary to add one or two more parts to the original harmonic progression before converting it into strata. The selection of functions which are to be added is a matter of harmonic style. But, in the field of transcriptions, paraphrases and arrangements, one style has not infrequently been transformed into another.

Inspecting the trends of existing music, we find that the development of harmony from the few parts into many, which happened in the course of the past few centuries, has relied on two fundamental devices:

(1) the acceptance of auxiliary tones as chordal functions;

(2) the addition of new chordal functions and groups.

Both devices undoubtedly derived from alternation of an auxiliary unit with the respective chordal function (tremolo, legato, trill). In slow motion the auxiliary units often formed suspended tones which later crystallized into chordal functions. In fast motion, alternation of the groups of auxiliary units with chordal functions produced psychological continuity of two *superimposed assemblages*. This gave birth to the simultaneous harmonic polytonality used intentionally by Stravinsky, Malipiero and myself. Facts reveal the systematic use of the strata technique, which I introduced in the United States about 1931, soon became the favorite style in the field of radio and motion-picture music. The sparkling quality of orchestrations, which can be immediately detected by listeners, is due primarily to harmonic factors: reharmonization and strata.

A bold example of the first device (crystallization of auxiliary units into chordal functions) is the cadence in the first movement of Prokofiev's *Piano Sonata* No. 5, which cadence sounds Mozartian when the auxiliary units are discarded.

The chordal functions of a given harmonic continuity (including the new functions or groups of functions, if such have been added) must be assigned before the original continuity is converted into strata. This is particularly important when the original continuity has a variable Σ. As the number of parts in the original remains constant, there is a constant set of letters corresponding to chordal functions. A function may change structurally in its interval value in relation to other functions, yet its functional meaning in the entire Σ assemblage remains constantly represented by the same letter.

For example, $\Sigma_1 S(5) = 1, 3, 5, 13$ and $\Sigma_2 S(7) = 1, 3, 5, 7$ can both be represented by the same assemblage of functions $\Sigma S = a, b, c, d$. However, while the a, b and c functions retain their structural meaning [which in this case is the diatonic $S(E_1)$] of 1, 3 and 5 respectively, function d changes its structural meaning, being the seventh in Σ_2 and the thirteenth in Σ_1. For this reason, transformations in the respective strata are performed by their functional and not by their structural meaning.

The superimposition of a whole new assemblage upon a given one (symmetric superimposition of strata) is equivalent to *harmonization of harmony by another harmony*. While the original sequence of assemblages in this case remains intact, the new added assemblage usually attributes a new quality, in which the original ingredient is still perceptible and yet appears as if it has been differently flavored. Its presence is often detected as timbre and not harmony. This explains why, in scores evolved through strata, the listener often mistakes harmony for orchestration.

Translation of a Given Harmonic Continuity into Strata.
Examples

The Original Progression with Functions Deciphered.

Transcription into Strata: $\Sigma = 4S = 8p$

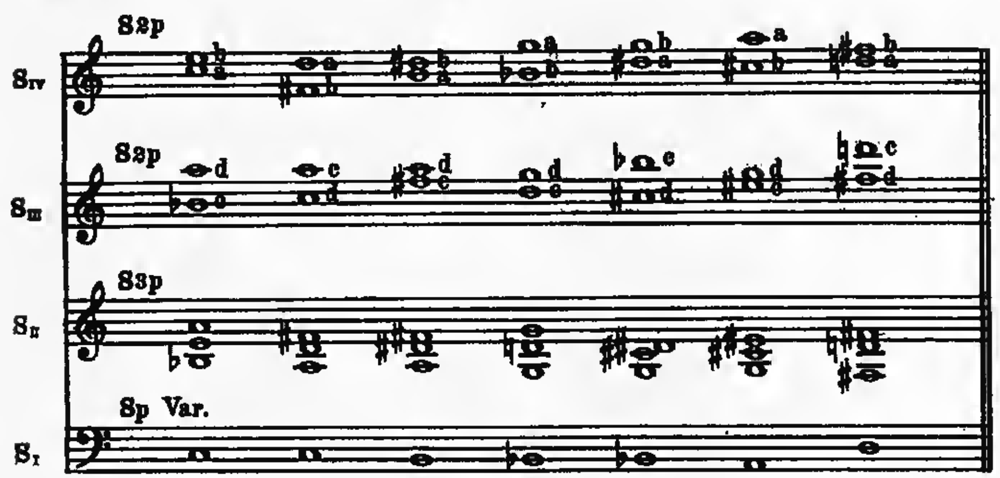


Figure 84. Harmonic continuity into strata. $\Sigma = 4S = 8p$.

The Original Progression with a New Function Added.

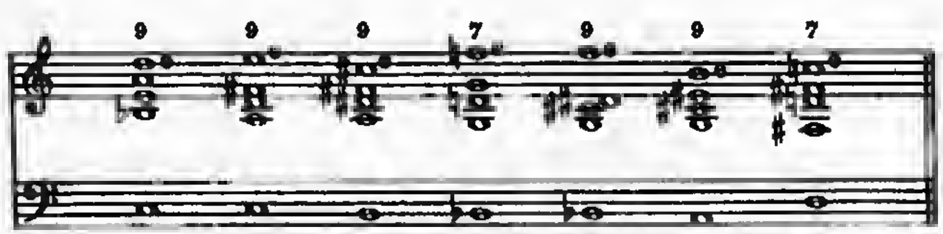


Figure 85. Harmonic continuity into strata. $\Sigma = 6S = 11p$ *(continued).*

Transcription into Strata: $\Sigma = 6S = 11p$.

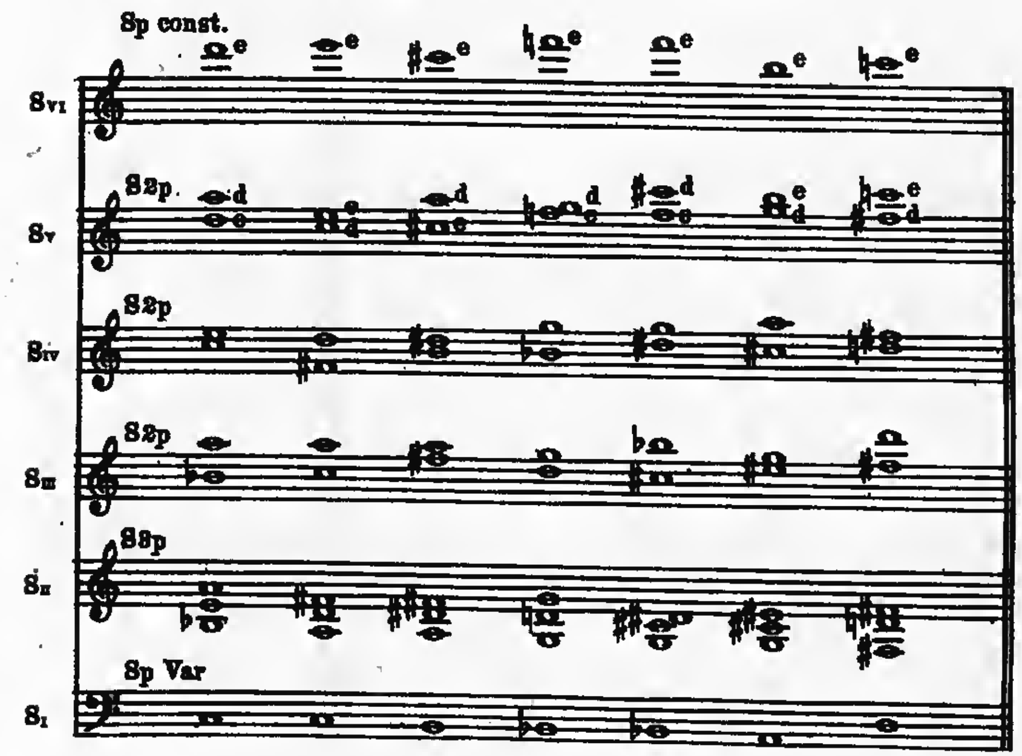
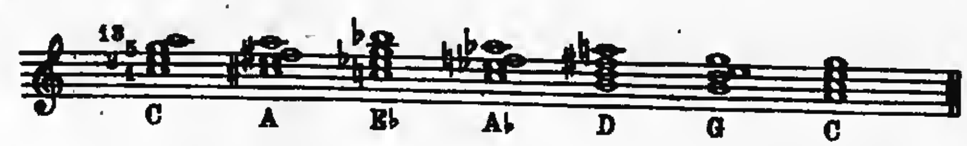


Figure 85. Harmonic continuity into strata $\Sigma = 6S = 11p$ *(concluded).*

The Original: $\Sigma = S4p$.



Harmonization of the Given Harmonic Stratum: $\Sigma = 2S = 7p$.

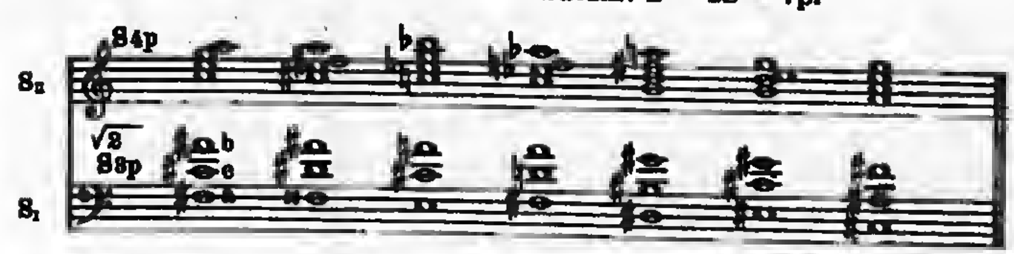


Figure 86. Harmonic continuity into strata. $\Sigma = S4p$ *(continued).*

Further Strata Development: $\Sigma = 4S = 10p$.

Addition of Two Harmonic Strata to the Original: $\Sigma = 5S = 12p$.

Figure 86. Harmonic continuity into strata. $\Sigma = S4p$ (concluded).

CHAPTER 8

GENERAL THEORY OF DIRECTIONAL UNITS

(Melodic Figuration)

ACCORDING to our analysis in the field of the Special Theory of Harmony, passing, suspended and anticipated units actually belong to the assemblage, i.e., they represent either a function present in the chord, or a function which is a potential unit of the sigma. Suspended and anticipated units can be obtained by mere rhythmic variation of harmony which we shall discuss in full detail in the Theory of General ("*Textural*") Composition.* Chromatic passing units are always to be regarded as elements (to be inserted *a posteriori*) of chromatic variation, applicable to any type of harmonic progression.

This leads us to the conclusion that the only authentic element of melodic figuration is the ***auxiliary unit.*** The latter is not bound to bear any relation either to Σ, or to any substructure of it. An auxiliary unit is selected to be the leading tone to a chordal function. The interval of the ***leading unit*** from the respective chordal function is limited by the arrangement of the adjacent chordal functions of one S. In the structures of wide expansions it may be 3i, 4i or even greater. However, for practical reasons it is advisable not to exceed $I = 2i$, as habits, partly inherited and partly developed of listeners, obstruct the association of remote pitch-units as leading tones. Our charts, for this reason, will be limited to two forms: $I = i$ and $I = 2i$.

From the viewpoint of melodic figuration, chordal functions will be considered ***neutral units*** and the auxiliaries will be considered ***leading units.*** The combination of both, developed into any repetitive form, will be considered a ***directional unit.***† A directional unit may start with either the neutral or the leading unit, but it must end with the neutral unit.

Thus the study of melodic figuration as a branch of the *General Theory of Harmony* is confined to the study of directional units in the various forms of S and the coordination of assemblages containing directional units.

A. Directional Units of Sp

(a, $a_{\rightarrow}$, $a^{\rightarrow}$)

Considering the neutral unit to be a special case of directional units we obtain the following three forms: a, $a_{\rightarrow}$ and $a^{\rightarrow}$, i.e., the neutral unit, the directional with the lower leading unit (ascending auxiliary) and the directional with the upper leading unit (descending auxiliary).

*See p. 1305.

†See **Kaleidophone** by Joseph Schillinger, M. Witmark and sons, 1940. Revised edition, 1945. (Ed.)

The last two forms may have the interval of ascending of i or 2i and the interval of descending also of i or 2i. Thus there are four forms of directional units of Sp:

(1) ╱ i : $\underline{b} \to \underline{c}$; (3) ╲ i : $\underline{d}\flat \to \underline{c}$
(2) ╱ 2i : $\underline{b}\flat \to \underline{c}$; (4) ╲ 2i : $\underline{d} \to \underline{c}$

Illustrations of Directional Units of Sp

Original: $2C_7 + C_{-8} + C_{-7} + C_8 + C_5$

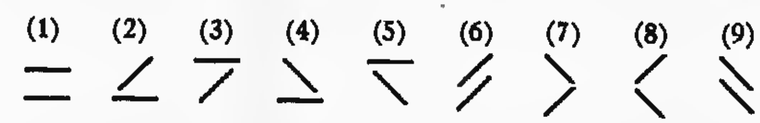
Figure 87. Directional units of Sp.

B. Directional Units of S2p

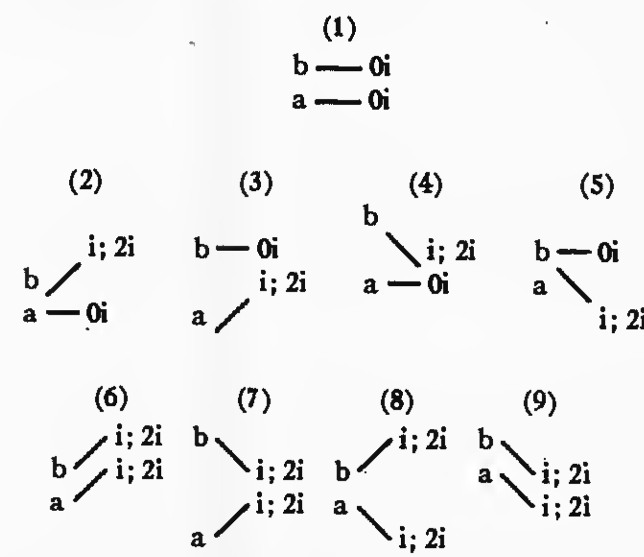
(a, b, $a_{\rightarrow}$, $b_{\rightarrow}$, $a^{\rightarrow}$, $b^{\rightarrow}$)

In tabulating the directional units and groups of S2p we shall resort to geometrical representation: neutral unit: —; ascending directional unit: ╱ and descending directional unit: ╲.

Using the terminology of the *Theory of Melody*, we can call these three forms: 0, a and b respectively. The three forms in combinations by 2, corresponding to S2p, give: $3^2 = 9$, as each form is combined with itself and with the two remaining forms. The first of these 9 forms represents neutral units in both parts.



The first form has no interval variation. The second, the third, the fourth and the fifth forms have an interval variation in one part. The remaining forms have an interval variation in both parts.



Thus the total number of directional units for any S2p is: one for (1), eight for (2 − 5) and sixteen for (6 − 9), since each of the latter has four variations, i.e., $1 + 8 + 16 = 25$.

In some structures, whose own interval-group is small (they usually belong to E_0 and seldom to E_1), some directional units containing inward motion have to be excluded in order to avoid crossing of the parts. In diatonic progressions, the semitonal precision of directional intervals is not compulsory.

An Exemplary Table of Directional Units Evolved to I(S2p) = 4i

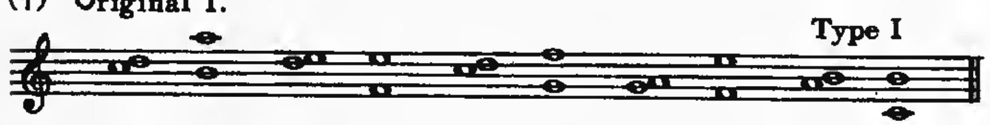
Figure 88. Directional units of I(S2p) = 4i.

Each of the 25 forms of directional units has its own distinct character. Various forms can be selected as a continuity-group of directional units, where each selected form has a definite coefficient of recurrence. It is very desirable, however, to restrict each case to one form, as only such a limitation guarantees perfect unity of style. For this reason our examples will be confined to one form at a time. The following variations should be considered as samples of different styles, and *not* as one sequence.

It is obvious that the directional units must be assigned to each structure when more than one structure is employed.

Directional Harmonic Continuity of S2p and Hybrid (through addition of Sp)

(1) Original I.



(2) Var. I. Form (7): $\frac{b\to}{a\to}$.

(3) Var. II. Form (8) and (4): $a\to + b\to$.

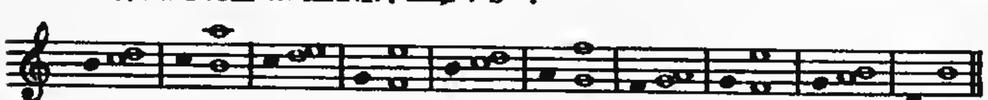


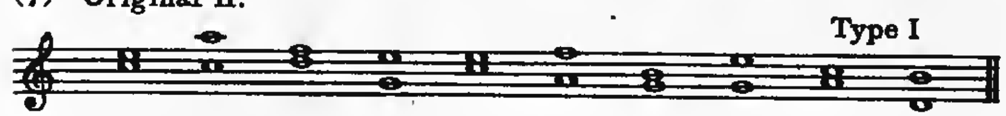
Figure 89. Directional harmonic continuity of S2p (continued).

(4) Original I. I(8) = 2i.

(5) Var. I. Form (7): $\frac{b\to i}{a\to i}$

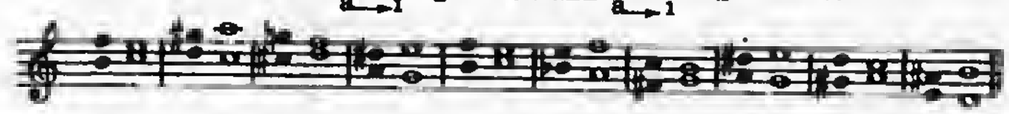
(6) Var. II. Form (7): $\frac{b\to i}{a\to 2i}$

(7) Original II.



(8) Var. I. Form (7): $\frac{b\to}{a\to}$.

(9) Var. II. Form (7): $\frac{b\to i}{a\to i}$ S_1 (I = 4i) and $\frac{b\to 2i}{a\to i}$ S_2 (I = 8i).



(10) Original II. I (8) = 4i.

(11) Var. I. Form (6): $\frac{b\to i}{a\to i}$.

Figure 89. Directional harmonic continuity of 2Sp (continued).

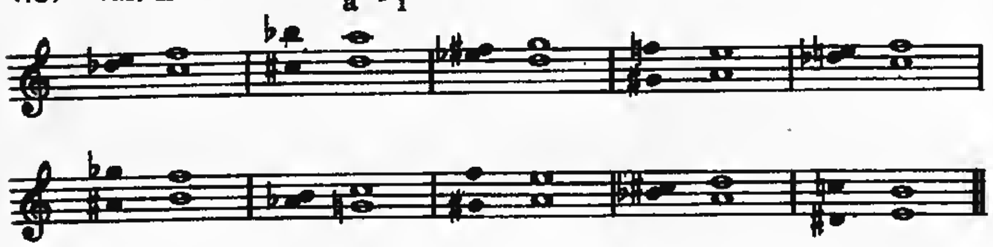
(12) Var. II. Form (8): $\frac{h \to i}{a \to i}$.

(13) Orignal III.

Type I

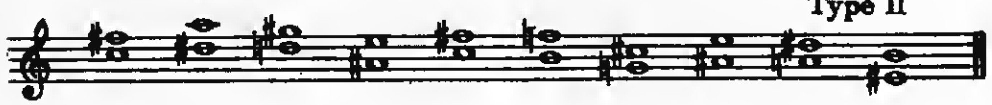
(14) Var. I. Form (8): $\frac{b \to}{a \to}$.

(15) Var. II. Form (8): $\frac{b \to i}{a \to i}$.



(16) Original III.

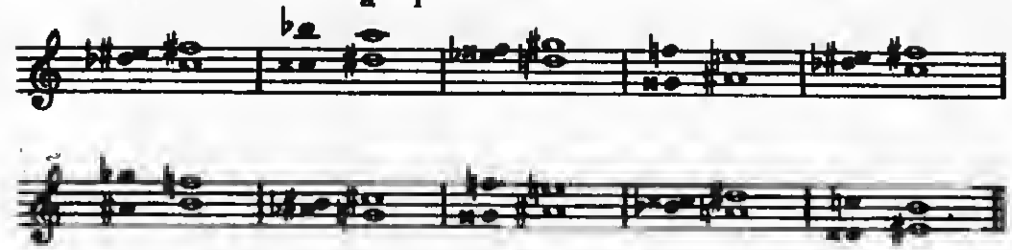
Type II



(17) Var. I. Form (6): $\frac{b \to i}{a \to i}$.

Figure 89. Directional harmonic continuity of S2p (continued).

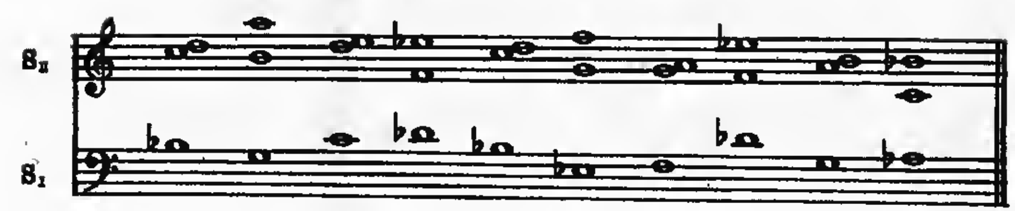
(18) Var. II. Form (8): $\frac{h \to i}{a \to i}$.



$\Sigma = 2S;\ S_I = p;\ S_{II} = 2p$

Figure 89. Directional harmonic continuity of S2p (concluded).

Original: (4) with added Sp.



Var.: (5) Form (7): $S_{II}\ \frac{b \to i}{a \to i}$; $S_I\ a \to 2i$.

Original: (10) with added Sp.

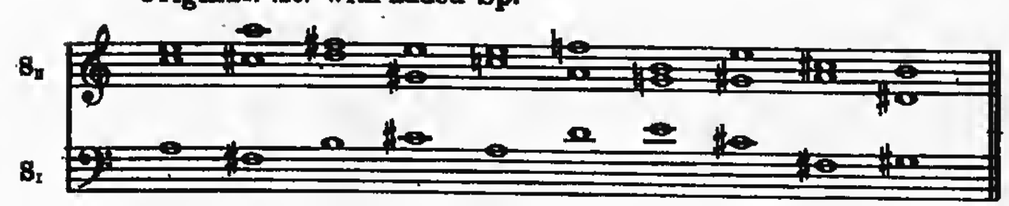


Figure 90. Hybrid harmonic continuity; addition of Sp to S2p (continued).

Var.: (11). Form (6): $S_{II}\ \frac{b_{\rightarrow} i}{a_{\rightarrow} i}$; $S_I\ a \rightarrow i$.

Original: (16) with added Sp.

Var.: (17) Form (6): $S_{II}\ \frac{b_{\rightarrow}}{a_{\rightarrow}}$; $S_I\ a \rightarrow i$.

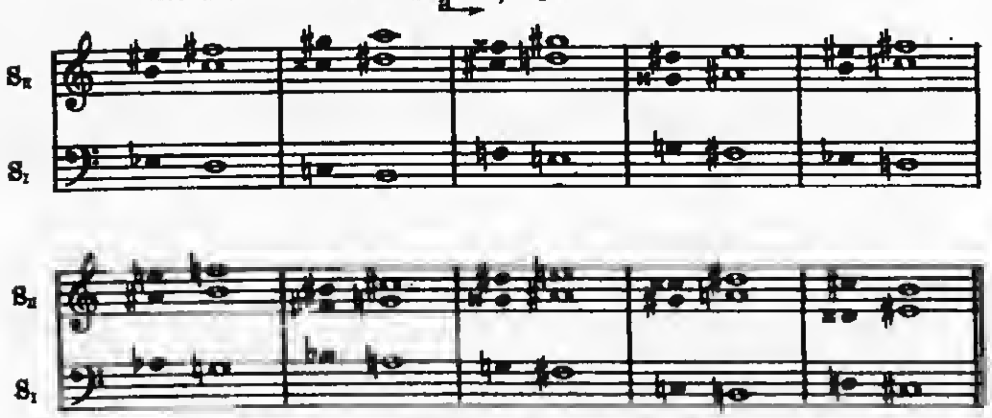


Figure 90. Hybrid harmonic continuity; addition of Sp to S2p (concluded).

Progression of mixed structures

$\Sigma^{\rightarrow} = 2\Sigma_1 + \Sigma_2 + \Sigma_1 + 2\Sigma_3.$
$C^{\rightarrow} = 2C_3 + C_7 + C_5 + C - 7.$
Scale of Roots = Nat. major d_4.

Original

Variation

Figure 91. Progression of mixed structures.

C. Directional Units of S3p

$(a, b, c, a_{\rightarrow}, b_{\rightarrow}, c_{\rightarrow}, a^{\rightarrow}, \overrightarrow{b}, c^{\rightarrow})$

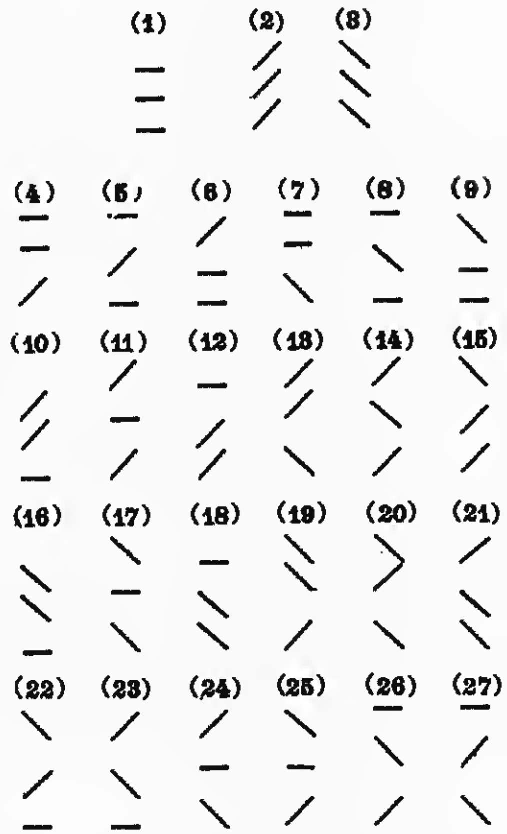
Directional units of S3p consist of the combinations of 0, a and b. As each form is combined with itself as well as with other forms, the total number of the forms of directional and neutral units is: $3^3 = 27$.

For convenience, these forms can be arranged into groups with three identical elements, with two identical elements and with no identical elements.

Table of Directional and Neutral Units of S3p

0 0 0	a a a	b b b

0 0 a	0 a 0	a 0 0	0 0 b	0 b 0	b 0 0
a a 0	a 0 a	0 a a	a a b	a b a	b a a
b b 0	b 0 b	0 b b	b b a	b a b	a b b
b a 0	a b 0	a 0 b	b 0 a	0 b a	0 a b



Of these 27 forms, (1) has no semitonal variations; the (2) and (3) have 8 variations each (i and 2i participating in each part); the (4), (5), (6), (7), (8) and (9) have 2 variations each; the (10), (11) and (12) have 4 variations each; the (13), (14) and (15) have 8 variations each; the (16), (17) and (18) have 4 variations each; the (19), (20) and (21) have 8 variations each; the remaining 6 forms have 4 variations each.

Thus the 27 forms together with the possible i and 2i variations produce 125 forms of intonation for each given S3p structure.

$1;\ 2\cdot 8 = 16;\ 6\cdot 2 = 12;\ 3\cdot 4 = 12;\ 3\cdot 8 = 24;\ 3\cdot 4 = 12;\ 3\cdot 8 = 24;\ 6\cdot 4 = 24.$
$1 + 16 + 12 + 12 + 24 + 12 + 24 + 24 = 125$

An Exemplary Table of Directional Units Evolved to $I(S3p) = 4i + 3i$

One movement 12 cases

Figure 92. Directional units. $I(S3p) = 4i + 3i$. One movement.

Two movements 48 cases

Figure 93. Directional units. $I(S3p) = 4i + 3i$. Two movements.

Three movements 64 cases

Figure 94. Directional units. $I(S3p) = 4i + 3i$. Three movements.

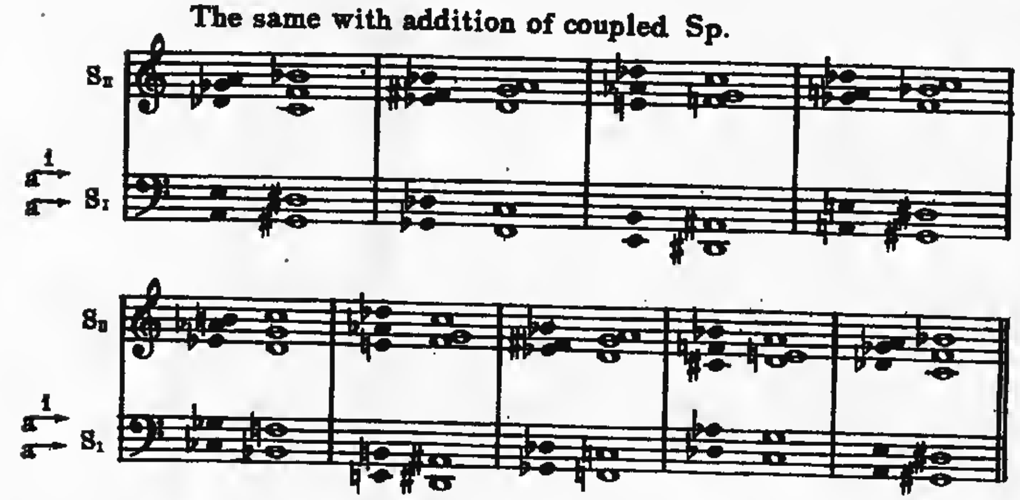
Similar tables can be devised for any other S3p structure. In spite of the abundance of resources, the composer will do well to assign just one combination to each structure used in a certain continuity. When the structures of one continuity differ in their form, an individual directional group must be assigned to each structure.

Examples of Application of Directional Units to S3p Progressions

Original

Var. = a→, b→, c→.

The same with addition of coupled Sp.



Original

Figure 95. Directional units to S3p progressions (continued).

Figure 95. Directional units to S3p progressions (concluded).

D. Directional Units of S4p

$(a, b, c, d, a_{\rightarrow}, b_{\rightarrow}, c_{\rightarrow}, d_{\rightarrow}, a^{\rightarrow}, b^{\rightarrow}, c^{\rightarrow}, d^{\rightarrow})$

Directional units of S4p consist of the combinations of 0, a and b. As each form is combined with itself as well as with all other forms, the total number of the forms of directional and neutral units is: $3^4 = 81$.

In the case of variable Σ, it is necessary to assign the directional units to each Σ individually.

Table of Directional and Neutral Units of S4p

0	a	b
0	a	b
0	a	b
0	a	b

0	0	0	a	0	0	0	b	a	a	a	0	a	a	a	b	b	b	b	0	b	b	b	a
0	0	a	0	0	0	b	0	a	a	0	a	a	a	b	a	b	b	0	b	b	b	a	b
0	a	0	0	0	b	0	0	a	0	a	a	a	b	a	a	b	0	b	b	b	a	b	b
a	0	0	0	b	0	0	0	0	a	a	a	b	a	a	a	0	b	b	b	a	b	b	b

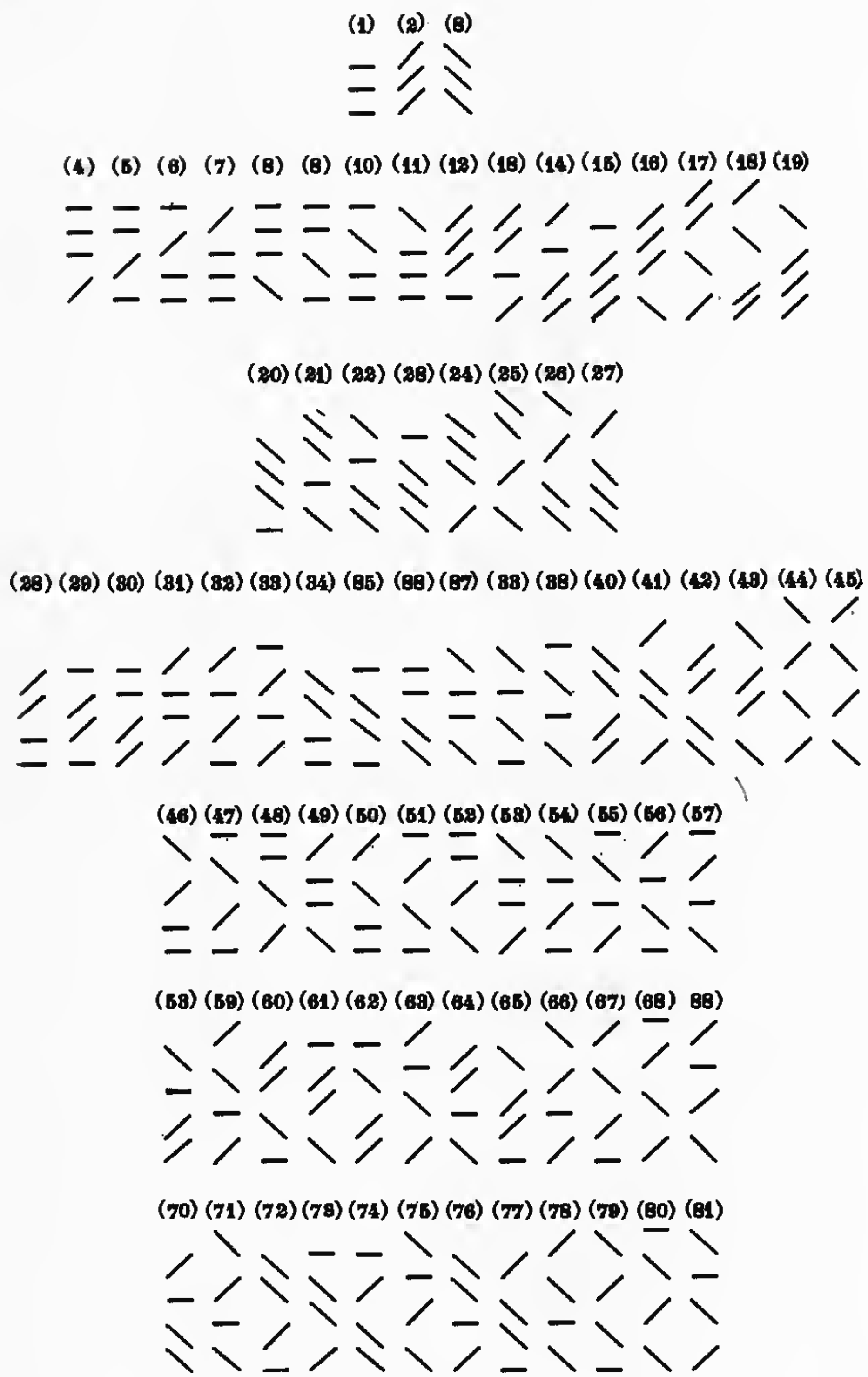
a	0	0	a	a	0	b	0	0	b	b	0	b	a	a	b	b	a
a	a	0	0	0	a	b	b	0	0	0	b	b	b	a	a	a	b
0	a	a	0	a	0	0	b	b	0	b	0	a	b	b	a	b	a
0	0	a	a	0	a	0	0	b	b	0	b	a	a	b	b	a	b

b	0	0	a	a	0	0	b	b	0	a	0
a	b	0	0	b	a	0	0	0	b	0	a
0	a	b	0	0	b	a	0	a	0	b	0
0	0	a	b	0	0	b	a	0	a	0	b

b	a	a	0	0	a	a	b	b	a	0	a
0	b	a	a	b	0	a	a	a	b	a	0
a	0	b	a	a	b	0	a	0	a	b	a
a	a	0	b	a	a	b	0	a	0	a	b

a	b	b	0	0	b	b	a	a	b	0	b
0	a	b	b	a	0	b	b	b	a	b	0
b	0	a	b	b	a	0	b	0	b	a	b
b	b	0	a	b	b	a	0	b	0	b	a

See graph presentation of above table on the following page.



Semitonal variations occurring in one p:

i	2i

2 variations

Semitonal variations occurring in two p's:

i	2i	i	2i
i	i	2i	2i

4 variations

Semitonal variations occurring in three p's:

i	i	i	2i	2i	2i	i	2i
i	i	2i	i	2i	i	2i	2i
i	2i	i	i	i	2i	2i	2i

8 variations

Semitonal variations occurring in four p's:

i	i	i	i	2i	2i	i	i	2i	2i	i	2i	2i	2i	i	2i
i	i	i	2i	i	2i	2i	i	i	i	2i	2i	2i	i	2i	2i
i	i	2i	i	i	i	2i	2i	i	2i	i	2i	i	2i	2i	2i
i	2i	i	i	i	i	i	2i	2i	i	2i	i	2i	2i	2i	2i

On the basis of the above table we find that the 81 forms, tabulated on pages 1183-4, produce the following number of semitonal variations:

(1) has 1 form;
(2) and (3) have . . . . 16 variations each
(4 — 11) have . . . . 2 " "
(12 — 15) have . . . . 8 " "
(16 — 19) have . . . . 16 " "
(20 — 23) have . . . . 8 " "
(24 — 27) have . . . . 16 " "
(28 — 39) have . . . . 4 " "
(40 — 45) have . . . . 16 " "
(46 — 57) have . . . . 4 " "
(58 — 81) have . . . . 8 " "

By multiplying the number of forms in each subdivision by its respective number of variations, we find the following:

1		
2·16	=	32
8·2	=	16
4·8	=	32
4·16	=	64
4·8	=	32
4·16	=	64
12·4	=	48
6·16	=	96
12·4	=	48
24·8	=	192

Total: $1 + 32 + 16 + 32 + 64 + 32 + 64 + 48 + 96 + 48 + 192 = 625$

The latter figure represents the number of possibilities for each S4p structure.

It is easy to evolve musical tables for any of the 165 possible S4p structures, taken in any of their three expansions (E_0, E_1 and E_2), by following the chart of 81 forms and the table of semitonal variations.

It is interesting to learn that the manifold of structures of S4p supplied with all the possible directional units produces: $165 \cdot 3 \cdot 625 = 309{,}375$ forms of the *c* -chord.

Examples of Application of Directional Units to S4p Progressions

Figure 96. Directional units of S4p progressions (continued).

Figure 96. Directional units of S4p progressions (concluded).

E. Strata Composition of Assemblages Containing Directional Units

Selection of directional units for a Σ depend on several factors:

(1) whether the number of parts is the same or different in the different strata;

(2) if the number of parts is the same in the different strata, it depends on whether the structures in the different strata are identical or not;

(3) whether it is desirable in each individual case to neutralize or to single out the character of the directional units in the different strata.

In case No. 3, the predominant characteristics of the groups of directional units accompanying assemblages are: the *identity* and the *reciprocity* of the patterns. The identity can be carried out with diatonic (with the precision up to 2i) or with general (with the precision up to i), i.e., chromatic precision. Reciprocity can be achieved by means of the axis of inversion. The axis of inversion of a Σ is located at the level of $\frac{\Sigma}{2}$. For example, if a $\Sigma = 2S3p$, it means that $\Sigma = 6p$. Hence $\frac{\Sigma}{2} = \frac{6}{2} = 3$, i.e., the axis is between the two strata.

Under the conditions of such reciprocity, the axis-inversion yields a symmetric arrangement of the directional units throughout the sigma.

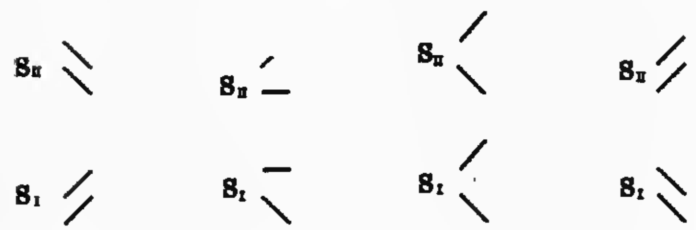
If the number of parts in a sigma is: $\Sigma = 2np + 1$ (i.e., an odd number), the part representing the center of the vertical arrangement in a sigma, becomes the axis of inversion.

In such a case, if perfect symmetry is desired in the distribution of directional units throughout the Σ, it is better to leave this part as a *neutral* unit.

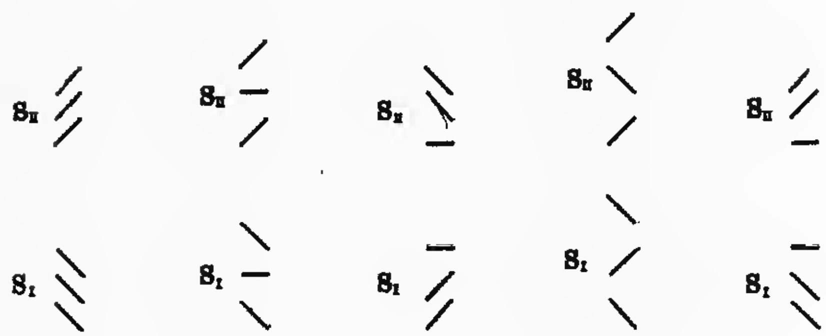
Thus, depending on whether $\Sigma = 2np$, or $2np + 1$, the axis of inversion is located between strata, or coincides with a p of the central stratum respectively

Examples of Composition of Directional Units in Strata; Graphic Representation

$\Sigma = 2S;\ S = 2p.$



$\Sigma = 2S;\ S = 3p.$



$\Sigma = 2S;\ S = 4p.$

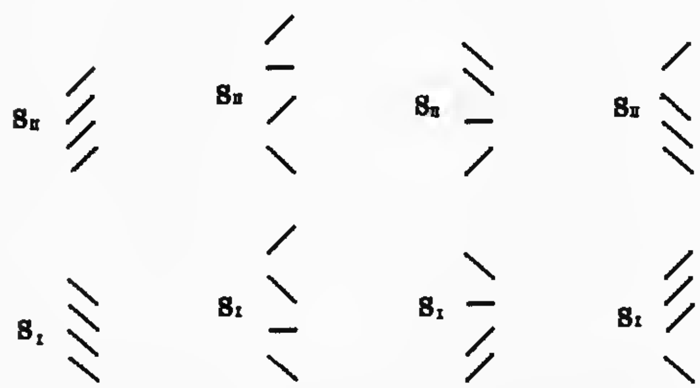
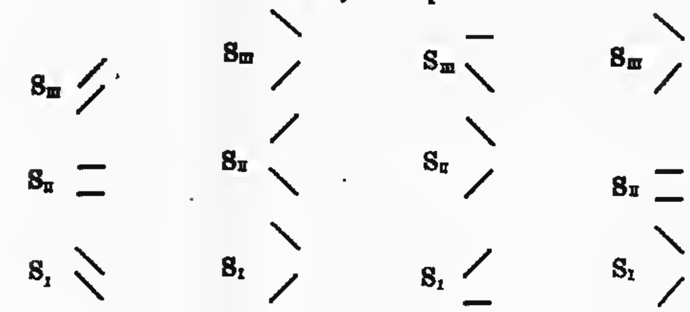
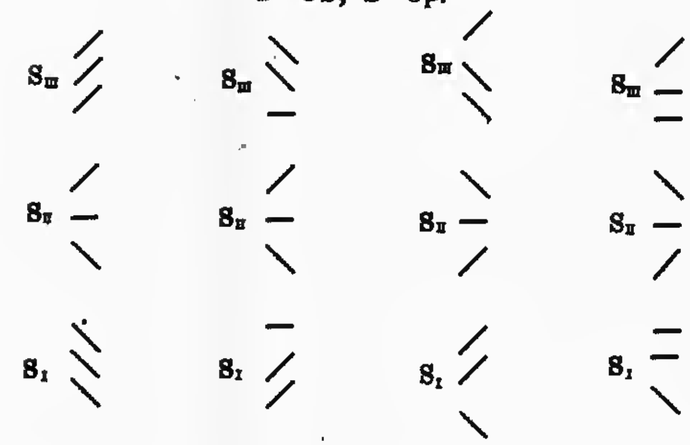


Figure 97. Directional units in strata (continued).

$\Sigma = 3S;\ S = 2p.$



$\Sigma = 3S;\ S = 3p.$



$\Sigma = 3S;\ S = 4p.$

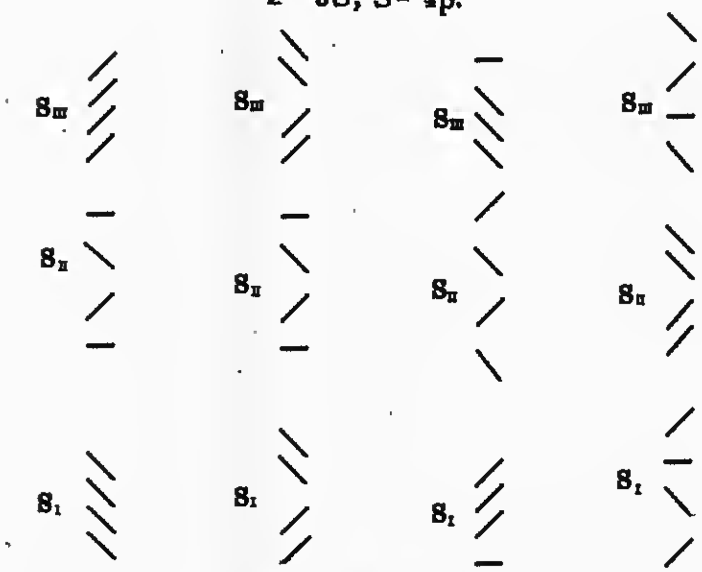


Figure 97. Directional units in strata (continued).

Musical Representation

Figure 97. Directional units in strata (continued).

Figure 97. Directional units in strata (concluded).

In the sigmae with a different number of parts in the different strata, the composer can use his discretion in attempting to evolve symmetric, or nearly symmetric forms, by assigning an axis of inversion.

In many instances directional units are reversible. Whether the structure S is of higher tension than the directional assemblage (i.e., the group of leading units) or vice-versa, both forms can be used. It is analogous in effect to moving from a consonance to a dissonance, or in reverse. As our harmonic progressions are always reversible, the reversal of directional units does not affect the quality of a progression but merely changes its character. Such progressions in position ⓐ and ⓑ are often useful as two themes of one composition.

Example of the Reversal of Directional Units

Progression

Directional progression

Figure 98. Directional units reversed (continued).

Directional progression reversed: (b)

Figure 98. Directional units reversed (concluded).

F. Sequent Groups of Directional Units
(*Leading Units of the Higher Orders*)

The leading units immediately preceding the respective chordal functions can, in turn, be preceded by other leading units which, in turn, can be preceded by still other leading units, etc. Since in most cases the interval of directional units is i, this technique attributes chromatic character to any strata progression to which it is applied.

There are certain limitations to this technique. When the intervals of chord-structures are small, only inward motion of successive leading tones can be achieved. The opposite is true, i.e. outward motion is preferable for widely-spaced intervals; otherwise, the sequent groups of leading units may interfere with adjacent chordal functions. Parallel motion of the sequent leading units is acceptable in all cases, except where such units are assigned to simultaneous chordal functions, undesirable in parallel motion.

The presence of the leading unit to the leading unit, i.e., the leading unit of the second order, can be expressed as follows·

a, a, b, b, c, c, d, d

Likewise the leading units of the third order can be expressed by introducing still another arrow:

a, a, b, b, c, c, d, d,

Examples of Leading Units of the Higher Orders

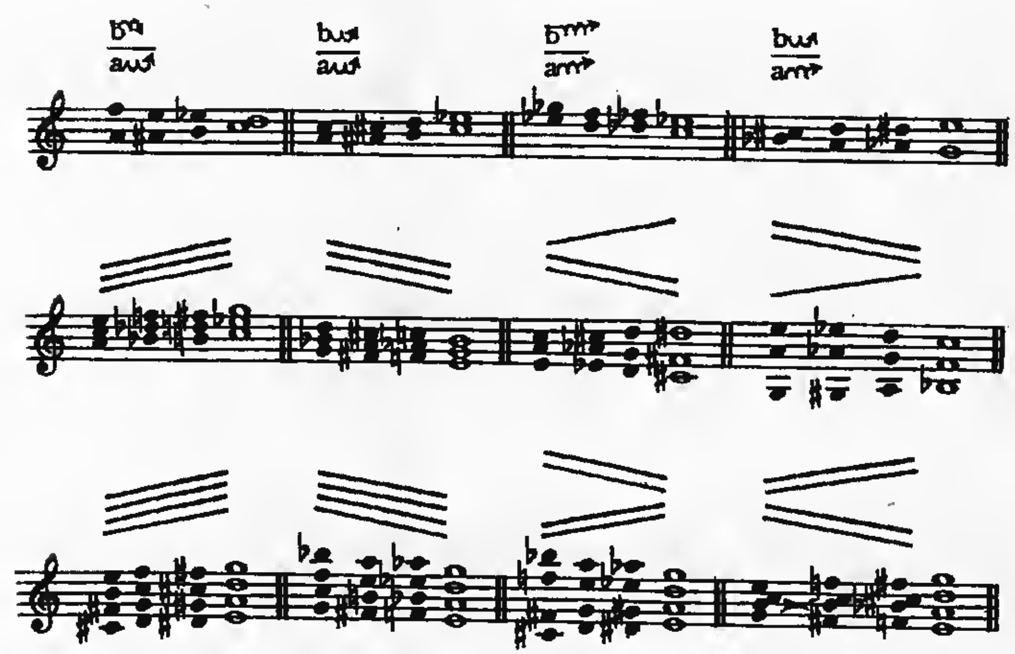


Figure 99. Leading units of the higher orders.

The conversion of harmony into melody will be discussed in ***Applications of Strata Harmony***.

CHAPTER 9

COMPOSITION OF MELODIC CONTINUITY FROM THE STRATA

EACH individual part of a stratum* can be used as part of a melodic continuity. When harmonic continuity forms a cyclic recapitulating progression, coefficients for the number of attacks on each change of a chord (H) may be set to any desirable type of interference. Thus there are three fundamental types of melodic continuity:

(a) where each part moves through the entire harmonic continuity from beginning to end, after which the next part begins.

(b) where coefficients can be set in such a way that the entire harmonic continuity will serve as a divisor to its own multiple. For example, if the number of chords equals 8, a distribution through $3 + 3 + 2$ produces one cycle. The above coefficients represent the number of H until p changes.

(c) coefficients set as an interference group in relation to the number of H. For example, if H equals 8, and the distribution-group is r_{5+2}, then $\frac{10}{8}$ interference will take place.

Theme

$\Sigma = 3S;\ S_1 = 4p;\ S_2 = 3p;\ S_3 = 2p;\ I^{\rightarrow} = 3+4+7+11-7-4-3$

Figure 100. $\Sigma = 3S;\ S_1 = 4p;\ S_2 = 3p;\ S_3 = 2p;\ I^{\rightarrow} = 3+4+7+11-7-4-3$

*Terminology and nomenclature of this branch corresponds to that used in the *Instrumental Forms* and *General Theory of Harmony*.

A. Melody from One Individual Part of a Stratum

Distributive forms of transitions from p to p and from S to S including $\Sigma^{\rightarrow}$ as a limit. The letters a, b, c, d correspond to chordal functions.

$$M = aS_1\Sigma^{\rightarrow}+bS_1\Sigma^{\rightarrow}+cS_1\Sigma^{\rightarrow}+dS_1\Sigma^{\rightarrow}+aS_2\Sigma^{\rightarrow}+bS_2\Sigma^{\rightarrow}+cS_2\Sigma^{\rightarrow}+aS_3\Sigma^{\rightarrow}+ + bS_3\Sigma^{\rightarrow}$$

Figure 101. Melody from one part of a stratum.

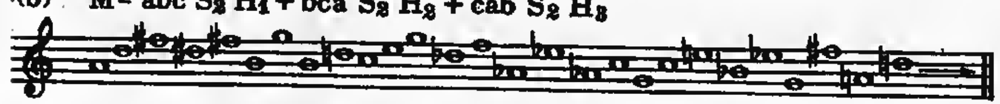
B. Melody from 2p, 3p, 4p of an S

Each section belongs to a pre-selected quantitative group. Distributive forms of transitions from S to S with respective pre-selected quantitative groups. Permutation of pitch-units within the groups.

$$a \equiv p_I;\ b \equiv p_{II};\ c \equiv p_{III};\ d \equiv p_{IV}.$$

(a) $M = ab\ S_1\ H_1 + dc\ S_1\ H_2$

(b) $M = abc\ S_2\ H_1 + bca\ S_2\ H_2 + cab\ S_2\ H_3$



(c) $M = aba\ S_3\ H_1 + bab\ S_3\ H_2$

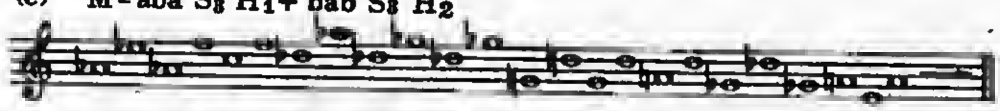
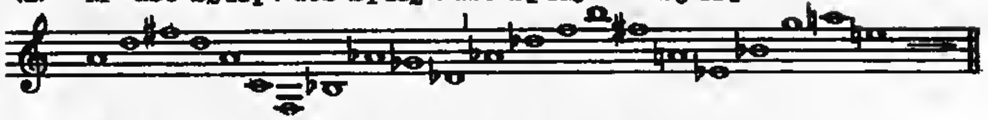


Figure 102. Melody from 2p, 3p, 4p, of an S (continued).

(d) $M = abc\ S_2 H_1 + dcb\ S_1 H_2 + abc\ S_1 H_3 + ba\ S_3 H_4$



(e) $M = ab\ S_1 H_1 + ba\ S_3 H_2 + ac\ S_2 H_3 + cb\ S_2 H_4$

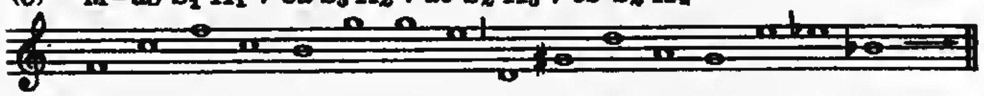


Figure 102. Melody from 2p, 3p, 4p, of an S (concluded).

C. Melody from One S

Permutation of pitch-units within one S. Distributive forms of transition from S to S.

a, b, c, d represent instrumental functions.

$M = ab\ S_3 H_1 + ba\ S_3 H_2 + dcb\ S_1 H_3 + cba\ S_1 H_4 + bab\ S_1 H_5 + bcd\ S_1 H_6 + abc\ S_2 H_7 + bca\ S_2 H_8$

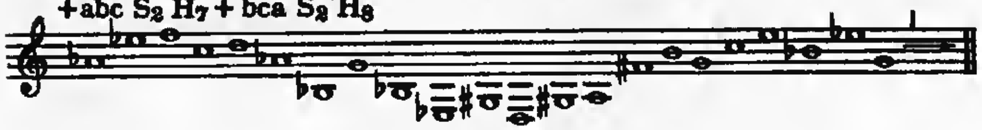


Figure 103. Melody from one S.

D. Melody from 2S, 3S

Each section of melody incorporates a pre-selected quantity and position of S. Permutation of pitch-units within a pre-selected group. Distributive forms of positions and transitions of the groups of S.

a, b, c, d represent instrumental functions.

$M = (abcab\ S_2 + ab\ S_3)\ H_1 + (bcabc\ S_2 + ba\ S_3)\ H_2 + (cabca\ S_2 + ba\ S_3)\ H_3$

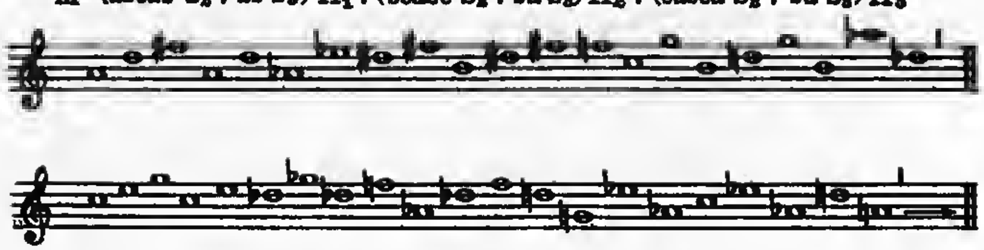


Figure 104. Melody from 2S, 3S . . .

E. Generalization of the Method

Σ as the limit becomes a pitch-scale or a melodic form. Permutation of the pitch-units. Transition to the following H occurs after a complete utilization of all p of the preceding H.

a, b, c, d represent instrumental functions.

$\Sigma \rightarrow = f, f\sharp, g\sharp, a, c, d, e\flat.$ $M = (bc\ S_1 + ba\ S_3 + bcab\ S_2 + da\ S_1)\ H$

Figure 105. Generalization of the method.

F. Mixed Forms

Derived through distributive combinations of Paragraphs A, B, C, D, E.

a, b, c, d represent instrumental functions.

$M = bd\ S_1\ 8H + a\ S_2\ 4H + dcb\ S_1\ 2H + b\ S_3\ 8H + (ab\ S_1 + cb\ S_2 + d\ S_1 + bc\ S_2 + ba\ S_3)\ H$

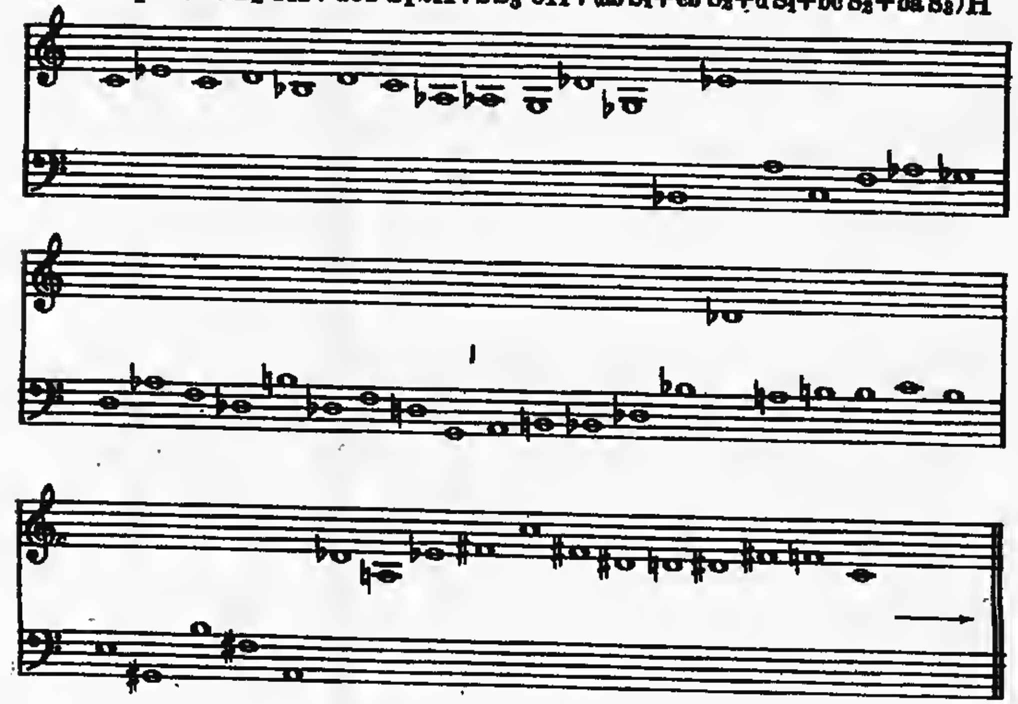


Figure 106. Mixed forms.

G. Distribution of Auxiliary Units Through p, S and Σ

Application of auxiliary units to sections A, B, C, D, and E. Permutation of directional units. Composition of continuity where some of the sections of melody contain directional units and some neutral units. Application of coefficients to the above two types of groups. Application of the directional unit technique to Paragraph F.

a, b, c, d represent instrumental functions.

$$M = a \to b \to S_2 3H + b \to a \to S_2 H + b \to a S_3 H + a \to b S_3 H + (bcS_2 + a \to b \to S_3 + c \to b \to S_2)\, 2H + d \to c \to abS_1 H + (ac \to b \to S_1 + ac \to S_2 + a \to S_3)\, 3H$$

Figure 107. Distribution of auxiliary units through p, S and Σ.

Directional units are one of the most customary forms of variation. Exposition of a theme, followed by variations of it, is a device which is particularly important from the standpoint of mobility. Increase in the number of attacks can be easily achieved through this device.

H. Variation of the Original Melodic Continuity by Means of Auxiliary Tones

a, b, c, d represent instrumental functions.

(Theme: see Fig. 104)

$$M = (abcab\, S_2 + ab\, S_3)\, H_1 + (bcabc\, S_2 + ba\, S_3)\, H_2 + (cabca\, S_2 + ba\, S_3)\, H_3.$$

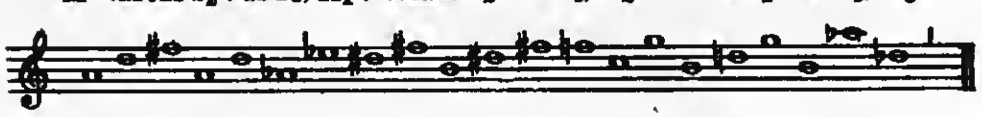


Figure 108. Variation by means of auxiliary tones (continued).

Variation:

$$M = (ab \to c \to a \to b\, S_2 + a \to b\, S_3)\, H_1 + (bc \to a \to b \to c\, S_1 + ba \to S_3)\, H_2 + (ca \to b \to c \to a\, S_2 + b \to a\, S_3)\, H_3$$

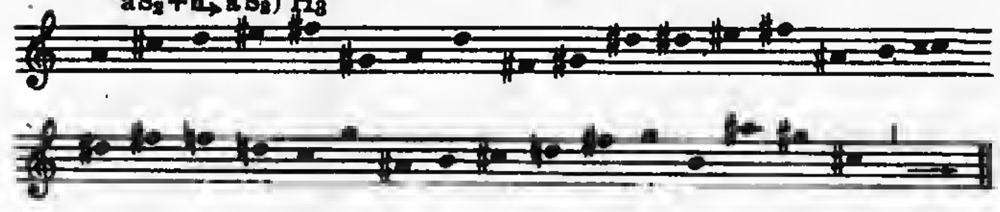


Figure 108. Variation by means of auxiliary tones (concluded).

There are two ways of assigning a duration-group to such a melodic continuity:

(1) each unit, neutral or leading, corresponds to one attack of the duration group;

(2) directional unit originally corresponds to one attack of the duration-group, afterwards is changed into a split-unit group.

See Theory of Melodization of Harmony: Composition of Durations to a Pre-set Attack-group.*

*See Vol. I, p. 649.

CHAPTER 10

COMPOSITION OF HARMONIC CONTINUITY FROM THE STRATA

COMPOSITION of harmonic continuity from a given $\Sigma^{\rightarrow}$ is primarily a method of selecting the different strata with regard to their quantity and the form of distribution. In applying this technique to orchestral writing, the different groups of instruments represent different strata, which permits one to obtain a superior flexibility of harmony with regard to ranges, registers and density. Composition of density as such is a matter of separate study and will be discussed later in this branch. For the time being, it is sufficient to assume that the density may vary gradually or suddenly as well as in an oblique fashion when one stratum alternates with the variable density of remaining strata.

A. Harmony from One Stratum (Any S of the Σ)

Distributive forms of transition from S to S within Σ as a limit. Circular continuity.

$H^{\rightarrow} = S_2 2H + S_1 H + S_3 H + S_2 2H$

Figure 109. Harmony from one stratum.

B. Harmony from 2S, 3S, . . .

Distributive method of selecting the groups of S and their sequence in time continuity. Circular continuity.

$H^{\rightarrow} = (S_1 + S_2)\ 2H + (S_2 + S_3)\ 4H + (S_3 + S_1)\ 2H$

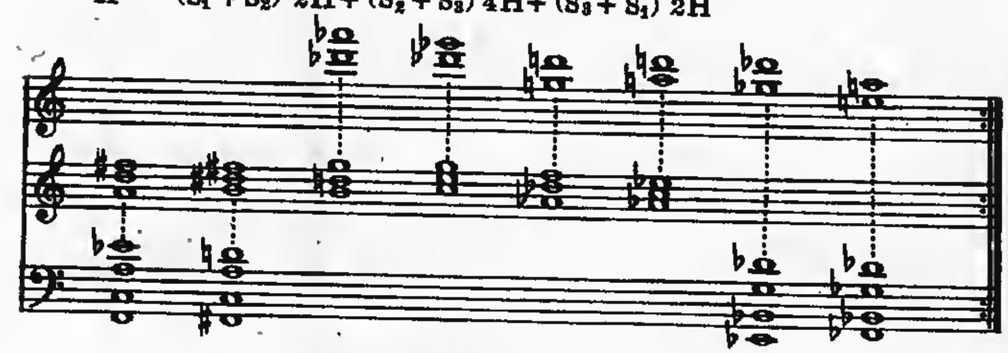


Figure 110. Harmony from 2S, 3S . . . .

C. Harmony from $\Sigma(\Sigma^{\rightarrow}$; the Original Layout)

$H^{\rightarrow} = \Sigma^{\rightarrow}$

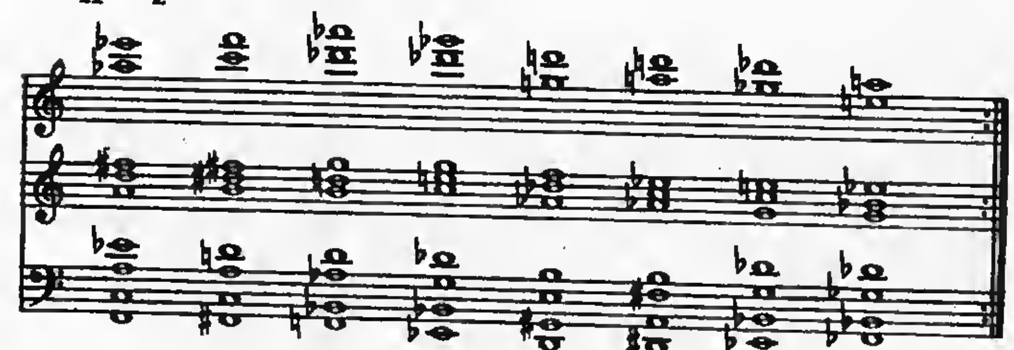


Figure 111. Harmony from Σ.

D. Patterns of Distribution (Variation of Density) from Sections A, B, C

Composition of continuity with variable density.

$H^{\rightarrow} = S_3 H + (S_1 + S_2) H + S_2 H + (S_2 + S_3) H + S_2 H + (S_1 + S_2 + S_3)\ 3H$

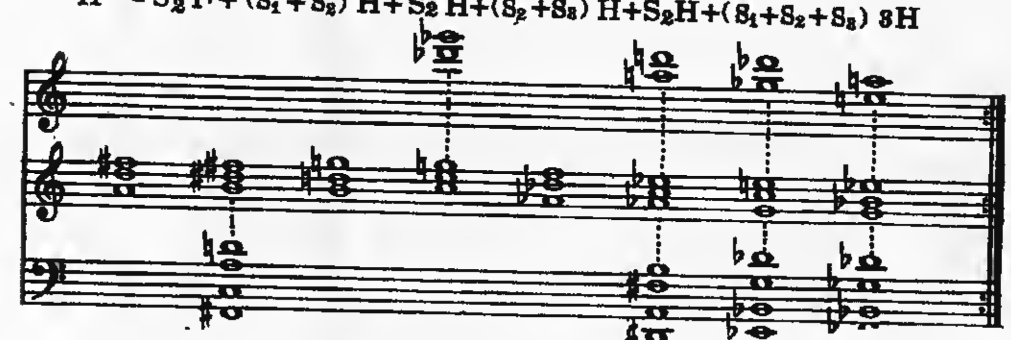


Figure 112. Continuity with variable density (continued).

$H^{\rightarrow} = S_1\,8H + (S_1+S_2)\,8H + (S_1+S_2+S_3)\,4H + (S_2+S_3)\,8H + S_3\,8H$

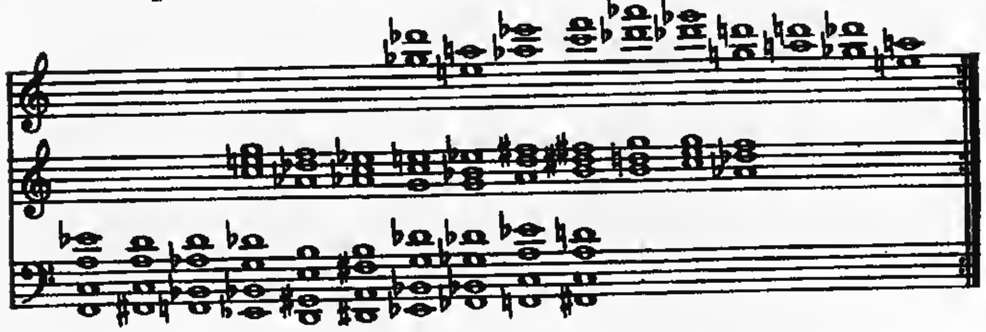


Figure 112. Continuity with variable density (concluded).

E. Application of Auxiliary Units and Instrumental Figuration of Harmony Through Any Of The Preceding Four Forms Of Harmonic Continuity Of The Strata

Hybrid forms of distribution of the auxiliary tones through strata.

F. Variation of the Original Harmonic Continuity Through Auxiliary Units

The following example illustrates both Sections E and F, as it may be used in place of a variation on the original theme.

$H^{\rightarrow} = I^{\rightarrow};\ S_1 = \begin{smallmatrix} d\rightarrow \\ c\rightarrow \\ b\rightarrow \\ a \end{smallmatrix} \rightleftarrows;\ S_2 = \begin{smallmatrix} c\rightarrow \\ b\rightarrow \\ a\rightarrow \end{smallmatrix}\ \text{const.};\ S_3 = \begin{smallmatrix} b\rightarrow \\ a\rightarrow \end{smallmatrix} \rightleftarrows$

a, b, c, d Represent instrumental functions.

Figure 113. Variations of original harmonic continuity (continued).

Figure 113. Variation of original harmonic continuity (concluded).

CHAPTER 11

MELODY WITH HARMONIC ACCOMPANIMENT

All illustrations are based on the theme of figure 100.

(1) One S becomes a melody; same S serves as harmony in a different octave.

$$\frac{M}{H} = S \text{ constant}$$

(a) $\frac{M}{H} = S_1$

(b) $\frac{M}{H} = S_2$

(c) $\frac{M}{H} = S_3$

. . . . . .

(n) $\frac{M}{H} = S_n$

Example: $\frac{M}{H} = S_1$

Figure 114. One S becomes a melody.

$H^{\rightarrow} = H_1 2T + H_2 T + H_3 T + H_4 2T$

Instrumental form: $(a+b)\,2T + bT + aT + (a+b)2T$

$T(M) = (\boxed{t}+t+t+t) + (t+\boxed{t}+t+t) + (t+t+\boxed{t}+t) + (t+t+t+\boxed{t})$

tied over: $T(M) = (\boxed{t}+t+t+t) + (2t+t+t) + (t+2t+t) + (t+t+2t)$

(2) Different individual S's of one Σ become melody with harmonic accompaniment $\left(\frac{M}{H} = S \text{ variable}\right)$.

$$\frac{M}{H} = S_1,\ S_2,\ S_3,\ .\ .\ .\ S_n$$

Example: $\frac{M}{H} = S_2H_1 2T + S_3H_2T + S_2H_3T + S_1H_4 2T$

Figure 115. $\frac{M}{H}$ *= S variable.*

(3) More than one S produce melody; one S produces harmony.

$$\frac{M}{H} = \frac{S_2+S_3}{S_1};\ \frac{M}{H} = \frac{S_2+S_3+S_4}{S_1};\ \frac{M}{H} = \frac{S_2+\ .\ .\ .\ +S_n}{S_1}$$

Example:

$$\frac{M}{H} = \frac{S_2+S_3}{S_1}$$

Rhythm: $\frac{6}{8}$ series: $T = (4+1+1) + (1+4+1) + (1+1+4) +$
$+ (1+1+2+1+1) + (1+2+1+1+1) +$
$+ (2+1+1+1+1)$

See Figure 116 on the following page.

Figure 116. More than one S produces melody.

(4) One S produces melody; more than one S produces harmony.

(5) Several S's produce melody and several other S's (of the same Σ) produce harmony.

(6) Distribution of tension for $\frac{M}{H}$ in the preceding cases ($H = S_1$, $M = S_2$; $H = S_1 + S_2 + \ldots + S_{n-1}$, $M = S_n$; $H = S_1$, $M = S_2 + S_3 + \ldots + S_n$).

$$\frac{M}{H} = \frac{Sa}{Sb}$$

Example: $\frac{M}{H} = \frac{S_2}{S_1}$

Rhythm: Foxtrot: $\frac{4}{4}$, $\frac{8}{8}$ and $\frac{8}{8}$ series

Instrumental form: $aH_12T + bH_2T + bH_3T + aH_42T$

See Figure 117 on the opposite page.

Figure 117. Distribution of tension for $\frac{M}{H}$.

(7) Variable distributive transition from one individual S to another for melody and a constant S or a group thereof for harmony.

(8) Constant S or a group thereof for melody and a variable distributive transition from one individual S to another for harmony.

(9) Σ for melody; variable density for harmony composed through distributive selection.

(10) Melody composed through distributive selection of scales derived from the individual S, the groups of S or the entire Σ; harmony from the entire Σ.

(11) All previous cases with application of auxiliary units. M and H without auxiliary units. M with and H without auxiliary units. M without and H with auxiliary units. Both M and H with auxiliary units.

(12) All the previous forms of harmonic accompaniment with instrumental figuration..

(13) Hybrid forms with respect to density of both melody (scale emphasis) and harmony.

(14) Hybrid forms with respect to the presence or absence of auxiliary units in both M and H.

(15) Forms of alternating transformations of M into H and vice versa with respect to the selective distribution of strata.

(16) Instrumental forms of melody used for the purpose of variation.

(17) Intercomposition of the instrumental forms of melody and harmony.

(18) Composition of continuity employing the previous devices.

Example:

$$\frac{M}{H} = \frac{\Sigma}{S_3} H_1 3T + \frac{S_2}{\Sigma} H_2T + \frac{S_3}{S_2} H_3T + \frac{S_3}{S_1 + S_2} H_3T + \frac{S_2}{S_1 + S_3} H_4T +$$

$$+ \frac{S_2}{S_3} H_4T + \frac{\Sigma}{0} H_5T + \frac{S_1}{S_1} H_6T + \frac{S_1 + S_2}{S_1} H_6T + \frac{S_1 + S_2 + S_3}{S_1} H_6T$$

Figure 118. Composition of continuity based on previous devices.

CHAPTER 12

CORRELATED MELODIES

CORRELATED *Melodies* (Transformation of Harmony into Counterpoint by Means of Strata).

The technique of correlating melodies consists of three fundamental processes:

(1) correlation of attacks and durations of two or more melodies;
(2) correlation of melodic forms (axial combinations) of two or more melodies;
(3) correlation of harmonic intervals between two or more melodies (distribution of tension).

The first two processes have been described in the *Theory of Correlated Melodies* (*Counterpoint*)* Here we shall deal with the third procedure as it evolves itself from the technique of strata.

The usual classical conception of a consonance and a dissonance, and the necessity of resolution must give way to the assortment and distribution of harmonic intervals through their respective degrees of tension.

From the harmonic point of view there are the following forms of matching intervals:

(1) a neutral unit against a neutral unit (or units);
(2) a neutral unit against a directional unit (or units);
(3) a directional unit against a neutral unit (or units);
(4) a directional unit against a directional unit (or units).

Taking as illustration the $\Sigma^{\rightarrow}$ used in the previous examples, we may enumerate the following possibilities:

$(1)\ \frac{CP_I}{CP_{II}} = \frac{aS_1}{aS_2};\ (2)\ \frac{CP_I}{CP_{II}} = \frac{bS_1}{aS_2};\ (3)\ \frac{CP_I}{CP_{II}} = \frac{cS_1}{aS_2};\ (4)\ \frac{CP_I}{CP_{II}} = \frac{dS_1}{aS_2};$

$(5)\ \frac{CP_I}{CP_{II}} = \frac{aS_1}{bS_2};\ (6)\ \frac{CP_I}{CP_{II}} = \frac{bS_1}{bS_2};\ (7)\ \frac{CP_I}{CP_{II}} = \frac{cS_1}{bS_2};\ (8)\ \frac{CP_I}{CP_{II}} = \frac{dS_1}{bS_2};$

$(9)\ \frac{CP_I}{CP_{II}} = \frac{aS_1}{cS_2};\ (10)\ \frac{CP_I}{CP_{II}} = \frac{bS_1}{cS_2};\ (11)\ \frac{CP_I}{CP_{II}} = \frac{cS_1}{cS_2};\ (12)\ \frac{CP_I}{CP_{II}} = \frac{dS_1}{cS_2};$

$(13)\ \frac{CP_I}{CP_{III}} = \frac{aS_1}{aS_3};\ (14)\ \frac{CP_I}{CP_{III}} = \frac{bS_1}{aS_3};\ (15)\ \frac{CP_I}{CP_{III}} = \frac{cS_1}{aS_3};\ (16)\ \frac{CP_I}{CP_{III}} = \frac{dS_1}{aS_3};$

$(17)\ \frac{CP_I}{CP_{III}} = \frac{aS_1}{bS_3};\ (18)\ \frac{CP_I}{CP_{III}} = \frac{bS_1}{bS_3};\ (19)\ \frac{CP_I}{CP_{III}} = \frac{cS_1}{bS_3};\ (20)\ \frac{CP_I}{CP_{III}} = \frac{dS_1}{bS_3};$

$(21)\ \frac{CP_{II}}{CP_{III}} = \frac{aS_2}{aS_3};\ (22)\ \frac{CP_{II}}{CP_{III}} = \frac{bS_2}{aS_3};\ (23)\ \frac{CP_{II}}{CP_{III}} = \frac{cS_2}{aS_3};\ (24)\ \frac{CP_{II}}{CP_{III}} = \frac{aS_2}{bS_3};$

$(25)\ \frac{CP_{II}}{CP_{III}} = \frac{bS_2}{bS_3};\ (26)\ \frac{CP_{II}}{CP_{III}} = \frac{cS_2}{bS_3}.$

Each of the above cases may be either a neutral or a directional unit.

*See Vol. I, pp. 730 and 753.

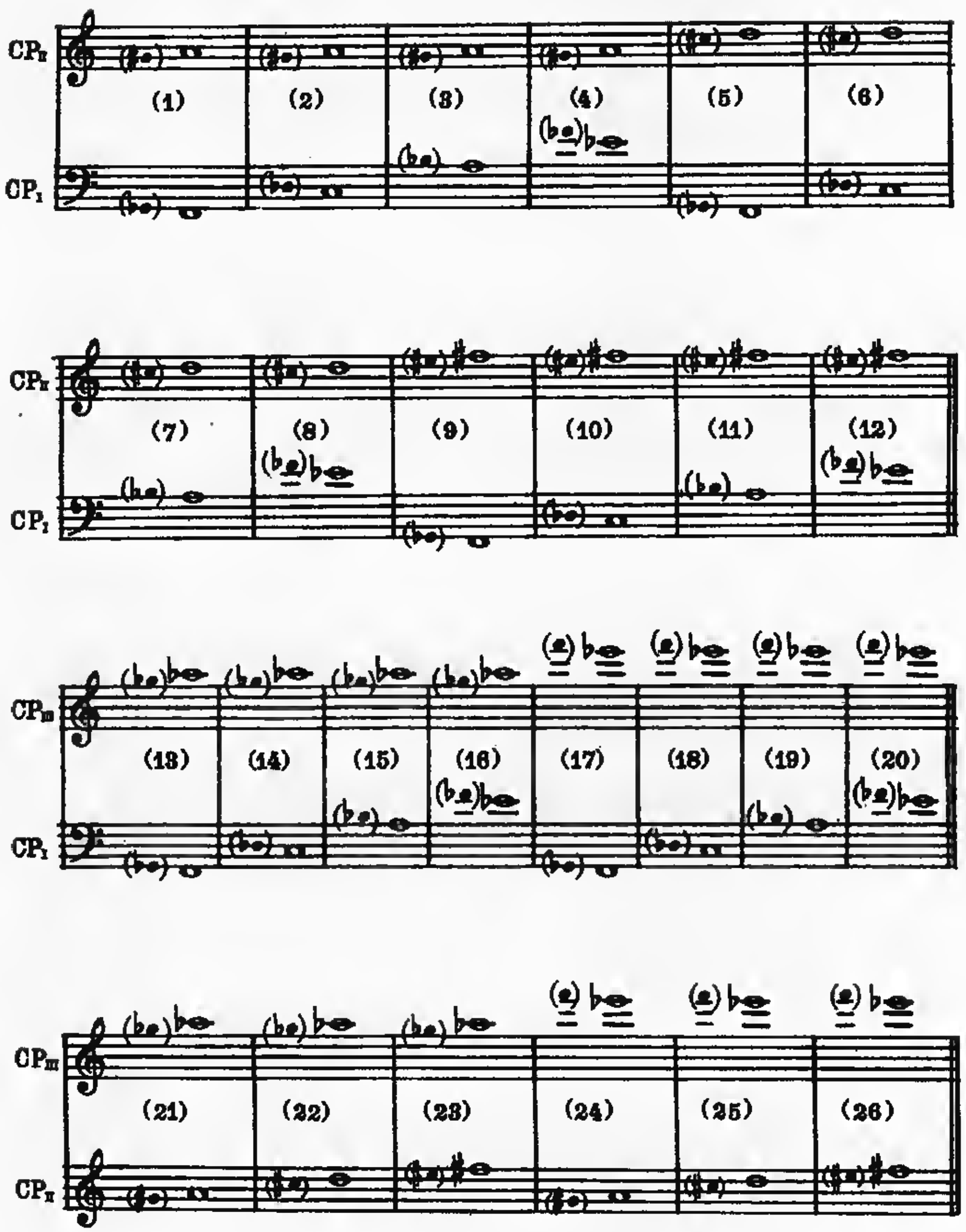


Figure 119. Forms of matching intervals.

Each stratum of harmony may be converted into a melody. The above case of $\Sigma^{\rightarrow}$ makes it possible to obtain a three part counterpoint. Distribution of attacks is the final factor in selecting matching units. Once the units are matched, the harmonic progression produces continuity.

Example:

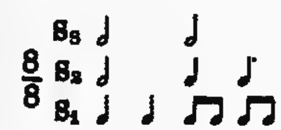
Composition of attacks (A — attack-group; a — individual attack):

$\underline{AS_3} = 2aT$
$\underline{AS_2} = 3aT$
$\underline{AS_1} = 6aT$

Composition of durations:

$\underline{TS_3 = 4t + 4t}$
$\underline{TS_2 = 4t + 2t + 2t}$
$TS_1 = 2t + 2t + t + t + t + t$

Time in musical notation:



Selection of matched units: (a, b, c, . . . designate chordal functions of the respective strata).

$\underline{CP_{III} = b4t + a4t}$
$\underline{CP_{II} = c4t + a_{\rightarrow}(2t + 2t)}$
$CP_{I} = c^{\rightarrow}(2t + 2t) + a^{\rightarrow}(t+t) + b^{\rightarrow}(t+t)$

Figure 120. Developing a Σ into 3 part counterpoint.

From this data we evolve the final form of continuity by applying the same selective pattern to the entire $\Sigma^{\rightarrow}$

etc.

Figure 121. Final form of continuity.

Composition of contrapuntal continuity can be accomplished from a theme such as the above progression (Figure 121) by means of various techniques. The most important of these are:

(1) vertical rearrangement of parts;
(2) variation of density;
(3) geometrical inversions.

Figure 122. Composition of contrapuntal continuity (continued).

Figure 122. Composition of contrapuntal continuity (concluded).

The example in Figure 121 is a case of the constant form of duration-groups correlated through three parts. Any other form of distribution deriving from the *Theory of Rhythm* on the basis of compensation or contrast is acceptable for this purpose. The contrasts are particularly effective when several synchronized power-groups are used. The neutral and the directional pitch-units can change their respective octave position. One H may correspond to any time equivalent. $H^{\rightarrow}$ may have any rhythmic distribution of its own. Contrapuntal parts which derive from strata may be coupled and subjected to instrumental variations.

CHAPTER 13

COMPOSITION OF CANONS FROM STRATA HARMONY

As we have seen, each stratum may become a contrapuntal part. In order to convert a Σ into canonic (continuous) imitation, it is necessary to fulfill the following requirements:

(1) Chord progression must be written in such a way as to permit regular occurrence of the identical chord positions, systematically moving from S to S. This can be accomplished by reciprocation of transformations. The latter must be either clockwise or counterclockwise throughout.
(2) Chord structures must be identical in all strata.
(3) Intervals between the roots of the different strata must be equidistant, i.e., only monomial symmetry is acceptable.
(4) The progression of chords must also be carried out in monomial symmetry of consecutive intervals, but not necessarily in the symmetry of simultaneous roots through which the Σ has been compounded.

So long as there is an interchange of symmetric roots of the same system of symmetry, canonic imitation remains unitonal. Beyond this, the form of imitation with respect to its harmonic correlation depends on the form of consecutive symmetry through which the chords progress.

The advantage of evolving a canon from $\overrightarrow{H}$ lies in the fact that such a canon possesses a definite harmonic characteristic set a priori, which it is impossible to obtain by means of purely contrapuntal technique.

A. Two-Part Continuous Imitation

Such an imitation is based on the reciprocation of the two functions a and b and on the reciprocation of the two symmetric roots of the $\sqrt{2}$.

The initial scheme of harmonic setting for a two-part canon is as follows:

$$CP_I\,(\equiv S_I) = \frac{b}{a}\,;\ CP_{II}\,(\equiv S_{II}) = \frac{a}{b}$$

The scheme of coordinated roots corresponding to the reciprocating positions is as follows:

$$\frac{CP_{II}}{CP_I} = \frac{C + F\#}{C + F\# + C}$$

The two schemes combined appear as follows:

$$\frac{CP_I}{CP_{II}} = \frac{C\left(\frac{b}{a}\right) + F\#\left(\frac{a}{b}\right)}{C\left(\frac{b}{a}\right) + F\#\left(\frac{a}{b}\right) + C\,\frac{b}{a}}$$

From this original scheme the canon follows any form of consecutive symmetry, resulting in a modulating canon. Other roots of symmetry can be used in similar reciprocation as well.

Figure 123. Two-part canon derived from $\Sigma 2S2p$.

A canon such as this can be further extended by means of quadrant rotation (geometrical inversions). It can be also coupled and subjected to instrumental variations. Any temporal scheme can be used, and T does not necessarily have to equal T″.

B. Three-Part Continuous Imitation

Such an imitation derives from a harmonic scheme, where either clockwise or counterclockwise transformations are applied to both the simultaneous arrangement of strata in the Σ and the continuous progression of $H^{\rightarrow}$.

The initial harmonic scheme must be arranged in the following way:

$$\begin{array}{cccccc}
S_{III} & \begin{matrix}a\\c\\b\end{matrix} & \begin{matrix}b\\a\\c\end{matrix} & \boxed{\begin{matrix}c\\b\\a\end{matrix}} & \left.\begin{matrix}'\,a\\'\,c\\'\,b\end{matrix}\right] & \left.\begin{matrix}b\\a\\c\end{matrix}\right) \\ \hline
S_{II} & \begin{matrix}b\\a\\c\end{matrix} & \boxed{\begin{matrix}c\\b\\a\end{matrix}} & \left.\begin{matrix}a\\c\\b\end{matrix}\right] & \left.\begin{matrix}'\,b\\'\,a\\'\,c\end{matrix}\right) & \begin{matrix}c\\b\\a\end{matrix} \\ \hline
S_{I} & \boxed{\begin{matrix}c\\b\\a\end{matrix}} & \left.\begin{matrix}a\\c\\b\end{matrix}\right] & \left.\begin{matrix}b\\a\\c\end{matrix}\right) & \begin{matrix}'\,c\\'\,b\\'\,a\end{matrix} & \begin{matrix}a\\c\\b\end{matrix}
\end{array}$$

This is a clockwise scheme where the sequence of imitation follows from S_I to S_{II} to S_{III}. Similar schemes can be devised for the remaining 5 forms of the sequence of imitation, as well as for the counterclockwise sequent transformations and the 6 forms of the sequence of imitation. Thus the total number of such schemes for Σ3S3p is 12.

The fundamental form of symmetric root-coordination is the $\sqrt[3]{2}$. However' other forms of symmetry may be used as well.

Examples of Schemes of Coordinated Roots

$$\begin{array}{cccccr}
 & & G\sharp & CP_{III} & & C + G\sharp + E + \underline{C} \\ \cline{4-4}\cline{6-6}
(1) & \sqrt[3]{2} & E & CP_{II} & = & C + G\sharp + E + C + \underline{G\sharp} \\ \cline{4-4}\cline{6-6}
 & & C & CP_{I} & = & C + G\sharp + E + C + G\sharp + E
\end{array}$$

$$\begin{array}{cccccr}
 & & B\flat & CP_{III} & & C + B\flat + F + \underline{C} \\ \cline{4-4}\cline{6-6}
(2) & \sqrt[12]{2^5} & F & CP_{II} & = & C + B\flat + F + C + \underline{B\flat} \\ \cline{4-4}\cline{6-6}
 & & C & CP_{I} & = & C + B\flat + F + C + B\flat + F
\end{array}$$

Figure 124. Schemes of coordinated roots.

The scheme of roots, after it is combined with the scheme of transformations, assumes the following form [Fig. 124 (1)]:

$$\begin{array}{ll}
CP_{III} = & C\begin{pmatrix}c\\b\\a\end{pmatrix} + G\sharp\begin{pmatrix}a\\c\\b\end{pmatrix} + E\begin{pmatrix}b\\a\\c\end{pmatrix} + C\begin{pmatrix}c\\b\\a\end{pmatrix} \\ \hline
CP_{II} = & C\begin{pmatrix}c\\b\\a\end{pmatrix} + G\sharp\begin{pmatrix}a\\c\\b\end{pmatrix} + E\begin{pmatrix}b\\a\\c\end{pmatrix} + C\begin{pmatrix}c\\b\\a\end{pmatrix} + G\sharp\begin{pmatrix}a\\c\\b\end{pmatrix} \\ \hline
CP_{I} & C\begin{pmatrix}c\\b\\a\end{pmatrix} + G\sharp\begin{pmatrix}a\\c\\b\end{pmatrix} + E\begin{pmatrix}b\\a\\c\end{pmatrix} + C\begin{pmatrix}c\\b\\a\end{pmatrix} + G\sharp\begin{pmatrix}a\\c\\b\end{pmatrix} + E\begin{pmatrix}b\\a\\c\end{pmatrix}
\end{array}$$

Figure 124A. Scheme of transformations combined with scheme of roots.

As the scheme of the sequence of imitation for Σ3S3p is sufficiently long by itself, such a scheme may be used as a canonic theme, and be extended further by quadrant rotation. This does not exclude the use of the technique applied to Σ2S2p where the original scheme of $H^{\rightarrow}$ was extended by some form of consecutive root-symmetry. The application of the latter produces a modulating canon.

Figure 125. Three-part canon derived from Σ3S3p (continued).

Figure 125. Three-part canon derived from Σ3S3p (concluded).

C. Four-Part Continuous Imitation

Such an imitation derives from a harmonic scheme, where either clockwise or counterclockwise transformations (which correspond to ↻ or to ↺ circular permutations of the chordal functions) are applied to both the simultaneous arrangement of strata in the Σ and the continuous progression of $H^{\rightarrow}$.

The initial harmonic scheme must be arranged in the following way:

S_{IV}	a	b	c	$\boxed{d}$	′	a	b	c
	d	a	b	$\boxed{c}$	′	d	a	b
	c	d	a	$\boxed{b}$	′	c	d	a
	b	c	d	$\boxed{a}$	′	b	c	d
S_{III}	b	c	$\boxed{d}$	a	′	b	c	d
	a	b	$\boxed{c}$	d	′	a	b	c
	d	a	$\boxed{b}$	c	′	d	a	b
	c	d	$\boxed{a}$	b	′	c	d	a
S_{II}	c	$\boxed{d}$	a	b	′	c	d	a
	b	$\boxed{c}$	d	a	′	b	c	d
	a	$\boxed{b}$	c	d	′	a	b	c
	d	$\boxed{a}$	b	c	′	d	a	b
S_{I}	$\boxed{d}$	a	b	c	′	d	a	b
	$\boxed{c}$	d	a	b	′	c	d	a
	$\boxed{b}$	c	d	a	′	b	c	d
	$\boxed{a}$	b	c	d	′	a	b	c

Figure 126. Initial harmonic scheme of four-part imitation.

This is a clockwise scheme where the sequence of imitation follows from S_I to S_{II}, to S_{III}, to S_{IV}. Similar schemes can be devised for the remaining 23 forms of the sequence of imitation, as well as for the counterclockwise sequent transformations and their own 24 forms (corresponding to the number of general permutations) of the sequence of imitation. Thus the total number of such schemes for Σ4S4p is 48.

The fundamental form of symmetric root-coordination is the $\sqrt[4]{2}$. However, other forms of symmetry can be used as well.

Example of a Scheme of Coordinated Roots

$$\sqrt[4]{2}\;\begin{array}{llcrl}
A & CP_{IV} & = & \overline{C + A + F\sharp + E\flat} & + \underline{C} \\
F\sharp & CP_{III} & = & \overline{C + A + F\sharp + E\flat + C} & + \underline{A} \\
E\flat & CP_{II} & = & \overline{C + A + F\sharp + E\flat + C + A} & + \underline{F\sharp} \\
C & CP_{I} & = & C + A + F\sharp + E\flat + C + A + F\sharp & + E\flat
\end{array}$$

Figure 127. Scheme of coordinated roots.

The scheme of roots, being combined with the scheme of transformations, assumes the following f (Fig. 127):

$$CP_{IV} = C\begin{pmatrix}d\\c\\b\\a\end{pmatrix} + A\begin{pmatrix}a\\d\\c\\b\end{pmatrix} + F\sharp\begin{pmatrix}b\\a\\d\\c\end{pmatrix} + E\flat\begin{pmatrix}c\\b\\a\\d\end{pmatrix} + C\begin{pmatrix}d\\c\\b\\a\end{pmatrix}$$

$$CP_{III} = C\begin{pmatrix}d\\c\\b\\a\end{pmatrix} + A\begin{pmatrix}a\\d\\c\\b\end{pmatrix} + F\sharp\begin{pmatrix}b\\a\\d\\c\end{pmatrix} + E\flat\begin{pmatrix}c\\b\\a\\d\end{pmatrix} + C\begin{pmatrix}d\\c\\b\\a\end{pmatrix} + A\begin{pmatrix}a\\d\\c\\b\end{pmatrix}$$

$$CP_{II} = C\begin{pmatrix}d\\c\\b\\a\end{pmatrix} + A\begin{pmatrix}a\\d\\c\\b\end{pmatrix} + F\sharp\begin{pmatrix}b\\a\\d\\c\end{pmatrix} + E\flat\begin{pmatrix}c\\b\\a\\d\end{pmatrix} + C\begin{pmatrix}d\\c\\b\\a\end{pmatrix} + A\begin{pmatrix}a\\d\\c\\b\end{pmatrix} + F\sharp\begin{pmatrix}b\\a\\d\\c\end{pmatrix}$$

$$CP_{I} \quad C\begin{pmatrix}d\\c\\b\\a\end{pmatrix} + A\begin{pmatrix}a\\d\\c\\b\end{pmatrix} + F\sharp\begin{pmatrix}b\\a\\d\\c\end{pmatrix} + E\flat\begin{pmatrix}c\\b\\a\\d\end{pmatrix} + C\begin{pmatrix}d\\c\\b\\a\end{pmatrix} + A\begin{pmatrix}a\\d\\c\\b\end{pmatrix} + F\sharp\begin{pmatrix}b\\a\\d\\c\end{pmatrix} + E\flat\begin{pmatrix}c\\b\\a\\d\end{pmatrix}$$

Figure 128. Scheme of transformations combined with scheme of roots.

Such a scheme can serve as a canonic theme, being further extended by quadrant rotation, or through continuation of $H^{\rightarrow}$ evolved through some form of consecutive symmetry. In the latter case, the canon becomes modulating.

For an obvious technical reason (4p is the limit of S), this method of evolving canons from strata is limited to four parts.

This does not exclude the possibility of writing correlated melodies in any desirable number of parts (corresponding to the number of S in a Σ) in the form of general counterpoint, or counterpoint of discontinuous imitations.

All canonic schemes of the type described in this chapter produce recapitulating canons or *rounds*, providing H⟶ does not extend itself beyond the original scheme of symmetric roots.

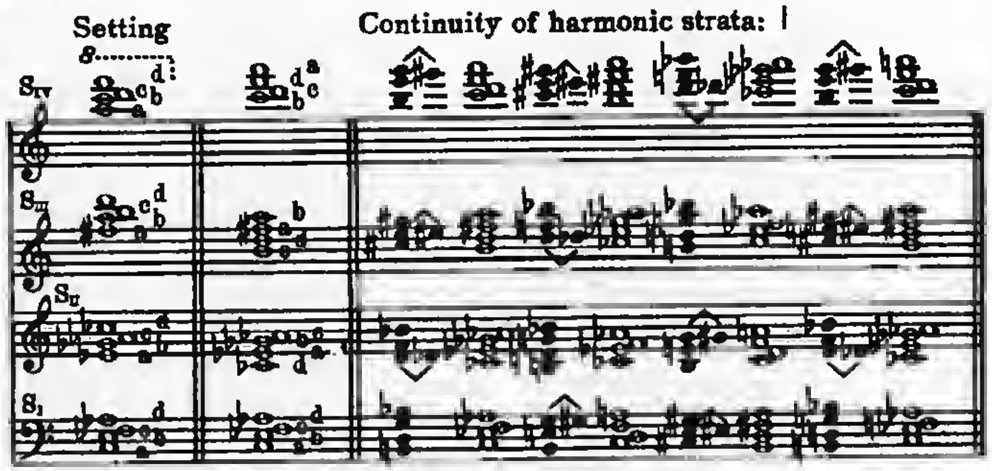


Figure 129. Four-part canon derived form Σ4S4p (continued).

Figure 129. Four-part canon derived form Σ4S4p (concluded).

CHAPTER 14

CORRELATED MELODIES WITH HARMONIC ACCOMPANIMENT

THE fact that harmony can be converted into melody makes it possible to develop such forms as correlated melodies with harmonic accompaniment. There are three main groups into which such techniques may be classified:

Group (1) in which counterpoint and harmonic accompaniment are selected on the basis of identity or non-identity of strata to which the counterpoint and the harmonic accompaniment belong;

Group (2) in which counterpoint and harmonic accompaniment are selected on the basis of neutral or directional units so that either the counterpoint has directional units and the accompanying harmony, neutral units, or vice-versa; or both counterpoint and harmony are based on the same kind of units (i.e., either neutral or directional);

Group (3) in which counterpoint and harmonic accompaniment are intercomposed on the basis of continuity and discontinuity so that either counterpoint or the harmonic accompaniment are either continuous (uninterrupted) or discontinuous (interrupted) , which, at certain times, leaves only one of the two components (i.e., either counterpoint or harmony) and also makes it possible to evolve dialogue-like alternating sequences between the two components; the harmonic accompaniment as well as the counterpoint itself become subject to variation of density (low, medium, high), which may be treated in various forms of reciprocation.

The following classification presents the most important forms of correlated melodies with harmonic accompaniment in their interrelation through the above described three groups:

(1) Correlated melodies with harmonic accompaniment whose strata derivation is identical with that of the counterpoint itself.
(2) Correlated melodies with harmonic accompaniment which derives from strata partly in common with the counterpoint.
(3) Correlated melodies with harmonic accompaniment which derives from strata not participating in the counterpoint.
(4) Counterpoint of constant density accompanied by harmony of constant density.
(5) Counterpoint of variable density accompanied by harmony of constant density.
(6) Counterpoint of constant density accompanied by harmony of variable density.

(7) Counterpoint of variable density accompanied by harmony of variable density:

(a) Counterpoint in increasing density, harmony in increasing density;
(b) Counterpoint in decreasing density, harmony in increasing density;
(c) Counterpoint in increasing density, harmony in decreasing density;
(d) Counterpoint in decreasing density, harmony in decreasing density.

(8) Continuous counterpoint with a continuous harmonic accompaniment.
(9) Discontinuous counterpoint with a continuous harmonic accompaniment.
(10) Continuous counterpoint with a discontinuous harmonic accompaniment.
(11) Discontinuous counterpoint with a discontinuous harmonic accompaniment.
(12) Rhythmic composition of the forms of continuity and discontinuity in both harmony and counterpoint.
(13) Relations of directional and neutral units in the correlated melodies with harmonic accompaniment:

(a) Neutral units in counterpoint, neutral units in harmonic accompaniment;
(b) Directional units in counterpoint, neutral units in harmonic accompaniment;
(c) Neutral units in counterpoint, directional units in harmonic accompaniment;
(d) Directional units in counterpoint, directional units in harmonic accompaniment; in this case the duration-unit of counterpoint has a different value from the duration-unit of harmony.

(14) Composition and coordination of the instrumental forms of harmony and counterpoint.
(15) Composition of continuity based on correlated melodies with harmonic accompaniment, and including the above described devices.

No musical illustrations are necessary, as previous chapters give sufficient guidance for executing these projects.*

*Schillinger expected his students to work out each of these suggested procedures as homework. Those who are using the present text as a study-book are urged to do so. (Ed.)

CHAPTER 15

COMPOSITION OF DENSITY IN ITS APPLICATION TO STRATA

WE have already encountered in the field of harmony and counterpoint certain elementary techniques pertaining to variation of density of the original texture. At the time we found it satisfactory to manipulate density by either employing some distinct degrees of it (like low, medium or high density), or by using harmonic parts as units of density.

Now, in view of the strata technique, with its potential abundance of parts and assemblages, we arrive at the necessity of generalizing a density technique so as to enable the composer to render the utmost plasticity to the density of texture, whether melodic, contrapuntal, harmonic, or combined.

In this branch we shall concern ourselves with the problems of *textural density* alone, as the technique of *instrumental density* belongs to the field of Orchestration.

The behavior of sounding texture in any musical composition is such that it fluctuates between stability and instability, and so remains perpetually in a state of unstable equilibrium. The latter is characteristic of albumen which is chemically basic to all organic forms of nature. For this reason, unstable equilibrium is a manifestation of life itself, and, being applied to the field of musical composition as a formal principle, contributes the quality of life to music.

NOMENCLATURE:

d —density unit $\equiv p, S$
D— simultaneous density-group $\equiv S, 2S, \ldots \Sigma$.
$D^{\rightarrow}$—sequent density-group (consecutive D)
Δ (delta) — compound density-group representing density limit in a given score (simultaneous $\Delta \equiv \Sigma$)
$\Delta^{\rightarrow}$ (delta) — sequent compound density group: general symbol for the entire consecutive composition of density: $\Delta^{\rightarrow} = \Sigma$
$\Delta^{\rightarrow}(\Delta^{\rightarrow})$ — the delta of a delta: sequent compound delta.
ϕ (phi) — individual rotation-phase:
$\phi\,\circlearrowleft$ and $\phi\,\circlearrowright$ in reference to t or T
$\phi\,\curvearrowleft$ and $\phi\,\curvearrowright$ in reference to p or P, or d or D
θ (theta) — compound rotation-phase, general symbol of the continuity of rotary groups in a given score; it includes both forms of ϕ.

A. Technical Premise

Depending on the degree of refinement with which the composition of density is to be reflected in a score, d may equal p or S. In scores predominantly using individual parts, either as melodic or harmonic parts, it is possible and advisable to make $d = p$. In scores of predominantly contrapuntal type, where each melody is obtained from a complete S, $d = S$ is a more practical form of assignment.

One of the fundamental forms of variation of the density-groups is rotation of phases.

The *abscissa* (horizontal) *rotation* follows the sequence of harmony ($\circlearrowleft$ or $\circlearrowright$); in it, all pitch-units (neutral or directional) follow the progression originally pre-set by harmony.

The *ordinate* (vertical) *rotation* does not refer to vertical displacement of p or S, but to thematic textures (melody, counterpoint, harmonic accompaniment) only; therefore there is *no vertical rearrangement of harmonic parts at any time.* Such displacement of simultaneously correlated S would completely change the harmonic meaning and the sounding characteristics of the original. Technically such schemes are possible only under the following conditions:

(1) identical interval of symmetry between all strata;
(2) identical structures with identical number of parts in all strata.

The above requirements impose limitations which are unnecessary in orchestral writing, as it means that each orchestral group would have to be represented by the same number of instruments, which is seldom practical.

The idea of *bi-coordinate rotation* (i.e., through the abscissa and through the ordinate) implies that the whole scheme of density in a composition first appears as a graph on a plane, then is folded into a cylindrical (tubular) shape in such a fashion that the starting and the ending duration-units meet, i.e., $\Delta^{\rightarrow} = \text{limit}_1 \leftrightarrow t_m$. Under such conditions the cylinder is the result of bending the graph through ordioate, and the cylinder itself appears in a vertical position. Variations are obtained by rotating this cylinder through abscissa, which corresponds to $\phi\,\circlearrowleft$ and $\phi\,\circlearrowright$.

Therefore: $\Delta^{\rightarrow} = \phi\,\circlearrowleft(t_1 \rightarrow t_m),\ \phi\,\circlearrowright(t_m \rightarrow t_1)$.

Folding the scheme of density (as it appears on the graph) in such a fashion that the lowest and the highest parts of the score meet, we obtain the limits for p, i.e., $\Delta = \lim p_1 \leftrightarrow p_m$. Under such conditions the cylinder is the result of bending the graph through abscissa, and the cylinder itself appears in horizontal position. Variations are obtained by rotating this cylinder through ordinate, which corresponds to $\phi\,\curvearrowleft$ and $\phi\,\curvearrowright$. Therefore: $\Delta^{\rightarrow} = \phi\,\curvearrowleft\begin{pmatrix} pm \\ \uparrow \\ p_1 \end{pmatrix},\ \phi\,\curvearrowright\begin{pmatrix} pm \\ \downarrow \\ p_1 \end{pmatrix}$.

Here delta is consecutive as physical time exists during the period of rotation.

B. Composition of Density-Groups

As we have mentioned before, the choice of p and t, or of S and T as density units, depends on the degree of refinement which is to be attributed to a certain particular score. For the sake of convenience and economy of space, we shall express dt as one square unit of cross-section paper. In each particular case, d may equal p or S, and T may equal t or mt. Yet we shall retain the dt unit of the graph in its general form.

Under such conditions a scale of density-time relations can be expressed as follows:

$$D = d,\ D = 2d,\ \ldots\ D = md$$
$$D^{\rightarrow} = dt,\ D^{\rightarrow} = d2t,\ \ldots\ D^{\rightarrow} = dmt$$
$$D^{\rightarrow} = dt,\ D^{\rightarrow} = 2dt,\ \ldots\ D^{\rightarrow} = mdt$$
$$D^{\rightarrow} = dt,\ D^{\rightarrow} = 2d2t,\ \ldots\ D^{\rightarrow} = mdnt$$

The above are monomial density-groups. On the graph they appear as follows:

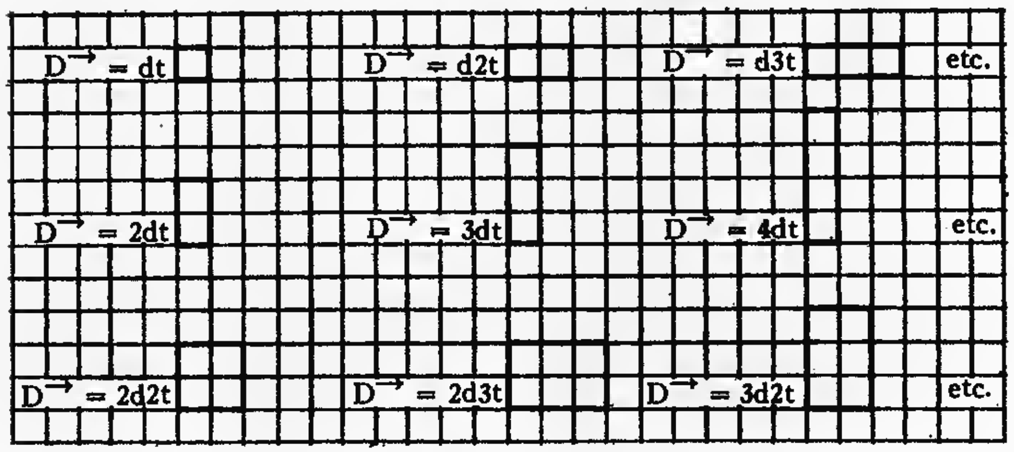


Figure 130. Monomial density-groups.

Binomial density-groups can be evolved in a similar way:

$$\Delta^{\rightarrow} = D_1^{\rightarrow} + D_2^{\rightarrow};\quad D_1^{\rightarrow} = dt;\quad D_2^{\rightarrow} = 2d2t;$$
$$\Delta^{\rightarrow} = dt + 2d2t$$

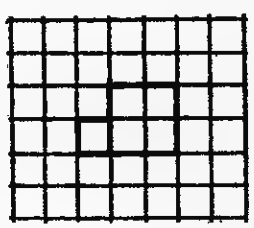
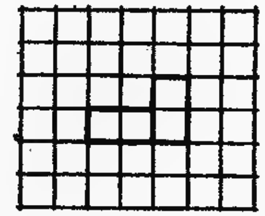


Figure 131. Binomial density-groups (continued.)

$$\Delta^{\rightarrow} = D_1^{\rightarrow} + D_2^{\rightarrow};\quad D_1^{\rightarrow} = d2t;\quad D_2^{\rightarrow} = 2dt;$$
$$\Delta^{\rightarrow} = d2t + 2dt$$



$$\Delta^{\rightarrow} = D_1^{\rightarrow} + D_2^{\rightarrow};\quad D_1^{\rightarrow} = 2d3t;\quad D_2^{\rightarrow} = 5d3t;$$
$$\Delta^{\rightarrow} = 2d3t + 5d3t$$

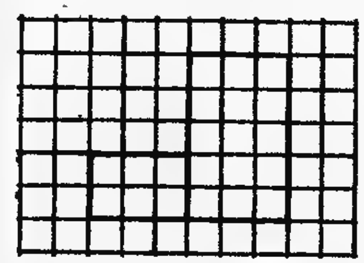
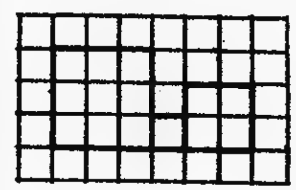


Figure 131. Binomial density-groups (concluded).

Polynomial density-groups may be evolved, depending on the purpose, from rhythmic resultants, permutation-groups, involution-groups, series of variable velocities, etc.

$$\Delta^{\rightarrow} = D_1^{\rightarrow} + D_2^{\rightarrow} + D_3^{\rightarrow};\quad D_1^{\rightarrow} = 3d3t;\quad D_2^{\rightarrow} = dt;\quad D_3^{\rightarrow} = 2d2t;$$
$$\Delta^{\rightarrow} = 3d3t + dt + 2d2t$$



$$\Delta^{\rightarrow} = 4D^{\rightarrow};\quad D_1^{\rightarrow} = 2d4t + 2d2t + 2d2t + 4d2t$$

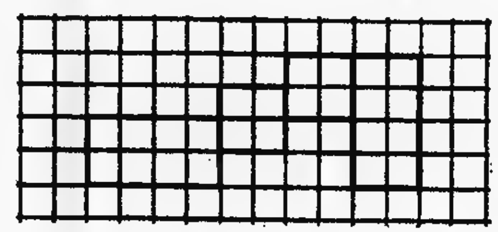
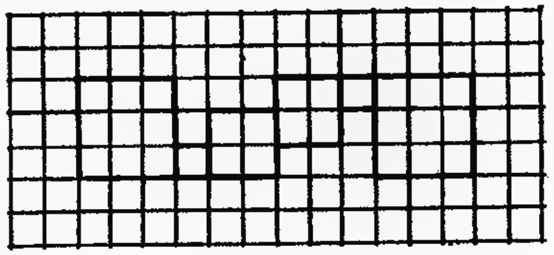


Figure 132. Polynomial density-groups.

As it follows from the above arrangement of density-groups, the latter may be distributed in any desirable fashion, preferably in a symmetric one within the range of D. In this particular case D = 4d.

$$\Delta^{\rightarrow} = 6D^{\rightarrow};\ D_1^{\rightarrow} = 3d3t;\ D_2^{\rightarrow} = dt;\ D_3^{\rightarrow} = 2d2t;\ D_4^{\rightarrow} = 2d2t;\ D_5^{\rightarrow} = dt;\ D_6^{\rightarrow} = 3d3t$$

$$\Delta^{\rightarrow} = 3d3t + dt + 2d2t + 2d2t + dt + 3d3t$$



$$\Delta^{\rightarrow} = 4D^{\rightarrow};\ D_1^{\rightarrow} = 4dt;\ D_2^{\rightarrow} = 2d2t;\ D_3^{\rightarrow} = 2d2t;\ D_4^{\rightarrow} = d4t;$$

$$\Delta^{\rightarrow} = 4dt + 2d2t + 2d2t + d4t$$

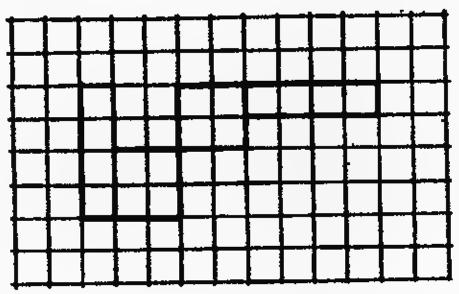


Figure 133. D = 4d.

$$\Delta^{\rightarrow} = 5D^{\rightarrow};\ D_1^{\rightarrow} = d8t;\ D_2^{\rightarrow} = 2d5t;\ D_3^{\rightarrow} = 3d3t;\ D_4^{\rightarrow} = 5d2t;\ D_5^{\rightarrow} = 8dt;$$

$$\Delta^{\rightarrow} = d8t + 2d5t + 3d3t + 5d2t + 8dt$$

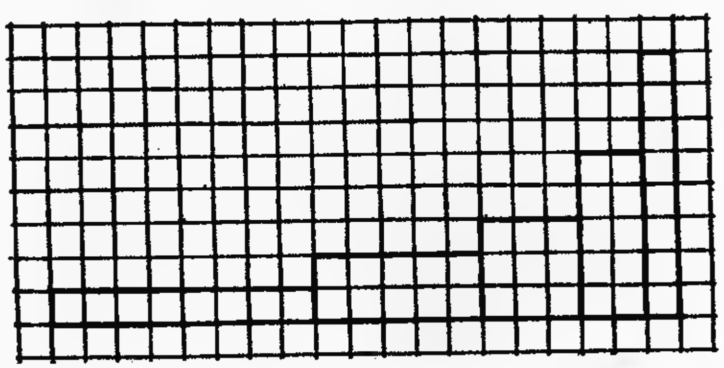
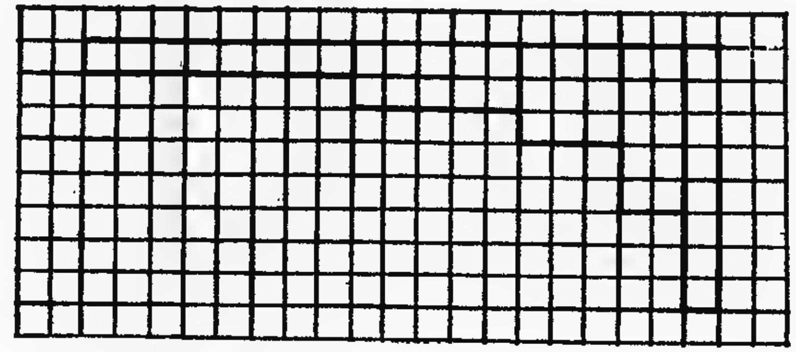


Figure 134. Variants of $\Delta^{\rightarrow} = 5D^{\rightarrow}$ (continued).

Another variant of the same scheme:



Another variant of the same scheme:

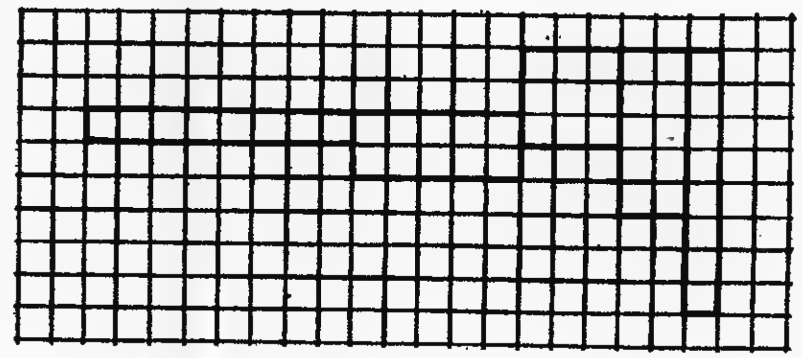


Figure 134. Variants of $\Delta^{\rightarrow} = 5D^{\rightarrow}$ (concluded).

In all the above cases $\Delta \ngtr D$, i.e., the compound density-group is not greater than any of the component density groups.

Density groups may be considerably smaller than Δ, in which case there are many more possibilities for the distribution of D's.

$$\Delta = 6D;\ \Delta^{\rightarrow} = 4D;\ D_1^{\rightarrow} = 2d2t;\ D_2^{\rightarrow} = dt;\ D_3^{\rightarrow} = dt;\ D_4^{\rightarrow} = 2d2t;$$

$$\Delta^{\rightarrow} = 2d2t + dt + dt + 2d2t$$

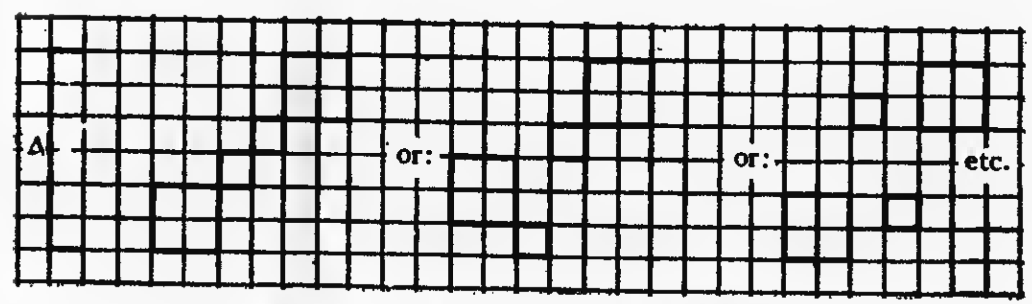
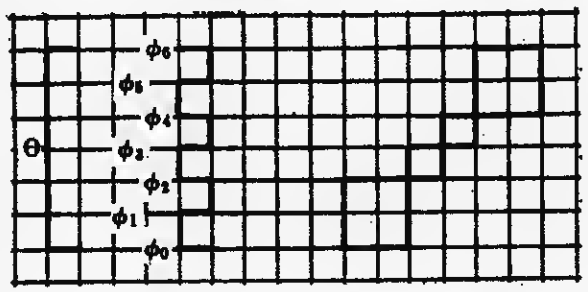


Figure 135. Density groups smaller than Δ.

The different distributions as in the above Figure can be specified by means of their phasic positions.

If we assume that the lowest d of Δ designates ϕ_0, i.e., the zero phase, then ϕ_1, ϕ_2, . . . designate all the consecutive phases. Thus the first variant of Figure 135 can be expressed as follows:

$$\Delta^{\rightarrow} = (2d2t)\phi_0 + (dt)\phi_2 + (dt)\phi_3 + (2d2t)\phi_4.$$



where: $\phi_6 = \phi_0$

Figure 136. First variant of figure 135.

It follows from the above that the first (ϕ_0) and the last (ϕ_6) phases are identical.

C. Permutation of Sequent Density-Groups within the Compound Sequent Density-Group

(Permutations of $D^{\rightarrow}$ within $\Delta^{\rightarrow}$)

Continuity where permutations of $D^{\rightarrow}$'s take place can be designated as a compound sequent group consisting of several other compound sequent density groups, the latter being permutations of the original compound group. Then such a compound density-group yielding n permutations of the original compound sequent density group can be expressed as follows: $\Delta^{\rightarrow}(\Delta^{\rightarrow}) = \Delta_0^{\rightarrow} + \Delta_1^{\rightarrow} + \Delta_2^{\rightarrow} + \ldots \Delta_n^{\rightarrow}$.

$$\Delta_0^{\rightarrow} = (3d3t)\,D_1^{\rightarrow}\phi_0 + (dt)\,D_2^{\rightarrow}\phi_0 + (2d2t)\,D_3^{\rightarrow}\phi_0 + (2d2t)\,D_4^{\rightarrow}\phi_1 + (dt)\,D_5^{\rightarrow}\phi_2 + (3d3t)\,D_6^{\rightarrow}\phi_0, \text{ where } \Delta = 3d.$$

$$\begin{aligned}\Delta^{\rightarrow}(\Delta^{\rightarrow}) \circlearrowleft = \Delta_0^{\rightarrow} + \Delta_1^{\rightarrow} + \Delta_2^{\rightarrow} + \Delta_3^{\rightarrow} + \Delta_4^{\rightarrow} + \Delta_5^{\rightarrow} = \\ = (D_1^{\rightarrow} + D_2^{\rightarrow} + D_3^{\rightarrow} + D_4^{\rightarrow} + D_5^{\rightarrow} + D_6^{\rightarrow}) + \\ + (D_2^{\rightarrow} + D_3^{\rightarrow} + D_4^{\rightarrow} + D_5^{\rightarrow} + D_6^{\rightarrow} + D_1^{\rightarrow}) + \\ + (D_3^{\rightarrow} + D_4^{\rightarrow} + D_5^{\rightarrow} + D_6^{\rightarrow} + D_1^{\rightarrow} + D_2^{\rightarrow}) + \\ + (D_4^{\rightarrow} + D_5^{\rightarrow} + D_6^{\rightarrow} + D_1^{\rightarrow} + D_2^{\rightarrow} + D_3^{\rightarrow}) + \\ + (D_5^{\rightarrow} + D_6^{\rightarrow} + D_1^{\rightarrow} + D_2^{\rightarrow} + D_3^{\rightarrow} + D_4^{\rightarrow}) + \\ + (D_6^{\rightarrow} + D_1^{\rightarrow} + D_2^{\rightarrow} + D_3^{\rightarrow} + D_4^{\rightarrow} + D_5^{\rightarrow})\end{aligned}$$

See Figure 137 on opposite page.

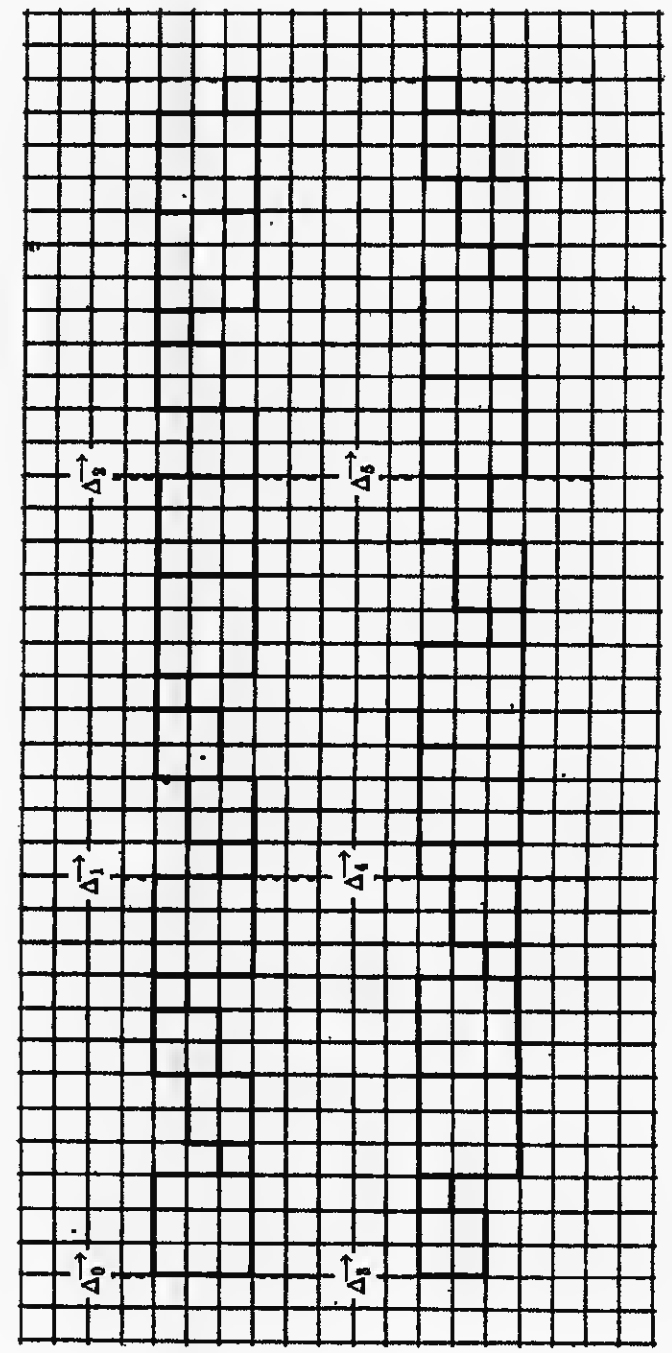


Figure 137. Compound density-group.

The same technique is applicable to all cases where $\Delta > D$, i.e., where delta is greater than any of the simultaneous density-groups.

D. Phasic Rotation of Δ and $\Delta^{\to}$ through t and d

Assuming $\Delta^{\to} = D^{\to} = dt$, we can subject it to rotation:

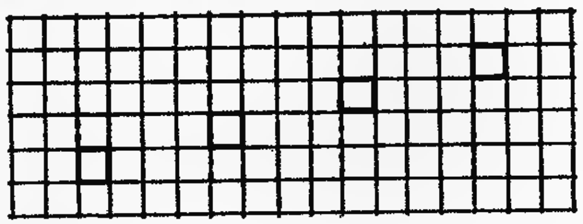
(1) $\Delta^{\to}\theta = t\phi_0 + t\phi_1 + \ldots$ and

(2) $\Delta^{\to}\theta = d\phi_0 + d\phi_1 + \ldots$

The following represents scales of rotation for $\Delta^{\to}T^{\to} = D^{\to}T = dt$; $\Delta = 4d$; $T^{\to} = 4t$; $\phi_4 = \phi_0$. $T^{\to}$ symbolizes the range of duration of D.

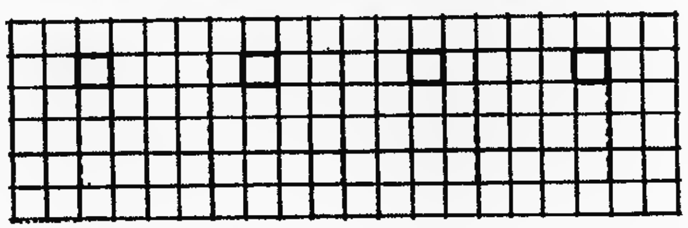
The original position: $d\phi_0\ t\phi_0$
The sequence of rotary phases of d:

$$\Delta^{\to}\theta = d\phi_0\ t\phi_0 + d\phi_1\ t\phi_0 + d\phi_2\ t\phi_0 + d\phi_3 t\phi_0:$$



The sequence of rotary phases of t:

$$\Delta^{\to}\theta = d\phi_0\ t\phi_0 + d\phi_0\ t\phi_1 + d\phi_0\ t\phi_2 + d\phi_0\ t\phi_3:$$



The sequence of rotary phases of dt:

$$\Delta^{\to}\theta = d\phi_0\ t\phi_0 + d\phi_1\ t\phi_1 + d\phi_2\ t\phi_2 + d\phi_3\ t\phi_3:$$

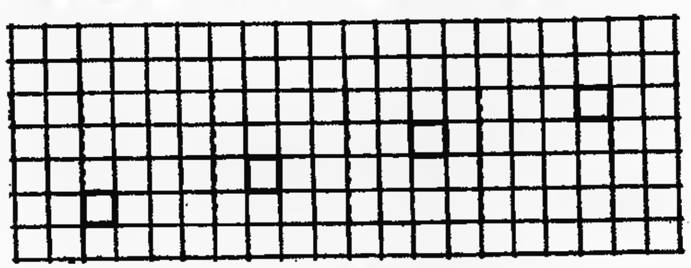


Figure 138. Phasic rotation of Δ and $\Delta^{\to}$.

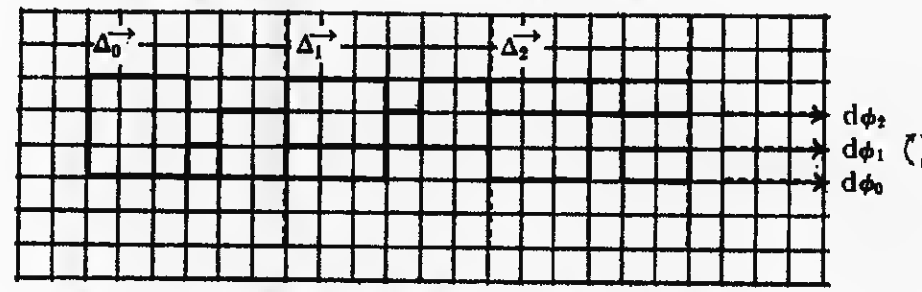
The same technique is applicable to a $\Delta^{\to}$ of any desirable structure.

For example: $\Delta^{\to} = 3D^{\to}$; $D_1^{\to} = 3d3t$; $D_2^{\to} = dt$; $D_3^{\to} = 2d2t$;

$\Delta = 3d \quad \Delta_0^{\to} = (3d3t + dt + 2d2t)\phi_0$;
$T^{\to} = 6t \quad \Delta_1^{\to} = \Delta_0^{\to}\phi_1$; $\Delta_2^{\to} = \Delta_0^{\to}\phi_2$; $\Delta_3^{\to} = \Delta_0^{\to}\phi_3$; . . .
θ and $\Delta_0 = (3d + d + 2d)\phi_0$;
$\Delta_1 = \Delta_0\phi_1$; $\Delta_2 = \Delta_0\phi_2$; $\Delta_3 = \Delta_0\phi_3$; . . .

$$\text{Let } \Delta^{\to}(\Delta^{\to})\,\theta = \Delta_0^{\to}(3d\phi_0 3t\phi_0 + d\phi_0 t\phi_0 + 2d\phi_0 2t\phi_0) + \\ + \Delta_1^{\to}(3d\phi_1 3t\phi_0 + d\phi_1 t\phi_0 + 2d\phi_1 2t\phi_0) + \\ + \Delta_2^{\to}(3d\phi_2 3t\phi_0 + d\phi_2 t\phi_0 + 2d\phi_2 t\phi_0).$$

Then, $\Delta^{\to}(\Delta^{\to})\,\theta = \Delta_0^{\to} + \Delta_1^{\to} + \Delta_2^{\to}$ appears as follows:



$$\text{Let further } \Delta^{\to}(\Delta^{\to})\,\theta = \Delta_0^{\to}(3d\phi_0 3t\phi_0 + d\phi_0 t\phi_0 + 2d\phi_0 2t\phi_0) + \\ + \Delta_1^{\to}(3d\phi_0 3t\phi_1 + d\phi_0 t\phi_1 + 2d\phi_0 2t\phi_1) + \\ + \Delta_2^{\to}(3d\phi_0 3t\phi_2 + d\phi_0 t\phi_2 + 2d\phi_0 2t\phi_2) + \\ + \Delta_3^{\to}(3d\phi_0 3t\phi_3 + d\phi_0 t\phi_3 + 2d\phi_0 2t\phi_3) + \\ + \Delta_4^{\to}(3d\phi_0 3t\phi_4 + d\phi_0 t\phi_4 + 2d\phi_0 2t\phi_4) + \\ + \Delta_5^{\to}(3d\phi_0 3t\phi_5 + d\phi_0 t\phi_5 + 2d\phi_0 2t\phi_5).$$

Then, $\Delta^{\to}(\Delta^{\to})\,\theta = \Delta_0^{\to} + \Delta_1^{\to} + \Delta_2^{\to} + \Delta_3^{\to} + \Delta_4^{\to} + \Delta_5^{\to}$ appears as follows:

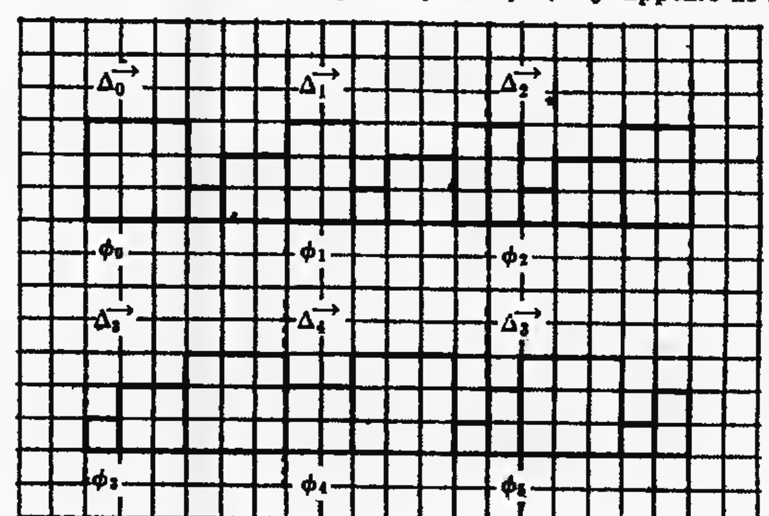


Figure 139. Phasic rotation of $\Delta^{\to} = 3D^{\to}$ (continued).

Now we shall combine the $\theta\circlearrowleft$ and the $\theta\circlearrowright$.

Let $\Delta^{\rightarrow}(\Delta^{\rightarrow})\,\theta = \Delta_0^{\rightarrow} D\theta_0 T^{\rightarrow}\theta_0 + \Delta_1^{\rightarrow} D\theta_1 T^{\rightarrow}\theta_1 + \Delta_2^{\rightarrow} D\theta_2 T^{\rightarrow}\theta_2 +$
$+ \Delta_3^{\rightarrow} D\theta_0 T^{\rightarrow}\theta_3 + \Delta_4^{\rightarrow} D\theta_1 T^{\rightarrow}\theta_4 + \Delta_5^{\rightarrow} D\theta_2 T^{\rightarrow}\theta_5$

Then $\Delta^{\rightarrow}(\Delta^{\rightarrow})\,\theta = \Delta_0^{\rightarrow} + \Delta_1^{\rightarrow} + \Delta_2^{\rightarrow} + \Delta_3^{\rightarrow} + \Delta_4^{\rightarrow} + \Delta_5^{\rightarrow}$ appears as follows:

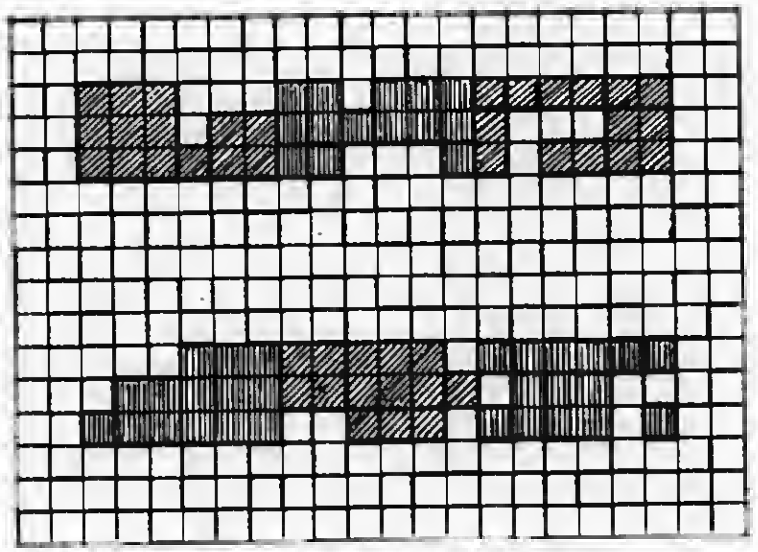


Figure 139. Phasic rotation of $\Delta^{\rightarrow} = 3D^{\rightarrow}$ (concluded).

The diagonal and vertical lines are inserted for clarity.

The addition of positive or negative phases of rotation to any given position of $\Delta^{\rightarrow}$ follows the rules of algebraic addition. Thus if the given position is ϕ_0, the addition of one $\phi\circlearrowright$ or $\circlearrowleft$ brings the density-group into position ϕ_1, or: $\phi_0 + \phi = \phi_1$. Likewise $\phi_0 + 2\phi = \phi_2$, $\phi_0 + m\phi = \phi_m$.

As the last phase equals the first phase, or $\phi_n = \phi_0$, negative quantities of phases, or the counterclockwise phases, i.e., $\phi\circlearrowright$ or $\phi\circlearrowleft$, must be added with the sign minus to the last phase. Thus if the given position is ϕ_0 and the number of phases is n, the addition of one negative phase brings the density-group into position ϕ_{n-1}; or, $\phi_n - \phi = \phi_{n-1}$. Likewise, $\phi_n - 2\phi = \phi_{n-2}$, $\phi_n - m\phi = \phi_{n-m}$.

Problem: find the phase ϕ after the following forms of rotation have been performed from the original ϕ_0, where $\theta = 8\phi$: $2\phi - 3\phi + 5\phi + \phi - 4\phi + 3\phi - \phi$.

Solution: $\phi_x = \phi_0 + 2\phi - 3\phi + 5\phi + \phi - 4\phi + 3\phi - \phi = \phi_0 + 11\phi - 8\phi = \phi_0 + 3\phi = \phi_3$, i.e., the density group appears in its third phase.

This is applicable to both ordinate and abscissa. It follows from the above reasoning that in order to obtain the original position θ_0, after performing a group of phasic rotations, the sum of the coefficients of ϕ must equal zero. As

we know from the *Theory of Rhythm*,* all resultants with an *even number of terms* have identical terms in both halves of the resultants. If such terms, used as coefficients of ϕ, are supplied with alternating "plus" and "minus", the sum of the whole resultant would be zero. This gives a perfect solution for the cases of variation of density groups, because resultants, being symmetric, produce a perfect form of continuity.

Examples:

$r_{4\div3} = 3+1+2+2+1+3$; changing the signs, we obtain:
$3-1+2-2+1-3 = 6-6 = 0.$

$r_{5\div4} = 4+1+3+2+2+3+1+4$; changing the signs, we obtain:
$4-1+3-2+2-3+1-4 = 10-10 = 0.$

$\theta(r_{7\div2}) = \phi_0 + 2\phi - 2\phi + 2\phi - \phi + \phi - 2\phi + 2\phi - 2\phi =$
$= \phi_0 + 7\phi - 7\phi = \phi_0 + 0 = \phi_0.$

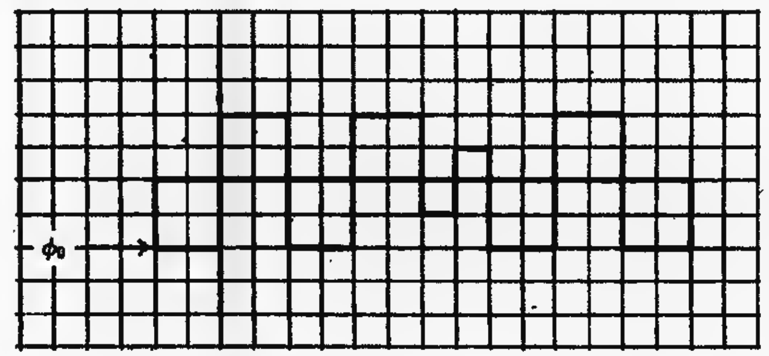


Figure 140. Applying resultants from the theory of rhythm.

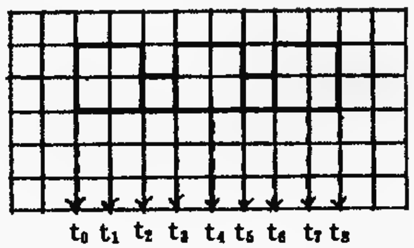
Computation of the phasic position θ_x, which is the outcome of a group of phasic rotation, can be applied to any position θ_m to which such rotations have been applied. The computation is performed through the use of same technique as before, i.e., through algebraic addition.

Problem: Let the original $\theta_0 = \phi_3$; find θ_x after the following group of rotations: $2\phi - 3\phi - \phi + 6\phi$, where $\theta = 8\phi$.

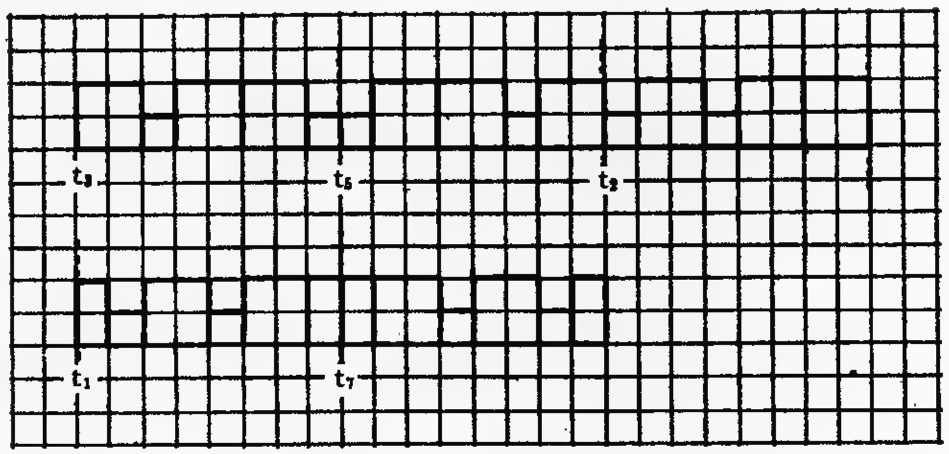
Solution: $\theta_x = \phi_3 + 2\phi - 3\phi - \phi + 6\phi = \phi_3 + 8\phi - 4\phi = \phi_3 + 4\phi = \phi_7.$

*See Vol. I, p. 10 ff.

The original $\overrightarrow{\Delta_0}$: $\overrightarrow{T} = 8t$



Variation of continuity: $\overrightarrow{\Delta_x} = t_8 + 2t - 3t - t + 6t = t_7$



The same group of conditions, as applied to D:

The original $\overrightarrow{\Delta_0}$: $\Delta = 8d$

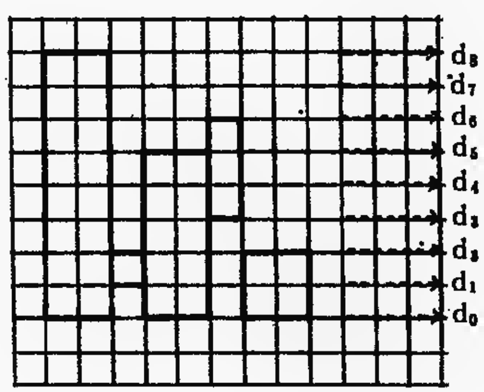
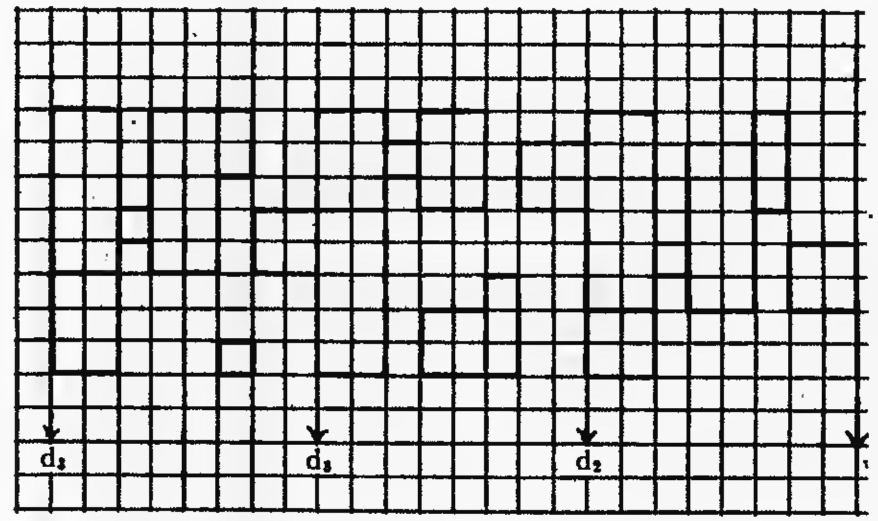


Figure 141. Rotations: $2\phi - 3\phi - \phi + 6\phi$ where $\Theta = 8\phi$.

Variation of continuity: $\overrightarrow{\Delta_x} = d_8 + 2d - 3d - d + 6d = d_7$



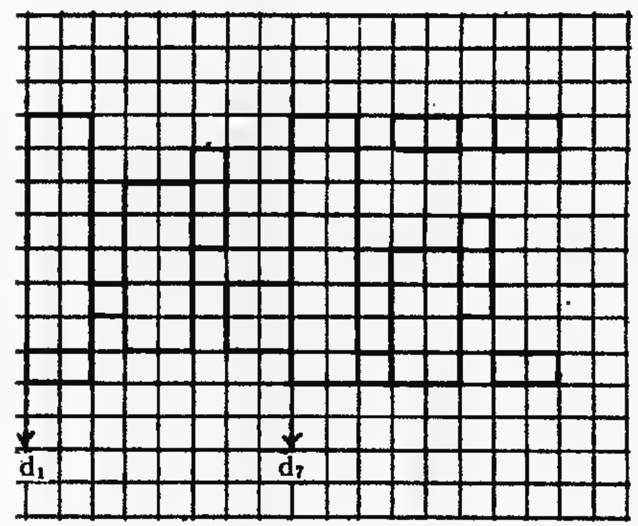


Figure 141. Rotations: $2\phi - 3\phi - \phi + 6\phi$ where $\Theta = 8\phi$ (concluded).

The next step is to combine the phasic rotation of both coordinates. Assuming first that both d and t are in their zero phases, we can express this as follows:

$\overrightarrow{\Delta_0}\Theta_0 = d_0t_0$, i.e., the rotary phase of a consecutive density-group for both coordinates (density and time) is zero. Now we can subject the $\overrightarrow{\Delta_0}\Theta_0$ to variations where the groups of phasic rotation are identical for both coordinates. Using the number values from the preceding example, we obtain the following:

$\overrightarrow{\Delta_x}\Theta_x = dt\phi_3 + dt2\phi - dt3\phi - dt\phi + dt6\phi = dt\phi_7.$

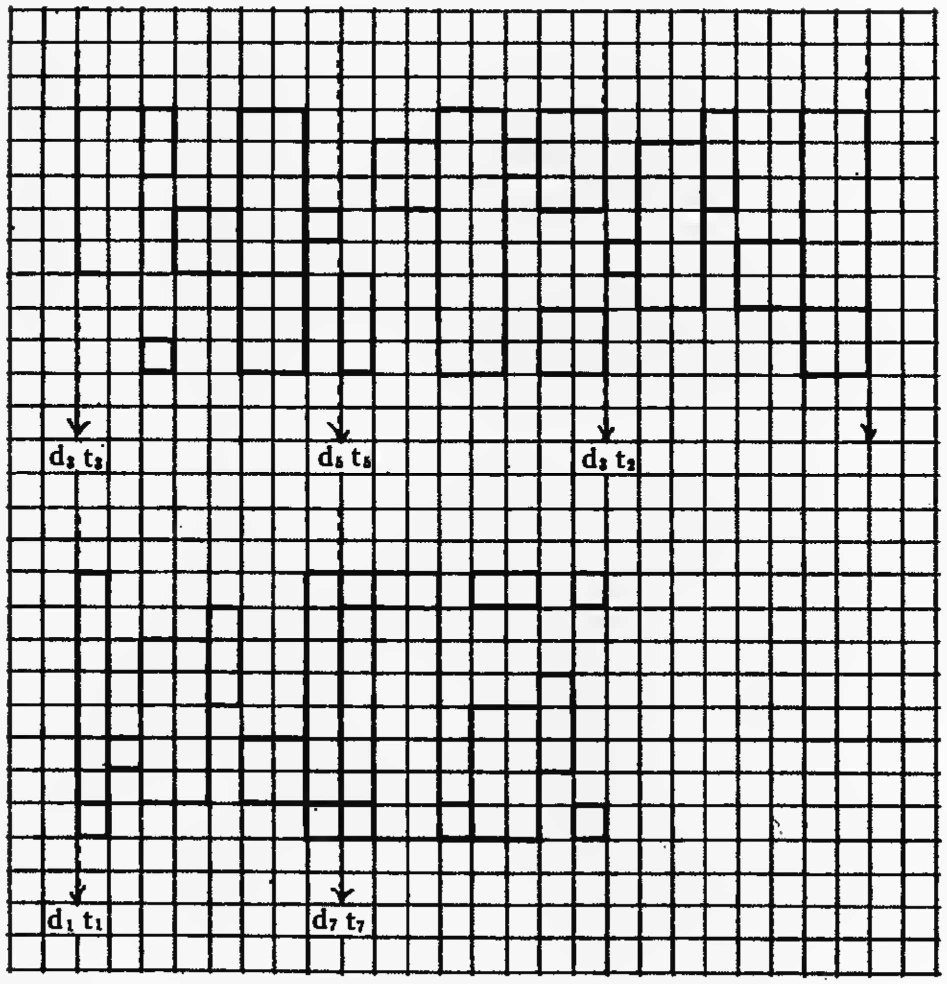


Figure 142. Combining phasic rotation of both coordinates.

When the rotation groups are different for both coordinates, the interference of phases of $\frac{d}{t}$ takes place. After a number of rotations has been performed through both coordinates, their respective resulting phases may be different.

Let $D = 8d$ and $T^{\rightarrow} = 8t$. Let further $\Theta_0 = d_3t_5$.

Now let us subject this group to the following form of phasic rotation: $d2\phi()t\phi\circlearrowleft + d4\phi()t3\phi\circlearrowleft + d2\phi()t3\phi\circlearrowleft$.

Then: $\overrightarrow{\Delta_x}\Theta_x = d_3t_5 + 2d - t + 4d + 3t - 2d - 3t =$
$= d_3t_3 + 4d - t = d_7t_4.$

Assuming Θ to be the limit of a rotation-group (cycle of rotation) and Θ^1 the sum of phasic rotations, we encounter the following conditions:

(1) $\Theta^1 \not> \Theta$ and (2) $\Theta^1 \not> \Theta$, $\Theta^1 \not> 2\Theta$, . . . $\Theta^1 > m\Theta$.

Under the first condition, the sum of phasic rotations does not exceed the total range of rotation, in which case all computations are carried out as shown before:

If $\Theta^1 \not> \Theta$, then: $\phi_0 + \Theta^1 = \phi_{\theta 1}$.

Under the second condition, the sum of phasic rotations does exceed the total range of rotation, in which case the relations of the sum of rotations with the range (limit) are as shown in (2).

In this case, computations must be carried out through use of the following formulae:

If $\Theta^1 > \Theta$, then: $\phi_0 + \Theta^1 = \phi_0 + (\Theta^1 - \Theta) = \phi_{\theta 1-\theta}$;
If $\Theta^1 > 2\Theta$, then: $\phi_0 + \Theta^1 = \phi_0 + (\Theta^1 - 2\Theta) = \phi_{\theta 1-2\theta}$;
. . . .
If $\Theta^1 > m\Theta$, then: $\phi_0 + \Theta^1 = \phi_0 + (\Theta^1 - m\Theta) = \phi_{\theta 1-m\theta}$.

Examples:

(1) $\overset{3}{\underset{0}{\Theta}}$ (read: theta to the limit from zero to three)
$\phi_0 + 2\phi = \phi_2$; $\phi_0 + 3\phi = \phi_3 = \phi_0$;
$\phi_0 + 4\phi = 4\phi - 3\phi = \phi_1$, where $\Theta^1 = 4\phi$;
$\phi_0 + 5\phi = 5\phi - 3\phi = \phi_2$, where $\Theta^1 = 5\phi$.

(2) $\overset{3}{\underset{0}{\Theta}}$
$\phi_0 + 6\phi = 6\phi - 2\cdot 3\phi = 6\phi - 6\phi = \phi_0$, because if $\Theta^1 = 6\phi$, $\Theta^1 > 2\Theta$;
$\phi_0 + 7\phi = 7\phi - 2\cdot 3\phi = 7\phi - 6\phi = \phi_1$, where $\Theta^1 = 7\phi$;
$\phi_0 + 8\phi = 8\phi - 2\cdot 3\phi = 8\phi - 6\phi = \phi_2$, where $\Theta^1 = 8\phi$.

(3) $\overset{3}{\underset{0}{\Theta}}$
$\phi_0 + 9\phi = 9\phi - 3\cdot 3\phi = 9\phi - 9\phi = \phi_0$, because if $\Theta^1 = 9\Theta$, $\Theta^1 > 3\Theta$;
$\phi_0 + 10\phi = 10\phi - 3\cdot 3\phi = 10\phi - 9\phi = \phi^1$, where $\Theta^1 = 10\phi$.

(4) $\overset{5}{\underset{0}{\Theta}}$; $\Theta^1 = 3\phi - \phi + 9\phi - 2\phi + 4\phi = 13\phi$;
$\phi_0 + 13\phi = 13\phi - 2\cdot 5\phi = 13\phi - 10\phi = \phi_3$.
$\phi_3 + 13\phi = 3\phi + 13\phi - 10\phi = 16\phi - 3\cdot 5\phi = 16\phi - 15\phi = \phi_1$;
as Θ^1 being added to ϕ_3 equals $3\phi + 13\phi = 16\phi$, in which case $\Theta^1 > 3\Theta$.

(5) $\overset{5}{\underset{0}{\Theta}}$; $\Theta^1 = -3\phi + \phi - 9\phi + 2\phi - 4\phi = -13\phi$;
$\phi_0 - 13\phi = -13\phi + 2\cdot 5\phi = -13\phi + 10\phi = -3\phi = \phi_{-3}$;
to locate ϕ_{-3} the latter must be subtraoted from Θ:
$\Theta - \phi_{-3} = \Theta - 3\phi = 5\phi - 3\phi = 2\phi = \phi_2$.

Figure 143. Sum of phasic rotations.

The technique of phasic rotation of the density-groups can be pursued to any desirable degree of refinement. The phases of d and t can be synchronized when they are subjected to independent rotary groups, in which case we follow the usual formula:

$$\frac{\theta d}{\theta t} = \frac{\theta^1 d}{\theta^1 t}; \qquad \frac{\theta^1 t\ (\theta d)}{\theta^1 d\ (\theta t)}.$$

In composing the original density-group ($\overrightarrow{\Delta_0}\overrightarrow{T_0}$), it is important to take into consideration the character of $\frac{D}{T}$ relations with regard to the effects such relations produce. In this respect we can rely on the three fundamental forms of correlation, which are mentioned for the first time in the *Theory of Melody*,* i.e., the parallel, the oblique and the contrary.

When they are applied to density-groups, these three forms must be interpreted in the following way:

(1) parallel: identical ratios of the coefficients of ϕd and ϕt;
(2) oblique: non-identical ratios of the coefficients of ϕd and ϕt, where—
 (a) partial coincidence of the coefficients takes place, and/or
 (b) the coefficient of one of the components (either d or t) remains constant;
(3) contrary: identical ratios arranged in inverted symmetry. When the number of coefficients in both coefficient-groups is odd, such case should be classified as *oblique*, due to partial coincidence of coefficients.

E. Practical Application of $\Delta^{\rightarrow}$ to $\Sigma^{\rightarrow}$.

(Composition of Variable Density from Strata)

In its complete form, this subject belongs to the field of Textural Composition and will be treated in this chapter only to the extent necessary in order to make the whole subject more tangible.

The first consideration is that $\Delta^{\rightarrow}$ can be composed to a given $\Sigma^{\rightarrow}$, or $\Sigma^{\rightarrow}$ can be composed to a given $\Delta^{\rightarrow}$. This means that either a progression of chords in strata or a density-group may be the origin of a whole composition. One harmonic progression may be combined with more than one density-group; the opposite is also true, i.e., more than one harmonic progression can be written to the same group of density. For this reason the composer's work on such a scheme may start either with $\Sigma^{\rightarrow}$ or with $\Delta^{\rightarrow}$.

It is practical to consider d = S as the most general form of the density-unit, leaving d = p for cases of particular refinement with regard to density. If d = S it means that one density-unit may consist of p, 2p, 3p or 4p. In actuality, however, harmonic strata acquire instrumental forms, in which case even S4p may sound like rapidly moving melodies. On the other hand, S may be transformed into melody, in which case we also hear one part. The implication

*See Vol. I, p. 275.

is that, in the average case, the density of a melodic line and the density of harmony subjected to instrumental figuration are about the same. Physically and physiologically, and therefore psychologically, ***density is in direct proportion to mobility.*** This means, for example, that a rapidly moving instrumental form of successive single attacks, which derives from S4p, is nearly as dense as a sustained chord of S4p; the extreme frequency of attacks makes an arpeggio sound like a chord, i.e., in our perception, lines aggregate into an assemblage.

In addition to this, it is important to realize that the insignificant difference, in the case just described, may be completely compensated for by the presence of directional units. In the above illustration, these would counterbalance sustained harmony of S4p by the highly mobile line which derives from 8 units (= 4 directional units, corresponding to 8 attacks, and in an average tempo acquiring high mobility).

As we have seen before, composition of density in its application to strata refers either to melody or harmony as thematic texture. Both melody and harmony can be present in the form of several coordinated parts. For example, there may be 3 correlated melodies and 2 harmonic accompaniments. Of course, any scheme of density may include correlated melodies alone or harmony alone.

The technique of superimposition of $\Delta^{\rightarrow}$ upon $\Sigma^{\rightarrow}$ consists of establishing correspondences between ϕd and p, and between ϕt and H, i.e., between the density-phase, or density-unit, and the number of harmonic parts; and between the duration-phase, or duration-unit, of the sequent density-group and the number of successive chords.

All subsequent techniques pertain to composition of continuity, i.e., to coordination of attacks and durations, instrumental forms, etc.

We shall now evolve an illustration of $\Delta^{\rightarrow}$ correlated with $\Sigma^{\rightarrow}$. To demonstrate this technique beyond doubt, we shall use the most refined form of it, where d = p and t = H.

If Nt = NH, then the cycle of $\Delta^{\rightarrow}$ and $\Sigma^{\rightarrow}$ are synchronized a priori; otherwise, (i.e., if $\frac{Nt}{NH} \neq 1$) they have to be synchronized. This shows that with just a few chords and a relatively brief scheme of density, one can evolve a composition of considerable length, since $\Delta^{\rightarrow}$ itself, in addition to interference with $H^{\rightarrow}$ of $\Sigma^{\rightarrow}$, can be subjected to rotational variations.

Let the original $\overrightarrow{\Delta_0} = \Delta = 8d$ (see Fig. 141).

Let $\overrightarrow{\Delta_0} = \Delta\ \phi_0 2t + d\phi_0 t + 5d\phi_0 2t + 3d\phi_3 t + 2d\phi_0 2t$.

As $\Delta = D = 8d$, Σ must equal 8p.

$T^{\to} = 8t$ and would require $H^{\to} = 8H$, unless we wish to introduce a case in which $\frac{T^{\to}}{H^{\to}} \neq 1$.

We shall introduce such a case.

Let $H^{\to} = 5H$. Then $\frac{T^{\to}}{H^{\to}} = \frac{8}{5};\ \frac{5\,(8)}{8\,(5)}$.
Hence, $T^{\to 1} = 8t \cdot 5 = 40t$.

As we intend to use 5 variations of $\Delta^{\to}$, the entire cycle will be synchronized (completed) in the form: $\Delta^{\to}(\Delta^{\to}) = 40t\ 40H$, where $H^{\to}$ (= 5H) appears 8 times.

For the sake of greater pliability of thematic textures, it is desirable to pre-set a directional sigma.

We shall choose the following sigma: $\Sigma = S_I 2p + S_{II} 3p + S_{III} 3p$ and $\Sigma^{\to} = 5H$.

Let $I^{\to} = 3i + 2i + 3i + 5i$ and $I(\Sigma) = \dfrac{\sqrt[12]{2^7}}{\sqrt[6]{2}}$

We shall now subject the $\Delta_0^{\to}\Theta_0\Sigma_0^{\to}$ to variations of density evolved in Figure 142.

Further elaboration of the above scheme into thematic textures will be discussed in the Theory of General (Textural) Composition.*

Similar schemes should be evolved by the student with the application of d = S.

It would not be entirely premature to convert the $\Delta^{\to}\Sigma^{\to}$ schemes into thematic textures, as the last nine chapters contain sufficient information on converting strata into melody and harmony, including instrumental treatment.

Figure 144. $\Delta^{\to}$ correlated with $\Sigma^{\to}$ (continued).

*See p. 1279.

Transcription into $\Delta_0^{\to}$.

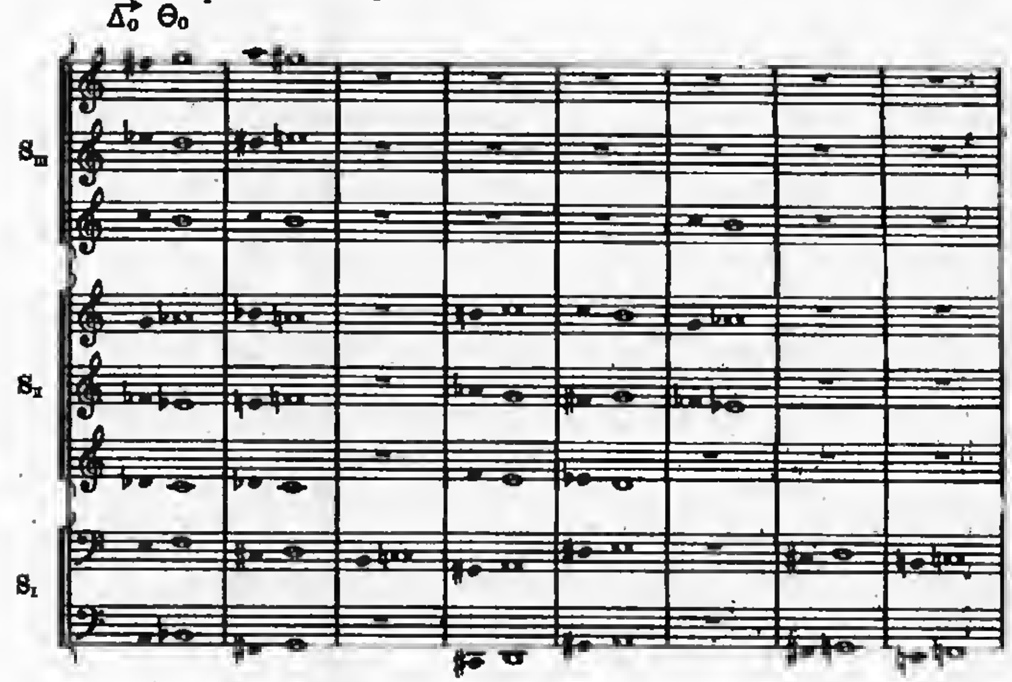


Figure 144. $\Delta^{\to}$ correlated with $\Sigma^{\to}$

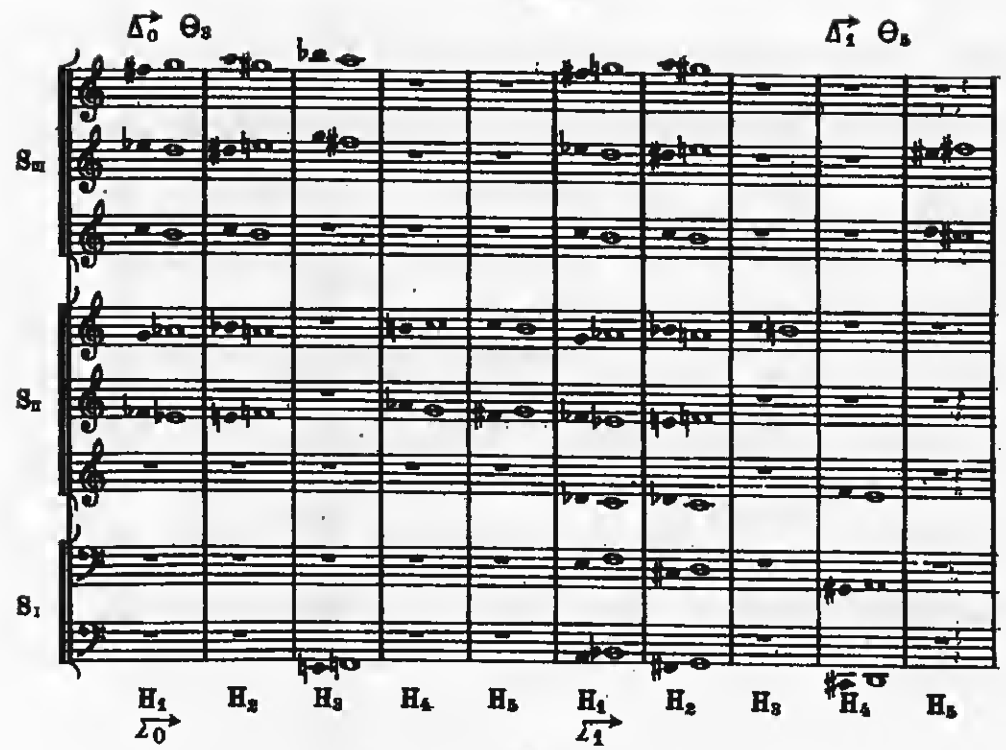


Figure 145. Variation of density of figure 142. $\Delta_0^{\to}\Theta_3$, $\Delta_1^{\to}\Theta_5$.

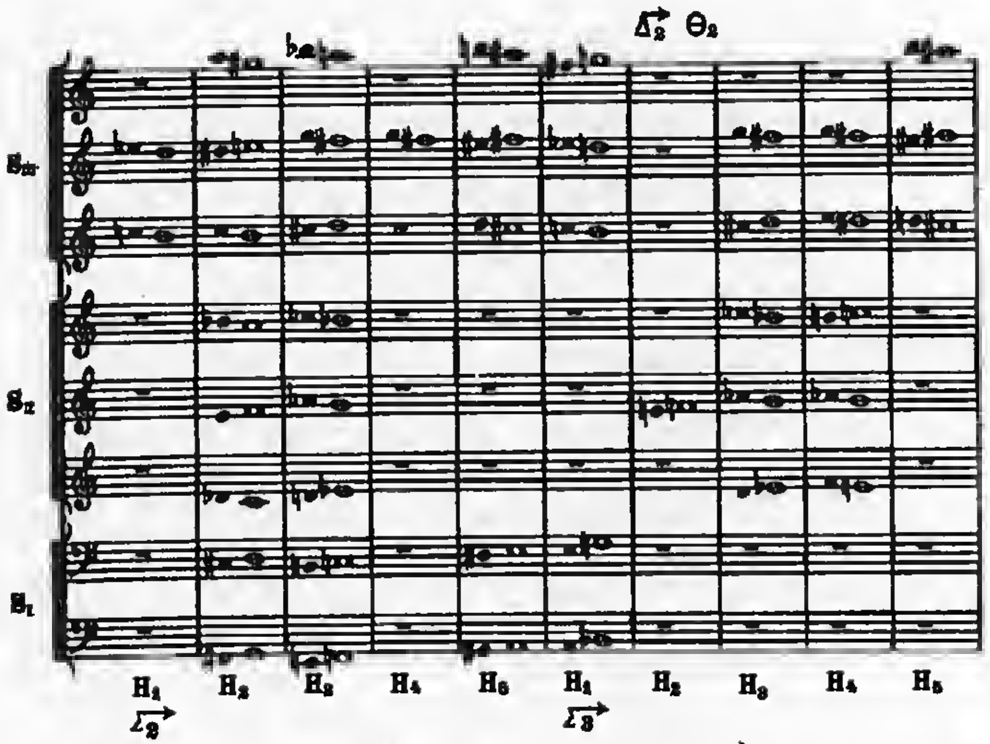


Figure 146. Variation of density of figure 142. $\overrightarrow{\Delta_2}\Theta_2$.

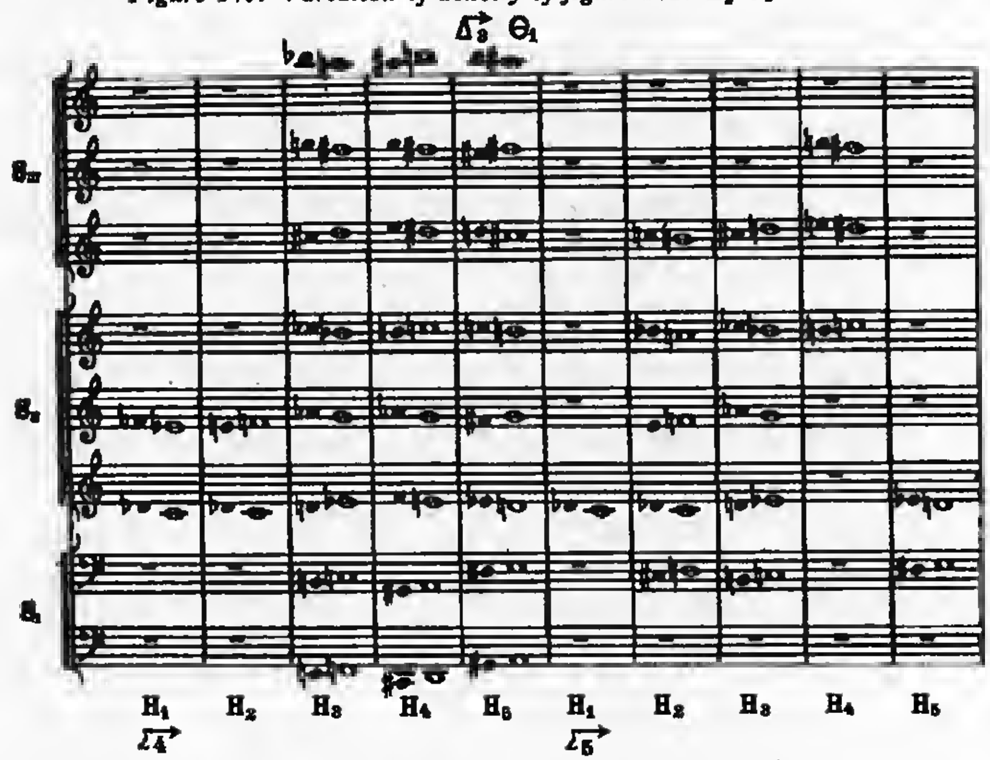


Figure 147. Variation of density of figure 142. $\overrightarrow{\Delta_3}\Theta_1$.

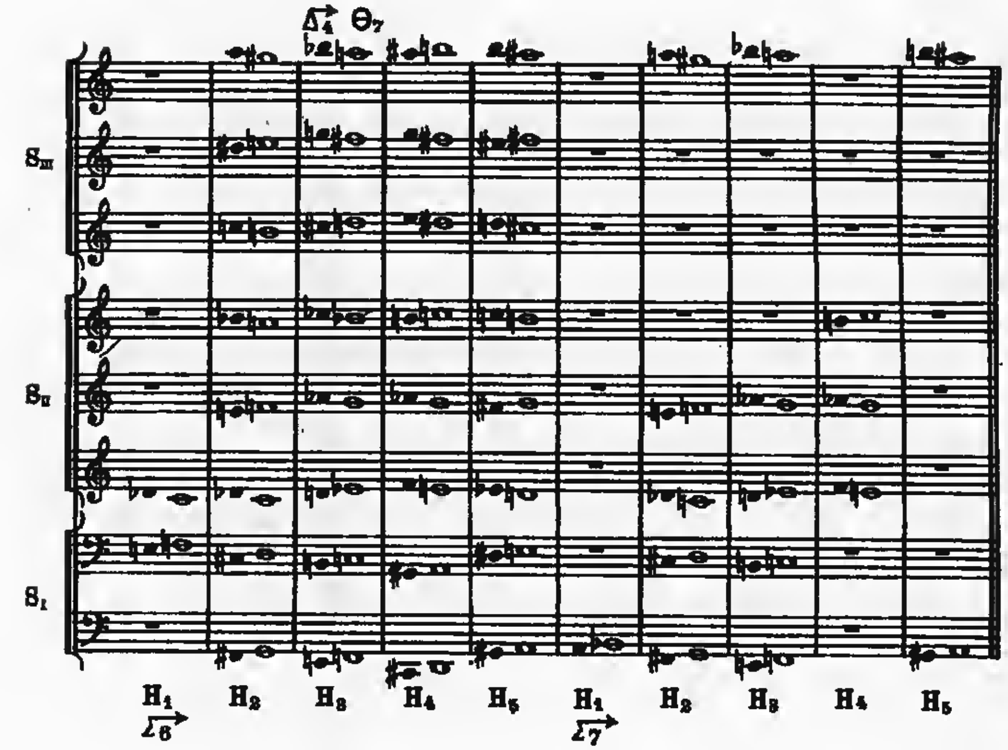


Figure 148. Variation of density of figure 142. $\overrightarrow{\Delta_4}\Theta_1$.

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK X

EVOLUTION OF PITCH-FAMILIES
(STYLE)

BOOK TEN

EVOLUTION OF PITCH-FAMILIES (STYLE)

INTRODUCTION

Unity of style evolved in one musical continuity or in a complete composition is, under ordinary circumstances, a task consuming most of a composer's life. To arrive at perfect auditory discrimination and orientation in any new material is a task of great difficulty. Only the greatest composers known were able to mold their own individual styles, and even in these cases, the crystallization of their own styles were actually prepared by a similar effort of their great predecessors.

The problem of unity of style in intonation, when approached from an analytical angle, becomes nothing but a methodological problem. If the factors which contribute to unity of style can be detected, then there is assurance that such unity can be achieved through scientific synthesis.

The factors determining that certain groups of intonation belong to one family are: (1) the identity of pitch-units, and (2) the identity of intervals.

CHAPTER 1

PITCH-SCALES AS A SOURCE OF MELODY

OFTEN styles of intonation can be defined geographically and historically. There may be a certain national style which, in due course of time, undergoes various modifications. These modifications, often associated with the progress of a civilization, can also be looked upon as modernization of the source.

The easiest way to illustrate this viewpoint is by demonstrating the source (*the true primitive*) and its stages of evolution (*the stylized* and *the modernized primitive*). For example, "Dixieland" improvised music of old New Orleans or plantation-songs of the Negroes of the South, or tribal songs of the American Indians, or ritual songs and dances and incantations of the Russian peasants in the sub-arctic north—are all true primitives. The various forms of "jazz" and "swing," the "Indian" music of MacDowell or Cadman or Stravinsky (*Les Noces*), are stylized or modernized primitives—each, of course, in its respective field.

Technically, the source of a *true primitive* is the First Group of Scales (see *Theory of Pitch-Scales*):* particularly, scales with few pitch-units.

The sources of *stylized primitive* are:

(1) derivative scales obtained through permutations of the pitch-units;
(2) derivative scales transposed to one axis;
(3) derivative scales obtained through permutation of the intervals;
(4) derivative scales obtained through direct transposition of the intervals of the original scale to its own consecutive pitch-units;
(5) directional units applied to the above 4 categories.

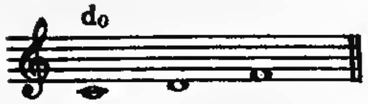
The sources of *modernized primitive* are:

(1) symmetric scales evolved from the primitive original, by assigning the interval between the extreme pitch-units as the interval of symmetry for the compound scale in which each sectional scale corresponds to the original; the family-scales of the original become the family-scales of the compound symmetric scale;
(2) symmetric superimposition of sectional scales and chords derived therefrom, in which progressions of chords derive from the same compound symmetric scale.

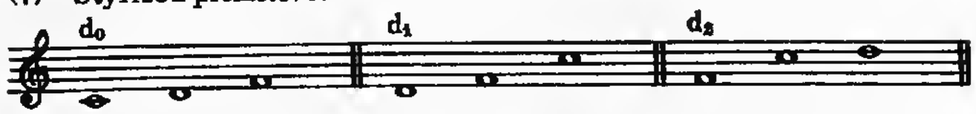
All the above resources are treated independently in their own sub-classifications, where the usual techniques, such as composition of melodic forms by permutation, superimposition of durations, etc., are used.

*See Vol. I, p. 103.

The original primitive:



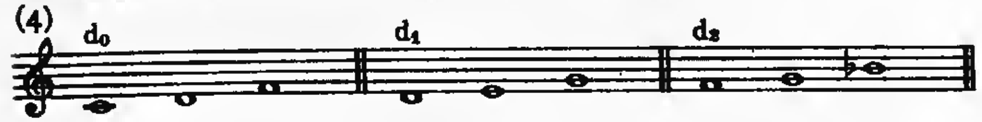
(1) Stylized primitive:



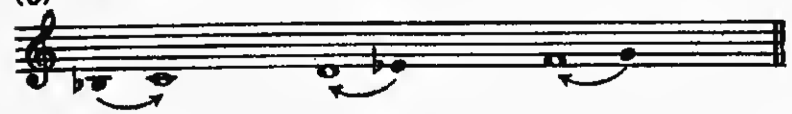
(2)

(3)

(4)

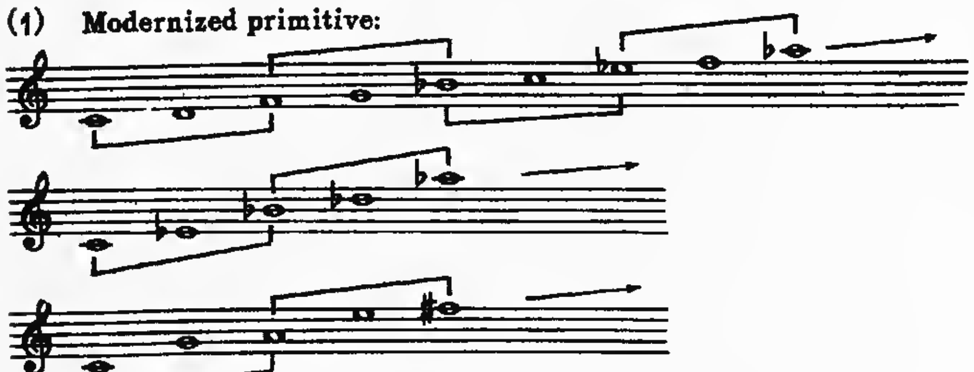


(5)

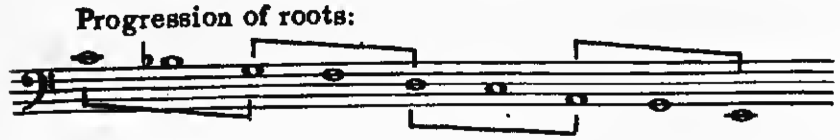


and others

(1) Modernized primitive:



Progression of roots:



etc.

Figure 1. Original, stylized and modernized primitives (continued).

(2)

etc.

Figure 1. Original, stylized and modernized primitives (concluded).

CHAPTER 2

HARMONY

A. Diatonic Harmony

DIATONIC structures, as well as diatonic chord progressions, derive from a pitch scale. Chord structures can be evolved by means of the first tonal expansion, thus serving as accompanying harmony to the original scale. Let us take the fundamental Balinese scale and designate it as E_0. Then E_1 represents the expanded scale. By placing this scale vertically and starting with each consecutive degree of the original E_0 as a root tone, we obtain $\Sigma(E_1)$ on all degrees of the original scale.

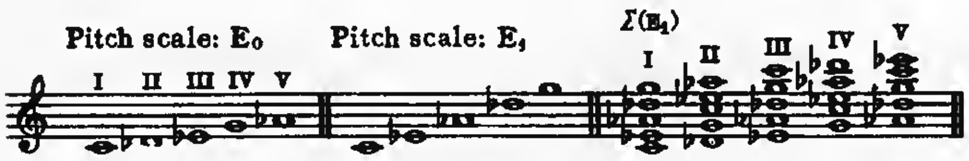


Figure 2. E_0, E_1 and $\Sigma(E_1)$

From these complete forms, partial forms of S2p, S3p or S4p can be obtained, thus offering structures for all degrees of the scale in the form of diads, triads and tetrads.

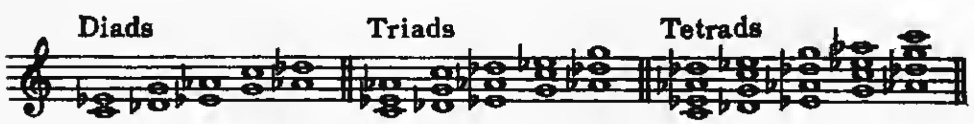


Figure 3. S2p, S3p and S4p.

The fundamental harmony scale is the E_1ⓑ of the original scale. As in all scales which do not contain all 7 musical names, expansions do not produce analogous musical intervals (such as 3rds or 4ths). Diatonic cycles cannot be determined by such names and will be indicated numerically. The fundamental harmony-scale will be referred to as the first cycle (C_1), the second cycle (C_2), and the third cycle (C_3). The second cycle represents the first tonal expansion of the original harmony scale, the third cycle represents the second tonal expansion of the original scale. Cadences are formed by tones adjacent to the tonics, thus producing directional units around the tonic.

In the following table the initial cadences are at the beginning, the final cadences are in the middle and the compound cadences are at the end. The left side of the table represents the fundamental positive cycles of the Balinese scale. By reading this table backwards we obtain the negative system of cycles.

Diatonic Cycles (Positive)

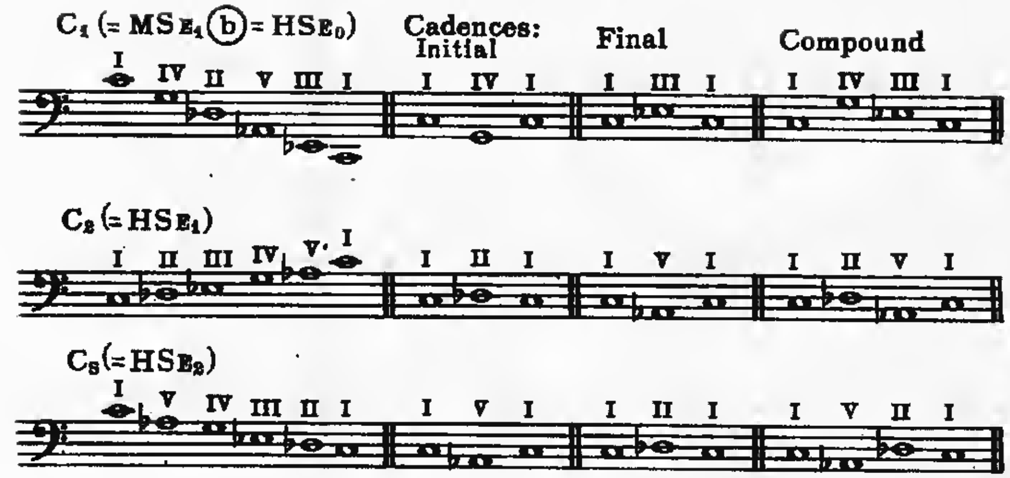
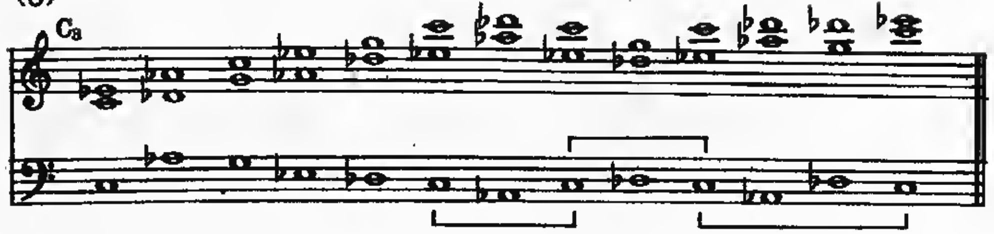


Figure 4. Diatonic cycles (positive). Read backward for negative.

Diatonic cycles and their mixtures are applicable to all types of harmony. The first illustration represents the 3 diatonic cycles used individually and in combinations. The form of harmony: hybrid 3-part harmony (functions a, b, and constant a in the bass).

Figure 5. Hybrid 3-part harmony; functions a, b, and constant a in bass (continued).

(3)



(4)

Figure 5. Hybrid 3-part harmony, functions a, b, and constant a in bass (concluded).

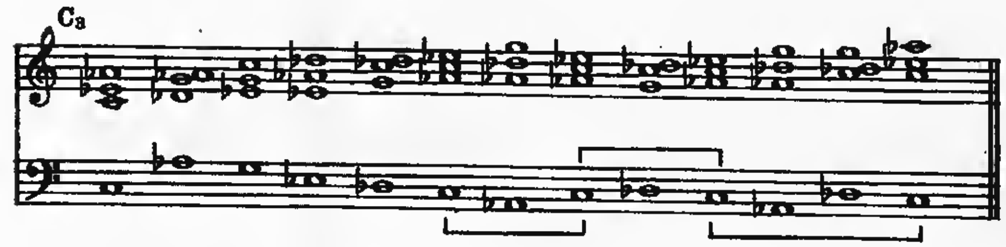
Progressions with more parts can emphasize any desirable choice of functions which may or may not include a coincidence of one function with the bass. For example, hybrid 4-part harmony may be constructed on the basis of a–function in the bass, and a, b, c, in the 3 upper parts; or a in the bass, and b, c, d, in the 3 upper parts.

In Figure 6, all progressions have the first form. The sequence of cycles corresponds to that of Figure 5. Transformations are chosen through the nearest pitch positions.

Progressions Type I (Diatonic)

$$\Sigma = \frac{S_{II}\,3p}{S_I\,p}$$

Figure 6. Progressions Type I (Diatonic). $\Sigma = \frac{S_{II}3p}{S_{I}p}$ *(continued).*



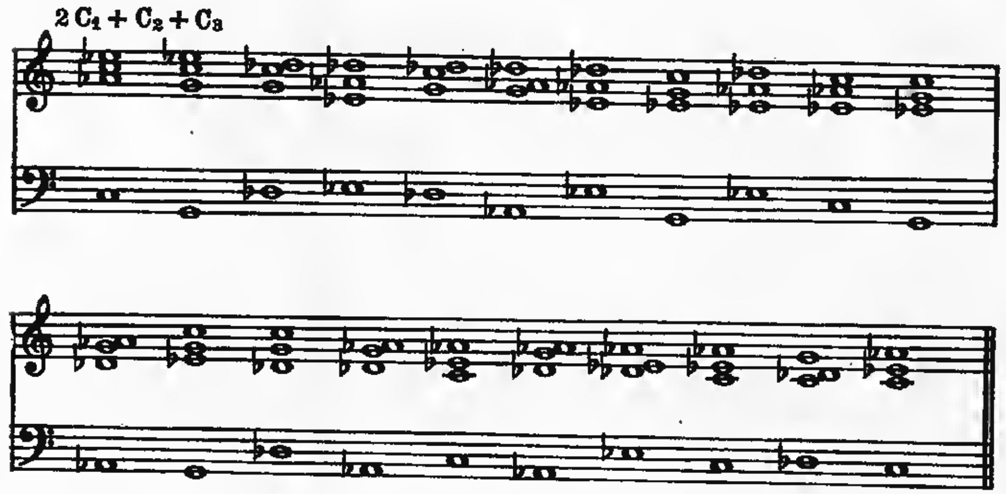


Figure 6. Progressions Type I (Diatonic). $\Sigma = \frac{S_{II}3p}{S_{I}p}$ *(concluded).*

B. Diatonic-Symmetric Harmony

Evolution of a diatonic-symmetric or a symmetric type of progression must be based on the following principles: the selection of chord structures must be confined to Σ produced by the scale itself; the number of each Σ and its sequence are a matter of selection and distribution.

The best procedure to follow is: first, make a table of all diatonic Σ, then transpose them to one axis. This produces a chart from which it is easy to draw comparisons between the different sonic structures. After an individual selection of structures, as well as their sequence, the coefficients of recurrence can be set, Progressions Type II (Diatonic-Symmetric)

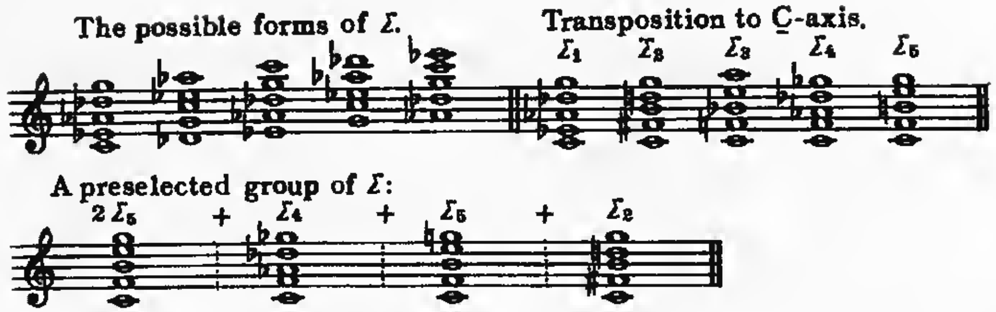


Figure 7. Progressions type II (Diatonic-symmetric).

In the following form of continuity, the sequence of root tones is the same as in the preceding examples. The functions of the upper 3 parts are b, c and d.

Figure 8. Functions of 3 upper parts are b, c, d. Root sequence is as in figure 6
$2C_1 + C_2 + C_3$.

C. Symmetric Harmony

Though the choice of intervals for the progression of root-tones in pure symmetric harmony is free, particular satisfaction is obtained when the intervals for the progressions of root-tones are present in the scale itself.

In the following example, the choice of the $\sqrt[4]{2}$ is justified by the relationship *c — eb* in the original pitch scale.

Progressions Type III (Symmetric).

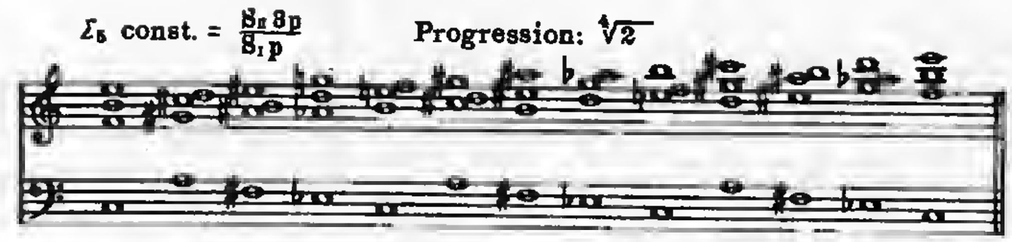


Figure 9. Progression type III (symmetric).

D. Strata (General) Harmony

For the sake of plasticity of voice leading, and when many voices are employed, it is practical to convert the entire Σ representing the expanded scale into strata. Progressions of strata harmony may be developed through the intervals producing Σ itself, or based on any other form of symmetry.

We shall convert the first expansion of the Balinese scale taken as a Σ into 3 strata where $S_I = p$, $S_{II} = 2p$, $S_{III} = 2p$, and where the progression is based on a descending scale of the Σ itself. Though other forms of symmetry for the chord progressions may be used as well, they represent a further stage of modernization of music.

The following example illustrates the strata described above.

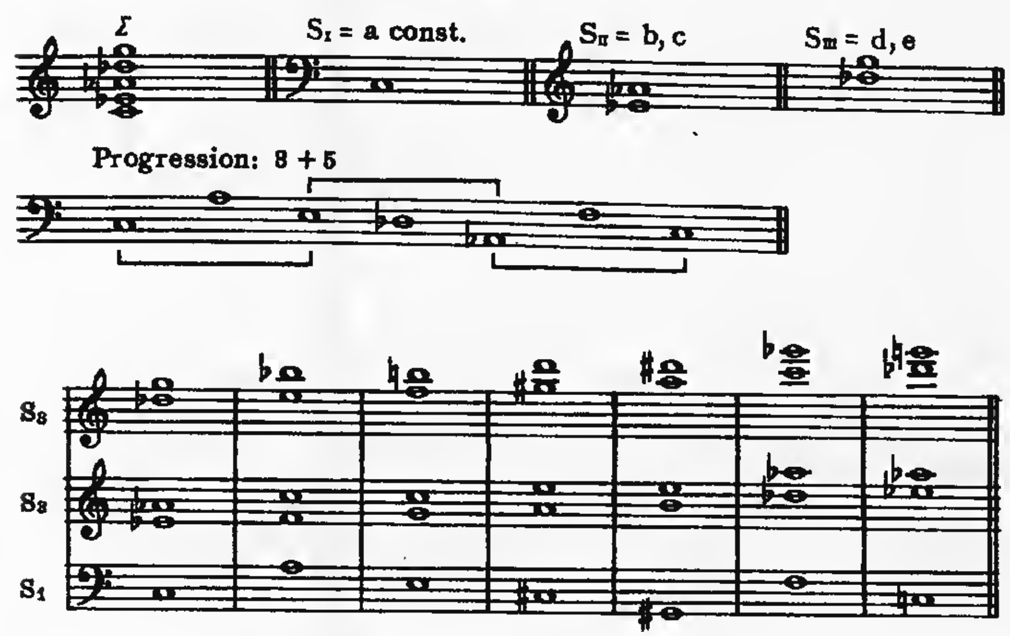


Figure 10. Strata harmony.

E. Melodic Figuration

The problem of melodic figuration, that is, the formation of directional units, is of utmost importance. When such units are set by chance, or free selection, from the general tables of directional units, they may destroy the inherent character of the music as expressed by the given scale. In order to get the proper type of directional units, which are derivative from the original scale, it is necessary to produce permutations either of the pitch-units or of the intervals in the given scale. The procedure is as follows: the original scale, designated as d_0, produces the respective number of derivative scales. These derivative scales furnish leading units to the given scale, after they have been transposed to one axis.

Pitch-units, which are not present in the original scale, become potential leading units. By selecting those nearest the chordal tones (neutral units), a variety of directional units may be secured from which the choice of actual units may be left to the composer.

The following example illustrates the entire procedure as it derives from permutation of pitch-units. See also Evolution of Scale-Families in the *Theory of Pitch-Scales** and in *Kaleidophone.***

Derivative scales obtained through permutation of pitch-units

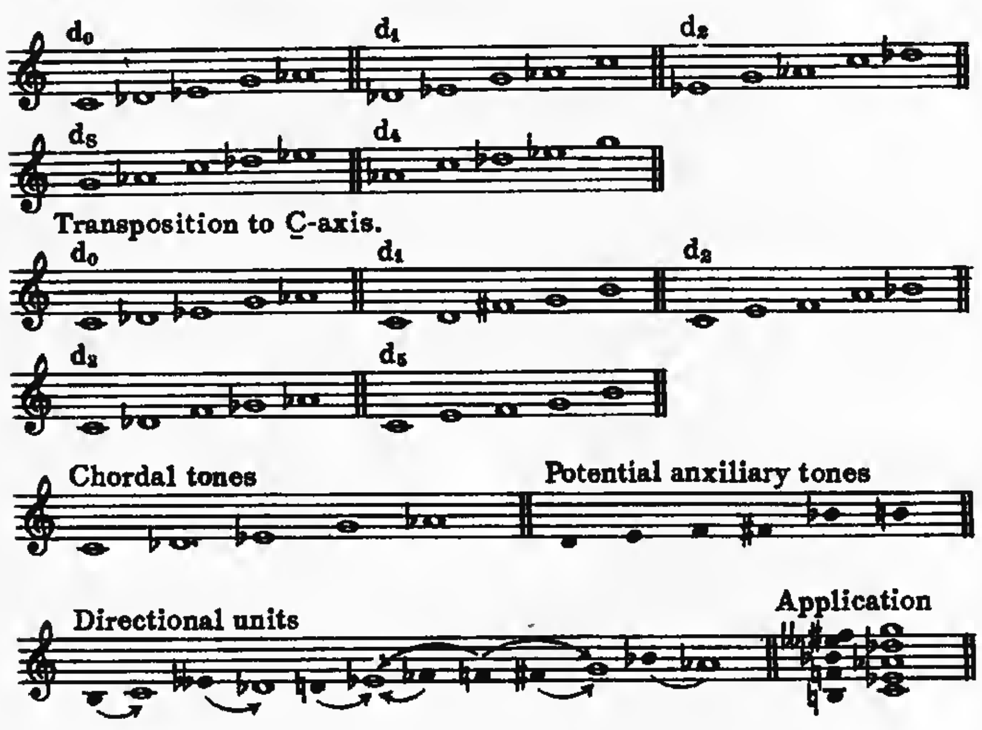


Figure 11. Melodic figuration (continued).

*See Vol. I, p. 115.

**Published by Witmark & Sons, 1940.

Figure 11. Melodic figuration (concluded).

In addition to this, another system based on permutation of intervals, will be found useful.

Derivative scales obtained through permutation of intervals.

Figure 12. Melodic figuration. Directional units derived from permutation of intervals.

F. Transposition of Symmetric Roots of Strata

Further modernization of the harmonic style of music may be achieved through transposing the symmetric roots developed from strata. This form is an adaptation of various native intonations to the ultimate development of equal temperament. This type of music is associated with modernity and is usually called "polytonal harmony." Casual and often incoherent examples of this type may be found in the works of Auric, Poulenc, Honegger, Stravinsky, Malipiero, Casella, and many others. Their attempts are in most instances inadequate owing to the fact that they have no definite technique of voice-leading in single strata, and their superimposition of strata is merely a device of placing different keys one above the other. They are unaware of the forms of pitch symmetry. There are a few consistent fragments to be found in Stravinsky's *Petrouchka*, *Le Sacre du Printemps* and *Les Noces*. Music of the Balinese scale, as developed through symmetric superimposition of strata, still retains its original character in spite of the extreme modernization of harmony.

The following figure illustrates a group of different settings obtained from the Balinese scale.

Figure 13. Further modernization through transposing symmetric roots of strata.

G. Compound Sigma

The development of a compound sigma follows the same procedure as the development of an individual Σ. The combination of two or more sigmae can be coordinated through any desirable form of symmetry.

In the following example, $\sqrt{2}$ is such a form of symmetry. The progression evolves through the form of symmetry used in Figure 10.

Figure 14. Compound Sigma.

CHAPTER 3

MELODIZATION OF HARMONY

A. Diatonic Melodization

The general principle of melodization in the diatonic system of harmony is based on an application of the chordal functions with the addition of a successive chordal function which is not present in the chord. Thus the entire system depends on the number of functions in a given harmony and on the number of pitch-units in a given scale.

In the case of two functions (a, b), melody may represent a, b and c functions. In the case of three functions in harmony, melody may consist of four functions. Thus, if harmony is a, b, c, melody consists of a, b, c, d. This is true of any type of harmony.

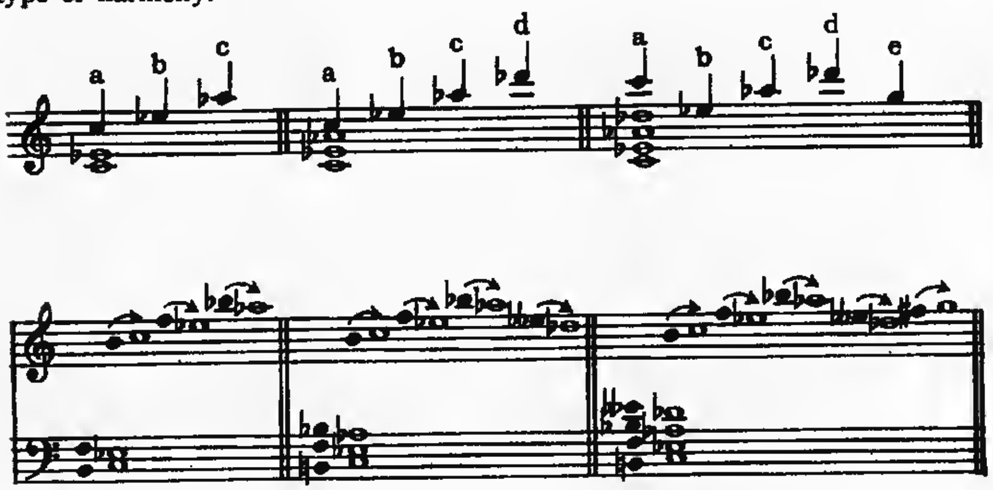


Figure 15. Where harmony has 3 functions, melody may consist of 4.

In the following example of diatonic melodization, the principle just presented is carried out. The duration-group selected for this example is in conformity with the characteristic Balinese rhythms and is a pure $\frac{3}{8}$ series.

Figure 16. Diatonic melodization (continued).

Figure 16. Diatonic melodization (concluded).

The character of music which is an equivalent of the chromatized pitch-scale of European music and can be recognized through the presence of auxiliary and passing units, can also be developed from scales containing less than seven pitch-units. Not all the steps are truly chromatic, but relatively speaking they are, as the scale has gaps between the pitches and the filling out of these gaps attributes a relatively chromatic character to this music. Some of the directional units which derive from permutation of the original scale are actually chromatic, i.e., they move in semitones.

In the following example of diatonic melodization, including melodic figuration, i.e., directional units, all the added functions are marked by the letter d as they appear in the melody. The chords consist of a, b, and c functions.

Figure 17. Diatonic melodization includes melodic figuration (continued).

Figure 17. Diatonic melodization includes melodic figuration (concluded).

B. Symmetric Melodization (Harmony Type II and III)

Symmetric melodization, in matching the sonority of the chord, develops on the same principles as diatonic melodization. The difference is that if one chord structure is carried out consistently, all the intonations conform to one pitch scale, which follows the sequence of chords in exact key-axis transposition.

This type of melodization accentuates the inherent character of intonations even more than in diatonic melodization where, owing to the different structures of chords, intonation varies in relation to the chord while the scale remains the same.

In the following example of symmetric melodization, harmony consists of 3 functions and melody is developed with the addition of the function d, which produces its own axis with every chord change.

Figure 18. Symmetric melodization.

Symmetric melodization which acquires chromatic character is analogous to the preceding form of melodization except for the use of directional units. When these two forms are distinctly dissociated, it is possible to use the first form (neutral units only) as an original melody, and the second form (directional units) as a variation of the original melody. The meaning of the second form is that it acquires a more chromatic character than in the case of the application of neutral units only.

In the following figure, the added function d is indicated. The melody itself is constructed through circular permutation of the original pattern.

Figure 19. Symmetric melodization with directional units.

C. Conclusion

By reversal of the above described procedures, coupled with the previous experience gained from the Theory of Harmonization, it is easy to obtain harmonizations of melody which are true to style in each specified category.

By combining the technique of the Third and Fourth Group of Pitch-Scales with the knowledge acquired in the branch of harmonic polytonality (*General Theory of Harmony*: S4p*), it is easy to obtain the whole either through harmonization or through transformation of harmony into melody in one of the strata.

Authentic counterpoint, true to style in each of our categories (primitive, stylized, modernized), cannot rely on the classical system of resolutions for these are technically impossible in the field of incomplete scales. The variety of harmonic intervals (within a pre-selected family-set) takes the place of that technique. Utilized with the coordination of attacks, durations and melodic forms (as expressed through axial combinations), pre-selected harmonic intervals in a family-set yield results which can truly be considered perfect.

*See p. 1141.

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK XI

THEORY OF COMPOSITION

BOOK ELEVEN

THEORY OF COMPOSITION

Part I

Composition of Thematic Units

Part II

Composition of Thematic Continuity

PART III

SEMANTIC (CONNOTATIVE) COMPOSITION

INTRODUCTION

AS the meaning of the word implies (cum + pono means "to put together"), composition is the process of coordinating raw materials and techniques into a harmonic whole. But the harmonic whole is the most difficult thing for a composer to create, and there are many good reasons for this.

There are three basic approaches in the actual work of a composer. One such approach requires the preparation of all or of most important themes in advance, but without the visualization of the whole. Such themes, being of good quality per se, may not fit into the whole. They may be improperly interrelated with one another as to their character, proportions, etc.

The second approach, typical of soloists and improvisers, requires the composition of a piece in finished form, step by step, from start to finish. In this case the composer can hardly anticipate the whole, as he does not even know what will happen in the next few measures before he gets there. The outcome of such a method of composition, or rather lack of it, is a lack of coherence, lack of proportion, excessive repetition and a generally loose structure.

The third approach involves a great deal of thinking first, the sketching of the whole, at least insofar as the general pace of temporal organization is concerned, and the elaboration of details thereafter. The third approach can be compared with the molding of a sculptured piece.

Each approach contains different ratios of the intuitive and the rational elements by which the process of composition is accomplished. Works of different quality may result from each of these three basic approaches. Often these forms of creation are fused with one another.

I have found, after a thorough and extensive analysis, that the degree of perfection in a work of art, and hence its vitality as a factor of the probability of survival, depends upon the *relation of a tendency to its realization*. If, for example, the tendency in a given work of art is toward a certain form of regularity, we may compute the degree of perfection on the basis of the percentage of adherence of the realized form to such regularity. Of all composers, J. S. Bach scored 100% in some instances. It is equally true that all composers recognized as "great" in the course of time, yield a high degree of perfection (scoring in many instances 80%-90%). In the music of mediocrities (who were in some cases eminent during their lifetime) such scores, on the contrary, are pitifully low.

If the degree of organization and the adequacy of the realization of a tendency constitute the vitality of a work of art, it is only reasonable to seek to evolve works which embody refinement of structural organization, mutual fitness of components and the complete realization of a tendency.

Such a process of reasoning leads us to the necessity of *prefabrication and the assembly of components according to a preconceived design of the whole*, i.e., to the scientific method of art production—in this case, of a musical composition.

PART ONE

COMPOSITION OF THEMATIC UNITS

CHAPTER 1

COMPONENTS OF THEMATIC UNITS

WE shall define a ***thematic unit*** as a ***variable quantity with a constant potential of quantitative aggregation.*** Variable quantity in this case refers to the duration of any component and its potential—the tendency by which such a component may grow. A thematic unit, in most cases, consists of more than one component. It evolves, however, from one basic original component, as if from a nucleus, around which all other components (participating in the formation of the thematic unit) develop. We shall call the basic component—major, and all other components—minor.

A major component may be evolved from any technical form. Technical forms, from which both major and minor components may be developed, may be described as follows:

(1) temporal organization (rhythm of factorial and fractional continuity);
(2) family-groups of pitch assemblages (pitch-scale developments);
(3) linear composition (plotted melodies);
(4) composition of simultaneous assemblages (chord structures and progressions);
(5) harmony as a source of melodization;
(6) correlated melodies (counterpoint of attacks, melodic forms, etc.);
(7) orchestral resources (tone-quality, dynamics, density, instrumental forms, attack-forms).

These technical forms correspond to the various branches covered in the present work:

(1) Theory of Rhythm (Book I);
(2) Theory of Pitch-Scales (Book II);
(3) Theory of Melody and Geometrical Projections (Books III and IV);
(4) Harmony, Special and General Theory (Books V and IX).
(5) Melodization and Harmonization (Book VI);
(6) Correlated Melodies (Counterpoint) (Book VII);
(7) Instrumental Forms and Orchestration (Books VIII and XII).

The selection of one or another technique for evolving the nucleus of a thematic unit, i.e., its major component, depends on the technical form which the composer wishes to have dominate over other components of the same thematic unit.

A certain thematic unit may evolve around the major component of temporal rhythm. In such a case temporal rhythm, even after the addition of other components, becomes the dominant characteristic of the thematic unit. In other cases, the dominance of melody may be desired. Then a plotted melody becomes the major component of the thematic unit. Such a melody may be later harmonized, in which case harmony becomes its minor component. On many occasions the dominance of harmony is so important that it becomes practical to evolve chord progressions first—in which case harmony becomes the major component of the thematic unit. A minor component may be evolved later by means of melodization. It is equally obvious that when continuous imitation is desired as the dominant characteristic of a thematic unit, it is best to start with the contrapuntal setting of a canon. The canon itself becomes the major component and its instrumental forms, harmonic accompaniment, etc., become minor components of the thematic unit.

Certain forms of musical expression have dynamics as a dominant characteristic. In such cases dynamic composition becomes the major component of the thematic unit. Harmony and melody in this case play merely a subsidiary role and, therefore, become the minor components. Much of such music is being written for radio-scripts, motion-picture and theatrical productions, and program music in general.

CHAPTER 2

TEMPORAL RHYTHM AS MAJOR COMPONENT

THE process of composing thematic units is both selective and coordinative.

When the decision is to make temporal rhythm the major component of one or more thematic units, the ***first selection*** refers to the ***series of style.*** As style can be pure or hybrid, such a correspondence must be established by selection of either a pure series or of a hybrid. The composer may evolve his own hybrids if such hybrids serve the purpose of musical expression best. In other instances, the selection of a hybrid is necessitated by the desire to carry out a certain style whose specifications require such a hybrid.

The second selection deals with ***concrete techniques.**** Among these are:

(1) Composition of attacks;

(2) Composition of durations to pre-set groups of attacks from the specified series of style;

(3) Direct composition of durations from the various resources** developed in the ***Theory of Rhythm***:

(a) the resultants of interference:

$a \div b$, $\underline{a \div b}$, $a \div b \div c$; . . .

(b) composition of balance, expansion and contraction;

(c) composition of instrumental interference;

(d) extension of the T — units by permutation: durations, rests, accents, split-unit groups and groups in general;

(e) extension of thematic units by permutations of the higher orders;

(f) composition and coordination of involution-groups belonging to the style-series;

(g) composition of groups of variable velocity, when the latter becomes the necessary characteristic of a thematic unit.

All the above techniques apply to both the factorial and the fractional forms of each thematic unit, wbose major component is temporal rhythm.

*Each of these techniques is illustrated below. Paragraph numbers and sub-letters are correlated with illustrations. (Ed.).

**For details concerning each of the resources (a) to (g) listed above cf. the page indicated after each of the following letters (all references are to Vol. I.: (a) p. 4; (b) p. 21; (c) p. 27; (d) p. 46; (e) p. 63; (f) p. 84; (g) p. 90.

Example: $\frac{1}{4}$ series; summation series: 1, 3, 4, 7, 11, . . .

(1) $A = aT_1 + 2aT_2 + 3aT_3 + 2aT_4$

(2) 

(3)

(a) $T^{\rightarrow} = r_{4\div3}$; $T'' = 3t$; 

$T^{\rightarrow} = r_{\underline{4\div3}}$; $T'' = 4t$; 

$T^{\rightarrow} = r_{7\div4\div3}$; $T^{\rightarrow} = r'_{7\div4\div3}$ | $T'' = 3t$; $T'' = 4t$; $T'' = 7t$

(b) $T^{\rightarrow} = B_{4\div3}$; $T'' = 4t$;
$T^{\rightarrow} = E_{4\div3}$; $T'' = 4t$;
$T^{\rightarrow} = C_{4\div3}$; $T'' = 4t$.

(c) I2p1 (A = 2a); $T^{\rightarrow} = r_{\underline{4\div3}}$

$$T^{\rightarrow\prime} = \frac{4t+3t+2t+3t+4t}{\boxed{3t}+2t+3t+2t+3t+3t}$$

Preliminary: 

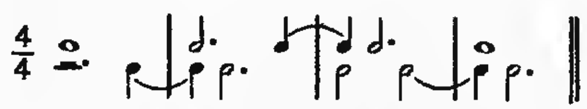
Final: 

Figure 1. ⓐ *Resultants of interference.* ⓑ *Composition of balance, expansion, and contraction.* ⓒ *Composition of instrumental interference.**

The above is applicable to harmony or any instrumental or density-group.

(d) $T^{\rightarrow} = (3t+t+2t+2t)$ ↻;

$$T^{\rightarrow\prime\prime} = (3t+t+2t+2t)T_1 + (t+2t+2t+3t)T_2 + (2t+2t+3t+t)T_3 + (2t+3t+t+2t)T_4$$

Figure 2. ⓓ *Extension of T-units by permutation (continued).*

*The letters appearing here are correlated with the letters given above under (3). (Ed.).

Applicable to melody with harmonic accompaniment, or to counterpoint.

Applicable to $\frac{M}{H\rightarrow}$ or $\frac{CP_I}{CP_{II}}$

Also general permutations of rests in 4p.

Applicable to $\frac{M}{H\rightarrow}$ or $\frac{CP_I}{CP_{II}}$

Combinations of the above and also general permutations of accents in 4p. Applicable to CP4p.

Figure 2. Extension of T-units by permutation (continued).

Applicable to $\frac{M}{H\rightarrow}$ and $\frac{CP_I}{CP_{II}}$.

Also $T^{\rightarrow} = 16T$, where $T'' = 4t$.

The same developed in 2p, 3p and 4p.

Figure 2. Extension of T-units by permutation (concluded).

(e) $T^{\rightarrow} = 4t_{2^o}$ (four T of the second order):

Applicable to $\frac{M}{H\rightarrow}$ or CP2p.

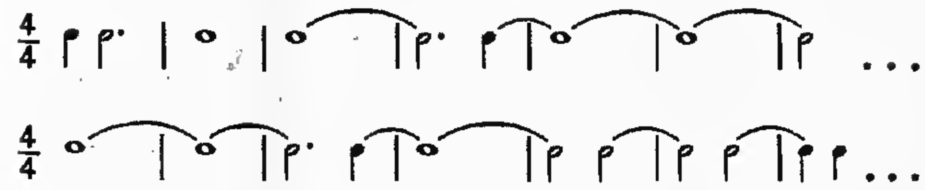
Figure 3. (e) *Extension of thematic units by permutations of higher orders.*

(f)

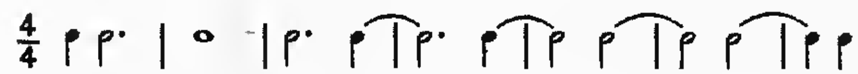
Applicable to: 43p; CP3p; $\frac{M}{H2p}$; $\frac{M}{\frac{M}{H}}$ (p, 2p, 3p, . . .np)

Figure 4. (f) *Composition of involution groups.*

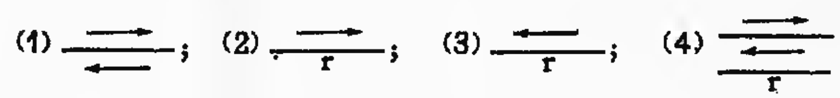
(g) $t + 3t + 4t + 7t + 11t \ldots \leftrightarrow$



$$r\left(\frac{1+3+4+7+11}{11+7+4+3+1}\right) = 1+3+4+3+4+3+4+3+1$$



The above forms can be combined into 2p and 3p.



Each combination gives the corresponding number of permutations.

Figure 5. (g) *Composition of groups of variable velocity.*

CHAPTER 3

PITCH-SCALE AS MAJOR COMPONENT

HERE, as in the case of temporal rhythm, the *first selection* refers to the *scale-family*. Such families, as we know from the *Theory of Pitch-Scales*,* can be evolved either on the basis of identity of the pitch-units or on the basis of identity of the interval-units. In the latter case, the sum of interval-units remains constant.

All other techniques, by which further modifications can be obtained, refer to the *second selection*. Among these techniques are the following:

(1) Permutation of pitch units in the selected scale for the purpose of producing MP (master-pattern);**
(2) Transposition of derivative scales to one axis;
(3) Transposition of MP to the consecutive units of the original MP (this also concerns scales as such);
(4) Further modification of MP by permutations of pitch-units, combined with (2) and (3);
(5) Tonal expansions applied to all the preceding techniques;
(6) Selection of the form of distribution of sectional scales through symmetric roots. (The form of symmetry must be constant for the entire family of thematic units used in one composition; such symmetry either is defined a priori, or is based on the limit-interval of the original sectional scale. All the preceding techniques are applicable to this technique.

It is desirable to specify, before composing the thematic units, whether such units will be diatonic or symmetric, as the two styles of intonation conflict. All thematic units of one composition, evolved on the basis of pitch-scales as major component, must be either diatonic or symmetric.

Since all major components require the presence of temporal rhythm as a minor component, it is important in this case to specify the attack-groups of MP in their relation to $T^{\rightarrow}$. Such a relation depends on the desirability of the interference of attacks, i.e., whether $\frac{a\,(M)}{a\,(T)} = 1$, or $\frac{a\,(M)}{a\,(T)} \neq 1$, or whether such an expression is a reducible fraction. Practically, it means that the repetitious character of MP, which may be due to the small number of pitch-units in the scale, can be eliminated by creating interference of $\frac{a\,(M)}{a\,(T)}$. The same is true for the brief duration-groups, whose repetitiousness can be eliminated by the use of MP with many attacks and a scale with many pitch-units.

The use of a pitch-scale as major component does not limiting the thematic unit to melody alone. Part-development can be evolved on the basis of Instrumental Forms, i.e., by reciprocating MP through its own

*See Vol. I, p. 101.

**In the illustration given below, figures 6-11, the numbers at the top of each figure are correlated with the paragraph numbers appearing here. (Ed.)

modifications, and may not require harmonization as a new minor component. The latter, in turn, may evolve either from scales or from harmony.

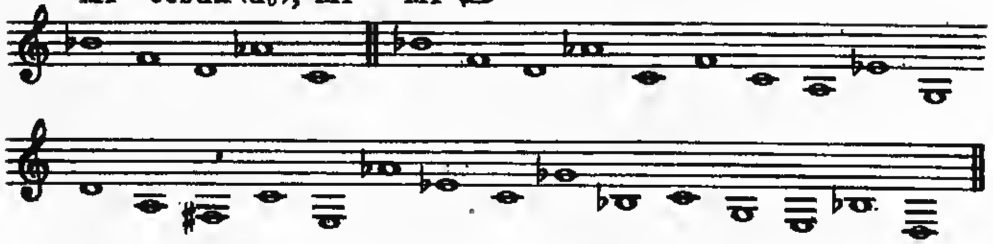
Example:

*Figure 6. Permutation of pitch units to produce MP.**

Figure 7. Transposition of derivative scales to one axis.

*The numbers at the top of each illustration in this figure and the succeeding one are correlated with the paragraph numbers on page 1286. (Ed.).

(3) MP= ecbda (d_0); MP' = MP



T= $\boxed{1}$ +1+1+4(¼)

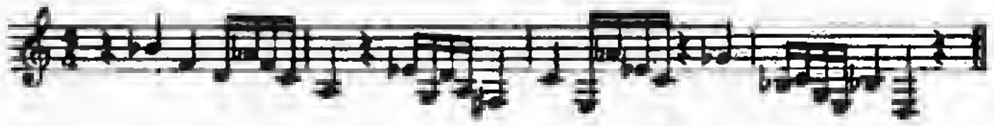
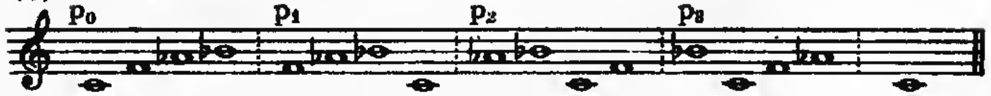


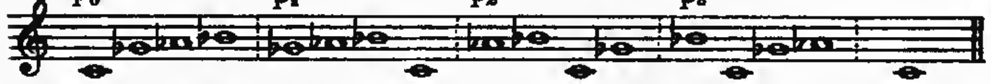
Figure 8. Transposition of MP to consecutive units of original MP.

(4) MP= acde (d_0)

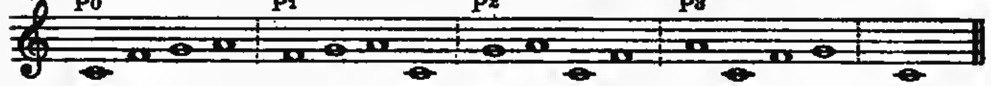
p_0 p_1 p_2 p_3



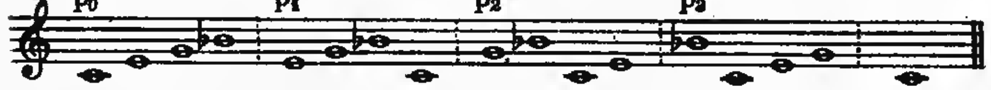
d_1 p_0 p_1 p_2 p_3



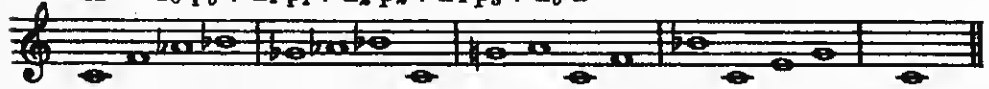
d_2 p_0 p_1 p_2 p_3



d_4 p_0 p_1 p_2 p_3



MP' = $d_0\,p_0 + d_1\,p_1 + d_2\,p_2 + d_4\,p_3 + d_0$ a



MP'' = MP' transposed to the consecutive units of $d_0\,p_0$.

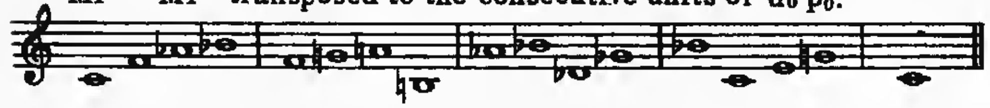


Figure 9. Modification of MP by permutation of pitch-units (continued).

T= $B_{4 \div 3}$

Figure 9. Modification of MP by permutation of pitch-units (concluded

(5) MP E_0 p_0 MP E_1 p_0

MP'= MP E_0 p_3 + MP E_1 p_0

T=(3+1+2+2); coupling: E_1

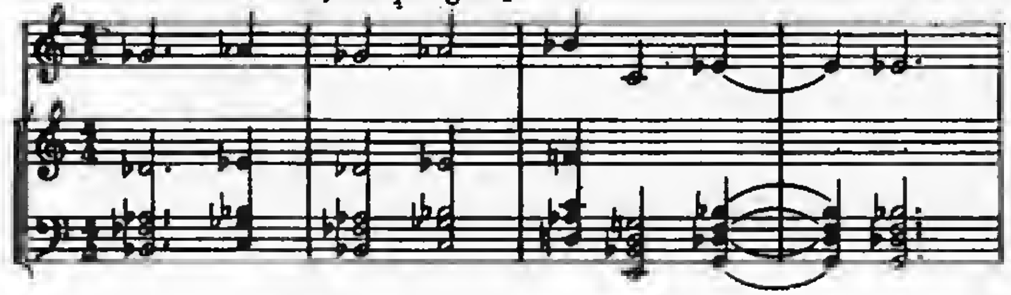


Figure 10. Tonal expansion applied to preceding techniques.

Figure 11. Using symmetric roots for the distribution of sectional scales.

CHAPTER 4

MELODY AS MAJOR COMPONENT

THE use of melody as major component is particularly advantageous when the configurational characteristics of a melodic line are of prime importance. These configurational characteristics correspond to the two forms of selection. The *first selection* refers to *axial combinations* and the *second selection* refers to *trajectorial forms.*

Melodic line as such becomes the dominant factor of a thematic unit. The customary minor components of a melodic line are: temporal rhythm, pitch scale, and, often, either harmonization or coupling.

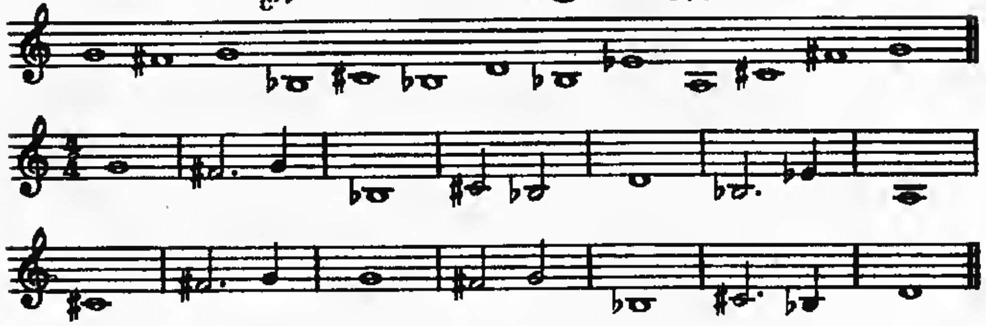
Composition of thematic units from melody can be accomplished either by plotting or by direct execution in musical notation. The latter requires an MP (master plan) which corresponds to the axial combination and to the intended trajectory. It also necessitates an *a priori* selection of the quantity of pitch-units in which such a trajectory can be realized. Composition of temporal rhythm usually follows this procedure. Finally, a system of accidentals can be chosen and superimposed upon the scale. After this is accomplished, harmonization or coupling may follow.

Example:

$MP = Ax = \frac{a}{a'} + \frac{o'}{b}$; $S^{\rightarrow} = h + 2i + h$ (Persian); $T = (\boxed{1} + 1 + 1 + 1)$.

Figure 12. Axial combinations in melody composition (continued).

$MP = Ax = O + \frac{c}{c'}$; $S^{\rightarrow} = (h+2i+h)\, d_4$ (d); $T = r_{7\div 4}$; $T'' = 4t$.



$MP = Ax = b+a+c$; $S^{\rightarrow} = (h+2i+h)\, d_1$; $T = (\frac{1}{2}+\frac{1}{2}+1+1+1)$

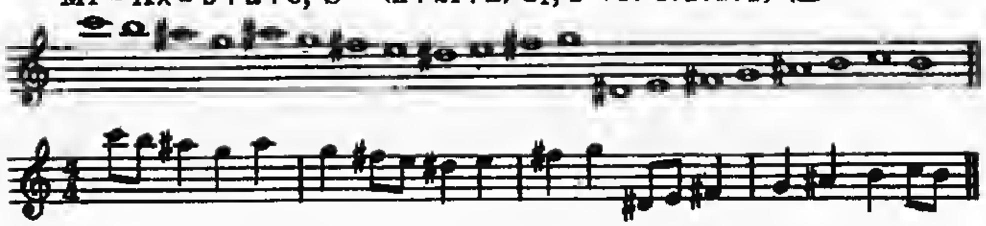


Figure 12. Axial combinations in melody composition (concluded).

$T' = T$ abcd

Figure 13. Melody and trajectorial forms.

To achieve unity of harmonic style for several thematic units in one composition all master-structures must be developed from one source. The composer is free to make his decision on the matter of the relationship of $S^{\rightarrow}$ and Σ.

If the general character of the entire composition is diatonic, then the system of progressions must also be diatonic. Once the style of harmonization becomes

symmetric, all types of such progressions are acceptable for chords. One may use type II, III, and the generalized. Once the symmetric style of harmonization is accepted, symmetric superimposition may be used as a form of harmonization. To illustrate this, we shall harmonize the thematic units of Figure 12.

In order to achieve contrasts in harmonization of all three thematic units and yet retain unity of harmonic style, we shall subject the first thematic unit to diatonic harmonization in hybrid five-part harmony; we shall employ one of the structures of $S^{\rightarrow}$ as the form of coupling for the second thematic unit; and we shall use the C_0 of the same Σ for the third thematic unit.

Under such conditions, the first thematic unit will have a moderate mobility of harmonic changes; the second thematic unit will have an extreme mobility of harmonic changes (as under couplings $\frac{M}{H} = a$); and the third thematic unit will have no harmonic changes at all.

As all units of the MP appear four times in permutations according to the structure of the first thematic unit, we shall use the corresponding number of harmonic changes, i.e., four. Under such conditions, any cycle would be acceptable. We shall have $C^{\rightarrow} = C_7$ const., and assume the first extended duration (d♭) to be 13. Then the first chord is F. Hence: $H^{\rightarrow} = H_1F + H_2G + H_3A\flat + H_4B\flat$.

The coupling of the second thematic unit will be the $\Sigma(S^{\rightarrow}d_0III)$, i.e., the third degree of the original scale. Each pitch-unit of the melody will become 13 owing to the downward coupling.

The constant structure of the accompaniment of the third thematic unit will be $S(7)S^{\rightarrow}d_0I$, which is a major seventh-chord.

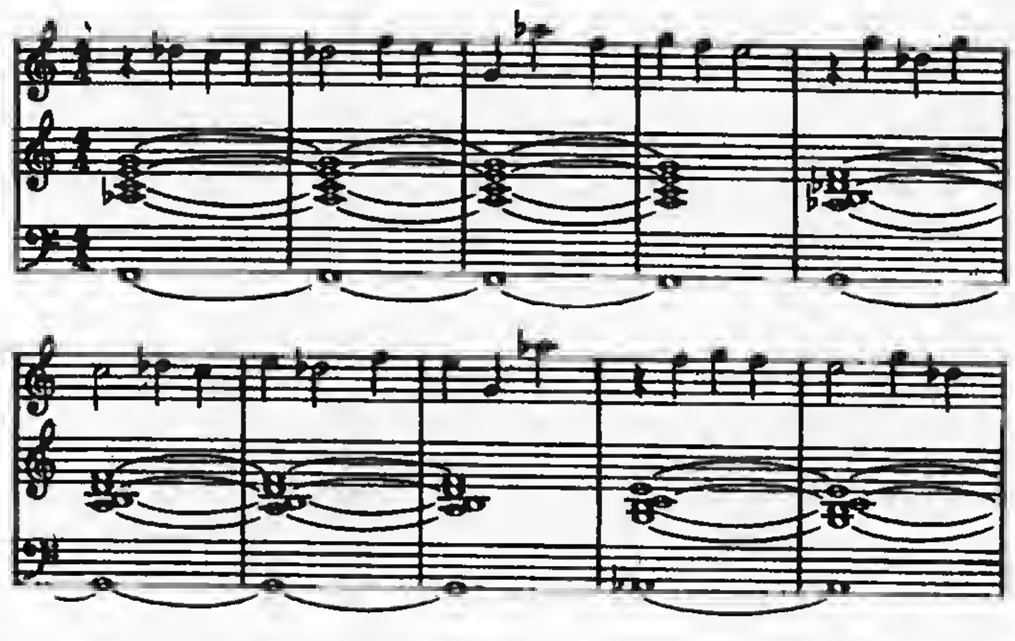


Figure 14. Diatonic harmonization in hybrid five-part harmony (continued).

Figure 14. Diatonic harmonization in hybrid five-part harmony (concluded).

Figure 15. Using one of the structures of $S^{\rightarrow}$ as a form of coupling.

Figure 16. Using C_0 of same Σ (continued).

Figure 16. Using C_0 of same Σ (concluded).

CHAPTER 5

HARMONY AS MAJOR COMPONENT

HARMONY can be a self-sufficient component not requiring melodization when combined with temporal rhythm. The simplest and most common form of the rhythmization of harmony occurs when each attack of a duration-group emphasizes all parts, resulting in rhythmic unison. Most hymns are composed in such a form. The musical interest of such self-sufficient harmony lies in the fact that, in reality, there is a dominance of one part over the others (usually soprano, sometimes bass, and, originally, tenor). Through the techniques of the *Special Theory of Harmony,** such a dominance of one part, which becomes a melody generated within harmony, can be obtained from various sources. Among them the most important are: groups with passing chords, generalization of the passing seventh, and chromatic variation of the latter.

Another way of evolving thematic units from harmony, and making such harmony self-sufficient consists of the distribution of a duration-group (T) through the instrumental (I) and the attack-group (A). This technique was fully described in the *Theory of Rhythm.*** In its application to harmony, this form of synchronization of attacks, durations and instrumental parts ($S = T \div \div I \div A$), can be accomplished in all cases of $A = ap$ (one attack per part) and in some cases of $A = 2ap$ (two attacks per part).

The selection of a duration-group must satisfy the following requirements:

(1) The number of terms of T must be even;
(2) T must consist either of reciprocating binomials, or of such binomials with an extra binomial (usually the central binomial of $r_{a \div b}$.) consisting of two equal terms.

It is easy to find such T among the various forms of $r_{a \div b}$. The most practical ones are the resultants whose generators have a negligible difference. For example: $3 \div 2$, $4 \div 3$, $5 \div 4$, $6 \div 5$, $7 \div 6$, $8 \div 7$, $9 \div 8$, . . .

The binomials whose first term is greater than the second produce *suspensions.* The binomials whose first term is smaller than the second produce *anticipations.* The extra binomials with two equal terms produce the balanced pace of chord changes. The latter is particularly practical for the bass part, though deviation from this principle is not always undesirable, particularly in variations of the original.

Temporal binomials in reciprocation can be taken also directly from the evolutionary series of rhythm. For example:

$\frac{8}{8}$	7+1	$\frac{9}{9}$	7+2	$\frac{12}{12}$	11+1
	1+7		2+7		1+11
	5+3		5+4		7+5
	3+5		4+5		5+7

Figure 17. Temporal binomials.

*See Book V.

**See Book I, Chapter 8, *Coordination of Time Structures.*

Such reciprocating binomials can be vertically re-arranged in any desirable fashion.

For example:

$\frac{8}{8}$	7+1	$\frac{9}{9}$	7+2	$\frac{12}{12}$	11+1
	5+3		5+4		7+5
	3+5		4+5		5+7
	1+7		2+7		1+11

Figure 18. Reciprocating binomials re-arranged.

If the series does not yield a sufficient number of reciprocating binomials, all its values can be multiplied by some common factor. For example: $\frac{3}{3}$ series. The original binomials are: 2+1 and 1+2. By multiplying this series by 2, we acquire two pairs of binomials: 5+1 and 1+5, 4+2 and 2+4. Thus:

5+1
4+2
2+4
1+5
3+3

In this case the extra binomial is 3+3. Of course, the result obtained in this way corresponds to $r_{6 \div 5}$, and could have been obtained from the latter directly.

The groups of reciprocating binomials are subject to permutations. This permits a sufficient variety for each individual part of harmony. It should be remembered that the balanced binomial does not participate in permutations affecting all other binomials.

Illustration:

S	(5+1) + (4+2) + (2+4) + (1+5)
A	(4+2) + (2+4) + (1+5) + (5+1)
T	(2+4) + (1+5) + (5+1) + (4+2)
B_I	(1+5) + (5+1) + (4+2) + (2+4)
B_{II}	(3+3) + (3+3) + (3+3) + (3+3)

Figure 19. Permutation of reciprocating binomials.

The principle of rhythmization of harmony by means of *reciprocating binomials* produces a condition under which every chord in the progression appearing in an odd place has a common attack in all parts. As a result there occurs a complete rectification of all suspensions and anticipations not extending beyond two successive chords. The attack in all parts falls on the succeeding chords of a harmonic progression: $\overrightarrow{H} = H_1 + H_2 + H_3 + \ldots$, between which points the suspensions and the anticipations take place.

The limiting of this principle to binomials is dictated by necessity. It would be difficult to discriminate the dependence of chord-units in a progression where complete rectification of the original structures extended beyond two chords.

Harmony, rhythmicized in such a manner, becomes a self-sufficient thematic unit and does not call for melodization. One of the reasons it appears to be self-sufficient is the presence of the resultants of instrumental interference in each part of the harmony. This attributes to each part an individual rhythmic character.

Rhythmization of harmony by means of reciprocating binomials works for any number of parts (including strata) and in any type of progression. However, it is particularly suited for progressions in which stationary parts are completely or nearly absent. The best progressions for this purpose are the various chromatic types, and particularly the automatic chromatic continuity with alterations of the individual parts. In the non-chromatic types, the constancy, or at least the dominance, of C_7 gives the most satisfactory solution.

Automatic chromatic continuity makes it possible to use T's in addition to those which consist of reciprocating binomials. This principle, when generalized, requires that alterations appear in sequence in each individual attack per part, or that there be an equal distribution of alterations and attacks per part. Thus, each part can have one or two or, on rare occasions, three attacks in succession.

The sequence in which the parts appear is subject to distribution *a priori*. If more than one part moves simultaneously, involving parallel alterations, such harmonic parts must be treated as one rhythmic part. Thus in the following sequence of attacks:

Only two rhythmic parts, i.e., 2pl (I) are necessary.

S		S			S
A			A	A	
	T	T		T	
	B		B		B

Figure 20. Sequence with 2pl (I).

Likewise in the following, only 2pl (I) are necessery:

	S	S			S		S
A			A		A		A
	T		T		T	T	
	B		B	B			B

Figure 21. Sequence with 2pl (I).

For the same reason, the following sequence requires 3 pl (I).

S			S					S		S	
	A			A		A					A
	T				T			T			T
		B	B				B		B		

Figure 22. Sequence with 3pl (I).

All alterations must proceed in one direction until the synchronization of all components (A, T, I, H) is complete.

Examples:

(1)

$T = (1+1+1+\boxed{1}) + (\boxed{1}+1+1+1)$

Figure 23. Rhythmization of harmony.

(2)

Figure 24. Rhythmization of harmony (continued).

Figure 24. Rhythmization of harmony (concluded).

(3)

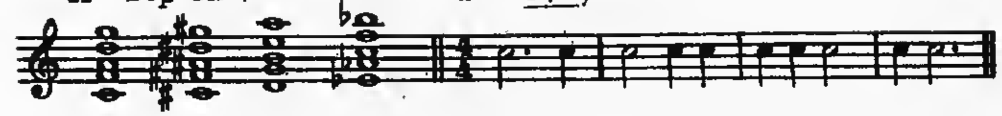
Var. I: $T(I)\,3p = r_{4 \div 3}$; $T'' = 3t$.

Var. II: Variable density by T.

Figure 25. Variable densities in rhythmization of harmony.

(4)

$H \rightarrow S5p$ Ch $\uparrow$ $T = r_{4 \div 3}$; $T'' = 4t$.



$I = 5$ pl.: $pl_{V} + pl_{IV} + pl_{III} + pl_{II} + pl_{I}$. $A = ap$; $A' = 5a$; $A(T) = 10a$.
$T' = 16t \cdot 2 = 32t$; $N(T'') = 8$.

Preliminary Scoring.

Final Scoring

Figure 26. Preliminary and final scoring.

(5) H→ S4 p Ch ↑

$I = 4$ pl.: $pl_I + pl_{II} + pl_{III} + pl_{IV}$. $A = 2ap$; $A' = 8a$; $A(T) = 20a$; $A(T') = 40a$; $T' = 32t$

Preliminary Scoring

Figure 27. Preliminary and final scoring (continued).

Final Scoring

Figure 27. Preliminary and final scoring (concluded).

(6)

H→ S4p Ch↓: $\frac{A}{T}$ + B + S T=3+1+2+2; T″=3t.

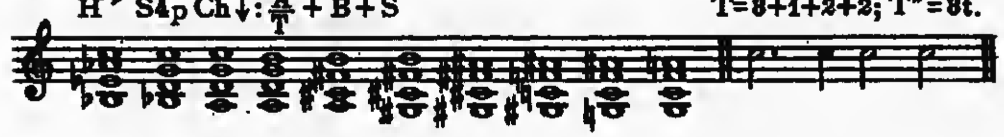


Figure 28. Preliminary and final scoring (continued).

$I = 3pl.; A = a; A' = 3a; A(T) = 4a; A(T') = 12a; T' = 8t \cdot 3 = 24t; N(T'') = 8.$

Preliminary Scoring

Final Scoring

Figure 28. Preliminary and final scoring ((concluded).

CHAPTER 6

MELODIZATION AS MAJOR COMPONENT

WHILE harmonic progressions, used as a source of melodization, are evolved in one style (one harmonic type) in order to give unity to the entire composition, such progressions may be given a desired amount of contrast in their different thematic units. This is achieved by varying the *type of melodization.*

Thus, for example, a progression evolved in type II (diatonic-symmetric) can be melodized diatonically (by the quantitative scale), symmetrically (by the usual method of Σ—transposition and modulations), or chromatically (by chromatization of either of the first two forms). For this reason, several different thematic units can be evolved from the same type of harmonic progression. Where the chief apparent characteristic becomes the type of melodization, we regard melodization as the major component.

Diatonic harmonic progressions produce their own types of melodization, i.e., the diatonic and the chromatized diatonic. Likewise chromatic harmonic progressions, of various forms and derivation, can be melodized through the two basic techniques assigned to that form, i.e., the acquisition of leading tones from the following chord and the device of quantitative scale.

As both of these techniques can be mixed, and as the diatonic (quantitative) melody can be chromatized, it is possible to devise a large number of thematic units (on the basis of chromatic melodization) which could participate in one composition, yet which would exhibit a noticeable degree of contrast with each other.

In the harmonic strata technique, transformation of a stratum (or strata) into melody is equivalent to melodization.

Examples:

(1)

Figure 29. Melodization of harmonic progression (continued).

(a)

Figure 29. Melodization of harmonic progression (continued).

(b)

Figure 29. Melodization of harmonic progression (concluded).

(2)

Quantitative scale

(a)

Figure 30. Melodization of harmonic progression (continued).

$H^{\rightarrow} = \sqrt[3]{2}$

(b) $T = {}^{r}4 \div 8$; $T'' = 8t$; $S^{\rightarrow}(M) = \Sigma$ XIII E_0. $A(M) = 4a$.

Figure 30. Melodization of harmonic progression (concluded).

(3) Quantitative scale

Figure 31. Melodization of harmonic progression (continued).

(a)

Figure 31. Melodization of harmonic progression (continued).

Figure 31. Melodization of harmonic progression (concluded).

CHAPTER 7

COUNTERPOINT AS MAJOR COMPONENT

THE dominant characteristic of counterpoint becomes particularly noticeable in such forms as imitation, ostinato of a ground melody, and in the contrasting forms of axial correlation. Thematic units evolved as correlated melodies furnish a major component in which the individualization of melodic lines is particularly prominent.

Here major component as such is not confined to any configuration-families. So long as the different thematic units contain their own configurational characteristics, sufficient for the purpose of detectable contrasts, no other requirements are necessary. Unification of style is accomplished through the selection of minor components, such as temporal rhythm, pitch-scale or master-structure (when counterpoint is evolved from strata).

Contrapuntal thematic units can be subjected to harmonization, in which case harmony becomes the unifying factor of style.

It is not advantageous to evolve contrapuntal thematic units by means of part-melodization, as in such a case the forms of correlation are greatly controlled by harmony and therefore force the counterpoint to become a minor component. Part-melodization may be used, however, when the second melody can play a subsidiary (obligato or background) role.

We shall illustrate now the composition of thematic units from counterpoint in its three characteristic aspects:

(1) axial coordination;
(2) canonic imitation;
(3) counterpoint to ground melody.

Example:

(1) $\frac{CP_2}{CP_1} = \frac{a}{o}$; Type IV.

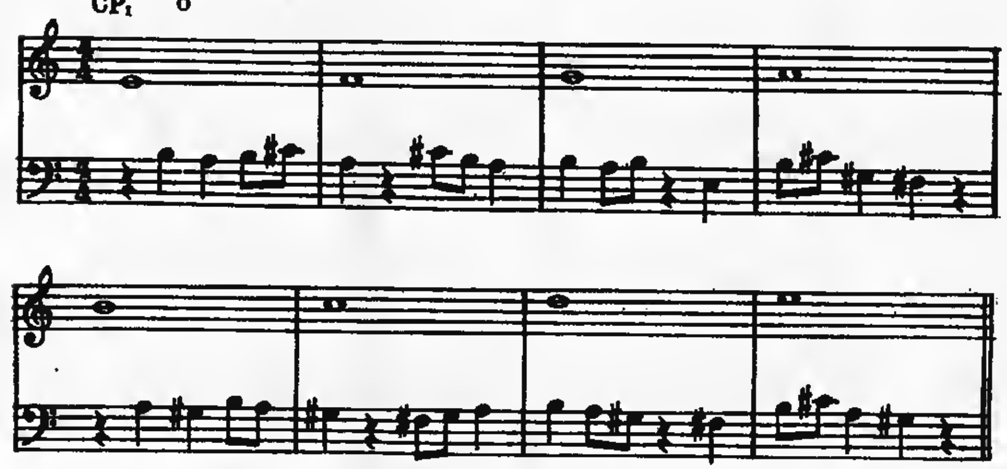


Figure 32. Axial coordination.

(2) Type III

Figure 33. Canonic imitation.

(3)

Figure 34. Counterpoint to ground melody (continued).

Figure 34. Counterpoint to ground melody (concluded).

All other techniques, such as inversions and expansions, constitute variation and therefore are not applicable to the composition of thematic units. They participate in building up the whole, and for this reason will be considered in the Composition of Thematic Continuity.* This statement concerns all six of the preceding chapters.

*See Part II of this Book. (Ed.).

CHAPTER 8

DENSITY AS MAJOR COMPONENT

DENSITY becomes a dominant factor of the thematic unit when the quantitative distribution of elements (parts) and groups (assemblages) becomes the chief characteristic of such a unit. Nevertheless, the greatest advantage offered by the Theory of Density lies in the composition of continuity from the original group of density by means of positional rotation. It is not difficult for an experienced composer to conceive one density-group as a thematic unit; but the instantaneous composition of melodies and harmonies as textural thematic groups in a considerable temporal extension cannot be solved satisfactorily except by the scientific method. This method which includes both the composition of a density-group and its positional rotation was fully described in Book IX, Chapter 15, Composition of Density in its Application to Strata.

Composition of thematic units or of thematic continuity from density-groups is of particular advantage where large instrumental combinations participate in the score. Chamber, symphonic, choral music, and ensemble music, in general, require such technique.

The student of this theory must realize that positional rotation, as was pointed out before, does not interchange the positions of harmonic strata or their parts, but refers solely and entirely to the positions of thematic textures, i.e., melodies and harmonies conceived as rhythmic and instrumental forms.

The practical outcome of this technique is the projection of thematic textures through harmonic strata of a $\Sigma^{\rightarrow}$, which in itself remains constant. Under such conditions, a certain melody M_I may appear in the different strata or parts of the strata accompanied by another melody M_{II}, which also may appear at different times in the different strata, and which may, in turn, be accompanied by harmony or several harmonies (which are detectable through their temporal and instrumental characteristics).

As positional rotation takes place, all these thematic textures undergo mutations, which change their positions, within strata and parts, individually and reciprocally.

In order to illustrate this technique more fully, we shall demonstrate not only the composition of thematic units from density as a major component, but also the respective form of continuity evolved from such units as the result of positional rotation.

As it was stated in the Theory of Density,* composition of Density-Groups may evolve either from a scheme of density or from a progression of harmonic strata.

*See p. 1227.

We shall use, for our illustration, the $\Delta^{\rightarrow}\Sigma^{\rightarrow}$ scheme offered in Figures 141, 142, 143 and 144 of the Composition of Density.*

Let us assume that the 3S of the Σ, superimposed upon the $\Delta_0\Theta_0$, are assigned in the following manner:

$$S_I \equiv M_I;\ S_{II} \equiv H;\ S_{III} \equiv M_{II}.$$

Then the respective textures controlling the corresponding parts and strata appear as follows:

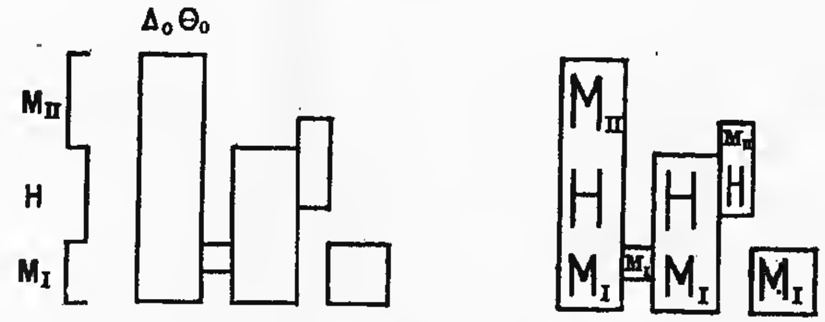


Figure 35. Textures: $S_I \equiv M_I;\ S_{II} \equiv H;\ S_{III} \equiv M_{II}$.

This represents a pattern consisting of two melodies and one harmonic accompaniment.

It is to be remembered that $\Delta^{\rightarrow}$ offered in Figure 142 begins with the third phase of dt. For this reason we acquire the following scheme of thematic continuity, based on the three original thematic textures.

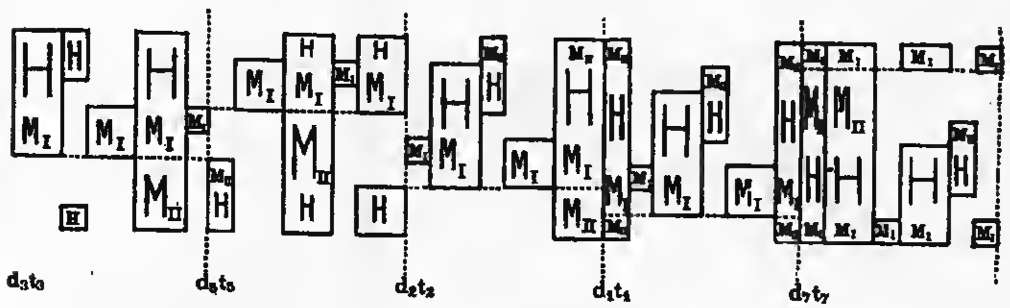


Figure 36. Scheme of thematic continuity. $\Delta^{\rightarrow}$ *begins with third place of dt.*

*See pp. 1238-9, 1240, 1241, 1244.

In order to individualize each thematic texture, we shall assign to each of the textures an individually selected T:

$$T(M_I) = (4+2+2) + (2+2+4)$$
$$T(M_{II}) = (1+1+1+1+3+1) + (3+1+2+2)$$
$$T(H) = 1+1+2+1+1+2$$

Instrumental characteristics may be added to this. We shall equip the harmonic accompaniment with a certain constant form of attacks.

$T(M_I) = (4+2+2)+(2+2+4)$

$T(M_{II}) = (1+1+1+1+3+1)+(3+1+2+2)$

$T(H) = 1+1+2+1+1+2$

Figure 37. Density as major component (continued).

Figure 37. Density as major component (continued).

Figure 37. Density as major component (continued).

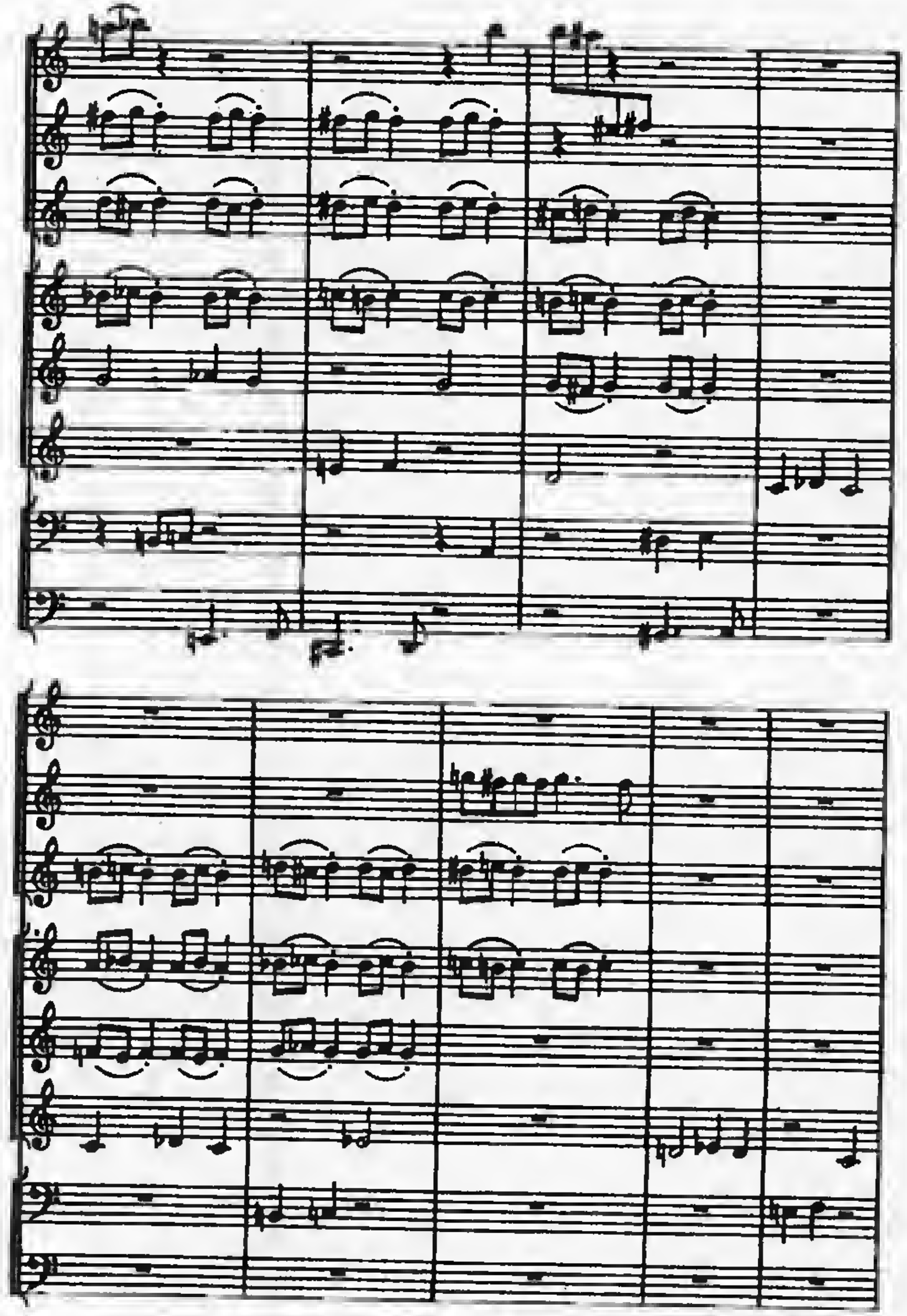


Figure 37. Density as major component (continued).

Figure 37. Density as major component (*concluded*).

When such a score is orchestrated, melodies which derive from *adjacent* parts of one or more strata are assigned to one or more instruments to play the *continuous* portion of melody in unison. However, for special orchestral effects, where extreme differentiation of tone-qualities is desired, orchestration can follow the fragmentary portions of one continuous melodic extension, assigning a different timbral participant to each fragment (no matter how brief) which derives from an individual part. It should be remembered that this extreme refinement is due, in our example, to the fact that d = p. This refinement associates itself, *ipso facto* with the style, where general economy of resources is the fundamental technical premise. Of all composers, Anton von Webern is probably the only one who went as far, as he did, in "splitting" thematic units.

It is easy to see that the effort required in orchestration can be reduced to a minimum by making a density-group the major component of thematic continuity.

CHAPTER 9

INSTRUMENTAL RESOURCES AS MAJOR COMPONENT

THE musical past shows that while some composers were capable of producing a real synthesis of textural and instrumental resources, the majority were not able to produce such a balance of the diversified techniques which constitute a musical composition as a whole. Thus, some creative artists, while in possession of numerous melodic and harmonic devices, were relatively (and sometimes completely) unsuccessful in handling instrumental techniques. At the same time others had very fruitful instrumental ideas and lacked sufficient technique in melodic and harmonic composition.

The dominating and impulsive types, among whom symphonic conductors are usually included, often write what may be called "conductorial music"—for which the Germans have an appropriate term, "Kappelmeister-musik." In many a composition by such men, orchestral versatility of device, coupled with proper use of instruments, usually helps offset the emptiness of intonational forms. The greatest representative of this creative type in the past is Hector Berlioz. At present, however, the sins of conductorial composers might be regarded as virtuous accomplishments with a number of our contemporaries, who, possessing great harmonic and sometimes rhythmic dexterity, lack this particular quality. In some cases, the crudeness of harmonic and melodic technique can be completely overshadowed by the expressiveness of orchestral resources, which in such cases become the major component of the thematic structure.

It is not so much a matter of refinement and skill in orchestration, as it is the simple fact of such a component being prominently present. Such is the case with Beethoven, in whose music, and practically for the first time, dynamics and forms of attack played such an important part.

At any rate, as the student of this system is most luxuriously equipped with all the imaginable techniques, it is time for him to become aware of the importance of *instrumental resources*, as these resources constitute one of the most powerful media of musical expression. The latter consideration makes us believe that a great deal of music written today, interesting as it may be for professionals, is sterile for lack of one of its most vital ingredients.

In writing functional music, i.e., music which is capable of stimulating associations, as most music written for the theatre, cinema, radio and television must be, in order to serve the purpose, instrumental resources most frequently are the major component of a thematic structure.

As details of this subject pertain to the field of orchestration, we shall confine ourselves to essentials for the present. The immediate goal of this discussion is to make the potential composer aware of such resources as instrumental structure offers.

Instrumental resources can be classified into the following fundamental components:

(1) *Density;* symbol: D (density);
(2) *Dynamics;* symbol: V (volume);
(3) *Tone-Quality;* symbol: Q (quality);
(4) *Instrumental Forms;* symbol: I (instrumental);
(5) *Forms of Attack;* symbol: A (attack).

The composer can start to conceive a certain thematic unit with one individual component or with any combination of the above components. But in order to conceive of anything as a definite form in music, two elements are necessary:

(1) configuration of time (T) and
(2) configuration of the special component (in this case: D, V, Q, I or A).

The behavior of the configuration of a special component during an assigned time-period is the basis of composition of thematic units. But in order to evolve configurations, it is necessary to have a scale from which such configurations can be evolved. Then the various degrees of the respective scales become basic units of the potential configurations.

We have seen before that the mere fact that a certain pitch-scale has two units already implies its configurational versatility. Such versatility is low as compared with that of a five-unit scale. But the latter is not nearly as versatile as a seven or eight-unit scale. Thus we arrive at the idea of *low*, *medium* and *high* versatility. We already have used such an approach more than once. In one instance, it was applied to low, medium and high density, in the branch of Part-Melodization.* Later, this elementary approach was changed to a detailed analytical study. Following this method, we shall deal with instrumental resources in an elementary manner for the present, leaving all strictly technical aspects to our analysis of orchestration.

Density has been thoroughly discussed in the Applications of General Harmony.** Instrumental Forms, likewise, received full attention in the respective branch dedicated to this matter.***

The meaning of composing thematic units, as instrumental forms first, implies the selection of instrumental configurations ruling over a certain simultaneity-continuity [the instrumental sigma: $\Sigma(I)$].

A certain unit, for example, may consist of Iap, while another unit may consist of $I\frac{a2p}{a3p}$, both implying definite time-periods in definite relations. For example:

$T^{\rightarrow}Iap = 4T$ and $T^{\rightarrow}I\frac{a2p}{a3p} = 12T$. In this case their ratio is $1 \div 3$. The harmonic content of both thematic units in this case becomes a minor component. This leaves for our discussion the three new components of instrumental resources: *dynamics*, *tone-quality* and *attack-forms*.

*See Vol. I, p. 700.

**See p. 1226 ff.

***See p. 883.

A. Dynamics

Dynamic scales can be composed within the range of intensity associated with music. As the excitor (amplitude) and the reaction (volume) are, according to the Weber-Fechner law, in logarithmic dependence ($y = \log x$), we can only approximate the designated degrees of volume in musical performance.

We may generally agree that a certain standard of *pp*, *p*, *mf*, *f* and *ff* can be established for practical purposes. But even such an allowance can be admitted only after we specify what instrument or group of instruments we mean. For *f* in a large symphony orchestra is quite different in volume from *f* in a string quartet. Nevertheless, even such a vague definition of dynamic degrees helps to a certain extent when the performer is confronted with interpretation of the composer's intentions.

We can hypothetically assume that the minimum of dynamic flexibility results from a one-unit (one-degree) scale. A one-unit scale can be any degree of the total range of volume. It can be an equivalent of *pp*, *p*, *mf*, *f* or *ff*. There are some forms of folk-music generally performed as *mf*. Then there are dance-bands in the U.S.A. that play everything *f*. When such dance-bands dare to produce two dynamic degrees, such as *p* and *f*, we witness the birth of a two-unit dynamic scale.

We shall replace (only for the composer's use) the customary symbols by the symbols of v and V. These symbols represent a dynamic unit (degree) and a dynamic group respectively. Thus we arrive at the following classification of dynamic scales:

(1) One-unit scales:

$V = v \equiv v_I, v_{II}, v_{III}, v_{IV}, v_V, \ldots,$ where
$v_I \equiv pp, v_{II} \equiv p, v_{III} \equiv mf, v_{IV} \equiv f, v_V \equiv ff.$

(2) Two-unit scales:

$$V = 2v \equiv v_I + v_{II}, v_I + v_{III}, v_I + v_{IV}, v_I + v_V,$$
$$v_{II} + v_{III}, v_{II} + v_{IV}, v_{II} + v_V,$$
$$v_{III} + v_{IV}, v_{III} + v_V,$$
$$v_{IV} + v_V.$$

(3) Three-unit scales:

$$V = 3v \equiv v_I + v_{II} + v_{III}, v_I + v_{II} + v_{IV}, v_I + v_{II} + v_V,$$
$$v_I + v_{III} + v_{IV}, v_I + v_{III} + v_V,$$
$$v_I + v_{IV} + v_V,$$
$$v_{II} + v_{III} + v_{IV}, v_{II} + v_{III} + v_V,$$
$$v_{II} + v_{IV} + v_V,$$
$$v_{III} + v_{IV} + v_V.$$

(4) Five-unit scales:

$$V = 5v \equiv v_I + v_{II} + v_{III} + v_{IV} + v_V.$$

Of these scales the most practical are the scales of symmetric structure, i.e., with an equidistant arrangement of the units within the total hypothetic range of $V = 5v$. The four-unit scales are entirely omitted, as our classification is based on so-called "normal series," which take place in crystal formations, and with which we are already familiar through the Evolution of Rhythm Series.*

Thus, the best selections from the above table are:

$V = v$	any selection
$V = 2v$	$v_I + v_V$; $v_{II} + v_{IV}$
$V = 3v$	$v_I + v_{III} + v_V$; $v_{II} + v_{III} + v_{IV}$
$V = 5v$	one scale

The transition from one dynamic degree to another can be either sudden or gradual. The first form of transition can be indicated in our symbols or in the customary musical symbols as the sequence of the different degrees within a specified time period. For instance:

$$v_I4t + v_{III}2t + v_V2t$$
or: $pp4t + mf2t + ff2t$

Gradual transition (leaving the form of such graduality to the performer) takes place in two directions: from a weaker degree to a stronger degree (known in musical terminology as *crescendo*), and from a stronger degree to a weaker (known as *diminuendo*). The latter are expressed in our notation as follows:

$(v_I \rightarrow v_{III})4t + (v_V \rightarrow v_{III})2t + (v_{III} \rightarrow v_I)2t$, or in musical notation:
$(pp < mf)4t + (ff > mf)2t + (mf > pp)2t.$

The dynamic groups V must be composed with a view to their potential correspondence with other components. For example:

$$T(V) = 16(2 + 1 + 1)$$
$$T(H) = 4(2 + 1 + 1)^2$$
$$T(M) = (2 + 1 + 1)^3$$

Let $V = 3v = v_I + v_{III} + v_V$. Then $V^{\rightarrow}$ (the dynamic continuity-group) can be composed as follows:

$$V^{\rightarrow} = v_V32T + (v_I \rightarrow v_V)16T + (v_V \rightarrow v_{III})16T.$$

*See Vol. I, p. 84.

B. Tone-Quality

By the same method, scales of tone-quality can be established. Each degree of quality (q) becomes a unit of the *quality-scale* (Q). From such scales, thematic units can be composed as timbral groups. Intonation and temporal structure can be devised later in order to conform to the temporal organization of the Q-group.

In evolving the scales of quality, we shall consider the following forms: $Q = q$, $Q = 2q$, $Q = 3q$, $Q = 5q$. In some special cases, as in writing for a stringed-bow ensemble, even $Q = 9q$ may be practical.

Since in this chapter we are concerned only with the general forms of timbral composition, our thematic units will be expressed only in terms of Q and not in any concrete selection of instruments. The latter can be superimposed upon any $\overrightarrow{Q}$, i.e., quality continuity-group, and technically belongs to a study of Orchestration. Neither shall we deal with acoustical matters now: they, too, belong to the field of orchestration.

For the present, without attempting to explain quality as phenomenon, we shall resort to the most obvious and, at the same time, the most fundamental conception of the quality-range.

The limits of perceptible quality-range can be defined as *open tone* (lower limit) and *closed* (muted) *tone* (upper limit). Open tone is characterized by a small quantity (sometimes none) of partials and is associated with the tuning fork, flute-stops of an organ, various types of flutes and the upper range of a French horn; also with the string-bow instruments, when played over the fingerboard (*sul tasto*).

Closed tone is characterized by an excessive aggregation of partials within a certain acoustical range and is associated with the heavily muted brass instruments and the string-bow instruments, when the latter are muted and played near the bridge (*sul ponticello*).

It is important to note that the tone production of the open tone is *immediate* (like blowing into the mouth-hole of a flute), while the tone production of the closed tone is *mediate* (as besides the mouthpiece, there is an "acoustical screen" produced by the mute).

In the quality-range of five degrees, the remaining three intermediate degrees appear as follows:

(1) The *single-reed* quality, which is associated with the clarinet and with string-bow instruments bowed in the customary manner. Some physical characteristics affecting this quality are: single-reed mouthpiece, cylindrical bore, and, as a consequence, the presence of *odd* partials.

(2) The *stopped* quality, which can be associated with stopped French horn and the slightly nasal character of the single-reed instruments with a conic bore, such as the saxophones.

(3) The *nasal* quality of prominence, which is associated with the double-reed instruments, such as oboes, bassoons, and with stringed bow instruments when played at the bridge but without a mute; it is also present, in the customary form of execution, in the high register of the 'cello.

Thus, we arrive at the basic quality-range of five degrees:

(1) open $\equiv q_{I}$;	symbol: ○
(2) single-reed $\equiv q_{II}$;	symbol: R
(3) stopped $\equiv q_{III}$;	symbol: ⊕
(4) double-reed (nasal) $\equiv q_{IV}$;	symbol: RR
(5) closed (muted) $\equiv q_{V}$;	symbol: ●

With extreme skill in orchestration, a certain degree of graduality, in transition from one q to another can be accomplished, but it is more practical as a rule, to conceive the $\overrightarrow{Q}$ schemes with the forms of sudden transitions only. In the year 1932, as a result of my collaboration with Leon Theremin, an electronic organ was constructed (at present it is in the possession of Gerald F. Warburg) on which the closing of an open tone could be accomplished as continuity by means of condensers controlled by pedals.

It is important to realize that the natural instrument of the human voice is capable of such continuous transitions from one q to another by modification of vowels and consonants. This topic will be discussed in orchestration.

Quality-scales can be classified in the same fashion as the dynamic scales, with the only difference that the symbols Q and q take the place of V and v respectively.

An example of $\overrightarrow{Q}$ evolved from $Q = 3q = q_{II} + q_{III} + q_{IV}$.

$$\overrightarrow{Q} = q_{IV}\,3T + \frac{q_{II}}{q_{IV}}\,T + q_{III}\,2T + \frac{q_{III}}{q_{II}}\,2T$$

As it follows from the previous explanation, such $\overrightarrow{Q}$ schemes can be coordinated with M and $\overrightarrow{H}$ on the basis of T, and all three of them, in turn, can be coordinated with $\overrightarrow{V}$.

There is a substantial amount of material which can be used for such ultimate forms of complex temporal coordination in the *Theory of Rhythm*, particularly in the Distributive Involution and in the Synchronization of Three and More Generators.*

It is to be remembered that the quality-scales must be evolved with the potential instrumental selection in view.

C. Forms of Attack

Forms of attack can also be classified by the method which we have applied to V and Q.

After the main classification of the attack-range is established, the composer may evolve a selective scale of attack-forms (A and a). These attack-forms, ultimately arranged into attack-form continuity-groups ($\overrightarrow{A}$) and coupled with T, constitute thematic units, conceived from the viewpoint of a and A.

*See Vol. I, p. 70 and p. 24.

The basis on which we establish the fundamental classification of attack-forms is the *durability of attack*. Again, as in the case of V and Q, this subject cannot be subjected to scientific scrutiny, for the present, as, in actuality, no attack-form can be dissociated from its dynamic characteristic. Nevertheless, the composer may derive important benefits from the concept and method of attack-scales, even in their elementary and approximate forms.

We shall define the lower limit of the attack-form range as uninterrupted continuity of sound resulting from one attack and extending over a certain time-period (*legatissimo*, in musical terminology). Then the upper limit of this range becomes a percussive form of attack with a minimum durability (*staccatissimo*, in musical terms, an equivalent of hard staccato, marked as ▼).

Between these two limits we find the basic three intermediate degrees of attack-forms, which are:

(1) the *legato* form, which minimizes the intensity of attack, like the detached (detaché) manner of bowing the stringed-bow instruments.

(2) The *portamento* form, which is discontinuous but not abrupt (marked for orchestral instruments, and for the Piano).

(3) The *staccato* form, which is abrupt and therefore percussive; but is associated with low dynamic degrees. It corresponds to soft staccato (usually marked . . .).

All further refinements of this range would be practical only for the stringed-bow instruments, and will be discussed in Orchestration.

As in the case of other instrumental resources, we can establish scales of attack-forms from which thematic units may be composed.

It should be remembered that this component is fairly new, as compared to the others. It usually has been left to the initiative of the performer.

Scales of attack-forms follow our usual classification: $A = a$, $A = 2a$, $A = 3a$, $A = 5a$, . . .

The respective correspondence of the attack-form degrees are:

(1) *legatissimo:* a_I

(2) *legato:* a_{II}

(3) *portamento:* a_{III}

(4) *staccato:* a_{IV}

(5) *staccatissimo:* a_V

As all known music is quite flexible with regard to the time-periods of ever-hanging a, time units can be specified either in t or T, depending on the degree refinement which the composer intends to impose.

An example of $A^{\rightarrow}$, evolved from $A = 3a = a_{II} + a_{III} + a_{IV}$.

$$A^{\rightarrow} = a_{II}2t + a_{IV}2t + a_{III}4t + a_{IV}4t + a_{III}2t + a_{II}2t.$$

This equipment is fully sufficient for the student-composer to proceed with composition of thematic units as they have been defined in this theory. He may choose any technical form as the major component. He must synchronize this major component with any minor components he may select. It is the coordination itself that is of prime concern. He is free otherwise in making his decisions and selecting his technical resources—choice being controlled only by the composer's decision to evolve this or that type of music.

The next stage of this technique consists of coordinating thematic units into an *a priori* planned musical whole.

PART II

COMPOSITION OF THEMATIC CONTINUITY

CHAPTER 10

MUSICAL FORM

THE term "musical form" is usually applied to casually contrived schemes of thematic sequence. Such schemes are both vague and dogmatic. They are devised solely on a trial-and-error basis. Schemes of thematic sequence usually include two components: the sequent arrangement of subjects and the sequence of keys in which the subjects must follow one another.

While the succession and the recurrence of subjects are intended to achieve some form of symmetry, without regard to the temporal relations of the subjects, the succession of keys in which such schemes are presented is based entirely on antiquated conceptions of tonality. The most convincing proof that such thematic schemes are unsatisfactory, with regard to both their symmetry and key-sequence, lies in the outstanding compositions of the classics. Even Beethoven, who followed these dogmatic schemes more closely than others, had to deviate from the schemes in order to get satisfactory results. This is true of his selection of recurrences, distributions and key-time relations.

Of course, there is no one form that is specifically "musical." Form, conceived as temporal structure evolved from thematic units, most obviously must possess the characteristics inherent in all forms of temporal regularity. But temporal regularity implies both the form of the sequence of units involved and the periodic relationship among such units.

So far as classical schemes of thematic sequence are concerned, we find a few prototypes of such schemes. The latter can be generally classified as ***monothematic*** (one subject) and ***polythematic*** (more than one subject). In the monothematic schemes, the subject (thematic unit) repeats itself several times and is usually subjected to variations. Such a form of thematic sequence is usually known as "theme and variations." Polythematic schemes consisting of two subjects usually appear in two forms: in the form of direct repetition: $(A + B) + (A + B) + \ldots$, like the "lower forms of rondo" and the "old sonata form"; or in the form of triadic symmetry: $A + B + A$, as in the "three-part song" or the "complex three-part song." In shorter compositions, and particularly in the structures of thematic units, a two-subject scheme adapts itself to the $\frac{4}{4}$ series binomial, i.e., $3+1$. It is usually known as a "two-part song" and may have the following scheme: $A_1 + A_2 + B + A_2$. This form of thematic sequence was commonly used for themes from which variations were to be devised, and as a complex structural unit in the form of triadic symmetry—in which case it was used either for A or for B.

It is interesting to note that none of the classical schemes contain symmetric inversion, except in the case of the old sonata form, where symmetric inversion of keys takes place—but not of the sequence of subjects. Designating the key of the tonic as T and the key of the dominant as D, we can represent this scheme as follows: $A(T) + B(D) + A(D) + B(T)$.

Certain schemes of polythematic sequence containing three subjects are referred to as "the higher forms" of the rondo. Whereas the "lower" rondo usually appears in the place of the slow movement in the larger forms (symphony, sonata, quartet, or other chamber ensemble), the "higher" rondo usually is employed for the finale in these forms. What I call the "higher" rondo usually conforms to the following scheme of thematic sequence: $A_1 + B_1 + C + A_2 + B_2$. It is generally agreed that C is longer than A or B taken individually, but not necessarily as long as $A + B$. The chief difference between A_1 and A_2, and B_1 and B_2, respectively, is in the key-relations. The main tendency is the conflict of keys between A_1 and B_1, and the reconciliation between A_2 and B_2. As these relations are workable only in certain forms of tonality, we shall not be concerned with further details pertaining to this matter.

This "higher" rondo may be looked upon as a pentadic form without an axis of inversion. Pentadic forms on a smaller scale are also to be found in some compositions of dance character. In Chopin's waltz No. 7, the scheme could have been a perfect pentadic symmetry, i.e., $A + B + C + B + A$, if not for the composer's passion for repetition, which spoiled the form by adding a "coda" (literally: "tail") consisting of an additional repetition of B.

A special version of triadic symmetry appears as the first movement of a symphony or sonata and is known as "the sonata form," or "the sonata-allegro form". This sectional scheme generally consists of $A_1 + B + A_2$. On the other hand the thematic scheme resembles the pentadic form of the "higher" rondo, with the difference that instead of subject C, there is the so-called "development." The development usually consists of harmonic transpositions of a continuously repeated fragmentary structure (or structures) borrowed from any of the subjects of section A_1. The latter often consists of a large number of subjects. There are, for example, eight of them in the piano sonata No. 4 by Beethoven. Section A_1 is usually known as "exposition"; section B, as "development"; and section A_2, as "recapitulation," which is an abbreviated exposition with a greater key-unification.

Contemporary composers have developed many individual schemes. What they call a "symphony" does not necessarily resemble the classical scheme. Unification of all movements of the sonata form was attempted by Liszt in his "cyclic" piano sonata in B minor. Contemporary composers often have more than one exposition, and the developments are often dissociated from their expositions. In some other cases, each theme undergoes immediately its own development, as in the later sonatas of Scriabine.

Polyphonic forms have schemes of thematic sequence similar to those of the homophonic forms. Thus, a fugue with one subject corresponds to a theme with variations or to a "lower" rondo. A fugue with two subjects is written in the same scheme; the only difference is that both subjects appear simultaneously and are temporarily treated as one subject. In other instances, a fugue with two subjects (see: *The Well-Tempered Clavichord*, Vol. II, No. XVIII) is evolved according to Hegel's triad, i.e., as thesis, antithesis, synthesis, or: $A + B + \frac{A}{B}$.

The manifold forms of thematic sequence in European musical civilization range from continuous repetition of brief thematic structures, which remind one of the repeat-patterns of visual arts, like tiles or wallpaper (Chopin wrote much "wallpaper" music); through continuously flowing broad linear design, constantly varied and syntactically dissociated by cadences, like the Gregorian Chant; to the temporal schemes containing no repetitions and embodying no syntactical cadencing before the end, devised by contemporary Germans (Schoenberg and Hindemith), recognized as a type of ***durchkomponierte Musik*** (through-composed music) and adopted by contemporaries of other nationalities (Shostakovich). The "through-composed" music is a logical development of the so-called "twelve-tone system," where any repetition of any pitch-unit is taboo until the whole chromatic set has been exhausted.

CHAPTER 11

FORMS OF THEMATIC SEQUENCE

Forms of thematic sequence can be classified into four main groups:

(1) groups of direct recurrence;
(2) groups of symmetric recurrence;
(3) groups of modified recurrence;
(4) groups of progressive symmetry.

Monothematic continuity can be evolved in the form of direct recurrence only. In this case the form of thematic sequence is ***monomial periodicity:***

$$A + A + A + \ldots$$

Polythematic continuity based on ***two subjects*** can be evolved through all four groups:

Group one: $(A + B) + \ldots$ ***binomial periodicity***
Group two: $A + B + A$
Group three: $(A + B) + (B + A)$
Group four: $A + (A + B) + B$

Polythematic continuity based on ***three subjects*** may assume the following forms:

Group one: $(A + B + C) + \ldots$ ***trinomial periodicity***
Group two: $A + B + C + B + A$
Group three: $(A+B+C) + (B+C+A) + (C+A+B)$
Group four: $A + (A+B) + (A+B+C) + (B+C) + C$

Polythematic continuity based on ***four subjects*** may assume the following forms:

Group one: $(A+B+C+D) + \ldots$ ***quadrinomial periodicity.***
Group two: $A + B + C + D + C + B + A$
Group three: $(A+B+C+D) + (B+C+D+A) + (C+D+A+B) +$
$+ (D+A+B+C)$
Group four:
(1) $A + (A+B) + (A+B+C+D) + (C+D) + D$, i.e., $1+2+4+2+1$
(2) $A + (A+B+C) + (A+B+C+D) + (B+C+D) + D$, i.e., $1+3+4+3+1$

Polythematic continuity based on ***five subjects*** may assume the following forms:

Group one: $(A+B+C+D+E) + \ldots$ ***quintinomial periodicity***
Group two: $A + B + C + D + E + D + C + B + A$
Group three: $(A+B+C+D+E) + (B+C+D+E+A) +$
$+ (C+D+E+A+B) + (D+E+A+B+C) +$
$+ (E+A+B+C+D)$

Group four:
(1) A + (A+B) + (A+B+C) + (A+B+C+D+E) + (C+D+E) + + (D+E) + E, i.e., 1+2+3+5+3+2+1
(2) A + (A+B+C) + (A+B+C+D+E) + (C+D+E) + E, i.e., 1+3+5+3+1

Polythematic continuity based on *six subjects* may assume the following forms:

Group one: (A+B+C+D+E+F) + . . . *sextinomial periodicity*
Group two: A+B+C+D+E+F+E+D+C+B+A
Group three: (A+B+C+D+E+F) + (B+C+D+E+F+A) + + (C+D+E+F+A+B) + (D+E+F+A+B+C) + + (E+F+A+B+C+D) + (F+A+B+C+D+E)
Group four:
(1) A + (A+B+C) + (A+B+C+D+E+F) + (D+E+F) + F, i.e., 1+3+6+3+1
(2) A + (A+B) + (A+B+C+D) + (A+B+C+D+E+F) + + (C+D+E+F) + (E+F) + F, i.e., 1+2+4+6+4+2+1

Polythematic continuity based on *seven subjects* may assume the following forms:

Group one: (A+B+C+D+E+F+G) + . . . *septinomial periodicity*
Group two: A+B+C+D+E+F+G+F+E+D+C+B+A
Group three: (A+B+C+D+E+F+G) + (B+C+D+E+F+G+A) + + (C+D+E+F+G+A+B) + (D+E+F+G+A+B+C) + + (E+F+G+A+B+C+D) + (F+G+A+B+C+D+E) + + (G+A+B+C+D+E+F)
Group four:
(1) A + (A+B+C) + (A+B+C+D) + (A+B+C+D+E+F+G) + + (D+E+F+G) + (E+F+G) + G, i.e., 1+3+4+7+4+3+1
(2) A + (A+B+C) + (A+B+C+D+E) + (A+B+C+D+E+F+G) + (C+D+E+F+G) + (E+F+G) + G, i.e., 1+3+5+7+5+3+1

The above forms of thematic sequence range from the most elementary recurrence to the most refined forms of progressive symmetry. Probably the most important characteristic of the latter is the *symmetric interpolation* of subjects. While one subject appears in its last phase, some other subject makes its first appearance. Under such conditions the interpolation of events is similar to that of interpolation of generations. While somebody is in his infancy, somebody else is fully mature, and somebody else is ready to die. The neighboring position of subjects of the different ages makes this form closer to the schemes of actuality than any other form of thematic sequence. It is also important to note that the subjects are symmetrically arranged and form their own hierarchy and ranks. In some of the more developed schemes, different subjects appear a different number of times in the course of entire continuity, the extreme subjects having higher ranks than the middle ones.

CHAPTER 12

TEMPORAL COORDINATION OF THEMATIC SEQUENCE

THE next step in evolving thematic continuity consists of coordinating thematic sequence with temporal forms of regularity. The latter represent the various forms of duration-groups discussed in the *Theory of Rhythm*.*

We shall look upon these groups as forms of the temporal organization of thematic sequence. They come from four main sources and serve different purposes respectively:

(1) The resultants of interference;
(2) The permutation-groups;
(3) The involution-groups;
(4) The acceleration-groups.

As any type of duration-group may be superimposed upon any form of thematic sequence, it is important in selecting a temporal group to consider the following points.

If the form of thematic sequence is simple and such simplicity is to be maintained in its temporal organization, the temporal group should be chosen from the simplest forms of temporal regularity, such as monomial or binomial periodicity. If, on the other hand, it is desirable to introduce temporal refinements into a simple form of thematic sequence, any other more complex form of temporal regularity may be used. In order to make such forms of temporal regularity detectable in a monothematic sequence, it is necessary to subject the thematic unit, in each of its appearances, to some form of variation. The latter may be based on quadrant-rotation, tonal expansion, density, instrumental form or modal transposition.

Under such conditions each appearance of the thematic unit is associated with one or another temporal coefficient. Even if the sequence is monothematic and the temporal group is a monomial, each appearance of the thematic unit is recognizable due to the above-described variations.

More complex forms of thematic sequence may be coordinated with the simplest forms of temporal groups when it becomes desirable to accentuate the complexity of thematic sequence by contrasting it with its temporal relations. Yet in some other instances the desired form of expression is such that both the thematic sequence and the temporal form require refinement and complexity.

This theory repudiates the academic point of view, according to which some themes are so unimportant that they function as mere bridges tying the main themes together. If a certain thematic unit is unimportant and insignificant and merely consumes time, it should not participate in a composition. Looking

*See Vol. I, p. 4 to p. 93.

upon thematic continuity as an organic form, the only viewpoint we can accept is: *thematic units have their relative temporal characteristics under which they appear in the sequence.* This implies that whereas in one portion of a composition a certain thematic unit may dominate over others owing to its high temporal coefficient, in another portion of the composition the same thematic unit may become subordinate owing to its low temporal coefficient and the relatively higher temporal coefficients of other thematic units; ultimately, the first thematic unit may vanish completely, being overwhelmed by other thematic units (this we have already witnessed in the groups of progressive symmetry).

Thus, through selection of temporal coefficients, we can vary the relative importance of any one of the thematic units in any portion of a composition. If the permanent subordination of certain thematic units is desired throughout the entire composition, such thematic units must be assigned to low temporal coefficients.

In the following applications of temporal groups to the forms of thematic sequence, $T^{\rightarrow}$ represents the entire period of a composition and T, coupled with the various coefficients, represents the relative time-values of thematic units in their individual appearances. Thus T is not necessarily one measure, but a unit by which the relative durations are represented. In translating the $T^{\rightarrow}$ into actual measures, additional coefficients are required. These coefficients (or coefficient, speaking of each individual case) are constant for any one $T^{\rightarrow}$.

For example, a temporal continuity-group $T^{\rightarrow}$ may originally have the following form: $T^{\rightarrow} = 3T + T + 2T + 2T$. At the same time, T may be equivalent to T'', $2T''$, $3T''$, . . . NT''. Then, in the actual realization of such a continuity group, we may have a variety of solutions, depending on the correspondence we establish between T and T''.

(1) $T = T''$, then: $T^{\rightarrow} = 3T'' + T'' + 2T'' + 2T''$;
(2) $T = 2T''$, then: $T^{\rightarrow} = 6T'' + 2T'' + 4T'' + 4T''$;
(3) $T = 3T''$, then: $T^{\rightarrow} = 9T'' + 3T'' + 6T'' + 6T''$;
. . . .
(4) $T = NT''$, then: $T^{\rightarrow} = 3NT'' + NT'' + 2NT'' + 2NT''$.

We shall now illustrate the coordination of $T^{\rightarrow}$ with the various forms of thematic sequence.

A. Using the Resultants of Interference

The simplest form of thematic continuity results from coordinating a monothematic sequence with a monomial form of temporal regularity.

$$T^{\rightarrow}(A) = A_1T + A_2T + A_3T + \dots$$

The same form of thematic sequence may be coordinated with a binomial form of temporal regularity. Then:

$$T^{\rightarrow}(A) = (A_1MT + A_2NT) + \dots$$

The values of M and N depend on the style-series of the respective composition. Thus, on being applied to $\frac{4}{4}$ series, such a form offers the following four basic variants:

(1) $T^{\rightarrow}(A) = (A_13T + A_2T) + \dots$
(2) $T^{\rightarrow}(A) = (A_1T + A_23T) + \dots$
(3) $T^{\rightarrow}(A) = (A_13T + A_2T) + (A_3T + A_43T) + \dots$
(4) $T^{\rightarrow}(A) = (A_1T + A_23T) + (A_33T + A_4T) + \dots$

Further refinement, variety and complexity can be achieved through the selection of resultants associated with the respective style of temporal organization. The following are a few examples which derive from the second summation-series, and therefore are associated with $\frac{4}{4}$ series in the evolution of style.

(1) $T^{\rightarrow} = r_{4\div3}$;
$T^{\rightarrow}(A) = A_13T + A_2T + A_32T + A_42T + A_5T + A_63T.$

(2) $T^{\rightarrow} = r_{\underline{4\div3}}$;
$T^{\rightarrow}(A) = A_13T + A_2T + A_32T + A_4T + A_5T + A_6T + A_7T + A_82T + A_9T + A_{10}3T.$

(3) $T^{\rightarrow} = r_{7\div4}$;
$T^{\rightarrow}(A) = A_14T + A_23T + A_3T + A_44T + A_52T + A_62T + A_74T + A_8T + A_93T + A_{10}4T.$

Figure 38. Derived from second summation-series.

B. Permutation-Groups

The following illustration refers to permutation-groups. Selecting $3 + 1 + 2 + 2$ as an appropriate form of temporal regularity, we shall subject it to circular permutations, thus quadrupling the original period of duration:

$$T^{\rightarrow} = (A_13T + A_2T + A_32T + A_42T) + (A_5T + A_62T + A_72T + A_83T) + (A_92T + A_{10}2T + A_{11}3T + A_{12}T) + (A_{13}2T + A_{14}3T + A_{15}T + A_{16}2T).$$

Longer forms of thematic continuity are hardly necessary under ordinary circumstances. In the above case, one thematic unit makes 16 appearances.

C. Involution-Groups

The use of involution-groups as forms of temporal regularity is of particular value, when the proportionate relations between thematic units become the chief characteristic of continuity.

Individual and synchronized involution-groups of different powers can be used in sequence as forms of temporal regularity of $T^{\rightarrow}$.

Examples:

(1) $T^{\rightarrow} = (3+1)^2$;
$T^{\rightarrow}(A) = A_1 9T + A_2 3T + A_3 3T + A_4 T$.

(2) $T^{\rightarrow} = (2+1+1)^2$;
$T^{\rightarrow}(A) = (A_1 4T + A_2 2T + A_3 2T) + (A_4 2T + A_5 T + A_6 T) +$
$+ (A_7 2T + A_8 T + A_9 T)$.

(3) $T^{\rightarrow} = (1+1+2)^2 + 4(1+1+2)$;
$T^{\rightarrow}(A) = [(A_1 T + A_2 T + A_3 2T) + (A_4 T + A_5 T + A_6 2T) +$
$+ (A_7 2T + A_8 2T + A_9 4T)] + (A_{10} 4T + A_{11} 4T + A_{12} 8T)$.

Figure 39. Involution-groups of different powers.

The use of various forms of acceleration (positive and negative) becomes necessary when temporal regularity expresses consistent growth or decline.

In monothematic continuity, a thematic unit either builds itself up with each consecutive appearance or goes into gradual decline.

Examples:

(1) $T^{\rightarrow} = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$;
$T^{\rightarrow}(A) = A_1 T + A_2 2T + A_3 3T + A_4 4T + A_5 5T + A_6 6T +$
$+ A_7 7T + A_8 8T$.

(2) $T^{\rightarrow} = 8 + 4 + 2 + 1$;
$T^{\rightarrow}(A) = A_1 8T + A_2 4T + A_3 2T + A_4 T$.

(3) $T^{\rightarrow} = 1 + 3 + 4 + 7$;
$T^{\rightarrow}(A) = A_1 T + A_2 3T + A_3 4T + A_4 7T$.

Figure 40. Growth and decline in monothematic continuity.

Each form of thematic sequence may assume various forms of temporal coordination, in the same way as indicated for the monothematic thematic sequence. In the form of sequence based on more than one subject, an additional technique may be used: interference between the number of terms of the temporal group and the number of terms of the thematic sequence. This technique is appropriate whenever it is desirable to obtain a rather long continuity for an entire composition, with relatively few thematic units and relatively few appearances of the latter.

We shall now apply some of the forms of temporal regularity to thematic sequences based on two subjects.

Thematic sequence: $(A + B) + \ldots$
$T^{\rightarrow} = 3 + 1$;
$T^{\rightarrow}(A + B) = A_1 3T + B_1 T + A_2 3T + B_2 T + \ldots$

In this case A is always 3 times longer than B.

$T^{\rightarrow} = r_{\underline{4 \div 3}}$;
$T^{\rightarrow}(A+B) = A_1 3T + B_1 T + A_2 2T + B_2 T + A_3 T + B_3 T +$
$+ A_4 T + B_4 2T + A_5 T + B_6 3T$.

In this case the period of A goes into decline, while the period of B grows.

Thematic sequence: $A + B + A$.
$T^{\rightarrow} = 3 + 1 + 4$;
$T^{\rightarrow}(A+B+A) = A_1 3T + BT + A_2 4T$.

If B is a bridge, such a temporal group is acceptable; otherwise a different variant of the same group would be preferable. For instance:

$T^{\rightarrow} = 3 + 4 + 1$; then:
$T^{\rightarrow}(A+B+A) = A_1 3T + B4T + A_2 T$, in which case
$T^{\rightarrow}(A_1 + A_2) = T^{\rightarrow}(B)$.

The use of interference necessitates the recurrence of the entire thematic sequence. For instance:

$T^{\rightarrow} = 3 + 1$;
$T^{\rightarrow}(A+B+A) = (A_1 3T + B_1 T + A_2 3T) + (A_3 T + B_2 3T + A_4 T)$.

The use of involution-groups, when applied to thematic sequences, yields the forms of proportionate temporal expansion or contraction for each subject. Thus $A + B$ coordinated with $T^{\rightarrow} = (3+1)^2$ produces the following result:

$T^{\rightarrow}(A+B) = A_1 9T + B_1 3T + A_2 3T + B_2 T$.

For opposite effects, the same scheme can be used in reverse, i.e., $T^{\rightarrow} = (1+3)^2$. Then: $T^{\rightarrow}(A+B) = A_1 T + B_1 3T + A_2 3T + B_2 9T$.

When more recurrences are desirable in proportionate distribution, involution of higher powers becomes necessary. For instance:

$T^{\rightarrow} = (3+1)^3$;
$T^{\rightarrow}(A+B) = (A_1 27T + B_1 9T + A_2 9T + B_2 3T) +$
$+ (A_3 9T + B_3 3T + A_4 3T + B_4 T)$.

It is important to study the effects of coefficient-groups upon schemes of thematic sequence in direct recurrence as compared to permutation-groups obtained from the same schemes.

For example:

Thematic sequence: $3(A + B + C)$;

$$T^{\rightarrow} = 2 + 1 + 1;$$
$$T^{\rightarrow}[3(A+B+C)] = (A_1 2T + B_1 T + C_1 T) + (A_2 2T + B_2 T + C_2 T) + $$
$$+ (A_3 2T + B_3 T + C_3 T).$$

Thematic sequence: $(A + B + C)\circlearrowright$;

$$T^{\rightarrow} = 2 + 1 + 1;$$
$$T^{\rightarrow}(A+B+C)\circlearrowright = (A_1 2T + B_1 T + C_1 T) + (B_2 2T + C_2 T + A_2 T) +$$
$$+ (C_3 2T + A_3 T + B_3 T).$$

Figure 41. Effect of coefficient groups.

There is a general way of selecting temporal groups, which becomes particularly practical for the symmetric schemes of thematic sequence consisting of many subjects. As the general characteristic of symmetric groups is reversibility from the center, temporal groups constructed on the same principle and with a corresponding number of terms fit the respective thematic sequence perfectly.

Example:

Thematic sequence: $A + B + C + D + E + D + C + B + A$

As this scheme has five subjects, it requires a five term temporal group. Let it be: $r_{\frac{4 \div 3}{2}}$. Then the temporal group assumes the following appearance:

$$T^{\rightarrow} = 3 + 1 + 2 + 1 + 1 + 1 + 2 + 1 + 3$$

Hence: $T^{\rightarrow}(A+B+C+D+E+D+C+B+A) =$
$= A_1 3T + B_1 T + C_1 2T + D_1 T + ET + D_2 T + C_2 2T + B_2 T + A_2 3T.$

Another important form of correlation of the groups of temporal regularity with the groups of thematic sequence consists of the application of involution-groups to the permutation-groups of thematic sequence. For comparison's sake, we shall offer an illustration of the application of involution-groups to both direct and modified recurrence.

Thematic sequence: $A + B + C$;

$$T^{\rightarrow} = (2+1+1)^2;$$
$$T^{\rightarrow}(A+B+C) = (A_1 4T + B_1 2T + C_1 2T) + (A_2 2T + B_2 T + C_2 T) +$$
$$+ (A_3 2T + B_3 T + C_3 T).$$

In this case: $A_1 = 4T$; $A_2 = 2T$; $A_3 = 2T$;
$B_1 = 2T$; $B_2 = T$; $B_3 = T$;
$C_1 = 2T$; $C_2 = T$; $C_3 = T$;

Thematic sequence: $(A + B + C)\circlearrowright$;

$$T^{\rightarrow} = (2+1+1)^2;$$
$$T^{\rightarrow}(A+B+C)\circlearrowright = (A_1 4T + B_1 2T + C_1 2T) + (B_2 2T + C_2 T + A_2 T) +$$
$$+ (C_3 2T + A_3 T + B_3 T).$$

In this case: $A_1 = 4T$; $A_2 = T$; $A_3 = T$;
$B_1 = 2T$; $B_2 = 2T$; $B_3 = T$;
$C_1 + 2T$; $C_2 = T$; $C_3 = 2T$.

Figure 42. Application of involution groups.

This case yields a greater temporal variability with respect to each individual subject.

D. Acceleration-Groups

Schemes of thematic sequence based on progressive symmetry are diversified enough to be used in temporal uniformity. In the cases of extreme refinement, however, other forms of temporal regularity may be used.

Example:

Thematic sequence: $A+(A+B) + (A+B+C) + (B+C) + C$.

Here each subject appears three times, and the total number of terms is 9. For this reason any temporal group consisting of 9 or 3 terms, and therefore not causing interference, can be used.

$$T^{\rightarrow} = T;$$
$$T^{\rightarrow}(A, B, C \text{ progressive}) = A_1 T + (A_2 T + B_1 T) + (A_3 T + B_2 T + C_1 T) +$$
$$+ (B_3 T + C_2 T) + C_3 T.$$
$$T^{\rightarrow} = 1 + 2 + 3;$$
$$T^{\rightarrow}(A, B, C \text{ progressive}) = A_1 T + (A_2 2T + B_1 3T) +$$
$$+ (A_3 T + B_2 2T + C_1 3T) + (B_3 T + C_2 2T) + C_3 3T.$$

In this case: $A_1 = T$; $A_2 = 2T$; $A_3 = T$;
$B_1 = 3T$; $B_2 = 2T$; $B_3 = T$;
$C_1 = 3T$; $C_2 = 2T$; $C_3 = 3T$.

$$T^{\rightarrow} = (2+1+1)^2;$$
$$T^{\rightarrow} = (A, B, C \text{ progressive}) = A_1 4T + (A_2 2T + B_1 2T) +$$
$$+ (A_3 2T + B_2 T + C_1 T) + (B_3 2T + C_2 T) + C_3 T.$$

In this case: $A_1 = 4T$; $A_2 = 2T$; $A_3 = 2T$;
$B_1 = 2T$; $B_2 = T$; $B_3 = 2T$;
$C_1 = T$; $C_2 = T$; $C_3 = T$.

Figure 43. Application of involution groups.

The last case offers a characteristic rank arrangement of the individual subjects.

This discussion gives the student sufficient information to enable him to use his own initiative in evolving more elaborate forms of temporal coordination of the thematic sequence. Forms of temporal regularity coming from different sources and different series should be thoroughly analyzed and studied.

CHAPTER 13

INTEGRATION OF THEMATIC CONTINUITY

In order to integrate thematic continuity in accordance with a temporally-coordinated form of thematic sequence, it is necessary to transform thematic units into subjects (themes) and their modifications, and to correlate such thematic groups with a group of key-axes.

Each of the above defined operations can be performed by means of special techniques.

A. Transformation of Thematic Units into Thematic Groups

An exposition of a subject (theme) or its modification constitutes a *thematic group*. The subject itself, or *theme*, can be defined as the *maximal thematic unit*, i.e., a thematic unit at its maximal duration.

As we have seen before, the period of a subject varies in its different expositions. If the subject is composed as the maximum of a thematic unit appearing in respective continuity, it can later be subjected to temporal modifications, such as shortening of its period.

In the thematic sequence with 3 subjects and 9 thematic groups in proportionate distribution, each subject has 3 thematic groups corresponding to 3 expositions. Thus, in a scheme: $(A_14T + B_12T + C_12T) + (A_22T + B_2T + C_2T) + (A_32T + B_3T + C_3T)$, A_1 constitutes subject A; B_1 constitutes subject B; and C_1 constitutes subject C—since all three subjects have their maximal temporal coefficient in their first exposition (indicated by the subnumeral 1). Hence: A_2 and A_3 are the shortened variants (temporal modifications) of subject A; B_2 and B_3 are the shortened variants of the subject B; and C_2 and C_3 are the shortened variants of the subject C.

Let us assume that the T of this scheme corresponds to 4 measures, or $T = 4T''$. Then subject A must be composed from its respective thematic unit as $A16T''$; subject B, from its respective thematic unit as $B8T''$; and subject C has, in this case, the period equivalent of B, i.e., $C8T''$.

If the case under discussion is evolved on the basis of $\frac{4}{2}$ series, then the actual transformation of thematic units into subjects may be realized in the following form:

$$A16T'' = 4(4T'');$$
$$B8T'' = 2(4T''); \; B8T'' = 4(2T'');$$
$$C8T'' = 2(4T''); \; C8T'' = 4(2T'').$$

Let us take another scheme having different proportions from the one we have just discussed. Let it have two subjects whose time ratio is 3. Then: $A_19T + B_13T + A_23T + B_2T$. Here, too, A_1 and B_1 constitute the original subjects (as having maximal coefficients of duration), and A_2 and B_2, their respective modifications. Thus A_2 and B_2 appear to be the temporal contractions of A_1 and B_1.

Assuming $T = 4T''$, we obtain the following thematic groups:

$$A = 36T''; \; A_2 = 12T'';$$
$$B = 12T''; \; B_2 = 4T''.$$

The transformation of thematic units into subjects may be realized in the following form:

$$A = 9(4T''); \quad B = 3(4T'')$$

Now we shall take a case where the full subject does not appear in its first exposition. Let the scheme of thematic continuity be: $(A+B+C) + (B+C+A) + (C+A+B)$, and the temporal coefficient-group be: $3 + 1 + 2$.

Then: $(A_13T + B_1T + C_12T) + (B_23T + C_2T + A_22T) + (C_33T + A_3T + B_32T)$.

In this case: $A_1 = 3T; \; A_2 = 2T; \; A_3 = T;$
$B_1 = T; \; B_2 = 3T; \; B_3 = 2T;$
$C_1 = 2T; \; C_2 = T; \; C_3 = 3T.$

Thus each thematic unit has a maximal duration in a different exposition. Therefore:

$$A_1 \equiv A; \; B_2 \equiv B; \; C_3 \equiv C.$$

This means that the full form of subject A appears in its first exposition, after which modification of this subject takes place; that the full form of subject B appears in its second exposition, and therefore B in its first and third expositions is a temporal contraction of the original B; that the full form of subject C appears only in its last (third) exposition, and therefore C in its first two expositions appears in the form of temporal contractions of the original C.

B. Transformation of Subjects into their Modified Variants

A subject can be modified with respect to its two basic components: time and pitch.

1. Temporal Modification of a Subject.

Temporal modification of a subject affects its period, but not the form of its temporal organization. The various thematic groups of a subject, corresponding to the different expositions of it in thematic continuity, are the temporally-contracted variants or portions of the original.

Our immediate technical problem lies in the definition of the method by which temporally-contracted versions of the original subject may be obtained. The shortening of the subject can be accomplished in two different ways:

(a) by reducing the value of the duration-unit;
(b) by dissecting the subject into its original thematic units, or even shorter fragments, and by using such units instead of the entire subject.

The first technique must be applied in full accordance with the style of temporal organization of the subject. If such style is associated with 2 or any multiple thereof, the coefficient of contraction must be 1/2, 1/4, . . . ; if it is associated with 3 or any multiple thereof, the coefficient of contraction must be 1/3, 1/9, . . . ; in the case of hybrid series, any of the multiples constituting such a series can be used as the coefficient of contraction. For example, if the set of coefficients controlling temporal organization of thematic continuity is associated with 6, not as the determinant of the $\frac{6}{8}$ series but as a product of 2 by 3, either 1/2 or 1/3 may be used as the coefficient of contraction. It is also appropriate to rely on any of the members of one summation-series if one of its members is the determinant.

For example, a temporal organization evolved from $r_{5\div3}$ can be contracted by means of such coefficients as 1/3, 1/5, and also 1/2 or 1/8, as 2 and 8 participate in the same (first) summation-series.

Empirically this form of temporal contraction can be performed directly in musical notation. In *Symphonic Rhumba** by Paul Lavalle, a student of this system, the entire subject consisting of 64T″, is contracted in its second exposition by the coefficient 1/2, thus resulting in a 32T″ structure. The style of the temporal organization of this composition is a hybrid of $\frac{8}{8}$ and $\frac{4}{4}$ series.

The second technique consists of *fragmentation of the subject*. Such fragmentation must be performed in accordance with the characteristics of the respective temporal structure. In some cases the thematic unit itself may be dissected further into the units of measures (T″). The most perfect results of fragmentation of the subject are obtained in all cases where temporal coefficients are the terms of an involution-group.

While performing the fragmentation of a subject, it is important to consider which particular fragment is most appropriate for a certain exposition. With regard to this, we offer the following method of selection: if a fragmentary exposition of the subject is at the beginning (or close to the beginning) of the entire thematic continuity, it is preferable to use its *first fragment;* if such an exposition of the subject is located in the center (or close to the center) of the entire thematic continuity, it is preferable to use the *middle fragment* of the subject; if a fragmentary exposition of the subject appears at the end (or near the end) of the entire thematic continuity, it is preferable to use the *last fragment*.

We shall demonstrate this technique in practical application. Let us apply it to the case of three subjects in direct recurrence where the temporal coefficients are $(2 + 1 + 1)^2$. Then:

$$(A_1 4T + B_1 2T + C_1 2T) + (A_2 2T + B_2 T + C_2 T) + (A_3 2T + B_3 T + C_3 T).$$

This calls for the following forms of fragmentation:

$$A2T = \frac{A4T}{2};\ BT = \frac{B2T}{2};\ CT = \frac{C2T}{2}.$$

*Performed by the N. Y. A. Symphony on WNYC 8-11-40 and later by the NBC Symphony under Frank Black, and by the NBC Symphony under Leopold Stokowski.

Then, considering the requirements stated above, we acquire the following scheme of fragmentation, related to the sequence of thematic groups:

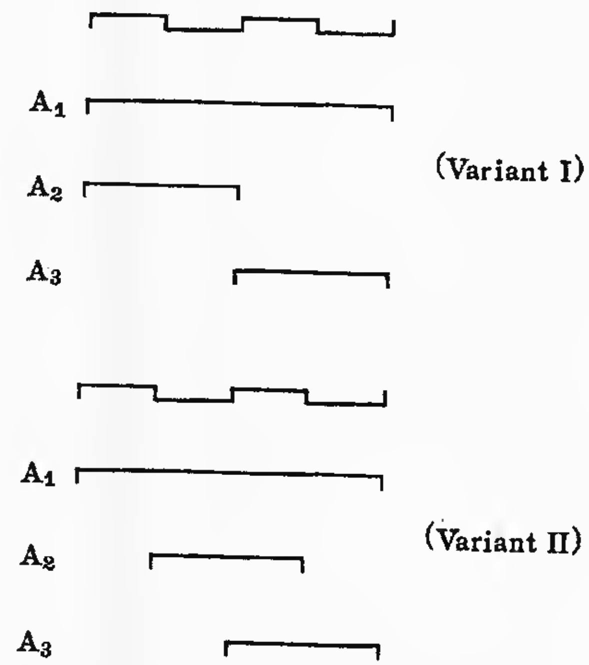


Figure 43. Fragmentation schemes.

This can be expressed as follows:

$$\text{Var. I: } A_1 4T + \ldots + A_2(T_1 + T_2) + \ldots + A_3(T_3 + T_4)$$
$$\text{Var. II: } A_1 4T + \ldots + A_2(T_2 + T_3) + \ldots + A_3(T_3 + T_4)$$

Variant I may be preferable because of its symmetry. Likewise:

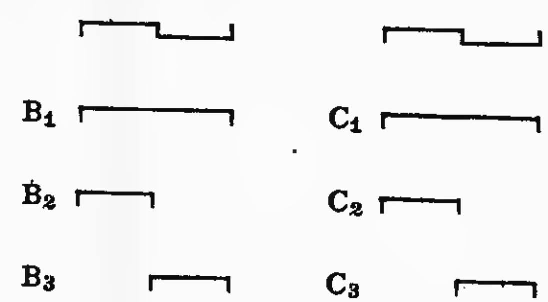


Figure 44. A symmetrical variant.

The entire thematic continuity (using Var. I for A) can be expressed as follows:

$$(A_1 4T + B_1 2T + C_1 2T) + (A_2 T_1 T_2 + B_2 T_1 + C_2 T_1) +$$
$$+ (A_3 T_3 T_4 + B_3 T_2 + C_3 T_2).$$

We shall now apply this technique to three subjects in circular permutations, and have $T = (1+2+1)^2$. Then: $(A_1 T + B_1 2T + C_1 T) + (B_2 2T + C_2 4T +$
$+ A_2 2T) + (C_3 T + A_3 2T + B_3 T)$.

This scheme requires the following forms of fragmentation:

$$AT = \frac{A2T}{2};\ BT = \frac{B2T}{2};\ CT = \frac{C4T}{4}$$

These forms of fragmentation may be graphically presented as follows:

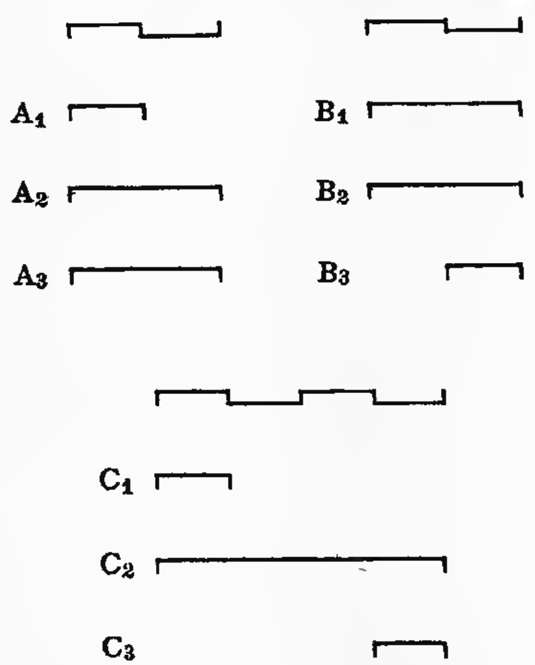


Figure 45. Forms of fragmentation.

The entire thematic continuity can be represented as follows:

$$(A_1 T_1 + B_1 2T + C_1 T_1) + (B_2 2T + C_2 4T + A_2 2T) + (C_3 T_4 + A_3 2T + B_3 T_2).$$

Forms of progressive symmetry yield perfect results from the subjects of equal period, like $T(A) = T(B) = T(C) = \ldots$; nevertheless fragmentation is applicable to such cases as well.

Let us take the form of progressive symmetry based on three subjects, and let us subject A, B and C to the same form of fragmentation: $2 + 1 + 1$. Then:

$$A_1 = 2T = T_1 + T_2;\ A_2 = T = T_3;\ A_3 = T = T_4;$$
$$B_1 = 2T = T_1 + T_2;\ B_2 = T = T_3;\ B_3 = T = T_4;$$
$$C_1 = 2T = T_1 + T_2;\ C_2 = T = T_3;\ C_3 = T = T_4.$$

The entire continuity assumes the following appearance:

$$A_1 T_1 T_2 + (A_2 T_3 + B_1 T_1 T_2) + (A_3 T_4 + B_2 T_3 + C_1 T_1 T_2) +$$
$$+ (B_3 T_4 + C_2 T_3) + C_3 T_4.$$

One of the most fruitful forms of fragmentation is the one which creates individual characteristics in the temporal behavior of the subject and, at the same time, offers temporal symmetry for the entire thematic continuity. For example:

$$A_1 = 2T;\ A_2 = T\ ;\ A_3 = T\ ;$$
$$B_1 = 4T;\ B_2 = 4T;\ B_3 = 4T;$$
$$C_1 = T\ ;\ C_2 = T\ ;\ C_3 = 2T.$$

This scheme of fragmentation can be arranged into a form of sequent temporal symmetry. For example:

$$A_1 T_1 T_2 + (A_2 T_3 + B_1 4T) + (A_3 T_4 + B_2 4T + C_1 T_1) + (B_3 4T + C_2 T_3) +$$
$$+ C_3 T_3 T_4.$$

In this case subject A, in its consecutive groups (expositions), undergoes an increasing fragmentation (decline); subject B remains constantly at its maximal duration (period) in all its consecutive groups (expositions); and subject C, in its behavior, reciprocates subject A, i.e., its consecutive groups (expositions) undergo a decreasing fragmentation.

Many other schemes of fragmentation can be evolved for the various forms of thematic sequence. The preceding illustrations are sufficient to start the future composer on his way to further exploration.

2. *Intonational Modification of a Subject.*

The process of fragmentation of a subject can be combined with intonational modification of it. Intonational modification takes place even in the absence of fragmentation. The experience of our musical past shows that when a subject has several expositions, in the course of the entire thematic continuity, it usually undergoes intonational modification. These modifications depend on the character of the musical culture, on its technical equipment in harmony, composition, etc. In plain chant, for example, intonational modification results in the modal variation; in the 18th Century—in "musical curls" of the Baroque, i.e., in the excessive use of melismas; in Wagner and his successors—in reharmonization; and even before Wagner—in modulatory key-changes. Thus, in the traditional sonata form of the early 19th Century, the second subject, usually appearing in the key of dominant in the exposition, reappears in the key of the tonic in the recapitulation.

With the technical equipment in the possession of a student of this theory, numerous intonational modifications can be applied to the successive expositions of a subject, thus making possible a sufficient variety even when the same subject reappears many times.

Among the techniques devised for the intonational modification of a subject, these are most essential:

(a) permutation of pitch-units within thematic units or within the entire subject; such permutations affect M, H and $\frac{M}{H}$, as well as CP;

(b) modal transposition and scale-modification in general; this is accomplished by direct change of accidentals;

(c) tonal expansion;

(d) quadrant rotation: geometric and tonal inversions;

(e) variations achieved by means of directional units and other resources of melodic figuration (chromatization of the original, which is not chromatic, is one of the most important techniques);

(f) variation of $\frac{M}{H}$ tension, which is equivalent to reharmonization.

It is advisable to use the above resources very sparingly in order not to overwhelm the listener, to whom an overabundance of technical devices may appear chaotic. The best path to follow within such limitations is to leave some of the subjects without any intonational modifications, and to subject others to individually specialized techniques. For instance, a certain theme A may be modified in the course of its various expositions with respect to quadrant rotation; another theme B may be left without any changes whatever; while a third theme C may be subjected, in its consecutive expositions, to modal variations, etc.

Besides the temporal and the intonational modifications of a subject, other forms of modification take place in the course of thematic continuity. These other modifications are based on the techniques of instrumental resources, and are inevitable in every composition. As this matter was sufficiently discussed in the *Composition of Thematic Units*,* and as we are now not discussing the technique of orchestration, initiative in using instrumental resources, as the technique for modifying a thematic group, must be left to the composer. He can make his decision on the matter of distribution of density, instrumental forms, dynamics, attack-forms, etc.

*See p. 1279 ff.

C. Axial Synthesis of Thematic Continuity

Axial synthesis corresponds to intonational coordination of thematic continuity on the basis of the selection of key-axes for all thematic groups as they appear in final continuity.

In classical music the key-axes followed what are known as tonic, dominant, subdominant, mediant, etc. Academic theorists prescribe such a key-selection. But the point is that a great many classical themes were based on the arpeggio forms of major and minor triads, i.e., $S_1(5)$ and $S_2(5)$ and their inversions, and for this reason such rules are of no consequence today when the forms of tonality are so diversified. Yet in the case of classical composers, rules or no rules, such a key-selection is *thematic;* and that is what really counts.

As an example we may refer to Beethoven's "Pathetique" piano sonata where the second subject of the first movement is based on $S_2\binom{8}{4}$; the first subject of the second movement has a harmonic arrangement in the three upper parts of an $S_1\binom{8}{4}$; and the first subject of the final movement is also based on an arpeggio of $S_2\binom{8}{4}$. This in itself would not be of any consequence. But it is to be noted, first of all, that it is very typical of Beethoven to build important melodic patterns on the instrumental forms of $S\binom{8}{4}$; and secondly, such a choice on his part is thematic and not based on any academic prejudice. Indeed, in the above mentioned Sonata in C-minor, the key-sequence of the first movement follows the pitch-units of $cS_2(5)$, i.e., c, $e\flat$ and g. The first subject is in C-minor (with c-axis as the pedal point in the bass); the second subject is in E♭-minor; the third subject is in E♭-major—and so is the following subject. At the end of what is usually called an exposition, there is a bridge in G-minor. The introduction is in C-minor.

This type of evidence leads us to the conclusion that to regard any system of key-axes as universal, is basically wrong. The only correct way to select a key-axis system is to derive such a system from the thematic material of intonation, which, being individually different in each particular case, results in an individual key-axis system for each individual composition. The intonational interdependence between some important thematic unit, or master-pattern of melody or harmony, and the sequence of key-axes is a necessary characteristic of intonational unity of style. An excellent example of such unity—overlooked by all critics and analysts—is the intonational interdependence between the key-sequence of the second section of the first subject and the master-pattern of the subject with which "Venusberg" music begins in Wagner's overture to *Tannhäuser*. The pattern is based on the symmetry of the $\sqrt[4]{2}$ (four tonics), or a diminished seventh-chord, if you wish.

To follow such a principle of thematic interdependence of intonations between the part and the whole, is to select a set of pitch-units from a characteristic thematic unit, and to assign such a set as a system of key-axes.

It is necessary to indicate at this point that the real key-axes do not always coincide with the officially established tonalities. If a subject or a thematic group appears in the d_0 of a natural C-major, and the next portion of continuity, or the next thematic group, represents the position ⓓ of that group, key-axes are on *c* in both cases, though the second thematic group acquires four additional accidentals (4♭). The reason for this is our definition of scale and axis-transposition, as offered in the *Theory of Pitch-Scales*.* We consider a scale with a *c*-axis, though read c — d♭ — e♭ — f — g — a♭ — b♭, which happens to be Phrygian (i.e., d_2), in the key of C.

So long as the composer adheres to the method of key-axis selection, through thematic interrelation of intonations, the concrete choice of an individual key-system is his.

*See Vol. I, p. 111 to p. 125.

CHAPTER 14

PLANNING A COMPOSITION

THE chief practical advantage of scientific planning over intuitive creation lies in the fact that, regardless of the value of intuition *per se*, scientific planning can be accomplished at any time and is independent of inspiration. For this reason, scientific method is more to be associated with professional performance, as such performance requires the achievement of high quality with regard to time consumed. Intuitive creation is beyond the artist's control. He cannot guarantee the amount of time which will be required in order to write a certain composition, nor can he guarantee the quality of the prospective work; moreover, even though the first two requirements may be satisfactorily fulfilled, the character of the work, when completed, may not possess the required characteristics.

The elements of an intuitively conceived composition, in actuality, are not elements, but *a priori* synthesized complexes. Their fitness is a matter of chance, and the remolding or fitting of such complexes, in order to meet specific requirements, usually calls for considerable effort.

Planning of a musical composition begins with "time." But "time" is one of the most elusive notions of humanity. Contemporary physics is lost in the maze of "times" it has created to solve its problems—the maze of definitions and classifications of time with respect to motion. But in addition to all these concepts of "subjective" and "objective" time (with which the student may acquaint himself through the lucid presentation by Sir Arthur Eddington in *The Nature of Physical World*) there is a concept of "psychological time" which we encounter in our daily existence.

Empirical time with which the composer has to deal, like everything else the composer has to deal with, has two sides. One aspect is physical, constitutes the excitor and is subject to measurement; the other, psychological, constitutes the reaction and is subject to experience. The quasi-objective, physical or clock-time, as encountered by the composer, is but an artificially isolated fragment of temporal manifold. This form of empirical time constitutes a concept by which events and their sequence are measured. It is conceived as one-dimensional and empirically irreversible.

When we think we reverse time, in actuality we only reverse the course of events. The direction of perceptible time remains irreversible. In the end, everything is subjective, as we cannot perceive time as such, but only as temporal configurations (events), taking their course in time. We perceive music as motion because there are in music continuous changes of temporal configurations. But we know from physics that motion perceived as "continuous" (in the mathematical sense) actually consists of an infinite number of phases, each of which, taken by itself, is stationary.

The physical constitution of music, as perceived temporally, is not the composer's concern, as he deals with the perceptive side of music, its psychological form, which is always a continuum. The only knowledge of practical importance to a composer, involves two concepts of time: the physical and the psychological. The physical time of musical composition is measured by the clock. The psychological time of musical composition is measured by the ***degree of saturation of physical time, by the temporal configurations of sound*** (*or sound in its relation to silence*).

We know from a study of psychology that the intensity of a reaction is in direct relation to the frequency of impulses stimulating such a reaction. Experience shows that it is equally true of reactions to impulses of a more complex form. If we look upon the subjects of a musical composition as complex impulses, the frequency of such "thematic impulses" has a similar effect upon the formation of psychological reactions. The effect of hearing a few thematic groups, each characterized by a relatively high temporal stability in a relatively long period of "physical" time, produces an effect of psychologically "empty" time, i.e., time during which few events take place, or "uneventful" time. The opposite may also be true. The effect of time being eventful is due to the presence of many thematic impulses in a relatively brief period of clock-time. In this case, time appears to be saturated with events.

It follows from this reasoning that the length of a relatively short musical composition (as measured by the clock) psychologically largely depends upon the ***degree of its thematic saturation***. This is the basis on which rests the quantitative characteristic of musical composition, i.e., the number of subjects and thematic groups necessary to produce certain effects contemplated by the composer.

Crowding of events into a relatively brief time-period was successfully accomplished in the polyphonic compositions of the 18th Century in the form of *stretto*, which is a form of thematic overlapping. In J. S. Bach's Fugue No. 5, *Well-Tempered Clavichord*, Vol. II, the entire composition consists of successive groups (expositions) with systematically progressive overlapping. The interval between the theme and the reply contracts itself in the following way: 12t + 8t + + 4t + 2t; this contraction is carried out in both major and minor, major preceding minor.

Temporal saturation achieved by means of overlapping of the thematic groups is quite an ancient device. It was successfully employed by the Roman, Lucius Apuleius in his novel "The Golden Ass."

Thirty years of my own life have been devoted to a study of temporal structures as they appear in various phenomena, including literature, plays, cinema, and music. I would like to refer to two examples of the temporal saturation of musical form, as they appear in two of my compositions for piano. One of them is "Heroic Poem" from "Five Pieces, Op. 12."* In this composition, which requires only three minutes of performance, events are so crowded that even experts greatly overestimate the actual clock-time of performance. In

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another composition, "Sonata-Rhapsody, Op. 17," events, besides being numerous, temporally overlap one another and merge one into another—one event taking another's place (like the "dissolve" in cinematic montage). This composition, in its temporal structure, more resembles a novel than a sonata. It attempts to project a whole epoch into 9 minutes of performance.*

I shall add to the observations above that it is a virtue to make a brief composition appear more eventful than its clock-time period would seem to permit, but that the opposite is the greatest sin a composer can commit.

The planning of a musical composition can be generally accomplished in 10 successive stages:

(1) Decision as to the clock-time duration of the entire composition.
(2) Decision as to the degree of temporal saturation.
(3) Decision as to the number of subjects and thematic groups.
(4) Decision as to the form of thematic sequence.
(5) Temporal definition and distribution of thematic groups.
(6) Organization of temporal continuity.
(7) Composition of thematic units.
(8) Composition of thematic groups.
(9) Intonational coordination (axial synthesis) of thematic continuity.
(10) Instrumental development.

A. Clock-time Duration of a Composition

Clock-time duration of a composition represents its dimensional aspect.

In architecture we define the space needed for a structure by the type of structure we plan to design. It may be an office building of many stories, it may be a cathedral, it may be a one-family house, or it may be a tent.

Likewise, in music, we define the necessary amount of clock-time, depending on the type of composition. An opera may occupy several hours of performance; a cantata or an oratorio may occupy a half or a whole of the concert program; a symphony usually lasts between 20 and 40 minutes; short instrumental or vocal compositions range from one to ten minutes; cues in radio-plays often are only a few seconds long. Thus, the first decision the composer has to make concerns the temporal dimension of a composition. If the form is "cyclic," i.e., consisting of several movements (like sonata, suite, symphony, oratorio, opera), the duration of the total composition must be determined first.

The next step consists of the definition of a common duration unit. It is my belief that in order to achieve perfect temporal coordination of the whole, it is necessary to work out the entire composition from homogeneous temporal units instead of the customary "tempo" modifications. It is over-optimistic for a composer to expect a performer or conductor to achieve the tempo he had in mind. Most performers are neglectful of the metronome indications provided

******Sonata-Rhapsody, Op.*** **17 for piano solo, has been widely acclaimed by critics (since its first performance in 1925) as 'a composition of overwhelming power, and has been performed on symphonic programs. (Ed.)**

by the composer. For this reason the most practical thing to do is to establish *one tempo* for the entire composition (even if it consists of several movements), and to produce the *apparent effects of mobility* by assigning different coefficients of duration to the common duration-unit (t). Thus, one subject, or movement will appear in a fast tempo because the coefficient of duration is one, i.e., $t' = t$; another subject will appear in an intermediate tempo due to the respective value of the coefficient of duration ($t' = 2t$; $t' = 3t$); . . . In the same way the effect of a very slow tempo can be achieved by using a still greater coefficient of duration, such as $t' = 4t$; $t' = 5t$; $t' = 6t$; . . .

I used this form of notation instead of the tempo changes in my Symphonic Rhapsody, *October* (1927), and found it very profitable. In this particular composition, the shortest $t' = t =$ 𝅘𝅥𝅰 and the longest $t' = 16t =$ 𝅗𝅥 . Of course, ultimately, t (the original duration-unit) has to be defined by the clock and, together with all forms of t' (derivative duration-units), translated into the appropriate symbols of musical notation. Thus, for example, if the total duration of a composition is 3 minutes, and $t = 1/4$ second, such a composition contains $180 \cdot 4 = 720$ t.

B. Temporal Saturation of a Composition

Temporal saturation is in direct relation to the *quantity of thematic groups*. This is true of both monothematic and polythematic continuity.

Thus, a monothematic composition consisting of one thematic group has a minimum of temporal saturation. Among the numerous compositions of this kind, J. S. Bach's *Aria on the G-string* for Violin can be mentioned as an outstanding example. Such forms belong to the category of "through-composed" music and have been extensively exploited by our outstanding contemporaries.

It must be obvious to students of this system that an extensive temporal form, which is monothematic and homogeneous, can be easily accomplished by means of various forms of interference. Its intonation can be evolved from any of the sources, such as MP designed from a scale or a set of scales, from a plotted melody (in this case being the entire composition), from melodization, rhythmicized harmony, counterpoint, etc.

Higher degrees of temporal saturation can be achieved either through the development of thematic groups, or through introducing more subjects.

Temporal saturation of a subject depends on the quantity of attacks. Subjects containing more attacks must be considered more saturated subjects. Thus, for example, if two adjacent movements of the same composition (as in a suite) have the same total period and are both monothematic, there is still a way to make one of them appear longer, i.e., by assigning to this particular movement a greater number of attacks.

In order to produce an effect of considerable saturation in a monothematic composition, it is necessary to evolve a number of thematic groups from the subject. This can be accomplished by various means, such as geometrical inversions, modal transpositions, tonal expansions, reharmonizations, instrumental variations, etc.

In polythematic compositions, a considerably higher degree of temporal saturation is due to the presence of a greater number of subjects. In this case, while each of the subjects may have a more limited number of thematic groups than is necessary in monothematic continuity, the appearance of a higher degree of temporal saturation may nevertheless be produced.

Further increase of temporal saturation in a polythematic composition can be accomplished either by increasing the number of subjects, or by increasing the number of expositions of each subject without increasing the number of the latter.

C. Selection of the Number of Subjects and Thematic Groups

After the composer has made his decision as to the form of temporal saturation of the prospective composition, his next step involves selection of the number of subjects and thematic groups. There are several situations which may be encountered in this selective process.

The first question is: shall all subjects have only one thematic group. The second question is: shall all subjects, or only some of them, have more than one thematic group. The next question is: how many thematic groups shall each subject have respectively. The last question implies the dominance of certain subjects over others, as the subject which has more thematic groups will *ipso facto* become a stronger thematic impulse.

Interrelations of the number of subjects and their respective thematic groups become a problem of temporal ratios. For example, the composer has decided to have two subjects: A and B. He wants A to dominate over B in $2 \div 1$ ratio. Then the number of thematic groups of A is 2, and the number of thematic groups of B is 1. Under the same ratio, however, the absolute quantity of thematic groups can be doubled, tripled, quadrupled, etc. Then the composer would have the following possible forms of selection:

2A, B;
4A, 2B;
6A, 3B;
8A, 4B;
. . . .

In each case there are several forms of distribution of the thematic sequence, but this we shall discuss later.

Now let us imagine that the composer has arrived at the decision to have four subjects: A, B, C and D. The next decision he has to make concerns the selection of a quadrinomial ratio. Suppose he chooses: $3 \div 1 \div 2 \div 2$ for A, B, C and D respectively. Then he may select any of the following schemes, representing the absolute quantities of thematic groups and equivalent to the above quadrinomial ratio:

3A, B, 2C, 2D;
6A, 2B, 4C, 4D;
9A, 3B, 6C, 6D;
12A, 4B, 8C, 8D;
. . . .

It is easy to see that either the number of subjects in a composition dominates the number of thematic groups, or the number of thematic groups dominates the number of subjects. In planning this particular aspect of a musical composition, we may arrive at various propositions which will prove valuable in different situations. For example, we may arrive at a condition (useful in a certain special case) in which the maximum number of thematic groups of one individual subject must not exceed the total number of subjects—so that in the event of three subjects, none of the subjects is allowed to have more than three expositions.

This is just an indication of the type of situation which the composer is compelled to work out for himself in each individual case. My system does not circumscribe the composer's freedom, but merely points out the methodological way to arrive at a decision. Any decision which results in a harmonic relation is fully acceptable. We are opposed only to vagueness and haphazard speculation.

Other illustrations of the conditions which control the relation of the number of subjects and their expositions.

(a) The quantity of exposition corresponds to the order of appearance of the subject:

Three subjects: A, B and C.

If the sequence is A + B + C for the subjects alone and does not involve the problem of temporal distribution of all thematic groups, then A, appearing first, is assigned to one exposition; B, appearing next, is assigned to two expositions; C, appearing last, is assigned to three expositions, i.e., A, 2B, 3C.

(b) The same proposition can be reversed: 3A, 2B, C.

(c) The quantity of exposition of each subject is one half of the total number of subjects:

Four subjects: A, B, C and D.
2A, 2B, 2C, 2D, as $\frac{4}{2} = 2$.

(d) The quantity of exposition for each respective subject is assigned on the basis of the arithmetical mean. Let us have 3 subjects, and let the number of expositions of the subject B be an arithmetical mean. Let A have four expositions, and let C have one half of this number; then B acquires 3 expositions, as: $\frac{4+2}{2} = 3$. Then the quantities of respective exposition are: 4A, 3B, 2C.

D. Selection of a Thematic Sequence

After the number of subjects and the quantity of their respective expositions have been defined, we obtain the total number of thematic groups. The next procedure deals with the form of thematic sequence into which all the thematic groups must be arranged.

Each individual case of the number of subjects and their respective expositions offers several possible forms of distribution. Let us start with a simple case first. Suppose the thematic selection is: 2A and B. In attempting to match the possible forms of distribution with the above quantities, we acquire the following solutions:

(a) $A_1 + B + A_2$;
(b) $A_1 + A_2 + B$;
(c) $B + A_1 + A_2$.

Obviously, case (a) is preferable because it offers a symmetric arrangement.

Under the same binomial ratio we may have: 4A and 2B. These can be distributed in the following manner:

(a) $(A_1 + B_1 + A_2) + (A_3 + B_2 + A_4)$;
(b) $(A_1 + A_2 + B_1) + (B_2 + A_3 + A_4)$;
(c) $(B_1 + A_1 + A_2) + (A_3 + A_4 + B_2)$;
(d) $A_1 + B_1 + B_2 + A_2 + A_3 + A_4$.

The first three cases are desirable since they are symmetric.

Let us discuss another case: 3A, B, 2C, 2D. This is a more elaborate quantitative group and requires a more elaborate distributive form. In order to evolve a symmetric form of distribution of the sequence, we must assign symmetric places to each letter individually:

```
A . . . A . . . A
        B
    C . . . . C
    D . . . . D
```

In this case perfect symmetry is impossible, as B has no recurrences to reciprocate. But forms of nearly perfect symmetry are possible:

(a) $A_1 + C_1 + D_1 + B + A_2 + D_2 + C_2 + A_3$
(b) $A_1 + D_1 + C_1 + B + A_2 + C_2 + D_2 + A_3$
(c) $A_1 + C_1 + D_1 + A_2 + B + D_2 + C_2 + A_3$
(d) $A_1 + D_1 + C_1 + A_2 + B + C_2 + D_2 + A_3$

As soon as these quantities are doubled, symmetry becomes possible: 6A, 2B, 4C, 4D.

(a) $(A_1 + C_1 + D_1 + A_2 + D_2 + C_2 + A_3) + B_1 + B_2 +$
$+ (A_4 + C_3 + D_3 + A_5 + D_4 + C_4 + A_6)$
(b) $(A_1 + C_1 + C_2 + A_2 + D_1 + D_2 + A_3) + B_1 + B_2 +$
$+ (A_4 + D_3 + D_4 + A_5 + C_3 + C_4 + A_6)$
(c) $(A_1 + B_1 + C_1 + A_2 + D_1 + D_2 + A_3) + C_2 + C_3 +$
$+ (A_4 + D_3 + D_4 + A_5 + C_4 + B_2 + A_6)$
(d) $(A_1 + C_1 + C_2 + A_2 + B_1 + D_1 + A_3) + D_2 + D_3 +$
$+ (A_4 + D_4 + B_2 + A_5 + C_3 + C_4 + A_6)$

Let us now distribute the case: 4A, 3B, 2C.

```
A . . . A . . . . . . . . . . . A . . . A
      B . . . . . . B . . . . . . . . B
                  C . . . C
```

From this we obtain the following forms:

(a) $(A_1 + B_1 + A_2) + (C_1 + B_2 + C_2) + (A_3 + B_3 + A_4)$
(b) $(A_1 + B_1 + C_1 + A_2) + B_2 + (A_3 + C_2 + B_3 + A_4)$

Inititative in constructing symmetric forms of distribution is the ability which composers must cultivate.

The composer can also make his choice directly from the forms of thematic sequence as they were presented in chapter 9 of Part Two of the Theory of Composition. Of course, such schemes have a pre-conceived quantity of exposition for each subject.

E. Temporal Distribution of Thematic Groups

The sum of durations of all thematic groups constitutes the duration of the entire composition. As each subject may have one or more expositions, the temporal coefficient of each subject, at first, must include all the expositions of such a subject. The temporal ratio consists of the number of terms corresponding to the number of subjects.

Thus, a thematic selection of A and B requires a binomial time ratio, regardless of the number of expositions of each subject. Temporal ratio in this case expresses the relation of the period of all expositions of A to the period of all expositions of B.

The simplest instance of temporal relations is that in which all the expositions of all subjects have the same period. In such a case, the dominance of some subjects over others is expressed solely through the number of expositions of such subjects in relation to other subjects.

In a scheme of 4A + 2B, with identical periods for all expositions of A and all expositions of B, the ratio of temporal dominance of the subject A over subject B is still 2. Assuming that the thematic sequence of this composition is symmetric, we obtain the following as one of the possible schemes of temporal distribution:

$$(A_1T + B_1T + A_2T) + (A_3T + B_2T + A_4T)$$

Such a scheme can be expressed as: $T^{\rightarrow} = 4A4T + 2B2T$. The realization of this scheme consists of division of the period of the entire composition by 6. Assuming that $T^{\rightarrow} = 3$ minutes, we obtain the following period for each exposition: $\frac{180}{6} = 30$ seconds.

As the total duration of this composition consists of 4A and 2B, we can define the total duration of A as 4/6 and the total duration of B as 2/6.

In other words, if the temporal ratio is 2÷1 for the two subjects, A takes 2/3 and B takes 1/3 of the entire composition. This reasoning is based on the fact that 2+1 = 3, and therefore the 2÷1 ratio belongs to $\frac{3}{3}$ series.

Now if we should decide that the total period of A equals the total period of B, such a decision would imply a different temporal distribution. In this case, then, $T^{\rightarrow}(4A) = T^{\rightarrow}(2B)$. Hence: $\frac{T^{\rightarrow}}{2} = \frac{180}{2} = 90$ seconds. Then the duration of each exposition of A is: $A = \frac{90}{4} = 22{\cdot}5$; the duration of each exposition of B is: $B = \frac{90}{2} = 45$. But this is true only if all the expositions of A have an identical period, and all the expositions of B have their own identical period. In some cases, the various expositions of one subject may have different temporal coefficients. Then the number of terms in such a ratio equals the number of expositions of its respective subject.

Let 3 + 1 + 2 + 2 be the temporal coefficient group for the four expositions of A, and 3 + 1, the coefficient group for the two expositions of B. As 3 + 1 + + 2 + 2 = 8 and the period of 4A = 90, we find the following periods for the individual expositions of A:

$$T(A_1) = \frac{90\cdot 3}{8} = \frac{45\cdot 3}{4} = \frac{135}{4} = 33\tfrac{3}{4};$$
$$T(A_2) = \frac{90}{8} = \frac{45}{4} = 11\tfrac{1}{4};$$
$$T(A_3) = \frac{90\cdot 2}{8} = \frac{45}{2} = 22\tfrac{1}{2};$$
$$T(A_4) = \frac{90\cdot 2}{8} = \frac{45}{2} = 22\tfrac{1}{2};$$

Likewise the periods of the individual expositions of B appear as follows:

$$T(B_1) = \frac{90\cdot 3}{4} = \frac{45\cdot 3}{2} = \frac{135}{2} = 67\tfrac{1}{2};$$
$$T(B_2) = \frac{90}{4} = \frac{45}{2} = 22\tfrac{1}{2}.$$

Now we can represent the entire temporal scheme of this composition in seconds:

$$T^{\rightarrow} = A_1 33.75 + B_1 67.5 + A_2 11.25 + A_3 22.5 + B_2 22.5 + A_4 22.5$$

As the period of 4A equals the period of 4B, we can represent this also in the ratio equivalents, by multiplying 3+1 by 2. $T^{\rightarrow} = A_1 3T + B_1 6T + A_2 T + + A_3 2T + B_2 2T + A_4 2T$.

In chapter 11 (Temporal Organization of Thematic Sequence), we discussed many possible approaches in translating thematic sequences into temporal ratios. In the present discussion, we are primarily concerned with subdividing the entire period of the composition into temporal sections corresponding to the individual expositions, subordinated to a certain form of temporal organization conceived *a priori*.

This makes it possible to proceed with the planning of a composition in a different sequence. For example, we can take some temporal group, assume its total duration to correspond to the duration of the whole composition and its single terms, to the successive expositions. After this we can proceed with the selection of the number of subjects and their expositions. The latter must be in some simple correspondence with the number of terms of the temporal group. Often such groups offer more than one practical solution. In such a case the decision of the composer must be based on the desired degree of temporal saturation.

Many of the resultants of interference, particularly the r′ which derives from ternary and quaternary synchronization, serve as practical temporal groups for such a procedure.

To illustrate this, let us take $r'_{3\div4\div7}$. This group consists of 12 terms, which permits numerous solutions for the different number of subjects and thematic groups. This resultant consists of the following terms: $r'_{3\div4\div7} = 12+9+3+4+8+6+6+8+4+3+9+12$.

Assuming that our composition consists of two subjects, and both subjects have the same number of thematic groups, we acquire 6 expositions for each subject since $\frac{12}{2} = 6$. Then two basic forms of continuity become possible:

(a) direct recurrence:

$$A_1 12T + B_1 9T + A_2 3T + B_2 4T + A_3 8T + B_3 6T + A_4 6T + B_4 8T + A_5 4T + B_5 3T + A_6 9T + B_6 12T;$$

(b) symmetric recurrence:

$$A_1 12T + B_1 9T + A_2 3T + B_2 4T + A_3 8T + B_3 6T + B_4 6T + A_4 8T + B_5 4T + A_5 3T + B_6 9T + A_6 12T.$$

The same temporal group can be applied to three subjects, in which case each subject acquires 4 expositions, as $\frac{12}{3} = 4$.

Two forms of thematic continuity:

(a) direct recurrence:

$$A_1 12T + B_1 9T + C_1 3T + A_2 4T + B_2 8T + C_2 6T + A_3 6T + B_3 8T + C_3 4T + A_4 3T + B_4 9T + C_4 12T;$$

(b) symmetric recurrence (taking the first two of the circular permutations and inverting them about the axis):

$$A_1 12T + B_1 9T + C_1 3T + B_2 4T + C_2 8T + A_2 6T + A_3 6T + C_3 8T + B_3 4T + C_4 3T + B_4 9T + A_4 12T.$$

As twelve is divisible by four, we can apply this temporal group to four subjects. This time, however, some subjects will dominate others.

We shall evolve our thematic sequences by arranging the letters in such a symmetry that four letters are supplemented by two of them, producing a group of six terms. By inverting this group about its temporal axis, we will obtain all 12 thematic groups for four subjects. In order to make A and B dominate over C and D, we shall repeat A and B after all four letters appear. This produces the following form of thematic sequence: (A+B+C+D+A+B) + (B+A+D+C+B+A), in which there are 4A, 4B, 2C and 2D.

This thematic continuity assumes the following form:

$$A_1 12T + B_1 9T + C_1 3T + D_1 4T + A_2 8T + B_2 6T + B_3 6T + A_3 8T + D_2 4T + C_2 3T + B_4 9T + A_4 12T.$$

In this case, the temporal dominance of A and B over C and D is due not only to the number of expositions of A and B but also to the total periods of these subjects:

$$\overrightarrow{T}(A) = 12T + 8T + 8T + 12T = 40T$$
$$\overrightarrow{T}(B) = 9T + 6T + 6T + 9T = 30T$$
$$\overrightarrow{T}(C) = 3T + 3T = 6T$$
$$\overrightarrow{T}(D) = 4T + 4T = 8T$$

An analogous treatment can be applied to the form of thematic sequence and continuity in which C and D become the dominant subjects. This requires the following arrangement of the thematic sequence: (A+B+C+D+C+D) + (D+C+D+C+B+A). In this case, thematic continuity assumes the following form:

$$A_1 12T + B_1 9T + C_1 3T + D_1 4T + C_2 8T + D_2 6T + D_3 6T + C_3 8T + D_4 4T + C_4 3T + B_2 9T + A_2 12T.$$

The temporal relations of the subjects appear as follows:

$$\overrightarrow{T}(A) = 12T + 12T = 24T$$
$$\overrightarrow{T}(B) = 9T + 9T = 18T$$
$$\overrightarrow{T}(C) = 3T + 8T + 8T + 3T = 22T$$
$$\overrightarrow{T}(D) = 4T + 6T + 6T + 4T = 20T$$

In this case the temporal dominance is more or less neutralized.

It is easy to see how such temporal groups, in their application of successive expositions, can be made useful in monothematic continuity—in which case they would influence the temporal relations of the different expositions of the same subject.

In all the above illustrations, T may represent any desirable duration-group.

One point concerning the general distribution of temporal groups remains to be discussed: *the distribution of climaxes.*

A musical composition may not contain any climaxes at all, or it may have one or more climaxes. Once it has one or more climaxes, proper distribution of the latter in thematic continuity becomes of utmost importance.

The problem of the distribution of climaxes is not limited to the climaxes appearing at the very beginning or the very end of a composition. It is the intermediate climaxes, appearing in the course of continuity, that require such distribution. The number of such climaxes (i.e., appearing in the course of continuity) is one less than the number of terms in the temporal ratio required for the respective distribution. Thus, a continuity containing one climax requires a binomial time ratio. The ratio itself must belong to the family to which the temporal structure of the entire composition belongs. Thus, in the $\frac{4}{4}$ series type of temporal structure, the position of the climax is determined by the ratio $3 \div 1$, i.e., the climax appears at the beginning of the last quarter of the entire composition. Likewise in a structure based on $\frac{8}{8}$ series, the ratio $5 \div 3$ determines the position of the climax, i.e., the climax appears at the beginning of the sixth eighth. It is always advisable to use the original form of the binomial—in which the first term has greater value than the second.

For the same reason two climaxes require a trinomial ratio, which ratio must be used in the form in which the values progressively decline.

In a structure based on $\frac{4}{4}$ series, the respective trinomial must be: $2 \div 1 \div 1$. Then the first climax appears at the beginning of the third quarter, and the second climax, at the beginning of the fourth quarter. Likewise, in a structure associated with $\frac{8}{8}$ series, the trinomial should be: $3+3+2$. The respective positions of climaxes in this case are: the first climax begins with the fourth eighth, the second climax, with the seventh eighth.

Indicating climax by the symbol Cl, we can express the two preceding cases as follows:

$$\text{(a)}\quad \frac{2T}{4} + Cl_1 + \frac{T}{4} + Cl_2 + \frac{T}{4};$$

$$\text{(b)}\quad \frac{3T}{8} = Cl_1 + \frac{3T}{8} + Cl_2 + \frac{2T}{8}.$$

The placing of the climax between the time-values means that the actual time for the climax (and its extension) is borrowed either from the preceding or the following term; and in some cases, the climax may itself extend over both (i.e., the preceding and the following) adjacent terms.

By taking temporal ratios with more terms, we can distribute more climaxes respectively. Thus, a temporal quintinomial becomes the tool for distributing four climaxes. For example, in the structure based on $\frac{8}{8}$ series, $2+2+2+1+1$ (which is one of the general permutations of the original quintinomial $2+1+2+$ $+1+2$ represented in declining values) offers the proper form of distribution of the four climaxes:

$$\frac{2T}{8} + Cl_1 + \frac{2T}{8} + Cl_2 + \frac{2T}{8} + Cl_3 + \frac{T}{8} + Cl_4 + \frac{T}{8}$$

Another basic form of the distribution of climaxes is based not on the individual ratios, but on proportions, i.e., *on the equalities of ratios.* This form of distribution of climaxes contributes the utmost temporal harmony to the entire composition.

Proportions are acquired in the forms of distributive involution-groups, i.e., squared and cubed binomials, trinomials, etc. In this case, the number of terms in the involution-group determines the number of climaxes.

In a temporal structure based on $\frac{4}{4}$ series, climaxes can be distributed as $(3+1)^2$, i.e., at $9T + Cl_1 + 3T + Cl_2 + 3T + Cl_3 + T$; in which case the common denominator of these values is 16.

In a temporal structure based on $\frac{8}{8}$ series, climaxes can be distributed as $(5+3)^2$, i.e., as $25T + Cl_1 + 15T + Cl_2 + 15T + Cl_3 + 9T$. Here the common denominator equals 64.

If the entire continuity is evolved from one or another type of acceleration-series, its climaxes can be distributed according to such a series. For example, a continuity consisting of 32T and evolved from the first summation-series can have its climaxes distributed as follows: $13 + 8 + 5 + 3 + 2 + 1$, i.e., $13T +$ $+ Cl_1 + 8T + Cl_2 + 5T + Cl_3 + 3T + Cl_4 + 2T + Cl_5 + T$.

This discussion leads us to the conclusion that the composer has to make his decision with regard to desirability of climaxes, their number and distribution before he completes the final planning of the temporal organization of continuity.

The means by which the composition of climaxes can be accomplished does not belong to this section and will be discussed later.

F. Realization of Continuity in Terms of t and t′

Subjects and their expositions vary not only in their temporal dimensions, but also in the dimensions of their duration-units. The dimension of duration-units may be in either *direct* or *inverse* relation to the dimension of the respective subject or its thematic groups. Nevertheless, once the dimension of a duration-unit for a certain subject is decided upon, it remains constant through all its expositions. As previously remarked, the duration-units from which the different subjects are constructed must be either identical or in simple relations with each other.

The original duration-unit (being at the same time the common denominator of the entire continuity) is designated as t, and all other duration-units of the same composition, as t′.

If the entire composition is associated with $\frac{2}{2}$, $\frac{4}{4}$, $\frac{8}{8}$, $\frac{16}{16}$, or other series of this class, the coefficients of duration for the various forms of t′ usually acquire such factors as 2, 4, 8, . . . If the composition is associated with $\frac{3}{3}$, $\frac{9}{9}$, $\frac{27}{27}$, or other series of this class, the factors of t′ usually are 3, 9, 27, . . . It is in this sense that one subject may be constructed from t as a duration-unit, another from $t' = 2t$ as a duration-unit, and still another from $t' = 4t$ as a duration-unit.

Once the respective t is translated into time equivalent, like 1/4 sec., all other forms of t′ can be relatively defined, and ultimately, all forms of t and t′ can be represented in musical notation.

Thus, for instance:

$t'(A) = t = 1/4$ sec. = 𝅘𝅥𝅯 ;
$t'(B) = 2t = 1/2$ sec. = ♪ ;
$t'(C) = 4t = 1$ sec. = ♩ .

Before composing any rhythmic patterns of duration-groups for the respective subjects, we have to know the total number of duration-units in each particular subject taken at its maximal period.

Let us take a new scheme of three subjects; e.g.,

$\overrightarrow{T}(A) = 16T;\ t'(A) = t = 1/4$ sec.
$\overrightarrow{T}(B) = 16T;\ t'(B) = 4t = 1$ sec.
$\overrightarrow{T}(C) = 16T;\ t'(C) = 2t = 1/2$ sec.

Then:

$$T(A) = 16t;$$
$$T(B) = 4t';$$
$$T(C) = 8t';$$

Hence:

$$T^{\rightarrow}(A) = 16t \,.\, 16 = 256t\,;$$
$$T^{\rightarrow}(B) = 4t' . 16 = 64t';$$
$$T^{\rightarrow}(C) = 8t' . 16 = 128t'.$$

In clock-time, all three subjects have the same period of 64 seconds; but psychologically the degree of temporal saturation varies with each subject, B representing the geometric mean of the remaining two subjects. Thus, psychologically, the most eventful subject (if we use the same rhythmic pattern of durations for all three subjects) is A and the least eventful is B.

To illustrate this in the simplest imaginable way, we shall assign $T = r_{\underline{4\div3}}$ to be the thematic duration-group for all three subjects. Then:

$$T'(A) = 16T$$
$$T'(B) = 4T$$
$$T'(C) = 8T$$

This means that A has a recurrence of the thematic rhythmic pattern 16 times. Such a recurrence can be of exact or modified form. The 16 modifications can be evolved on the basis of four circular permutations of the second order:

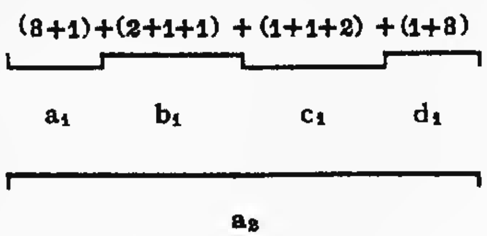


Figure 44. T′(A) = 16T.

Subject B has only 4 recurrences of the thematic rhythmic pattern. The latter may have either 4 direct recurrences or four circular permutations of the first order.

Subject C has 8 recurrences of the thematic rhythmic pattern. The latter may have either 8 direct recurrences, or four circular permutations of the first order with symmetric duplication:

abcd, bcda, cdab, dabc,
dabc, cdab, bcda, abcd Or:
abcd, bcda, cdab, dabc,
cbad, badc, adcb, dcba.

If we count the number of impulses (which correspond to the individual terms or attacks of the thematic rhythmic pattern) in each subject individually, we acquire the following comparative table of temporal saturation for A, B and C.

As $T = 10a$, the respective quantities of attacks (impulses) appear as follows:

$$T'(A) = 10a \cdot 16 = 160a;$$
$$T'(B) = 10a \cdot 4 = 40a;$$
$$T'(C) = 10a \cdot 8 = 80a.$$

Similar reasoning can be applied to subjects constructed from different thematic rhythmic patterns as well.

G. Composition of Thematic Units

We have had a thorough discussion of this matter in chapter one of this book.* Here we simply approach the subject from a different angle. In our first discussion of the composition of thematic units, we put stress on the flexibility and the adaptability of thematic units to temporal expansion and contraction. When we approach the temporal structure of thematic groups from the viewpoint of the entire continuity of the composition, we have to evolve rhythmic patterns of duration-groups in such a manner that they will satisfy the total duration of a respective thematic group as it is expressed in terms of t′. This means that the form of a duration-group, in its total period, must equal the total time-period assigned for the respective thematic group. The same concerns the possible number of attacks which may result from the application of a thematic duration-group expressed in definite t′-units.

For example, if a thematic group consists of 20t′, only certain forms of duration-groups are satisfactory. The easiest way to find such duration-groups is by finding the possible multiples producing 20 as a product. Such multiples are:

(a) $5 \cdot 4 = 20$; hence: $r_{5\div4}$;
(b) $2(5 \cdot 2) = 20$; hence: $2r_{5\div2}$;
(c) any 5T of the $\frac{4}{4}$ series;
(d) any 4T of the $\frac{5}{8}$ series.

The composer may also use his initiative in modifying various duration-groups in such a way that the sum of duration-units satisfies the case.

For example:

$$r_{5\div3} = 3+2+1+3+1+2+3 = 15;$$

The modified version achieved by the addition of 5, and distributed symmetrically $(2 + 1 + 2)$:

$$T = 3 + 2 + 3 + 4 + 3 + 2 + 3;$$
or: $r_{7\div2} = 2 + 2 + 2 + 1 + 1 + 2 + 2 + 2 = 14;$

*See p. 1279 ff.

the modified version achieved by the addition of 6, and distributed symmetrically $(2 + 1 + 1 + 2)$:

$$T = 4 + 2 + 2 + 2 + 2 + 2 + 2 + 4.$$

Examples of section F may serve as additional illustrations.

In a composition which contains climaxes and in whose scheme of temporal organization such climaxes are distributed *a priori*, it becomes necessary to pre-compose such climaxes for the respective subjects.

We have seen in the *Theory of Melody** that the melodic climax is a pitch-time maximum and is preceded by a resistance. We shall discuss here other important resources which produce climaxes.

As the main forms of resistance consist of rotary or centrifugal patterns, such patterns may be conceived as the bass of harmony (expressing some constant harmonic function and, thus, defining the tonal cycles, for instance); they either remain as bass or may be transferred into soprano (after the respective harmonization is completed). Another form of resistance produced by harmony consists of a group of tensions and releases, with an ultimate tension for the climax. Since centrifugal forms represent some of the most powerful forms of resistance, harmonic climaxes can be achieved by using such progressions as produce the respective configurations—for example, all cases of upper harmony ascending against a descending bass, or a pair of diverging harmonic strata. This, by the way, is one of the favorite devices of Beethoven, an example of which can be found in the third theme (E♭-major) of the first movement of the piano-sonata, *Pathetique*. In such forms, harmonic climax is represented by the maximum interval between the strata, usually coupled with a high dynamic degree (f or ff).

In addition to growth of tension of the harmonic structure and the diverging patterns of strata in motion, density plays an important role as a climax-builder. As the form of resistance, density either grows consistently or with delays, but reaches its climax at its maximum (i.e., the highest degree of density corresponds to a climax). As tension can be expressed not only through the growing complexity of harmonic structures, which appear in sequence, but also through harmonic intensification of melodic climaxes (i.e., by making such climaxes become higher harmonic functions), the latter also become the climactic resources of the entire thematic texture.

Dynamics as such is a powerful tool for building resistances and climaxes. The first is accomplished by the progressive or delayed growth of dynamic degrees (such as crescendo or $pp < mf + p < f + mf < ff$); and the second, by sustaining the highest dynamic degree reached (ff in this case) by the resistance.

The ultimate climactic effect can be achieved through a combination of the above described devices, such as high tension of harmonic structure accompanying melodic climax (which in itself represents a high degree of harmonic tension) coupled with high dynamics and high density.

*See Vol. I, p. 279.

Counterpoint can be successfully used for developing resistances in the form of a group of diverging melodic trajectories.

The oblique patterns in both harmony and counterpoint are useful in building the intermediate, secondary climaxes.

The period of a climax must represent a definite portion of the respective thematic group, and may even occupy the entire group.

To sustain a climax means to sustain the climactic conditions. Nevertheless, it is psychologically unavoidable that the intensity of climax goes into decline, as the receiving apparatus accommodates itself to the respective degree of the impulse relatively quickly. For this reason prolonged climaxes, in actuality, cannot be continuously climactic.

Since time periods, preceding climaxes, usually contract with each successive climax, the climactic periods themselves, though gaining in power, must necessarily contract.

H. Composition of Thematic Groups

In a thematic continuity which is planned from the duration of an entire composition, the chief problem of the composition of thematic groups lies in the distribution of intonational modifications. These, as stated before, are approached from the point of view that each subject, in its successive expositions, is varied through one specified technique. The main point to be discussed here is that the planning of the number and forms of intonational modifications depends upon the pre-set form of thematic continuity.

If each subject appears in the entire composition only once, no modifications have to be planned at all. If a certain subject has three expositions, another subject has two, and still another subject has one, the planning of intonational modifications concerns only the first two subjects—and even then there are only two modifications to be planned for the first subject and only one for the second.

Thus, the composition of thematic groups for a pre-planned form of thematic continuity can be carried out with a minimal expenditure of the composer's time and energy.

When intonational modifications require an increase of attacks in certain expositions of a subject, the individual duration-values of the original become a split-unit group. This permits retention of the rhythmic characteristic of the respective subjects.

All the necessary techniques by which intonational modifications can be performed have been fully described in the preceding chapters.

The sequence of modifications of each subject as it appears in its successive expositions must grow from simple to complex.

I. Composition of Key-Axes

This subject, having been previously discussed, concerns us for the present only insofar as the number of key-axes has to be defined and distributed.

The number of thematic groups does not have to equal the number of key-axes. After the number of thematic groups is established and the form of continuity specified, the composer has to make his decision about the number of key-axes. Such a decision should be based on some one of the three fundamental approaches: either (a) vary the key-axis with each Thematic Group; or (b) vary the key-axis with each recurrence of the same Thematic Group; or (c) change the key-axes at points determined by some structural subdivision of the entire continuity; for example, according to the symmetric groupings of the various recurrences (i.e., expositions) of the various Thematic Groups.

Although its exploitation so far by composers has been very limited, the technique of symmetric recurrence of key-axes produces effective results. Examples of such symmetry would be: (a) Key I + Key II + Key I; or (b) Key I + Key II + Key II + Key I; or (c) Key I + Key II + Key III + Key II + + Key I. All that is required is that the recurrences of the same key be symmetrically arranged.

In composition of systems of key-axes all the methods previously discussed for use in handling thematic sequences are applicable: direct recurrence, symmetric recurrence, modified recurrence, and progressive symmetry. Note that such thematic sequences applied to key-axes need not be the same schemes (although they *may* be) as those controlling recurrence of other Thematic Groups; it is only the *method* of composition of sequences that need be the same.

Here are some examples of the ways in which conditions of the three basic approaches mentioned above can be met by synchronizing a sequence of Thematic Groups with a sequence of Key-Axes:

(a) The key-axis changes with the entrance of each Thematic Group

(Supposing that the sequence of Thematic Groups is one of direct recurrence of A + B:)

A in Key I + B in Key II + A Key III + B Key IV + . . . (etc.)

(Supposing that the sequence of themes were a modified recurrence scheme, or (A + B + C) ↻ (permuting clockwise), the requirement could be met by either of the following variations: Var. I below, changing to a new key each time; Var. II, exhibiting five different keys recurring symmetrically; that is to say, the sequence of keys is symmetric, while the sequence of themes is one of modified recurrence, and the two are superimposed:)

Var. I: (A Key I + B Key II + C Key III) + (B Key IV + C Key V + A Key VI) + (C Key VII + A Key VIII + B Key IX);

Var. II: (A Key I + B Key II + C Key III) + (B Key IV + C Key V + A Key IV) + (C Key III + A Key II + B Key I).

(b) The key-axis does not change until a Thematic Group reappears

For a direct recurrence sequence of themes, A + B + C:

Var. I: (A + B + C) Key I + (A + B + C) Key II + (A + B + C) Key III + . . .;

Var. II: (A + B + C) Key I + (A + B + C) Key II (A + B + C) Key I;

For a progressive symmetric scheme of thematic sequence: A + (A + B) + (A + B + C) + (B + C) + C:

Var. I: A Key I + (A + B) Key II + (A + B + C) Key III + (B + C) Key IV + C Key V;

Var. II: A Key I + (A + B) Key II + (A + B + C) Key III + (B + C) Key II + C Key I.

(c) The key-axis changes at points determined by some structural subdivision of the whole continuity

For a modified recurrence group, (A + B + C) ↻, the key-changes might be pre-set for every Third Thematic Group, for instance, and the pattern of key-change could either be a single series (Var. I below) or exhibit symmetry (Var. II below):

Var. I: (A + B + C) Key I + (B + C + A) Key II + (C + A + B) Key III;

Var. II: (A + B + C) Key I + (B + C + A) Key II + (C + A + B) Key I.

J. Instrumental Composition

Instrumental composition of the entire musical piece in detail depends a great deal on its purpose and instrumental combination. For example, the idea of a composition being an "etude" or a "concerto" implies that a high degree of virtuosity is required of the performer (whether individual or collective). A composition written for a beginner must be instrumentally simple. The idea of an ensemble or orchestra implies richness and diversity in the utilization of instrumental resources.

The basic approaches to instrumental composition of the entire continuity are as follows:

(a) the degree of complexity of the instrumental form of a subject is in *direct* relation to the complexity of its other components;

(b) the degree of complexity of the instrumental form of a subject is in *inverse* relation to the complexity of its other components;

(c) the degree of complexity of the instrumental form of a subject is in *oblique* relation to the complexity of its other components.

The meaning of the term "oblique," as it is used in paragraph (c) of this discussion, represents a variable instrumental form (variable with respect to its complexity) applied to a subject whose other components are of constant complexity.

The degree of instrumental complexity can be determined by the number of attacks, by the variety of their forms and by the general diversity of instrumental resources. The degree of complexity of other components is determined by the complexity of temporal and intonational forms in a broad sense.

Empirically, it is never difficult to determine whether the subject is simple or complex, and to what degree, when we make such an evaluation on the basis of comparison with other subjects participating in the same composition.

It is highly desirable to specify the characteristics of instrumental forms with respect to each subject and its successive expositions individually. Then we may arrive at highly diversified schemes of instrumental composition where one subject, being simple, acquires a progressively increasing instrumental complexity; another, being of intermediate complexity, acquires instrumental forms of corresponding complexity; still another subject, being complex, acquires a progressively decreasing instrumental complexity, etc.

Much of the success in composing depends upon the extension of the general method used in all branches of this theory, i.e., the method of *regularity and coordination*, and that is what the Theory of Rhythm basically represents.

The next two chapters are devoted to practical applications of this theory to monothematic and polythematic composition.

CHAPTER 15

MONOTHEMATIC COMPOSITION

A MONOTHEMATIC composition, having one subject and one or more expositions, can be evolved from any technical source, which in this case contributes its major component. The major component must be looked upon as the dominant characteristic of the subject. The selection of minor components, their style and form of coordination with the major component, is subject to the composer's choice.

A monothematic composition, with one exposition constituting the entire piece, hardly requires the use of any elaborate form of variations. A monothematic composition with more than one exposition obviously depends on the variations. Variations, as such, come from different sources, and may influence temporal, intonational or textural patterns. The selection of quantities and types of variations, as well as the distribution of the latter, are left to the composer's discretion, as by now the potential composer is sufficiently equipped to use his own initiative in selection.

I shall illustrate the final synthesis of monothematic composition in such a way that the student will be supplied with samples based on different technical sources. For such illustrations, I shall use my own compositions which have been produced through the use of this system. I shall supply the student with technical data only to the extent to which it is necessary in each individual case, as these compositions should serve also as material for analysis.

My own compositions will be supplemented by reference to the works of my students, whose compositions were also produced through the use of this system.

A. "Song"* from "The First Airphonic Suite" (1929)

(Composed for the space-controlled theremin with sound amplification and a large symphony orchestra. It had its premiere in November 1929 in the Masonic Hall in Cleveland and was later performed in Carnegie Hall in New York. Both performances were given by the Cleveland Symphony under Nicolai Sokoloff, with Leon Theremin as soloist.)

The "Song" is a monothematic composition, with one exposition and a partial recurrence of the beginning of the subject. The subject is "through-composed" music, for it is based on one continuous melody, originally plotted, and then harmonized. Here melody is the major component.

*In the original sketch for piano and the version for Thereminvox and Piano, this is called "Melody" instead of "Song". (Ed.)

MELODY
for Thereminvox and Piano

Joseph Schillinger

Figure 45. Monothematic composition with melody as major component.

New York, Aug. 28, 1939

Figure 45. Monothematic composition with melody as major component (concluded).

B. "Mouvement Électrique et Pathétique" (1932)
(Composed for the space-controlled theremin and piano).

This piece is a monothematic composition, whose subject derives from a plotted melody. Later the melody was subjected to harmonization. The features of melodic structure are: pedal points, successive climaxes, temporal expansions and contractions of the thematic rhythmic patterns and a few geometrical inversions.

Pedal points have special significance in this case, as the theremin provides a tone of infinite duration without renewal of attack.

MOUVEMENT ÉLECTRIQUE ET PATHÉTIQUE

for Thereminvox and Piano

Joseph Schillinger

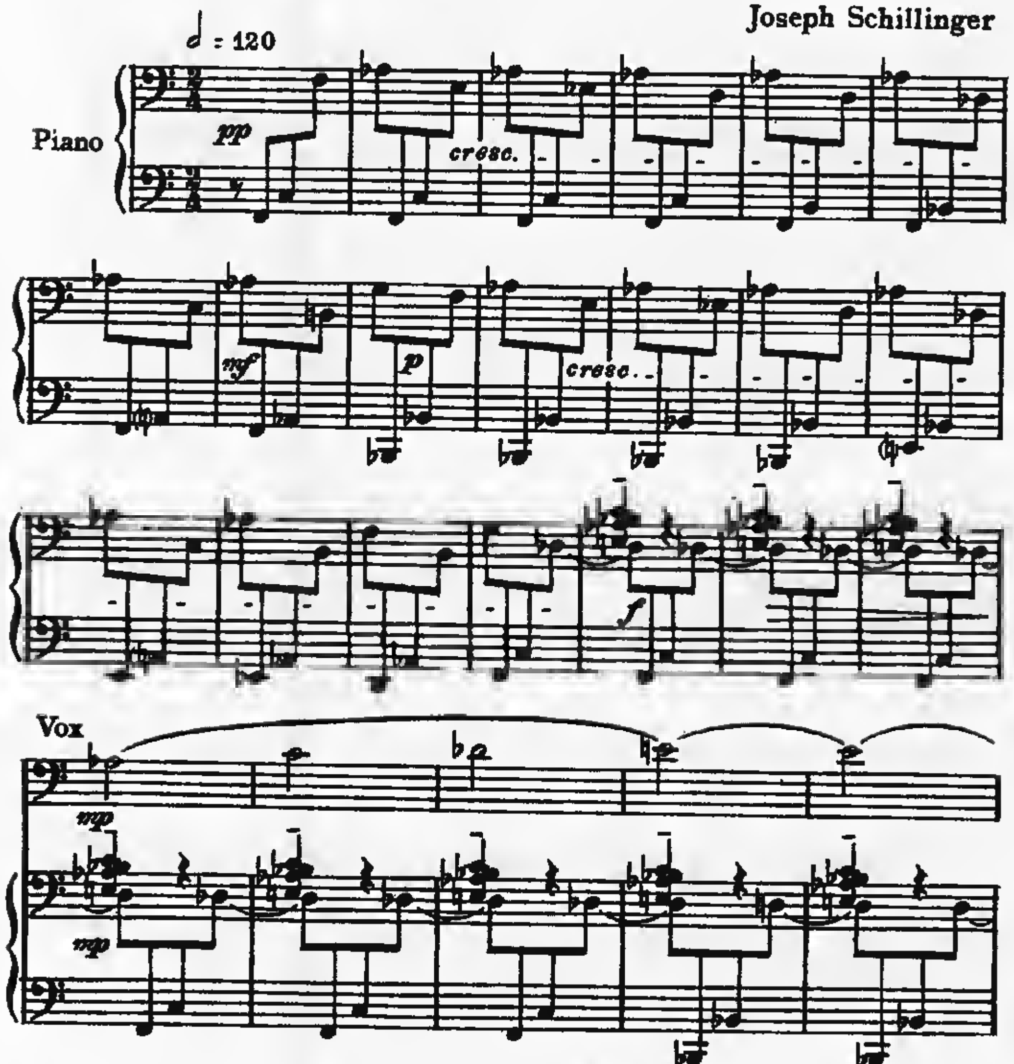


Figure 46. A monothematic composition whose subject derives from a plotted melody.

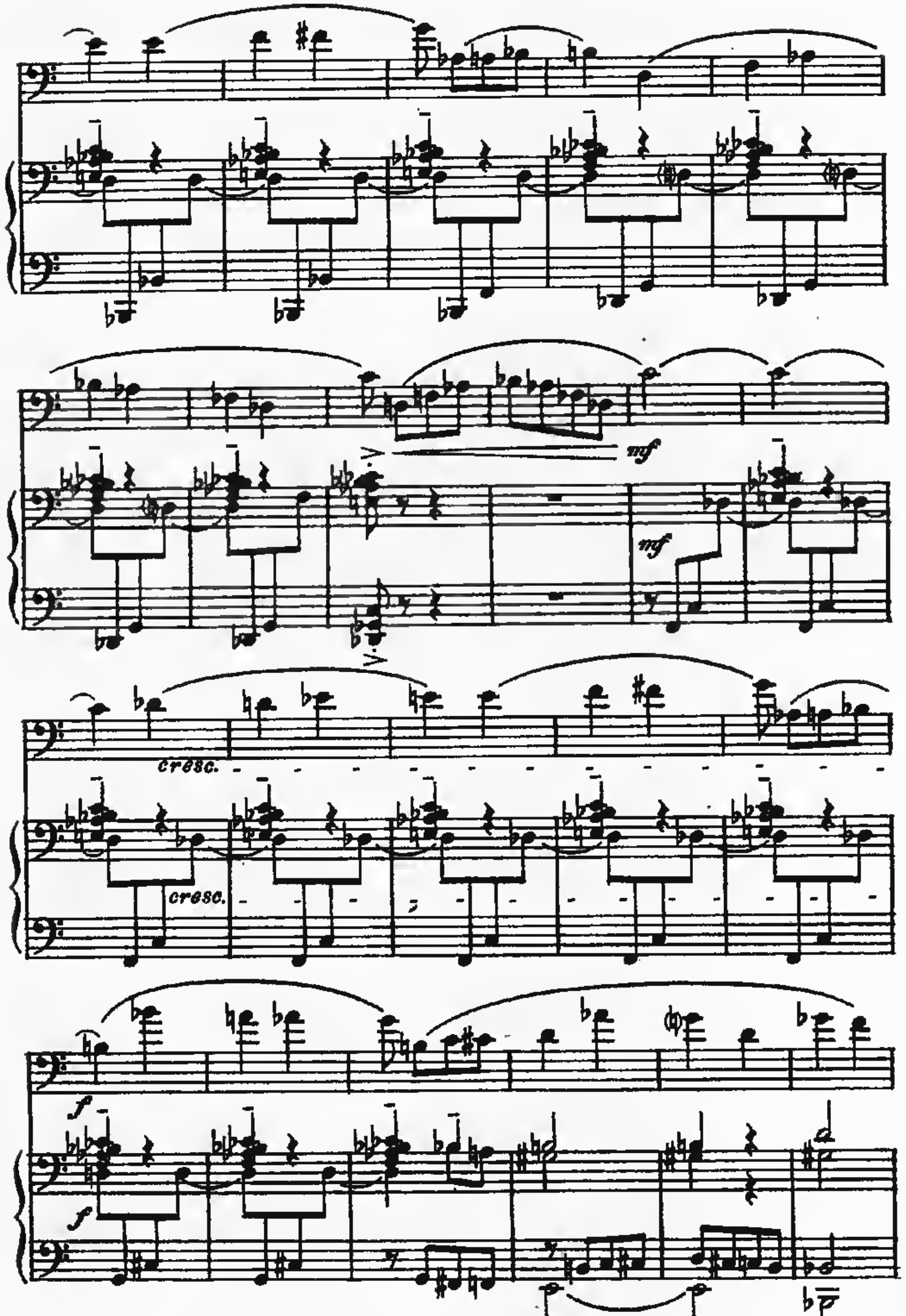


Figure 46. A monothematic composition whose subject derives from a plotted melody (continued).

Figure 46. A monothematic composition whose subject derives from a plotted melody (continued).

Figure 46. A monothematic composition whose subject derives from a plotted melody (continued).

Figure 46. A monothematic composition whose subject derives from a plotted melody (continued).

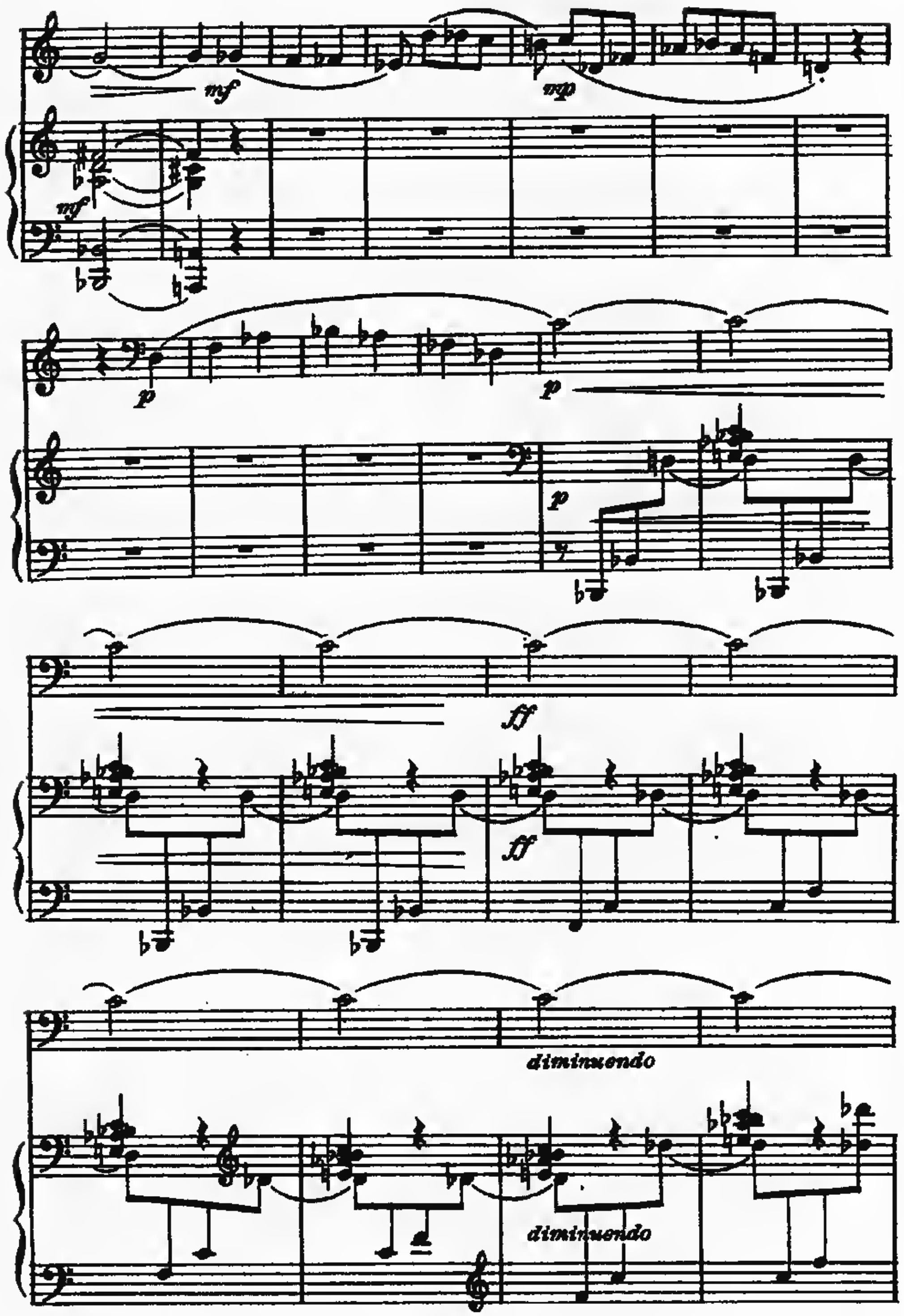


Figure 46. A monothematic composition whose subject derives from a plotted melody (continued).

March 15, 1932, New York

Figure 46 A monothematic composition whose subject derives from a plotted melody (concluded).

C. "Funeral March" for Piano (1928)
(American premiere by the League of Composers in 1930).

In this monothematic composition, the major component of the subject is harmony. There is no independent melody. What appears to be the melody is a combination of instrumental and melodic figuration. There is a partial recapitulation of the beginning, only in a climactic form. The harmonic structure itself is a symmetric superimposition of the $\sqrt[4]{2}$: S_I is B♭ and S_{II} is C♯. The building up of the strata occurs gradually thus giving the listener an opportunity to adapt himself to the Σ. For this reason, the beginning, based on S_I, seems to be in B♭ and the very end, based on S_{II}, seems to be in C♯ Minor.

MARCHE FUNEBRE

Joseph Schillinger

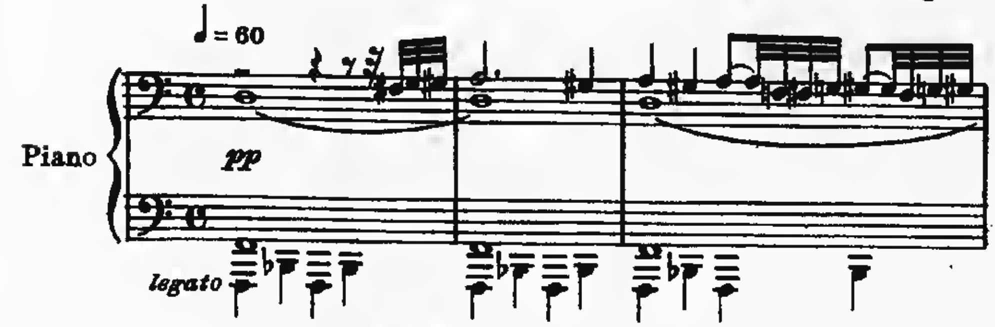


Figure 47. A monothematic composition with harmony as major component (continued).

Figure 47. *A monothematic composition with harmony as major component (continued).*

Figure 47. *A monothematic composition with harmony as major component (continued).*

Figure 47 A monothematic composition with harmony as major component (continued).

Figure 47. A monothematic composition with harmony as major component (concluded).

D. "Study in Rhythm I" for Piano (1935)

This monothematic composition is based on a subject consisting of 12 measures in 7/8 time, and has four expositions evolved in quadrant rotation: ⓐ + ⓓ + ⓒ + ⓑ. All expositions have the same period.

In spite of the title, the subject's major component is strata-harmony: $\Sigma = 2S$. The structure of the lower stratum is: $S_I = 4+3$ (used in clockwise positions); the structure of the upper stratum is: $S_{II} = 5+5$. The progression consists of a random arrangement of 4i and 3i, made to produce 12 H: $1^{\rightarrow} = 4+3+3+4+$ $+3+4+4+4+3+4+4$. The transformations in S_I are consistently clockwise, and the transformations of S_{II} consist of binomial regularity of the clockwise and the counterclockwise alternation.

The chords, reading by the lower stratum, are: F + D♭ + B♭ + G + E♭ + + C + A♭ + E + C + A + F + D♭.

Quadrant rotations were obtained from F as the axis of inversion.

T_1 represents an introduction consisting of H_1; the next 12 Tⓐ represent the first exposition; the following three expositions ⓓ 12T + ⓒ 12T + ⓑ 12T are followed by a coda, which consists of 5T and represents a repetition of the preceding measure in a slowing down pace; it is based on one H, which is the first chord of the subject.

The temporal thematic pattern of this composition is evolved from the simplest elements of $\frac{7}{\div}$ series. Melody, which in its first three expositions uses only the chordal functions of S_{II}, is based on $T = (4+2+1) + (2+1+4)$. If we designate the first trinomial as a and the second as b, the whole subject appears as follows: (a+b+b+a) + (b+a+a+b) + (b+a+a+b). The harmonic accompaniment follows the same scheme as melody. Its temporal thematic pattern is: $T = (\boxed{1} + 1 + 1 + 2 + 1 + 1) + (1 + 1 + 2 + 1 + 1 + \boxed{1})$; its instrumental form is based on single attacks throughout the entire composition. In the third exposition, it is varied by the split-unit groups. Melody has three instrumental forms. The first form, consisting of single-attack sequence of S_{II}, is followed by the second form, which is a double-attack sequence, combined with octave-coupling. These two forms are evenly distributed in the first exposition. The second exposition is based on the first form. The third exposition is based on the second form. The fourth exposition is a variation combining the first two forms with the split-unit groups. The tones which appear as auxiliary, in reality are the chordal functions of S_I; the presence of leading units combined with splitting of durations attributes to the last exposition the character of melodic figuration.

(1) $\Sigma^{\rightarrow}$ of the subject.

Figure 48. $\Sigma^{\rightarrow}$ of the subject.

"STUDY IN RHYTHM" I

Joseph Schillinger

Figure 49. Major component is strata-harmony $\Sigma = 2S$ (continued).

Figure 49. Major component is strata-harmony Σ = 2S (continued).

Figure 49. Major component is strata-harmony Σ = 2S (continued).

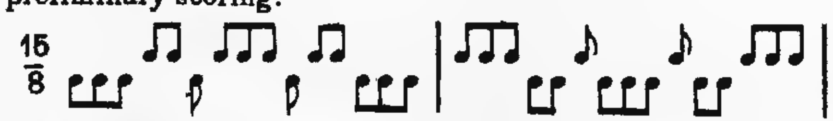
Nov. 16, 1936, New York

Figure 49. Major component is strata-harmony $\Sigma = 2S$ (concluded).

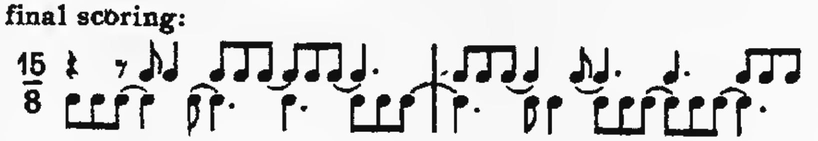
E. "Study in Rhythm II" for Piano (1940)

A two-part instrumental interference is the major component of the subject in this monothematic composition. Its source is the $r_{5\div3}$. Each term of the resultant is broken into single t-units. Thus the attack-group appears as follows: $A = 3+2+1+3+1+2+3$. By distributing the attacks through two parts and through the durations, we obtain a double cycle of $r_{5\div3}$; $\frac{7}{2}$ interference makes the 7 terms of r appear twice:

(a) preliminary scoring:



(b) final scoring:



The entire continuity is based on the circular permutations of single terms:

$$T^{\rightarrow} = 7r_{5\div3} \circlearrowright = (3+2+1+3+1+2+3) + (2+1+3+1+2+3+3) + (1+3+1+2+3+3+2) + (3+1+2+3+3+2+1) + (1+2+3+3+2+1+3) + (2+3+3+2+1+3+1) + (3+3+2+1+3+1+2).$$

$$T^{\rightarrow\prime} = 7r_{5\div3}\cdot 2 = 7\cdot 15\cdot 2 = 210\ t.$$

$$T'' = 15\ t;\ \text{hence: } NT'' = 210 \div 15 = 14.$$

In the form of two-part instrumental interference, this continuity appears as follows:

Figure 50. Source $r_{5\div3}$. Attack distributed through 2 parts (continued).

Figure 50. Source $r_{5 \div 3}$. Attack distributed through 2 parts (concluded).

After accomplishing this, I came to the decision that the rhythmic exuberance of this setting is apt to horrify almost any performer. I then re-wrote the same setting into 3/4 time with t = ♩, thus extending this setting to 70 measures. This gave the scheme an "easy" optical appearance:

Figure 51. Material of figure rewritten in 3/4 time (continued).

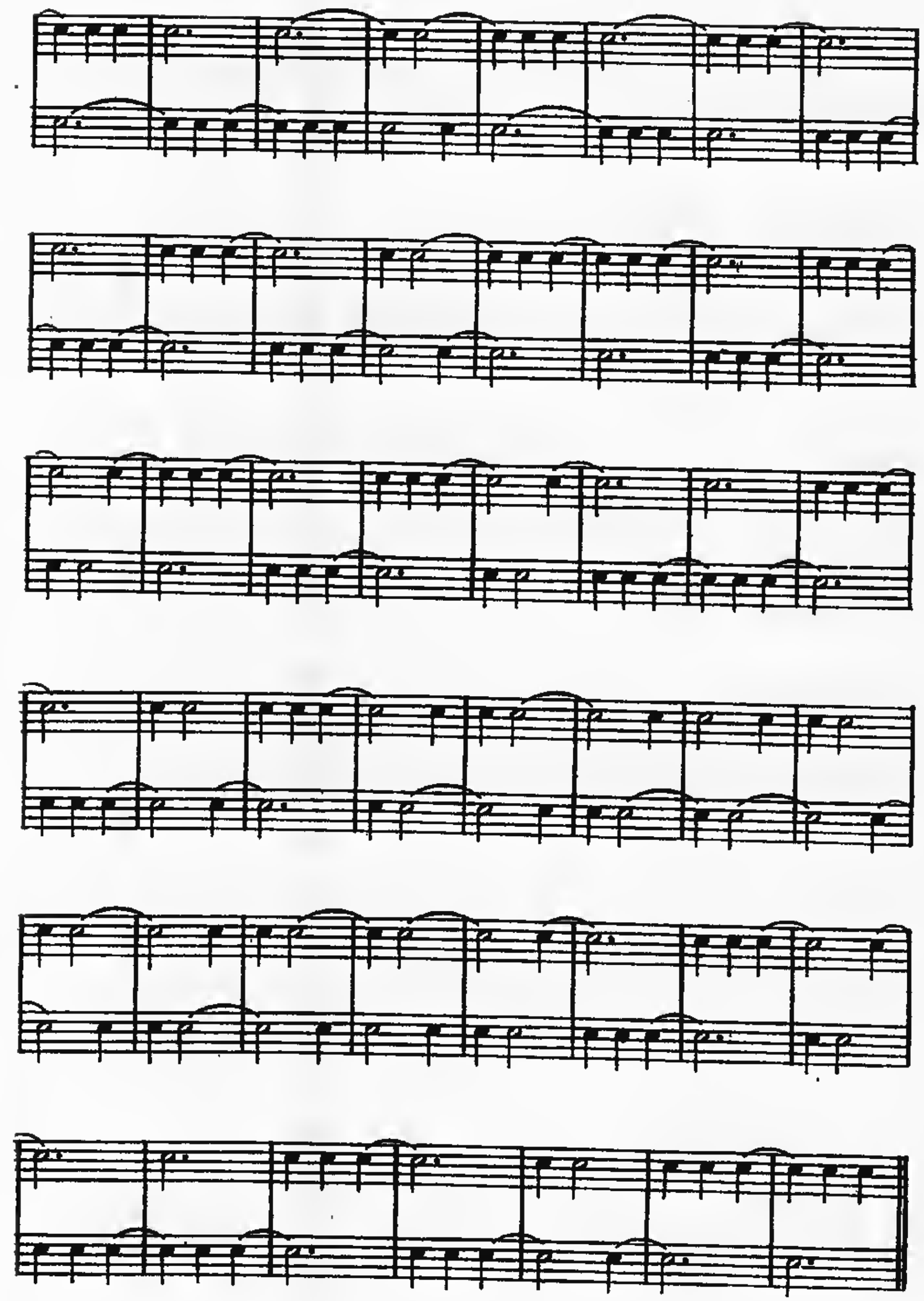


Figure 51. Material of figure 50 rewritten in 3/4 time (concluded).

This two-part setting was transformed later into continuous two-part counterpoint. The same pitch scale was used in both parts, but in the $\sqrt[3]{4}$ relation to each other, thus producing counterpoint of type III. The axis of the upper part was fixed on *c* and the axis of the lower part, on *e*.

After the counterpoint was written, couplings were added. The fundamental scheme of couplings (four to each part) was used in systematic permutations, employing one coupling at a time.

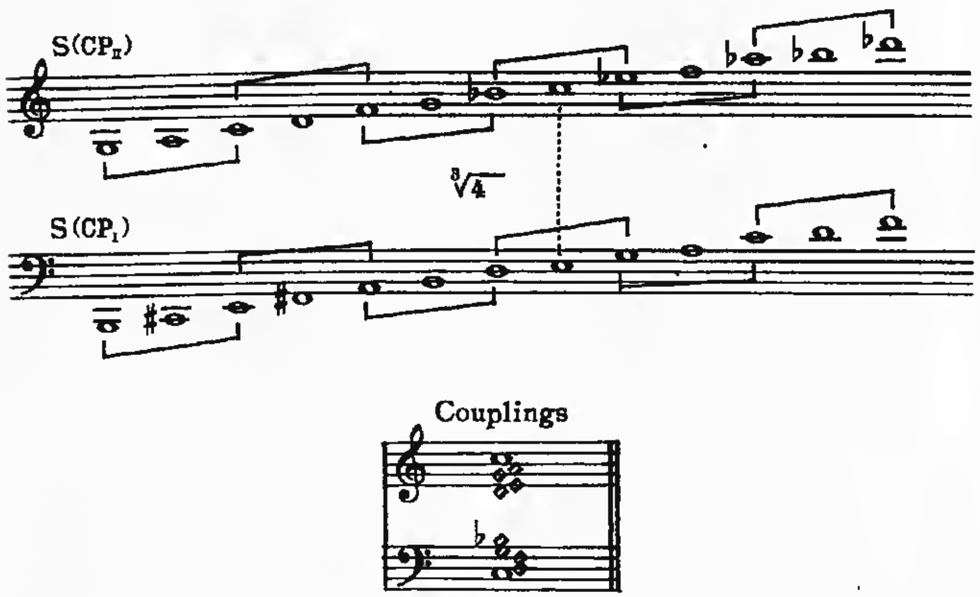


Figure 52. Two-part setting transformed into continuous two-part counterpoint.

Figure 53. Coupling (continued).

Figure 53. Coupling (continued).

Figure 53. Coupling (concluded).

The rhythmic scheme itself serves as an introduction and consists of 10 T. This is followed by the first exposition, based on the entire rhythmic scheme. Instrumental forms change in each 10 T subdivision of the subject. The last 10 T of the first exposition are used as a rhythmic modulation to the second exposition. This is accomplished by introducing split units progressively. The second exposition, lasting as long as the first, is based on juxtaposition of couplings in the upper part, in the original rhythm, and couplings transformed into single-attack instrumental forms (by means of split-unit groups) in the lower part.

"STUDY IN RHYTHM" II

Joseph Schillinger

Figure 54. Study in Rhythm II (continued).

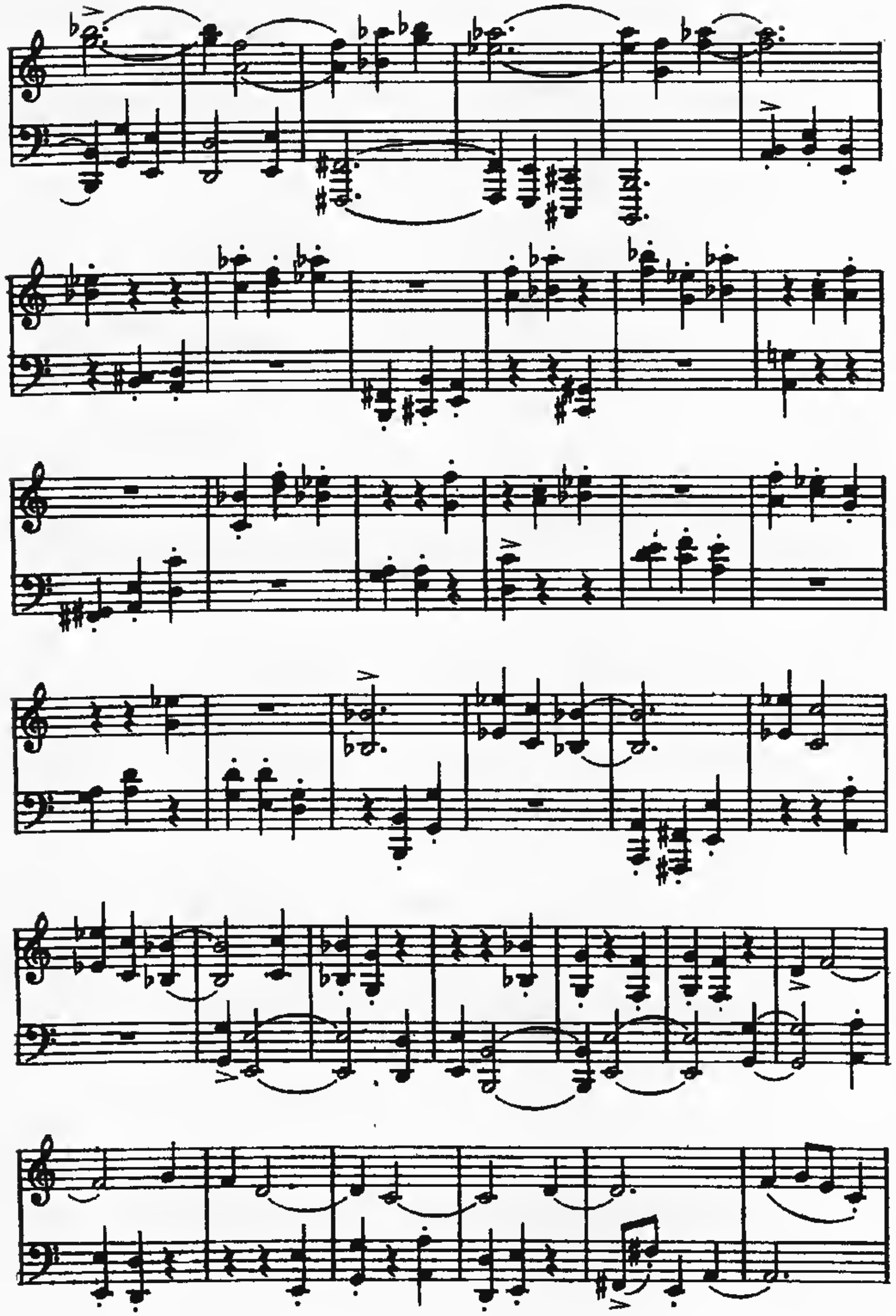


Figure 54. Study in Rhythm II (continued).

Figure 54. Study in Rhythm II (continued).

Figure 54. Study in Rhythm II (continued).

Nov. 10, 1940, New York

Figure 54. Study in Rhythm II (concluded).

Other examples of *monothematic composition* written by the students of this system:

Will Bradley: *Nocturne* for flute and piano. The subject is based on symmetric melodization and has one exposition. The melody has no recurrences and is of exceptional quality.

Edwin Gerschefski: *Solfeggietto*, etude for piano. This piece represents a through-composed music evolved from $\Sigma 4S4p$ in the form of unaccompanied melody. It was meant to be, according to the composer's intentions, a modern counterpart of Karl Ph. E. Bach's *Solfeggietto*.

Paul Lavalle: *Symphonic Rhumba* (one version is written for a 23-piece radio-orchestra; another, for a full symphony orchestra). The subject is based on symmetric melodization of harmonic ostinato: Phrygian descending tetrachord in S(9) connected in sequence in identical progressions three times through the $\sqrt[4]{2}$ and producing four groups 16 T each. The total length of the subject is 64 T. In the second exposition, the whole subject is accelerated twice. The introduction is a build-up of instrumental interferences in $\frac{8}{8}$ series. The middle section consists of a fugal exposition, whose theme is the basic rhythmic trinomial of $\frac{8}{8}$ series, used in circular permutations. Melodically, it is identical with the bass of the Phrygian harmonic ostinato.

Rosolino De Maria: (a) *Prelude* No. 1 for piano.* The subject consists of several sections of different instrumental form, and strictly speaking, has only one exposition. Coupled two-part counterpoint was used as major component (type II). The attacks of the original counterpoint were $\frac{A(CP_{II})}{A(CP_{I})} = \frac{4a}{a}$.

(b) *Etude* in C for piano.** This composition represents an elaborate use of harmonic strata, combined with high mobility. It is a sample of virtuosity in composition, and a challenge to the virtuoso performer.

Nathan Van Cleave: (a) *Improvisation and Scherzo**** for string orchestra. The improvisation is a through-composed subject, based on harmony and melodic figuration.

(b) *Etude for Orchestra*****. This composition is evolved from a three-unit scale, its modifications and derivative scales for the family. The through-composed melody is coupled by a full Σ 13. Rhythmic modifications are achieved by doubling the speed. Intonational modifications are achieved by quadrant rotation. A counterpart was included *a posteriori*. There is a great variety of instrumental modifications. The thematic sequence is based on the triadic progressive symmetry, but the three thematic groups are merely quadrant-modifications of the same subject.

*Published by Ricordi.
**Published by Ricordi.
***Performed on CBS on May 27, 1940 by string ensemble conducted by Alexander Semler. Available at Boosey & Hawkes, Inc. on rental.
****Performed by Robert Russell Bennett and his orchestra on June 13, 1941 on "Robert Russell Bennett's Notebook." Available at Boosey & Hawkes, Inc. on rental.

CHAPTER 16

POLYTHEMATIC COMPOSITION

We shall illustrate the process of assembling a polythematic composition with materials presented in the preceding chapters.

Our first decision will concern the style of temporal organization. We shall assign $\frac{4}{4}$ as the determinant of the series.

The style of intonation will be based on the Persian (Double-Harmonic) scale.

We shall select three subjects, and plan our composition in such a way that the degree of mobility will be highest for A, lowest for B, and intermediate for C.

Let the thematic unit of A be: Fig. 8.

Let the thematic unit of Fig. 15 (melody with couplings) be assigned to B, and let the thematic unit of Fig. 16 be assigned to C.

Our next step will be to define the form of thematic sequence. Let it be evolved in the form of progressive symmetry, as we learned it in reference to three subjects:

$$A_1 + (A_2 + B_1) + (A_3 + B_2 + C_1) + (B_3 + C_2) + C_3.$$

Next comes the temporal organization of continuity. We shall arrange it in such a way that A, in the course of its expositions, will be a growing subject; B will be the dominant subject appearing in its maximal period through all three expositions; C will be, in the course of its expositions, a declining subject.

We shall assume the maximal period to be equal for A and C, and designate this value at 16 T''.

We shall select the form of growth and decline for A and C to be in $1 \div 2 \div 4$ ratio. Then we acquire the following temporal scheme for all three subjects:

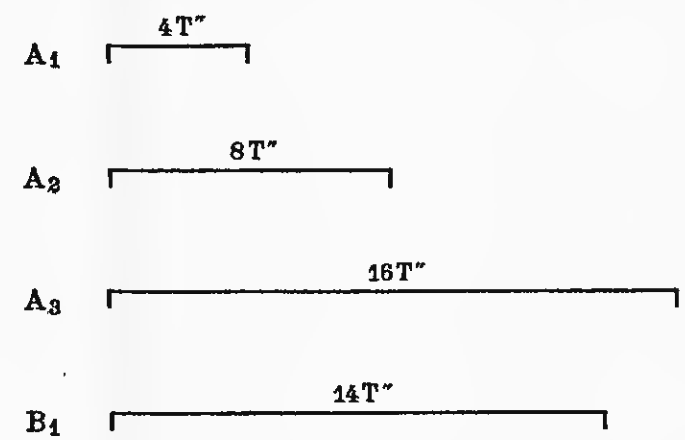


Figure 55. Ratio $1 \div 2 \div 4$ (continued).

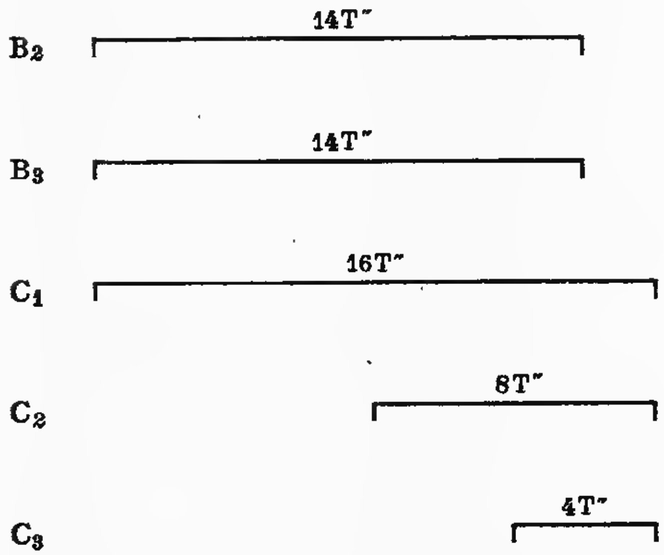


Figure 55. Ratio 1÷2÷4 (concluded).

From this we find: $T^{\rightarrow}(A) = 4+8+16 = 28\,T''$
$T^{\rightarrow}(B) = 14 \cdot 3 = 42\,T''$
$T^{\rightarrow}(C) = 4+8+16 = 28\,T''$

Hence: $T^{\rightarrow}(A, B, C) = A28T'' + B42T'' + C28T'' = 98T''$

Temporal relations of the subjects appear as follows:

$$\frac{B}{A+C} = \frac{42}{56} = \frac{3}{4};\ \frac{A}{B} = \frac{28}{42} = \frac{2}{3};\ \frac{C}{B} = \frac{28}{42} = \frac{2}{3}.$$

Hence: $T^{\rightarrow}(A) \div T^{\rightarrow}(B) \div T^{\rightarrow}(C) = 2 \div 3 \div 2.$

Assuming $t = 1/8$ sec. = 𝅘𝅥𝅯 , we obtain the following duration-units for all three subjects:

$t'(A) = t = 1/8$ sec. = 𝅘𝅥𝅯
$t'(B) = 4t = 1/2$ sec. = 𝅘𝅥
$t'(C) = 2t = 1/4$ sec. = 𝅘𝅥𝅮

Then:
$T''(A) = 16t = 2$ sec.
$T''(B) = 4t' = 2$ sec.
$T''(C) = 8t' = 2$ sec.

The quantities of the respective duration-units in each subject appear as follows:

$T^{\rightarrow}(A) = 28T'' = 448t$
$T^{\rightarrow}(B) = 42T'' = 168t' = 672t$
$T^{\rightarrow}(C) = 28T'' = 224t' = 448t$

Since every $T'' = 2$ sec., the duration of the entire composition is: $T^{\rightarrow} = 96 \cdot 2 = 196$, or 3 minutes, 16 seconds.

The form of continuity of this composition appears as follows:

$$A_1(T_1 - T_4) + [A_2(T_1 - T_8) + B_1 14T] + [A_3 16T + B_2 14T + C_1 16T] + [B_3 14T + C_2(T_9 - T_{16})] + C_3(T_{13} - T_{16}).$$

We shall distribute the key-axes in such a way that:

(a) they will be symmetric;

(b) they will change with each term of pentadic symmetry, which this form of thematic sequence represents.

Let the sequence of key-axes be based on the E_1 ($S^{\rightarrow}d_0$): C − E − G − E − C Further, let each term of pentadic symmetry appear in the different geometric positions; and let these positions be: a − d − a − c − b.

We shall select our dynamic forms in the following way:

A_1P; A_2MF; A_3F ;
B_1F; B_2P ; B_3MF;
C_1F; C_2MF; C_3P .

Under such a form of selection, A and C reciprocate dynamically in time-continuity, while B changes from one extreme degree to another and balances itself on an intermediate degree.

Now we can express the entire continuity with respect to intonational, axial and dynamic synthesis: $A_1 C\,a\,P + (A_2MF + B_1F) E\,d + (A_3F + B_2P + C_1F) G\,a + (B_3MF + C_2MF) E\,c + C_3 C\,b\,P.$

We shall select the instrumental forms in such a way that A and C will have the same form in their respective expositions, while B will appear in a different instrumental form in each exposition.

Example of Polythematic Composition

Figure 56. Polythematic form (continued).

Figure 56. Polythematic form (continued).

Figure 56. Polythematic form (continued).

Figure 56. Polythematic form (continued).

Figure 56. Polythematic form (concluded).

The miniature form in which we evolved the above composition must serve as a sample for such exercises. The student will find it more expedient to get acquainted with the various forms of composition by executing the most ambitious tasks in miniature form. These miniature forms will serve him as models for future works of greater temporal and instrumental dimensions. This method is comparable with the execution of stage models before the actual sets are constructed. It saves the artist's time, develops his initiative and technique, and helps him to visualize projects of a greater scope.

I shall refer for analytical purposes to a few compositions in *polythematic form* composed by students of this system.

Will Bradley: (a) *String Quartet*

(b) *Duet for Two Clarinets and Piano.*

Carmine Coppola:

(a) *Quintet for Wind Instruments:**
(b) *Concerto for Oboe;*
(c) *Pagan Dance* for Orchestra.**

Edwin Gerschefski: *Fanfare for the New York World's Fair of 1939* for Brass Septet.†

Rosolino De Maria: (a) *String Quartet;*

(b) *As I Remember* Symphonic Impressions for large orchestra.

Of my own works, two may serve as examples of unusual and diversified forms of polythematic composition:

(a) *Sonata-Rhapsody* for Piano (1925);††
(b) *October*, Symphonic Rhapsody (1927) for large orchestra.‡

This concludes Part Two of the *Theory of Composition.*

*Chamber work for flute, oboe, clarinet, bassoon and horn in three movements. Performed by Detroit Symphony Woodwind Quintet in Detroit, March 17, 1941. (Ed.).

**Written in the summer of 1938, it was performed by the Rochester Philharmonic, Jose Iturbi conducting, on January 19, 1939. It has also been performed by the Michigan Symphony and by the Detroit Symphony. (Ed.)

†Performed over the CBS network in January 1939.- (Ed.).

††Performed in Russia by Alexander Kamensky, in Berlin by Irene Westermann, in Milan by Antonio Russolo, in New York by Nicolai Kopeikine, and in Washington, California, Mexico City by Keith Corelli (Ed.).

‡Performed in Moscow in 1927 and in Leningrad in 1928. Premiere in the United States by Leopold Stokowski and the Philadelphia Orchestra in 1929 (Ed.).

PART THREE

SEMANTIC (CONNOTATIVE) COMPOSITION

CHAPTER 17

SEMANTIC BASIS OF MUSIC

A. Evolution of Sonic Symbols

THIS discussion requires on the part of the student complete familiarity with the semantics of melody and the connotative meaning of configurations, which information can be found in the Theory of Melody.*

Our present task is to include all the technical resources of composition in the field of connotative music. This inclusion of all forms of musical expression increases the range of admissible associations, thus enriching music as the ***language of sonic symbols.***

All symbols are configurations. Graphic symbols are perceived by sight; sonic symbols are perceived by hearing. Sonic symbols are modifications of frequency and intensity. At the early stage of human evolution, there was only one language of sonic symbols. Later on it gradually differentiated into two sonic languages: speech and music. Early forms of speech greatly rely on the intonation (modification of frequency) as an idiomatic factor: words of the same etymological constitution, spoken with a different intonation, acquire different meanings, i.e., they become new symbols. Music, i.e., what we know now as music, emancipated itself from the fore-language of sonic symbols through the dominance of intonation over other sonic forms and through the crystallization of fixed frequency units.

Modifications of frequency and of durations are the basic components of sonic configurations. Further refinement of symbols is achieved by modification of intensity (which also includes the form of attack) and quality (which is physically the product of frequency and intensity). All other configurations, such as those produced by modifications of density, take place only when complex sonic symbols participate.

Sonic semantics is altogether possible because of the configurational interdependence of the activating (stimulative) and the reactive patterns. All components of sound work in similar patterns, and these patterns are similar in all sensory experience. Identical patterns exhibit a tendency of mutual attraction, and the latter stimulates association. The meaning of music evolves in terms of physico-physiological correspondences. These correspondences are quantitative, and quantities express form.

We can easily imagine that at its early stage the language of sonic symbols existed in the form of larynx reflexes, caused by certain forms of physiological activity. As these sounds, stimulated by somewhat similar experiences, repeated

*See Vol. I, p. 279.

themselves in somewhat similar reactive forms, these reactive forms eventually began to crystallize. The crystallized sonic patterns could be intentionally repeated. Being associated with definite stimuli, they became symbols. As the response to sonic forms exists even in so-called inanimate nature in the form of sympathetic vibrations, or resonance, it is no wonder that even primitive man inherited highly developed mimetic responses. From this we can conclude that a great many of the early sonic symbols probably originated as imitation of sonic patterns, coming as stimuli from the surrounding world. We must not forget that echo (as a physical pattern-response) existed on this planet before any auditory receptor was developed. It is also true that tactile responses to pressure in general, as patterns of compression and rarefaction coming from the generalized cutaneous, i.e., skin-receptor, preceded the development of a more specialized auditory receptor.

The next stage of the evolution of sonic symbols is characterized by the use of intonational patterns as symbols of ideas and concepts. We find such use of musical symbols in ancient China, just as we find graphic symbols in the sand-paintings of the Navajo. At this stage, both sonic and graphic symbols are in competition with linguistic, i.e., etymological symbols. *Oxford History of Music* (Vol. 6, p. 111) defines this stage in the following way: "Program music is a curious hybrid, that is, music posing as an unsatisfactory kind of poetry."

Finally, we arrive at the stage where the forms of musical expression become confined to their purely configurational meaning. In this aspect sonic symbols may be looked upon as generalized pattern-stimuli. The first formulation of this meaning of music comes from Aristotle: "Rhythms and melodious sequences are movements quite as much as they are actions." "Musical motion," when projected into spatial configurations, possesses characteristics similar to that of motion, action, growth, or other "eventual" processes. It particularly resembles the mechanical trajectories and the projections of periodic phenomena, i.e., the processes which are characterized by a high degree of regularity. As mechanical trajectories are the inherent patterns of "musical motion," ***music is capable of expressing everything which can be translated into form of motion.***

B. Configurational Orientation and the Psychological Dial

The interaction which we call "association," and which permits the formation of reactions, sensations, and emotional and mental attitudes, is based on an inherent capacity which we may term "configurational response." This capacity appears to us to be a special case of configurational responses in general. It extends itself to the entire range of the knowable, including physical and chemical reactions, various types of reflexes (including articulatory responses of speech), emotional, mental and even telepathic reactions.

The associative power of musical configuration depends upon three basic conditions: first, the selection of a configuration which is in proper correspondence with the configuration of the state or process to be expressed; secondly, the selection of a musical form which adequately corresponds to the selected configuration and which operates in media comprehensible to the listener; and finally, a favorable state of reactivity on the part of the listener, i.e., his disposition to react and comprehend at a particular moment.

So far we have dealt with sonic symbols in the form of linear configurations taken individually (*Theory of Melody*)* or in combinations (*Theory of Correlated Melodies*).** In the latter case the component melodies still remained linear patterns. The extension of configurational semantics into other simple and complex components of music constitutes the subject of the present discussion.

We shall consider linear configurations as simple and group configurations as complex. Linear configurations consist of individual components. Group configurations represent assemblages of conjugated components. Simple configurations are produced by melody and possess a greater configurational versatility than the complex configurations. The latter are produced by harmony and, compared to melody, are relatively inert.

The degree of configurational versatility of melody depends on the technique employed. When configurational versatility becomes the chief factor of expression, the Theory of Melody (plotting technique***) must be preferred to the Theory of Pitch-Scales† (variation technique).

The degree of configurational versatility of harmony partly depends on the number of conjugated parts in the respective assemblage, partly on the number of simultaneous assemblages, and partly on the number of transformations employed. Harmonic progressions, as they derive from the permutations of intervals or from direct transposition of pitch-scales, have relatively limited configurational possibilities. In comparison with this, transformations and cycles employed in the *Special Theory of Harmony*†† offer a great many configurations. Configurational versatility of harmonic progressions reaches its maximum with the use of all transformations of the *General Theory of Harmony*.†††

The versatility of expression depends on the number and the forms of configurations. While the number of conjugated parts in an assemblage defines the possible number of transformations (which grows from p to 2p, 3p, 4p to Σ), it is also true that the stability of configuration, under all conditions being equal, grows with the increase of configurational elements: *the denser the assemblage, the greater its configurational inertia*. The patterns of S2p are more alert than the patterns of S3p, and the latter are more alert than that of S4p.

The discussion of this subject, i.e., the spatio-temporal patterns of simple and complex trajectories, brings us closer to an understanding of music in terms of motion and action. We have already seen that pattern stimuli activate *configurational response*. We shall use this term as a complex concept emphasizing all partial responses of the entire reactive chain. It includes *physico-physiological* (chemical, neurological, psychonic) and *psychological* reactions (associational, emotional, mental). Since we react as a unit, dissociation of the partial responses is impossible in actuality. For this reason there is a great advantage in using one concept which can emphasize reflexes, associations and judgments. We shall

*See Vol. I, p. 228; also p. 1432. **See Vol. I, p. 708. ***See Vol. I, p. 299. †See Vol. I, p. 115.
††See Vol. I, pp. 368, 378, 382. †††See pp. 1068, 1106 and 1127.

define *judgment* as the self-evaluating partial response of the entire reactive group. Its function consists of associating current configurational responses with past configurational responses, with which it has pattern-similarities. The association itself is a form of attraction (like that of sympathetic reactions) existing between the pattern-similarities. Thus, judgment may be looked upon as a form of *configurational orientation*. The evaluation of event, as a process, in terms of mechanical efficiency, i.e., in terms of performance (action), crystallizes the configurational response into an *attitude*. The relativity of forms (standards) of mechanical efficiency corresponds to the relativity of forms (standards) of configurational orientation and results in the corresponding attitudes.

To illustrate this, we shall demonstrate the translation of events into actions; and to accomplish this, we shall resort to the *scale of configurational responses*, as it was presented in the *Theory of Melody*,* i.e., in the form of a *psychological dial*.

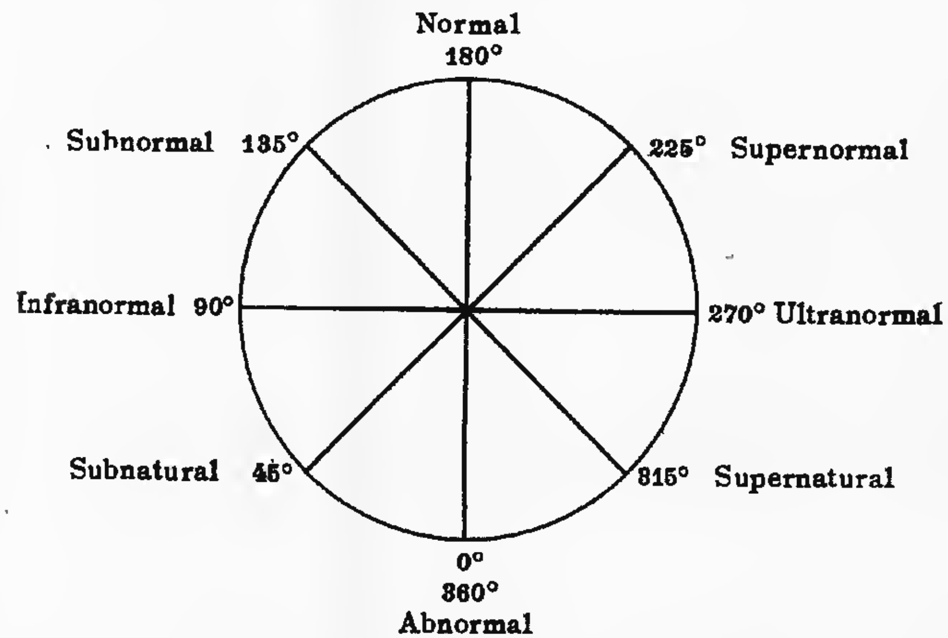


Figure 57. Psychological dial.

Here, infranormal represents the lower limit of normality and corresponds to ultimate depression; ultranormal represents the upper limit of normality and corresponds to ultimate ecstasy. The 0°, when arrived at by counterclockwise motion on the circumference, represents the lower limit of performance; the 360°, when arrived at by clockwise motion, represents the upper limit of performance. These two coinciding points are both in the range of the improbable.

*See Vol. I, p. 232 ff.

To facilitate our further discussion, we shall use a graphic representation for each response by the respective hand-position on the dial:

Figure 58. Hand-positions on the dial.

This scale, if necessary, can be developed to a further degree of refinement by introducing the intermediate hand-positions, in addition to those offered above.

The psychological dial may be looked upon as a form of bifold symmetry, having the ordinate for its axis. There is a configurational reciprocation of patterns symmetrically located on both sides of the axis. The reciprocating pairs are: , i.e., normal-abnormal; , i.e., subnormal-supernormal; , i.e., infranormal-ultranormal; , i.e., subnatural-supernatural. This implies that the reciprocating responses are activated by stimuli of mutually converse patterns.

The left half of our dial represents the differentiated forms of the original *defense-response;* the right half, the differentiated forms of the original *aggression-response.* The first response is characterized by *contraction-patterns;* the second, by *expansion-patterns.* Either of the two may be *active* or *passive*, depending on the presence of "resistance," which psychologically is the effortful feeling of striving. The presence of resistance in the activating pattern intensifies the configurational response. It "dramatizes" the response and is based on the amplitude-evaluation.

As we have seen in the "Semantics of Melody," this resistance in the response-pattern has its counterpart in the mechanical resistance-pattern. As aggression psychologically corresponds to *inducement*, and defense, to *submission*, we shall consider the right half of the dial as *positive* and the left as *negative.* The positive zone is associated with the *gain of energy and growth*, the negative, with the *loss of energy and decline.* The inducement in association with resistance becomes *dominance*, or the active form of inducement. The submission in association with resistance becomes *compliance*, or the active form of submission.

Now, we come back to the *evaluation of performance.* It is not difficult to see how mechanical performance can be put on a quantitative basis. Application of such-and-such amount of energy is expected to produce such-and-such result. When the result of the application of energy is what we expect, the response-pattern is *normal.* When the result is below our expectations, i.e., *less* than we expected, the response-pattern is *subnormal.* When the result is above our expectations, i.e., *more* than we expected, the response-pattern is *supernormal.* Further extension of the performance beyond these limits produces *subnatural* for the first group and *supernatural* for the second group. The final limit for both groups, merging one into another, produces the response-pattern of *abnormal* (absurd).

The concepts of normal, subnormal, supernormal, etc., are integrated and crystallized response-patterns, and as such, are capable of stimulating associations with other response-patterns and their integrated and cystallized conceptual forms. It is on the basis of these associations that it becomes possible to translate the original configurational response of *accumulation-discharge* into *tension-release, anticipation-fulfillment*, etc. It also becomes possible to form an attitude in each case on the basis of evaluation. The evaluation refers to pattern-similarities by associating them with past experiences, and the response-pattern becomes an attitude as the result of evaluation. The attitude demonstrates whether the outcome of a certain process (course of events) is below or above expectations, or is exactly what was expected. Thus, the fulfillment of anticipation may be *equal* or *smaller* or *greater* than is expected.

C. Anticipation-Fulfillment Pattern

In the chapter on "Climax and Resistance" (*Theory of Melody*),* we analyzed the various responses to the discus-thrower. These responses formed attitudes as a result of evaluation of the athlete's mechanical performance. We shall analyze now a group of responses and attitudes on events which do not contain any apparent motion or action, but merely the anticipation-fulfillment pattern.

For our first illustration, we shall employ a case which involves quantities, that is, a case in which evaluation can be based on some obvious quantitative relations.

Let it be a man who comes to a drugstore to buy an article for which he expects (owing to his previous experience) to pay one dollar and ninety-eight cents, and, perhaps, two cents tax. Suppose the customer gets the article he wants for the price he expects to pay. The response-pattern in this case is , and the customer's attitude is either indifference or acknowledgment of the fact: he got neither more nor less than he expected, i.e., the response-pattern is *normal.* Now let us move to the negative zone. It is an assumed increase in price that would produce it. Suppose the price went up to $2.49. It undoubtedly would disappoint the customer and, whether he bought the article or not, it would stimulate the response of regret. Now if we continue our venture further into the negative zone, we might set the price for the same article at $2.98 or even more. If the article is of great importance to the customer, and if the customer is poor and cannot afford the purchase, depression would be the response. He may not be inclined to commit suicide in this case. But imagine a father whose beloved child has to undergo a surgical operation, for which the poor man cannot afford to pay, because of the increased cost. In this case, depression reaches its maximum and the response-pattern is (infranormal). The continuation of this venture through the negative zone may suggest such illustrations as one of an imaginary customer coming to a drugstore, or even better to a "five and ten cent" store, to buy a fountain pen, and being told that the pen costs several hundred dollars.

*See Vol. I, p. 279.

As this is in the category of loss that is so incredible, the response-pattern will be not that of disappointment or depression, but rather of humor. The owner of the store may even tell the customer that the pen is made of platinum, or maybe even studded with diamonds. Still the price would appear ridiculous to the customer, as he is conditioned to definite expectations at the drugstore, which are quite different from those which, let us say, he would expect at Tiffany's. Such a situation puts the response-pattern somewhat like this: (subnatural).

To bring this case to the pattern of the abnormal or the absurd, we shall imagine that an ordinary fountain pen at Woolworth's, which the imaginary customer picks from the counter, is not for sale and belongs to the Maharaja of Jodpur. The response-pattern to be expected in this case is (abnormal) and represents astonishment.

Let us resume our purchasing venture from the balance point and move into the positive zone. In that zone, the pattern of gain will reciprocate the loss-patterns which we have already described with respect to the negative zone.

Coming back to the prospective purchaser of the $1.98 article, we find him at a drugstore on a day of a special sale: the current price is then only $1.49. The customer buys the article and enjoys the acquisition of it at such an advantageous price. He would hardly jump for joy, but there would be a response-pattern of satisfaction, expressing the dial position at (supernormal) or less. The relative limit of satisfaction, which theoretically is at the point of esctasy, might occur for the same article at a "penny-sale"; that is, when by paying one more cent, the customer acquires two identical articles for the price of one. Carrying this incident further into the positive zone, somewhere around (supernatural), we must imagine a case where the store owner says to the customer: "I value your patronage of many years and I wish to give you a present. Choose anything you like within the range of $25." The situation is quite improbable, of course, but not entirely impossible. However, it exceeds all the possible expectations of the unsuspecting customer.

To bring this case to the point of the absurd in gain we might imagine something which would be diametrically opposite to the zero position with the Maharaja's pen at Woolworth's. Such a situation might occur when the store owner offers his entire store and a good sum of cash to the astounded customer who expected merely to get a pen and to pay for it. Thus, gain may extend itself to the degree of the absurd, in which case the pattern-response is one of astonishment. After the subject recovers from this stage, he undoubtedly fluctuates into one or another of the adjacent zones. But the latter are associated with humor; therefore, the subject will accept the incredibly-generous offer as a joke.

In this group of episodes, or imaginary events, we have based the evaluation process on the tangible figures of quoted price in relation to expected price.

We shall now present a case in which no obvious quantities are involved. We shall base this illustration on a *moral* instead of *material* evaluation. And in this case, the evaluation will be based on moral loss or gain.

Mr. A knows Mr. B for many years as an honest wage earner. One day Mr. A discovers to his regret that Mr. B is a petty thief. Mr. A does not find it tragic; but his response-pattern is sorrow and can be located on the psychological dial as (subnormal). Mr. A does not believe any human being is perfect and with regret, makes allowances for such a weakness. One fine day however, Mr. A learns that Mr. B had actively participated in a bank robbery. This comes to Mr. A as a very depressing bit of news and his response-pattern becomes . But Mr. A is positive that Mr. B cannot be a killer. Yet, at a later date, Mr. B is accused of murder. This comes to Mr. A as a great surprise and his pattern-response becomes (infranormal). As everything beyond this point of the negative zone is associated with an incredible loss (moral loss in this case), we would have to compel poor Mr. B to assassinate at least one or two families—and we can't afford to spare even the little children.

In order to find a proper response-pattern for the deeds of Mr. B, we would have to move the dial-hand to position (subnatural). Remember that Mr. B is not known to be a maniac; otherwise, such actions might have been expected. To conclude the unfortunate venture of Mr. B, we shall collect sufficient evidence in order to prove beyond doubt that Mr. B, during his absence from town, exterminated cold-bloodedly and methodically the complete population of several small and remote communities.

The response pattern of Mr. A to such actions of Mr. B, and we sympathetically join Mr. A in his reaction, is (abnormal); his attitude can be described as complete astonishment, from which it is not easy for him to recover. The concepts associated with such gruesome and cruel actions of Mr. B are: incredible, unbelievable, impossible, insane, nonsensical, etc.

Now to cheer up Mr. A and ourselves, we shall start a new life for Mr. B. Mr. B has just moved into a new neighborhood, where he makes new acquaintances, one of whom is Mr. A. The latter thinks he is "all right," but does not suspect what a nice fellow Mr. B really is. One day Mr. B pays a visit to Mr. A and brings him a gift. As time goes on, Mr. A. learns that the present was a token of true friendship and that Mr. B did not expect anything in return. The evaluation of such an action on the part of Mr. B can be expressed as moral gain. It places the response-pattern in position. (supernormal). To put Mr. A into a state of real ecstasy, we shall compel Mr. B to perform an act of real sacrifice in favor of Mr. A. We may let Mr. B save Mr. A's drowning child, in which action he subjects himself to real danger. Thus, we reach the stage at which the response-pattern becomes (ultranormal). Beyond the heroic action in saving his friend's child from drowning lies the field for incredible and fantastic actions that call for a superman.

Mr. B is not a superman, and for this reason his attempt to save a whole family of dogs from a burning house, which he succeeds in accomplishing, causes our response-pattern to become (supernatural), as the whole affair seems in our evaluation to be incredible and fantastic. An unnecessary self-sacrifice on the part of Mr. B would also do as well to illustrate this response-pattern.

Such a case might have occurred if Mr. B had started some sort of business enterprise, and Mr. A, at some later date, had entered into competition with him. We find both competitors at a stage where Mr. B, who had established himself first, makes his enterprise a success while Mr. A fails to accomplish anything. Then, one day, Mr. B feels it such a pity to see Mr. A struggling and decides not only to move to another town, in order to get out of Mr. A's way, but even gives his whole enterprise and all the accumulated profits to Mr. A. The response-pattern to such actions of Mr. B, naturally, would come close to position . To make the situation completely absurd, we would have to induce Mr. B, in addition to the sacrifices he has just made, to commit suicide. This type of action on his part certainly would be unnecessary and abnormal. Our gain from this ridiculous action is that we obtain an illustration of the response-pattern associated with position (abnormal).

As all our evaluations are relative, some of Mr. B's actions in the negative zone would have been quite normal for a criminal or a maniac. Likewise, were Mr. B a saint, we might evaluate his dog-saving expedition as quite normal. In other words, all our evaluations and response-patterns have been based on the assumption that Mr. B is an ordinary (i.e., 180°) man.

These illustrations are offered in order to demonstrate that response-patterns and the resulting evaluations depend upon the estimated realization of event; that the evaluation itself evolves from associations of loss and gain; that such loss and gain may pertain to either physical or moral actions.

These illustrations may serve the student as analytical examples for finding the corresponding response-patterns of events and locating them on the psychological dial. The next stage, after this has been accomplished, is to translate the response-patterns into geometrical configurations, i.e., trajectories.

D. Translating Response Patterns Into Geometrical Configurations

The configurations, which represent both stimuli and responses, and are identical for both with a certain degree of approximation, form two basic groups. These two configurational groups correspond to negative and positive zones of the psychological dial. They also can be arranged in a bifold symmetry, in which case there is always a reciprocal pattern in one zone for any given pattern in another. Hence these patterns are geometrically convertible.

The balance point corresponds to simple harmonic motion, i.e., to a *sine-wave* pattern, which at its zero amplitude is a straight horizontal line. Any oscillation about the point of balance increases the amplitude accordingly. Thus, absolute balance is a motionless state. All other forms of balance appear in the form of slight oscillation (compare this with the behavior of a magnetic needle).

The forms expressing gain (the positive zone) are directed away from balance. All forms expressing loss (the negative zone) are directed toward balance. Here the amount of gain or loss corresponds to the value of the angle of the axis-pattern in its relation to the line of absolute balance (that is, the abscissa, or horizontal line, which corresponds to primary axis).

We can now represent the scale of configurations in their correspondences with the dial positions.

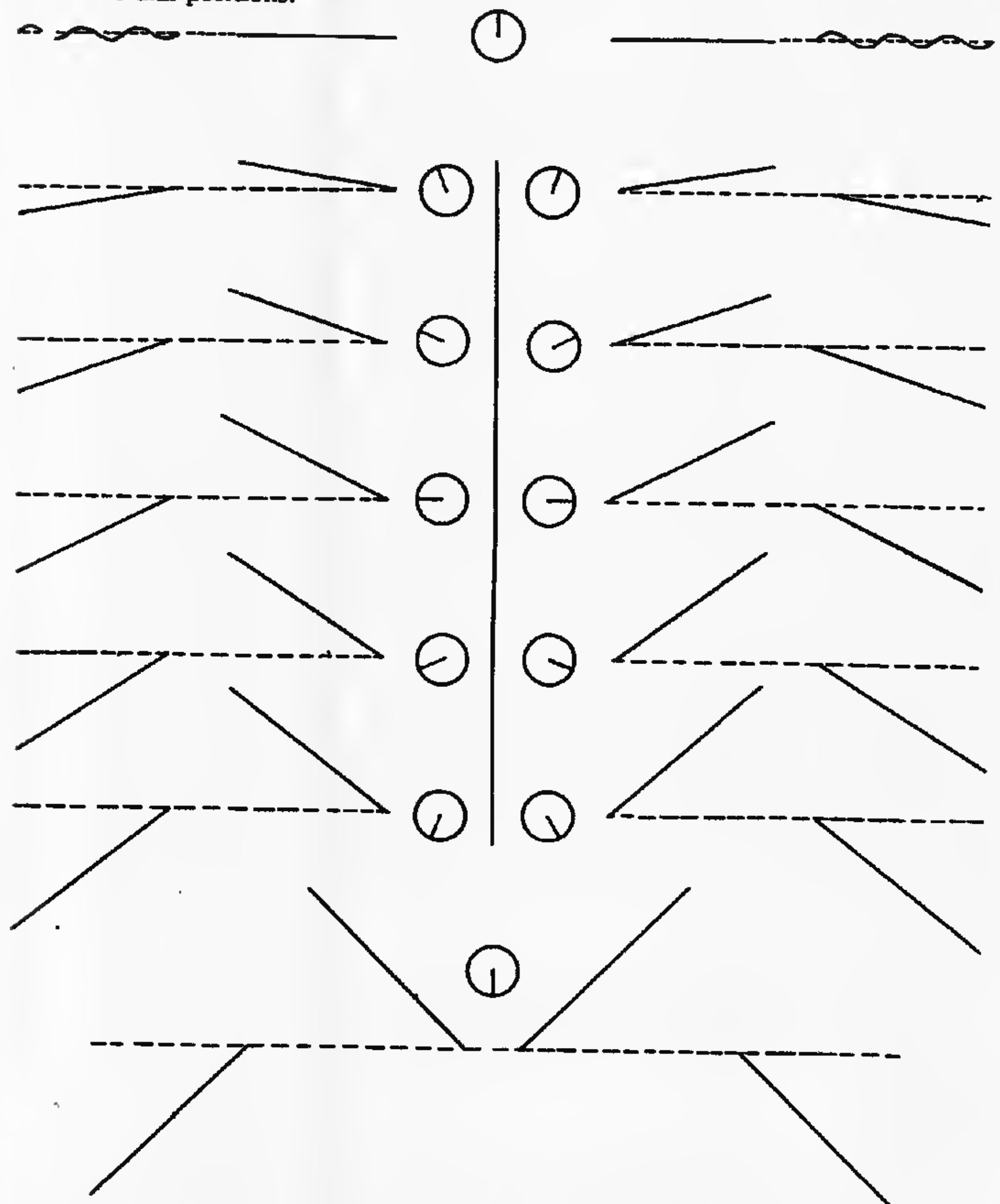


Figure 59. Basic scale of stimulus-response configurations.

The above scale represents configurations not containing resistance. Earlier, in the *Theory of Melody*, we defined the resistance-pattern as a geometrical projection of rotary motion. Its trajectory is that of a sine-wave (originally a circle; later extended into a cylindric spiral and, finally, into a sine-wave). By combining each of the above patterns with oscillatory motion in a sine-wave projection, we obtain the resistance forms of the stimulus-response configurations.

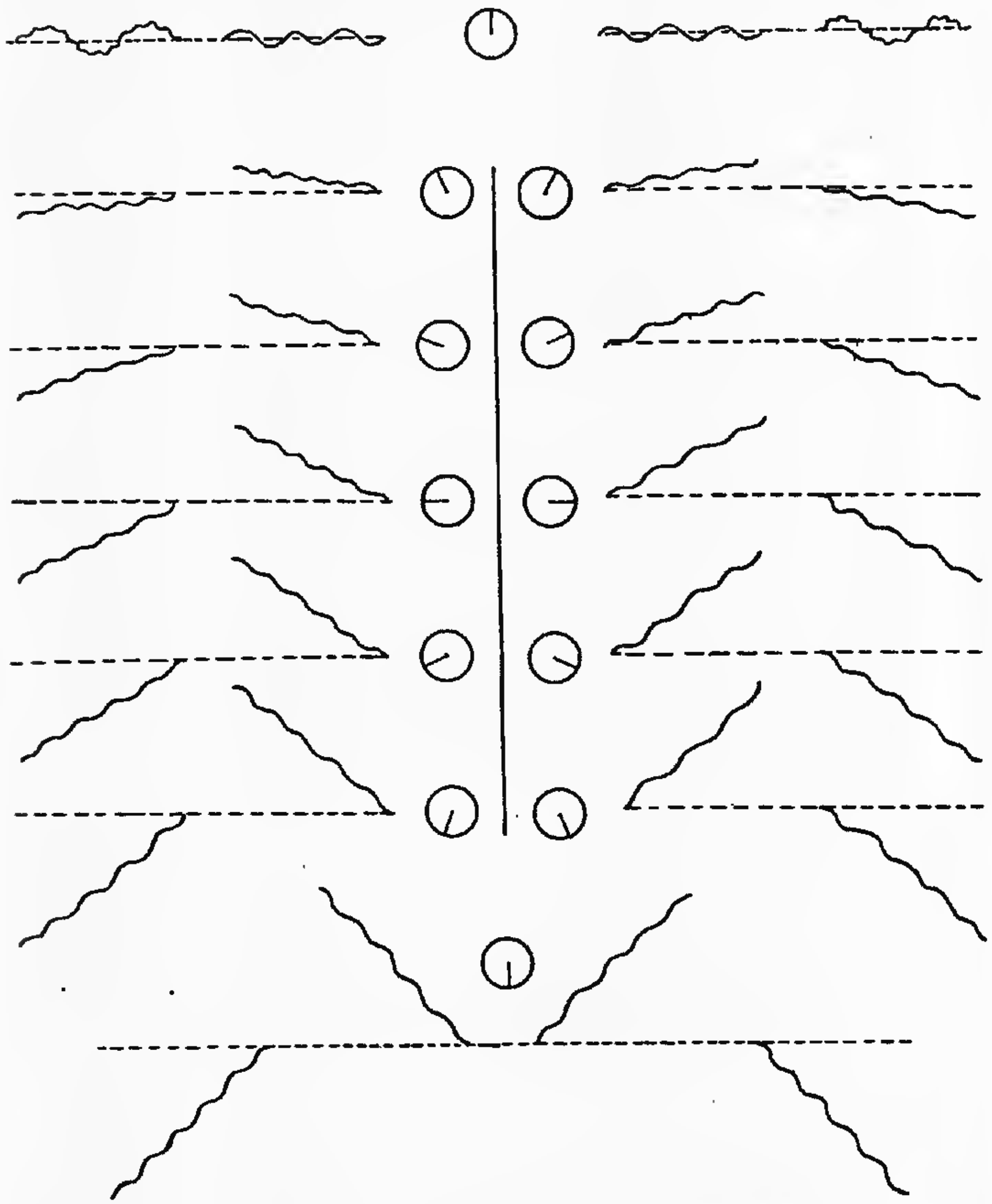


Figure 60. Resistance forms of the stimulus-response configurations.

E. Complex Forms of Stimulus-Response Configurations

Further intensification of resistance forms derives from combinations of the basic patterns. All positive forms become diverging and all negative, converging. We shall consider the basic patterns to be *fundamental* and the auxiliary patterns, *complementary*. Thus in the $\frac{a}{0}$ axial combination, a is fundamental and 0, complementary. In all oblique patterns, i.e., where 0 participates with the converging or diverging axes, the 0-axis is always complementary. In the case of a pair of converging or diverging axes, the axis which leads to a climax becomes fundamental.

A certain amount of intensification can be obtained by two or more parallel patterns, which in this case act as forces of the same direction; the addition of such forces increases the energy. This is true of mechanical phenomena; the intensification (growth or increase) of the amplitude which results from the addition of two or more identically directed phases (like the addition of sines or cosines) offers a purely physical illustration. Complex patterns resulting from several parallel configurations may be designated as $\frac{a'}{a}$, $\frac{a}{a''}$, $a \div a' \div a''$, $a' \div a \div a''$, $a' \div a'' \div a$, etc., in which cases they represent intensified variants of a.

The intensification of a pattern as a stimulus of configurational response depends on two main factors:

(1) the numbers of axes;

(2) the value of angles between the axes and the abscissa (primary axis).

The greater the number of axes employed simultaneously to produce one complex configuration, the more intense the response. An increase in the value of the angle (increase of its obtusity) stimulates an increase in the intensity of the response.

1. Parallel Forms

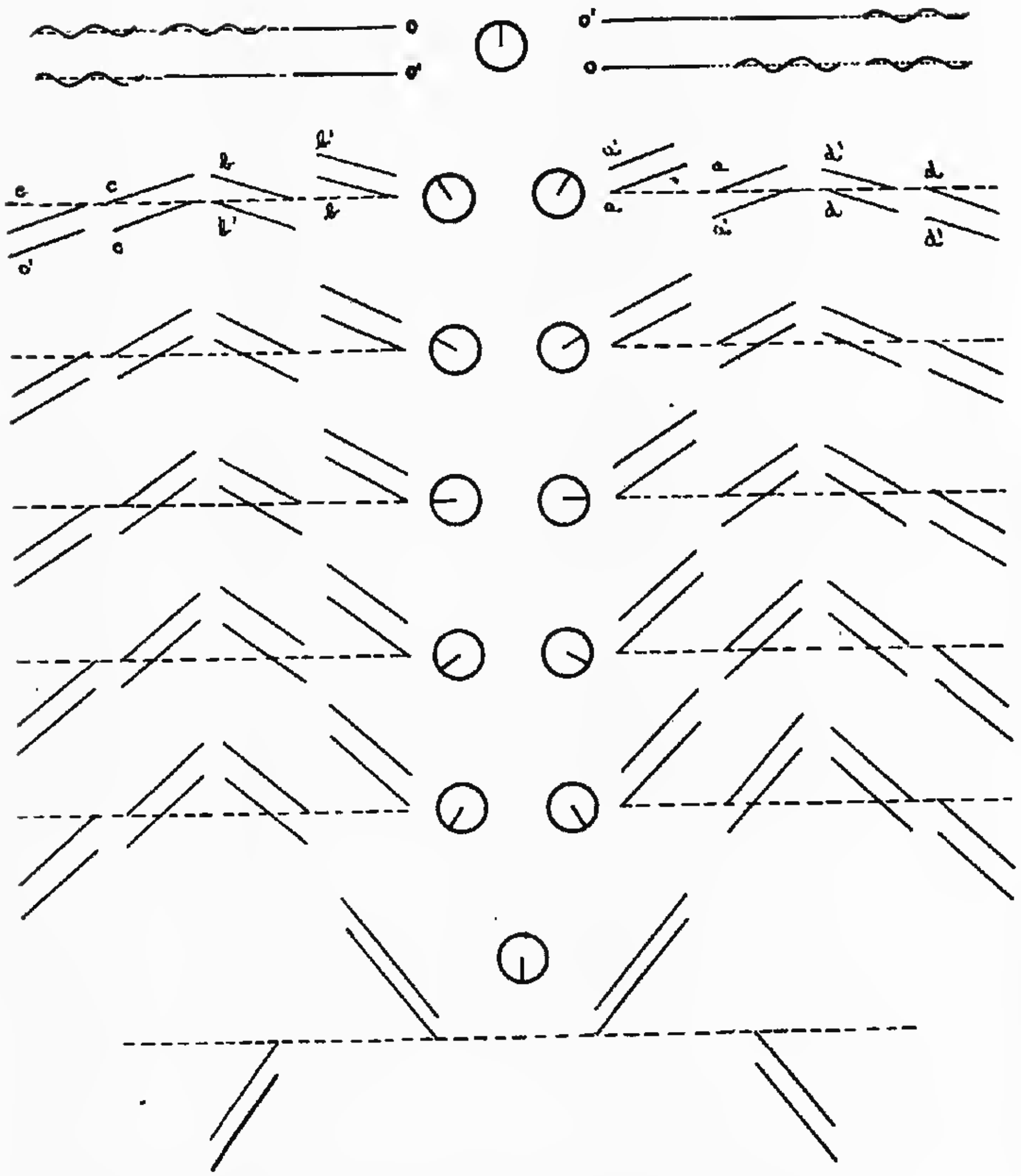


Figure 61. Binary forms of the stimulus-response configurations.

2. Oblique Forms

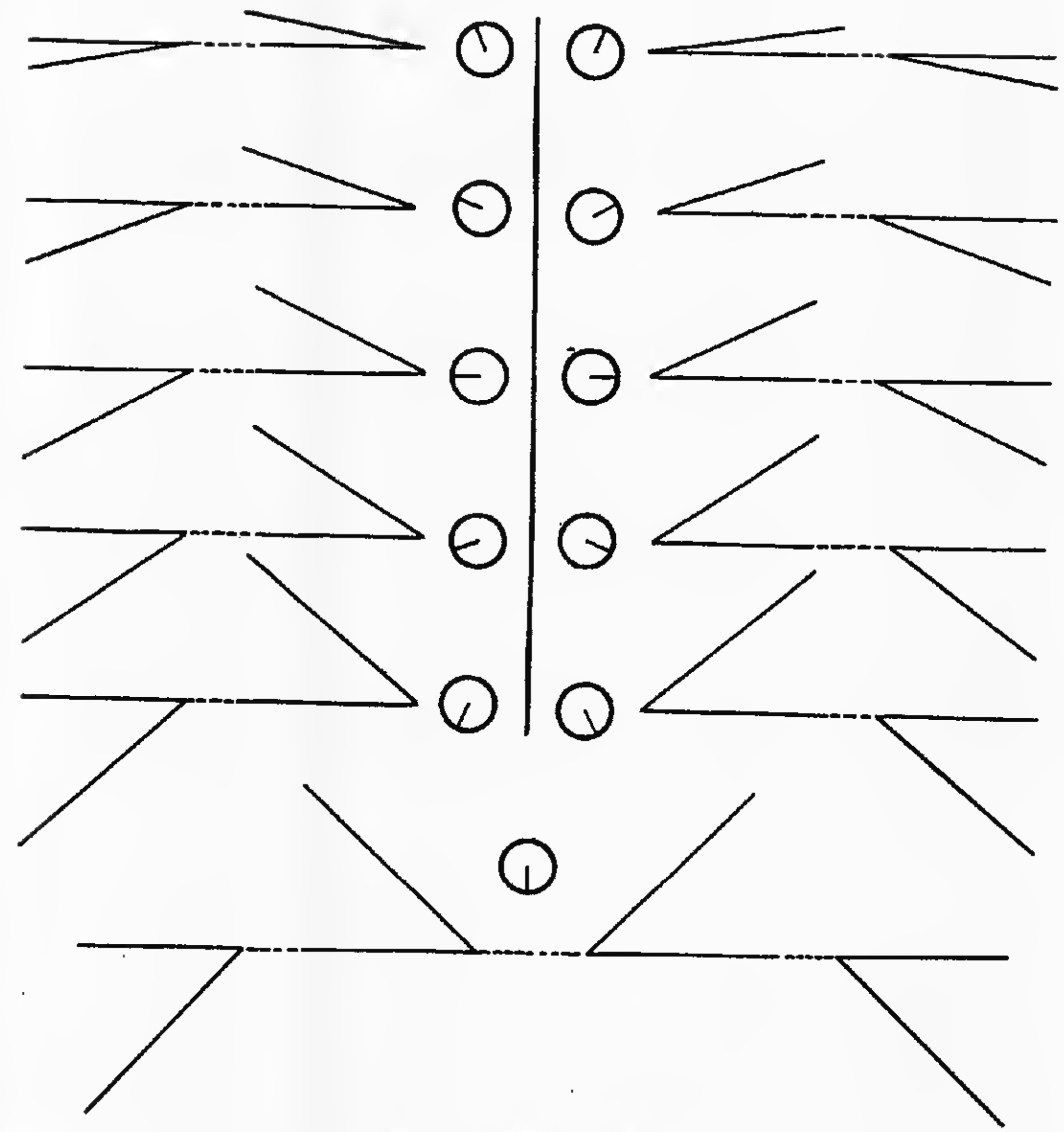
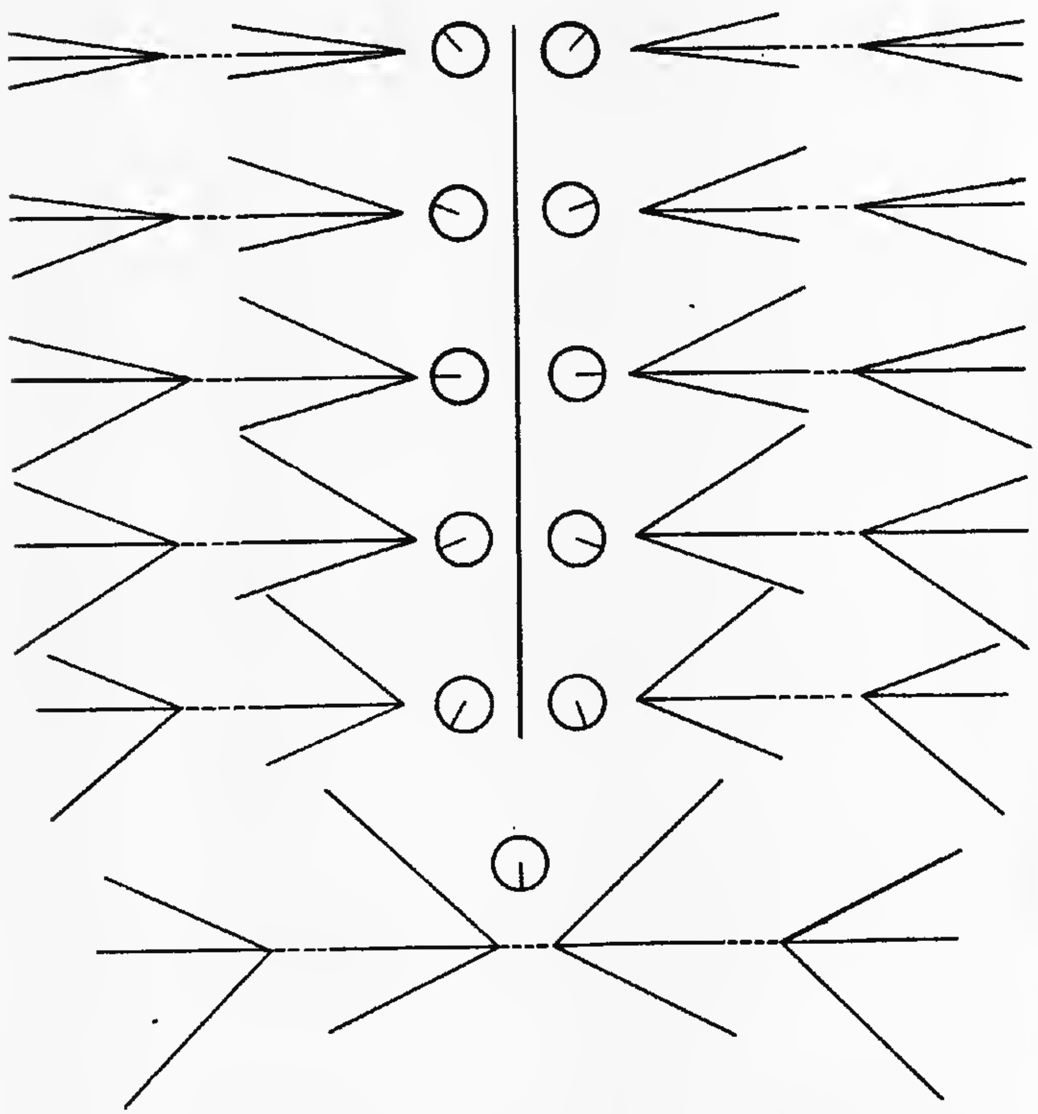


Figure 62. Oblique forms.

3. Diverging-Converging Forms

The angle of divergence or convergence between the fundamental secondary axis and the primary axis is equal to or greater than the angle of divergence or convergence between the complementary secondary axis and the primary axis: $\alpha \geqslant \alpha'$.



To all the above patterns of Fig. 61 (A, B and C) further resistance may be added by means of oscillatory sine-wave motion.

Figure 63. Diverging-converging forms.

Only diverging-converging forms are included in this table.

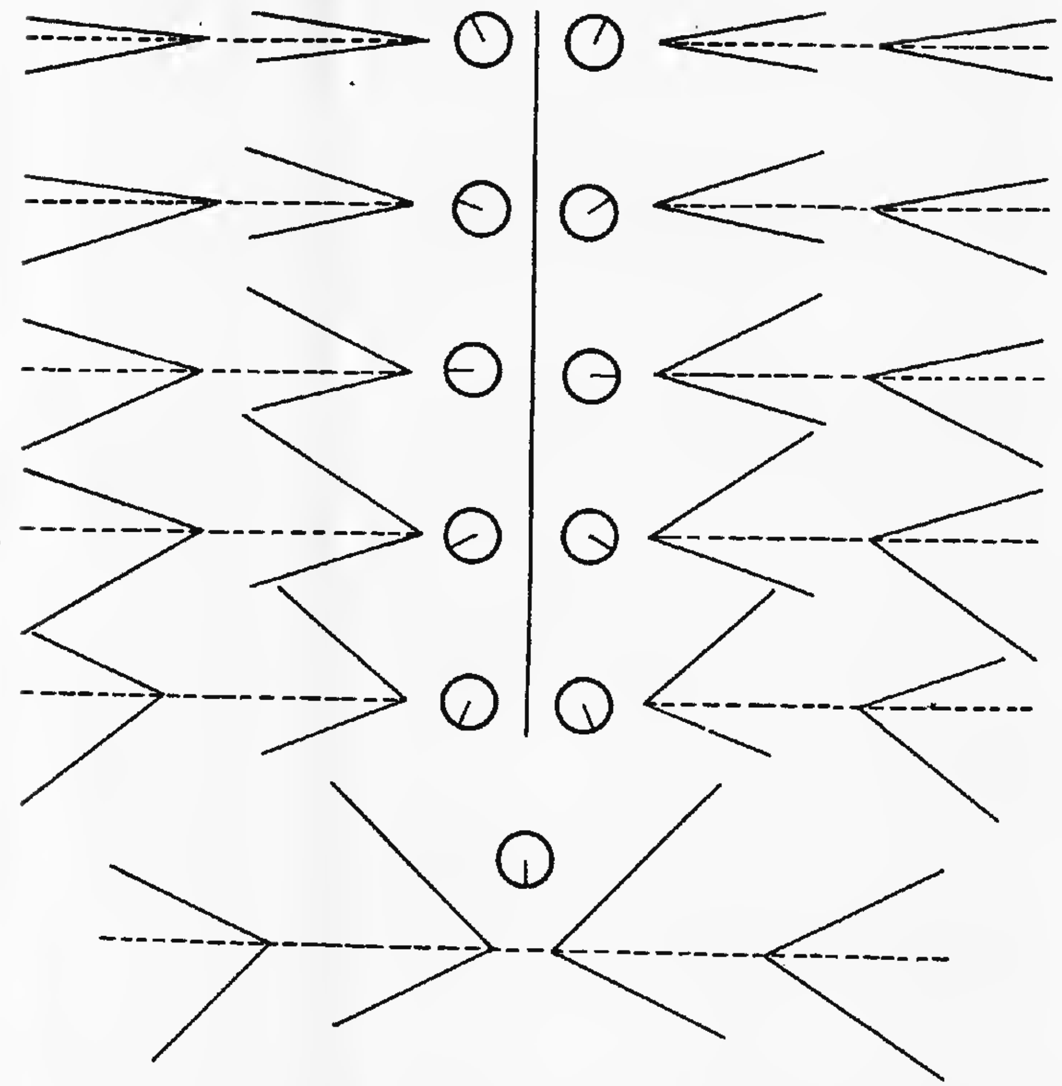


Figure 64. Ternary forms of the stimulus-response configurations.

To all the above patterns, further resistance may be added by means of oscillatory sine-wave motion.

Still more complex stimulus-response configurations can be included by means of a group of converging-diverging secondary axes which, in this case, produce a variety of angle values with the primary axis. Such configurations are of the radiating type. The angle value decreases with proximity to primary axis. This permits avoidance of overlapping when such configurations are transformed into harmonic strata. The outer strata must be expressed through S2p; the intermediate, through S3p; the closest to primary axis, through S4p.

All such complex diverging-converging patterns are characterized by extreme intensity and are applicable mostly to the last quadrants of the negative and the positive zone of the psychological dial.

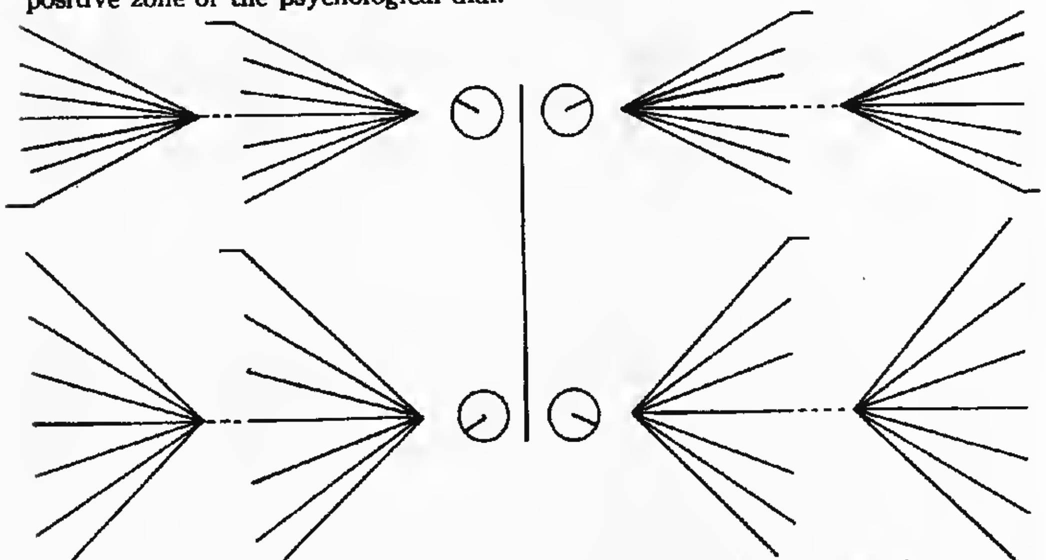


Figure 65. Some complex forms of stimulus-response configurations.

Further resistance may be added to all the above forms by means of oscillatory sine-wave motion. This form of resistance can be realized by means of combined (upper and lower:) directional units.

The degree of angular divergence-convergence can be modified by the respective selection of tonal cycles for Sp, S2p, S3p and S4p, as each cycle has a different divergence-convergence tendency. The horizontal segments define the position of a climax for each configuration.

F. Spatio-Temporal Associations

The responses we have dealt with thus far are of the inherited type. Other responses are inherited only as a tendency or inclination. These can be cultivated further. New uninherited associations can be conditioned and cultivated. These associations enter into the response-system by means of sense-organs. The latter may be activated simultaneously by the different stimuli entering the system, and also by self-stimuli already present in every sense-organ. The impulse-groups combine themselves in some fashion with those which are already present and which are integrated with self-stimulated groups.

The responses we have dealt with thus far are of the *eventual* (i.e., pertaining to event or process) type. We shall discuss now the semantics of responses of the *essential* (i.e., non-eventual) type. These are primarily associated with intensity and quality.

One group of such responses belongs to spatio-temporal associations. These responses associate the auditory with the visual in terms of dimension, distance, form, luminosity and visible texture. These essential characteristics of the visual are mutually convertible with their auditory counterparts; that is, if a certain visual *symbol* has its auditory *counter symbol*, both symbols become interchangeable.

Another group of essential responses pertains to extra-visual-auditory perception. Sensations like the olfactory (smell), the gustatory (taste) and the thermic (temperature) belong to this group. These sensations can also be cultivated to a high degree of discrimination and, eventually, become hereditary, like the olfactory discrimination of canines. However, at present human beings in most cases are equipped with greater discriminatory power over the visible and the audible. All other sensations are less developed and therefore less crystallized.

Responses which are less concentrated than in the case of the visual and the auditory have a lower, i.e., a weaker associative capacity. In such cases the response-patterns and the associations they stimulate are often not capable of crystallizing beyond the "unpleasant-pleasant" diad. Individually, this is sometimes true of auditory discrimination as such. There are still some listeners who are not capable of auditory pattern-discrimination beyond the two biologically basic forms of response, which are characteristic of the undifferentiated universal fore-sense, i.e., the unpleasant (or unsatisfactory) and the pleasant (or satisfactory).

We shall return now to an analysis of the spatio-temporal associations. These include: dimension, direction, density of structure, form and luminosity.

Texture, i.e., the molecular structure of matter, as it can be perceived, is partly in visual and partly in tactile perception. Dimension, which in perception is closely associated with distance, corresponds to the intensity (volume) of sound. The close (near) appears to be loud, and the remote (far) appears to be soft. Including the dimensional characteristics, we acquire the following parallelisms of association:

close — big — loud
remote — small — soft

These associations are basic because they correspond to the space-perception of sounds in which intensity decreases with distance.

The association of spatial directions with sound seems to be pretty well established so far as high and low, or up and down are concerned. There may be a number of sources and reasons for the development of such associations in our civilization. But it may be questionable that such associations are either basic or rigid. For example, our basic pitch-scale motion is associated with increasing frequencies, in spite of the fact that in vocal execution it is the direction of increasing effort. On the other hand, the same pitch-scale motion among the primitive and past civilizations represents just the reverse, i.e., it is associated with decreasing frequencies (the biological pattern of exhaling or sighing). At any rate, for our purposes the following associations are acceptable:

high (above the point of observation) corresponds to high frequency of sound;

low (below the point of observation) corresponds to low frequency of sound.

Variation between the two opposites corresponds to respective frequency variation:

ascent—increasing frequency;
descent—decreasing frequency.

There are no right-left associations with any component of sound. Immediate associations with the *direction of the source of sound* can be obtained through the positioning of such sources, through the positioning of loud-speakers, as was done, for example, in the first presentation of Walt Disney's *Fantasia* in New York. Under such conditions the source of sound can be projected from any direction in relation to the listener. This possibility, however, has nothing to do with the expression of direction by any configuration of the components of sound.

Density of structure corresponds to the density of musical texture, which includes tone-quality, and instrumental and harmonic density. There is a general correspondence between the dimensional quality, i.e., size, and density. Large spatial extensions correspond to large frequency-ranges; small spatial extensions correspond to small frequency ranges. As the density of matter corresponds to the density of musical texture, different degrees of the density of matter, which have the same dimensional range, can be expressed by corresponding variations of textural density.

For example, S3p, distributed through two octaves, would associate itself with matter of lower density than S6p distributed through the same range. Thus, a wide area of cumulus clouds can be associated with the middle-high and high register of relatively low density. On the other hand, the sinister dark rain clouds can be associated with the middle and middle-low register of a considerably higher density.

As we have seen before, some spatio-temporal associations are mutually convertible. One of such mutually convertible associations is the association of continuity and discontinuity of space with the continuity and discontinuity of time. This capacity permits us to associate continuous durations with continuous extensions, and discontinuous durations with discontinuous extensions. Thus we arrive at the following correspondences:

continuous extension — curvilinear spatial form — continuous durations (smooth attack followed by legato);

discontinuous extension — rectilinear spatial form (angular form) — discontinuous durations (accented attack followed by non-legato or portomento);

discontinuous configuration — configuration consisting of dissociated elements — abrupt durations (abrupt attack corresponding to staccato).

It follows from the above group of correspondences that the musical expression of smooth or round or spheric configurations such as sky, domes, rolling hills, lakes, cotton-like clouds, etc., must assume the form of *legato*; that buildings, bridges, elementary rectilinear geometrical patterns, partitioned interiors, squarely landscaped grounds or gardens, etc., must assume the form of *portamento* which corresponds to broad strokes; that stars, raindrops, snow-flakes, birds in flight, planes in group-formation and other patterns produced by dissociated elements must assume the form of *staccato*, whose attack form, i.e., whose durability and intensity, corresponds with the dimensions of the elements producing the respective configuration.

The *texture* of matter can be defined as its molecular structure. The perception of textures is partly visual, partly tactile. They appear to our senses as a group of gradations from smooth to rough. In a smooth texture the structural units are imperceptible. Such a texture associates itself with sound whose physical components of the partials are also imperceptible. It can be associated musically with a pure "tone". An exceptionally good tone-quality on such instruments as flute, french horn, clarinet, violin corresponds to the sensation of smooth. In a rough texture, on the contrary, the structural units are perceptible. Such a texture associates itself with sound where either a vibrato is present, or certain partials noticeably stand out (as in the double reed instruments), or a certain harshness of tone-quality is due to the presence of inharmonic elements (such as noises produced by the friction of the bow over the strings as in mediocre violin playing).

Smooth and rough, when associated with the pleasant and the unpleasant, may also be expressed by the degree of musical harshness, which is one or another form of tension, i.e., of dissonant quality. A melody coupled in octaves or other simple harmonic relations appears smooth; on the other hand, a melody coupled in dissonant or complex harmonic relations appears rough. In all cases, there is a scale of gradations between the two extremes.

The associations of *luminosity* (the intensity of light) have a basic correspondence with the frequency-intensity components of sound. The intensity of light, its brightness, generally corresponds to high frequencies, i.e., to high register or a timbre composed of high partials, which render the brightness of tone quality. Flutes, french horns (in their high register), chimes and harmonics as such belong to this group. Concentrated light associates itself with intense sound, and diffused light, with a moderate or low intensity combined with the same bright tone-quality.

Light of low luminosity, dimmed light, sombreness and darkness correspond generally to low frequencies, i.e., to low range and to sombre timbres composed of middle-range or low partials. All brass in the low register, all double-reed woodwind instruments in the middle-low or low register, all single-reed woodwind instruments in the low register and all stringed-bow instruments, either in the low register or muted (if the register is not high), belong to this group. Concentrated light of low intensity can be best expressed by instruments with saturated timbre, such as trombones and particularly tuba. Diffused light of low intensity or darkness can be best expressed by the low register of saturated string timbres, such as 'celli and particularly basses.

So far as *color* associations are concerned, such associations with musical pitch (or tonalities), as verified by serious investigation, belong entirely to the individual type of conditioning and, therefore, cannot be generalized. However, the inherent luminosity of the different spectral hues (for example, yellow is more luminous than red at the same intensity) generally associates itself with the respective high and low frequencies of sound (i.e., the higher the luminosity, the higher the sound-frequency). In other words, there is apparently an *intensity-frequency correspondence*. It is easy to understand such a correspondence, if we take into account the data of psychology, which show that the *intensity of a sensation* is the result of the *number of impulses*. As this holds true for any sensation, we can produce an intensification of response by increasing the number of identical impulses, i.e., by repetition. As we have seen before, in our applications of this process to melody, the repetition of an impulse produces resistance and intensifies the anxiety-response.

Saturation is another factor of intensity. A saturated tone-quality is the result of the addition of several components (partials in this case). As identical phases of many components add up, this addition increases the amplitude. For this reason a dense sound is at the same time a loud sound.

Intensity of *all sensations* parallels the amplitudinal intensity of sound, i.e., stronger responses associate themselves with the louder sound. The sensation of high pressure, for instance, associates itself with high intensity of sound. Of course, the reverse is also true. The reason for such correspondences is that pressure is in direct association with force. We respond to pressure as a sort of "passive force".

In tactile form pressure appears as a kinaesthetic sensation of the "opposition" type, which comes from the receptors in the muscles. The latter permits us to judge the relative hardness or softness of an object and associates itself with the corresponding forms of attack (hard: pesante, portamento; soft: non-legato, legato).

Thermic sensations have not yet crystallized into any rigid associations with sonic forms. A general tendency may be observed, however, to associate "warm" with saturated tone-qualities and middle or middle-low register and particularly with the tone of brass instruments; and "cold", with unsaturated tone-qualities and high register. "Vibrato" also produces the effect of "warm" just as the "non-vibrato", that of "cold". Of course, some of the thermic forms can be associated with sonic forms through association with other sensations, in which case the latter become pattern-stimulating impulses. For example, an impression of boiling may be associated, not with temperature, but with the kinetic characteristics of the process of boiling and the sound pattern it produces.

The non-cutaneous sensations (i.e., the sensations not originating in the skin, which may be considered the basically biological sensations, such as hunger, thirst, pain, sexual urge) associate themselves with sonic patterns through the parallelism of pleasantness-unpleasantness. Some extreme forms of non-cutaneous sensations become so intense that they stimulate resistance associations. Then their configurations fall into the general class of kinetic patterns (as expressed in our psychological dial), i.e., the patterns of motion and action, as they include the striving for a goal of relief and satisfaction.

The *kinetic* patterns are immediate and self-sufficient symbols, and have a universal significance as elements of language. Nevertheless, as was stated before, the intonational forms which the kinetic patterns assume, are of local significance. In this sense, for example, a certain state of melancholy, corresponding to a certain response-pattern, may assume the intonational form of Chinese or of Roumanian music, precisely in the same way as "I feel sad" can also be said in the Chinese or Roumanian language, stimulating the same association-pattern.

The non-kinetic, or rather "extra-kinetic" patterns are not self-sufficient and therefore, not as universal in their association stimulating intensity. As in the case of gestures, when words alone do not seem to be sufficient, the extra-kinetic patterns have their value only as an auxiliary stimulus parallel to some other form of stimulation which is more universal as a symbol.

Such patterns are a good supplement to a script or a program, and serve as intensifiers of the basic symbols. Sonic symbols of music usually supplement verbal symbols, and as such are universally used in the theatre, cinema, radio and television.

CHAPTER 18

COMPOSITION OF SONIC SYMBOLS

THE principles disclosed in the preceding chapter constitute an application of my General Theory of Configurational Semantics to music. Now we arrive at the practical application of this theory to the composition of sonic symbols.

The maximum success with which such an application may be met depends upon the optimum of response, which is a reactive pre-disposition, and geometrically corresponds to congruence, i.e., configurational identity (or at least to a close approximation to it). This congruence exists between the stimulus and the response configurations and, in turn, is conditioned by the congruity of the dial of stimuli (i.e., phasic stimuli) with the dial of responses (i.e., self-stimuli and reactive pre-disposition). Thus the response optimum is achieved when all points of the response-dial adequately correspond (i.e., geometrically coincide) with the respective points of the stimulus-dial.

Such a condition exists when the listener is in a state of balance (180° position on the dial) before he is subjected to musical stimulation.

For the individual whose normal state of balance is a state of depression (to any extent), the stimulus which would bring him to what we would generally consider normal, must be above normal, i.e., in the positive zone, at an angle which equals the individual's deviation from normal in the negative zone.

For an under-stimulated Mr. Hypochondriac whose normal is at 150°, that is, 30° below normal, the stimulus which would appear to him as normal and which would bring him to our balance at 180°, would have a pattern corresponding to 180° + 30°, i.e., of 210°. On the contrary, an over-stimulated Mr. Highstrung, whose normal is at 210°, would require 180° − 30°, i.e., 150° stimulus-pattern, in order to bring him to our balance. In other words, the corresponding dial-adjustment must be made for each individual case deviating from normal. That is, the stimulus-dial must be turned to the right or to the left (clockwise or counterclockwise), on the angle of deviation from normal, and in the *opposite* direction from the individual's point of balance.

Indicating the response-clock by R and the stimulus-clock by S, we can illustrate the two cases discussed above as follows.

First, we have the case of the under-stimulated individual and the stimulus-clock adjusted to produce the intended response of balance:

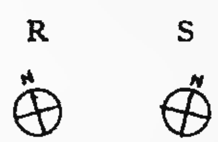


Figure 66. Stimulus-clock adjusted to understimulated individual.

Next, we have the case of an over-stimulated individual and the stimulus-clock adjusted to produce the intended response of balance:

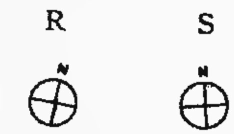


Figure 67. Stimulus-clock adjusted to overstimulated individual.

In both cases, N indicates the point of normal for the respective individual and the stimulus which would affect him as our normal.

This process of the stimulus-dial adjustment for each individual case of response which does not coincide with this dial, may be looked upon as psycho-physiological coordination, or synchronization, of the two dials.

Each component produces its corresponding configuration for each dial-point. However, it is not necessary to have *all* components of one sonic symbol in exact correspondence with one another. For example, a melodic trajectory corresponding to may have a pitch-scale corresponding to , and still produce the general character of . Naturally, an exact correspondence of several components of one sonic symbol intensifies the latter. But such an intensity of pattern is not always necessary.

A. NORMAL:

Associations: Balance, Repose, Quiescence, Passive Contemplation, Uniformity, Eventlessness, Inactivity, Monotony.

The stimulus-patterns of this group tend themselves toward uniformity, which must be expressed through all the participating components. As the point of absolute balance is imaginary rather than actual, most of the patterns of this group have a certain degree of oscillatory tendency and fluctuate to a certain degree about the balance point. The direction to the right (clockwise) from the balance point expresses the tendency of unbalancing and the direction to the left (counterclockwise) from the balance point expresses the tendency toward balancing. It is correct to think of the patterns of this group as trajectories of a pendulum or a magnetic needle.

Technical Resources:

(1) **Temporal Rhythm:** Durations ranging from very long to moderately long, depending on the degree of activity, in uniform or nearly uniform motion. Alternation of such durations with rests possessing similar characteristics. Uniform or nearly uniform attack-groups.

(2) **Pitch-Scales:** Scales with a limited number of pitch units and a fairly uniform distribution of intervals. In extreme forms of inactivity, one-unit scale.

(3) **Melodic Forms:** Only stationary and regularly oscillating forms, within a moderate pitch-range for associations with small dimensions, and a

wide range for associations with large dimensions. A good example of the latter is one-unit oscillation through three octaves in Verdi's scene on the Nile from *Aida*. The violins play *g* on all four strings, through an oscillating instrumental form of single attacks and uniform durations.

The typical trajectories are:

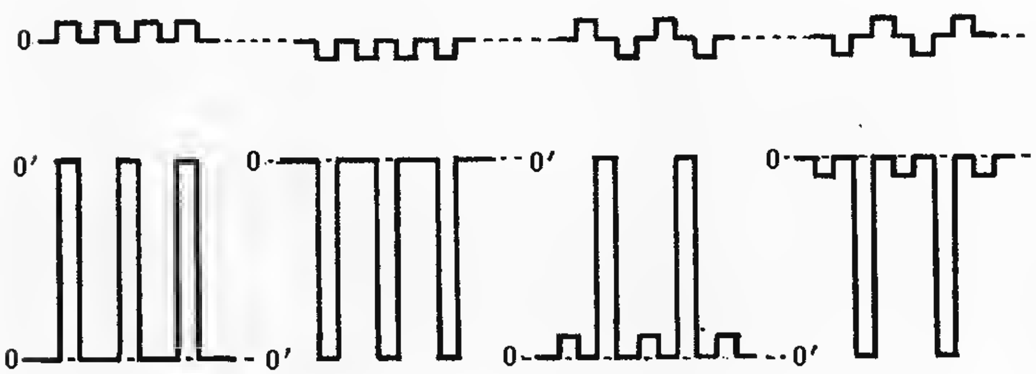


Figure 68. Trajectories of typical melodic forms for ⏀.

(4) **Harmonic Forms:** Either a complete absence of harmony or one H which remains constant. The instrumental form is either sustained (stationary) or slightly oscillating in uniform durations. The most suitable structures are tonal expansions of the participating pitch-scale. Only one harmonic stratum should be employed. If harmony is employed without melody, its structures must consist of fairly uniform and consonant intervals. The latter is necessary in order to secure tranquility.

(5) **Contrapuntal Forms:** None, as the presence of a group of trajectories suggests activity.

(6) **Instrumental Resources:**

(a) *density:* uniform density which is conditioned by the

(b) *range:* which depends on the dimensional associations;

(c) *dynamics:* uniform and either low or medium; no sporadic accents;

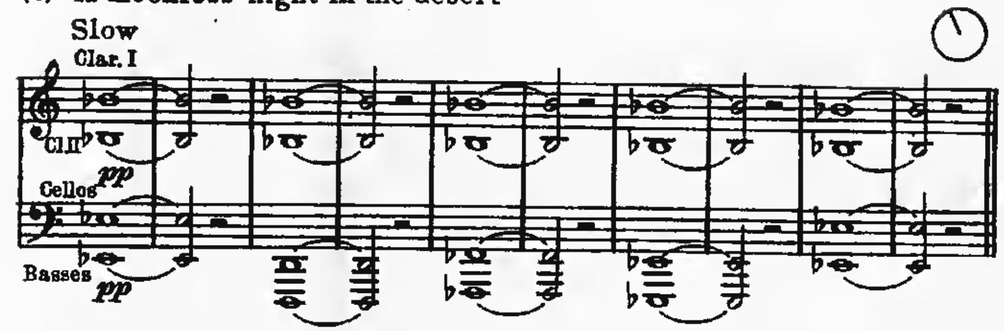
(d) *attacks:* smooth; legato, non-legato and light staccato (uniform and continuous) are appropriate;

(e) *tone quality:* open, i.e., approaching the sine-wave form as far as possible: Flute, Violin (non-vibrato), particularly its harmonics, high French Horn (pp), sub-tone Clarinet, Double-Bass on open strings and particularly harmonics;

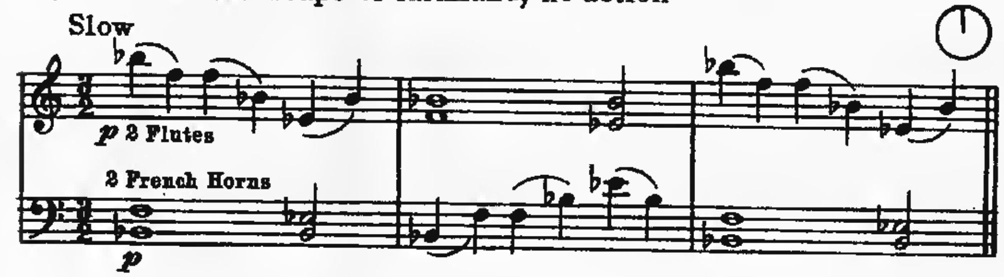
(f) *register:* depends on luminosity associations; night, darkness—low; sunrise, shining moon or stars—high; for neutral associations, like a peaceful landscape or quiet lake—middle register.

The following fragments have a recapitulating construction

(1) A moonless night in the desert



(2) Summer landscape of farmland; no action



(3) Starry sky over Grand Canyon

(4) Contemplation

Figure 69. Musical examples for normal clock stimulus

B. Upper Quadrant of the Negative Zone:

Associations: Dissatisfaction, Melancholy, Weakness, Sadness, Depression, Pain, Suffering, Despair.

The stimulus-patterns of this group tend themselves toward loss of energy and balancing. In their extreme and intense form, they assume anticlimactic configurations. The basic patterns of this group are *b* and *c* axes.* The degree of intensity of the stimulus-form corresponds to the amplitude of the respective configuration. The degree of dramatic tension corresponds to the respective form of resistance.

Technical Resources:

1 Temporal Rhythm:

Uniform and fairly uniform duration-groups, followed by one or two long durations; the weariness effect is achieved by frequent cadencing; moderate or slightly animated tempo. Many waltzes and mazurkas of Chopin will serve as suitable illustrations. Slow syncopation and upbeat-groups; also false syncopation produced by rests.

Configurations corresponding to the loss of momentum: (a) decreasing number of attacks in the successive groups; (b) increasing duration-values. The latter may correspond to either rhythmic (i.e., containing resistance) or progressive (i.e., direct) rallentando. Moderate tempo.

These characteristics, when they increase progressively, ultimately lead to an anticlimax:

This stage is the conclusive form of the preceding development. It signifies ultimate despair, exhaustion, loss of power and, finally, death. Use extremely long durations, often dissociated from one another by long rests, and obtained as a result of direct or indirect (delayed, i.e., rhythmic) rallentando. Slow tempo.

2 Pitch-Scales:

Uniform or fairly uniform intervals, arranged in such a way that the smaller intervals are below the larger ones. For example:

1 + 2 + 1 + 2 + 1 + 2 + 1 + 2, i.e.,
c − *d*♭ − *e*♭ − *f*♭ − *f*♯ − *g* − *a* − *b*♭ − *c*;
c − *e*♭ − *g* − *b*♭ − *d* − *f* − *a* − *c* − . . .; (3 + 4) + . . .,
c − *f* − *d*♭ − *g*♭ − *d*♮ − *g* − *e*♭ − *a*♭ − *e* . . . (5 + 8) + . . ., i.e.

Also use the above scales combined with descending directional units.

Further increase of contrast between the upper and the lower interval placed adjacently. For example:

(1 + 5) + . . ., i.e., *c* − *d*♭ − *f*♯ − *g* − *c*;
(3 + 8) + . . ., i.e., *c* − *e*♭ − *b* − *d* − *b*♭ − *c*♯ − *a* − . . .;
(1 + 3) + . . ., i.e., *c* − *d*♭ − e − *f* − g♯ − *a* − *c*.

*The b and c axes are balancing axes. The b axis is the descending direction toward the primary axis. The c axis is the ascending direction toward the primary axis. See Vol. I, p. 252 (Ed.)

Also several small intervals appearing in succession and followed by one large interval. For example:

(1 + 1 + 3) + . . ., i.e., *c* − *c*♯ − *d* − *f* − *f*♯ − *g* − *b*♭ − *b*♮ − *c* − *e*♭ − . . .; (3 + 3 + 5) + . . ., i.e., *c* − *e*♭ − *g*♭ − *b* − *d* − *f* − *b*♭ − *d*♭ − *f*♭ − *a*
(1 + 1 + 2 + 6) + . . ., i.e., c − c♯ − *d* − *e* − *b*♭ − *b*♮ − *c*♮ − *d* − *a*♭ − . . .

Such scales usually represent a combination of the scales referred to in and their crystallized descending directional units, in which case the latter become neutral units.

As the predominant configuration of this zone is one of decline, and is associated with descending tones, it is practical to think of scales belonging to this zone as being constructed downward (as in the primitive and archaic civilizations).

3 Melodic Forms:

Balancing axes (*b* and *c*). Balancing binary parallel axes $\left(\frac{b'}{b}, \frac{b}{b''}, \frac{c'}{c}, \frac{c}{c'}\right)$. Only weak forms of resistance.

Balancing axes with a strong form of resistance; often beginning with a climax and evolving into anticlimactic forms. Binary converging axes $\left(\frac{b}{c}\right)$. All forms have resistance. Ternary converging axial combinations (b ÷ 0 ÷ c and c ÷ 0 ÷ b). Longer time-period is necessary for more extreme forms.

4 Harmonic Forms:

(a) *Structures:* consisting of balanced or nearly balanced consonant intervals, with smaller intervals being placed below the larger ones (downward gravity effect). These structures are similar to or identical with the pitch-scales of this zone and can be used in any tonal expansion. Also balanced structures of the consonant type, with one lowered function (descending alteration), like the minor ninth in a diminished S(9). Casual descending directional units used in moderate quantities.

(b) *Progressions:* containing moderate downward motion. For example: Sp in C_{-7}; S2p in C_{-3}, C_3 (this pattern contains a certain amount of resistance); S3p in C_{-3}, C_5; S4p in C_3, C_5, C_7.

(a) *Structures:* of the lower gravity type, containing ***dissonant*** descending alterations (one or more). Such structures can be obtained by altering some of the functions in a balanced or nearly balanced structure. For example, a balanced structure 4 + 3 + 4, i.e., *c* − *e* − *g* − *b*, altered into 3 + 3 + 4, i.e., *c* − *e*♭ − *g*♭ − *b*, by lowering its gravity, i.e., by aggregating smaller intervals in the lower part of the structure. Likewise, 5 + 5 + 5, i.e., *c* − *f* − *b*♭ − *e*♭, can be altered into 4 + 6 + 5, i.e., *c* − *e* − *b*♭ − *d*♯, or into 4 + 5 + 6, i.e., *c* − *e* − *a* − *d*♯. Also strata consisting of structures possessing lower gravity. Needless to say, the intensity of the pattern grows with the addition of the respective characteristics.

(b) *Progressions:* containing extreme (i.e., rapidly progressing) downward motion, or delayed downward motion with resistance; the latter is caused by the motion-pattern of transformations, which is inherent in some structures. Examples of cycles and transformations: SpC_3, C_5; $S2pC_7$; $S3pC_7$ ↻, S4p all general transformations producing rapidly descending or delayed descending patterns. For extreme effects: aggregations of rapidly descending strata; converging strata.

(5) $\frac{M}{H}$ **Tension Forms** (i.e., forms pertaining to harmonization and melodization: functional relations of melody and harmony):

Descending directional units whose neutral units represent lower chordal functions. For example, in $S_2(5)$ such melodic steps as: a♭ → g, f → e♭, d → c, against *C*-chord; in $S_1(7\flat)$, i.e., large, such a melodic step as: d♭ → c, against *C*-chord.

Descending directional units whose neutral units represent higher chordal functions. For example, in $S_1(7\flat)$ such melodic steps are: e♭ → d♭, b♭♭ → a♭, against *C*-chord. In extreme cases, symmetric superimpositions of $\frac{M}{H}$, where both M and H have the lower gravity characteristics; also the same type, combined with descending directional units. For example, $\frac{M}{H} = \sqrt[12]{2}$:

$$\frac{S_{II}}{S_I} = \frac{d\flat - f\flat - a\flat}{CS_2(5)}\,;\quad \frac{M}{H} = \sqrt[12]{2}:\ \frac{S_{II}}{S_I} = \frac{d\flat - g\flat - c\flat}{b\flat}\quad S: \frac{\overline{f}}{c}$$

6. Contrapuntal Forms:

Oblique balancing forms: $\frac{b}{0}$ and $\frac{c}{0}$; binary parallel forms leading to balance: $\frac{b}{b}$ and $\frac{c}{c}$. The same in slightly converging angles:

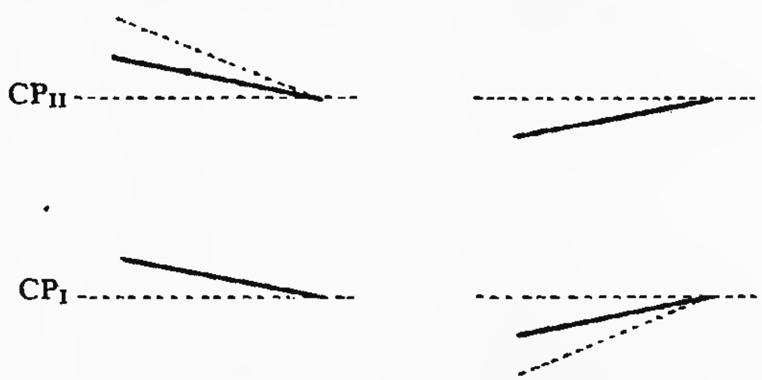


Figure 70. Oblique balancing axes and binary parallel axes.

Identical balancing axes in a more extreme convergence:

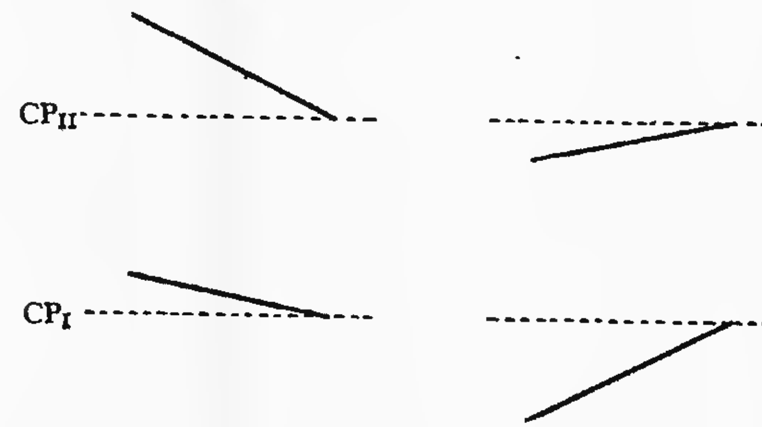


Figure 71. Identical balancing axes.

Simple versus complex axial groups of the same (balancing) direction:

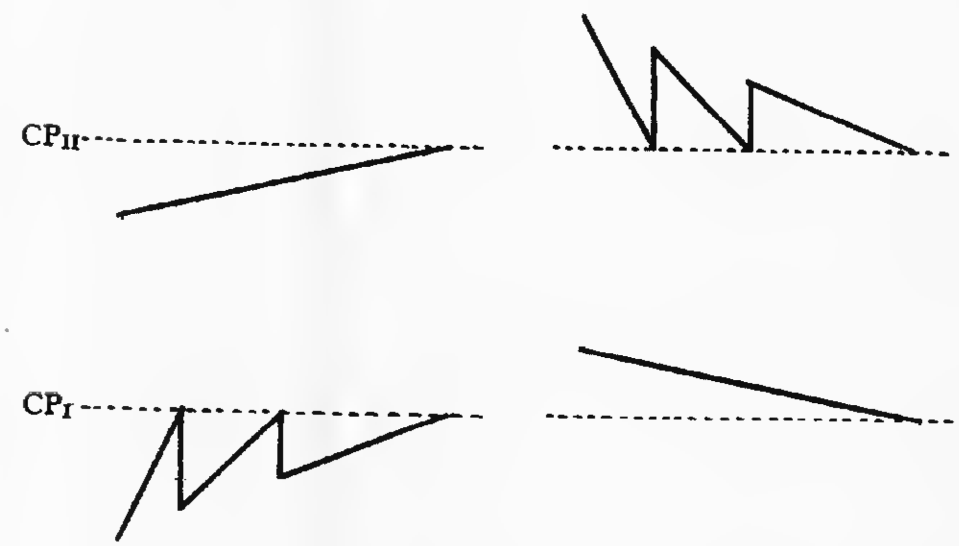


Figure 72. Simple versus complex balancing axial groups.

Non-identical converging axes, often containing resistance: $\frac{b}{c}$ and b ÷ 0 ÷ c

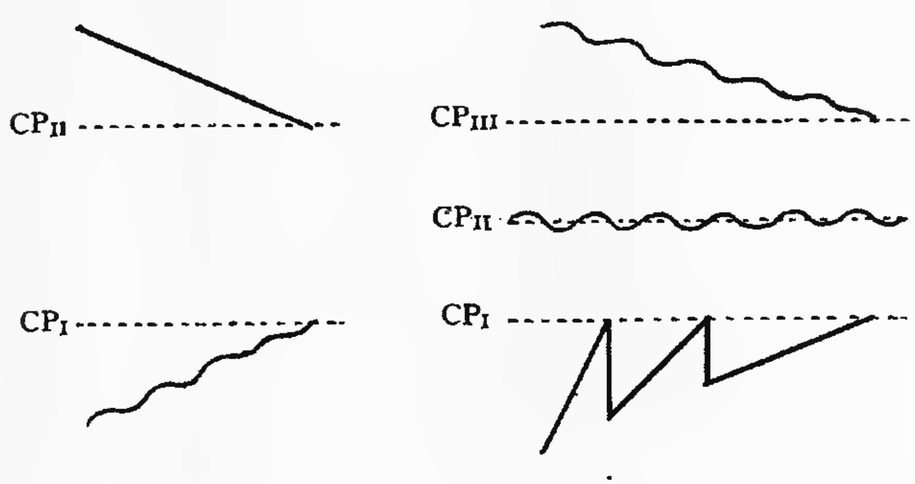


Figure 73. Non-identical converging axes.

For more extreme cases, convergence of many parts.

7. Instrumental Resources:

(a) *density:* low; medium; high; in extreme cases, variable density of the following forms:

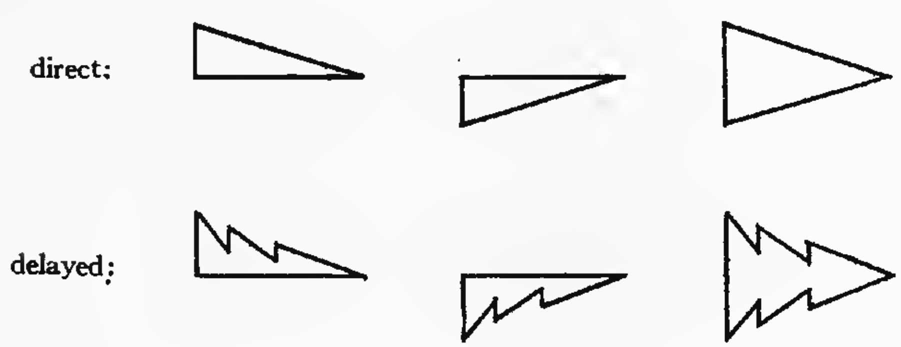


Figure 74. Variable densities.

(b) *range:* partly depends on the dimensional associations; more generally is associated with the intensity of the stimulus-pattern:

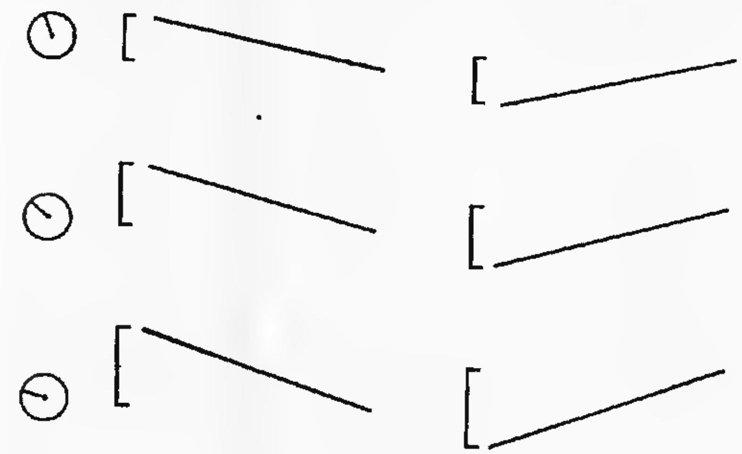


Figure 75. Stimulus-patterns.

(c) *dynamics:* either low (p, pp) or decreasing; the intensity of the stimulus-pattern is associated with the period of its diminuendo and with its dynamic range:

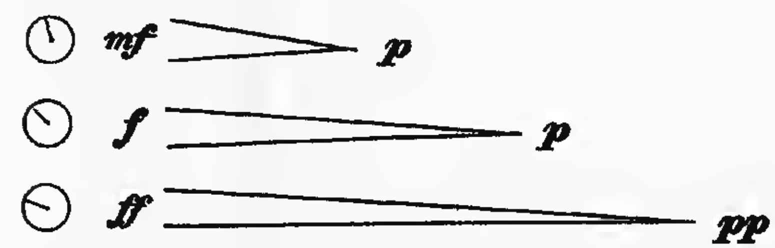


Figure 76. Dynamic ranges.

also groups of *sfp* with a gradual decline: *sfmf* + *sfp* + *sfpp*; the initial dynamic energy derives from the preceding climax;

(d) *attacks:*

short legato groups starting with an accent; short staccato groups starting with an accent; mixed short legato-staccato groups starting with or without an accent: minimal scale of attacks;

alternate legato and portamento groups in which portamento follows legato, particularly when combined with rallentando; groups of an average length;

successive groups of legato, followed by portamento, followed by staccato, particularly when combined with rallentando: maximal scale of attacks; groups of considerable length;

(e) *tone-quality:*

the single-reed quality: clarinet, violin (vibrato), French horn in its middle register, a mellow trombone at low intensity and in its high register (as in Tommy Dorsey's performance of "I'm Getting Sentimental Over You");

the double-reed (nasal) quality in the middle or the high register: violin on the G-string, viola in general, 'cello (high or middle register), high oboe and high bassoon, muted trumpet;

the muted quality: all stringed-bow instruments muted, low oboe, English horn, low bassoon, low trombone (also muted, the entire range) low and middle register of the bass clarinet, low French horn (also stopped), tuba, gong;

(f) *register:* the intensity of the stimulus-pattern in relation to range:

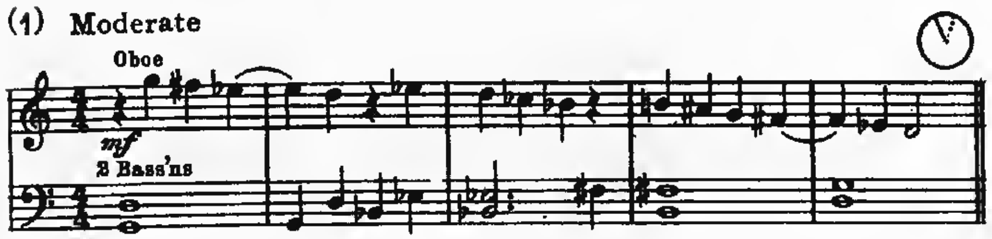
middle or middle-low;

middle and middle-low;

middle, middle-low and low;
the basic characteristics of the stimulus-pattern in relation to register:

middle; middle-low; low.

(1) Moderate



(2) Slow

Figure 77. Musical illustrations for upper quadrant of negative zone (*continued*)

(3) Moderate

(4) Moderate

Figure 77. Musical illustrations for upper quadrant of negative zone (*concluded*)

C. Upper Quadrant of the Positive Zone:

Associations: Satisfaction, Well-being, Strength, Accomplishment, Happiness, Joy, Gaiety, Challenge, Aggression, Conquest, Success, Triumph, Exuberance, Elation, Exaltation, Jubilation, Ecstasy.

Technical Resources:

1 Temporal Rhythm:

Uniform or fairly uniform duration-groups; groups of longer durations followed by groups of shorter durations: binomials with stress on the first term ($2+1$; $3+1$; $5+3$; . . .). Duration-groups characteristic of regimental marches and folk-dances. Down-beat patterns and down-beat accentuation. Only the simplest forms of syncopation, such as $1+2+1$, or $1+2+2+1$, or $1+2+2+2+1$. Fairly animated tempo.

Configurations corresponding to the gain of momentum: (a) increasing number of attacks in the successive groups; (b) decreasing duration-values. The latter may correspond to either rhythmic (i.e., containing resistance) or progressive (i.e., direct) accelerando. Animated tempo.

These characteristics, when they increase progressively, ultimately lead to a climax:

This stage is the conclusive form of the preceding development. It signifies a climax, i.e., the state of ultimate joy, exuberance, jubilation, and finally, ecstasy. It is characterized by an energetic overabundance resulting in groups which consist of many attacks and minimal durations. The total effect is vibrant and scintillating. The approach to short durations (often in the form of a rapid arpeggio, or tremolo, or frulato*) is accomplished by direct or indirect accelerando. A climax cannot be sustained for any appreciable length of time as the response automatically goes into decline (defense-reflex of the sense-organs and of the entire response-system is the probable cause of it; refer to the Weber-Fechner psycho-physiological law). Fast tempo.

2 Pitch-Scales:

Uniform or fairly uniform intervals arranged in such a way that smaller intervals are above the larger ones. For example: (4 + 3) + . . ., i.e., *c* – *e* – – *g* – *b* – *d* – *f*♯ – *a* – *c*♯ – e – *g*♯ – *b* – . . .; (5 + 4) + . . . i.e., *c* – – *f* – *a* – *d* – *f*♯ – *b* – *d*♯ – *g*♯ – *c*♯ – . . .; (8 + 7) + . . ., i.e., *c* – *a*♭ – – *e*♭ – *c*♭ – *g*♭ – *d* – *a* – . . . These scales may be combined with ascending directional units.

Further increase of contrast between adjacent upper and lower intervals. For example: (3 + 1) + . . ., i.e., *c* – *d*♯ – *e* – *f*× – *g*♯ – *b* – *c*; (4 + 1) + . . ., i.e., *c* – *e* – *f* – *a* – *b*♭ – *d* – *e*♭ – *g* – *a*♭ – *c* – *d*♭ – . . .; (8 + 3) + . . ., i.e., *c* – *g*♯ – *b* – *g*♮ – *b*♭ – *g*♭ – *a* – *f* – *a*♭ – . . .

Also several small intervals appearing in succession and following one large interval. For example: (3 + 1 + 1) + . . ., i.e., *c* – *d*♯ – *e* – *f* – *g*♯ – *a* – – *b*♭ – c♯ – *d* – *e*♭ – . . .; (5 + 3 + 3) + . . ., i.e., *c* – *f* – *g*♯ – *b* – *e* – – *g* – *a*♯ – *d*♯ – *f*♯ – *g*× – . . .; (6 + 2 + 1 + 1) + . . ., i.e., *c* – *f*♯ – – *g*♯ – *a* – *a*♯ – *e* – *f*♯ – *g* – *g*♯ – *d* – *e* – *e*♯ – *f*♯ – . . .

Such scales usually represent a combination of the scales referred to in and their crystallized ascending directional units, in which case the latter become neutral units.

Since the predominant configuration of this zone is one of growth and is associated with ascension, it is practical to think of scales belonging to this zone as being constructed upward (in terms of the conception of civilized musical contemporaries).

*See p. 1458.

3 Melodic Forms:

Unbalancing axes (*a* and *d*). Unbalancing binary parallel axes $\left(\frac{\mathrm{a}'}{\mathrm{a}}, \frac{\mathrm{a}}{\mathrm{a}''}, \frac{\mathrm{d}'}{\mathrm{d}}, \frac{\mathrm{d}}{\mathrm{d}'}\right)$. Weak forms of resistance.

Unbalancing axes exhibiting a strong form of resistance and leading to a climax. Binary diverging axes $\left(\frac{\mathrm{a}}{\mathrm{d}}\right)$. All forms have resistance. Ternary diverging axial combinations (a ÷ 0 ÷ d and d ÷ 0 ÷ a). A longer time-period is necessary for more extreme forms. In extreme cases, a development of several successive climaxes.

4 Harmonic Forms:

(a) *Structures:* consisting of balanced or nearly balanced consonant intervals, with smaller intervals being placed above the larger ones (upward gravity effect). These structures are similar to or identical with the pitch-scales of this zone and can be used in any tonal expansion. Also balanced structures of the consonant type, with one raised function (ascending alteration), like the augmented fifth in an augmented S(7♮). Casual ascending directional units used in moderate quantities.

(b) *Progressions:* containing moderate upward motion. For example: Sp in C_7; S2p in C_3, C_{-5} (this pattern contains a certain amount of resistance); S3p in C_3, C_{-5}; S4p in C_{-3}, C_{-5}, C_{-7}.

(a) *Structures:* of the upper gravity type, containing *dissonant* ascending alterations (one or more). Such structures can be obtained by altering some of the functions in a balanced or nearly balanced structure. For example, a balanced structure 4 + 3 + 4, i.e., *c* – *e* – *g* – *b*, altered into 4 + 4 + 3, i.e., *c* – *e* – *g*♯ – *b*, by raising its gravity, i.e., by aggregating larger intervals in the lower part of the structure. Likewise, 5 + 5 + 5, i.e., *c* – *f* – – *b*♭ – *e*♭, can be altered into 6 + 5 + 4, i.e., *c* – *f*♯ – *b*♮ – *d*♯. Also strata consisting of structures possessing upper gravity. The intensity of the pattern grows with the addition of these characteristics.

(b) *Progressions:* containing extreme (i.e., rapidly progressing) upward motion, or delayed upward motion with resistance; the latter is caused by the motion-pattern of transformations, which is inherent in some structures. Examples of cycles and transformations: SpC$_{-3}$, C$_5$; 2SpC$_{-7}$; 3SpC$_{-7}$; S4p all general transformations producing rapidly ascending or delayed ascending patterns. For extreme effects: aggregation of rapidly ascending strata; diverging strata.

(5) $\frac{M}{H}$ **Tension Forms** (functional relations of melody and harmony):

Ascending directional units whose neutral units represent lower chordal functions. For example, in $S_1(5)$ such melodic steps as: $b \rightarrow c$, $d\sharp \rightarrow e$; $f\sharp \rightarrow g$ against *C*-chord; in $S_2(7\natural)$, i.e., the first augmented, such melodic steps as: $f^{\times} \rightarrow g\sharp$, $a\sharp \rightarrow b$, against *C*-chord.

Ascending directional units, whose neutral units represent higher chordal functions. For example, in $S_2(7\natural)$ such melodic steps are: $c^{\times} \rightarrow d\sharp$, $g^{\times} \rightarrow a\sharp$, against *C*-chord. In extreme cases, symmetric superimposition of $\frac{M}{H}$ where both M and H have the upper gravity characteristics; also the same type, combined with ascending directional units. For example,

$$\frac{M}{H} = \sqrt[4]{2}: \quad \frac{S_{II}}{S_I} = \frac{d\sharp - f^{\times} - a^{\times}}{CS_3(5)};$$

$$\frac{M}{H} = \sqrt[3]{2^2}: \quad \frac{S_{II}}{S_I} = \frac{g\sharp - b\sharp - d\sharp}{CS_1(5)}; \qquad \frac{M}{H} = \sqrt{2}: \quad \frac{S_{II}}{S_I} = \frac{f\sharp - b - e}{b\flat} \quad S: \frac{f}{c}$$

6 Contrapuntal Forms:

Oblique unbalancing forms: $\frac{a}{0}$ and $\frac{d}{0}$; binary parallel forms leading away from balance: $\frac{a}{a}$ and $\frac{d}{d}$. The same in slightly diverging angles:

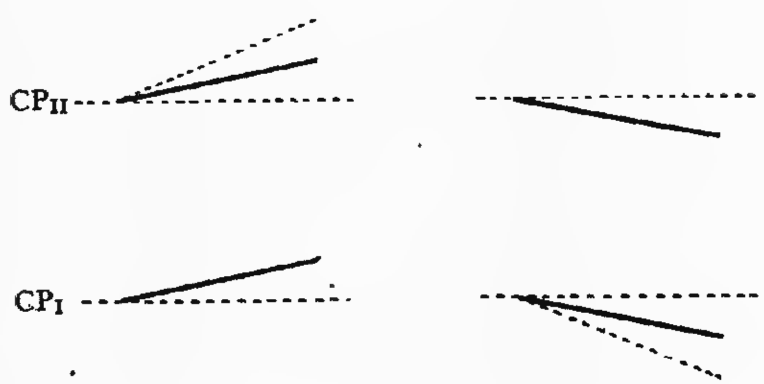


Figure 78. Oblique unbalancing axes and binary parallel forms leading away from balance.

Identical unbalancing axes in a more extreme divergence:

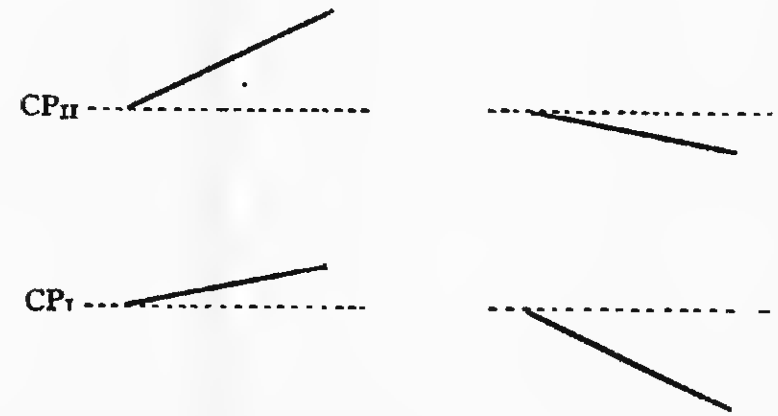


Figure 79. Identical unbalancing axes.

Simple versus complex axial groups of the same (unbalancing) direction:

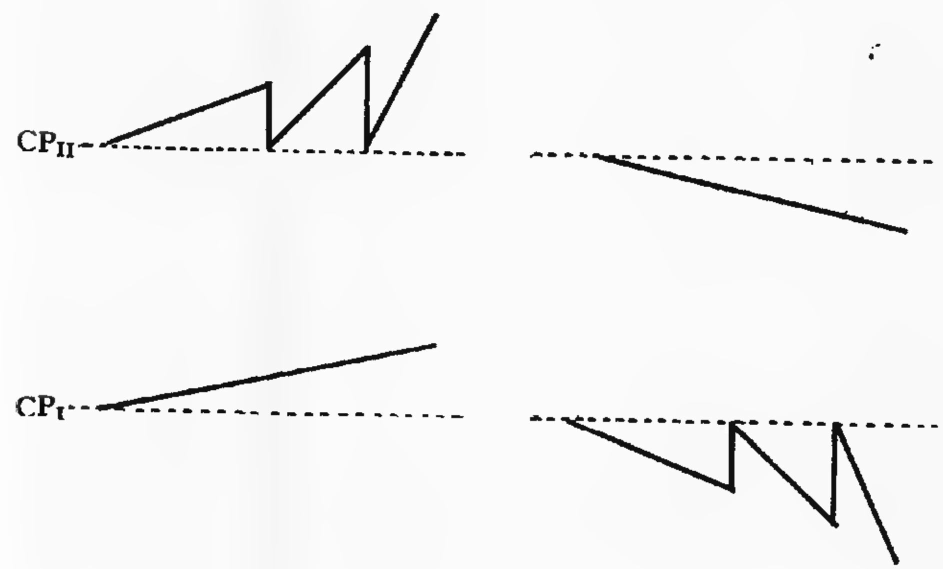


Figure 80. Simple versus complex unbalancing axial groups.

Non-identical diverging axes, often containing resistance: $\frac{a}{d}$ and a ÷ 0 ÷ d.

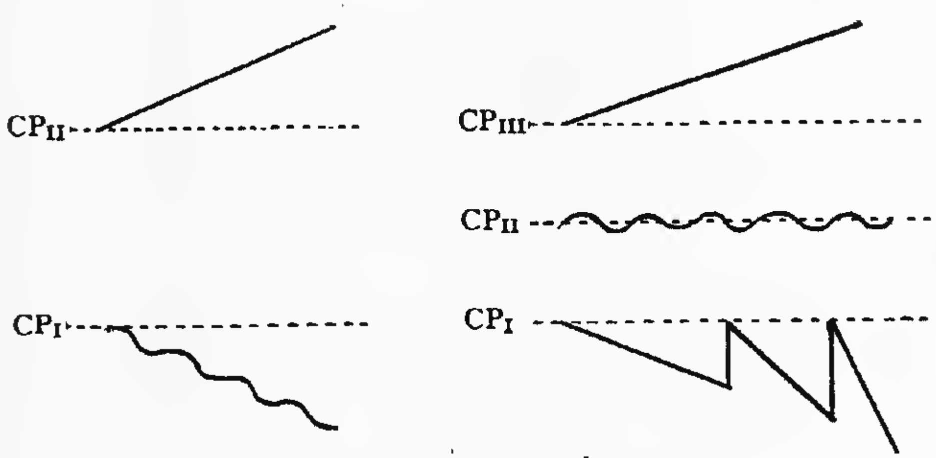


Figure 81. Non-identical diverging axes.

For more extreme cases, divergence of many parts.

7 Instrumental Resources:

(a) *density:* low; medium; high; in extreme cases, variable density of the following forms:

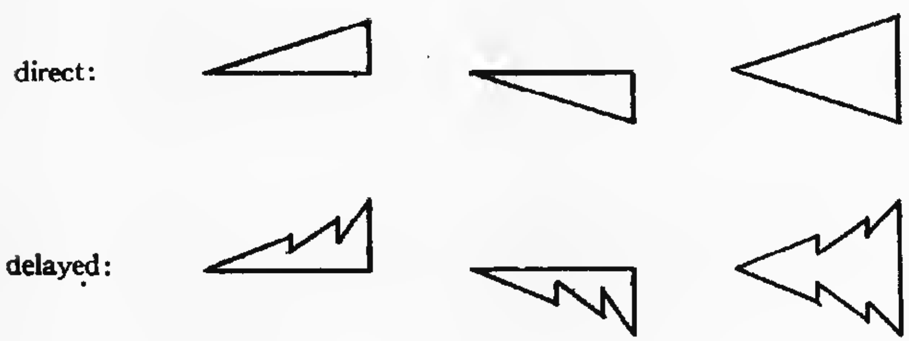


Figure 82. Variable densities.

(b) *range:* partly depends on the dimensional associations; generally is associated with the intensity of the stimulus-pattern:

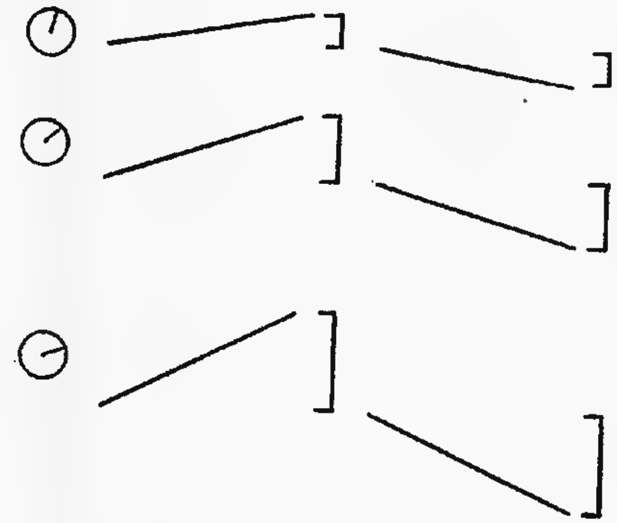


Figure 83. Stimulus patterns.

(c) *dynamics:* either high (*f*, *ff*) or increasing; the intensity of the stimulus-pattern is associated with the period of its crescendo and with its dynamic range:

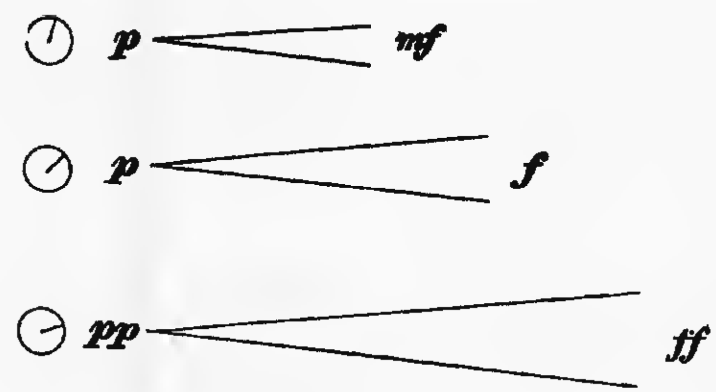


Figure 84. Dynamic ranges.

Also groups with rapid crescendo in a gradual growth: pp*f* + p*f* + m*f*-*f*; mp <m*f* + p <*f* + pp <*ff*; the dynamic energy grows through resistance.

(d) *attacks*

short legato groups ending with an accent; short staccato groups ending with an accent; mixed legato-staccato groups (often two-attack groups of legato-staccato: ; also upbeat-downbeat two-attack groups: etc.); generally, minimal scale of attacks;

alternate portamento and legato groups in which legato follows portamento, particularly when combined with accelerando;

successive groups of staccato, followed by portamento, followed by legato (which often falls on the climax point) and combined either with accelerando (momentum gain) or with rallentando (suspension of a discharge, immediately preceding the climax); the portamento forms often become marcato or pesante in this case; maximal scale of attacks; groups of a considerable length;

(e) *tone-quality:*

open and single-reed quality: flute, clarinet, violin, French horn; piano, harp, celeste, chimes, high-pitched drums, castagnets, wood-blocks, orchestra bells, tamburin;

brilliant and open brass quality: mixtures of high stringed-bow and wopdwind instruments; open trumpets and trombones; high register of 'celli for "passionate" effects; cymbals (*mf*) and kettle-drums;

scintillating quality: tremolo, trills and rapid arpeggio forms on stringed-bow and woodwind instruments; extreme high register of trumpets and trombones; xylophone (also with abundant glissando and multiple attacks); frulato, trills and multiple tongue of flutes; multiple tongue (also sustained, for the climax) on trumpets; chimes and cymbals *ff*; kettle-drums, tremolo; brilliant qualities obtained by superimposition of harmonics;

(f) *register:* the intensity of the stimulus-pattern in relation to range:

middle or middle-high;

middle and middle-high;

middle, middle-high and high;
The basic characteristics of the stimulus-pattern in relation to register:

middle; middle-high; high.

Figure 85. Musical illustrations for upper-quadrant of positive zone. *(continued).*

Figure 85 Musical illustrations for upper-quadrant of positive zone (concluded).

D. The Lower Quadrants of Both Zones:

1. Negative Zone

Both lower quadrants represent an exaggerated version of the respective patterns of each of the upper quadrants. As the negative (left) zone corresponds to decline, its patterns are that of decomposition. When such decomposition exceeds its natural maximum, the pattern begins to appear discontinuous and the formation of an image, greatly retarded. The normally unobservable details become apparent and begin to obstruct perception of the pattern as a whole. Such an effect is comparable to the extremely magnified optical images seen through a microscope. As we can observe only an insignificant part of an image, which part is greatly magnified, we cannot reconstruct the image as a whole.

For example, a very small portion of a man's arm, appearing as skin surface with some hair growing on it, under magnification may look like a fantastic jungle forest. It would be difficult to stretch the observer's imagination so far as to reconstruct the image of the entire arm, as the dimensional scale is too large. A similar situation exists with regard to the so-called "slow motion" of cinematic projection, in which an image, photographed at 128 frames per second, is cast on the screen at 24 frames per second. In temporal phenomena this extreme magnification of time-period obscures the image, or the process, as a whole, bringing out too many details, and dissociating the observable links of the image, or the process. With an increase in the number of images in recording (taking), the projected image becomes more and more stationary. Imagine a pugilist delivering a blow to his opponent at the rate of 2 minutes per blow. Such a rate, according to standards with which it would be associated, would appear subnatural. Thus it would be perceived as either fantastic or humorous.

As it follows from the above illustrations, in the temporal projections of an image or a process, the rate of mechanical speed is the basic source of extending a time-period. For this reason, sound images, recorded for performance at a certain rate of speed and played back at a considerably lower rate, are bound to produce an effect analogous to cinematic "slow motion". In both cases (i.e., optical and acoustical) of projection, the perceived image appears to be psychologically (i.e., as an associative group) more discontinuous, but physically approaches continuity, even in observation, as more intermediate points or events become noticeable.

As a consequence of this, phonograph records, made to be played at 78 R.P.M. and performed at 33.3 (i.e., taking mechanical speeds which are standard for the phonograph turntable at the present time) appear to be subnatural in effect. Depending on the association with the anticipated stimulus-pattern, such a performance activates the response of fantastic or humorous. The obvious character of "mechanical inefficiency" taking place during the process of formation of an image, makes such an image appear humorous. Further disintegration of perceptible image, caused by a still lower rate of projection, makes such an image appear fantastic.

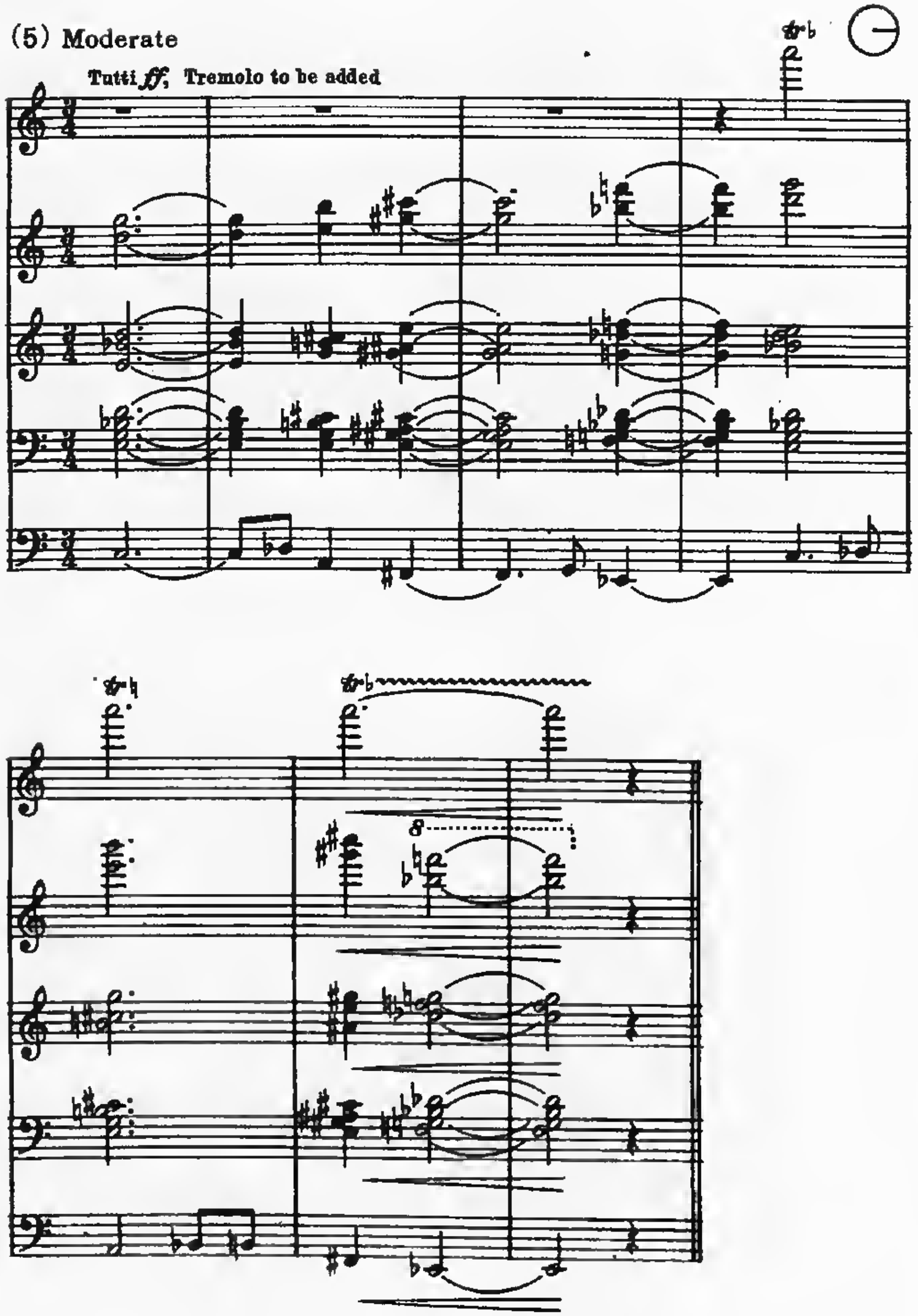


Figure 85 Musical illustrations for upper-quadrant of positive zone (concluded).

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Different rates of speed for a phonograph record using the standard turntable speeds, can be obtained through duplication, triplication and even quadruplication of the original from one speed to another, i.e., from 78 R.P.M. to 33.3 R.P.M., or vice versa. Beyond this, further variation of speed becomes impractical, as the sound frequencies extend beyond the range of audibility, or beyond physical continuity.

Many ordinary recordings of fairly animated music, when recorded at 78 R.P.M. and played back at 33.3 are, for any practical purpose, humorous. However, the presence of certain physical characteristics in the musical performance, when exaggerated by "slow motion" of the acoustical (i.e., mechanical) performance, sometimes increases the effect of humor. For example, "The Mad Scene" from *Lucia di Lamermoor* by Donizetti, as performed by Amelita Galli-Curci (a Victor red-seal record), appears incredibly hilarious when played at 33.3. The special reasons in this case are a lack of rhythmic unison (synchronization) with the accompaniment (normally associated with beginners and not with accomplished artists), and the other, a considerable deviation from the required intonation (also associated with beginners who are not able to exert sufficient control over their vocal apparatus), which is also a form of mechanical inefficiency pertaining to the control of frequency.

Nevertheless, the basic effect of humor in this case is mainly due to the straining of the anticipation-fulfillment chain conditioned by the pre-conceived image of an accomplished coloratura, from whom a high degree of alertness and mechanical efficiency are expected.

In another case, the effect of humor arises from: an exaggerated form of vibrato, appearing at a low rate of speed, and a quick fading of sound (the decrease of amplitude) following each attack and combined with the aforementioned vibrato. Such an effect can be observed in Bing Crosby's performance of *My Honey's Lovin' Arms* (accompanied by Mills Brothers on a Decca record), when the record is played at 33.3 R.P.M.. A secondary association contributing to the effect of humor is the anticipation-fulfillment chain, as, under such conditions of performance, Mr. Crosby's voice acquires the characteristics of piano or Hawaiian guitar (i.e., strong attack, quick fading, exaggerated vibrato, which characteristics are non-vocal, but generally typical of jazz.)

Music of slow pace and of middle-low or low register, triplicated from 33.3 R.P.M. performance to 78 R.P.M. recording and played back at 33.3, gives approximately 1/5 of the original speed. Under such conditions, this type of music appears to be so extended in time as to produce an extreme effect of the subnatural, i.e., fantastic. Something like the beginning of the Overture to *Tannhäuser* by Wagner is apt to produce this effect at 1/5 of its normal speed.

Among my own numerous experiments in this particular field, a record of singing canaries, played at 1/13 of its normal speed (quadruplication of the original from 33.3 R.P.M. performance to 78 R.P.M. recording), is as fantastic and unimaginable as any effect of music can be.

2. Positive Zone

We shall return now to an analysis of the lower quadrant of the positive zone. As the positive zone (right) corresponds to growth, its patterns are that of composition. When such a composition exceeds its natural maximum, the pattern appears not only continuous but extremely precipitated. At extreme velocities, the whole is more observable than the details. A comparable effect may be observed in the case of an extremely reduced optical image (seen through the reverse side of a binocular or a telescope for instance) or in the appearance of extremely remote images, however big in size (the moon, the planets, remote details of a landscape, etc.). In cinematic projection, such a situation occurs when a film exposed at 8 frames per second (as in the early days of the cinema) is projected at 16 or 24 frames per second. Today the old films (or similar use of "accelerated motion" for special effects) infallibly produce a humorous effect when projected at 24 frames per second, and the photographed motion itself appears to be fantastic.

The period of a given movement becomes so short that the observer can only see its initial and its final phases, and misses all the intermediate ones. As such speed is inconceivable for human beings, automobiles, trains and even airplanes, the "accelerated motion" appears supernatural. For this reason, it is perceived as either fantastic or humorous. Animated cartoons use a great deal of this technique of over-efficiency, as it means, besides the intended effect, economy in the number of individual drawings required to represent individual phases of given movements.

Thus, varying the rate of speed of the temporal projection of an image is, in this case, the basic device for contracting a time-period. For this reason, sound images recorded for a performance at a certain rate of speed and played back at a considerably higher rate, are bound to produce an effect analogous to the cinematic "accelerated motion". Phonograph records made to be played at 33.3 R.P.M. and performed at 78 R.P.M. appear to be supernatural in the effect of mechanical over-efficiency, and activate responses of the fantastic or humorous. In case of music, these effects are associated with the performing skill of individual artists. For example, no pianist can move his fingers at a speed which is several times greater than the known speed of a virtuoso pianistic performance, yet music recorded at 33.3 R.P.M. and played back at 78 R.P.M. gives all details of musical images, and including all the individual attacks, with full clarity. This translation of speed produces a miracle of technical accomplishment for even an unaccomplished performer. A smile is the usual form of response to such a speed translation: the performance, as it appears to the listener, is too good to be true.

Other forms of acceleration achieved by triplication and quadruplication from 78 R.P.M. to 33.3 R.P.M., each duplicated version to be played at 78 R.P.M., become incredibly fantastic. Besides exceeding any imaginable mechanical efficiency, such versions change pitch and tone-quality to a considerable extent. Any male speech in the first duplication becomes that of the Disney character "Donald Duck; any female speech, that of "Minnie Mouse". Male singers, particularly in choirs, produce a hilarious impression which cannot be verbally described. Female singing in its triplicated version (approximately a quintuple speed when played back at 78 R.P.M.) approaches very closely the singing of birds. However, my experiment in slowing down chirping of canaries produces a grotesque effect of howling wolves rather than female singers. This merely shows that our discrimination of tone-qualities of very high frequencies is quite poor, because physically such forms should be reversible.

The late tenor Enrico Caruso sounds at half speed like a cow (particularly when the consonant "m" is combined with an open vowel). I did not have an opportunity to convert a cow into Caruso by reversing the process. In my quadruplicated version of the Overture to *Tannhäuser* (approximately 13 times the speed of the original), the entire composition runs one minute. The incredibly fantastic character of this version is due to three factors: first, the unbelievable mechanical efficiency of performance *per se*; second, reversal of the anticipated dignified character of this composition in its original form (all versions were made from my own recording of Arturo Toscanini conducting the National Broadcasting Symphony Orchestra); third, the physical image of frequencies, which episodically vanish beyond the audible range.

A study of music written by the recognized experts of the humorous, such as Modeste Moussorgsky, and of the responses of listeners to such music, show that the problem of creating humorous music has not been solved. Music combined with words (i.e., vocal music) in some cases stimulates the response of humor not by virtue of the music, but by virtue of the words which activate humorous associations. This is easily proved by playing such music on some instrument (or instruments) to somebody who has never heard it before and is not familiar with the accompanying text. Likewise instrumental music which is programmatically humorous, does not generally appear as such to a listener unaware of the program.

On the other hand, in many public performances we have witnessed audiences laugh, and laugh very heartily, at music which was not intended to be humorous. Such is the case on occasion of the first performances of new and very original, i.e., unconventional, compositions. Just about ten years ago, at a chamber concert sponsored in Town Hall by the League of Composers of New York, a Chamber Suite by Anton von Webern (for 14 instruments) was performed by a group of very skilful musicians under the direction of Eugene Goossens. Such concerts are generally attended by an audience which can appreciate and often enjoy extreme contemporary creations. Yet in the case of Mr. von Webern's Suite, the audience rolled in laughter as if it were extremely humorous.

On the basis of the theory, which I have advanced, it is easy to explain why a certain composition which is intended to be humorous does not appear

as such at all, and why a composition, as serious as possible, may make people laugh. The explanation is very simple: ***music is humorous when it gives the impression of extremely low or extremely high efficiency.*** In von Webern's case, both these forms were present. On the one hand, the durations were very long or staccato, followed by very long rests; on the other, there were so few attacks to each movement of the Suite that each movement lasted only a few seconds, during which very few things happened; finally, the range was extremely wide, while the frequencies followed the course of extremely sudden changes from one end of the whole range (low pitches of Bassoon) to the other (high pitches of the Flute and Piccolo). The reaction to this piece as being humorous, of course, was the result of previous conditioning. From a philosophical standpoint, there is nothing inherently humorous in a vacuum. And this piece was a vacuum, since very few material sound-particles, or sound-images, appeared in a very broad range. But then the whole astronomical universe, which is a greater vacuum than we can produce artificially in any laboratory, must appear to be still more hilarious. Yet there is a reason why this does not happen. While the vacuumatic quality of the von Webern's Suite is immediately apprehensible auditorily, the vacuumatic quality of the universe is not immediately apprehensible visually. Besides we are not conditioned by any previous experience to a less vacuumatic universe.

What composers of supposedly humorous music have missed is that effects of the humorous and of the fantastic are primarily *agogic* (i.e., pertaining to speed) and cannot be expressed by purely intonational devices, such as melody, harmony, counterpoint or even tone-quality, unless such tone-quality is an imitation of sounds associated with the humorous (like the wow-wow trumpet effect in jazz music, or Rubinoff's laughing violin), or is a product or a result of an agogical process. By the latter, I mean tone-quality which appears to be humorous, owing to the excessive speed of projection, as in the case of a bass-clarinet performed at double speed.

It is true that a melody which is overloaded with resistances and does not move to a considerable climax, as well as a melody having extreme climaxes not adequately prepared by resistances (or, better, by any resistance at all) does appear humorous—but only to a slight degree and only to a highly discriminating audience.

The real sources of stimuli activating spontaneous responses of the humorous or the fantastic are, as we have seen, purely agogic. As our frequency-response (we mean the regular auditory response to sound-frequencies) is at the same time an intensity-response, the loudness of perceptible sound becomes an important component of the lower half of the psychological dial.

The right approach in composing sonic symbols, which are intended to stimulate reactions of the fantastic and the humorous, is to reproduce characteristics associated with extreme forms of acoustical projection. It is for this reason that we discussed the subject of recording and reproducing speed.

3. Technical Resources:

Still considering the mechanical extremes of acoustical projection to be the best means for this purpose, we offer, nevertheless, a parallel table of the common technical resources which, in the absence of technical facilities, will serve as the next best choice.

Low register. Extremely low speed. Low intensity. Suitable instruments: Double-Bass; Contrafagot; Tuba; low register of the Harp or Piano; low register of the electronic instruments, or of the pipe-organ.	High register. Extremely high speed. High intensity. Suitable instruments: Flute Piccolo; high register of the Clarinet Piccolo; high register of the electronic instruments, or of the pipe-organ.
The lowest audible register. Still lower speed and longer durations. An extremely slow vibrato, artificially obtained either by producing slow beats in the low register or by very slow semitone trills. Very low intensity. Suitable instruments: the 32′ pedal of the pipe-organ or its electronic equivalent; the lowest register of Double-Bass, Contrafagot and Tuba. Percussion: gong. Rests, when inaudibility is to be represented. This may affect only the lower parts of musical texture, such as harmony.	The highest audible register. Still higher speed and faster durations. Abundance of staccato and accents. An excessive and extremely fast vibrato, artificially obtained by trills or frulato (flutter-tongue); in some cases by beats caused by minor seconds in high register. Very high intensity. Suitable instruments: highest pipe-organ registers or their electronic equivalents; the highest register of Flute, Piccolo, Harp and Violin (for vanishing sounds approaching the high limit of audibility). Percussion: triangle, clavis (Cuban), etc. Partial inaudibility effect can be achieved by eliminating the upper parts of musical texture, leaving the bass as the only audible part.
0° Music almost stops altogether. One pitch unit is formed in the form of a trill, which is extremely slow and alternately stops and moves.	360° Intonations changing with ultimate velocity (such as scalewise grace-note groups on Flute, Piccolo): glissando of the highest Violin positions; also glissando of the highest positions on the space-controlled Theremin, or its equivalent; rapid passages in the highest ranges of the pipe-organ or electronic organ.

Figure 86. Table of resources for producing humorous and fantastic effects.

All other sonic symbols pertaining to the lower quadrants of both zones are produced on the basis of *association by contrast*. Such contrast can be achieved only through another stimulus pattern, executed in a different medium from sound. The most immediate forms of the basic stimuli are optical images and verbal symbols to which sonic symbols are composed as counterstimuli.

The basic associations by contrast, with respect to the lower quadrants of both zones, pertain to mechanical efficiency, power, dimension and density. The selection of a counterstimulus to a given stimulus must be performed on the basis of dial-reciprocity. For example, if the symbol can be located on the psychological dial as , the countersymbol must be , i e., it must have the same angle as the symbol only in the opposite quadrant. This proposition controls both quadrants in their entirety until they reach the 0° and 360° point.

The basic symbols appear as images on the stage or the screen (cinema, television), or as ideas stimulating imaginary optical forms, as in the play, the poem, the narrated story received through a broadcast.

The humorous effect results from the anticipation conditioned by an actual or imaginary situation, and the conflict created by fulfillment. For example, the symbol of a giant conditions anticipation of a powerful, low voice, for the dimensions of a giant, by previous conditioning, suggest large vocal cords. Therefore, in order to create an effect of humorous, it is necessary to compose a counterstimulus of the opposite character, i.e., a high frequency and low intensity sonic symbol. This sonic countersymbol would create conflict between anticipation and fulfillment, i.e., between the optical symbol and the inversely corresponding sonic countersymbol.

As the quadrant positions are reversible for symbol and countersymbol, we can put a mouse in place of a giant, and supply the tiny creature with a powerful basso, in which case the resulting effect will still be humorous.

In both these illustrations, the associations were based on *dimension-frequency and dimension-intensity inverse correspondence*. As a result, the density association is also affected, since a powerful basso is more saturated sound (by implication of its physical characteristics) than a weak high-pitched sound. We could also introduce an association by contrast on the basis of mechanical efficiency by adding agogical characteristic to both our illustrations: the giant has a weak high-pitched voice and, besides, speaks very slowly or stutters; the mouse, on the other hand, in addition to having a powerful low-pitched voice, speaks at such a high speed that it makes his speech almost incomprehensible.

More remote forms of association producing an effect of more subtle humor, require a considerably greater degree of refinement in the responses of an audience. For example, the counter symbol for a giant raging in furious violence, is quiet and pleasing lyrical music; or in a scene suggesting serenity, peace, silence and contemplation, the accompanying sonic countersymbol is crude, harsh and noisy music. But such countersymbols are effective only in the case of an audience equipped with highly developed associational responses.

Not only frequency and intensity can serve as a medium for creating humorous effects through association by contrast, but also the forms of attack in their inverse correspondence to movements and actions. For example, angular and abrupt movements can be accompanied by sonic symbols of extreme fluidity (legatissimo). Such an approach can be successfully applied to the staging of a humorous dance. The opposite, i.e., fluent movements accompanied by music with abrupt (staccatissimo) attacks, would produce an equally humorous effect.

Associations by contrast can be applied with the same amount of success to effects of the supernatural. For example, a poor and simple herdsman gets a horn or a pipe as a present from a stranger. The pipe looks like a very ordinary one, but in actuality it is enchanted. When the poor man begins to play, it sounds like a large and glorious orchestra with harps, human voices and an organ. Thus the conflict between anticipation and fulfillment is created on the basis of inverse correspondence between the primitive crudeness of the pipe and the sonic countersymbol of rich and glorious music .

All other forms of associations by contrast, which do not pertain to the lower quadrants, will be discussed in the following exposition.

Figure 87. Varying the tempi.

The above must sound one octave higher and may be accompanied by a high pizzicato of strings, with a fill-in by the glissando of xylophone, also high.

CHAPTER 19

COMPOSITION OF SEMANTIC CONTINUITY

SONIC symbols, acting as associational stimuli, may assume numerous forms of simultaneous and sequent coordination. In many instances, contrasting and even conflicting patterns may become simultaneously or sequently adjacent. Under no circumstances should this deprive the continuity of its stylistic unity. The conflicting character of patterns does not imply conflicting systems of intonation and temporal organization of durations. Just as tranquility and excitement may be expressed in a poem written in one language, unity of the forms of musical expression is a necessary esthetic condition. In cases where the very nature of association requires hybrid forms, such hybrid forms must be unified by some one component. For example, in a rapid transition of reminiscences associated with different countries and nationalities, it may be desirable to express the different corresponding intonational forms through melody; yet a sequence of such melodies, bearing no resemblance to each other, i.e., based on the pitch-scales belonging to totally different families, may be stylistically unified by a certain form of harmonization applied to the entire continuity. As we have learned before, symmetric harmonization provides such a unifying technical resource.

The form of semantic continuity may be either *uninterrupted* or *interrupted.* The first takes place in a program composition, such as an opera or a symphonic poem or background music written for the stage, screen, radio or television production; the second is characteristic of fragmentary and often isolated sonic symbols serving as musical cues in the same types of production.

As the temporal organization of the plot of a play or a script is in the hands partly of the playwright and partly of the director, there is very little that the composer can do in this particular direction. In most cases the composer is called to do his job when it is too late, as the temporal organization of a plot is in the hands of people who know too little, if anything at all, about such matters. On the basis of principles evolved and disclosed in my major work, *Mathematical Basis of the Arts,** it is possible to evolve the temporal structure of a plot and to coordinate it with the temporal structure of music into one organic whole by a purely scientific method. As such a luxury is not to be found in contemporary production-units yet, the composer can only try to do his best under the circumstances. For this reason we shall not discuss the technique of the temporal coordination of plot-music at present.

For the composers who intend to write music to their own program, we would like to offer a few basic suggestions.

Select a plot. Distribute the plot over a group of events (episodes). Analyze the sequence of episodes on the basis of our semantics (i.e., establish the relationship of episodes to balance, tension and release, anticipation and fulfillment, climaxes, etc.). Classify the episodes according to their importance. Give the

*To be published shortly.

episodes of primary importance the longest time-periods. Give the secondary and tertiary episodes shorter time-periods. Organize the entire temporal scheme according to such a selection. Write a continuity of sonic symbols to satisfy the temporal scheme of the plot.

Our main problem lies in the field of techniques pertaining to modulation and coordination of sonic symbols, by which the production of semantic continuity is accomplished.

A. Modulation of Sonic Symbols

The character of transition may be either sudden or gradual, and its technical forms either temporal, intonational or configurational.

Sudden transition introduces adjacent contrasts, characterized by the lack of commonness. Technically, such a transition is the negation of graduality. Gradual transition represents the transformation of one sonic symbol into another. The degree of graduality depends on timing. Modulation of one symbol into another can be accomplished through any technical component (i.e., temporal, intonational, configurational), by means of a common or a neutral form, i.e., such a form which is common or neutral, with respect to pre-modulatory and post-modulatory character of the respective sonic symbol.

1 Temporal Modulation

Transition of one stimulus-pattern to another often requires a change from one temporal pattern to another in the respective sonic symbol. Sudden transition implies only negative requirements: the absence of common characteristics. Gradual transition necessitates either neutralization of the preceding duration-group by introducing *uniformity* of t for the modulatory period, or by introducing a recurrence of the last *duration-pattern* of the pre-modulatory temporal group if such a pattern can be accepted as common for both (i.e., pre-modulatory and post-modulatory) groups. The commonness of a duration-pattern does not necessarily mean the commonness of T″.

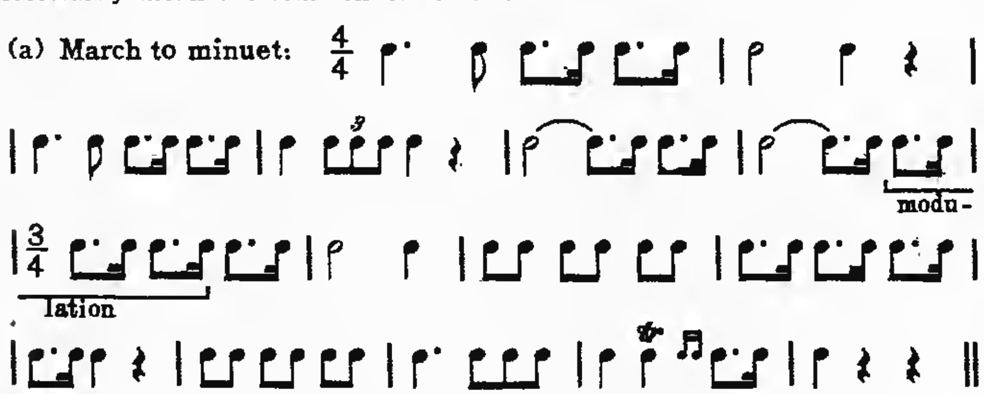


Figure 88. Temporal modulation (continued).

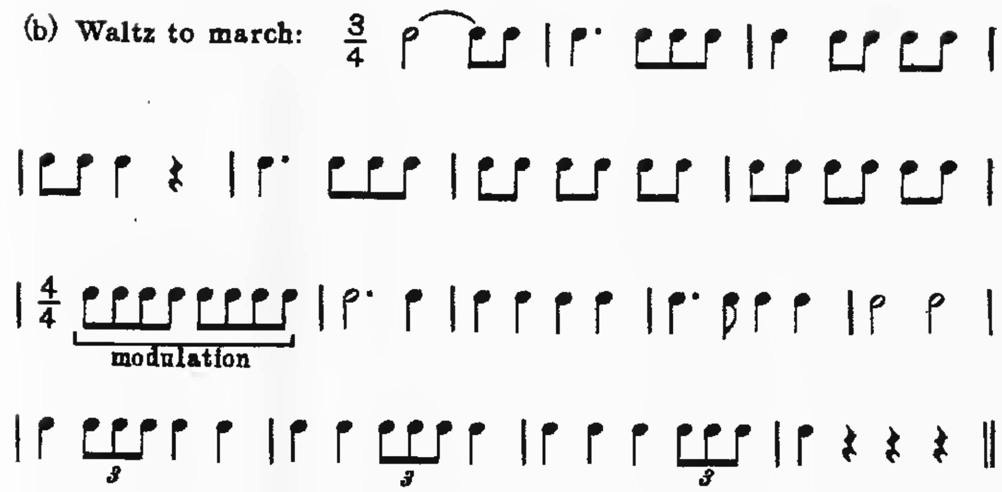


Figure 88. Temporal modulation (concluded).

2 Intonational Modulation

All problems pertaining to intonational modulation received full attention in the respective chapters of various branches of this theory.

Modulatory forms of melody are accomplished either by modal transposition, or by permutation of intervals, or by one of the modulatory techniques proper (common tones, chromatic alterations, identical motifs). Modulatory forms of correlated melodies (counterpoint) are obtained either by harmonic or by contrapuntal technique of modulation (sequent modulatory coordination of melodies). Modulatory forms of harmony consist of the following techniques: computative technique (see: Application of the Generalized Symmetric Progressions to Modulation)* applied to the distribution of the interval-group between the pre-modulatory and post-modulatory harmonic groups; direct modulation by altering the units of a modulatory chord, as described in the chapter on modulation in the *Special Theory of Harmony*;** indirect modulation (i.e., a modulation containing intermediate keys), as described in the same chapter.

Since in the modulations from one sonic symbol to another, axis-modulations are less essential, in most cases, than the modulations of the structural pattern of intonation (i.e., the modification of a chord-structure achieved by the re-distribution of intervals), the latter are accomplished mostly by means of a C_0. Direct transitions are based on the uncommonness of pitch-units in the adjacent assemblages. For this reason, C_7 and C_{-7} is one of the most suitable resources and particularly in the symmetric forms ($\sqrt[6]{2}$ and $\sqrt[12]{2}$). Instantaneous change from positions ⓐ and ⓑ to ⓒ and ⓓ is another excellent device for a sudden transition.

3 Configurational Modulation

Sudden change from one pattern to another does not require any technical considerations, as each pattern is a definite associative stimulus, and we are conditioned to produce instantaneous changes in our responses when such changes take place in the stimulus.

Gradual modulations from one configuration to another are based on two fundamental techniques: (1) neutralization of a pattern and (2) introduction of a common pattern.

The *first technique* consists of gradually depriving the pre-modulatory pattern of its individual characteristics, such as the axial combination and the trajectory. This technique is based on the assumption that the neutral pattern is that of balance, i.e., of uniform periodic motion, associated with the 0-axis and the sine-wave. Thus, the growing dominance of the 0-axis constitutes a neutralization of any other form of stimulus. In this sense 0-axis is a neutralizer of all characteristics but repose, and for this reason is expedient as an "inter-eventual" link. The most gradual forms of neutralization are those in which the effect of the 0-axis is such that it influences the decrease of amplitude in the pre-modulatory axis (or axial combination), whatever such combination might be. In this case the pre-modulatory pattern (mostly its last axis) repeats itself with decreasing amplitudes (a fading effect).

*See Vol. I, p. 492. **See Vol. I, p. 524.

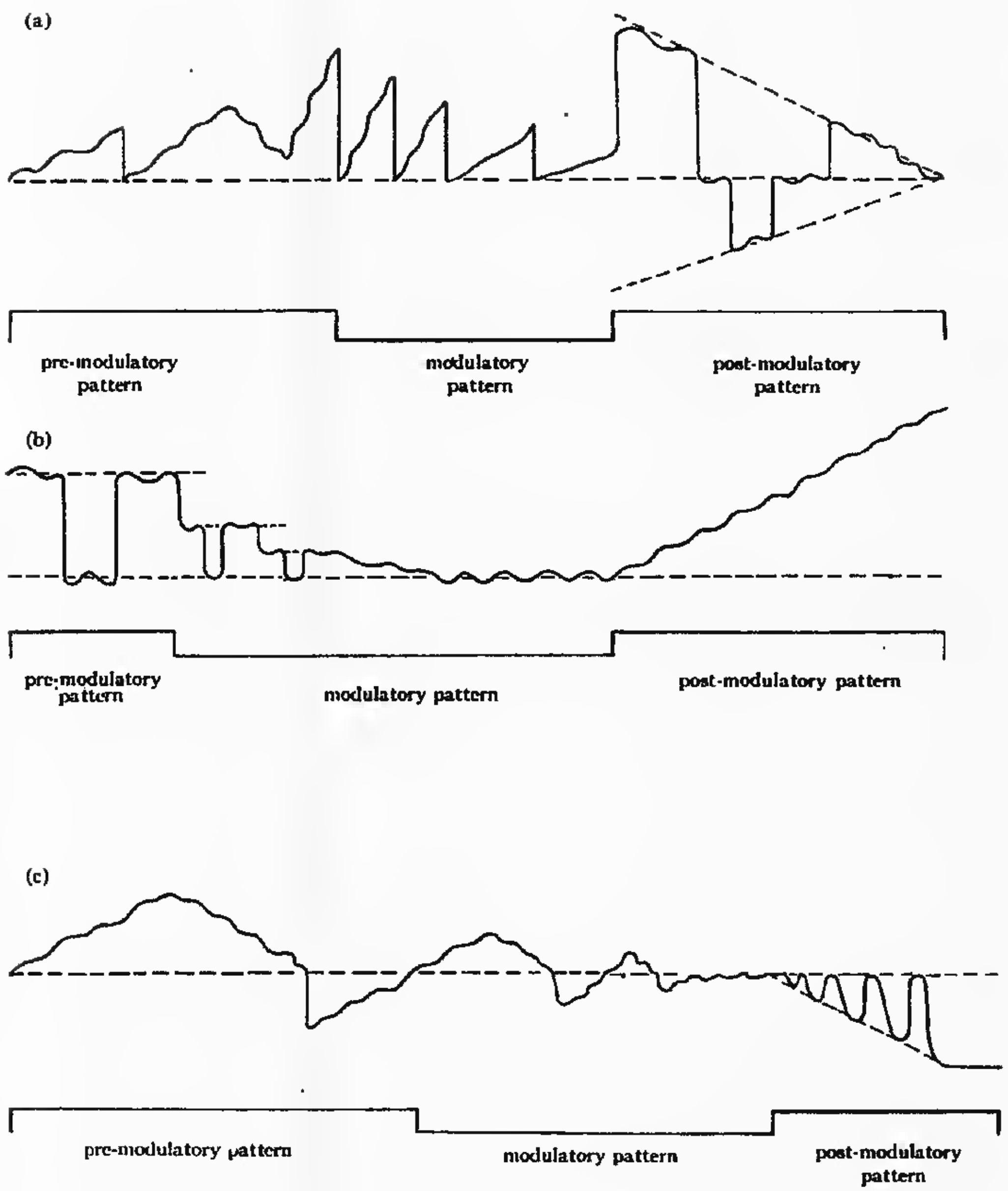


Figure 89. Neutralizing a pattern.

The *second technique* is based on recurrence of the pattern which is identical for both, the ending of the pre-modulatory and the beginning of the post-modulatory group. Such a common pattern is an axis (or axial combination) and a trajectory.

Examples

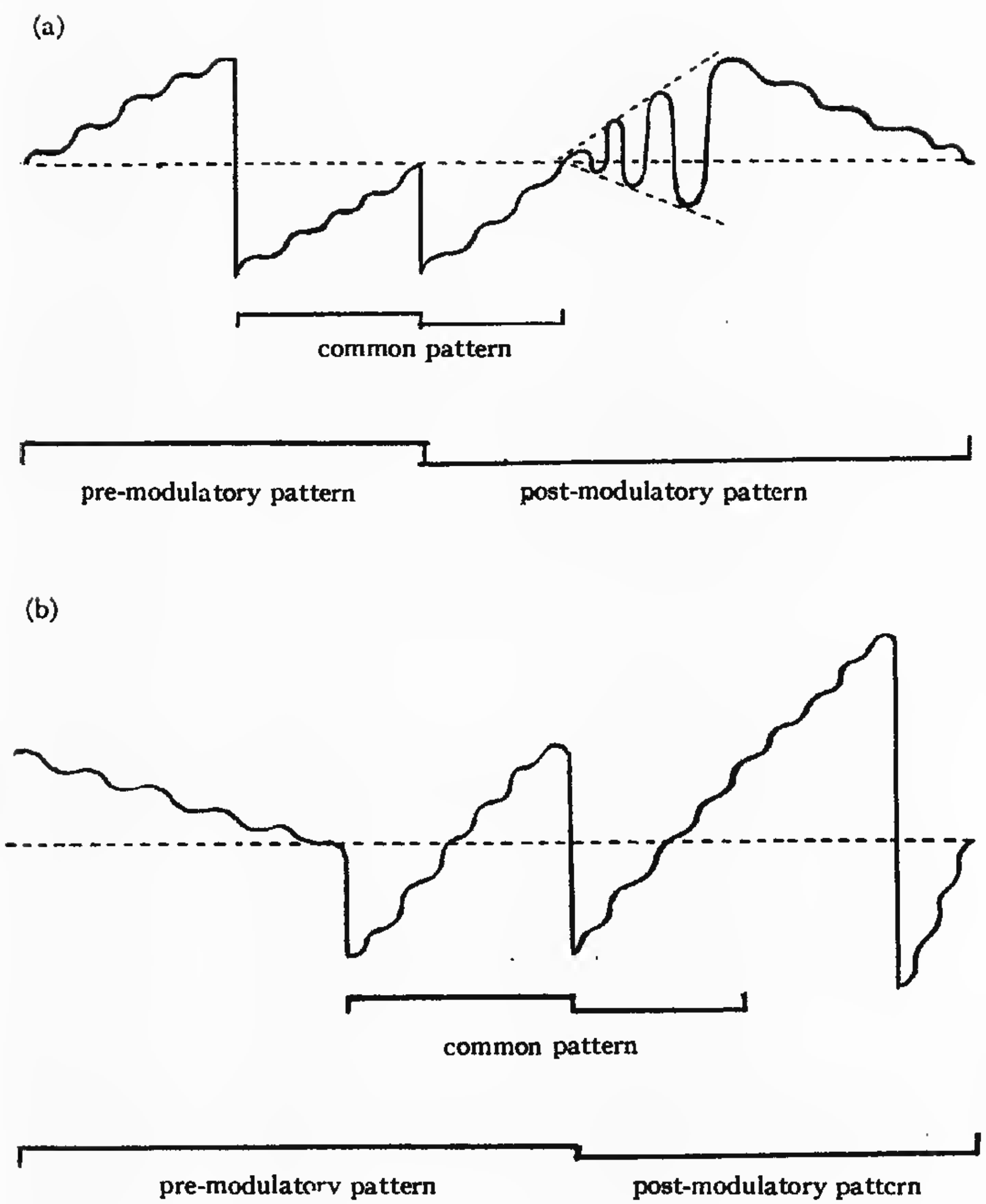


Figure 90. Introduction of a common pattern.

Configurational modulation is a resource by which one stimulus-pattern can be changed into another through the increasing dominance of one pattern over another. We have encountered such situations in the *Theory of Melody*,* where rhythmic resultants were applied as coefficient-groups controlling the rise of one axis and the decline of another.

For example: 4a + b + 3a + 2b + 2a + 3b + a + 4b produces the declining dominance of a and the increasing dominance of b, which situation illustrates a configurational modulation from the stimulus "a" to the stimulus "b":

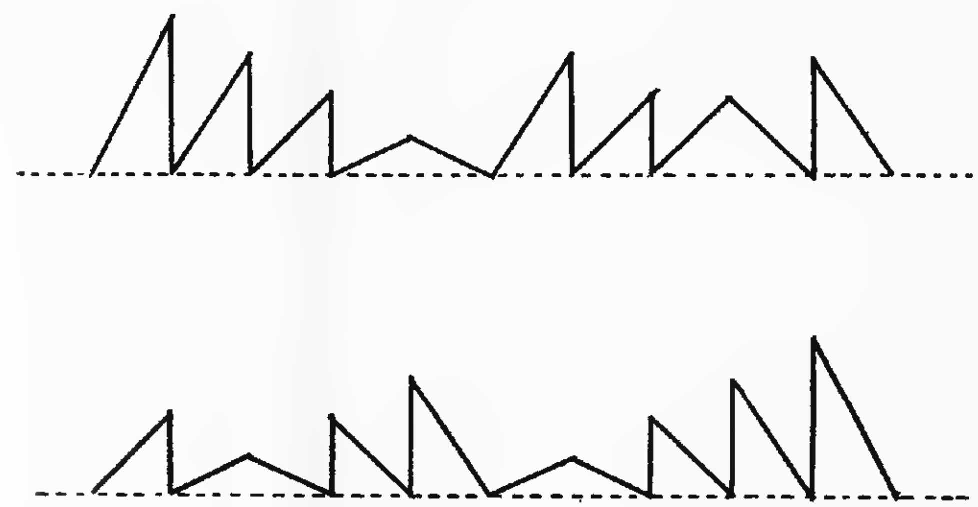


Figure 91. Graphic representation of 4a + b + 3a + 2b + 2a + 3b + a + 4b.

This case may psychologically correspond to a transition from dominance to compliance.

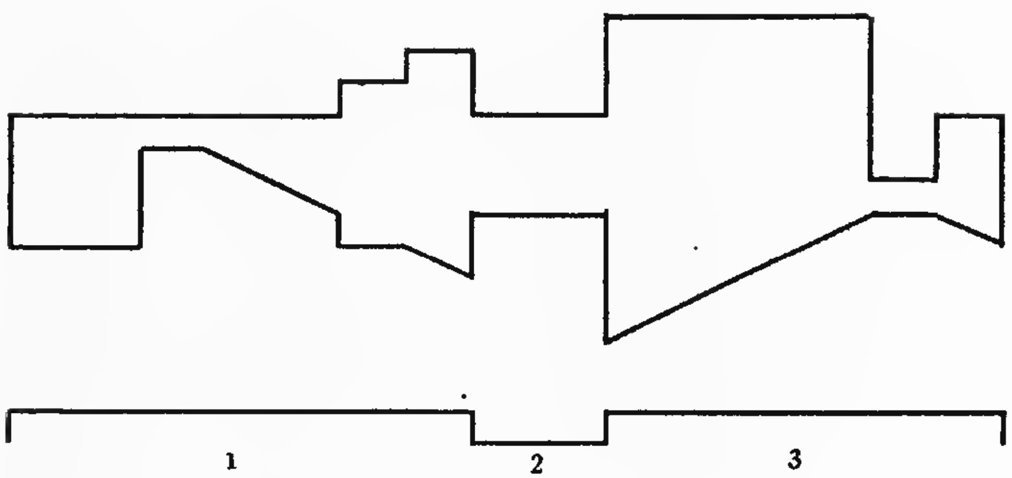
Configurational modulation is applicable in its respective forms to melody, harmony, counterpoint. In all these cases, patterns correspond to melodic trajectories, whether self-sufficient as in melody, or conjugated as in harmony and counterpoint. Configurational modulation can also be applied to the patterns of density and dynamics. The method of application remains the same as in the intonational patterns, but the meaning of balance or neutral configurational equilibrium lies in the center between the two extremes of density and dynamics. Neutralization of the extreme forms of density (low or high) is accomplished by resorting to *medium density*, which forms a general primary axis of the density-patterns. Neutralization of extreme dynamic forms (pp and *ff*) is also accomplished by the use of the *intermediate dynamic degree* (m*f*) acting as a neutralizer.

Common patterns of density and dynamics, linking together the otherwise contrasting or conflicting pre-modulatory and post-modulatory configuration-groups, serve as another technique of transition from one stimulus to another.

*See Vol. I, pp. 261 and 275.

Examples of neutralization

(a) density:



1. pre-modulatory pattern
2. modulatory pattern
3. post-modulatory pattern

Figure 92. Neutralization of density.

(b) dynamics:

sfp + *sfpp* + *pp* < *f* + *f* > *mf* + *mf* + *ff*

1. pre-modulatory pattern
2. modulatory pattern
3. post-modulatory pattern

Figure 93. Neutralization of dynamics.

Examples of a common pattern

(a) density:

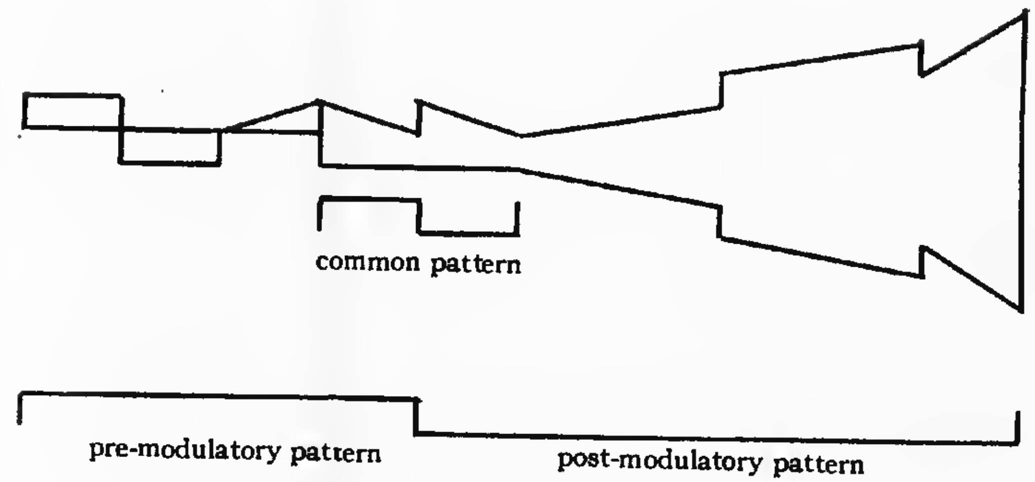


Figure 94. Common pattern in density.

(b) dynamics:

pp + *sfpp* < *f* + *sfpp* < *f* + *ff* > *mf*

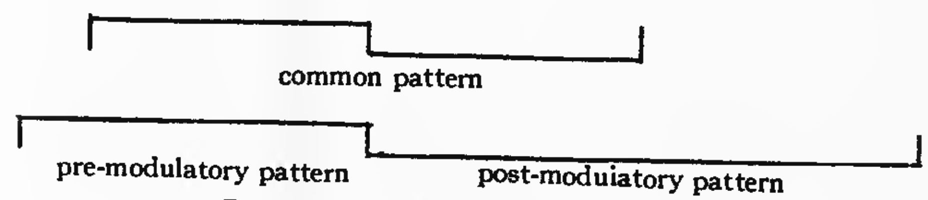


Figure 95. Common pattern in dynamics.

Finally, configurational modulation can be applied to tone-quality, instrumental forms and attack-groups. Here, too, either neutralization of extreme patterns or configurational similarity serves as a modus of transition from one stimulus-pattern to another.

Instrumental forms which I view as essentially generalized arpeggio forms combine melodic and density configuration. In them a modulation from one stimulus-pattern to another is performed either by neutralization of the arpeggio form by changing it gradually into a sustained chord, after which a new post-modulatory form begins, or by transition through a common pattern. If the arpeggio forms are alike in the pre-modulatory and the post-modulatory groups, the modus of transition is confined to amplitudinal variation: technically it corresponds to the transition from one form of tonal expansion to another. Modulations representing a variation of density in an arpeggio form are performed by a gradual increase or decrease of the number of simultaneous attacks in the arpeggio.

Tone-quality modulations are also configurational modulations in the physical sense, as they represent a transition from one pattern to another. These modulations therefore are subjected to the same principles, as all other configurational modulations which we have already discussed. Empirically speaking, the change in the configuration of a tone-quality is accomplished, not by a physical transformation of one pattern into another* as they can be seen on the screen of an oscillograph, but by the pure techniques of orchestration, i.e., by the increase of timbral ingredients of one kind and by the decrease of timbral ingredients of another kind.

For example, a gradual transition from q_I (on a 5q scale) to q_V may be illustrated as follows:

$$2\ \text{Fl.}\ (q_I) + \frac{\text{Fl}}{\text{Cl}}\ \frac{\text{Cl}}{\text{Cl}}\ (q_{II}) + \frac{\text{Cl}}{\text{Fr. H}} + \frac{\text{Fr. H}}{\text{Fr. H}}\ (q_{III}) +$$
$$+ \frac{\text{Fag}}{\text{Fr. H}} + \frac{\text{Fag}}{\text{Fag}}\ (q_{IV}) + \frac{\text{Fag}}{\text{Tromb}\ (\bullet)} + \frac{\text{Trump}\ (\bullet)}{\text{Tromb}\ (\bullet)}\ (q_V).$$

A greater graduality as we have seen before can be accomplished by the human voice, where the modulation of a pattern may be performed by the modification of vowels.

Finally, configurational modulation of the attack-forms (embracing the legatissimo-staccatissimo scale) can also be performed, either through neutralization of the patterns possessing extreme characteristics (like legato or staccato) through introducing the neutral pattern of portamento, or by connecting the pre-modulatory and the post-modulatory groups by means of a common pattern. In addition to this, as in the case of amplitudinal variations, a gradual transition through the scale of attack-forms, from one extreme pattern to another, constitutes a modulation. For instance, a pre-modulatory pattern being staccato may gradually be transformed into legatissimo: $a_{IV} + a_{III} + a_{II} + a_I$ (i.e., staccato, portamento, legato, legatissimo), in which case legatissimo is the form of the post-modulatory pattern.

Range and register, as configurational stimuli, provide their own forms of transition and may serve as links connecting otherwise different patterns. For example, the commonness of range or register may bridge different intonational or timbral forms. On the other hand, gradual trransitions from one register to another, as well as amplitudinal range-variation, can serve as modulatory techniques.

*This will undoubtedly be done in the near future; I did it in 1932 by means of a special electronic organ built by Leon Theremin. —J.S.

B. Coordination of Sonic Symbols

As all our differentiated sensations developed in the course of biological evolution from one general tactile fore-sensation, so did our music develop from monody into the complexity of contemporary scoring. And though at one time or another a certain sensation may dominate over another, in actuality we have no pure sensations. The dominant sensation stands out by its intensity though it is conjugated with other sensations. The gamut of sensations can be compared to a certain extent with the acoustical phenomenon of timbre, where one frequency dominates over other frequencies with which it is conjugated and which are its partials. Of course, the sensational mechanism is much more complex than this, as it contains not only simultaneous and sequent processes, but also overlapping ones. As one sensation is in progress, another may be just activated and still another may be in its decline.

It is only natural that the art music, which, in its present state of complexity, is employed as a connotative language of sonic symbols, should be flexible enough to produce a worthy counterpart of human sensations, not only in their isolated but also in their combined forms. Combined pattern-stimuli activate combined responses. And such stimuli may be created by simple or complex conjugated sonic symbols, each symbol being represented by the individual or by the group-components. Complex stimuli may also be produced through coordination of various art media where music, in all its complexity, becomes only one (simple or complex) component of the whole.

It is not our purpose to discuss here the semantics of other arts than music and their possible forms of correlation with music. For this reason our analysis of processes involving complex stimuli shall be confined solely to music. Let us suppose that the source of sonic symbols, simple or complex, is a text. The complexity of combined stimuli would derive from a certain treatment of the same text. For example, we may choose two dissociated events from a scenario and present them as two simultaneous conjugated sonic symbols. In this case, one symbol may parallel the present event, while the other may stimulate the presaging or premonition of another event to come. A scene of gaiety, taking place on the stage or screen, may be combined with a group of parts in the musical score, which have the same gaiety pattern. At the same time, a certain thematic counterpart of the same score may reflect the impending disaster, of which there is no sign in the respective scene.

As scripts and scenarios nowadays cover almost any imaginable situation, the composer must be so well equipped that he would never be caught unawares. Instead of the romantic "love in moonlight" he may face the problem of connoting a "day in an insane asylum" or "rush-hour at the Times Square shuttle", which cases call for all the dial positions to be employed simultaneously, as the gamut of associations in such cases ranges from normal to abnormal.

The correlation of sonic symbols pertaining to various pattern-stimuli, first, implies the selection of such stimuli as a combination and, second, discovering of the conditions under which such coordination can be performed.

In actual application, the stimulus-response scale, as represented on our psychological dial, can be greatly increased by our usual method of inserting intermediate forms between the basic forms already represented on the dial. Thus, the configurational scale can be extended to 12 or more patterns. Such details must be left to the initiative of the composer. Our present problem is to classify as simply as possible the combinations of the different patterns with each other in order to establish the basic procedure for producing scales of the combined complex stimulus-patterns and their corresponding combined complex sonic symbols.

For this reason we shall confine ourselves to the eight-pattern scale, which corresponds to an eight-point dial. Thus, mathematically, the entire problem is to compute the number of combinations possible out of 8 elements.

Table of Combinations of the Sonic Symbols Evolved in Accordance with an Eight-Point Stimulus-Response Dial.

$${}_8C_1 = \frac{8\,!}{1\,!\,(8-1)\,!} = 8$$

$${}_8C_2 + \frac{8\,!}{2\,!\,(8-2)\,!} = 28$$

$${}_8C_3 = \frac{8\,!}{3\,!\,(8-3)\,!} = 56$$

$${}_8C_4 = \frac{8\,!}{4\,!\,(8-4)\,!} = 70$$

$${}_8C_5 = \frac{8\,!}{5\,!\,(8-5)\,!} = 56$$

$${}_8C_6 = \frac{8\,!}{6\,!\,(8-6)\,!} = 28$$

$${}_8C_7 = \frac{8\,!}{7\,!\,(8-7)\,!} = 8$$

Figure 96. Combinations of sonic symbols.

Thus there are 8 cases when 1 pattern out of 8 is used at a time; 28 cases when 2 patterns out of 8 are used as a combination; 56 cases when 3 patterns out of 8 are used as a combination; 70 cases when 4 patterns out of 8 are used as a combination; 56 cases when 5 patterns out of 8 are used as a combination; 28 cases when 6 patterns out of 8 are used as a combination; 8 cases when 7 patterns out of 8 are used as a combination. Then the total of all these combinations which are at the composer's disposal, when he is limited to an eight-point dial, amounts to: $8 + 28 + 56 + 70 + 56 + 28 + 8 = 254$. To this, we can add one combination of all 8 elements, thus making the total: $254 + 1 = 255$.

Of course more complex forms of the above combinations are used rather seldom. Strata harmony is the most suitable technical resource for evolving more or less complex combinations of sonic symbols.

Our next stage is the method of classifying configurational characteristics as they appear in combined patterns.

All patterns belonging to one quadrant must be considered as ***identical patterns of different intensities.*** The growth of intensity follows the clockwise direction in the positive zone and the counterclockwise direction in the negative zone. Thus, for example, the pattern of is identical with, but more intense than, that of . Such reasoning is applicable to all quadrants.

Patterns belonging to different quadrants may also vary in intensity, but they are to be considered ***non-identical.***

C. Classification of the Stimulus-Response Patterns, to be Represented as Combined Sonic Symbols, on the Basis of their Intensity of Configurational Identity

(1) identical patterns of identical intensities;
(2) non-identical patterns of identical intensities;
(3) identical patterns of different intensities;
(4) non-identical patterns of different intensities.

Illustrations:

(a) Two craftsmen who are partners in the trade and are not rivals have a different degree of skill in making, let us say, Christmas tree ornaments. The problem of the composer is to produce a combined sonic symbol of two identical patterns of different intensities. Superior accomplishment corresponds to the pattern of greater intensity. Thus expressing the less accomplished craftsman as A and the more accomplished craftsman as B, we can establish the following correspondence: $\frac{A}{B} =$

The resulting symbol B may acquire a wider range, higher mobility and higher intensity (of sound) than symbol A. Both may be expressed as self-sufficient but correlated melodies, or as an accompanied counterpoint.

(b) The old story about a poor young man in love with a rich young girl: the girl's parents in a united coalition of the entire family clan create unsurmountable obstacles and the marriage is called off; both boy and girl are in despair. The positive pattern of love and hope (i.e., if our sympathy is on the side of the young couple) is counteracted by a more powerful negative pattern of the family's opposition. The composer's problem is to produce a combined sonic symbol of two non-identical patterns of different intensities. Let the young couple be A and the family—B. Now we can establish the following pattern-intensity correspondence: $\frac{A}{B}$ = . The evil forces (i.e., evil from our viewpoint) pull counterclockwise, overpower and win, bringing A into the negative zone. Here are the considerations for composing the corresponding sonic symbols for A and B.

Symbol A modulates from the positive into the negative zone under the pressure of B, which retains its constant characteristics of the negative pattern of high intensity, high dynamics and density (coalition). The B pattern is negative in the sense that it is a destructive and not a creative force. By virtue of its characteristic, B pattern is a counteracting force and for this reason must have the axial characteristic opposite to that of A. As the A pattern obviously corresponds, in its initial phases, to a-axis, the B pattern must be expressed by b-axis of greater amplitude than the first phase of A. The effect of the B pattern upon the A pattern is such that a-axis gradually loses its momentum and goes into decline, transforming itself into b-axis.

This can be represented diagramatically as follows:

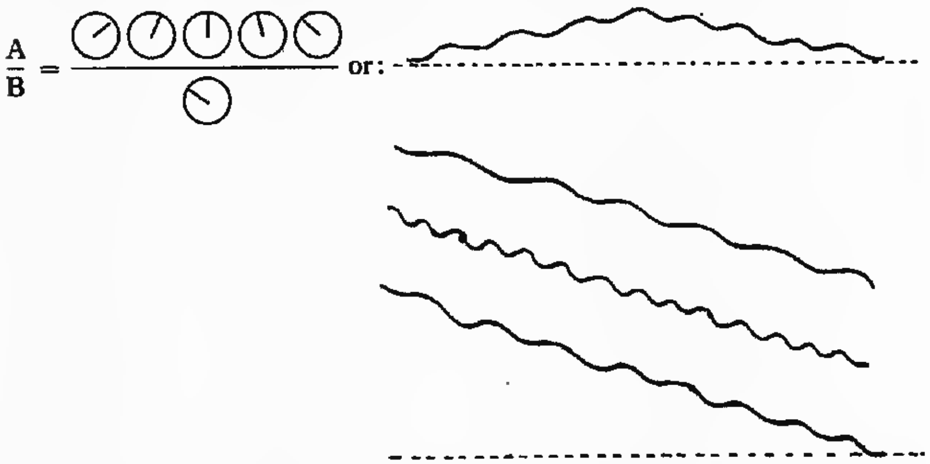


Figure 97. Two non-identical patterns of different intensity.

The above identity-intensity pattern classification must now be supplemented further by the characteristic of *constancy* or *variability* of the pattern. Then the original 4 forms become, in turn, quadrupled: $4^2 = 16$. The basic classification represents the original phase from which the departure is made.

(1) identical patterns of identical intensities:
  (a) const. identity, const. intensity;
  (b) var. identity, const. intensity;
  (c) const. identity, var. intensity;
  (d) var. identity, var. intensity;

(2) non-identical patterns of identical intensities:
  (a) const. identity, const. intensity;
  (b) var. identity, const. intensity;
  (c) const. identity, var. intensity;
  (d) var. identity, var. intensity;

(3) identical patterns of different intensities
  (a) const. identity, const. intensity;
  (b) var. identity, const. intensity;
  (c) const. identity, var. intensity;
  (d) var. identity, var. intensity;

(4) non-identical patterns of different intensities:
  (a) const. identity, const. intensity;
  (b) var. identity, const. intensity;
  (c) const. identity, var. intensity;
  (d) var. identity, var. intensity.

In this table "constant" means a "constant form of relationship" with regard to the identity or intensity of the conjugated symbols; likewise, "variable" means a "variable form of relationship" in the same sense.

In the case of the unfortunate couple in love, the hope for marriage and happiness, represented by A, was a pattern of variable identity and variable intensity, while B was a pattern of constant identity and constant intensity. The basic relation of $\frac{A}{B}$ was that of non-identical patterns of different intensities. The fact that A was variable in both respects, made their relationship appear variable in the same respects.

The ultimate number of variations of all kinds, possible for each original relationship, depends on the number of individual symbols, conjugated into a combined complex symbol.

When sonic symbols are combined with script symbols or with each other into a combined complex symbol (the latter ultimately acquires the form of a score), their correlation assumes one of the following forms:

(1) parallel;

(2) contrary (inverse);

(3) oblique.

Parallel implies identity of symbols and is the most obvious and, for this reason, the most generally used form of association.

Contrary implies an association by contrast or juxtaposition, such as gay music to a sad scene or vice-versa; or two conjugated sonic symbols, each stimulating one of two contrasting associations.

Oblique implies either a deviation from identity to non-identity, or from non-identity to identity. This can be graphically illustrated as follows:

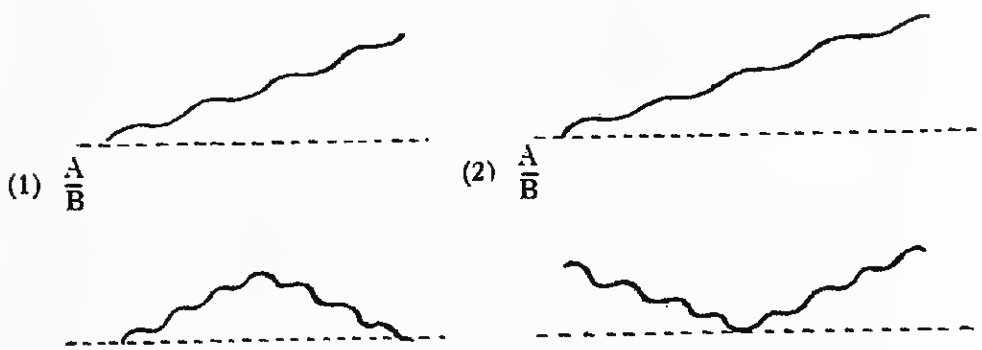


Figure 98. Parallel and contrary correlation.

Such is the case when friends gradually become enemies or enemies become friends; also when a gay scene is accompanied by music, modulating from gay to sad, or vice-versa.

In addition to all the previous relational classifications, there still remains the general category of ***temporal congruence.***

Temporal congruence may emphasize either complete events, represented by sonic symbols, or their individual phases. Each event generally consists of five basic phases: generation (beginning, origination), growth, climax (goal, maximum), decline (anticlimax, balancing tendency), degeneration (completion, end).

As combinations of events take place in the interaction, ***simultaneous*** (synchronized) ***associations*** constitute only one form of temporal congruence. The two other forms represent the ***anticipated*** and the ***delayed associations.***

The anticipated associations (i.e., that of presaging, premonition, etc.) represent the event to come, at a time when another event takes place (this other event may be executed as a different thematic component of the same score and can be carried out either in the same [music] or in a different [words, action] medium).

The delayed associations (i.e., that of recollection, reminiscence, etc.) represent a past event at a time when another event takes place.

From a technical standpoint, all forms of correlation of sonic symbols into their conjugated combined complex forms can be executed by identical, partly-identical, or non-identical groups of musical components.

Materials which illustrate the processes analyzed and systematized in this exposition are profusely scattered throughout all the program and operatic music, written by the competent composers of all ages. In my opinion, most of the so-called "great composers" produced in many instances impressive music

because they had a high intuitive notion of configurational semantics, that is, they ***felt music in terms of patterns***—which ability is lacking in most of our contemporaries. At the same time these men of the past had, in most cases, a very crude technique in handling special components, such as harmony, orchestration, etc., in which field our contemporaries are much more accomplished. Yet many of the present creations are born dead, as they lack the necessary qualities of associational stimuli.*

The field of connotative music is so broad and its applications so numerous that in this course of study we are only able to direct the student's attention toward the problems and the method by which they can be solved.

*The use of this system by Schillinger (as a staff composer of the Academic State Theatre of Drama, the Experimental Theatre of the State Institute of the History of Arts, both in Leningrad, and of the State Theatre for Children in Kharkov) and by his students in the U.S.A., who are active as radio, theatre, cinema and television composers, brought extremely fertile results . . . Among Schillinger students who made noteworthy use of the system are such men as Leith Stevens ("Columbia Workshop", "Tish", "Alice in Wonderland", "Big Town" and others); Paul Sterrett, Nathan Van Cleave (both at CBS, in the "Columbia Workshop" and other productions), Oscar Levant ("Nothing Sacred", "Charlie Chan at the Opera" and other motion pictures); Bernard Mayers ("Basin Street Chamber Music Society", NBC, where he made some very effective scores in the semantic sense: "Three Blind Mice", "Mary Had a Little Lamb", "The Bullfrog and the Robin" and others); Jesse Crawford ("Valiant Lady", CBS); Carmine Coppola ("Pictures in Music", CBS, where he wrote "G. B. Shaw"); Lyn Murray ("The Adventures of Ellery Queen", "26 by Corwin", opera "Esther", "This is War" and other CBS programs); Charles Paul ("City Desk", "The Adventures of Ellery Queen" and other NBC programs); and Rudolf Schramm [music to radio and cinema (U.S. government productions) in the field of education], just to mention a very few. (Ed.)

THE SCHILLINGER SYSTEM

OF

MUSICAL COMPOSITION

by

JOSEPH SCHILLINGER

BOOK XII

THEORY OF ORCHESTRATION

BOOK TWELVE

THEORY OF ORCHESTRATION

PART I

INSTRUMENTS

PART II

Instrumental Techniques

INTRODUCTION

WHAT has been known for the last couple of centuries as a "symphony orchestra" is a heterogeneous aggregation of antiquated tools. Wooden boxes and bars, wooden pipes, dried sheep's guts, horse hair and the like are the materials out of which sound-producing instruments are built.

The evolution of musical instruments, during their history of several millenia, followed the course of individual craftsmanship and of the trial-and-error method.

The instruments themselves are not scientifically conceived and not scientifically combined with each other. Some of the orchestral groups participate with others by virtue of tradition (like brass and string instruments which, in most cases, do not blend) and not by necessity. Nobody ever asks the basic question: why should there be such a combination as the stringed-bow, the wood-wind, the brass-wind and the percussive instruments; and why should the respective groups be used in the unjustified ratios which are considered standard?

It takes a long time to force upon the average normal human ear such combinations as piano and violin or strings and brass. And this imposition of unblendable combinations upon the selector called the human ear is termed "cultivation of musicianship". But eventually people begin to like it, as they begin to like smoking tobacco, which suffocates them at first. It is even possible to condition the human ear to hear the sound at a sustained intensity, while the sound is fading at its source. Such is the case with the piano. Ordinarily we are not aware of the fact that the piano tone fades very quickly. I once intentionally subjected myself (at the age of 30) to a forced isolation from the piano for three full months. The only sounds I heard during the time were that of an organ and of choral singing (i.e., durable sounds). I lived among peasants. When I returned to the city, the piano sounded to my ear as it really sounds, i.e., as a percussive instrument with exaggerated attack and quick fading. It took me fully two weeks to "recover" from this unconditioned modus of hearing.

The implication is that many of the orchestral tone-qualities and blends are gradually assimilated by our ear. Many of them are highly artificial and do not possess the appeal of natural beauty, as many natural forms and natural colors do.

The musician's argument against better balanced, more uniform tone-qualities, which are possible on the electronic instruments, is that they have not the individuality the old instruments have. But what *they* call "individuality" is often a group of minor defects and imperfections. A trombone, due to its acoustical design, has several tones (certainly, at least one) missing. While the composer can easily imagine those missing tones and imagine them in the trombone quality, he cannot use them in his score, since they cannot be executed. Now, take a bassoon. Its low b♮ is of a quality inferior to that of the surrounding tones. Why should one particular pitch be defective? No one knows.

A composer, due to his experience, can also imagine certain tone-qualities beyond the ranges of the respective standard instruments. He cannot use these qualities because there are no instruments to perform them.

Under such conditions, the art of orchestration amounts to a constant (and in most cases unsuccessful) struggle of the composer's imagination and inventiveness against the actuality of instrumental limitations and imperfections. The way things stand today, the composer must compose not in terms of tone-qualities, intensities, frequencies and attack-forms (if he does not want to live in a fool's paradise), but in terms of concrete instruments, each designed with no regard to any other instrument—each. therefore, having peculiarities of its own.

Musicians also have a sentimentally-childish attachment to the craftsmanship of executing a "beautiful" tone on a violin or other instruments. Very few performers, indeed, can execute such a tone. But why is this self-imposed difficulty and struggle necessary? Such an attitude has the flavor of sportsmanship and competition. Why not liberate the performer from the necessity of struggle to obtain the proper tone-quality, when such tone-quality can be achieved, and has been achieved, by means of electronic sound production?

The answer is that many good performers, once relieved of this struggle, would feel lost since, to them, the production of tone-quality is half of the problem of interpretation.

In 1918 I published an article ("Electrification of Music") in which I expounded my own ideas (at that time completely new and original) on the inadequacy of old musical instruments and on the necessity of developing new ones, where *sound could be generated and controlled electrically*. I thought it would be desirable to have tone-qualities, attack-forms, frequencies (tuning) and intensities under control, to be able to vary each component through continuous or discontinuous (tempered) scales, suddenly or gradually, and to determine the degree of the graduality of transition as well.

Though there is no universal use of electronic music yet, it is progressing very rapidly. Most of my dream has already come true. In 1920 Leon Theremin demonstrated his first primitive model of an electronic instrument before a convention of engineers in Moscow, Russia. On this model, pitch was controlled by movement of the right hand in free space (in actuality, in an electro-magnetic field) and volume, by a specifically designed pedal; the form of attack was controlled by a knob; the timbre was constant.

After a number of years of my collaboration with this inventor, the early history of electronic music culminated in 1930 in two Carnegie Hall (New York) performances in which participated a whole ensemble of 14 improved space-controlled theremins, manufactured by Radio Corporation of America on a mass production scale at the plant in Camden, New Jersey.

That first decade of electronic music, in which I am proud to have played the part of a musical pioneer, started the art of music on an entirely new road, which is in keeping with the engineering accomplishments of our industrial era

of applied science. There is no turning back from this road, regardless of the absolute value of today's models of electronic instruments. The fact is that *a new principle of sound production and control* has been established, and this principle will bring further improvements and perfection.

It is important to realize that existing musical instruments and their combinations are not stabilized but ever-changing accessories of musical expression; that absolute knowledge of the functioning of the keys of a clarinet is of no basic value, as the design of such an instrument varies and the whole family of such instruments may vanish.

Thus, though in my description of standard instruments all the necessary information is given, the composer must not overrate the importance of it, as the entire combination of a symphony orchestra, with all its component instruments, may soon become completely outmoded and eventually obsolete. It will be a museum combination for the performance of old music. New instruments and combinations will take its place.

The moral of this *Introduction* is that it is more important for the composer to know the physical aspects of tone-qualities, frequencies, intensities and attack-forms *per se*, rather than the resultant forms as they appear on certain types of old Instruments. It is a warning not to attach too much importance and confidence to certain types of instruments, simply because they are so much in use today.

In the *Acoustical Basis of Orchestration*, the student will find the type of knowledge which is *basic and general* and, therefore, can be applied to any special case. This system is devised with a point of view which will give lasting service and will not become antiquated with the first turn the history of this subject takes.

In order to broaden the student's outlook on the existing instruments, I am supplementing this Introduction with a chronological table borrowed from one of my other works, *Varieties of Musical Experience.*

Two items of this table deserve particular attention: (1) the chronological precipitation of progress and (2) the age of the new "electronic" era.

Scheme of Evolution of Musical Instruments

From Prehistoric Time

I. Man utilizes his own organs: voice, palms, feet, lips, tongue, etc.

From 10-20 Thousand Years Ago Until Our Time

II. Man utilizes finished or almost finished objects of the surrounding world: bamboo pipes, shells, bones of birds, animal horns and antlers, etc.

From 5 - 10 Thousand Years Ago Until Our Time

III. Man processes raw material, giving it a definite form: from a piece of terra cotta and hunter's bow up to the Steinway piano and modern organ.

From 18th Century A.D.

A. Man constructs automatically performing instruments: from 18th Century, mechanical musical instruments: from 19th Century, recording and reproducing musical instruments.

From the End of 19th Century

B. Man develops transmission of sound waves over long distances: radio.

From the Beginning of 20th Century

C. Man devises sound production by means of:

1. Electro-magnetic induction
2. Interference in electro-magnetic field.

CHAPTER 1

STRING-BOW INSTRUMENTS

CONTEMPORARY string-bow instruments have as their immediate ancestor the viol family. When the treble-viol, in the hands of Italian craftsmen, achieved its ultimate degree of perfection, it became the dominant member of the viol family: the treble-violin emancipated itself into the plain "violin". In this sense, the evolution of the violin family followed the downward (in the way of frequency) trend, i.e., the perfecting of the violin was followed by the perfecting of violas, 'celli and string double-basses (or contrabasses). This course of evolution was somewhat contrary to the development of the viol family, where bass-viol (later, violone) was the dominant instrument of the group, the patriarch of the family. Thus "violoncello" originated as the diminutive form of the "violone".

The more remote ancestor of this family is the Arabian "rebab", a primitive type of string-bow (often having only two strings, however, tuned in 3 ÷ 2 ratio, i.e., in a perfect fifth) and having a resonating chamber. This ancient instrument leads us back to the "monochord", a one-string bow instrument with a resonating chamber, and, finally, to the actual source of the violin, which is the bow and arrow.

This remarkable evolution of a defense weapon into a musical instrument of high degree of perfection consumed not only millenia of astronomical clock-time, but also an incalculable amount of human energy lavishly spent by generations of craftsmen and musical performers.

But with so much said and written about violin-making and violin-playing, certain facts remain obscure. Since most of the time (between and during the eras of mutual mass-extermination), is spent by humanity in *creative mythology*, the history of the violin discloses a constant struggle between the glorification of violin-makers and violin-players. The fundamental question is: which factor is more essential in achieving perfection, the instrument or the player? Nobody would deny the importance of both. However, I am entitled to state, on the basis of experiments performed with Nathan Milstein and another highly accomplished, but not extraordinary, representative of the Leopold Auer school (which also contributed Heifetz, Zimbalist, Elman, Piastro, Seidel and many other virtuosi), that the player is a more important factor than the instrument. I draw this comparison particularly in reference to the quality of tone-production. In my experiment both performers were tested on the same two instruments: one was a violin made by Antonio Stradivarius and the other, a mediocre sample of mediocre craftsmanship. Milstein's tone-quality was superior on both violins and with less individual difference between the two instruments than that of the other violinist. This may be a good lesson to some parents and teachers: only a mediocre violinist needs a very expensive instrument.

As the best musical organizations of today have at their disposal some of the best string-bow performers (usually the potential soloists rejected by the market's maintenance of only the few best performers), the composer of our civilization may indulge in scoring which requires, on the part of the performer, a highly developed and versatile technique.

A. VIOLIN

1. TUNING

The entire range of the violin is written in treble clef.

The four strings are named g, d, a, e. From the physical standpoint all four strings have a different timbre. The timbre of the g-string is particularly different from the three upper strings. In the hands of an accomplished performer this timbral variance is greatly minimized. However, good playing does not affect the variance of the g-string with the three upper strings. This difference is due to the fact that g-string is a sheep's gut wrapped around with a metal wire, while d-string and a-string are sheep's guts which remain unwrapped. E-string only about three decades ago underwent a transformation: sheep's gut was replaced by a metal wire.

The violin is tuned in perfect fifths, i.e., in $3 \div 2$ ratio. The tuning begins with the a-string. Thus the ratios of the remaining strings are:

$$e = \frac{3}{2}; d = \frac{2}{3}; g = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

As the above ratios noticeably deviate from the corresponding pitches of the twelve-unit equal temperament, some of the more discriminating composers (Hindemith, for instance, makes it a rigid rule) avoid the use of open strings altogether, except in chords.

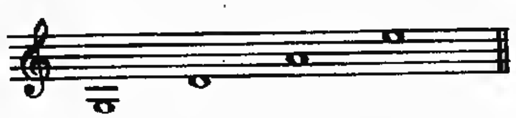


Figure 1. Tuning of the violin.

2. PLAYING

The Left Hand Technique

Intonation is controlled on the violin by means of shortening its strings, which is accomplished by pressing the string against the fingerboard. For this purpose the fingers of the left hand are employed. Strings vibrate between the two fixed points (nut and bridge) and transfer their vibrations to the bridge. The vibrations of the bridge stimulate sympathetic response from the body of the violin, which is a resonating chamber.

Four fingers of the left hand (thumb is excluded) participate in producing intonations. The various distances which the left hand occupies on the fingerboard (while supporting the violin) in relation to the nut are called *positions*. Each position on each string emphasizes *four* pitch-units of the common diatonic scales. The positions begin with an *open* string. Such a position might be called the *zero position*.

Figure 2. The zero position.

Arabic numerals indicate the fingers employed. Major tetrachords are used here merely for convenience: other accidentals can be employed as well.

The first position begins with a whole tone from the open string.

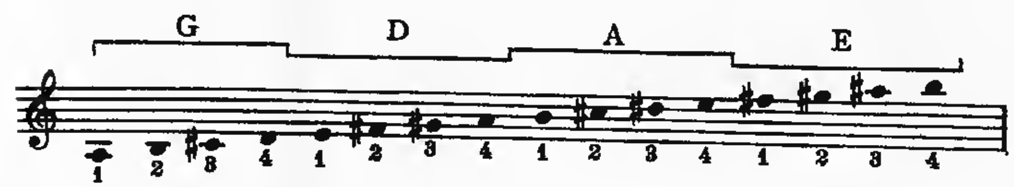


Figure 3. The first position.

If the first pitch-unit is only a semitone away from the open string, then such a position is called *half-position* or *semi-position*.

Figure 4. The half-position.

From here on, violinists do not discriminate any semi-positions, but consider only the *Second*, the *Third*, the *Fourth* and so on, positions, regardless of whether they are tone-and-a-half or two tones, two-and-a-half or three tones from the open string.

Figure 5. Positions above the first (continued).

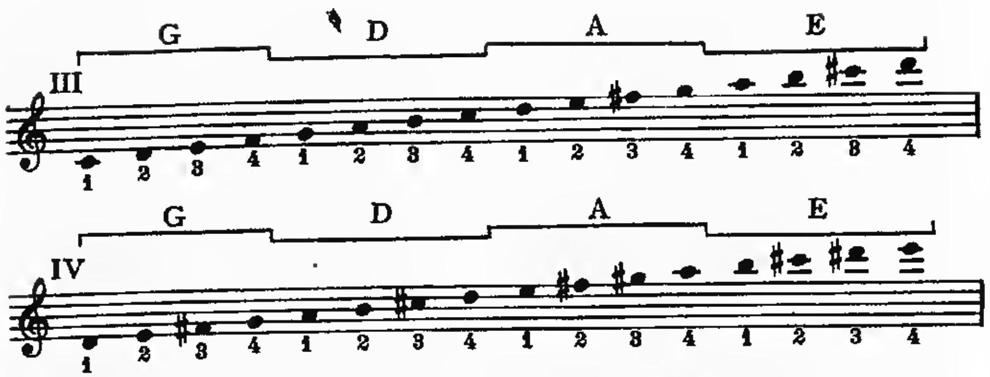


Figure 5. Positions above the first (concluded).

The three lower strings (G, D, A) are seldom used beyond the *eighth* position; the e-string is used even in orchestra-playing up to the fifteenth position (the beginning of Rimsky-Korsakov's opera *Kitesh.*)

All violin-playing is accomplished in most cases, including double-stops and chords, by means of standard fingering. Chromatic alterations are performed by moving the same finger a semitone up or a semitone down.

Insofar as the precision of intonation is concerned, it is always easier to move the fingers in the same position, making transitions from one string to another, than to change positions rapidly, particularly when such positions are not adjacent. It is to be remembered that though the use of the four fingers is analogous on all four strings and in all positions, the actual spatial intervals on the fingerboard contract logarithmically while moving upward in pitch. This means that a semitone in the first position is spatially wider than a semitone in the second position; the latter is wider than the semitone in the third position, and so on.

Musical intervals from the open strings can be defined in terms of positions, and positions can be defined in terms of musical intervals.

Position, where a given note is produced by the first finger, equals the number of the corresponding musical interval, minus one. For instance:

Figure 6. Position.

The given note g♯ to be played on a-string with the third finger requires the hand to be in such a position where *e* can be played on a-string with the first finger. As the musical interval from *a* to *e* (up) is a *fifth*, the position can be defined as 5 − 1 = 4 (i.e., it is the fourth position). This is so because the *first* position is produced by the interval of a *second* (i.e., 2) from the open string.

This proposition can be reversed. For example: what note is played by the *second* finger in the *sixth* position on the e-string?

The first finger in the sixth position produces an interval of a seventh (i.e., 6 + 1 = 7); therefore the second finger, in the same position produces an octave. Thus the note to be found is *e*, one octave above the open string.

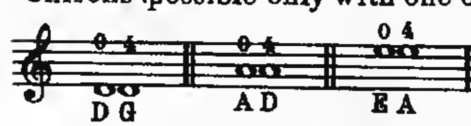
Figure 7. Example of fingering. Single notes.

Playing of S2p

The so-called "double-stops", i.e., couplings, harmonic intervals and two-part harmonies belong to this category.

S2o are played by means of standard fingering. Left hand is considered in an *open position* if the finger on the lower of the two pitches corresponds to a smaller number than that of the higher of the two pitches. The reversal of this proposition corresponds to a *closed position*. Open positions are easier to play. Closed positions can be used in double stops without particular difficulties, but preferably in a tempo that is not too fast.

Unisons (possible only with one open string):



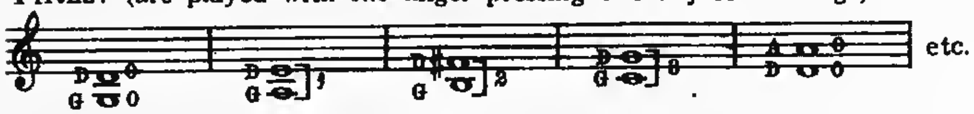
Seconds:

Thirds:

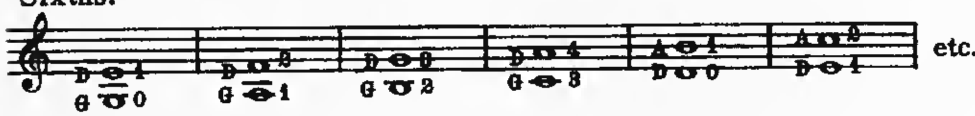
Figure 8. Fingering of S2p (continued).

Fourths:

Fifths: (are played with *one* finger pressing two adjacent strings):



Sixths:



Sevenths:

Octaves (quite difficult on account of the stretch between the first and the fourth finger; easy, with one string open):

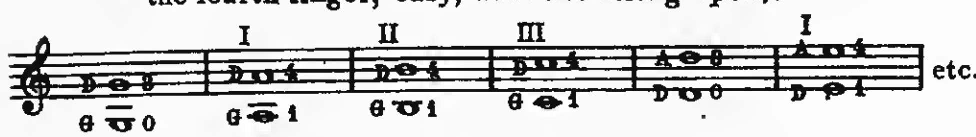


Figure 8. Fingering of S2p (concluded).

Octaves are used mostly in solo playing. As a perfect acoustical octave (i.e., 2 ÷ 1 ratio) sounds quite empty, soloists usually resort to playing an imperfect octave (somewhat more narrow in stretch than the acoustical octave), which sounds fuller. In scoring for an orchestra, octaves of violins are usually written *divisi* (i.e., both pitches are played by the different parts).

As octaves without participation of an open string require a stretch between the first and fourth finger, it becomes obvious that intervals wider than an octave can be performed only if the use of at least one open string is possible.

A special double-stop effect should not escape the attention of the orchestrator: passages on one string combined with another string remaining open. For example:

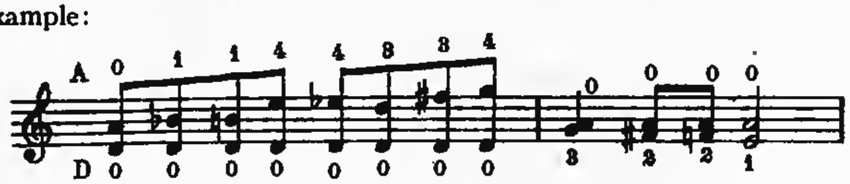


Figure 9. Special double-stop effect.

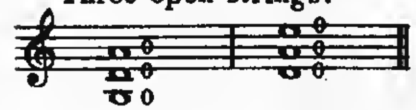
Such passages can be played at considerable speed.

Playing of S3p

Playing of triple-stops includes melody with two couplings and three-part harmony.

When employing 3 fingers at a time (i.e., without participation of open strings), only open position of the left hand can be used. In all other cases, previous considerations hold true.

Three open strings:



Two open strings:

One open string:

No open strings:

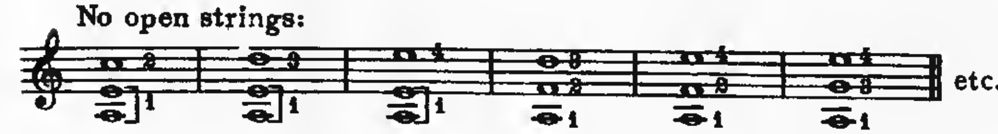


Figure 10. Fingering of S3p

Playing of S4p

Playing of quadruple-stops includes melody with three couplings and four-part harmony. There is only one quadruple-stop with four open strings:

All other cases include 3, 2, 1 or no open strings. All left hand positions must be open. Such chords as S(5) in open harmonic (⟳) positions are quite easy because only 3 fingers participate (as the perfect fifth is played with only one finger).

Three open strings:

Two open strings:

Figure 11. Fingering of S4p. Three and two open strings (continued).

Figure 11. Fingering of S4p. Three and two open strings (concluded).

One open string:

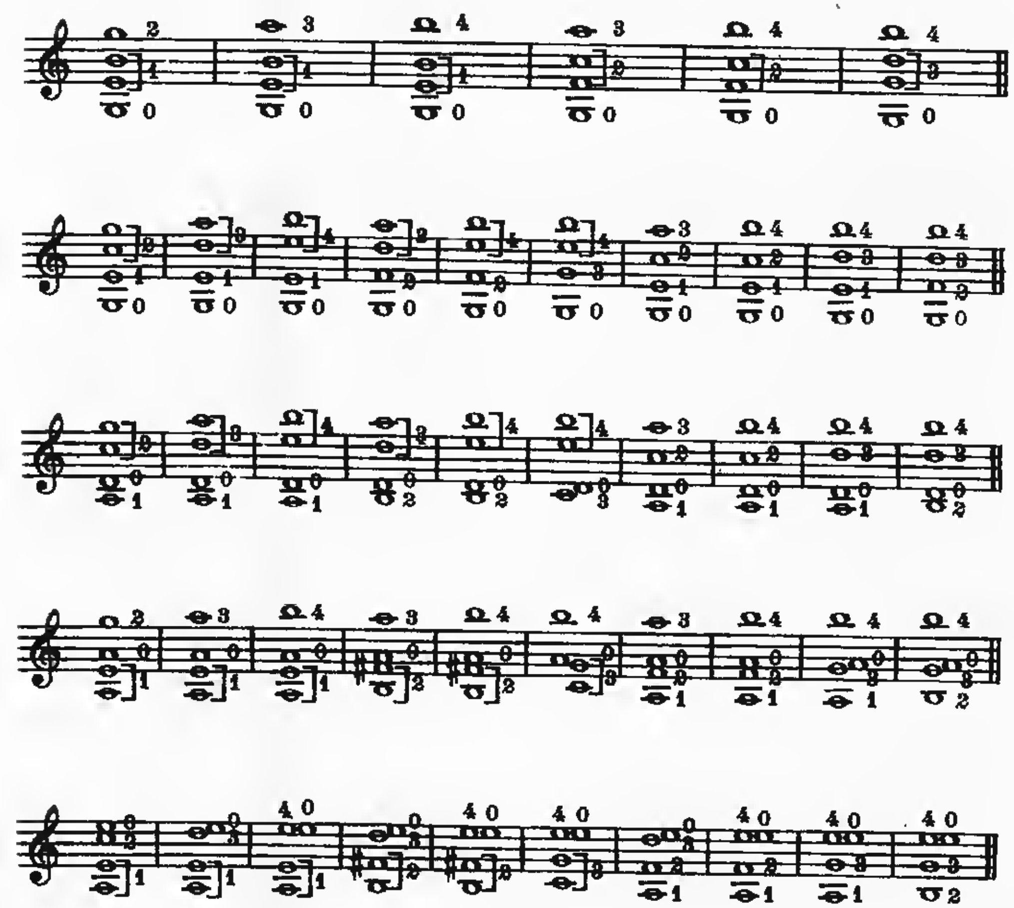


Figure 12. Fingering of S4p. One open string.

No open strings:

Figure 13. Fingering of S4p. No open strings.

The above tables are merely samples of the systematization of the material on fingering; they can be extended to higher positions (with or without participation of the open strings). These forms of fingering are applicable to various instrumental forms.

As the bow can move simultaneously over not more than *two* strings (some exceptional virtuosi can bow three strings simultaneously in *forte*; but such an accomplishment is exceptional and we cannot count on it in writing orchestral parts for the violins), we see that:

1(2p) can be performed as:

ap and a2p in sequent combinations;

1(3p) can be performed as:

ap and a2p in sequent combinations;

1(4p) can be performed as:

ap and a2p in sequent combinations.

Figure 14. Examples of instrumental forms suitable for the violin (continued).

Figure 14. Examples of instrumental forms suitable for the violin (concluded).

B. The Right Arm Technique

Bowing is a process by which friction is produced between the horse-hair of the bow and the string. The various techniques by which strings can be made to oscillate in different patterns, constitute the bowing attacks. Heavy bowing attacks cause large amplitudes, and light bowing attacks, small amplitudes. In order to produce a continuous sound, without a renewal of attack, the bow must move in one direction. The duration of a period depends upon the pressure of the bow on the string. Thus the period of continuous bowing in one direction in *piano* is greater than in *forte.*

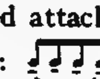
We shall now classify the forms of bowing as the forms of attack in relation to the durability of sound. We shall assume that the total scale of attacks lies between the two limits: the lower limit corresponds to the most continuous form of attack, and the upper limit, to the most discontinuous, i.e., abrupt, form of attack.

The movement of the bow in the direction from g-string to e-string is considered *downward* and, when necessary, is indicated as ⊓ ; the movement in the opposite direction is considered *upward* and is indicated as V . The upbeat groups are usually played V and the downbeat groups are usually played ⊓ . Otherwise a composer must indicate the direction of the bowing which expresses his desire.

The Scale of Bowing Attacks

(1) *legato:* a group of notes united by a slur represents continuous bowing in one direction; *large legato* pertains to a long group, and *small legato*, to a short group;

(2) *non-legato* (detaché) or *detached* is indicated by the absence of slurs or any other signs: each note corresponds to an individual *smooth* bowing attack, i.e., the bow must be turned in the opposite direction after each note;

(3) *portamento* (in bowing) represents a group of slightly accentuated attacks, while the bow moves in one direction; it is indicated as follows:  ;

(4) *spiccato:* abrupt bowing for each attack, while the bow moves in one direction: ; it sounds somewhat lighter than staccato;

(5) *staccato:* abrupt bowing for each attack and changing the direction of the bow after each attack: (no slurs);

(6) *martellato* (hammering): a vigorous downward or upward stroke indicated like this: (no slurs; bow changes its direction after each attack, unless specified otherwise);

(7) *saltando* (jumping): a bouncing group of attacks obtained by one stroke (usually two, three or four attacks, which can be described as throwing the bow from above; bouncing is caused by the resilience of the string and the bow; *saltando* has a light percussive character and is usually employed in accompaniments of the character of Spanish dances: this effect is a mild version of castanets; saltando is indicated like this: ;

(8) *col legno* (with the wooden part of the bow) is marked by these words above the part; no other indications are necessary; this effect is still more percussive in character than *saltando:* it is performed by an individual thrust of the bow downward upon the string, each throw corresponding to an individual attack; the general effect of *col legno* is that of pianissimo.

To continue the abrupt forms of attack, we may add, at this point, the various forms of plucking the strings.

From the orchestrator's viewpoint there are two basic forms of *pizzicato*: (1) *pizzicato legato*, where the respective finger of the left hand is moved on a small interval (usually a semitone or a whole tone), after the string is plucked (this effect resembles the so-called "Hawaiian guitar"); (2) *pizzicato* (the usual form), where each attack, single (one string) or compound (several strings; this sounds like an arpeggio) is produced by individual plucking. The regular pizzicato is marked *pizz.* and the pizzicato legato is indicated by a *pizz.* and a *slur:* pizz. . From the violinist's standpoint, there is also a distinction between the right-hand pizzicato and the left-hand pizzicato (the latter is indicated by a cross [+] above the note; it is mostly used on open strings, and can be easily executed amidst rapid passages of bowing).

Bowing positions in relation to the sections of the bow

Insofar as the manner of playing is concerned, the bow may be regarded as consisting of three sections: the nut (lower part), the middle section, and the head (upper part), which, in international musical terminology, corresponds respectively to: (1) *du talon*, (2) *media* (or: modo ordinare) and (3) *a punta d'arco.*

When specific sections of the bow are to be used, the composer must make corresponding indications. However, *du talon* is associated with *martellato*; *a punta d'arco* is associated with high-pitched *bowing tremolo* in pianissimo; and *media* simply serves as a symbol for cancellation of one of the previous special forms of bowing.

Bowing positions in relation to the fingerboard and the bridge

There are three such basic positions: (1) over the fingerboard (usually at its widest part), known and marked as *sul tasto;* this effect produces a delicate flute-like quality; (2) in the usual place between the fingerboard and the bridge (usually slightly closer to the bridge), indicated also as *media* or *modo ordinare*, used mostly for cancellation of the preceding or the following effect; (3) very close to the bridge, marked as *sul ponticello*, which is mostly used in *bowing tremolo;* this produces a nasal "double-reed" quality.

It is possible, while performing the bowing tremolo, to move the bow *gradually* from *sul tasto* to *sul ponticello* or back. This is a neglected but very valuable technique, by which a gradual modification of quality (*tasto* corresponds to flute; *ponticello*, to double-reed) can be obtained on all the stringed-bow instruments.

Bowing tremolo (i.e., rapid forward-backward movement of the bow) must not be confused with *tremolo legato*, which is a finger-tremolo (like the trill, only in a different pitch-interval).

3. RANGE

The range of the violin, as employed by composers, *grew upward* during the 18th and 19th centuries. It was the desire of some of the outstanding composers to extend violin pitch beyond the range known to their predecessors. This evolution of range must be considered now to be completed, so far as the known type of violin is concerned. The reason for this is that Rimsky-Korsakov employed (as a pedal point), at the very opening of his opera *Kitesh*, *b* of the *third octave* (the highest *b* on the piano keyboard), which happens to lie (that is, the point of finger-pressure) at the very end of the fingerboard. During Beethoven's time, the upper limit was at *c* of the same octave.

Only the e-string is used in such a wide range; all other strings are used within the range of a *ninth* (14 semitones); however, the range of g-string is frequently extended to a *twelfth* and even more (the purpose of this is to obtain the specific quality of high positions on that string).

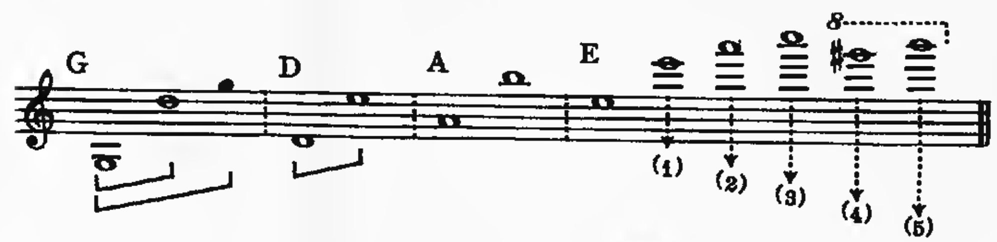


Figure 15. Range of the violin.

On the E string note the following:

(1) represents the limit for *cantabile* in unsupported unison (i.e., without octave doubling) and corresponds to the upper limit of the highest human voice, i.e., coloratura soprano; it is also the limit for pizzicato, after which limit the sound becomes too dry;

(2) Haydn's limit;
(3) Beethoven's limit; also the limit of *free orchestra-playing*, beyond which only easy passages in single notes and sustained notes (single or double) can be used;
(4) the limit in the early scores of Wagner reached *e* below this g♯; the latter was introduced in the *Ring;*
(5) Rimsky-Korsakov's *Kitezh;* no fingerboard beyond this point.

4. QUALITY

The basic resources (besides those which we have already described) of special tone-qualities on the string-bow instruments and decidedly contrasting with each other are the *mute* (double-reed quality, marked *con sordino*) and the *harmonics* or *overtones* (purest quality: sine-wave; no vibrato). The mute can be put on (con sordino) or taken off (senza sordino) wherever the composer desires, providing he gives enough time to the performer to make such a change.

Harmonics are produced on the violin by touching instead of pressing the string. The scale of harmonics can be only approximated in our system of musical notation. Harmonics are a natural phenomenon corresponding to what is known in mathematics as "natural harmonic series", i.e., 1, 2, 3, 4, 5, 6, 7, 8, 9, ...

The sound of harmonics corresponds to simple ratios of frequencies and to the partial distribution of a sounding body. In the case of strings, harmonics correspond to the division of a string into uniform sections. These sections are in inverse proportion to the order of a harmonic.

Thus, in order to get the fundamental (which is considered the first harmonic), it is necessary to let the *entire string* vibrate. In order to get the second harmonic, it is necessary to let the two halves of the string vibrate separately. The zero point between the two halves is known as "node". The finger must touch (not press) at the point of the node. The higher the harmonic, the shorter the partial division of the string (and the higher the frequencies).

The correspondence between divisions of the string and the order of harmonics is as follows:

Division of the string	Order of the harmonic
1	1
$\frac{1}{2}$	2
$\frac{1}{3}$	3
$\frac{1}{4}$	4
$\frac{1}{5}$	5
$\frac{1}{6}$	6

Beyond this limit, harmonics produced on the string-bow instruments become impractical, except perhaps for the double-bass seventh harmonic. What

violinists usually do not know, and what the composer should know is that *every node* in the same subdivision (denominator) produces identical harmonics.

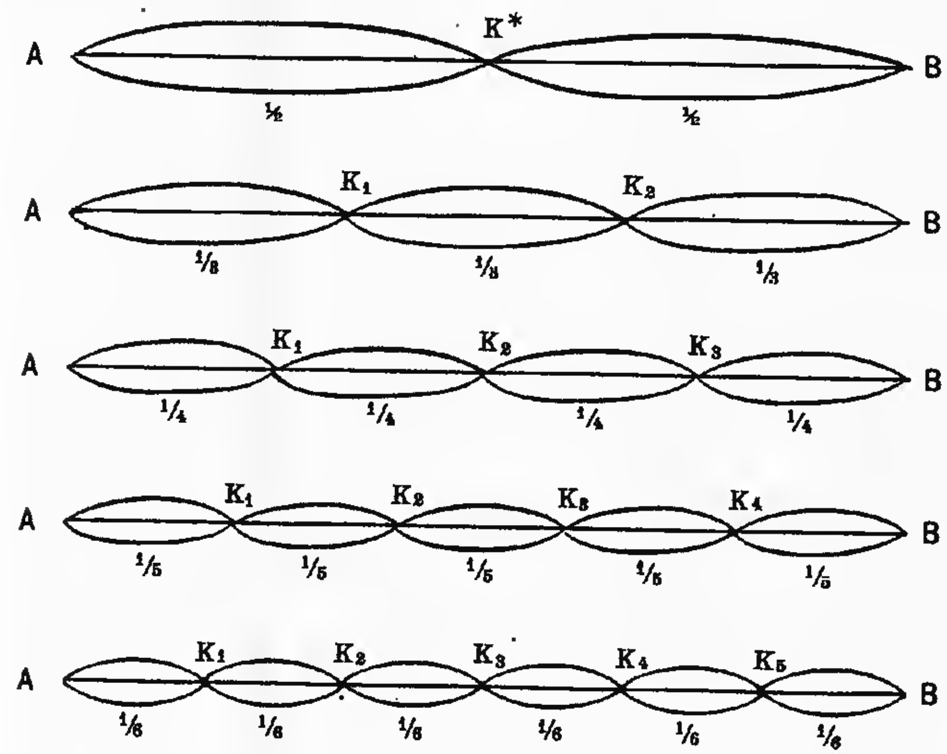


Figure 16. String nodes.

The practical consequences of this situation are the diversified ways of getting a harmonic in a passage where a violinist may think it impossible. Imagine a regular rapid passage which brings you to the upper (close to the bridge) part of the fingerboard. Now assume you want to use the third harmonic. A violinist might try to reach the point K_1 in Fig. 16 (2), while touching the string at the point K_2 would produce the same harmonic.

As more careful composers (Wagner, for instance) indicate in musical notation by a diamond-shaped note (, , , , etc.) the point of the finger-contact with the string, it is possible to carry out the above principle to a practical end.

Each string is subject to the same physical conditions, so far as harmonics are concerned. The longer the string, the more pronounced the harmonics. Thus, the *quality of harmonics increases* in the following order of instruments:

(1) Violin
(2) Viola
(3) Cello
(4) Bass

*Schillinger employed the word "knot" instead of "node." He therefore used the letter "K" as a symbol. (Ed.)

The lower the order of the harmonic, the richer it sounds. This means that lower harmonics still form physically their own harmonics (or the harmonics of the second order). Thus it is correct to state that, let us say, the third harmonic on the bass is denser than a third harmonic on the 'cello, and that the latter is denser than a third harmonic on the viola, etc. But a sixth harmonic on the 'cello may not be as dense as a second harmonic on the violin.

Here is a complete table of harmonics for the string tuned in c, which can be transposed to any other tuning. The large notes indicate the sound of the open string, the diamond notes indicate the point of finger-contact with the string and the small notes indicate the resulting pitch of the harmonic.

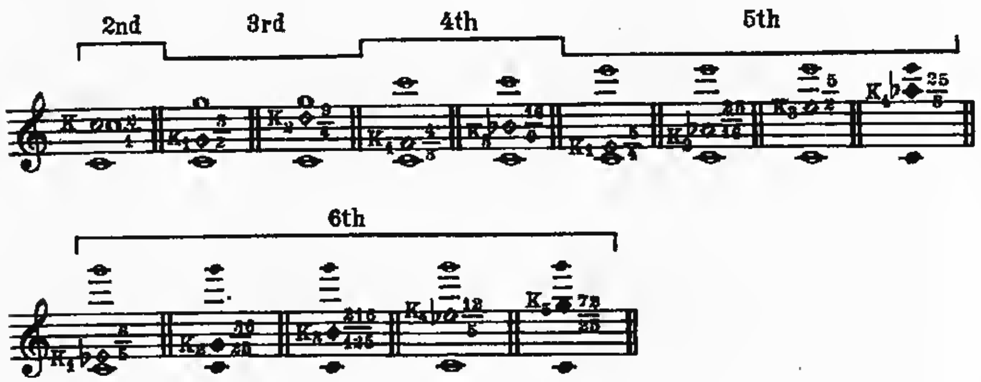


Figure 17. Table of harmonics for "c" string.

Fractions indicate the frequency ratios. All black notes indicate impractical cases.

With regard to equal temperament, the corresponding contact points (K) are practically exact:

$\frac{2}{1}$, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{16}{9}$

$\frac{16}{9}$ is very slightly lower

$\frac{5}{4}$, $\frac{25}{16}$, $\frac{10}{4}$ are slightly lower

$\frac{6}{5}$ and $\frac{12}{5}$ are slightly higher

In addition to all these harmonics, usually called "natural harmonics", there are harmonics produced by pressing the string with one finger and touching with another. The latter are called by the violinists "artificial harmonics". In reality harmonics cannot be artificial. What would you think of an "artificial sunset"?

The pressing finger shortens the string, and the touching finger produces the respective partial subdivision. There is only one harmonic which is practical under such conditions: *the fourth harmonic. The pressing finger is always the first finger and the touching finger is always the fourth.* The practical advantage of this device is its chromatic universality, which permits the performance of any melodies in the form of harmonics.

Figure 18. Melodies in the form of harmonics.

B. Viola

The viola differs from the violin mainly in its tone-quality and in the possibilities for virtuosity. Its tone quality is "somber" as compared to that of the violin. The technique of performance is more difficult than on the violin. The reason for this is that though the dimensions of the viola are greater, the system of fingering remains the same. Thus, playing the viola requires greater stretching of the fingers. In most cases, the unsuccessful but broad-handed and practical minded violinists become violists. It is interesting to note that one of the best composers of today, Paul Hindemith, is one of the best violists of today. For many years he was the leader and the violist of the excellent "Amar-Hindemith Quartette". He has composed works for this neglected instrument in the form of a concerto, sonata and unaccompanied suite.

The tuning of the viola is one fifth lower than that of the violin. The alto and the treble clefs are used in the notation of viola parts.

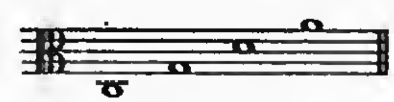


Figure 19. Tuning.

The range of viola in orchestral use does not exceed a ninth from each of the lower three strings (C, G, D) and not more than a twelfth from the upper string (A). In writing for viola solo, the upper string can be used within a range of two octaves.

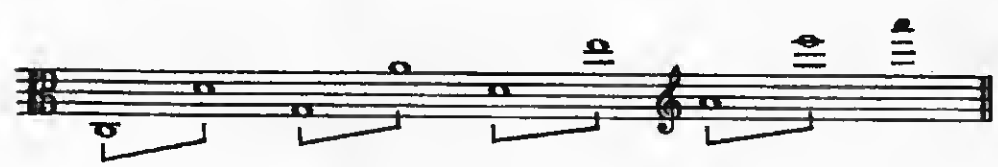


Figure 20. Range.

It is correct to say that the *viola is related to the violin as two to three.*

All forms of technical execution correspond to that of the violin. Except with regard to range, the parts written for the viola need not be limited in any respect in which the violin parts are not limited.

C. Violoncello

Violoncello means a small *violone*, which was the *bass viol* of the viol family. This is why it has a diminutive name in spite of its size. This instrument is commonly called *cello*, which does not make any sense, but conveys the association through the established use of this word. It is more correct to write " 'cello" (with an apostrophe in front).

Being held in a different position from the violin and the viola, and exceeding the latter in size (the 'cello is related to the viola as one to two and to the violin as one to three), the 'cello requires a different type of technique in fingering. The intervals on the fingerboard are wider, and the stretching is greater. Though the thumb does not have to support the instrument, it seldom participates in playing and is used on special occasions only (mainly for pressing the string while playing harmonics). The thumb is indicated as "φ". All other fingers are numbered in the same way as on the violin.

The 'cello is tuned in fifths and one octave lower than the viola. Bass (F), tenor (C) and treble (G) clefs are commonly used. Contemporary composers in most cases have abolished the tenor clef; but the 'cellists have to know it well because most composers of the past have used it in their scores.

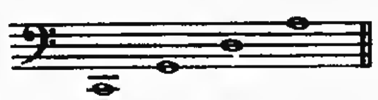


Figure 21. Tuning.

The range of the 'cello in orchestral use does not exceed a ninth from each of the lower three strings (C, G. D) and a twelfth from the upper string (A). In solo playing, however, the latter can have a two-octave range.

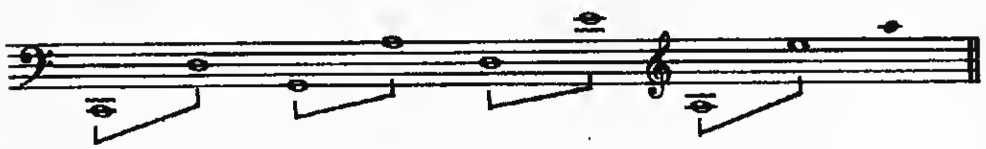


Figure 22. Range.

It is customary in ordinary passage-playing to make transitions from string to string in one position, rather than to change positions on one string. In case of chromatic scalewise passages, positions are frequently changed.

The usual fingering for the lower positions is based on the following principle:

(1) semitones are played by adjacent fingers;
(2) whole tones by alternate fingers;
(3) chromatic scales are played with continuous changes of positions, each position emphasizing three fingers: the first, the second and the third;

(4) all executions of double-stops, chords and rapid arpeggio are based on the above forms of normal fingering; as a consequence, the chords which are easy to play are either in open positions or contain open strings;
(5) perfect fifths are played with one finger on two adjacent strings;
(6) all "artificial harmonics" are played with the thumb (pressing) and the third finger (touching).

Figure 23. Examples of fingering.

All the forms of bowing, practical for the violin, are practical for the 'cello. As the bow of the 'cello is proportionately shorter than that of the violin, the composer must use long durations of single notes and of passages emphasized by the bow moving in one direction with discrimination.

One of the 'cello's features are harmonics. Owing to long strings, they are very sonorous. For the same reason pizzicato is richer on the 'cello than on the violin. Pizzicato glissando (marked: *pizz.* and a slur over the two bordering notes), produces a colorful effect similar to Hawaiian guitar. (See *Four Hindu Songs* for voice and orchestra by Maurice Delage).

Glissando of harmonics is another effect to which the 'cello is particularly suited. In order to execute it, touch the string at the nut and move the finger quite fast toward the central knot of the string. This causes a sequence of harmonics from high to low ones. Moving in reverse, i.e., from the central (node) knot to the nut, causes the reversal of the sequence of harmonics. There is no need to move the finger beyond the central knot (node) as the string has an axis of symmetry for all the knots (nodes), and such a finger movement would produce the same harmonics as when moved from the central knot (node) back to the nut. The resulting effect has great color value and has been used by the best orchestrally-minded composers. It sounds like a rapidly moving arpeggio of a large seventh-chord.

A combination of such harmonics glissando played by several 'cellists on different strings, and also in different directions if desired, produces a shimmering effect of fantastic harps, subtle and fragile.

The adopted notation of this effect is as follows (black notes show the *main points* of the actual sounds as all the points cannot be expressed in our musical notation).

Figure 24. Harmonics glissando.

See Rimsky-Korsakov's opera *Christmas Night.*

D. Double-Bass (Contrabass)

The double bass (corresponding to the antiquated violone) has *four* strings usually. They are tuned in fourths.

Figure 25. Tuning.

In the 18th and 19th centuries when a lower note was required, the bassists re-tuned the lower string to E♭ or to D. In the 20th century the problem has been solved by the addition of a fifth string (below the fourth regular string), which is tuned in C. All large symphonic and operatic organizations have at least half of their string basses equipped with five strings.

High positions are used less frequently on the string bass than on any other string-bow instrument.

The range, practical for orchestral use, is as follows: Double bass always sounds one octave lower than the written range.

Figure 26. Range.

All forms of bowing and effects, including the use of mutes, *pizz.*, glissando, harmonics and harmonics glissando, are perfectly suitable for the bass, though they are sometimes unjustly neglected.

Fingering technique and intonation are the chief difficulties of this instrument. The fundamentals of fingering are as follows:

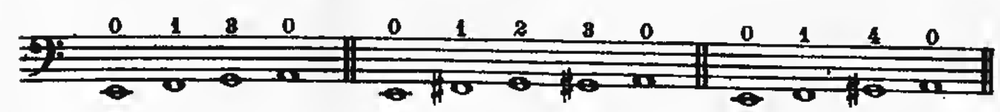


Figure 27. Fingering.

The last case is quite difficult and must be avoided, unless absolutely necessary.

As higher positions require closer spacing, it is easier to play the bass in the higher positions. The purity of intonation increases, but it becomes more and more difficult to get a pleasing tone. It is best not to use the double-stops at all as they sound muddy in low register anyway. However, certain forms of pedal and strata can be used.

Figure 28. Forms of pedal and strata.

Chords are impractical even when possible. Some composers have written solo passages and phrases for the bass, and have exceeded on such occasions the established orchestral range. See Rimsky-Korsakov's opera *Coq D'Or* in which a bass solo is written in the alto (C) clef.

There are very few outstanding bassists who appear as soloists. Probably the best of all bassists in the whole history of this instrument is Sergei Koussevitsky (at present the conductor of the Boston Symphony Orchestra). When Koussevitsky was younger, he frequently gave recitals on the double bass as well as played concertos with his own orchestra (which was known as the Koussevitsky Orchestra in Moscow, Russia). As bass literature is limited, Koussevitsky often played his own transcriptions of concertos written for some other neglected instruments. Thus, one of his favorites was Mozart's concerto for a bassoon (Fagotto) with orchestra. Another accomplished bassist (at present with the Radio City Music Hall Orchestra in New York) also comes from Russia. His name is Michel Krasnopolsky.*

When used as a solo instrument, the double bass must be tuned a tone higher and read a minor seventh down. It really becomes a bass in D. Some of the outstanding violin-makers in Italy made a few excellent basses, which are slightly smaller in size and permit the tuning one tone higher. They are better in tone too.

In jazz, the double bass is used mostly as a percussive instrument: it is plucked (pizzicato) and slapped. It is interesting to mention that in jazz playing, where virtuosity on some orchestral instruments leaves the classical way of playing far behind, the development of the performer's technique influenced mostly the right and not the left hand and, even then, not in bowing. This particular form of virtuosity produced some proficient performers. There are two duets for piano and double bass on Columbia records: *Blues* and *Plucked Again* (Columbia, Jazz Masterwork, 35322), with Jimmy Blanton (bass) and Duke Ellington (piano).**

*In Russia, he played among other things Schillinger's own "Suite for Double-Bass and Piano" composed in 1921. (Ed.)

**The catholicity of Schillinger's interest and the range of his information always astounded Schillinger's students. (Ed.)

CHAPTER 2

WOODWIND INSTRUMENTS

A. The Flute Family

1. FLAUTO GRANDE (FLUTE)

This instrument, known as a "large flute" in contrast to the smallest member of this family known as a "small flute," or Flauto Piccolo, or just plain "Piccolo" (which is as bad as "cello"), is a D-instrument without transposition. This means that, whereas its natural tones, i.e., the tones produced by modification of blowing and not by using holes and keys, have D as their fundamental, the tones sound as they are written. Tones which are not in the acoustical scale are produced by means of six holes and a number of keys (depending on the make). Opening of the holes from the foot-joint up shortens the air column and produces the tones of the natural major scale in D, i.e., d, e, f#, g, a, b, c#. The following d is the second natural tone (harmonic) from which the scale can be executed further in a similar fashion. All chromatic intervals are filled out by means of keys. The two (in some makes, three) tones below the fundamental d are executed by extending the bore with a pair of specially designed keys, which close instead of opening the holes.

Being cylindrical on the outside, the bore of a flute may be an inverted cone inside although with a very slight deviation from a cylinder. The shape of the bore and the form of exciting the air column directly (through an open hole), instead of through a mouth-piece of any kind gives the flute its whistle-like tone-quality.

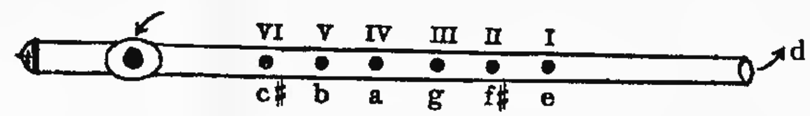


Figure 29. Harmonics.

As a consequence of this construction, the easiest keys for the flute are D, A, G, etc., i.e., keys adjacent to D through their signatures.

The flute is particularly suited for scalewise passages (which can be played at any practicable speed) and close forms of arpeggio (E_1). Finger technique is highly developed among flutists. All forms of *tremolo legato* (arpeggio of couplings), *trills, rapid grace-note scalewise passages* are typical of the flute.

Another flute specialty is the multiple-tongue effects: double, triple and quadruple, which as the name shows, are accomplished by a rapid oscillatory tongue movement. There is no special notation for this effect, and every flutist knows it should be used when there is a rapidly repeating pitch.

It must be understood that the term "legato" (indicated by a tie), as applied to flute as well as to *all wind instruments* (including woodwind and brass), means a group of notes executed in one breath. As non-legato, staccato, etc., are also executed in one breath for a group of notes, legato means one breath without a renewal of the tongue-attack.

Increase in the number of attacks augments the volume of the instrument and should be used in all cases when the natural volume is weak; yet harder blowing may produce the next natural tone. As a special device for both increasing the volume and giving the tremolo effect, *frulato* (flutter-tongue) is used. In order to execute *frulato* (which is only practical in the high register), it is necessary to pronounce (in a whispering manner) a continuous rolling of frrr. The notation for *frulato* is: ～～ for the period of duration of the note.

Because blowing the flute is immediate, the air column in the bore is quite unstable. This causes great sensitivity of registers. Each register has its own dynamic characteristics. Consideration of the latter is of the utmost importance in orchestration. Contemporary manufacturers are constantly seeking a scientific solution for equalization of registers. To put it plainly, each register, unless very skilfully handled, sounds like a somewhat different instrument. When one melodic group occupies more than one register, the contrast between the registers becomes very undesirable. Some old-fashioned minds think it desirable to have nearly each tone in a different flavor, because they believe it attributes individuality to the instrument. This assumption is psychologically wrong because each sound does not sound *per se*, but in connection with preceding and following sounds. Imagine a book where each character is printed in a different type. It certainly attributes individuality to each letter, but at the same time makes the process of reading far from pleasurable.

Uniformity of tone-quality throughout the entire range is the main weapon of attack against electronic instruments because such instruments have a much greater qualitative stability than woodwind instruments. In other words, electronic instruments are condemned by the reactionaries while great string instrumentalists try hard to conceal bow changes from one string to another (which is equivalent to the change of registers).

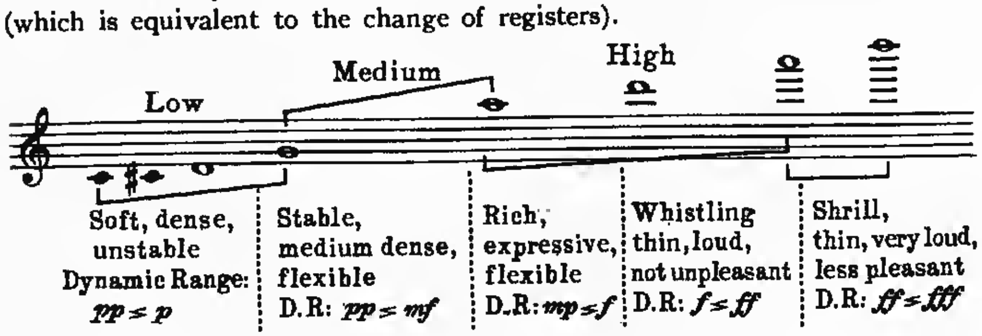
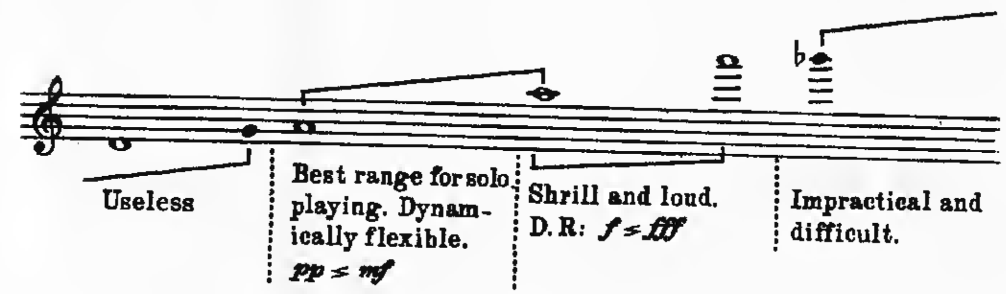


Figure 30. Range and registers of the flute.

2. FLAUTO PICCOLO (PICCOLO)

This is a diminutive flute and possesses all the main characteristics of the large flute. Its acoustical scale is also in D, but its range is much more limited. The lower register is practically useless, except for some humorous effects. The agility of this instrument is truly remarkable, and particularly so in the scalewise passages.



(Sounds one octave higher than written)

Figure 31. Range and Registers of the Flute Piccolo.

3. FLAUTO CONTRALTO (ALTO FLUTE)

This comes in two sizes (or types):

(1) Fl. Contralto in G.
(2) Fl. Contralto in F (used less than the one in G).

Both types are used a great deal in operatic and symphonic scoring.

The main value of the alto flutes lies *not in extending the range below the ordinary flute, but in giving a better quality and a more stable range* corresponding to the low register of flauto grando.

Fl. Contralto in G sounds a perfect fourth (5 semitones) lower than the written range.

Fl. Contralto in F sounds a perfect fifth (7 semitones) lower than the written range.

The first of the two has a better tone quality.

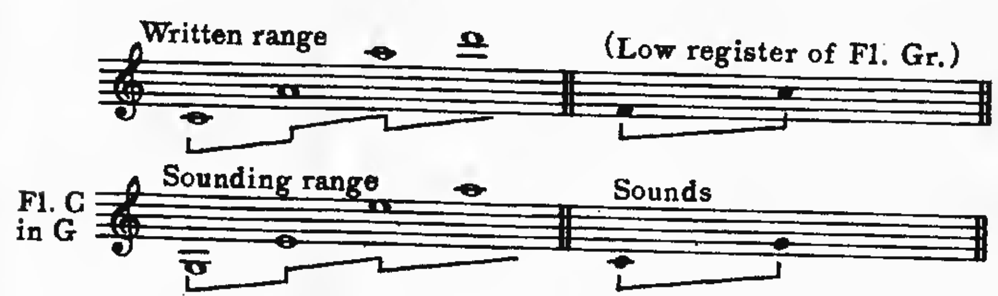


Figure 32. Range and registers of the Alto Flutes (continued).

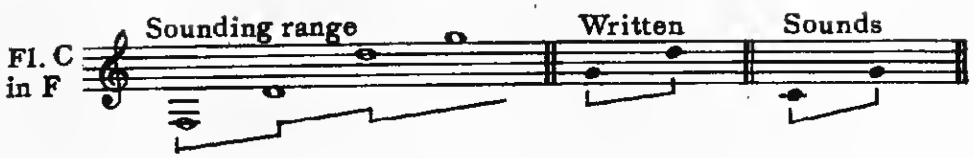


Figure 32. Range and registers of the Alto Flutes (concluded).

There is no need to use the high register of alto instruments as the regular type gives a better tone-quality.

Other types, such as Bass Flutes, are obsolete nowadays. They produce tones in quality somewhere between the *ocarina* and an *empty bottle.*

B. The Clarinet (Single-Reed) Family

1. CLARINETTO (CLARINET) IN B♭ and A

This instrument has a cylindric bore, which causes, according to Helmholtz, the appearance of only odd (1, 3, 5, 7, 9, . . .) harmonics. The even-numbered harmonics are absent. This situation creates a gap of 18 semitones between the fundamental and the next (i.e., the third) appearing harmonic. Somehow the designers of this instrument succeeded in reducing the number of holes and keys considerably (usually to 13) though theoretically 18 holes are necessary in order to produce a chromatic scale covering the gap.

From the performer's angle, the clarinet is a difficult instrument to master. However, this should not worry the composer as accomplished clarinetists are really in abundance. The main consideration for the composer to bear in mind is that while approaching the third harmonic, the tone of the clarinet weakens for about the last 6 semitones. The register between the fundamental and the third harmonic is known as *chalumeau* (French, from Latin "calamus"—reed; originally, a single reed instrument with a built-in reed, now obsolete; probably the ancestor of clarinet). A special tone-quality, in addition to the usual one, and one which is hard to produce, corresponds to the chalumeau register and is known as *subtone* (soft, delicate and tender). Starting with the third harmonic and going up, the tone-quality of the clarinet changes noticeably. Of course, it is the task of an accomplished performer to neutralize this difference.

The sound on the clarinet is produced by blowing into a detachable mouthpiece to which a reed is attached. A complete chromatic scale is produced by means of various types of keys and by holes which are covered by the fingers (special keys on the bass clarinet). The clarinettists of American dance orchestras are able to produce a *glissando* (i.e., continuous pitch modulation between two frequencies). This is accomplished by the *embouchure* (which usually means "the assumed position of lips combined with lip-pressure"). Symphonic and operatic clarinettists are not trained to play glissando.

The scale of natural tones on the clarinet is *written* as follows (and sounds as written when played on a clarinet in C):

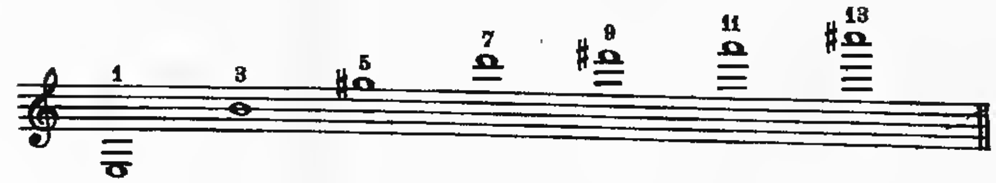


Figure 33. Natural tones of the clarinet.

The clarinet in C was discarded a long time ago because its tone quality was not as satisfactory as that of the clarinets in B♭ and in A (some contemporary manufacturers make an extra hole and key to compensate the lower semitone on the B♭-clarinet; thus it can play the parts written for the A-clarinet; in other instances, mechanical adjustments have been made in order to obtain a combined version of the B♭ and the A clarinets).

Though some individual performers get far beyond the common range, there is an unwritten international code of ethics by which composers limit themselves to the written *g* of the second octave.

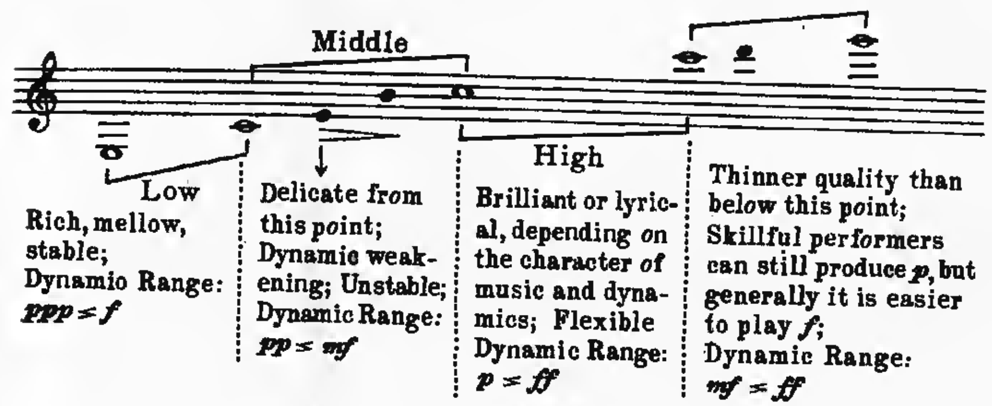


Figure 34. Range and registers of the clarinet.

For the clarinet in B♭ the above table *sounds one tone lower.* This means that the composer must write his parts for the B♭ instrument *one tone higher* than he expects to hear the actual sounds. For instance, the part which sounds in the key of *C* must be written in the key of *D*. Thus, the clarinet in B♭ acoustically is a *D* instrument, as its fundamental tone (by sound) is *d*.

Likewise, parts for the clarinet in A must be *written three semitones higher* than they are expected to sound. Thus the above table sounds three semitones lower. Parts expected to sound in the key of *C* must be written in the key of *E♭*. Thus clarinet in A acoustically is a C# instrument, as its fundamental tone (by sound) is *c#*.

It was believed in the 19th century that the B♭-clarinet represented the masculine quality, and that it was more substantial but less delicate than the feminine quality of the A-clarinet. However, today skilful performers can obtain both characteristics on the B♭-clarinet.

Considering the quality of manufacture and the skill of contemporary performers, we may say that the clarinet can play practically everything. Its specialties are: rapid diatonic and chromatic passages, tremolo legato and trills. Staccato is preferable in its soft form. Arpeggio of the E_1 form is very grateful both ascending and descending.

2. CLARINETTO PICCOLO IN D AND E♭

The first instrument (D) is used in symphonic and operatic orchestras and the second (E♭), in military bands. Both these instruments are inferior in their tone quality as compared with the clarinets in B♭ and A.

The acoustical range of the D-clarinet is in F♯. It is *written one whole tone lower* than it is expected to sound. The parts which are written in the key of *B*♭ sound in the key of *C*.

The acoustical range of the E♭-clarinet is in G. It is *written three semitones lower* than it is expected to sound. The parts which are written in the key of A sound in the key of C.

Except for tone-quality, the piccolo clarinets can be favorably compared with the regular clarinets: their mobility is as high.

3. CLARINETTO CONTRALTO (ALTO CLARINET) and CORNO DI BASSETTO (BASSETHORN)

Clarinetto contralto is usually an E♭, but sometimes an F instrument. Thus its part should be written a *major sixth* and a *perfect fifth* higher, respectively, than the sounding keys. The F instrument is so constructed that its lowest written note is *c* below the usual *e*. The tone-quality of each of these instruments can be described as more "hollow" than the tone of a regular clarinet.

Corno di bassetto has a smaller bore than the clarinet. It looks somewhat like a miniature version of the clarinetto basso (bass clarinet). Its tone-quality is more "reedy" than that of the clarinet. The bassethorn is an instrument in F: it is *written a perfect fifth higher* than it sounds. Today the bassethorn is becoming more and more obsolete: the alto clarinet in E♭ takes its place.

4. CLARINETTO BASSO (BASS CLARINET) in B♭ and A

The A instrument is seldom used outside of Germany. Both these instruments sound one octave below their respective regular clarinets. This means that the B♭-basso is written a major ninth higher than it sounds; A-basso is written a minor tenth higher than it sounds when the treble clef is used. In German scores, both treble and bass clef are often used.

The rule is that in using the bass clef, *write one octave below* the corresponding note of the treble clef: that is, the transposition of sound from the bass clef is only a whole tone, or a tone-and-a-half down instead of the major ninth or minor tenth as in the treble.

Both these instruments are manufactured with and without the lower extension from *e* to *c*. The B♭-basso without lower range extension is used by dance orchestras, whereas the B♭-basso which reaches the lower c (*b*♭– by the sound) is used in symphonic and operatic scoring. These instruments have quite a sinister tone in their lower register. It is wise not to write for the bass-clarinet above *d* of the second octave. The bass-clarinet possesses somewhat less mobility than the smaller clarinets.

There is also a *contrabass* or *pedal clarinet*, a monstrous affair which has to be suspended on special stands and which is very hard to play. Richard Strauss used one in his *Electra*, but apparently only the Germans could play it. It sounds one octave below the bass clarinet (it is also in B*♭*) and has an awe-inspiring quality.

C. The Saxophone (Single-Reed) Family

The saxophone is one of the numerous creations of Adolf Sax, an eminent instrument designer of the 19th century. This instrument is a crossbreed between the oboe (owing to its conic bore) and the clarinet (owing to its single-reed mouthpiece).

Very few composers used this instrument in the 19th century (one of them was Georges Bizet) and eventually it became quite obsolete, with the exception of its use by military bands in France and Belgium, which have employed saxophones widely.

The original saxophone family consisted of instruments in C and in F.

Soprano	Saxophone	in	C
Alto	"	"	F
Tenor	"	"	C
Baritone	"	"	F
Bass	"	"	C

American manufacturers rejuvenated interest in this instrument. They succeeded in constructing saxophones of a more improved design. American saxophones as played by American saxophonists have introduced a whole new style of music and musical execution.

American-made saxophones are so flexible that any type of part can be written for them. Rapid scales, arpeggio, tremolo legato, trills, staccato, glissando are all possible and grateful on this instrument. The last two or three decades have produced a number of outstanding virtuosi, many of whom are Negroes and many of whom are skilful improvisers. It is due to the wide influence of jazz and jazz-playing that saxophone manufacture has become a considerable industry.

Standard dance-band combinations customarily use 4 or 5 saxophones. In some instances this number varies. It is quite common for a saxophonist to double as a clarinettist. Some performers are equally good on both instruments.

In the earlier days of American jazz (and also in some instances in Europe) there were some ensembles consisting only of saxophones, but they have not survived.*

The American family of saxophones is tuned in B♭ and E♭.

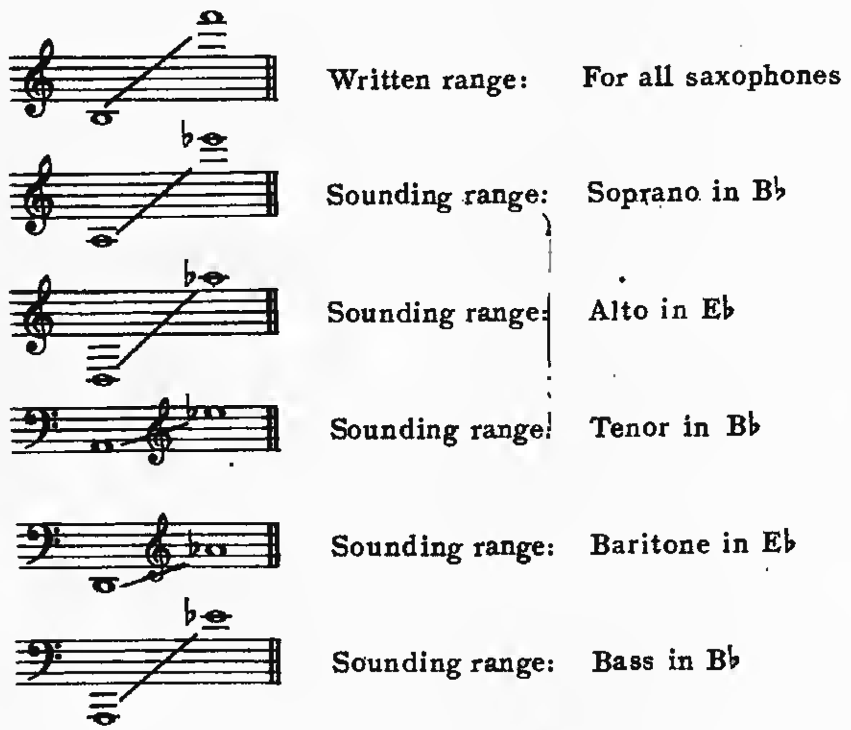


Figure 35. Saxophones.

The soprano and the bass are seldom used today. All saxophone parts are written in the treble clef. There is no noticeable difference of registers in a good performance, and it is for this reason that we have omitted range subdivisions.

D. The Oboe (Double-Reed) Family

1. OBOE

The oboe is an instrument of ancient origin. In its primitive form it was in wide use throughout Asia. One of the oboe's ancestors was the Hellenic *aulos*, which was used for the expression of passion.

Blowing through the narrow opening of the flatly folded reed (usually called double reed) requires strong lungs and a peculiar technique of breathing. Some of the Asiatics (Persians, for example) can play the oboe-like double-reed instruments with uninterrupted sound (like the Scottish bagpipe). These performers

*Recently a well-known band leader, Shep Fields, organized a large, successful dance orchestra which consisted, apart from several percussion instruments, wholly of saxophones. (Ed.)

usually hold a reserve supply of air in one cheek, which is exhaled, i.e., blown into the reed, while the lungs are inhaling a new supply of air.

The contemporary oboe has a conic bore, which characteristic permits the appearance of the full scale of natural tones (harmonics).

Without additional keys, the oboe acoustically can be considered an instrument in D, like the flute. The oboe, like the flute, is not a transposing instrument. Most oboes of European manufacture have *b* of the small octave as their lowest tone. American-made oboes reach *b*♭, immediately below it. It is customary not to use the oboe above *f* of the second octave. Owing to its construction the oboe is a slow-speaking instrument. Only passages of moderate speed are possible on this instrument. The oboe is valued mainly for its characteristic tone-quality, which can be described as "nasal" and "warm."

All types of passages are possible, including tremolo legato and trills, providing they are executed at a speed which seems moderate compared to flutes and clarinets. One of the most valuable characteristics of the oboe is the versatility and distinct character of the attack forms. The legato, the portamento, the soft and particularly the hard staccato appear on the oboe with clear distinction.

The density of the oboe's tone decreases considerably in the upper part of its range. The low register is somewhat heavy and has a natural volume increase in the direction of decreasing frequencies. The most flexible and expressive part of the range is the middle register. High tones are thin and shrill.

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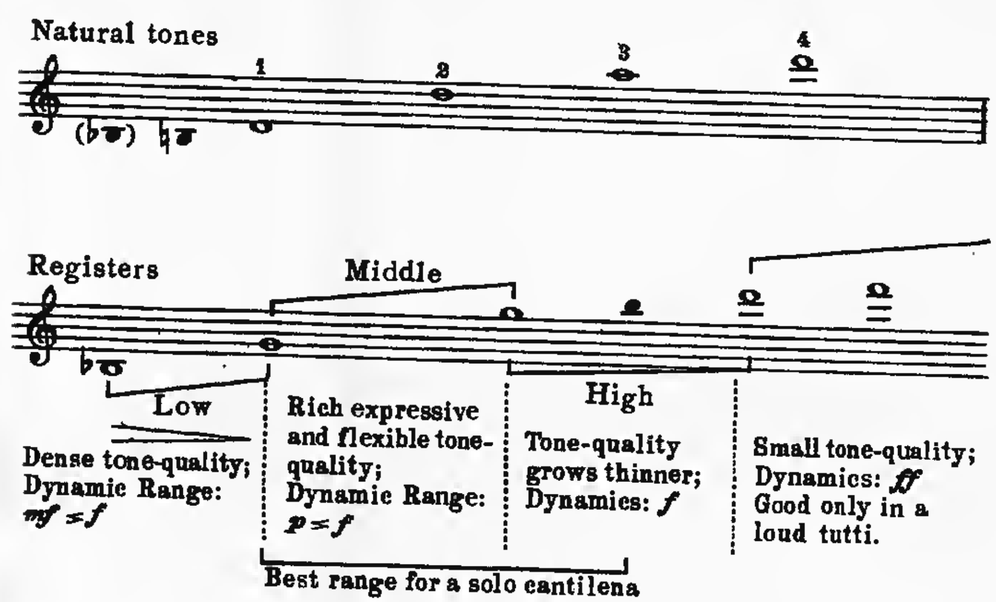


Figure 36. Range and registers of the oboe.

2. OBOE D'AMORE

A mezzo-soprano type of oboe which is now rarely used. J. S. Bach used it in his *Christmas Oratorio*. It was revived by Richard Strauss in his *Sinfonia Domestica*.

This is a transposing instrument in A♭.

Figure 37. Range of Oboe d'Amore.

3. CORNO INGLESE (ENGLISH HORN)

The immediate predecessor of this instrument is the *oboe da caccia* (hunting oboe), now obsolete. The contemporary version of corno inglese (also known as *oboe contralto*) represents an instrument similar in most respects to the oboe, but sounding a perfect fifth lower. It is a transposing instrument in F.

The middle octave is its best register for an expressive solo. The low register is denser and heavier than that of an ordinary oboe. The high register is seldom used beyond the written *d* (sounds *g*) of the second octave. All other characteristics correspond to oboe. It is still a somewhat slower-speaking instrument than the oboe.

The English horn is exceptionally suitable for the expression of passion and suffering. In orchestral scoring it is often given a solo. One of the famous solos is in Wagner's *Tristan and Isolde* (Prelude to the third act).

Figure 38. Range of corno inglese.

4. HECKELPHONE (BARITONE OBOE)

The baritone oboe is an instrument of German manufacture (made by Heckel) which, in its perfected form, was introduced about 1905. The tone has a quality of overwhelming richness and expressiveness. Richard Strauss used it first in his opera *Salome;* Ernst Krenek also employed it in his opera *Sprung Ueber den Schatten* ("Leap Over the Shadow"). It is an instrument well deserving wide use together with the oboe and English horn.

The heckelphone is made to sound one octave below the oboe; it sounds one octave below the written range. Its size is so big that the bell of the instrument rests on the floor, while the performer is playing it from a sitting position.

The key and hole system is designed to resemble that of an ordinary oboe, which construction makes it easy for an oboist to master the heckelphone.

As the range and the registers of this instrument correspond exactly to that of an oboe (the lowest tone is *b*♮), but sound one octave lower, there is no need for a table of range and registers.

E. The Bassoon (Double-Reed) Family

1. FAGOTTO (BASSOON)

The name "fagotto" derives from "faggot": a bundle of sticks; the name "bassoon" from the association with bass register.

The bassoon is an instrument with a very long conic bore (about eight feet), which is folded upon itself, somewhat in the manner of the letter "u". This u-shape makes it possible to have a system of accessible holes and keys. Some of the key-holes produce only one tone (the lower keys) and some, two (octave variation is easily produced by lip-pressure).

Being an instrument with a conic bore and a double-reed mouth-piece, the bassoon may be considered a bass of the double-reed group, i.e., it is a natural bass to the oboes.

The main difference between the oboe and the bassoon lies in the fact that the latter has an additional section, which extends its low register.

Under the same conditions of fingering (with the basic six holes closed), the bassoon is a perfect twelfth below the oboe, i.e., under the conditions which produce the middle *d* on the oboe, the bassoon produces the *g* one twelfth below.

The range of the bassoon (for all practical purposes) begins with the *b*♭ of contra-octave and ends with *d* of the first octave. The b♮ tone at the lower end is of somewhat poorer quality than all the other tones of the low register.

This instrument is capable of mobility, noticeably greater than that of an oboe. Various forms of arpeggio (practically in all expansions), octaves and leaps in general, as well as rapid scalewise passages, tremolo legato and trills constitute the versatile technique of this instrument. The attacks are distinct. Legato, portamento, soft and hard staccato (the latter being the bassoon's specialty and possible at a considerable speed) can be executed with quick changes.

Bassoon parts are written in the bass and the tenor clefs (though alto-clef may be used as well). It is not a transposing instrument.

The dynamic peculiarities of the bassoon require particular attention on the part of the composer. The low register (from *b*♭ of the contra-octave to *c* of the small octave) is the most powerful part of the bassoon's range. It weakens slightly toward the middle register (this begins with *c* of the small octave and ends with *c* of the middle octave), which is considerably weaker than the low register. The high register, from *c* to *g* of the middle octave, is somewhat harsh; it becomes very mellow from *g* of the middle octave to *d* of the first octave. Stravinsky is one of the few composers who has utilized this upper region effectively (the opening bassoon solo at the beginning of the *Rites of Spring*).

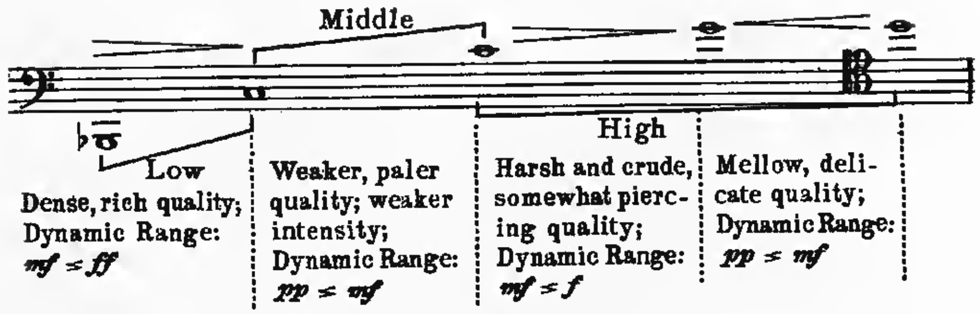


Figure 39. Range and registers of the bassoon.

2. FAGOTTINO (TENOROON, QUINTFAGOTT, TENORFAGOTT)

This instrument (now practically obsolete) was built a perfect fourth and a perfect fifth above the regular bassoon. Both types are transposing instruments: tenoroon in E♭, sounding one perfect fourth higher than written and tenoroon in F, sounding one perfect fifth higher than written. The tone-quality of these instruments is inferior to that of the regular bassoon.

3. CONTRAFAGOTTO (DOUBLE-BASSOON, CONTRA-BASSOON, CONTRAFAGOTT)

This instrument, still of greater dimensions, is meant to be the lower octave-coupler to an ordinary bassoon. The engineering quality of this instrument, being inferior to that of a bassoon, causes inferior tone-quality and less exacting intonation. The tone of this instrument is somewhat dry and does not sound as healthy as the tone of the bassoon. Its alertness is also somewhat lower.

As the contrabassoon is an instrument built mainly to produce low frequencies, it must not (except for some special purposes, such as creating associations of a "humorous" or "painful" nature) be used beyond its regular middle register.

The contrabassoon is a favorite instrument with many composers. Its sounding range is one octave lower than written. Its lower register is considerably weaker than that of a bassoon.

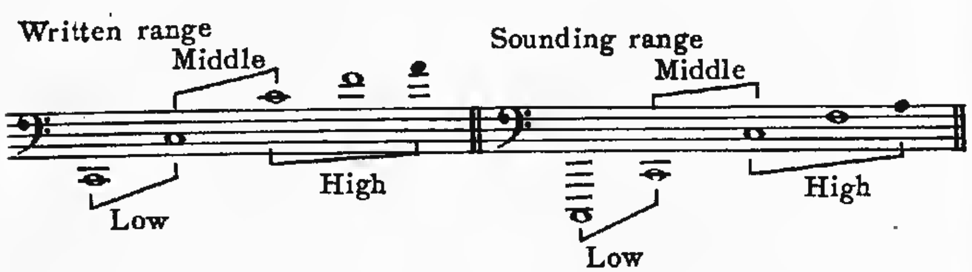


Figure 40. Range and registers of the contrabassoon.

CHAPTER 3

BRASS (WIND) INSTRUMENTS

A. Corno (French Horn)

THE horn is an instrument with a long and rich history. The immediate predecessor of the contemporary three-valve chromatic French horn was the so-called natural horn, capable of producing only the natural tones. All other tones on the natural horn were obtained by putting the fist of the left hand into the bell and varying the depth of its position within the bell. The deeper the fist is set, the lower the sound of the respective natural tone. This manner of altering natural tones is based on the physical principle of *open* and *closed pipes:* an open pipe sounds one octave higher than the same pipe closed. As the gradual conic pipe (which is coiled around itself) extends in a horn to about seven feet, the partial closing of this pipe by a fist, at the bell, lowers the respective natural tone only by one or two semitones. This device does not cover all steps chromatically, as the acoustical gaps between the second and the third, and between the third and the fourth tones are too great. It is for this reason that the parts written in the 18th and early 19th centuries were predominantly fanfare-like.

Eventually natural horns became practically obsolete. Rimsky-Korsakov used natural horns in his opera *May Night* (when chromatic horns were universally in use) for the sake of his own amusement, which he called "self-discipline".

In order to read scores by such composers as Mozart and Beethoven, not to mention Bach or Händel, it is important to have at least some basic information about the sizes and the transposition-keys of the various horns which were used in not such a remote past.

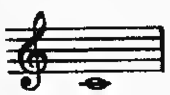
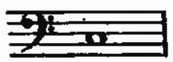
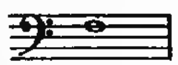
Natural horns were constructed in two main size-groups: the *alto* horns and the *basso* horns. All horns transpose downward, i.e., they sound below the written range. Alto horns transpose directly to a designated interval, indicated by the transpositional name of the instrument. Basso horns, in addition to the alto transposition, sound one octave lower (compare with the clarinet in B♭ and the bass-clarinet in B♭).

The alto horns were constructed in all chromatic keys except *G♭*. The selection of a particular horn was in correspondence with the key in which a certain piece was written. Basso horns were used, where it was essential to reach the lower register. Basso horn parts are known only in three transpositions: the B♭ – basso, the A– basso and the A♭– basso. There was no octave confusion in interpretation of the scores because it was the alto horns that were usually meant. The use of basso horns was quite exceptional. An instance of their use may be found in Beethoven's *Fourth Symphony* (written in the key of B♭).

Except for the use of valves, which secure the entire chromatic scale, there is no noticeable difference in the construction of the present day French horns (including the conic mouth-piece).

Blowing through a long narrow channel creates conditions under which it is easy to "overblow" the fundamental tone of the scale. That is, the air-column tends to break into two halves. For this reason, the officially recognized range of the horn begins with the second tone. From there on, all pitches are practical up to and including the twelfth natural tone. The sixteenth tone is seldom used nowadays. As the frequency increases, the tone-quality becomes brighter.

We shall represent now the scale of natural tones for the hypothetical French horn in C. As the chromatic horns used today are in *F*, the actual sounds appear a perfect fifth below the written range when the part is written in the treble clef; in using the bass clef, write the parts a fourth below the intended pitch, or, to state it differently, one octave below the treble clef. Thus the transposition of the French horn, when written in the treble clef, is exactly the same as that of an English horn.

Thus:  and  sound alike: 

This cumbersome octave-variation as well as the whole idea of pitch-transposition is a survival of an old tradition. The sooner it is abolished, the better; no one gains by this transpositional technique, which is a constant source of complications and confusion.

During Wagner's time and later, chromatic French horns in E were used together with those in F. They are abolished today, because of the superior tone-quality obtainable on the horns in F.

Figure 41. Scales of natural tones of the French horn.

Only in very exceptional cases is the French horn part written one or two semi-tones above the twelfth tone. The best tone-quality for solos lies between the fourth and the twelfth natural tones. The French horn provides a direct continuation of the tuba's timbre in the lower portion of its range. From the fourth to twelfth tones it acquires a gradually growing characteristic of lucidity; in its upper range, the French horn blends well with clarinets and particularly with flutes; in its lower range, with trombones, tuba and bassoons. In this sense, the French horn is an intermediary between the wood-wind and the brass groups.

The chromatic scale, as already stated, is obtained by operating the three valves. All three-valve instruments are designed on the same general principle.

The first valve (operated by the upper key) lowers the natural tone by two semitones.

The second valve (the middle key) lowers the natural tone by one semitone.

The third valve (the lower key) lowers the natural tone by three semitones.

Valves are indicated by the respective Roman numerals:

I lowers by 2 semitones
II " " 1 semitone
III " " 3 semitones

These indications are not used in scores or parts, but merely for reference, when necessary.

The operation of valves is such that while blowing the written middle *c*, for example (the 4th tone which sounds *f*), and pressing key I, one obtains b♭ (sounds *e*♭); blowing the same tone and pressing key II, one obtains b♮ (sounds e♮); blowing the same tone and pressing key III, one obtains a♮ (sounds d♮).

All other intervals, by which a natural tone can be lowered, are obtained by a combined use of keys controlling the operation of valves. Thus: I + II lowers the natural tone by 3 semitones;

I + III lowers the natural tone by 5 semitones;
II + III lowers the natural tone by 4 semitones;

the combination of all three keys lowers the natural tone by 6 semitones.

In the French horns of old make, there were some deficiencies of intonation when the combined valves were used. They are abolished in present manufacturing by a special interlocking of air columns in the valves, which device rectifies the corresponding frequency-ratios.

Valves themselves are additional short pipes, connected with the main channel by operation of the keys. The latter affect the *pistons* or the *rotary cylinders*. Cylinders are more common on the present French horn. So far as tone quality is concerned, it does not make any difference which particular mechanism is used. Thus keys open the valves, thereby connecting them with the main channel, which results in the lengthening of the air column and, for this reason, lowers the pitch of a given natural tone.

Since the change of *embouchure* (lip condition with respect to form and pressure) is never as alert as finger technique, it is preferable to write rapid passages when they can be produced mainly through the operation of keys. It is for this reason that the composer must have an exact knowledge of the key-valve operations. Even trills and tremolo legato are possible when they are obtained through the use of keys.

It follows from the above that the valve system is acoustically opposite to the whole system used on all wood-wind instruments (i.e., on the brass instruments natural tones are lowered; on the wood-wind instruments they are raised).

The French horn is a slow-speaking instrument, and for this reason speed is not limited by the finger-technique but rather by slow tone-production. All forms of legato and staccato, as well as portamento, are available and distinct. The breathing process, as applied to this instrument, is normal and healthy. It is possible for this reason to execute sustained tones or passages of considerable period in one exhaling. Contrary to the double-reed practice, playing the French horn is a healthful occupation.

Owing to the conical shape of the mouth-piece, double-tonguing is not within the scope of this instrument. One of the French horn's specialties is the dynamic effect of *sforzando-piano* (sfp). This can be performed at any point from the 3rd harmonic upward. The French horn has a wide dynamic range but its lower part weakens considerably.

The French horn is played either *open* (indicated as o) or *stopped* (indicated +). The first indication is not used, except as a cancellatinn of the "stopped." Stopping is usually indicated above each attack.

Mutes are generally applicable to French horns, but used by performers only under compulsion: they think the stopping "will do".

In volume (intensity), the French horn occupies an intermediate position between the brass (in relation to which it is weaker) and the wood-wind instruments (in comparison with which it is louder, particularly when played high and ff).

B. Tromba (Trumpet)

The trumpet is a chromatic three-valve instrument. Depending on manufacture, either cylinders or pistons are used.

Of all types of trumpets, the soprano (ordinary) type in B♭ and A is used more universally than the alto trumpet in G and F, the piccolo trumpet in D and E♭, and particularly the bass trumpet in E♭ and B♭.

1. TROMBA (SOPRANO TRUMPET) in B♭ and A

Of these two designs, preference is given to the B♭ trumpet in the U.S.A., while in Europe both tunings are used for the respective parts. American dance-bands use the B♭ trumpet exclusively.

Some of the B♭ trumpets can be converted into A trumpets, by drawing a special telescopic slide which lowers the range of the instrument by a semitone.

The trumpet part as written sounds two or three semitones lower respectively, as in the case of the clarinets.

Its scale begins with the second natural tone and ends, for all normal purposes, with the eighth. Outstanding trumpeters are able to blow the ninth, the tenth and even the twelfth tones. In this case the use of the piccolo trumpet becomes unnecessary as the tone of the regular soprano trumpet is preferable. On the other hand, the composer must not rely on the presence of a virtuoso in every orchestra, even the performer playing the part of the first trumpet.

Natural tones are produced by the embouchure, and the pitches between them by fingers, i.e., by pressing the keys which control the valves. The trumpeters

of American dance-bands produce many chromatic variations and glissando by the embouchure. These virtuosi very frequently go beyond the eighth tone. In writing "improvised" solos (which in most cases are actually written out and studied), it is best to test the individual performer's range first.

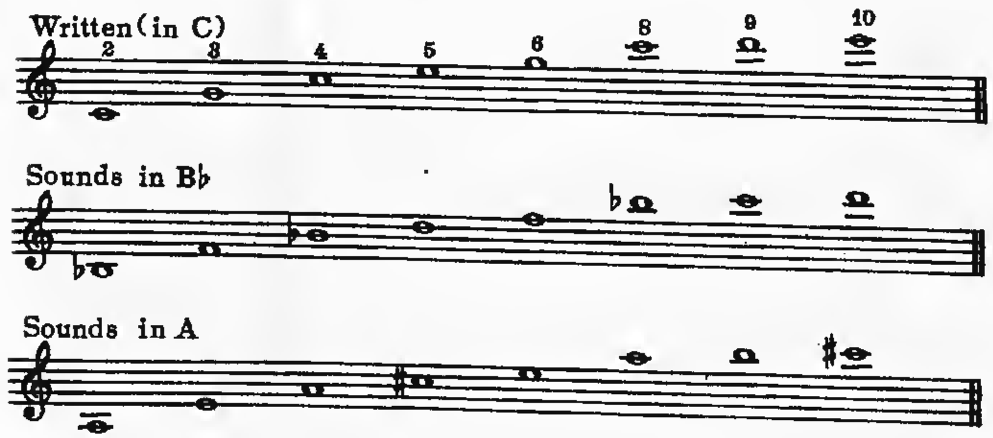


Figure 42. The range of the trumpet.

With the combined use of all three valves, the lowest tone of the trumpet is: f♯ (in C), e (in B♭), e♭ (in A). Tones below the second natural tone are generally weak. The natural intensity grows with the increase of frequency, but skilful performers have a considerable control over the dynamic range of this instrument.

The cup-shaped mouth-piece of the trumpet, the shape of the bore (slightly deviating from a cylinder to a cone) and the length of the bore make the transmission of tongue attacks more immediate. For this reason double and multiple tongue attacks become one of the main assets of the trumpeter's virtuosity (as in the case of a flute).

Rapid finger-work on the keys permits execution of trills and tremolo legato at a high speed, providing both component pitch-units are executed through the same natural tone (both pitch-units may be keyed, or one of them may be natural).

All forms of attacks are well defined on a trumpet: legato, portamento, soft and hard staccato. Scales and even arpeggio can be executed at a considerable speed. At one time the trumpet was considered as mostly suitable for a performance of signal-like and fanfare-like music, but this viewpoint (considered even by Rimsky-Korsakov) is completely outmoded. The prestige of this instrument has been amazingly restored and heightened by jazz.

2. CORNETTO (CORNET) IN B♭ and A

This instrument, also known under the French name of *cornet a pistons* (i.e., a cornet with pistons; the name implies chromatic possibilities) does not, strictly speaking, belong to the trumpet family. Its bore is more conical than that of a trumpet; this makes its tone-quality more mellow. For this reason it is considered a more lyrical instrument than the trumpet. Today, however, the skill of performers is so great that accomplished artists are able to imitate the sound of a cornet on a trumpet and the sound of a trumpet on a cornet.

In most cases, American cornettists use the B♭ instruments. It is also customary for a trumpeter to play both trumpet and cornet. The scale of natural tones, the range and the whole mechanism of execution are practically identical with that of a trumpet. The cornet is generally considered to be somewhat less alert than the trumpet. Tone-quality on both trumpet and cornet can be altered by means of a mute inserted into its bell. The use of the mute is marked "con sordino"; the cancellation of this effect, "senza sordino".

American jazz created a real mute-o-mania, resulting in a great variety of new mutes (straight mute, cup-mute, harmon-mute, etc.). Another device, closely related to mutes is the "hat" (usually made out of metal, in the shape of a trench helmet or a derby). It is used for glissando "wow-wow" effects (acoustically, a transformation of the open pipe into the closed pipe).

This instrument is the prima donna of the brass band, but it has found its way into symphonic, operatic and particularly dance scoring.

3. TROMBA PICCOLA (PICCOLO TRUMPET) in D and E♭

This instrument is considerably smaller in size than the ordinary trumpet. The D-type is mostly used in symphonic scoring (for example, Stravinsky's *Sacre*), but relatively very seldom. The E♭ type is much more common in brass bands.

The tone-quality of both is decidedly inferior to that of a regular trumpet.

The transposition of this instrument is analogous to clarinet piccolo, i.e., two and three semitones up respectively. Thus the eighth natural tone (c) sounds d and e♭ respectively. As this instrument requires an excessive lip-pressure, it is very difficult to produce any tone above the eight harmonic. For this reason there seems to be no practical advantage in the further use of this instrument.

4. TROMBA CONTRALTA (ALTO TRUMPET) in G and F

This is a very useful instrument not only for the extension of the regular trumpet's range downward, but also (and mainly so) for obtaining better quality tones within the low register (from the third natural tone down) of a regular trumpet. Rimsky-Korsakov made a very extensive use of this instrument in his operas. It is a softer instrument compared to the B♭ and A trumpet.

The lowest possible pitch on the alto trumpet is the written f♯ (three keys pressed: all valves open), which sounds c♯ on the G trumpet and *b* on the F trumpet respectively, i.e., it transposes down like the alto flute.

It is quite customary for the performer of the third trumpet part to double on the alto trumpet.

5. TROMBA BASSA (BASS TRUMPET) in E♭ and B♭

Strictly speaking, this instrument is not a trumpet but a miniature tuba and, therefore, belongs to the so-called saxhorn family (the dominant brass group of the military bands). It is also known as tenor tuba or Wagnerian tuba.

In many instances the parts written for this instrument are played by the French horns. The E♭ instrument sounds eight semitones below the written range; the B♭ instrument, one octave below the soprano B♭ trumpet. Undoubtedly this instrument will become obsolete. There is also a bass trumpet in C basso which is very seldom used. It sounds one octave below the written range.

C. Trombone (Trombone)

The trombone is one of the most remarkable instruments in the orchestra. Its design is based on an ingenious yet very simple principle: it has an air column, whose length can be varied by means of a *slide*, which is a part of the instrument proper. As a result of this construction the trombone produces a complete chromatic scale consisting of *natural tones only*.

The pulling out of the slide increases the volume of the air-column and thus produces the additional six standard *positions*. When the slide is pulled all the way in, the trombone is considered to be in the *first position*. The opposite position, with the slide pulled out (to the limit, but still producing a continuous bore or air-column, as the slide can be pulled out completely, disjoining the instrument into two individual sections) is considered the *seventh position*. All other positions are between the two extreme positions. Thus the slide actually converts seven natural instruments into one chromatic instrument. As different positions possess different acoustical characteristics, we shall describe each position individually.

The first position has the following natural scale:

Figure 43. First, second, third and fourth positions of the trombone (continued).

The second, the third and the fourth positions have similar acoustical characteristics. See following page.

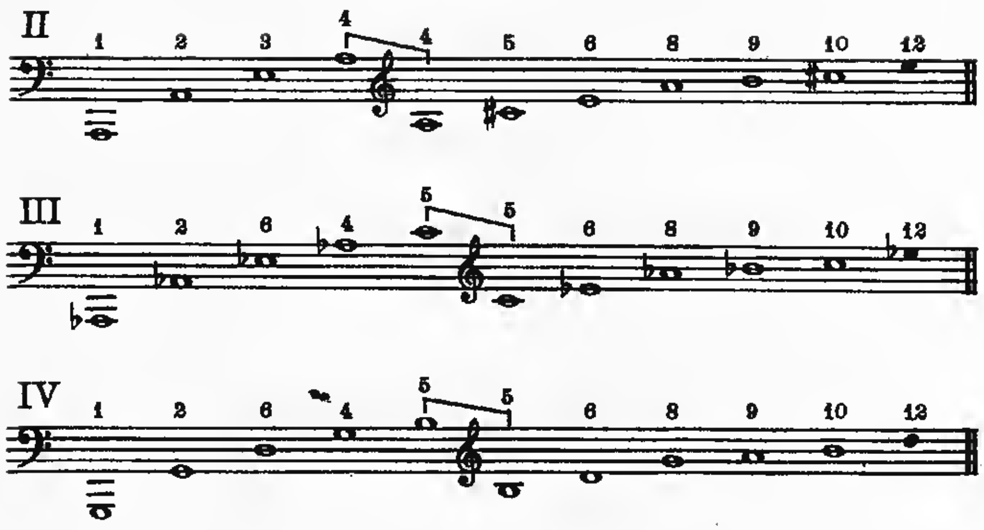


Figure 43. First, second, third and fourth positions of the trombone (concluded).

Tones produced by the fundamental are often called *pedal tones.*

Beginning with the fifth position, the air-column breaks up into two halves, thus making the production of the fundamental impossible.

The fifth, the sixth and the seventh positions have the following natural scale:

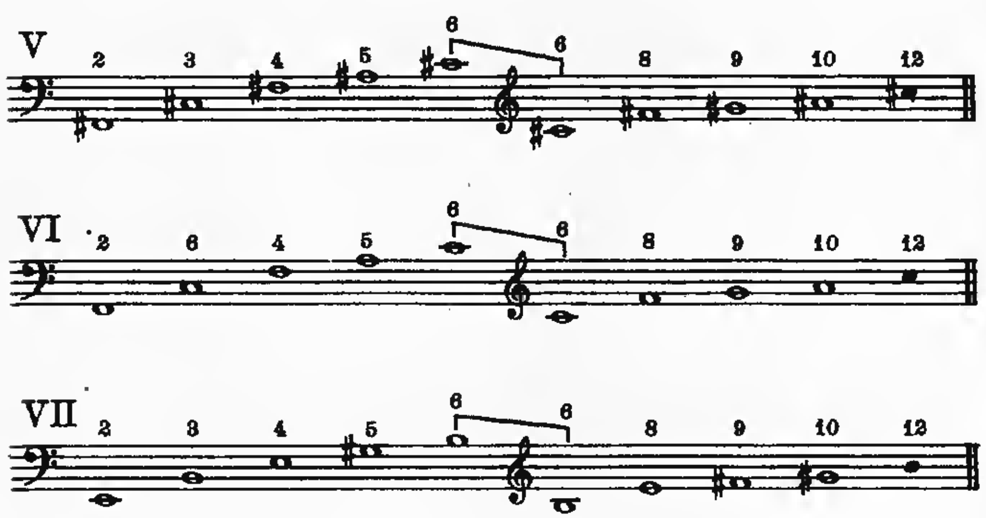
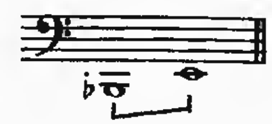


Figure 44. Fifth, sixth and seventh positions of the trombone.

It is easy to see that after the natural tones of all seven positions are combined, there appears to be a gap between the second natural tone of the seventh position and the fundamental of the first position:



Thus, the following pitches are not available on the trombone of this type:

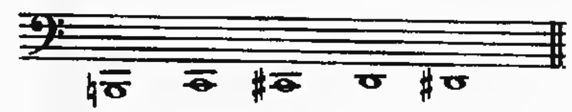
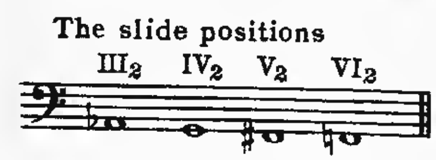


Figure 45. Unavailable pitches on the trombone.

The ability to produce the natural tones above the tenth depends upon the skill of the performer. It is advisable, in writing orchestra parts, not to exceed the eighth harmonic, reserving the use of the ninth and the tenth for exceptional effects.

To compensate for the absence of pitches within the gap, an instrument with a special valve has been designed. This valve, operated by a string attached to a ring controlling the opening of the valve, lowers any natural tone by five semitones (perfect fourth). For this reason a trombone supplied with such a device is known as a *trombone with a valve.*

By means of this device, d♯, d♮, c♯ and c♮ can be obtained from the second natural tones of the III, IV, V and VI positions respectively.



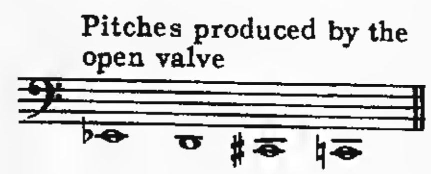


Figure 46. Pitches produced by the open valve and slide.

The lowest pitch of the gap (b♮) still remains impractical owing to the fact that the air-column of the seventh position, augmented by the valve, becomes so great that it breaks itself into three thirds of the total volume, thus causing the third natural tone:

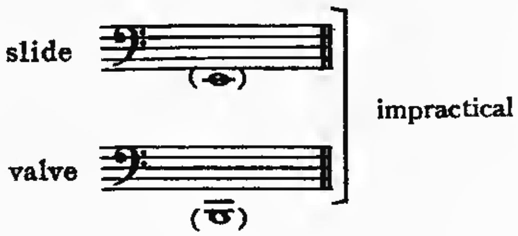


Figure 47. B impractical

It follows from this description that not only the entire chromatic scale is available, but that some of the pitches are even duplicated: they appear as the different natural tones of the different slide-positions. The preference in such cases depends on two conditions:

(1) the positional distance from the preceding to the following pitch; if such positions are too remote and there is a possibility of obtaining the same pitch on a different natural tone of a nearer position, it is the positional distance that becomes a decisive factor;

(2) the difficulty of producing higher natural tones in the lower positions as compared to lower natural tones in the higher positions; for example

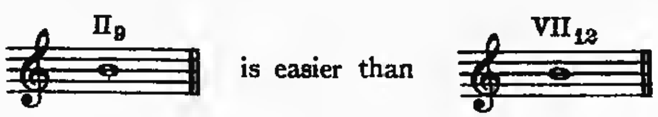


Figure 48. II_9 is easier than VII_{12}.

The trombone has a cup-shaped mouth-piece. Its tone-quality greatly depends on the manner of playing. Some trombonists have a bold, powerful tone; some have a mellow lyrical tone; and some have both. The character of the tone greatly depends on the form of vibrato (tremulant). All forms of vibrato on the trombone are vibrato by pitch (obtained by oscillating the slide within a small pitch interval as on the stringed-bow instruments). In comparison, trumpet vibrato is vibrato by intensity and is caused by variation of embouchure.

The trombone is an instrument of a sliding pitch par excellence, easily comparable to the 'cello. For a long time composers misunderstood the true nature of this instrument. American jazz recaptured the real meaning of the trombone, though in many instances dance-band trombonists overdo both the vibrato and the sliding, which renders a sugary character to the whole performance.

Glissando, which was first regarded (in the hearing of Stravinsky's scores) as an innovation, in reality is very basic on a trombone and today has become not only a commonplace resource, but also a source of annoyance. From the technical standpoint a true glissando can be executed *only on the same natural tone*, while the slide is being gradually moved through its *continuous* points (that is, not only the positional but also interpositional). All other forms of glissando are made by variation of embouchure and are not standardized.

A glissando can be performed either up or down. It is sufficient to indicate a glissando by showing the starting and the ending pitch of it, and to connect the two by a straight or a wavy line:

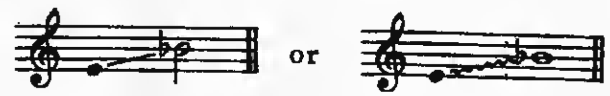


Figure 49. Glissando.

The term *gliss.* may also be added above the part, if desirable.

The passage just illustrated is executed on the eighth natural tone, while pulling-in the slide from the VII to the I position gradually. If a passage falls on the different natural tones, it is impossible to execute it in continuous, i.e., glissando, form. For example:

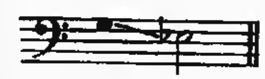


Figure 50. No glissando on different natural tones.

The execution of this passage is impossible because *e*♭ can be only III_3, while if *g* is the third natural tone, its fundamental would be *c* and there is no such position on the trombone.

Mutes were very seldom used on the trombones in the symphonic music of the past. However, the development of jazz has led to a very extensive and diversified use of mutes (including "hats") in the same manner as they are being used on the trumpets.

Besides raising the standards of performance on this instrument, jazz has also created some outstanding virtuosi, among whom the greatest artist is Tommy Dorsey, particularly because of his unsurpassed tone-quality.

Some trombonists are capable of producing (as a special effect in the higher positions) simultaneously the fundamental and the third harmonic (actually sounding as a harmonic). In addition to this, some jokers sing the fifth harmonic, thus obtaining a whole triad.

Trombone parts are usually written in the bass and the alto clefs; 19th century composers preferred the tenor clef. Today it is practical to use treble clef for the higher register, as all trombonists can read these four clefs.

The trombone with a valve is usually employed as the third trombone in symphonic scoring, but is seldom used by dance-bands. All other types of trombones, such as alto trombone (in E♭, sounding a perfect fourth higher than written) or bass trombone (in F, sounding a perfect fourth lower than written) have become completely obsolete. The old three-valve trombones of various types were found unsatisfactory in their tone-quality, which was decidedly inferior to that of the natural (slide) trombone.

D. Tuba (Tuba)

This instrument is also known as bass-tuba and belongs to the sax-horn family, which is fully represented in the large brass bands. The tuba, which is used as a standard instrument in symphonic and operatic scoring, seldom appears in the dance bands. Dance bands mostly use the E♭ sousaphone bass (a three-valved instrument commonly used in the infantry).

The tuba, acoustically, is an instrument in F, but does not require transposition. Its parts sound exactly as written. Due to traditional use of a quartet consisting of three trombones and a tuba (usually the tuba part is written on the same staff as the third trombone), composers developed a habit of associating the tuba with trombones. However, the tuba comes closer to the French horn than to the trombone. Its pipe is conical, like that of a French horn, while the trombone's pipe is cylindrical until it reaches its bell. The mouth-piece of the tuba is closer in shape to that of the trombone than of the French horn.

The scale of the natural tones of the tuba is as follows:

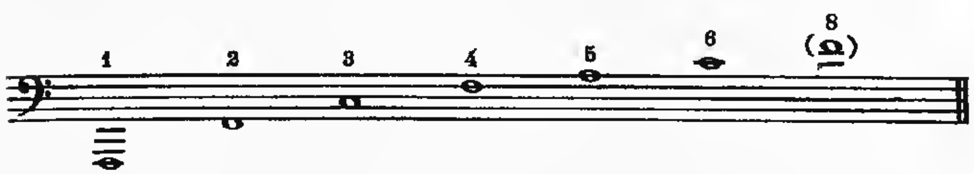


Figure 51. Natural tones of the tuba.

It is advisable to use the first six natural tones, and to resort to the eighth tone only in exceptional cases. The tone-quality of the French horn is preferable to that of the high register of the tuba and it bears a close resemblance to the latter.

Tones below the fundamental are difficult to execute as there is a constant danger of overblowing the fundamental. It is best not to write below *d* which lies three semitones below the fundamental.

There is an interval of a whole octave between the fundamental and the second tone and the design of the tuba requires four valves. These four valves are evolved according to the standard three-valve principle, the fourth valve being capable of lowering a natural tone by 5 semitones. In addition to this, tubas used in symphonic and operatic orchestras have a fifth valve. The purpose of his valve is to give an acoustically more satisfactory semitone-valve for the lower egister, as the second valve is not sufficiently large. Tubas of the type being

described here have a valve operation on cylinders. Pistons are to be found in an instrument serving similar purposes in infantry bands, the ophicleide, which is carried over the shoulder while being played.

Thus the valve arrangement on the five-valve tuba is as follows:

I lowers the natural tone by 2 semitones
II lowers the natural tone by 1 semitone
III lowers the natural tone by 3 semitones
IV lowers the natural tone by 5 semitones
V lowers the natural tone by 1 large semitone

Combined application of these valves produces any desirable interval between the first and the second tones.

The tuba is a slow-speaking instrument. Good intonation is one of the main difficulties of this instrument. The main asset of the tuba is its rich tone quality. All forms of attack are available, but the tuba is particularly suited for long sustained tones and slow passages in general. No mutes and no special effects are used on the tuba.

The Russian composer Shostakovich used, in his First Symphony, two tubas, instead of the customary one. As intonation on the tuba is usually less precise than on the other brass instruments, this score, at least when being performed in Russia, created considerable difficulties during rehearsal: one tuba is bad enough but two become unbearable.

CHAPTER 4

SPECIAL INSTRUMENTS

A. Arpa (Harp)

THE origin of the harp leads back to antiquity. In the bas-reliefs of ancient Egypt, dated as far back as 2700 B.C., court orchestras are represented which consist mostly of pipes and harps. In the last two or three centuries the harp has undergone many modifications. Some manufacturers have built chromatic harps and some, diatonic. Contemporary harps are diatonic instruments with a triple tuning.

The contemporary harp is originally tuned in a natural major scale in c♭. There are seven strings to each octave. All octaves are identical. The main feature of the contemporary harp is a set of seven pedals which control the tension of strings. The mechanism of the pedals is devised in such a manner as to produce modification of the same-named strings throughout all octaves. Thus, through the first step pressure-position of the c♭ pedal, the pitch of all the c♭-strings becomes c♮. Through the second pressure-position of the c♭-pedal, the pitch of all c♭-strings becomes c♯. A similar mechanism affects the remaining six name-strings. The step-pressures are independent for each pedal. While one pedal is put into its first pressure-position, another pedal may be put into its second pressure-position. This is possible because all pedals have an independent operation. Pressure-positions are retained by the instrument until they are changed by the performer. This is possible because each pedal has a locking arrangement in the form of two inverted steps:

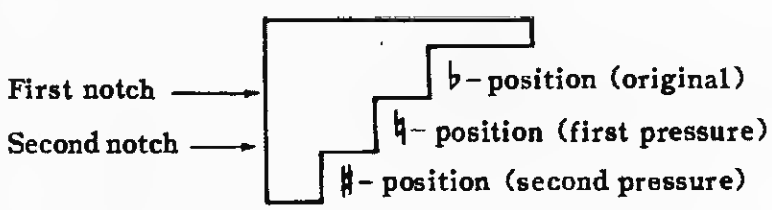


Figure 52. Pedal notches.

Looking at the harp from above, the pedals appear in the following arrangement:

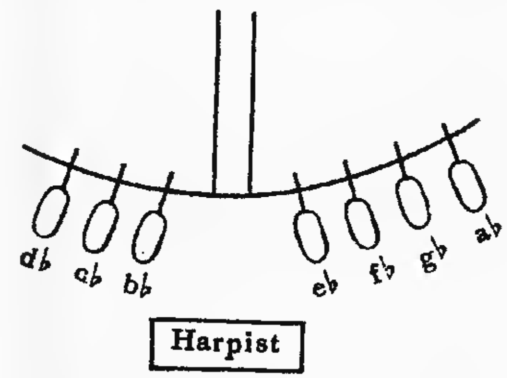


Figure 53. Pedals of the harp.

Accomplished harpists manipulate the pedals with great dexterity and can rearrange up to four pedals per second.

Harpists, as in the case of pianists, find the different strings by tactile distance-discrimination. However, in some cases, strings of red color are used for all the c♭'s, and of blue color, for all the f♭'s. This helps one to find the remaining strings.

The harp is played by either plucking a string, or a group of strings, with the individual fingers:

(1) in sequence (arpeggiato), which is the normal form of execution of chords on a harp;

(2) simultaneously (non-arpeggiato).

In addition to this, the harp is often played glissando, which is always a chord-glissando and is executed by sliding one of the fingers over the strings. As glissando affects *all* strings within its range, the problem of tuning glissando-chords becomes of major importance. Glissando can also be executed in octaves and other simultaneous intervals.

As a special effect, octave-harmonics can be used on a harp. This is executed by touching the string at its nodal point (geometrical center) with the palm and plucking with a finger of the same hand. If the interval is relatively small, each hand can produce harmonics in simultaneous intervals.

Dynamically the harp is a delicate instrument. It gains in volume considerably through the use of glissando. This effect can be executed in various degrees of the dynamic range (from pp to ff), depending on the pressure exerted on the strings and the speed of sliding over the strings: increase in speed and pressure results in the increase of volume.

It is important for the composer to understand that when pressure-positions are alike for all the strings, only natural major scales in the following three keys result therefrom: C♭, C♮, C♯.

Original position: c♭ — d♭ — e♭ — f♭ — g♭ — a♭ — b♭
First pressure-position: c♮ — d♮ — e♮ — f♮ — g♮ — a♮ — b♮
Second pressure-position: c♯ — d♯ — e♯ — f♯ — g♯ — a♯ — b♯

All other scale-arrangements require rearrangement of the pressure positions.

It would be of great advantage to the composer to know that all the 36 forms of Σ (13), tabulated in the *Special Theory of Harmony*,* are at his disposal. And any tonal expansions which derive from the above master-structures do not require any rearrangement of the pressure positions. This is possible because none of the above Σ (13) contains intervals greater than 4 semitones, which satisfies the pedal mechanism of the harp when tuned in E_0.

As the harp is a strictly diatonic instrument, it is desirable to use it as such. Quick modulations, containing several alterations, are quite impossible on this instrument. Many large scores contain two harp parts (used alternately for this purpose) in order to accomplish groups of modulating chords.

The part for the harp, like that of the piano, is written on two staves joined by a figured bracket. The clefs in use are the common bass (F) and treble (G):

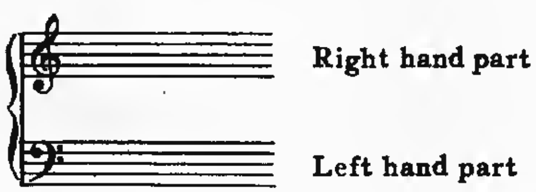


Figure 54. Harp clefs.

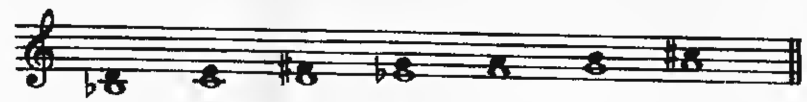
Instrumental forms suitable for the harp are quite similar to piano forms. Octaves in each hand can be executed only at moderate speed. Chords with wide intervals for both hands are more difficult than on the piano. Close positions are preferable to open ones, though the bass can be detached from the upper structures. Many effective passages can be accomplished by alternation of hands. Here the composer's inventiveness may bring many fruitful developments.

From the viewpoint of thematic texture, the harp can be looked upon as an instrument similar to piano, i.e., it can perform melody (in its various instrumental forms), harmony, accompanied melody, correlated melodies, and accompanied counterpoint.

In the orchestra it is frequently used as a coloristic instrument, which is due particularly to its capacity to execute effective and diversified forms of glissandi (upward, downward, combined, rotary, etc.)

*See Vol. I, p. 654.

There is a wide selection of structures which can be executed glissando (such structures often contain repeated pitches produced by the adjacent strings enharmonically tuned; but the speed of the slowest practical glissando is sufficiently great not to make these repeated pitches apparent to the ear). There is an easy way to determine whether a certain structure permits the performance of a glissando: ***if the structure does not contain major thirds, built on the degrees of a natural major scale in B♭, then glissando is possible.*** In other words, the structure in question cannot contain the following simultaneous intervals:



Thus the following chords are *possible* in glissando:

Figure 55. Glissando chords on the harp.

because they do not contain the major thirds referred to in Figure 55.

On the other hand the following chords are *impossible* since they contain such major thirds as are classified in Figure 55.

Figure 56. Impossible glissando chords on the harp.

The principle of major thirds of the B♭ – scale saves the composer the trouble of empirical verification. For example, let us see why d — f♯ — a — c is impossible in glissando:

d♭ — d♮
e♭ — impossible to stretch to f♯.

In other words, the e♭ – string would be in the way, even if other strings could be tuned to the given chord.

On the other hand a chord like c — d — f — a♭ is possible:

c♭ — c♮
d♭ — d♮
e♭ — e♯ (f♮)
f♭ — f♮
g♭ — g♯ (a♭)
b♭ — b♯ (c♮)

There are several different forms in use, by which a glissando can be indicated. Here are the most common:

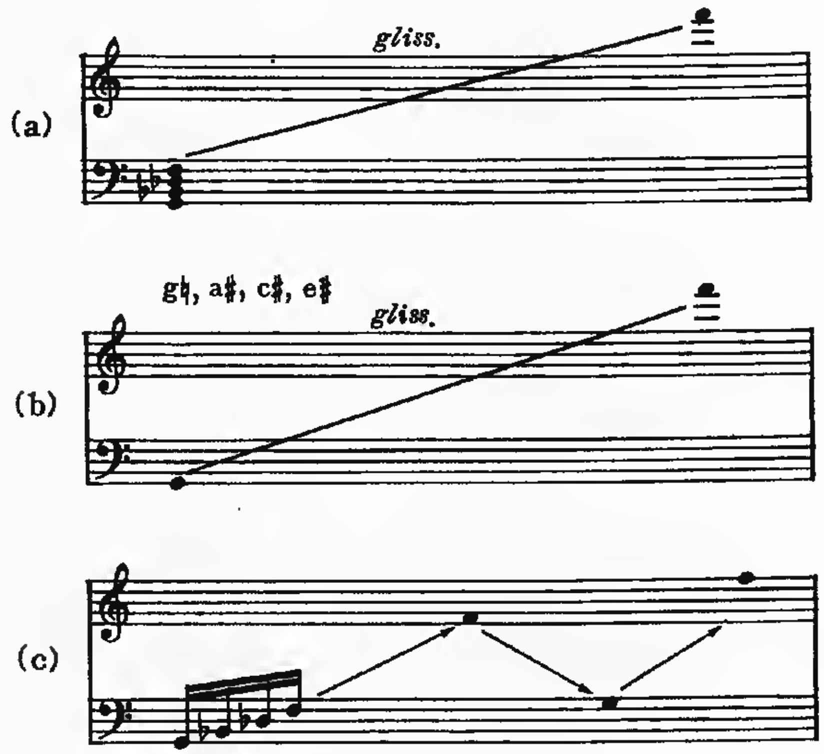


Figure 57. Notations for glissando.

The tuning of pedals in general, particularly when parts are harmonically simple, does not require any indication. Cautious composers, however, often indicate the pedal changes. For example:

Figure 58. Notation of pedal changes.

In the fourth measure b♮ and e♮ do not require any changes in tuning as b♮ = = c♭ and e♮ = f♭.

Octave harmonics, which are the only ones used on this instrument, are indicated by zeros above the notes, which notes should sound as harmonics in the same octave as written.

Figure 59. Notation of octave harmonics.

The forms of attacks on the harp correspond to that of a piano, i.e., legato, portamento, staccato, but the difference is less distinct than on the piano.

The basic timbre of the harp resembles the clarinet, owing to the method of playing (i.e., finger-plucking, instead of a hammer-attack, as on the piano; piano strings when played by fingers, without the medium of keys and hammers, also sound like the harp). The harp blends well with flutes and clarinets. The composer must not forget that the harp is a self-sufficient solo instrument of a diatonic type.

In the orchestra, of course, it is mostly used as an accompanying and coloristic instrument. It is also extremely effective as a semi-percussive rhythmic instrument.

Sometimes the harp, doubling wood-wind instruments, produces a more transparent equivalent of the pizzicato of stringed instruments.

Carlos Salzedo, who is probably the most accomplished and the most versatile harpist of all times, has invented a number of new effects for this instrument. He and some of his accomplished students (at the Curtis Institute in Philadelphia) are capable of executing these effects.

B. Organ (Pipe-Organ or its Electronic Substitutes*)

The pipe-organ is a more self-sufficient instrument than any other instrument known. This is due to the number of tones which can be produced simultaneously and to their timbral variety.

The number of different tone-qualities depends upon the number of *stops*, which can be used individually or in combinations. More expensive organs usually have more stops, price also determines the quality. Organs range from two-manual to five-manual models, in addition to which every organ has a pedal keyboard, generally used for production of the lower pitches. The dynamic range of a pipe-organ is fully comparable with that of a full symphony orchestra.

This instrument underwent many evolutionary changes. The latest and most spectacular type of pipe-organ is the large theatrical organ. This type of instrument is furnished with a very diversified selection of stops (including many

*See p. 1544.

percussive effects like xylophone, chimes, etc.) not excluding all the essential stops of an ecclesiastic organ. There are a number of pipe-organs in the world which can be justly considered masterpieces of acoustical engineering.

As organs vary widely in design, number of manuals, selection of stops, etc., it is impractical to give a detailed description of a ***pipe-organ.*** Basically, however, all pipe-organs possess certain general characteristics in common. It is essential for the composer to know some of these common characteristics:

(1) The amount of pressure exerted by the performer on the keys has no effect on the intensity or character of the sound.

(2) Forms of attack are effective: legato, non-legato, staccato are quite pronounced.

(3) Physically, the tone is generated by a pipe or a group of pipes, which are often built-in at a considerable distance from the console; this produces an effect of ***delayed action:*** a very important detail to bear in mind in using the organ in combination with other instruments.

(4) Tone-qualities are classified into groups, representing timbral families: the strings, the flutes, the reeds, the chalumeaux, etc. Each family has a number of distinctly different stops (i.e., tone-qualities).

(5) Each stop has a set of pipes covering a definite range; organists look on ranges and registers as represented by the length of respective pipes. Thus they say: a 4′ string stop, or an 8′ reed stop, or a 32′ pedal stop. The longer the pipe, the lower the pitch. Certain timbres are available only in certain registers, while others cover the entire (or nearly the entire) range.

(6) The massive tone-qualities characteristic of the pipe-organ are due to single, double, triple, etc. octave-couplings. These couplings are executed by pushing coupler-keys. Under these conditions, an organist can produce a powerful and massive tone by using only one finger.

(7) Volume (the intensity of sound) is controlled in part by special pedals. Thus gradual dynamic changes are possible. A sforzando-piano (sfp) effect is also available on most organs.

(8) Composition of stops for the performance of a given piece of music is known as ***registration.*** Notation for the latter is seldom provided by the composer (unless he is an organist). Even when the composer or the editor of organ parts indicates the registration, it is quite traditional for the performer to change the indicated registration to one of his own choice.

(9) It is customary to mix the stops belonging to different timbral families as well as to couple them through several octaves.

(10) In addition to this, there are so-called organ-"mixtures", which are built-in combinations of ***various couplings.*** When such mixtures are used, one key pressed by a finger produces a whole chord structure of one or another type. Thus, melodies may be played directly in parallel chords. In some of the organs built in Germany in the second decade of this century, mixtures producing some less conventional chords were introduced (in one instance, the mixture added to *c* produced c — d# — f# — — b).

It is important for the composer to realize that as a consequence of couplings and mixtures accompanying each individual note, what reaches the ear of the listener (including the organist himself) is quite different from what is written on paper. Not only the respective octaves and registers (in the general sense of this word) can be different than in writing, but they also can be accompanied by whole sets of new pitches which do not even appear in parts. Often symphonic, operatic, oratorio and cantata scores contain an organ part.

The above-described characteristics of this instrument make it very difficult for the composer to use the organ in an orchestra or in a mixed vocal-instrumental combination properly, since the principle of clarity as a necessary quality of instrumental and vocal scoring often conflicts with the natural tendencies of the instrument. For this reason the organ is either misused in most scores, or it plays a purely decorative part. In the old scoring, the organ was used, according to ecclesiastic tradition, as a duplication of the choir, and its part was often written merely as a figured bass, which the organist had to fill in. This can be found in the scores of leading composers of the 19th century.

Another important characteristic of the organ is its tone-quality with respect to vibrato. The organ can produce non-vibrato or a ***vibrato by intensity*** (some instruments, particularly in the string-stops, have also a mechanical vibrato of beats, produced by simultaneous pitches which are set at slightly different frequencies). For this reason, organ vibrati are mostly of a different type from orchestra vibrati. Simultaneous use of both often creates conflicts and discrepancies.

Organ parts are generally written on three staves: the two upper staves refer to one or more manuals, and the lower to the pedal. Other manuals are ***choir, solo, echo, etc.***

Figure 60. The three staves of the organ.

CHAPTER 5

ELECTRONIC INSTRUMENTS

THIS group of instruments is more diversified than all other groups combined. The term "electronic musical instrument" can be used to describe any instrument where electric current generates sound directly or indirectly. There are two basic subgroups of electronic instruments.

A. First Subgroup. Varying Electromagnetic Field

The first subgroup consists of instruments whose sound (i.e., sonic frequencies) is generated by varying the capacity of an electromagnetic field created by two currents. The instruments invented and constructed by Leon Theremin are based on this principle. They include three basic models:

(1) Space-controlled Theremin (also known as Victor-Theremin; later: R.C.A. Theremin).
(2) Fingerboard-Theremin.
(3) Keyboard-Theremin.

Of these types, the first acquired far greater popularity than the other two models. Recitals are being given by various performers on this instrument. I was the first composer to use this instrument in a solo (concertizing) part with a symphony orchestra. The composition was called *The First Airphonic Suite* and was performed by Leon Theremin as soloist with the Cleveland Orchestra in Cleveland and New York in 1929. In 1930 a realization of an early dream came through. I scored, rehearsed and produced together with Leon Theremin and 13 other performers, two programs presented at Carnegie Hall in New York, in which an ensemble of 14 space-controlled theremins was presented for the first time.

1. SPACE-CONTROLLED THEREMIN

Each musical instrument displays some characteristics of its own. The chief characteristic of the space-controlled theremin is its extreme adaptability not only to pitch and volume variation, but also to the different *forms of vibrato*. In this respect it is so sensitive that the pleasantness or beauty of tone depends largely on the performer. In order to obtain a "beautiful tone" on this instrument, the performer must know what physical characteristics make a tone "beautiful". These can be briefly described as a combination of vibrato frequency and the depth of vibrato, i.e., pitch variation between vibrato points. As this text is meant for the composer or orchestrator, there is no need to elaborate on this matter further. In 1929 I wrote *A Manual for Playing Space-Controlled Theremin*, where these matters are discussed in detail.

Pitch on the theremin is controlled by the right hand, which is moved toward and away from a vertical rod (antenna). The spatial dimensions of pitch intervals vary with respect to total space range, which is adjustable either individually or for each performance. In other words, pitch is varied within the spatial boundaries of the electro-magnetic field. Depending on the stature of the performer and the length of his arms, spatial range may be practically adjusted (tuned by a knob control) somewhere between one and three feet.

The electro-magnetic field can be imagined as a three-dimensional invisible fingerboard. It is so sensitive that even the slightest move on the part of the performer affects the pitch. Spatially, intervals contract with the increase in frequencies, i.e., by moving the hand toward the right antenna (which is a physical generality; it works the same way on the regular fingerboards, air columns, etc.). Not having a fixed-length fingerboard, the thereminist faces, as it proved itself to be the case in many individual instances, much greater difficulty in pitch control than any string-bow performer. Yet some performers, who were not even professionals on any instrument, could master the pitch-control problem in about two weeks. Their reaction was that you control pitch mostly by "feeling distances", that you play as if you were singing.

I am not offering any description of the basic timbre of this instrument, as each model has a timbre of its own. Vaguely they all resemble a combination of a string-bow instrument (when bowed) at its best, if not better, and of an excellent human voice singing every tone on the consonant "m", which, of course, has its own basic acoustical characteristics.

The left antenna of this instrument serves the purpose of controlling the volume. The left hand moves vertically toward (decrease of volume) or away from (increase of volume) the loop-shaped left antenna. The intensity range can be also spatially adjusted by turning a knob, just as in the case of pitch-control. This permits any degree of subtlety in varying the volume, as in the case of the right antenna with respect to pitch.

Playing this instrument is a task in the coordination of both hands and arms moving through two space-coordinates. It would be just to say that this instrument is much more delicate and sensitive than any human being who has played it up to now. People with good coordination and sufficient sense of relative pitch turned out to be better performers than eminent musicians. Leon Theremin and his assistant, George Goldberg (also an engineer), proved this to be so.

The composer can have at his disposal the entire audible range, if necessary, and any volume, as sound is amplified electrically. All forms of attacks are available. The space-controlled theremin is a monodic (i.e., producing a single tone at one time) instrument par excellence and, therefore, particularly suitable for broad sustained cantilenas, pedal points, etc. Rapid passages of any kind can be executed by an accomplished performer at speeds comparable to that of an oboe. One of the first models of this instrument had a knob contact for producing attacks. By pushing the knob with a finger of the left hand abruptly, one could produce the most abrupt forms of staccato at any desirable speed.

The Philadelphia Orchestra, through the initiative of Leopold Stokowski, its music director, used a specially built model of the theremin. This instrument served the purpose of coupling and reinforcing orchestral basses of various groups. It had a pure (that is, sinusoidal) tone and immense volume amplification.

It is best not to compare the theremin with any other standard orchestra instruments, but to look upon it as the first instrument of the coming electronic era of music, having its own characteristics and being conceived and designed along entirely new principles of sound-production and sound-control. It is the first child of the electronic musical dynasty. Its first model dates back to 1921, when Leon Theremin demonstrated it in Moscow before a conference of electrical engineers and inventors. At that time it was in its early experimental stage. In the U. S. A. it was manufactured by R. C. A. Manufacturing Co., Camden, New Jersey.

2. FINGERBOARD THEREMIN

This instrument was designed and constructed for the purpose of supplying violinists and 'cellists with an electronic instrument, which they could learn to play in a very short time. Some violinists and 'cellists have played it with great success.

This instrument's main part is a cylindrical rod, about as long as the 'cello's fingerboard. While being played, it is held in position similar to the 'cello. The part which is touched by the fingers of the left hand (to which procedure all string-bow performers are accustomed) is covered by celluloid. Production of tone results from the contact of a finger with the celluloid plate. Thus pitch-control is very similar to that of a 'cello. Volume is controlled by a special lever, resilient and operated by the right hand. The greater the pressure on the lever, the louder the tone. This form of dynamic control allows not only gradual variations of intensity but also accents and sforzando-piano. All forms of attacks are available through direct contact with the fingerboard. Though the manner of playing this instrument more resembles the 'cello than the violin, violinists have found it easy to play.

The range of this instrument is adjustable, i.e., the same model can be tuned in high, low, or both registers. The tone quality of the fingerboard theremin resembles an *idealized* 'cello tone (i.e., one which is deprived of inharmonic sounds, usually resulting from the friction of horse hair over sheep's guts while bowing) and is more of a constant than on the space-controlled model. The usual type of 'cello vibrato gives a perfectly satisfactory result. The basic timbre is quite close to the double-reeds (nasal).

Of course timbre and other characteristics of this instrumeut could be easily modified. Some engineers in Europe, after Theremin, constructed instruments whose outer design resembled the violin, 'cello, or bass. Leon Theremin thought this pointless, because the dimensions and the shape of an electronic fingerboard instrument have nothing to do with its range or registers.

The fingerboard theremin is a *monodic* instrument. One of the advantages of having such instruments in the orchestra is tone-quality, which can be literally "made to order" by the engineer or manufacturer; another, is its range which offers a great economy: a passage, which generally starts on the 'celli and is completed by the violins, can be executed on one instrument and by the same performer.

3. KEYBOARD THEREMIN

Keyboard theremin is a *monodic* instrument, with a standard piano keyboard. It is a direct predecessor of the *solovox*, manufactured by the Hammond Organ Company today. Physically, though, the solovox does not belong to the *first subgroup* as piano strings, electromagnetically inducted, are the original sound-source. Nevertheless, the keyboard theremin operates physically on the same principle as other theremin instruments, i.e., by variation of the capacity of an electromagnetic field.

This instrument was designed with the purpose of supplying keyboard performers with an instrument which they could play without much additional training, yet which would possess such features as economy of space, any pre-conceived tone-quality, well expressed forms of attack, regulated forms of tremulant, fading effects with vibrato automatically performed (as on the Hawaiian guitar), and automatically pre-set varied degrees of staccato etc.

The business end of the Theremin enterprise was not properly handled. As a result there are not many space-controlled models to be found today, more than a decade since they were first built, not to mention the fingerboard and the keyboard theremins, of which there are very few, if any, left.

Leon Theremin built a number of other electronic instruments, among them various types of organs with micro-tuning and variable timbre-control (in the design of which I participated), but these instruments mostly served the purpose of research and have never reached the attention of the public at large.

The purpose of my directing the attention of the composer to these short-lived models is to show the direction in which lie the future stages of the field of orchestration, as there has never been any doubt in my mind that the present standard (non-electronic) instruments will soon be outmoded and superseded by perfected electronic models.

In this regard the composer will be confronted with new approaches and techniques of orchestration. He will have to think acoustically and not in terms of violins, clarinets, trumpets etc. This is just a note of warning.

B. Second Subgroup; Conventional Sources of Sound.

The *second subgroup* of electronic instruments uses conventional sources of sound (strings, bars, oscillating membranes, etc.), but they are excited by means of *electro-magnetic induction* and amplified through a loud-speaker system.

While Theremin's models were entirely revolutionary and constituted a decidedly radical departure from all existing ideas of designing musical instruments, the instruments which I refer to as the second subgroup are decidedly a result of compromise, lack of vision and immediate commercial considerations. It is just to say that the theremin instruments are more refined as an idea though not sufficiently perfect in actual operation, while the existing models of the second subgroup are well designed, well-built, and are reliable in operation but are based on old-fashioned and often erroneous notions as to what a musical instrument should be. For this reason the instruments of the first subgroup eventually will be resurrected and will last longer in improved forms, while the instruments of the second subgroup will be considered too crude in comparison, and will die out the way the player-piano did when the perfected radio left no room for its existence. The instruments of the second subgroup are manufactured and sold on a mass production-consumption basis. They are widely used today, particularly in the field of radio and dance music.

The instruments of the second subgroup are generally called by their old original names, with the addition of the definitive "electrified". Thus we speak of such models as the electrified piano, electrified organ, electrified guitar, etc. The history of these instruments leads far back to Thaddeus Cahill, who in 1897 constructed the "Sound Staves", a clumsy instrument with oscillating membranes, effected by electric current.*

As electronic instruments of all types are in an early stage of development, and as the present models may soon become outmoded and obsolete, I shall offer only a brief description of the models which are most in use today, and only such a description as will provide the composer with information and ideas valuable *per se*.

1. ELECTRIFIED PIANO

This instrument consists of an ordinary piano and a system of electromagnetic inductors connected with an amplified sound system. There are different designs of this instrument, but the resulting sounds have most characteristics in common. This instrument is usually known as electronic piano. In the U. S. A. the Miessner piano is better known; in Germany, the Bechstein (after the famous firm manufacturing the best pianos ever built). Some of the electronic Bechsteins are also in use in the U. S. A.

The main feature of all such instruments is the conversion of a regular piano into several different instruments. This is accomplished by a system of various pre-set forms of induction. The two characteristic extreme forms are: one, in which the duration of a tone is prolonged indefinitely and the volume of it can be increased even after the respective key has been released, and another, in which a pre-set form of quick fading, the sound of which resembles harpsichord, is produced. There are usually various intermediate effects between these two extremes. At the same time this instrument can be used without electrification, which is of great practical advantage. Any accomplished pianist or organist can master this instrument in a very short time.

*For more historical detail see my article "Electricity, the Liberator of Music" in the April issue of *Modern Music Quarterly*, published by the League of Composers in 1931 (Vol. 8). (U. S.)

2. SOLOVOX (manufactured by the Hammond Organ Co.)

Solovox is a monodic instrument, devised in the form of a piano-attachment. In fact, it is a monodic version of an electrified piano. The purpose of this instrument is to execute melody of a durable and, if desirable, tremulant tone directly from the piano (with the right hand playing the solovox) and the accompaniment being played by the same performer on the same piano (with the left hand). Whether such a combination is desirable, is a debatable matter. But this will be discussed in "Acoustical Basis of Orchestration".*

3. THE HAMMOND ORGAN

This instrument (manufactured by the same company and designed by Lawrence Hammond) is the most universally accepted of all the larger types of electronic musical instruments. The Hammond organ is a fairly complex piece of electrical engineering without being bulky.

The name "organ" is applicable to this instrument only insofar as the production of sustained simultaneous sounds is concerned. Otherwise, every organist or any experienced musician can tell, without seeing the instrument, whether he is hearing a pipe-organ or a Hammond organ. There is undoubtedly a general difference in the tone-qualities of the two instruments throughout their ranges, particularly in the pedal. The Hammond Co. expected to sell most of its instruments to cathedrals, churches and chapels. The instrument, however, approaches the theatre organ more closely than it does the church organ (particularly when used with a special tremulant speaker which, by the way, is not manufactured by the Hammond Co.). Today this instrument is widely used for dance music and "swing".

There are certain basic principles on which the instrument is designed and built, and they are important for the composer to know. The following information is not available elsewhere.

The first fact of importance is that this instrument does not sound like a pipe-organ in its tone-qualities. There are two reasons for this. The first is that the type of speaker and the whole sound system do not permit the high frequencies (the real partials of a tone) to come through. I verified this fact by connecting the Hammond speaker with a turntable. Good high fidelity recordings sounded completely "muffled". The second reason is that the Hammond instrument is not designed to include certain inharmonic sounds, which are the constants of many organ pipes. Whether such inharmonic sounds are desirable *per se*, is another matter.

The second fact, which is inseparable from the first, is that this instrument does not sound like a pipe-organ in its emission of sound. In a pipe-organ, the emission of sound is not instantaneous (particularly in old church organs) owing

*Vibratone, manufactured by Brittain Sound Equipment Co., Los Angeles.

to the necessary time interval required in transmission of an impulse from the keyboard of the console to the pipes and then to the ear of the listener. In the Hammond organ the transmission of sound is instantaneous, owing to the speed of electric contact. This particular characteristic adds one advantage to the Hammond organ, namely, the hard staccato of extreme abruptness. Organists complain that on the Hammond instrument "the souod appears before you touch the key".

The two factors are closely interrelated. The lack of real high partials on this instrument is due to the mechanical design of the Hammond organ which does not permit the use of better speakers and of a better sound system; high-frequency response would make the *key-contacts audible* (they would click loudly). Hence, the "muffled" tone, as the lesser of the two evils.

The speed of sound transmission could be easily modified by a special mechanism for delayed action. The inharmonic tones could be introduced electronically (such devices were used with success in the electronic instruments of the first subgroup type built by Dr. Trautwein in Berlin in 1928).

A valuable factor in applying electro-magnetic induction to oscillating membranes or revolving discs (as in the case of the instrument under discussion) is the stability of frequencies. So long as the electric current is relatively stable, i.e., of a constant voltage, the instrument, no matter how long it is in continuous use, remains in tune. This is not true of the instruments of the first subgroup, where warming up of the tubes eventually affects the pitch.

The Hammond organ, evaluated *per se* and not in comparison with other musical instruments, must be considered a valuable self-sufficient or auxiliary instrument. The chief asset of this instrument is its acoustical system of timbre variation.

The Hammond organ produces pitches of a twelve-unit equal temperament in simple (sinusoidal) waves. These simple components can be mixed at random at different intensities, which results in different tone qualities. The simple components are called by the names of the nearest tones of the natural scale. Each component is controlled individually and has *eight* graduated degrees of intensity. Actual control is exercised by pulling out the respective levers. There are *nine* levers corresponding to the nine components of each tone quality.

Figure 61. The nine levers of the Hammond organ.

The numbers in the circles indicate the levers as they appear from left to right.

1 corresponds to the subfundamental, i.e., one octave below the fundamental;
2 the subthird harmonic, i.e., one octave below the third harmonic;
3 the fundamental;
4 the second harmonic;
5 the third harmonic;
6 the fourth harmonic;
7 the fifth harmonic;
8 the sixth harmonic;
9 the eighth harmonic.

We shall consider such a set to be an acoustical system of components for production of one tone-quality at a time. All present models have two such systems for each of the two manuals. A special two-lever (two-component) system (the fundamental and the subthird) controls the pedal.

Once the levers of one system are pulled out into a certain pre-arranged position, such a position mechanically corresponds to a certain push-button. That is, the pre-arranged combination, producing a certain tone-quality, can be obtained instantaneously, by pushing the corresponding button. On the model E of the Hammond organ, the two systems correspond to push-buttons 11 and 12. The push-buttons are the same for both manuals.

All other push-buttons, numbered from 1 to 10, control pre-set combinations. The pre-set combinations are the most common stops of a church pipe-organ. However, these too can be re-arranged by changing some of the wire connections within the console.

The total number of tone-qualities for each manual individually (which would also absorb any of the pre-set combinations) equals the sum of all combinations by 2, by 3, ... by 9 out of nine elements (since there are nine levers). Each combination can be modified according to the different positions of intensity for each lever (of which there are eight). Thus, if it is originally one-lever setting, each of such settings has to be multiplied by 8. There are thus $9 \cdot 8 = 72$ one-lever settings. For a combination of two levers, the value 8 must be squared; for a combination of three levers, the value 8 must be cubed, etc.

There is no need to make a complete computation of all tone-qualities thus obtainable, as it would take several centuries to play them through. However, from a musical standpoint (i.e., from the standpoint of imperfect auditory tone-quality discrimination), there are not so many really distinctly different combinations since many modifications of the same combination sound quite similar to the ear.

Though components of tone-qualities on the Hammond organ are the tones which approximate harmonics in the twelve-unit equal temperament—but not real harmonics—the very principle of composing tone-qualities from elements and not from complexes (like the timbres of standard instruments) has a great educational value for any student of music in general and for the orchestrator in particular.

The Hammond organ is supplied with some controls adopted from the pipe-organ. Among these are the various couplers, the dynamic control swell-pedal, the tremulant-control, the "chorus", etc. The range of model E is from *c* of contra-octave to f♯ of the fourth octave (it has the frequency of approximately 6000 cycles and corresponds to lever ⑨ for f♯ of the first octave on the keyboard, which pitch is half an octave higher than the highest piano *c*).

Besides being a very diversified self-sufficient instrument, the Hammond organ is frequently used in small instrumental combination to supply the missing timbres.

The composer will make the best use of this instrument by realizing that the Hammond organ is an instrument whose specialty is production of controllable and highly diversified tone-qualities, combined with sufficiently versatile forms of attacks and an enormous dynamic range, without sacrifice of dynamic versatility. The Hammond organ keyboard has a very light action, which permits the production of rapidly repeating tones.

In order to assist the orchestrator with a method by which he can find the *basic timbral families* out of the enormous number of possible combinations, I have devised a simple system by which such families can be instantaneously arranged and easily memorized. This system is based on the *patterns of intensity* of the different components in relation to their lever-scale position (which *approximately* corresponds to the frequency position).

Scale of Basic Timbral Families on the Hammond Organ

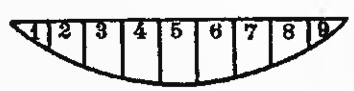
Families: Patterns:

1. Uniform intensity of all participating components

2. Scalewise increase of intensity of all participating components.

3. Scalewise decrease of intensity of all participating components

4. Convex arrangement of intensities of all participating components



5. Concave arrangement of intensities of all participating components

Figure 62. Basic timbral families on the Hammond organ (continued).

6. Selective pattern of partials of uniform intensity based on odd-numbered levers.

7. Selective pattern of partials of uniform intensity based on even-numbered levers.

Figure 62. Basic timbral families on the Hammond organ (concluded).

This system helps the orchestrator to associate timbral families with the corresponding scale of visual patterns:

Figure 63. Visual patterns of the timbral families.

Verbal description of these basic qualities is highly inaccurate. For this reason I shall eliminate it altogether. The best way to get acquainted with these timbral families is by practical study of this system of timbral selection at the instrument. This practical study should be accompanied by further investigation of the dynamic variations within each timbral pattern. For instance, in the second family we may vary the angle representing intensities:

In the fourth family, we may modify the form of its convexity:

This study will be of great practical benefit to any composer or orchestrator, and particularly with regard to his study of my Theory of Orchestration.

4. THE NOVACHORD

The Novachord, another Hammond development, is a keyboard electronic instrument on which simultaneous sounds can be produced. The name is somewhat misleading, as "chorda" means "string", and, of course, there are no strings on this instrument.

The Novachord has the range of a combined string-bow group. It has one keyboard of the piano type. It is supplied with numerous timbre-controls and attack-controls. This instrument can be justly considered an improved and developed version of the keyboard theremin. One of the specialties of the Novachord is attack-forms whose fading periods can be automatically pre-set. The forms of vibrato can also be automatically controlled. Dynamic variation is controlled by pedal.

The timbres of the Novachord resemble closely (owing to the selective system of attack-forms) many of the standard orchestra instruments. Some Novachord timbres are of such high quality that only the very best performers on the original instruments can rival them.

The Novachord is a very valuable instrument as a substitute for missing standard instruments in an ensemble or orchestra. As a self-sufficient instrument, which it is meant to be, it is not quite satisfactory. The reason for this is that it is a simultaneously monotimbral instrument: only one tone-quality can be produced at a time. As the result of this characteristic, melody and accompaniment sound in the same tone-quality and, in addition to this, at the same volume. Thus, when melody is played with an accompaniment, it can be singled out by one means only: playing the accompaniment staccato.

CHAPTER 6

PERCUSSIVE INSTRUMENTS

WE shall adhere to the following definition of percussive instruments: ***all instruments*** whose sound is produced on a string, a membrane or a bar (often built of different materials) by direct attack and not by electro-magnetic induction. As a consequence of this characteristic, all percussive instruments naturally (and automatically, unless extended by some special devices) have a fading sound. Therefore the period of fading is in direct proportion to the intensity of sound,i.e., to the amplitude of its attack.

Since all the inharmonic (i.e., noise producing) instruments will be described as percussive instruments, though some of these really are not percussive, one distinction must be made clear: while on the percussive and inharmonic instruments sound is basically produced by ***attack***, it is also produced by ***friction***. For example, a drum can be played not only by a stick or a hand attack, but also by rotary motion of the palm of the hand over the skin of the drum. The same is true of the rubbing of surfaces of two emery boards, etc.

Some of the instruments known as self-sufficient, will be described here ***specifically as orchestral and, therefore, as coloristic instruments***. Particular attention will be paid to their percussive possibilities, which are so often neglected.

Percussive sounds of the instruments, which originally are not meant to be percussive (such as string-bow instruments, when played pizzicato, col legno, etc.), will be discussed in the Technique of Orchestration*, in the chapter devoted to the Forms of Attacks.

We shall classify all percussive instruments in four groups:

Group one, where the source of sound is a string or a bar (metal or wood);
Group two, where the source of sound is a metal disc;
Group three, " " " " " is a skin membrane;
Group four, " " " " " is various other materials.

A. GROUP ONE. SOUND VIA STRING OR BAR.

1. PIANO

Piano (grand and upright) is a self-sufficient instrument, most universally used in our musical civilization. The range of a piano varies from concert grand manufactured by Bluethner in Germany (whose range extends from *g* of the subcontraoctave to *c* of the fourth octave) to American made five-octave miniature uprights. The standard range, however, can be considered 7¼ octaves, which comprises 88 keys (it extends from *a* of the subcontraoctave to *c* of the fourth octave).

The timbre of the piano, strictly speaking, cannot be uniform, as its strings are made of different materials, differently shaped and attacked by somewhat

*See Editor's note at the end of this book.

differently designed hammers. The upper and the middle registers consist of straight steel strings used for each tone in groups of three. The middle-low and the low registers have coiled copper strings coupled in pairs. The lowest register has single copper strings for each tone.

It is to be remembered that the piano is a strictly percussive instrument, as strings are excited by the stroke of a hammer. The tone of the piano fades very quickly, as the oscillograph shows. It is our cultivated auditory imagination that extends the duration of a piano tone. Physically, a piano tone has a sharp attack and quick fading. The depressing of the right pedal extends the duration of a tone, as this releases the string, permitting it to vibrate. This, however, does not exclude the fading of a tone, but merely extends the time period of the fading. In musical terms this can be stated as: diminuendo is a constant of a piano tone.

The piano gets very quickly out of tune because its system of double and triple strings for each tone makes it physically difficult to maintain perfect unisons.

We had the description of piano possibilities in the Theory of Instrumental Forms*. Here we are primarily concerned with the unconventional uses of the piano as a percussive and coloristic orchestra instrument.

Igor Stravinsky made an interesting use of four pianos combined with an ensemble of percussive instruments in his *Les Noces*.

The real use of the piano as a percussive instrument comes mainly through the explorations of Henry Cowell, an American composer, who himself is an excellent exponent of his own techniques. Cowell has developed an exact and thoroughly developed system of playing piano with forearms and fists. Harmonically this device involves the use of "tone-clusters" (Cowell's term). Under such conditions the piano is capable of producing an amazing volume, uncommon to this instrument. My record library includes my own recordings of various Cowell devices, as they appear in his own compositions performed by himself. Unfortunately these are not on the market at present. There is, however, one Victor record of Ravel's *Bolero* arranged and played by Morton Gould. In this arrangement Cowell's forearm technique was employed. For more details on this subject see Henry Cowell's *New Musical Resources*.**

Apart from this specialized field of piano execution, rapid alternating tremolos of both hands involving the use of three or more fingers in each hand can be employed very effectively as a percussive device.

Another device which Henry Cowell uses and which is generally not unknown, consists of plucking the strings or sliding over them (with the right hand) while pressing the keys silently (with the left). Sliding over a group of strings permits the sound to come only from the strings whose keys are pressed. This produces a delightful harp-like tone.

Henry Cowell has also developed a highly coloristic effect which, so far as I know, he is the only one able to execute. It consists of sliding over the strings (in the back of the piano; somebody has to press the right pedal down continously) and sometimes plucking them. The sliding is done across an individual string

*See p. 1043 ff. **Published by A. A. Knopf, N.Y.

and produces a most fantastic sound. Cowell often touches the nodal points of a string in order to get harmonics. He holds the string at a node with one hand and slides across it with the other. He has some compositions, like *Banshee*, entirely written for this technique. This device can be used with great success for wind and storm effects, as well as for fantastic and ghost-like effects.

The use of regular piano harmonics was made in Arnold Schoenberg's and my own compositions. Harmonics are particularly interesting as a variable timbre effects. By silently pressing a key (or a group of keys) which corresponds to the respective harmonic and by striking the fundamental (or a group of fundamentals) we obtain an actual harmonic. This is due to the sympathetic vibrations of the open string in response to the partial vibrations of the fundamental (which is executed staccato). The effect is that of an abrupt attack, followed by an extended fading harmonic. It is very interesting to note that under such conditions, each harmonic has a different timbre.

(a) Second harmonic (b) Third harmonic

Figure 64. Second and third harmonics.

Cases (a) and (b) in the above figure have different timbres. Higher harmonics (preferably the ones which are used on a trumpet) can also be achieved.

The piano is also capable of producing *vibrato* (in single tones or chords). Not the imaginary vibrato, where a pianist is vibrating his finger while pressing the key (which is physically meaningless, as after the hammer strikes the string, no manipulation of the key has any effect upon the string), but a real physical vibrato by pitch.

This is my own device at which I arrived by the following reasoning. If we silently press the eleven lower keys (which is easily done with the palm of the left hand), then any keys we strike at an interval of an octave or more would stimulate the respective partials on the lower open strings. As the actual partials differ slightly in pitch with the corresponding keys we strike, the differences in frequencies produce *beats*, i.e., vibrato by pitch. I used this device with great success in the piano part of my Symphonic Rhapsody *October*. All sounds must be produced either portamento or staccato. They come out with special prominence on a concert grand piano, as there the strings are correspondingly longer and, for this reason, their partials are louder. This device can be applied either in long durations or in rapid arpeggio passages. For this effect the pedal must not be used.

The piano can be turned, for some special effects, into a harpsichord and other plucked instruments. For these effects, it is necessary to use paper (particularly wax-paper), placed right on the strings. When the hammer strikes a string covered with paper, it produces a buzzing effect. For a more drastic percussiveness, plywood boards may be used instead of paper. I made use of the latter in background music to *Merry Ghost*, a Japanese play by Kitharo Oka; this effect was used to produce a sound resembling that of the shamisen (a Japanese plucked instrument).

Finally the piano can be used as a sympathetic resonating (echo) system. The piano, when its right pedal is pressed, is able to reproduce sympathetically any sounds which are in its vicinity, i.e., any such sounds whose air waves can reach the strings with sufficient intensity. This concerns both the harmonic and inharmonic (noises) sounds.

Whistle into the piano and the response is the same pitch and the same tone quality. Sing, and the same sound continues as an echo. This device can be used specifically as an *echo generating device*. It is a natural phenomenon based on the physical pattern-response. It existed in nature before any animals inhabited this planet. Nobody can lay any claim to discovering the echo.

I suggested this device to all my students of orchestration, and it was Nathan L. Van Cleave who effectively used it in scores made for the Kostelanetz orchestra. This device can be utilized practically, in the alternation of staccato of an instrument or a group of instruments (preferably identical ones) and its echo; both should follow in uniform durations.

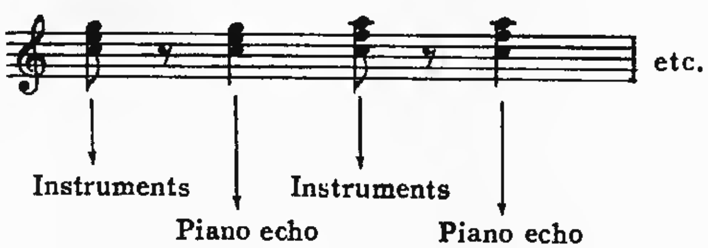


Figure 65. Piano echoes.

The alternation of such durations must not be too fast.

Many spectacular effects in orchestration can be achieved by a combined use of these piano devices. The *Harp*, *Novachord*, *Harpsichord*, *Guitar*, *Hawaiian Guitar* in many cases may be looked upon and utilized as percussive instruments. This does not require any additional description.

2. CELESTA

Celesta ("divine") is a keyboard instrument with soft hammers striking metal bars. These bars are made of precious and semi-precious metals. This instrument has a tone unsurpassed in delicacy and tenderness.

The most common design of this instrument includes four octaves (the small, the middle, the first and the second, usually starting from *c*).

Figure 66. Range of celesta. It sounds an octave higher.

The parts for this instrument are written on a two-staff system, the same as for piano. Standard bass (F) and treble (G) clefs are used.

It is a miniature self-sufficient instrument, on which melody, harmony, or both, can be executed simultaneously. Chords in their various instrumental forms, are frequently used on the celesta, as it produces a very delicate accompaniment suitable for melody played on the flute, the clarinet (particularly the subtone register), or in combination with the harp.

This instrument may be looked upon as a still more delicate version of chimes. It can be employed only in transparent (low density) textures and amid low dynamics (p, pp).

Debussy and Ravel used this instrument extensively in their scores. Chaikovsky made some effective use of it in his *Nutcracker Suite*.

3. GLOCKENSPIEL (Orchestra Bells; Campanelle)

This instrument is known in two basic models: the hammer and the keyboard types.

Hammer orchestra bells are played somewhat like the xylophone, i.e., by striking the bars with two hammers (usually made of wood) held in both hands. The bars, of semiprecious and common metals, are built in a portable closing box. The bars are arranged in two rows, similar to the arrangement of black and white keys on the piano. Often even the musical names of the individual pitches are engraved on each bar. This makes it easy for the performer to strike the right bar. Glockenspiels of both types are chromatic instruments. The keyboard model is of piano design.

The hammer instrument has a superior tone-quality to the keyboard model, which is clumsy and produces a less brilliant tone. Generally, the tone of this instrument is a harsher version of the tone of a celesta. The attacks, owing to unsoftened hammers, are more pungent. Some musicians describe it as having a "metallic" timbre.

The range commonly used for both models of orchestra bells is as follows:

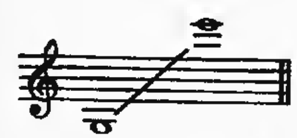


Figure 67. Range of orchestra bells. Sounds two octaves higher.

Parts are usually written on one staff, in treble clef, but can be written, when necessary, on two staves.

As the sound of this instrument has a relatively long durability, it is not desirable to write rapid passages, unless such passages represent instrumental forms of one harmonic assemblage. However, the glockenspiel is a commonly used instrument. Its brilliance is due to the dominance of high partials.

4. CHIMES (Campane)

"Campane" means bells; the English term is "chimes". This instrument is used in large orchestras. It has a group of cylindrical metal bars suspended from a frame. The bars are struck by a wooden hammer (sometimes two hammers are used). This instrument has the sound of the church carillon and represents a more compact version of the latter. It is used for similar climactic or jubilant episodes, or, in some cases, for stimulating associations with a real carillon. The carillon, of course, is a totally different instrument, consisting of church bells and bars and played by fists, striking specially designed large keys.

Chimes usually have a set of bars covering one chromatic octave from *c* to *c*. The parts are written in the middle octave (treble clef) but there is such a predominance of higher partials that, strictly speaking, the pitches do not belong to one particular octave. Chimes blend well with the brass instruments.

5. CHURCH BELLS

This instrument is actually a group of several suspended church bells, matched in their pitches for each individual score. Such a set was used in Chaikovsky's overture "1812" where the church bells represented some of the standard Russian-Orthodox carillons and conveyed the idea of jubilation over the retreat of Napoleon Bonaparte from Moscow.

6. VIBRAPHONE (also known as Vibra-Harp)

This is a relatively new instrument, designed and manufactured in the United States. It is widely used at present in dance-bands. There are already several very eminent virtuosi, who appear as soloists with the dance-bands and small ensembles playing dance music (Adrian Rollini, Lionel Hampton and others).

This instrument is built on the general principle of the xylophone, but its bars, quite large in size, are made of metal, have resonating tubes under them and an extension of tone. The latter is achieved by means of electro-magnetic induction (which not only extends the durability of the tone, but also supplies it with an automatic vibrato by intensity); this effect is controlled by pressing a special pedal, built for this purpose. The execution of various dynamic effects, like sforzando-piano, is thus possible.

The vibraphone has a rich "golden" tone and differs from chimes in its timbral components: it has some similarity, in its basic timbre, with the "chalumeau" of the clarinet. Vibraphones, depending on their size, vary in range. Large concert vibraphones usually have the following range:

Figure 68. Range of vibraphones.

This instrument is played by special hammers, often even of a different design (to achieve different types of attacks). Some vibraphonists hold two, three and even four hammers in each hand. This permits execution of some self-sufficient solos in block-harmonics, following one another at a considerable speed.

7. MARIMBA and XYLOPHONE

The marimba and xylophone are essentially the same kind of instrument. The difference between the two is chiefly in the resonating cylindrical tubes which are part of the marimba and are absent on the xylophone. Both types have the same kind of wooden bars and are played with special hammers. The xylophone is more traditional with the symphony orchestras, while the marimba is more used in dance-bands. It is interesting to note that many truly primitive African tribes use the marimba, i.e., even they have arrived at the necessity of using a resonating medium. The resonating tubes give the marimba a richer and a more sustained tone than the xylophone.

Music written for this instrument in a dance band is considerably more complex technically than parts written for the xylophone in symphonic scoring. One of the reasons for this is that in symphony orchestras one of the percussioners plays the xylophone part, but he is not expected to be a xylophone virtuoso. In the dance bands, quite the contrary, the marimbaist is a specialized soloist (often also playing the vibraphone) and is even capable of handling two, three and as many as four hammers in each hand. Some of these virtuosi handle the xylophone or the marimba as a very delicate instrument. This is accomplished by the use of special soft hammers. Some of such performers give a very refined rendition of Chopin's piano compositions. One very versatile xylophonist even built a dance band around the xylophone as a leading solo instrument. His name is Red Norvo, and recordings of his performances are available.

The range of the xylophone and the marimba varies. In writing for symphony orchestra, it is best to adhere to the following range.

Figure 69. Xylophone range.

In writing for the xylophone or the marimba used in present-day American dance-bands, the range can be extended as follows:

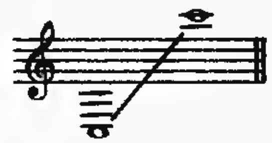


Figure 70. Range of xylophone in dance-bands

Full chromatic scale is available in both cases.

The alternate tremolo (like the plectrum tremolo on the mandolin) of both hands on the same bar (which is equivalent to the same note) is a common way of playing long notes on this instrument. All shorter durations are bound to sound staccato. It is an excellent instrument for execution of 1S2p in any form and at practically any speed.

Glissando either on the naturals (c,d,e,f,g,a,b) or the sharps (c#, d#, f#, g#, a#) is another common device on this instrument. Combinations of both glissando forms, and in both ascending and descending directions may also be used.

Both the xylophone and the marimba have a wide dynamic range. The xylophone blends well with the flute; the marimba, either with the low register of the flute or with the "chalumeau" of the clarinet. Good combinations are also obtained by using the xylophone with the piano.

Parts for these instruments are usually written on the staff in the treble clef (G). In many French scores xylophone parts are written one octave higher than they sound. The reason for this is, probably, the dominance of upper harmonics which, in some cases, produces an impression that a certain tone sounds one octave higher. Many interesting effects may be achieved when parts for this instrument are written with full knowledge of the Theory of Instrumental Forms*.

The following percussive instruments of this group can be looked upon as more primitive or more simplified versions of the instruments already described.

8. TRIANGLE

This instrument consists of one long metal bar of cylindrical form and of relatively small diameter and is bent into an isosceles or an equilateral triangle (hence the name), not quite closed at its vertex. It is usually suspended on a string and is played by striking it with another straight metal bar of about the same length as each side of the triangle itself and of about the same (or smaller) diameter.

The triangle is rather like a single bar of a glockenspiel. Its high partials dominate to such an extent that it is considered to be an instrument "without definite pitch". Thus, the triangle can be used with any harmonic assemblage whatsoever.

*See Vol. I, p. 881 ff.

There are only two ways of using this instrument:

(1) individual attacks (all staccato) arranged in any desirable form of temporal rhythm;
(2) tremolo, which is accomplished by attacking alternately two adjacent sides of the triangle.

It is an instrument of limited dynamic range (generally mf) but can be made to sound very loud in tremolo. The latter also offers crescendo-diminuendo effects. The tone-quality of this instrument is very prominent and very "metallic". It blends well with all higher registers, as at such frequencies tone-qualities lose their timbral characteristics (owing to weakness or inaudibility of the high partials). Parts for this instrument are written on a single line. No clefs are used.

9. WOOD-BLOCKS

Wood-blocks are made in the form of a parallelepiped (rectangular solid) or, more frequently, in the form of a spheroid (eliptic solid). In both cases, some portion of the solid is carved out, and the hollowness thus formed contributes to the resonating quality of this instrument. Wood-blocks are made in different sizes to secure a selection of pitches, but these pitches are not too distinct.

A wood-block may be looked upon as a simplified version of the xylophone. The blocks are struck with sticks or hammers. Often (in dance combinations) an apparatus consisting of three, four or five wood-blocks is added to the usual combination of traps so that they can be handled by one performer. The wood-block is a purely rhythmic instrument. However, if a set of several is used, their parts may be notated on the regular five-line staff, where the pitches can be represented by the closest notes.

10. CASTAGNETTE (Castanets)

Castanets are an instrument of Spanish origin, and in most cases are used in music which is, if not truly Spanish, somehow associated with Spain. By tradition, castanets are an accompanying rhythmic instrument, played by the dancer and not by an outside performer.

Castanets are two small hardwood plaques (with the shape of the sole of an infant's shoe) loosely joined by a cord. They are held within the palm of a hand, with the string pulled over the middle finger. The actual execution of sounds is produced by finger attacks. Fingers strike one of the castanets and this, in turn, strikes the other. This produces a clicking and very brilliant high-pitched inharmonic sound. In some cases, two pairs of castanets are used (one pair for each hand). Some of the Spanish and Flamenco dancers are real virtuosi of this unpretentious instrument.

It is a highly developed (by tradition) rhythmic resource in orchestration and may be looked upon as a simplified version of the xylophone. It is particularly useful for animated high-pitched figures; wood-blocks are considerably lower in pitch and cannot be maneuvered at such a high speed.

The score for each hand occupies one line. Thus for two pairs of castanets, two lines must be used. The advantage of writing on two lines lies in the fact that it is simpler to score many complex interference rhythms, which are easily executed by two hands. It is well worth the time to make a study of traditional Spanish castanet rhythms.

11. CLAVES

The claves is a Cuban instrument consisting of two fairly thick sticks made of hardwood. The performer hits one stick with another. Both sticks are alike. This instrument is commonly used today as a rhythmic ingredient of Afro-Cuban dance forms (Rhumba, Conga, etc.) by our dance orchestras.

The sound of the claves is high-pitched, inharmonic and piercing. In rhumbas it usually performs the $\frac{8}{8}$ series trinomial (i.e., $3+3+2$, $3+2+3$, $2+3+3$). The part for the claves occupies one line.

The claves is ordinarily used with the so-called rhumba bands, but can be introduced into symphonic scoring, when Cuban character is desired in the music.

B. Group Two. Sound via Metal Disc.

1. GONG

This instrument comes from Hindustan and China. It is made in two shapes: a circle or a rectangle. It is made of metal. It is usually very massive and large, at least the type used by the symphony orchestras. It is suspended from a frame to which it is attached by a pair of strings.

Figure 71. Two types of gongs.

It is the lowest-pitched inharmonic percussive instrument of the metal disc group. It is struck with a stick with a round soft end. The sound is very rich in quality and has a great dynamic range, combined with long durability of tone. It blends well with the low register of brass instruments.

The gong must be very moderately used, as it is the last resource of main climaxes. Too frequent use of this startling tone-quality neutralizes its character in the listener's impression. If the sound of the gong must be shorter than its natural fading period at a given intensity, it is damped out by the hand. Otherwise the term commonly used is written out above the note: "laisser vibrer" (let vibrate).

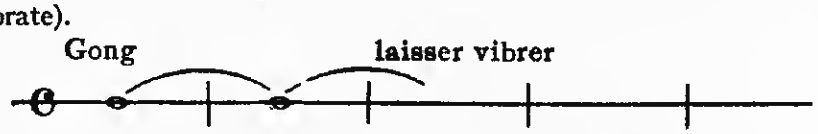


Figure 72. Allowing the gong to fade naturally

As the gong has a slow fading sound, successive attacks require considerable time-intervals between them.

2. PIATTI (Cymbals)

Cymbals consist of a pair of discs approximately 18" in diameter. They are made of semi-precious and precious metals. Each disc has a leather handle in the form of a short loop, by which it is held.

Cymbals are played in two basic ways:

(1) by striking one cymbal over the other (for louder and more prolonged sounds, with a certain amount of friction);

(2) by making a tremolo of alternating attacks over one suspended cymbal (held in horizontal position); for this purpose either hard drumsticks (result in harsher tone-quality and higher-pitched) or kettle-drum sticks (which are soft, and render lower-pitched softer tones) are used.

The range of the cymbals, the tone of which consists of rich inharmonic sound-complexes, varies depending on the form of attack. When the friction surface is small, the sound is higher-pitched. The partials cover approximately the range of trombones (excluding their pedal tones) and trumpets, with which they blend very well.

When cymbals are to be struck by one another, it is usually not necessary to indicate anything other than the temporal values and the dynamics wished. That a suspended cymbal is to be played tremolo is indicated by placing the sign ∼∼∼∼ over the note. The use of hard sticks is marked: *colla bacchetta da tamburo*. The use of soft sticks is marked: *colla mazzuola*, or *colla bacchetta da timpano*.

This standard terminology is notoriously clumsy. I recommend that students use my own nomenclature, which is simple and economical, and permits a much more diversified use of the different types of attack:

a suspended cymbal:

(a) hard sticks: —●
(b) soft sticks: —○

two cymbals in hands: ○

I usually make footnotes at the beginning of my scores explaining the meaning of these symbols. I made the first use of this nomenclature in 1921.

An instrument which at once belongs to Group Two (discs) and Group Three (membranes) is the well-known

3. TAMBURINO (Tamburin)

This instrument consists of a circular wooden frame over which a skin membrane is stretched, covering one side of it. Thus the form of the membrane is a circle. In addition to this, there are small (about 1.5" in diameter) metal double discs, loosely mounted on perpendicular pegs in the frame of the tamburin. The tamburin viewed from above appears as follows:

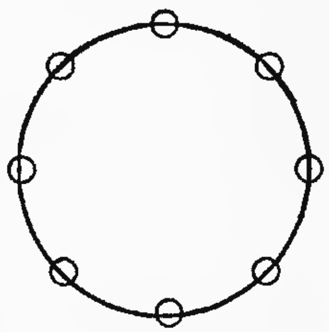


Figure 73. Tamburin

This instrument, associated with Italian and Spanish folk dancing, is played either by striking the skin with the palm, which at once produces a high-pitched inharmonic drum sound and the jingling of the discs (high-pitched "metallic" inharmonic sound); or by shaking the tamburin in the air (held by the left hand), which produces the jingling of the discs alone; or by producing an oscillatory frictional movement over the skin, with the thumb of the right hand, which results in a scintillating type of tremolo. Often these ways of playing the tamburin are combined in effective dynamic and rhythmic sequences. Much initiative in varying the attack forms is left to the performer.

The parts are commonly written on one line, and simply indicate durations and dynamics. Tremolo is marked as usual by:

C. Group Three. Sound via Skin Membrane.

1. TIMPANI (Kettle-Drums)

Kettle-drums are the first percussive instrument to occupy a lasting place in symphonic scoring. It was Josef Haydn, who introduced them [*Sinfonie mit Paukenschlag* (Symphony with kettle-drums)]. Since that time, they have become a standard ingredient in symphonic and operatic scoring.

Kettle-drums are ordinarily used in groups of three and four. The original selection of three kettle-drums usually furnished the tonic, the subdominant and the dominant. Today they are used in any pitch-group combination that satisfies the harmonic need.

The kettle-drum consists of a hollow copper hemisphere, with a skin membrane stretched over its equatorial circumference. The tension of the membrane is adjustable: in other words, kettle drums can be tuned. This is accomplished by screwing in and out the handles (of which there are several around the skin surface) controlling the tension of the membrane. Tuning calls for a keen sense of pitch since it may have to be done quietly while the orchestra is playing. Kettle-drummers (or tympanists) usually know the parts of the neighboring instruments, from which they borrow the necessary pitch.

Each kettle-drum produces one pitch at a time. To obtain many pitches at a time would require as many kettle-drums. Berlioz, in one of his scores, used as many as 16 of them. Considering the usual equipment of the large symphony orchestra, it is advisable not to use more than four. In some instances two simultaneous tympanists can be used, in which case one may count on four or five instruments.

The three standardized sizes usually allow tuning within the following ranges:

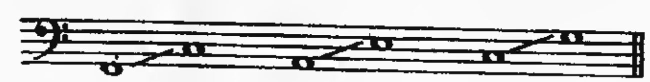


Figure 74. Ranges of the three standardized kettle-drums.

The total range may be considered practical even if one semitone is added at each end:

Figure 75. Total range of kettle-drums.

Rimsky-Korsakov ordered for his opera-ballet *Mlada* a small kettle-drum, which could be tuned up to *d*♭ of the middle octave. He called it "timpano piccolo".

Contemporary American-made kettle-drums have a pedal device for automatic tuning.. This device is supposed to stretch the membrane at all points at an equal tension; it is not too reliable in actual practice. Performers still have to rely on their pitch-discrimination.

The tuning of kettle-drums is marked at the beginning of the score as follows, for instance: Timpani in F, B♭, C. When the tuning changes, the performer is warned by the composer in advance, as a certain amount of time is necessary for tuning of the instrument (the actual time period required largely depends on the performer's experience and skill). It is indicated like this, for example: *muta* in G, B♭, D.

The parts are written in the bass clef [F] on a regular five-line staff; two staves can be used if necessary. Kettle-drums are played by two special sticks having soft spheroid-like ends. The whole technique consists of individual and rolling attacks (i.e., alternating tremolo attacks; the latter may affect one or two instruments).

This instrument has an enormous dynamic range and in ff can pierce the entire tutti of an orchestra. Big crescendi are particularly effective in tremolo (marked:).

Sometimes, though very seldom, delicate sounds are obtained by muting. Flannel or other soft cloth is put over the skin of a kettle-drum. The use of such mutes is indicated by: timpani coperti (i.e., covered kettle-drums). To restore the normal effect, "modo ordinare" is used as a term.

The pitches of the kettle-drums leave something to be desired with respect to precision. This is due to the abundance of lower inharmonic tones. The instrument has a quickly fading tone. The pitch, owing to the presence of low inharmonics, seems lower to the ear than it is written.

2. GRAN CASSA (Bass-Drum)

This instrument usually has a cylindrical frame of very large diameter. The skin-membranes are stretched on both sides. It is considered to be an instrument without definite pitch, as the inharmonic tones predominate and all frequencies are very low.

The bass-drum is usually played with a special stick made for this instrument. The part is usually very simple, is written on one line, and consists of merely individual attacks. Of course other sticks can be used, and the execution of tremolo is also possible. Some of the bass-drums used by dance-bands have a narrow frame and only one membrane. The bass-drum blends naturally with low pitches.

3. TAMBURO MILITARE (Snare-Drum)

This is the most alert instrument in the entire third group. Although in shape it is the same as the bass-drum, it is considerably smaller in size. While the bass-drum is played in vertical position, the snare-drum is played in an almost horizontal position (there is a small angle to the horizon). It is played by a pair of hard sticks known as drum-sticks. The snare-drum derives its name from the snares, a pair of thin gut strings stretched across its lower head which produce a rattling sound. Sometimes the tamburo is used without snares (it is quite customary with dance bands), in which case a notation is made: "no snares".

This instrument produces middle-high inharmonic sounds. It has a wide dynamic range. The speed of rolling is the main feature of this instrument. Even the equivalent of grace-notes is often extended into rolls (marked: , i.e., the small note is the roll and the large note is the attack). It is suitable for the most intricate rhythmic patterns, which can be executed practically at any speed.

The jazz era has created many outstanding drummers in America. Yet the patterns of their improvised rhythms are still very one-sided and limited, as compared to their cannibal colleagues in Belgian Congo. The snare-drum has long been in use in military organizations. Its martial character by now is an inherited association. The part is written on one line. For students of this system, there are many opportunities to utilize the snare-drum as a two-part instrumental interference medium.

4. BONGO DRUMS

This usually consists of a pair of drums. The shape of the frame is a hollow inverted cone (it can be played on either side), which has skin membranes at its open ends. One of the drums is somewhat larger than the other, but there is no fixed ratio. Bongo drums are played by hand. Though probably of African origin, they are widely used in Cuban rhumbas and congas. Rhythmic patterns executed by the Cuban performers are often extremely intricate (mostly based on splitting of the $\frac{8}{8}$ series).

5. TOM-TOM

This instrument consists of a small cylindrical frame, which is relatively wide for its size. It has one skin membrane over its frame. It is ordinarily used (one or more) in jazz bands and played with a stick. Its inharmonic sound blends with the middle register of the ensemble.

D. GROUP FOUR. SOUND VIA OTHER MATERIALS.

No instrument can be considered standard in this group. All *special sound-effect* instruments belong to this group. It is neither necessary nor possible to describe all such instruments, as new types are being developed and introduced every year. Some of these instruments have a brief popularity, after which most of them become obsolete.

The purpose of bringing sound-effect instruments to the composer's attention is to stimulate his resourcefulness and to suggest that he too can use special materials for sound effects. It is also advisable for him to study the history of instruments and to attend the music departments of museums, as this will help him develop proper perspective and orientation in the subject.

One of the more commonly known instruments of this group is an ordinary *sheet of iron* (usually termed in French: *feuille de fer*). By holding such a sheet at one end and shaking it, one obtains thunder-like sounds. Single strokes and tremolo also can be executed on the suspended iron sheet, using different types of standard sticks.

Cow bells are used sometimes as a musical instrument for descriptive music of bucolic character. The bells can be either shaken or struck with a hard stick. Their tuning is unimportant, as the use of them is supposed merely to suggest the rural.

Emery boards (I used them in my Symphonic Rhapsody *October* to produce a steam engine effect) are sometimes used in symphonic and dance scoring. Rubbing of the surfaces of two emery boards (i.e., the sound is obtained by friction and not by attack) produces a powerful sound. It is an excellent descriptive medium for locomotive or train effects.

The *musical saw* was once very popular. It was used as an instrument of the melodic type. Two methods of playing were used: striking it with a stick or a small hammer, or stroking it with the bow (usually a long stroke ending with staccato). It is an extremely effective instrument, whose tone-quality resembles an idealized soprano voice and whose vibrato can be controlled by the performer. The handle is held rigidly between the knees and the end of the saw is supported by the middle finger of the left hand. While the finger presses the end of the saw, the entire saw bends: the greater the curvature, the higher the pitch. Bow or hammer produce attacks and either is held in the right hand.

Today composers have begun to use phonograph records with sound effects (birds, animals and other sounds of the surrounding nature); the latter are included as component parts of a score. Program and background music in radio and cinema utilize such recordings and often simply transfer them to a sound-track. There are several sound-effect renting record libraries containing any imaginable sound effects (there are more than 10,000 items now). The firms are located in New York, but they supply the entire country.

1. HUMAN VOICES (Vocal Instruments)

The human voice is one of the original natural musical instruments. It is by no means standardized. There are too many types of voices and too many ways of using them. Each national culture has different types of voices and different methods of singing. Even different styles of music within one national culture often call for totally different manners of execution. Just to use a bold illustration, compare the bel canto style of operatic vocal art with popular crooning or "torch-singing" of today. The contrasts in singing of different nations are at least as great. Compare, for instance, French folk singing with Siamese folk singing or with Abkhasian choral singing (some of the Black Sea Caucasian shore; the mythical land of the Golden Fleece [Jason]) which has a unique instrumental character of its own.

Even in so-called European musical culture, we find such different styles as the Italian bel canto, the Russian vocal style (as in Chaliapine), the German lieder-singing, etc. Then we find such contrasting styles as vocal jazz ensembles and the plain chant of the Catholic Church. No doubt new styles will appear in the future.

Beside the necessity of considering all these stylistic and national differences in the voice as musical instrument, there are also biological differences and modifications, which take place as time goes on. One of such modifications is the appearance of greater differentiation of ranges and characters. Some time ago male voices were mostly tenor and bass. Later it became necessary to single out

the intermediate type: baritone. Now we have bass-baritones, tenor-baritones etc. Standard parts of the classical repertoire are not written for them; so they have either to sing the parts which are too high or too low for them, or else to look for composers who would write for these new vocal instruments.

Sometimes we also encounter biological aberrations producing such voices as *altino*, which is not only higher than the male tenor, but also has a peculiar quality of its own, not to be confused with a boy's alto or a female's contralto. Rimsky-Korsakov even wrote a part for an altino (the astrologer in *Coq d'Or*), for which Russia found only two performers.

There are also other cases of vocal travesty, like the Russian Gypsy singer, Varia Panina, who possessed a genuine baritone; or another Russian singer, Anna Meichick, who had such a massive and wide-ranged contralto that she sang the part of the Demon, in Rubinstein's opera of the same name. Anna Meichick was the first contralto at the Metropolitan Opera House in New York for many years.

With all this in view, the problem of describing standard human voices seems insoluble. What the composer has to be aware of is that in writing for an oboe, he has a pretty well-defined auditory image in his mind, whereas in writing for a tenor, he can not know what he is likely to get in actual performance.

There are other considerations of equal importance. One of them is the effect of language upon the style of vocal execution. And this often involves such important considerations that the very nature of the Italian language (i.e., the type and the distribution of vowels and consonants) makes singing easy and natural and articulation clear, as compared with the English language. A number of good singers whose native tongue is English, *sing better* in Italian. Certain English sounds, like *th*, do not permit a proper air impact. On the other hand, the entire manner of singing in French, owing to its phonetic and articulatory nature, acquires a nasal character (on, en, un, in, etc.). All this naturally cannot be neglected by the composer. Thus, in order to present a somewhat practical description of human voices as orchestral instruments, I have to resort to somewhat specialized generalities.

Among these are the standard choral ranges, as they are traditionally used in our scoring for *a cappella* or accompanied chorus. Soloists sometimes have wider ranges. But it is not always the case. Another generalization can be drawn with respect to basic timbres of vowels, in which case I shall use the Latin pronunciation of vowels.

No other components can be generalized, as all tone-qualities are individual; their forms of vibrato are also individual. Physically, each sound produced by the same voice on different vowels of the same pitch, or on the same vowel differently pitched, not to speak of different vowels differently pitched, has a different character. But this we cannot take into consideration, as even the violin changes its character (and in many instances even timbre) on different strings.

Among components which cannot be generalized is dynamics. The volume of voice and its dynamic range varies individually. Powerful voices, if combined with pleasing quality, are considered valuable, as such voices can produce a powerful impression by their dynamic versatility. Nowadays timbre, character and volume can be considerably modified either by using a microphone or by acoustical modification of the sound-track, which is constantly done in the radio and the cinema field.

Neither can individual articulating quality be generalized, (which, strictly speaking, belongs to the field of vocal attacks), even when we consider only one particular language. Some outstanding singers have magnificent articulation in addition to their vocal quality and general technique. I can mention two, as examples of perfect articulatory technique, though these singers belong to two different national cultures: one is Mattia Battistini (an Italian baritone); another, Feodor Chaliapine (the Russian basso).

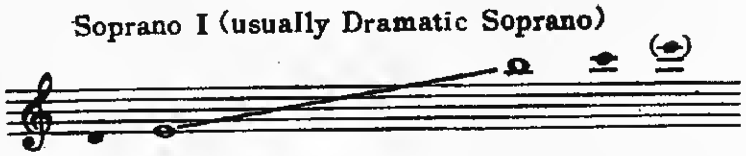
Now after making all these necessary warnings, I can proceed with the description of choral ranges and basic timbres of the latinized vowels.

In some cases composers write certain solo, or even choral parts for a definite performer or a definite organization of performers. In such a case, of course, he can do a better job, as his parts are likely to fit the individual charateristics of the soloist or the ensemble.

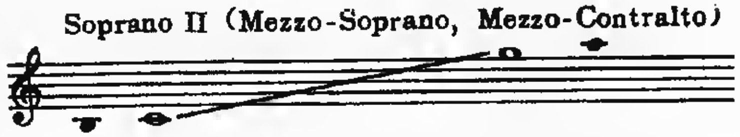
Standard Choral Ranges

Female ranges:

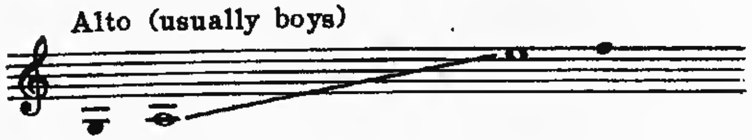
Soprano I (usually Dramatic Soprano)



Soprano II (Mezzo-Soprano, Mezzo-Contralto)



Alto (usually boys)



Contralto

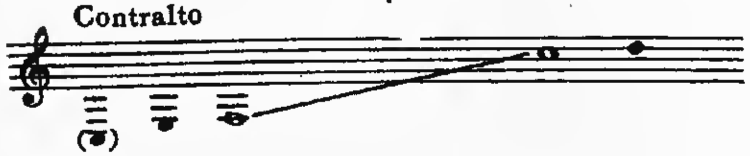
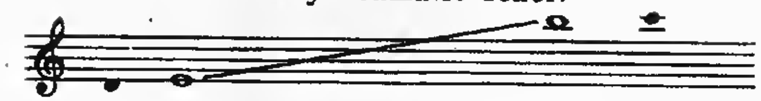


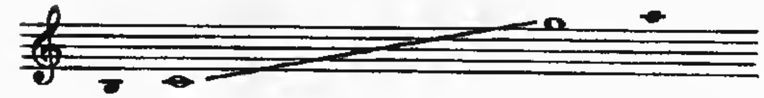
Figure 76. Standard choral ranges (continued).

Male Ranges:

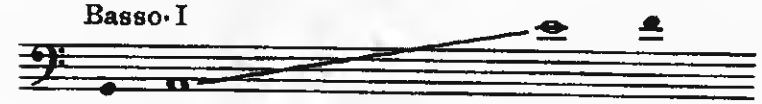
Tenor I (usually Dramatic Tenor)



Tenor II (Baritone)



Basso I



Basso II (usually Basso Profundo)

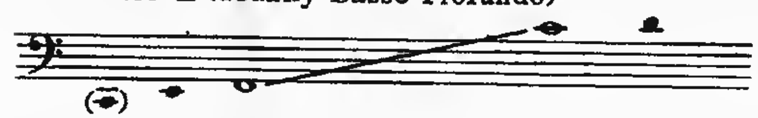


Figure 76. Standard choral ranges (concluded).

The parts for male voices, when written in treble clef, sound one octave lower than written. The so-called lyric sopranos and tenors usually have the range of soprano II and tenor II respectively, but with less developed lower register.

Latin	English Phonetic		Timbre
u	oo	open	O
o	oh	reed	R
a	ah	stopped	⊕
e	eh	double reed	RR
i	ee	closed	●

Figure 77. Timbral scale of the five basic Latin vowels.

This scale relates the vowels to five basic timbral groups, with which each vowel blends itself respectively. Thus, O corresponds to flutes, R to clarinets, ⊕ to horns, RR to oboes and bassoons, ● to nasal timbres and muted instruments (muted brass, celli, muted stringed instruments in general).

This scale can be extended to nine units, by means of combined vowels. The latter can be obtained by mixing the adjacent vowels of the basic scale. A nine-unit scale may be extremely helpful in evaluating general timbral characteristics of the English, French, German and Scandinavian vowels.

Latin	English	Timbre
u + o	u (up)	O + R
o + a	o (cod)	R + ⊕
a + e	a (as)	⊕ + RR
e + i	i (it)	RR + ●

Figure 78. Timbral scale of the four combined (intermediate) vowels.

Further supplements, which may still be necessary, derive from combinations of the non-adjacent vowels. The most important of these are somewhat common to Latin, English, French, German and Scandinavian.

Latin	English	(Phonetic)	French	German	Timbre
oe	e (alert) i (bird u (fur)	oe	eu	ö	R + RR
y			u	ü	O + ●

Figure 79. The two additional combined vowels.

All other u-vowels, as in the English word "you". or the sound of Russian character "Ю" (pronounced: you), have an attack of the attack of the English "y" (as in "yoke"), or German "j" (yot), or Russian "ŭ" (brief "ee" [in Russian: ee kratkoye]) and the duration of the Latin "u", or English "oo".

This information is sufficient to guide the student in the field of basic vowel characteristics and to help him understand the reason for selecting one or another instrumental timbre in the accompaniment to vocal parts. Selection is based on coincidence (similarity) or juxtaposition (contrast) of the basic timbral characteristics, such as "u" (Latin), for flute, "o" (Latin), for clarinet, etc.

PART TWO: *INSTRUMENTAL TECHNIQUES*

CHAPTER 7

NOMENCLATURE AND NOTATION

THE following symbols represent a new system of notation, whose compactness and clarity may be of assistance in orchestral analysis and synthesis.

We shall use this system only if and when there seems to be a decided advantage in doing so. In the meantime, such notation will educate the composer to think of orchestral techniques through the medium of a unified system of concepts, thereby reducing his associational effort to a minimum.

The field covered by this system of symbols is as follows:

(1) orchestral forms (generalities);
(2) orchestral components (resources);
(3) orchestral tools (instruments);
   (a) groups;
   (b) families;
   (c) members;
   (d) auxiliary members

Some of the symbols, such as the last character of the Greek alphabet "ω" (omega) which is an equivalent of the Latin "o", are employed to designate the final stage of musical synthesis: orchestral form; thus, a symbol of finality is employed.

Other symbols are abbreviated idioms, like "q" for quality and "a" for attack. Still other symbols are simplified pictographs, like for gong and for tamburin.

In some cases it was necessary to resort to somewhat more complex symbols. Sometimes they are combinations of abbreviated idioms, such as ("H" superimposed upon "O") for Hammond organ; and sometimes they are combinations of idioms and pictographs, as in for xylophone, for marimba and for vibraphone, where the pictograph of a bar is combined with the abbreviated idioms of X (xylophone), M (marimba) and V (vibraphone) respectively.

One group of symbols plays a particularly important part in the process of unifying the system. This group relates orchestral components (resources) to orchestral tools (instruments of execution). It is in this manner that the symbol of a simple open tone "O" becomes associated with the flute family, as its basic representative. The same concerns "R", the symbol of a single-reed quality, which becomes associated with the clarinet family.

Finally, the general use of horizontal lines added at different levels to basic idioms establishes range-register associations. Thus is the flute quality, is the highest-sounding member of the flute family (piccolo), is the basic type (grande) and is the lowest-sounding member (contralto). Similarly, the violin family is designated by V for violin, V for viola etc.

Greater accuracy in designating members of one family is hardly possible, as the range-register correspondences for the different families vary.

The nomenclature and symbols of *orchestral components* are identical with that of *instrumental resources* (see *Theory of Composition*, Part One*). For the present purpose this group of symbols must naturally be more complete. To make it more useful, each group is represented in the form of three- and five-unit scales. The latter can be extended still further if necessary.

It may be added that this system of nomenclature and notation is well worth studying as, quite apart from its use in this Theory of Orchestration, it has a methodological value *per se*, as the first system capable of designating, by means of its symbols and numerical coefficients, any situation to be encountered in the planning and execution of an orchestral composition.

A. Orchestral Forms (Generalities).

Musical synthesis results from three operational stages:

(1.) harmonic forms;
(2.) instrumental forms;
(3.) orchestral forms;

These three stages are interrelated through their density forms.

Symbols:

(1) harmonic:

p —part, a unit of an assemblage
S —structure of an assemblage, stratum
Σ —sigma, compound structure
$\Sigma(\Sigma)$ —compound sigma

p —a neutral unit
$p^{\rightarrow}$ —a descending directional unit
$p_{\rightarrow}$ —an ascending directional unit
$p^{\rightarrow}_{\rightarrow}$ —a two-directional unit

$S^{\rightarrow}$ —sequent assemblage, stratum
$\Sigma^{\rightarrow}$ —compound sequent assemblage, sigma
$\Sigma^{\rightarrow}(\Sigma)$ —compound sequent sigma

H —harmony, a group-unit of harmonic continuity, chord
$H^{\rightarrow}$ —sequent harmony, chord progression

*See p. 1323.

(2) instrumental:

a —an attack-unit
A —an attack-group
I —simultaneous instrumental form
$I^{\rightarrow}$ —sequent instrumental form
p —simultaneous part
$p^{\rightarrow}$ —sequent part

(3) orchestral:

p —simultaneous part, simultaneous unit
$p^{\rightarrow}$ —sequent part, sequent unit
ω —simultaneous instrumental group, orchestral group (small omega)
$\omega^{\rightarrow}$ —sequent instrumental group, orchestral group
Ω —simultaneous instrumental combination, orchestra, tutti, (capital omega)
$\Omega^{\rightarrow}$ —sequent instrumental combination, orchestration, orchestral score

Density:

d —density unit
D —simultaneous density-group
$D^{\rightarrow}$ —sequent density-group
Δ —compound density-group
$\Delta^{\rightarrow}$ —sequent compound density-group
$\Delta^{\rightarrow\prime}(\Delta^{\rightarrow})$ —sequent compound delta
ϕ —phi, individual rotation-phase
$\phi\circlearrowright$ and $\phi\circlearrowleft$ in reference to t or T
$\phi(\circlearrowright)$ and $\phi(\circlearrowleft)$ in reference to p or P, or d or D
θ —theta, compound rotation-phase

Density forms relating the three stages:

(a) harmonic density:

$d(H)$ —simultaneous harmonic density-unit
$d^{\rightarrow}(H)$ —sequent harmonic density-unit
$D(H)$ —simultaneous harmonic density-group
$D^{\rightarrow}(H)$ —sequent harmonic density-group
$\Delta(H)$ —simultaneous compound harmonic density-group, harmonic density
$\Delta^{\rightarrow}(H)$ —sequent compound harmonic density-group, sequence of harmonic density

likewise:

$d(H^{\rightarrow})$, $d^{\rightarrow}(H^{\rightarrow})$, $D(H^{\rightarrow})$, $D^{\rightarrow}(H^{\rightarrow})$, $\Delta(H^{\rightarrow})$, $\Delta^{\rightarrow}(H^{\rightarrow})$

(b) instrumental density

$d\,(I)$ —simultaneous instrumental density-unit
$\vec{d}\,(I)$ —sequent instrumental density-unit
$D\,(I)$ —simultaneous instrumental density-group
$\vec{D}\,(I)$ —sequent instrumental density-group
$\Delta\,(I)$ —simultaneous compound instrumental density-group, instrumental density
$\vec{\Delta}\,(I)$ —sequent compound instrumental density-group, sequence of instrumental density

likewise:

$d\,(\vec{I}),\ \vec{d}\,(\vec{I}),\ D\,(\vec{I}),\ \vec{D}\,(\vec{I}),\ \Delta\,(\vec{I}),\ \vec{\Delta}\,(\vec{I})$.

(c) orchestral density:

$d\,(\Omega)$ —simultaneous orchestral density-unit
$\vec{d}\,(\Omega)$ —sequent orchestral density-unit
$D\,(\Omega)$ —simultaneous orchestral density-group
$\vec{D}\,(\Omega)$ —sequent orchestral density-group
$\Delta\,(\Omega)$ —simultaneous compound orchestral density-group, orchestral density
$\vec{\Delta}\,(\Omega)$ —sequent compound orchestral density-group, sequence of orchestral density

likewise:

$d\,(\omega),\ \vec{d}\,(\omega),\ D\,(\omega),\ \vec{D}\,(\omega),\ \Delta\,(\omega),\ \vec{\Delta}\,(\omega)$
$d\,(\vec{\omega}),\ \vec{d}\,(\vec{\omega}),\ D\,(\vec{\omega}),\ \vec{D}\,(\vec{\omega}),\ \Delta\,(\vec{\omega}),\ \vec{\Delta}\,(\vec{\omega})$
$d\,(\vec{\Omega}),\ \vec{d}\,(\vec{\Omega}),\ D\,(\vec{\Omega}),\ \vec{D}\,(\vec{\Omega}),\ \Delta\,(\vec{\Omega}),\ \vec{\Delta}\,(\vec{\Omega})$

Generalization:

harmonic forms:	$p,\ S,\ \Sigma$
density forms:	$d,\ D,\ \Delta$
instrumental forms:	$a,\ A,\ I$
orchestral forms:	$p,\ \omega,\ \Omega$

density forms relating the three stages: $\Delta\,(H),\ \ \Delta\,(I),\ \ \Delta\,(\Omega)$

Musical Synthesis:

transformation of harmonic density into instrumental density and, finally, into orchestral density: $\Delta\,(H) \rightarrow \Delta\,(I) \rightarrow \Delta\,(\Omega)$

B. Orchestral components (Resources)

Five orchestral components constitute omega (Ω)

D —orchestral density
V —orchestral volume (loudness)
Q —tone-quality
I —instrumental form of orchestration
A —instrumental form of attack

$$\Omega = D \div V \div Q \div I \div A$$

Scales of units in relation to their groups:

$D = d,\ 2d,\ 3d,\ \ldots\ nd;\ d_I\ ,\ d_{II}\ ,\ d_{III}\ ,\ \ldots$
$V = v,\ 2v,\ 3v,\ \ldots\ nv;\ v_I\ ,\ v_{II}\ ,\ v_{III}\ ,\ \ldots$
$Q = q,\ 2q,\ 3q,\ \ldots\ nq;\ q_I\ ,\ q_{II}\ ,\ q_{III}\ ,\ \ldots$
$I = i,\ 2i,\ 3i,\ \ldots\ ni;\ i_I\ ,\ i_{II}\ ,\ i_{III}\ ,\ \ldots$
$A = a,\ 2a,\ 3a,\ \ldots\ na;\ a_I\ ,\ a_{II}\ ,\ a_{III}\ ,\ \ldots$

Scales of orchestral components

(a) Scales of D:

$D = 3d$:

d_I	low	solo
d_{II}	medium	group
d_{III}	high	tutti

$D = 5d$:

d_I	low	solo
d_{II}	medium-low	solos
d_{III}	medium	group
d_{IV}	medium-high	groups
d_V	high	tutti

(b) Scales of V:

$V = 3v$:

v_I	low	*pp*	*p*
v_{II}	medium	*mf*	*mf*
v_{III}	high	*ff*	*f*

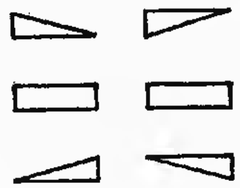
$V = 5v$:

v_I	low	*pp*
v_{II}	medium-low	*p*
v_{III}	medium	*mf*
v_{IV}	medium-high	*f*
v_V	high	*ff*

(c) Scales of Q:

Q = 3q:

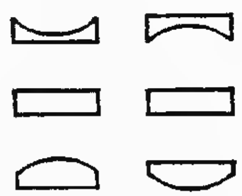
A.P. (*Amplitudes of Partials*) H.L. (*Hammond Organ Levers*)



q_I	low	○	open
q_{II}	medium	⊕	stopped
q_{III}	high	●	closed

Q = 3q;

A.P. H.L.



q_I	medium-low	R	single-reed
q_{II}	medium	⊕	stopped
q_{III}	medium-high	RR	double-reed

Q = 5q:

A.P. H.L.

q_I	low	○	open
q_{II}	medium-low	R	single-reed
q_{III}	medium	⊕	stopped
q_{IV}	medium-high	RR	double-reed
q_V	high	●	closed

(d) Scales of I:

I = 3i:

i_I	low	ap	ap
i_{II}	medium	aS	anp
i_{III}	high	aΣ	aS

I = 5i:

i_I	low	ap
i_{II}	medium-low	anp
i_{III}	medium	aS
i_{IV}	medium-high	anS
i_V	high	aΣ

(e) Scales of A:

A = 3a:

a_I	low	legatissimo	legato
a_{II}	medium	portamento	portamento
a_{III}	high	staccatissimo	staccato

A = 5a:

a_I	low	legatissimo
a_{II}	medium-low	legato
a_{III}	medium	portamento
a_{IV}	medium-high	staccato
a_V	high	staccatissimo

C. Orchestral Tools (Instruments)

Groups:

SB	stringed instruments bowed
SP	stringed instruments plucked
W	wood-wind instruments
B	brass-wind instruments
P	percussive instruments

Families, members, auxiliary members:

Stringed Instruments:

(a) Violins:

violin
viola
violoncello
double-bass

open: ; muted:

bowing (arco):

bowed		
head	(punta)
middle	(media)
nut	(talon)

bowing in relation to fingerboard:

fingerboard	(tasto)
middle	(media)
bridge	(ponticello)

plucking, striking, slapping:

plucked	(pizzicato)
struck	(col legno)
slapped	

(b) other instruments:

- piano (grand or upright)
- piano (electronic)
- harp
- guitar (Spanish)
- guitar (electronic)
- mandolin
- balalaika

Wood-Wind Instruments:

(a) flutes:

- piccolo
- grande
- alto
- basso

(b) clarinets:

- piccolo
- soprano
- alto
- basso
- contrabasso
- bassethorn

(c) saxophones:

- soprano
- alto
- tenor
- baritone
- bass

(d) double-reed instruments:

- oboe
- English horn
- heckelphone
- fagotto (bassoon)
- contrafagotto (contrabassoon)

Brass-Wind Instruments:

(a) horns:

- horn (French)

(b) trumpets:

- piccolo
- soprano
- alto
- basso

(c) trombones

- trombone
- trombone (extra crook: fourth ↓)

(d) tuba:

- tuba (contrabassa)

Closing:

- open
- stopped
- muted

Organs:

pipe-organ

electronic organ(in general)

Hammond organ

Electronic Instruments:

N novachord
V solovox
T theremin (space-controlled)

Percussive Instruments:

(a) bars:

celesta

bells (chimes)

orchestra bells (glockenspiel)

xylophone

marimba

wood-blocks

vibraphone

(b) plates:

gong

cymbals

FF iron sheets (feuilles de fèr)

(c) skins:

kettle-drums (timpani)

snare-drum

tamburin

snare-drum without snares

bass-drum

(d) rods and others:

triangle

castanets

clavis

(e) auxiliary percussive instruments:

drumstick (hard stick)
soft stick (kettle-drums)
soft stick (gong)
brush (brushes)

Human Voices:

S soprano
SA mezzo-soprano, mezzo-contralto
A alto, contralto
AT altino
T tenor
TB baritone
B bass; $\bar{B}$ bass-baritone; $\underline{B}$ basso profundo

CHAPTER 8.

INSTRUMENTAL COMBINATION

IN this age of progressively precipitating mutations of forms, it becomes necessary to think in terms of present mutations and of mutations to come. One of the attributes of current progress is the plurality of the individual. This concept implies versatility of a self-contained unit. While it has been considered a virtue for a creative artist to develop one particular style from which he could be recognized, it is no longer so—since the composer equipped with a scientific method of production (such as is offered by this *System of Musical Composition*) can afford to master a multitude of styles, and be equally proficient in all of them. We have accumulated sufficient factual evidence to this effect to substantiate this claim.

In view of this consideration it becomes apparent that a certain style not only may become outmoded and obsolete, but the very idea of a composer being confined to one style no longer holds true. The character of progress affects not only the creators but also their tools. Musical instruments as types become outmoded and obsolete not only with regard to their design in general, but also with regard to the type of functions they are called upon to perform. It is not only important that a new method of sound-production has been discovered and put to use, but also that this new method transforms an instrument of a certain individual type into a versatile self-contained unit.

Until very recently the piano was "just a piano". Now we have an electronic piano, an instrument with a versatile functionality. It may be percussive, yet it may have a sustained tone; it may sound like a harpsichord and again it may sound like an organ. Not only its attack-characteristics become variable, but also its tone-qualities. It was formerly impossible to control the tone after a stroke of the hammer. This, in the case of an electronic piano, is no longer true.

Mutations affect not only individual instruments alone, but also the ways in which they are selected and combined in an instrumental combination. In view of this, hardly any combination can be considered standard, as what appears to be "standard" today, eventually may become an obsolete model of the vogue 1942.

This situation, over which we have no control, requires a broader basis for selecting individual instruments (though some of them may be of the plural type) and for combining them into groups.

A. QUANTATIVE AND QUALITATIVE RELATIONS BETWEEN INDIVIDUAL MEMBERS AND THE GROUP IN AN INSTRUMENTAL COMBINATION.

1. *Quantitative relations of members belonging to an individual timbral group or family.*

Members of the flute family are represented either by an individual instrument or by a pair of identical instruments, in which case the characteristics of both members with respect to timbre, intensity, attack-forms and range are identical, providing that both members are used in unison or at least in close intervals. Opening of a harmonic interval destroys correspondences of registers and partly of intensities.

The addition of a third flute, which is usually the piccolo, adds an upper octave-coupler which, for practical purposes, has a satisfactory correspondence of components with the large flutes.

In certain rare cases symphonic and operatic scores include an alto flute which, in some instances, alternates with the piccolo. It is important to realize that the range of the alto flute is located a fourth or a fifth below the large flute. It is in such relations that this instrument corresponds to the large flute. Thus the quantitative and the range relations within the flute family may be represented as follows:

$$\text{Fl.};\quad \text{2Fl.};\quad 8\left[\begin{array}{l}\text{Picc.}\\ \text{2 Fl.};\end{array}\right.\quad 5\left[\begin{array}{l}\text{2 Fl.};\\ \text{Alto}\end{array}\right.\quad \begin{array}{rl} 8 & \text{Picc.}\\ & \text{2 Fl.}\\ 5 & \text{Alto}\end{array}$$

Figure 80. Flute family

Such quantitative relations are quite different in the oboe family. As there is no piccolo type of oboe, there are only the following possible combinations:

$$\text{Ob.};\quad \text{2 Ob.};\quad 5\left[\begin{array}{l}\text{2 Ob.}\\ \text{E.H.}\end{array}\right.\quad 8\left[\begin{array}{l}\text{2 Ob.}\\ \text{B.O.}\end{array}\right.\quad \left.\begin{array}{rl} 5 & \text{2 Ob.}\\ & \text{E.H.}\\ 4 & \text{B.O.}\end{array}\right]8$$

Figure 81. Oboe family

The clarinet family, on the other hand, besides the lower octave-coupler (B.C.) has a special type of diminutive clarinet (in D and in E♭), which is an upper second-or third-coupler. The quantitative relations of this family appear as follows:

Cl.; 2 Cl.;	8 [2 Cl. / B.C.]	3 [Picc. / 2 Cl.;]	3 [Picc. / 2 Cl.] 8 [B.C.]

Figure 82. Clarinet family

The bassoon family uses only two types at present. In this case there is only the basic type and the lower octave-coupler.

Bssn.; 2 Bssn.; 8 [2 Bssn. / C.B. (Contrabssn.)]

Figure 83. Bassoon family

Thus we find no identical relations in four families of the wood-wind instruments, unless the basic types are used alone and only in even quantities.

The comparative tuning-range characteristics of the wood-wind group appear as follows:

Flutes	Oboes	Clarinets	Bassoons
8		3	
5	5	8	8

Figure 84. Wood-wind tuning range.

In the absence of tuning-range correspondences, composers select the quantity and the type of supplementary instruments at random. In some cases an upper octave-coupler is added, in some a lower; in some other cases, the lower fifth-coupler is added, without adding any other couplers. More pretentious scores include four and even five members belonging to one group so that the quantitative relations of types vary greatly. Thus, we see that the quantitative relations within the wood-wind group are not based on any definite system of correspondences, unless only the basic types are used in equal quantities in each family.

The lack of a system of quantitative correspondences is equally as noticeable in the group of brass-wind instruments. There are 2, 3 or 4 French horns ordinarily used. In spite of the fact that they have identical tuning-range, they are

often used as mutual octave-couplers. The quantitative and the range relations of the horns appear as follows:

H.; 2 H.; 3 H.; 4H.

Figure 85. French horn family

As we are not discussing the use of instruments at present, we shall not include horns as mutual octave-couplers. A natural octave-coupler to this group is the tuba. It is customary to couple two horns with one tuba because the quality of the latter is so dense.

Trumpets, in their quantitative and tuning-range relations, represent a mixture of the flute and the clarinet family. The piccolo coupler is located a major second or a minor third above the basic type (as in the clarinets) and the lower coupler is an alto type (as in the flutes).

The quantitative and range relations of trumpets are as follows:

Tr.;	2 Tr.;	3 Tr.;	3 [Picc. / 2 Tr.;]	3 [Picc. / 3 Tr.;]
	5 [2 Tr. / Alto]	3 [Picc. / 2 Tr.] 5 [Alto]		

Figure 86: Trumpet family

The trombone family consists of identical type-instruments only. Their quantities vary but their tuning-range relations are identical, though variable:

Tromb.; 2 Tromb.; 3 Tromb.

Figure 87. Trombone tuning range

In the customary type of symphonic scoring, 3 trombones are generally used —and ordinarily in association with the tuba as lower octave-coupler of the third trombone.

The comparative tuning-range characteristics of the B.-W. group appear as follows;

Horns	Trumpets	Trombones	Tuba
	3		
0		0	0
	5		

Figure 88. Brass tuning range.

These relations apparently do not disclose any system.

Quantitative and tuning-range relations in the string-bow group possess their own characteristics. It is customary to join groups of instruments of one type for the unison playing of one part. Thus from the composer's standpoint, one flute or one clarinet usually corresponds to a whole group of violins playing in unison. Whether such a method is justified is another matter.

It is customary to arrange the S.-B. instruments into four-part harmonies with a coupled bass (octave coupling). Their actual tuning-ranges, however, appear as follows:

1st Vlns. + 2nd Vlns.
5
Violas
8
Cellos
8
Basses

Figure 89. String-bow tuning ranges.

In actual use, however, 1st Vlns. are frequently placed at some interval with 2nd Vlns. Inasmuch as string-bow instruments are of identical design and identical sound production, they can be considered to be of one type, though of a different tuning-range.

2. *Quantitative relations between the different timbral groups or families.*

We shall consider our classification on the basis of single, double, triple, etc., participation of each type of instrument in its respective group.

Coefficients of coupling are not used ordinarily with the lower octave-couplers—and very seldom with other couplers.

In the single combination there is only one representative of each family for each tuning-range. The assortment for a single instrumental combination, including the three orchestral groups (S., W.-W. and B.-W.) assumes the following form.

Fl. Ob. Cl. Bssn.	Quartet	Quartet
Horn Tr. Tromb. Tuba	Quartet	Trio 8 Coupler
Vlns. Violas Cellos Basses	Quartet	Trio 8 Coupler

Figure 90. The single instrumental combination.

The second reading is considered because it is commonly used.

2 Fl. 2 Ob. 2 Cl. 2 Bssn.	Octet	Octet
2 Horns 2 Trump. 2 Tromb. Tuba	Septet	Sextet 8 Coupler
1st Vlns. 2nd Vlns. Violas Cellos Basses	Quintet	Quartet 8 Coupler

Figure 91. The double instrumental combination.

Here all types appear in two's, except for the tuba. Of course, the subdivision of violas and cellos into two parts each (but not the basses) is also acceptable and in some cases is actually used.

3 Fl. (Picc., Alto) 3 Ob. (E.H.) 3 Cl. (Basso, Picc.) 3 Bssn. (C.-B.)	12 parts
3 Horns 3 Trump. (Alto, Picc.) 3 Tromb. Tuba	10 parts
1st Vlns. 2nd Vlns. Violas Cellos Basses	5 parts

Figure 92. The triple instrumental combination.

Here W.-W. may have two or three octave-couplers (C.-F., Cl.-B., Fl. Picc.); B.-W. may have one or two octave-couplers (Tuba, Horn or Tromb.) S.-B., one octave-coupler (Basses).

4 Fl. (Picc. and Alto)	
4 Ob. (E.H. and B.O.)	16 parts
4 Cl. (Picc. and B. Cl. [Bassethorn])	
4 Bssn. (Contrafag.)	
4 Horns	
4 Trump. (Alto, Picc.)	13 parts
4 Tromb.	
Tuba	
1st Vlns.	
2nd Vlns.	
Violas	5 parts
Cellos	
Basses	

Figure 93. The quadruple instrumental combination.

Upper and lower octave-couplers can be used as in the previous combination.

These classified instrumental combinations do not always correspond to the actual selections of instruments, which sometimes are a matter of tradition and routine, and sometimes the result of a random selection by the composer himself. As a result of this, many combinations used during the last century are of the intermediate, mixed type. In the latter, some groups contain only one member, while other groups consist of two, three and even four members.

One of the most standardized instrumental combinations of symphonic scoring for a large orchestra is as follows:

Fl. Picc.	
Fl. I	
Fl. II	
Ob. I	
Ob. II	9 parts
(E.H.)	
Cl. I	
Cl. II	
(B. Cl.)	
Bssn. I	
Bssn. II	
4 Horns	
Tr. I	
Tr. II	
Tromb. I	10 parts
Tromb. II	
Tromb. III	
Tuba	
1st Vlns.	
2nd Vlns.	
Violas	5 parts
Cellos	
Basses	

Figure 94. The standard symphonic combination.

In some cases the English Horn and/or the Bass Clarinet are added to this standard combination. Extra players may be required for these instruments but more often the second oboist is left free to play the English Horn as the second clarinettist plays the Bass Clarinet.

Radio orchestras are often reduced and modified versions of the symphonic combinations. They are by no means standardized. However, certain instrumental combinations are preferred by leading radio stations. We refer to the following combination merely as a prevailing one:

Fl. I	
Fl. II (Picc.)	
Ob. I	
Ob. II (E.H.)	8 parts
Cl. I (Sax)	
Cl. II (Sax)	
B. Cl. (Sax)	
Bssn.	
2 Horns	
3 Trump.	7 parts
2 Tromb.	
Violins	
Violas	4, 5 parts
Cellos	
Basses	

Figure 95. Radio orchestras.

Since the development of jazz, doubling on a saxophone has become quite customary. In addition to the plural aspects of an individual instrument, performers begin to develop plurality in mastering several instruments. All accomplished saxophonists are expected to play clarinets of various types, and some of them play also the double-reed instruments.

The distribution of groups in a score has undergone a number of modifications. It is somewhat standardized in each type of scoring, but different for the different types.

In symphonic scoring, at present, the parts for the wood-wind instruments are written at the top of the score; brass-wind parts appear below these; the percussive and solo parts (harp, piano, voices) follow; the lowest section is reserved for the string parts. The customary distribution is shown in Figure 94.

It is easy to see that the quantitative diversity of instrumental combinations poses a great many problems for the orchestrator or the composer. Since combinations vary, it is not sufficient simply to master any specific combination, as is required in the existing academic training. It becomes more and more important, as the diversity of instrumental combinations grows, to master the principles of this art.

3. *Qualitative relations of members and groups or families.*

In addition to *quantitative diversity*, there is a great *qualitative diversity* which is noticeable even in one instrument of a certain type, not to mention the different types and, particularly, the different families of instruments. We shall now discuss these *qualitative relations* which concern correspondences of *timbre, intensity, attack-forms* and *pitch-range.*

In the wood-wind group, we find a close timbral similarity between the members of one family. The density of the timbre varies with the individual types, the lower instruments being denser than the higher. This, of course, is due to the fact that when more partials are within the audible range, the resulting quality appears denser. Timbral density, as a consequence, decreases in all instruments as frequencies increase.

The following subgroups of the wood-winds are those that are most homogeneous:

flutes and clarinets;
oboes and bassoons;
clarinets and oboes;
clarinets and bassoons.

There is a greater timbral similarity between the two families of the double-reeds than between any other combinations. We can establish, for purely methodological reasons, a *scale of decreasing timbral similarities* for combinations of wood-wind families by two:

oboes and bassoons;
clarinets and bassoons;
flutes and clarinets;
clarinets and oboes;
flutes and bassoons;
flutes and oboes.

Timbral characteristics of the brass-wind group are more homogeneous than that of the wood-winds.

Trumpets have at least as much timbral similarity with trombones, as oboes with bassoons. In addition to this, it is more common to find several brass instruments of one type (like 3 trumpets, 3 trombones, 4 horns), than it is to find several wood-wind instruments of one type, except in very large combinations. The French horns used today are all of one type. Their timbral characteristics can be considered as corresponding with trumpets and trombones to at least the same extent as that of flutes when combined with clarinets. The tuba bears a great timbral similarity to horns, but its quality is considerably denser. Thus we acquire two naturally blending subgroups:

trumpets and trombones;
horns and tuba.

The scale of decreasing timbral similarities, which is less pronounced in the case of brass instruments, appears as follows:

horns and tuba;
trumpets and trombones;
trombones and tuba;
horns and trombones;
trumpets and horns;
trumpets and tuba.

Of course, in the case of brass-wind instruments, timbral similarities are often variable, since they largely depend on execution. As mentioned in the description of instruments, trombonists can produce a very mellow tone, which remaining rich in the content of its partials, approaches the timbre of a French horn. The same is true of trumpets, which can be made to sound like cornets.

Though the individual timbral differences between the different strings of one string-bow instrument exist, they are not sufficiently pronounced to produce an undesirable timbral heterogeneity. Although the different strings are differently tuned in the sense that the degree of the tension in a string varies, depending on whether its material is gut, metal, or metal-wrapped gut, these different string-bow instruments can be accepted as members of one timbral family.

It follows from this discussion that though there is a relative timbral correspondence between the various families of one instrumental group, such correspondence is very remote between the three basic orchestral groups, i.e., the strings, the wood-winds and the brass-winds.

But then such a lack of similarity or correspondence may be very beneficial for producing contrasts. It is not only a matter of basic timbral characteristics but also of the manner of tone production. In this respect there are really two basic groups: the wind instruments and the string instruments. Both groups of the wind instruments give closer blends with each other than they give (particularly the brass group of higher register) with strings.

B. Correspondence of Intensities.

The next problem to discuss is the correspondence of intensities within families, groups and instrumental combinations.

At this point we are not interested in the physical aspect of intensities, but merely in their basic relations which are conditioned by the various types and families of instruments.

The general characteristic of intensity in the flute family is such that there is a gradual increase of intensity in the direction of increasing frequencies and a broader dynamic range available in the middle register.

In the clarinet family there is an increase of intensity in both frequency-directions, with a sufficiently broad dynamic range. The exception is the upper part of the chalumeau where the sound is weak and weakening toward the upper end of that register.

The oboes must be described apart from the bassoons as these two types of double-reeds have different dynamic characteristics. The lower register of oboes has naturally increasing dynamics in the direction of decreasing frequencies. From the middle range upward the dynamic range is quite flexible, but that flexibility gradually disappears in the higher register which is loud, though the sound loses its density.

Bassoons have a powerful and dynamically flexible low register; they weaken gradually toward the higher register, which again becomes fairly strong, though lower in density and harsh in quality; this harshness disappears toward the upper end of the whole range, where the dynamics are quite narrow in range and of a low intensity.

It is to be remembered that outstanding performers succeed in neutralizing the registral differences of dynamics.

The dynamic correspondences of the wood-wind group as a whole appear as follows:

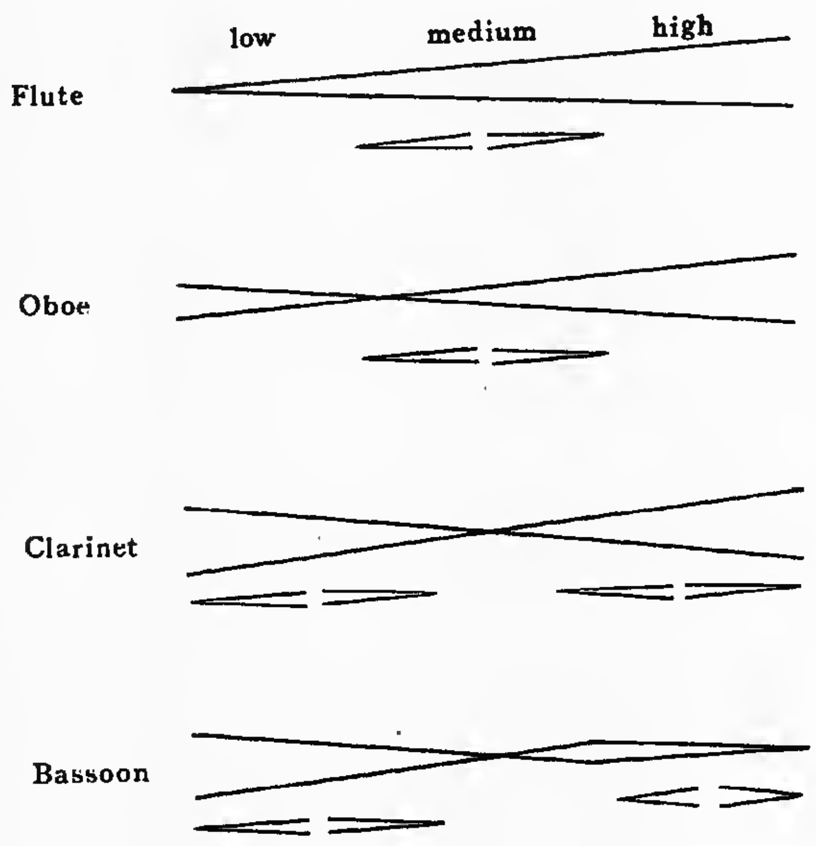


Figure 96. Dynamic correspondences of wood-wind group

There is a greater dynamic correspondence among the different types of brass-wind instruments.

Trumpets have low intensity in their lower register with a fairly wide dynamic range in the middle register and a high intensity in the high register. Thus the general tendency of the range is increasing intensity in the direction of increasing frequencies, with a fairly stable middle register and a fairly wide dynamic range.

Trombones grow in natural intensity with the increasing order of natural tones. The dynamic range of the middle register is fairly wide and stable. The pedal tones are weaker than the rest of the range.

French horns have the same natural tendency of increasing dynamics in the upward pitch direction. It is the upper half of the range that is dynamically most flexible.

The tuba has similar characteristics. Its lower register appears to be relatively loud, but this impression is really due to the high density of its tone in the lower register.

Summing up the dynamic characteristics of the brass-wind instruments, we obtain the following group of correspondences:

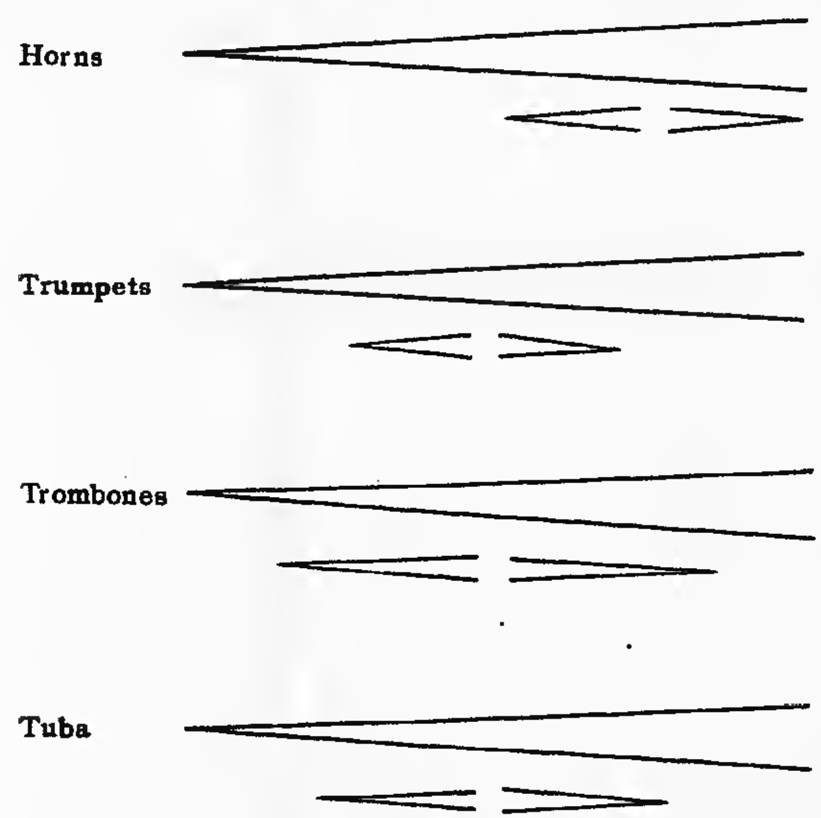


Figure 97. Correspondences of intensity in brass-wind group

As in the case of wood-winds, a great deal depends on the performer's skill.

All types of string-bow instruments have generally corresponding dynamic ranges, which give in all registers the same degrees of intensity and the same dynamic range. This statement, of course, is a simplification of the actual physical situation, but it is sufficiently accurate for the purposes of orchestration. In practice, the dynamic balances of string parts are often accomplished by selecting identical instruments (playing one part) in appropriate quantities. Here, however, we are chiefly interested in the correspondences of dynamic characteristics and not their equivalence with regard to the composition of balances.

It follows from the above discussion that string-bow instruments dynamically constitute the most homogeneous group. Strings, in homogeneity of dynamic correspondences, are followed by the brass-winds; the wood-winds, in this respect, occupy the last place.

C. Correspondence of Attack-Forms.

Next we shall be concerned with correspondences of attack-forms which exist between the different families of one group and among the groups.

Different families of the wood-winds have different attack characteristics. The flutes have legato, portamento and staccato. The latter is of one kind but can be obtained in piano and in forte, thus approaching only to some extent the distinctly different soft and hard staccato of the double-reeds. Two other special forms of attacks are available on the flute: the flutter-tongue (frulato) and the multiple tongue (double, triple etc.) The latter are not found in common with any other wood-wind instrument.

Clarinets have a perfect legato, a well-expressed portamento and a good soft staccato. The hard staccato is not characteristic of this instrument. It is more pronounced on the saxophone.

Oboes and bassoons have identical attack-characteristics but different mobility. Oboes are generally slower than bassoons. All double-reeds have an excellent legato, a perfect portamento and two distinct forms of staccato: the soft and the hard.

The attack characteristics of the wood-wind group may be summarized as follows:

Flutes: legato, portamento, staccato, frulato, multiple tongue
Clarinets: legato, portamento, staccato
Oboes: legato, portamento, soft stacc., hard stacc.
Bassoons: legato, portamento, soft stacc., hard stacc.

In the brass-wind group we find the following attack characteristics.

French horns have an excellent legato, a perfect portamento and a staccato which is closer to soft than to hard. The latter is due to the time period necessary for the transmission of attack through the long air column.

Attack-forms available on the trumpet are similar to those available on the flute; in addition to legato, portamento, and staccato (the trumpets somewhat emphasizing the distinction among these forms) trumpets, like flutes, can execute multiple-tonguing and flutter-tongue attacks.

Trombones offer only the first three of these forms, i.e.; legato, portamento, and staccato; the individuality and distinction they bring to these attack-forms is analagous to that of the trumpets.

Attack-forms for tuba resemble those available on the horns.

In tabular form, attack-forms available from the brass and wood-wind instruments may be summarized as follows:

Horns: Legato, portamento, staccato.
Tuba: the same as above:
Trombones: the same as above except that the subdivision between the soft and hard staccato is more pronounced.
Trumpets: the same as above except that, in addition, multiple-tonguing and flutter-tongue (*frulato* or *flatterzunge*) are also available.

Richest of all in attack-forms is the string group, all instruments of which afford the same attack-forms. In nearly all cases, each attack-form available from the brass and wood-wind instruments is paralleled by ***more than one*** attack-form available from the strings—it is as if it were a question of two different languages, one of which might have but one word for a certain concept, while the other would have more than one word in order to describe minor shadings of meaning.

String attack-forms* were classified in the chapter on the violin, the whole manifold forming a series that can be arranged into a decreasing scale with respect to tone duration: starting with the legato and proceeding through the detached (detaché, or non-legato), the portamento, spiccato, staccato, martellato, saltando, and sometimes the col legno to the pizzicato at the extreme of minimum duration. Strings can imitate all the attack-forms available from the brass and wood-wind instruments (although the imitation of the frulato is least exact); certain attack-forms available from the strings, on the other hand, cannot be obtained from the wind instruments. In establishing correspondences, then, between the attack-forms available from the strings on the one hand and the wind instruments on the other, the strings will exhibit a greater variety of form and of terminology than the wind instruments. It should be useful to establish a table of these correspondences, listed as to general characteristics.

Legatissimo in general: obtained from the strings (S) wood-wind (W) and brass (B) playing very *legato*.
Legato: obtained from the S, W and B by producing several notes with the same bow or breath.
Detached (or detaché) available from S, W and B with a separate attack on every note.
Portamento: obtainable from S, W and B.
Staccato (soft): from the S in spiccato, mezzo-staccato, saltando or col legno; also from the W and B.
Staccato (hard): from the S in staccato, martellato, pizzicato; also from the W and B.
Multiple-tongue effects: from the S by measured tremolo in mezzo-staccato; from the W (flute) or B (trumpet) as double, triple or multiple-tonguing.
Flutter-tongue effects: the nearest approximation on the S is obtained from an unmeasured tremolo; from the W (flute) by flutter-tonguing or frulato and from the B (trumpet) by frulato.

Figure 98. Correspondence of attack-forms

*See p. 1499.

D. Correspondence of Pitch-Ranges.

The last form of correspondences to be discussed concerns instrumental pitch-ranges.

The individual range-characteristics of the different instruments, families and groups are the main source of difficulties encountered by the composer or the orchestrator in his work on a score. If all instruments had been designed to produce the same range (in different zones of the general acoustical range, of course) and had the same register-distribution characteristics, such difficulties would be completely eliminated, and the composer would have felt greater freedom in conceiving an orchestral work. But with present instrumental combinations, such is not the case.

To get a clearer picture of ranges, we shall represent them in *semitones*. We shall confine all ranges to the practical limits in which the respective instruments are used.

Flutes:

Alto	Grande	Piccolo
31	38	27

Clarinets:

Bass	Alto	Soprano	Piccolo
38 (34)	34	39	39

Saxophones:

Bass	Baritone	Tenor	Alto	Soprano
30	30	30	30	30

Oboes:

Baritone	Alto	Soprano
27	28	31

Bassoons:

Contrabassoon	Bassoon
34	40

Figure 99. Semitone range of wood-winds

As we can see, only the saxophones have a balanced assortment of ranges. No other family gives such a correspondence and there is no definite correspondence between the families.

Horns:

44

Trumpets:

Bass	Alto	Soprano	Piccolo
25	25	32	30

Trombones:

Tenor-Bass (with valve)	Tenor-Bass (without valve)
3 + (-1[gap]) + 38	3 + (-5[gap]) + 34

Tuba:

39

Figure 100. Brass-wind ranges in semitones

As we can see, no obvious correspondences of ranges exist in this group.

The stringed-bow instrumental group, though homogeneous in other respects, is entirely heterogeneous with regard to the instrumental ranges of its members.

	Orchestra	Solo	Pizzicato
Violins:	52	52	33
Violas:	33	40	28
'Cellos:	40	47	40
Basses:	(31) 27	41	(28) 24

Figure 101. Range of string-bow group

It follows from the above three tables that a definite range-correspondence is not to be expected when instruments are combined in families and groups as they appear in a standard instrumental score.

It is particularly important to note the extreme difference between the violins and the basses; however, since basses are primarily used as octave-couplers to cellos, they have very little range similarity with the latter.

E. Quantitative and Qualitative Relations Between the Instrumental Combination and the Texture of Music.

1. Quantitative relations

One of the chief obstacles the composer encounters in translating his music into orchestral form is the lack of *quantitative correspondences between the harmonic and the density forms* of music, on the one hand, and the *instrumental combination*, on the other.

In scoring of the Mozartian type, where harmony consists of two parts with added bass, the problem of quantitative correspondences is very simple. Instruments of identical type are matched by pairs, thus supplying the two harmonic functions. On the other hand, the same pairs, when functioning in the low register, are assigned to represent the harmonic bass.

Unfortunately this happy situation does not exist in more developed forms of orchestral writing. Existing forms of harmony seldom correspond to the selection of members in a family, to combinations of families and groups. Often double instrumental combination is used to represent so-called four-part harmony, which, as we know, in actuality is 3p + p. Such a harmonic structure basically requires a homogeneous instrumental combination of three with the addition of one instrument which is of the same or of a different timbral family from the first three.

My purpose in discussing this matter now is to call the student's attention to the fact that harmonic forms of music have developed independently of the quantitative aspect of instrumental combinations. It is natural for this reason to expect all kinds of quantitative discrepancies in translating music into orchestral language. From the subjective view of the composer, this discrepancy becomes a source of never-ending struggle. The elimination of quantitative discrepancies and the establishment of quantitative correspondences between the harmonic and the density forms of music, on the one hand, and the instrumental combinations, on the other, is one of the major tasks of my Theory of Orchestration.

All such problems in this system are solved by a different methodological approach, which in this case is the *translation of special harmony into a specified form of general* (strata) *harmony* in correspondence with the selected instrumental combination. This methodological approach also allows for the use of new harmonic forms, as well as new instrumental combinations. Thus the problem is solved both for the orchestration and the orchestral composition. This method gives a fully satisfactory solution to all the situations concerned with the balance within harmonic groups (i.e., the balance of p's within S) and between such groups (i.e., the balance of S's within Σ).

2. Qualitative Relations

Qualitative relations between the harmonic and the density group, on the one hand, and the instrumental combination, on the other, often compel the composer to draw his musical texture from the instrumental combination instead of from its own components.

For example, a high density harmonic group which would at its best be represented by a homogeneous timbral group, cannot be properly assigned because a certain (more or less) standard instrumental combination does not contain as many members as are necessary in a certain specified timbral group.

The reverse is true: in some cases there may be more members in a homogeneous timbral group than is required by the status of a harmonic structure and its presupposed density. Of course such situations are easily solved by employing only some of the members belonging to one timbral family. But suppose this situation is more or less prevalent in a given score. Then it would result in an unjustified waste of instruments and performers, which just does not agree with the universally accepted idea of the economy of resources necessary in artistic expression.

To refer to one of the previous discussions of composition (Theory of Composition: Part Two) it is important to estimate the qualitative relations between the musical and the orchestral texture. This means that any composition achieves its optimum only under a certain group of corresponding conditions, which affect both the musical and the orchestral textures. Otherwise it may happen that the selected instrumental combination is not capable of expressing a certain tonal texture. For example, it may not be possible for the French horns to execute a highly mobile fugato even if such timbre is desirable; or it may not be possible for a small instrumental combination of monodic instruments of any type to execute a diversified texture of high harmonic density.

Generally speaking, musical textures and instrumental combinations are closely interrelated. And though nearly every piece of music can be adapted, arranged or orchestrated with varying degrees of skill, the optimum of a synthesis can be achieved only under a certain specified group of conditions for nearly every instrumental combination that has been more or less standardized.

CHAPTER 9

ACOUSTICAL BASIS OF ORCHESTRATION

The problem before us is to define and describe the *material of orchestration.* But in what terms can such a definition and description be adequately accomplished? Certainly not in terms of violins, clarinets, trumpets and drums. To do so would mean to repeat once more the methodological error made by music theorists of the past. The description of orchestral devices in terms of musical instruments leads to the dogmatism of assorted recipes which presuppose a certain type of musical texture. If the composer is fortunate enough to apply them to just such a texture, he may meet with success. In all other cases he is bound to be a failure. Should then the elements of orchestration be described, perhaps, in *physical terms*, i.e., as frequencies, amplitudes, phases, energies, air compressions, and rarefactions? In a way such a description would be highly accurate. Yet it would be erroneous to assume that frequencies, amplitudes and air compressions constitute the material of orchestration. Such a viewpoint would be very one-sided, and description based on it would be insufficient.

On the receiving end, phasic stimuli produced by instruments encounter a *metamorphic auditory integrator.* This integrator represents the auditory apparatus as a whole and is a complex interdependent system. It consists of two receivers (ears), transmitters, auditory nerves, and a transformer, the auditory brain-center. The response to a stimulus is integrated both quantitatively and selectively. The neuronic energy of *response* becomes the psychonic energy of *auditory image.* The response to stimuli and the process of integration are functional operations and, as such, can be described in *mathematical terms*, i.e., as synchronization, addition, subtraction, multiplication, etc. But these integrative processes alone do not constitute the material of orchestration either. The auditory image, whether resulting from phasic stimuli of an excitor or from self-stimulation of the auditory brain-center, can be described only in *psychological terms* of loudness, pitch, quality, etc. This leads us to the conclusion that the *material of orchestration* can be defined only as a *group of conditions under which an integrated image results from a sonic stimulus subjected to an auditory response.* This constitutes an interdependent tripartite system, in which the existence of one component necessitates the existence of two others. The composer can imagine an integrated sonic form, yet he cannot transmit it to the auditor (unless telepathically) without sonic stimulus and hearing apparatus. The transmission of a sonic form by an instrumental stimulus necessitates the existence of such a form and of a hearing apparatus capable of reacting to it. Finally, the hearing apparatus itself produces an auditory image out of a sonic form executed by an instrument.

The study of sound and hearing is not complete for the present. Since Helmholtz's work on *Tone-Sensations*, certain facts have had to be added and certain others rectified. Some aspects of acoustics require further experimental study and verifications. No final clarity has been achieved in the matter of subjective and objective. Yet too many processes of importance pertain to this field. Among them are the striking tone, the combination tones, the auditory illusions, etc. Under these circumstances our definition, classification and description of the components of an auditory image, must necessarily lack the precision which might have been attained if the physical and the psychological study of sound were more complete. Nevertheless, even with what can be offered now, the acoustical basis of orchestration can be considered established for any practical purpose. Our main achievement is a methodological one. The definition of auditory image as a function of physical stimulus and the integrative process of hearing, with further progress of physical and psychological knowledge, will become more accurate and will permit a more adequate description of the material of orchestration. The present description of this material and the deductions based upon it are true only within the scope of the present knowledge of sound.

EDITORS' NOTE:

The original manuscript of the *Schillinger System of Musical Composition* does not end at this point. But the editors, after consultation with the publisher, deem it wise to terminate at this point because the material that follows is not complete and because much of the material on orchestration has already been presented by Schillinger in earlier books.

In Book I, for example, the application of resultants to Instrumental Forms (Chapter 7) foreshadows the procedure for developing scores of unprecedented richness and complexity from rhythmic raw material. In Chapter 8, *Coordination of Time Structures*, Schillinger describes the synchronization of an attack group with an instrumental group. In Book VII, Chapter 5, Schillinger considers the composition of a counterpart to a given melody by means of axial correlation—a technique indispensable in modern "arranging" and in virtually all good orchestration.

Book VIII, *Instrumental Forms*, covers comprehensively one of the most important aspects of orchestration. As Schillinger himself described the purpose of this book: "Instrumental forms will mean, so far as this discussion is concerned, a modification of the original melody and/or harmony which renders them fit for execution on an instrument . . . Depending on the degree of virtuosity which can be expected from singers, instrumental forms may be applied to vocal music as well as orchestral." An examination of chapter headings in this book quickly reveals how basic the material is for orchestration: Chapter 5. Strata of Four Parts; Chapter 6. Composition of Instrumental Strata; Chapter 8. The Use of Directional Units In Instrumental Forms of Harmony, etc.

Book IX, *General Theory of Harmony*, likewise is concerned with matters fundamentally orchestral. "My general theory of harmony," Schillinger writes, "denotes the whole manifold of techniques which enable the composer to write *directly* for groups of instruments or voices . . ." In this book, Schillinger develops two of his most original orchestral techniques: the Σ concept as it relates to orchestral strata (Chapters 2, 7) and the composition of density as it relates to strata (Chapter 15). These techniques were largely responsible for the rich and arresting arrangements made by Schillinger students.

From Schillinger's notes, it is clear that he planned to integrate the ideas of Books VIII and IX in Book XII. Other matters which he planned to consider and had in part written down include: instrumental media for achieving variation of true color; forms of attacks (such as durable, abrupt, bouncing, oscillating, etc.) as they relate to instrumental forms; curvature of a melodic line in instrumental performance; and other related material. In addition to *Acoustical Basis of Orchestration*, Schillinger had planned to include a section called *Theory of Interpretation*.

As previously stated, the original manuscript contains some of these materials in incomplete form. The editors also had before them notes taken by students who had studied personally with him. These notes covered various topics included in a table of contents prepared by Schillinger for Book XII.

For a time the editors and the publisher considered the possibility of reworking all of these materials and including them in the present volume. Since the major aspects of Schillinger's theory of orchestration are covered in earlier books and in Book XII as it appears here, it was decided to confine the published work in its first edition to Schillinger's manuscript as he had completed it.

[L. D.-A. S.]

GLOSSARY

Compiled by LYLE DOWLING and ARNOLD SHAW

(Terms appearing in bold face within a definition are explained elsewhere in this glossary and will be found in alphabetical order. Students should consult the index in order to discover the pages on which terms appear in the text itself.)

A

A Symbol ordinarily used for **Attack.**

$A^{\rightarrow}$ Symbol for **Attack Continuity.**

a Denotes an *a* axis; see **Axes.**

a See **Attack.**

a Denotes one of the positions in **Quadrant Rotation.**

a ÷ b Symbol for **Resultant** of **a** and **b**. See the footnote: Vol. I, p. 4.

<u>a ÷ b</u> Symbol for another **Resultant** of *a* and *b*, fractioned with symmetry around an axis.

a, b, c, These letters frequently denote **Chordal Functions** when the exact name of the function need not be specified.

ABSCISSA. In the **Graphing** of music the measurements horizontally, left to right, denoting the time dimension.

ACCELERATION SERIES. Any series in which there is an increasing or decreasing differential between successive terms. The decreasing series is sometimes known as a **Retardation Series.** Prime number series, summation series, etc., are ordinarily used for this purpose. A positive series may be synchronized with its reverse to produce a **Resultant.**

ACOUSTICAL CLARITY. In **Orchestration,** the result of (a) differentiation of overlapping **Strata** by timbral and/or **Attack-Form** variety, and (b) proper relationship of **Clockwise** and **Counterclockwise** positions of strata.

ACOUSTICAL EQUIVALENTS. Intervals differently named in diatonic nomenclature but consisting of an identical number of semitones, hence sounding the same.

ACOUSTICAL PROPERTIES OF INTERVALS. The critical properties are **Density** and **Tension,** as applied to **Harmonic Intervals.** See Vol. I, p. 700.

ACOUSTICAL RANGE. The range of an instrument as it actually sounds.

ACOUSTICAL SET. A distribution of tones corresponding exactly or approximately to the series of **Harmonics.**

ALIEN MEASURE-GROUPING. The grouping of a durational continuity (especially, a **Resultant**) by measures consisting of a number of units which does *not* correspond to any generator or product of generators, used in making the resultant.

AMPLITUDE. A measure of intensity (loudness) of sound. When a sound wave is graphed, the amplitude is the distance between the highest and lowest points of the track of the sound wave, and the intensity of the tone is related to this measurement in a logarithmic ratio.

ANTICIPATED TONE(S). In harmony, a tone of one structure caused to sound before other tones of the structure, and while tones of the preceding structure are still sounding.

ANTI-CLIMAX. Not to be confused with *negative* climax; refers to a segment of a composition in which the tension or magnitude of a climax is relaxed; see **Climax.**

ARITHMETICAL MEAN. An average found in the ordinary way by adding a series of numbers, then dividing the total by the number of terms in the series.

ARITHMETICAL PROGRESSION. A series in which each term is the previous term plus some constant number, *n*. For example, 1, 3, 5, 7, 9 is an arithmetical progression in which the constant is 2.

ASCRIBED MOTION. A type of melodic movement produced by constructing the melodic steps on a graph so that they are *outside* the **Secondary Axis,** that is, so that the secondary axis is between the pitch-line and the primary axis. When the steps are constructed *inside,* the motion is called **Inscribed Motion.** More strictly, ascribed motion is *sine* motion; inscribed *cosine* motion.

ATTACK. In this system, a very general term meaning both an instance of some musical event and the moment in time when the event begins. It is not to be confused with **Attack-Form,** which is an instrumental matter. When one says "three attacks per measure," one means that there are three events—of whatever kind—occurring in the measure, without specifying in exactly what rhythm they occur. The term need not refer always to tonal material; two *attacks* of O_1 would mean "two instances or occurrences of orchestral group number one," for example. The abbreviation for attack is *A* or some form of it, often *a*. Attacks may be grouped into **Attack-Groups,** consisting of various numbers of attacks in series. Such attack-groups may further be grouped into **Attack Continuity.**

ATTACK CONTINUITY. A **Continuity** composed of Attack-Groups which are, in turn, composed of **Attacks.**

ATTACK-FORM. The pattern of tonal material assigned to an instrument: for example, an *arpeggio* is one of many attack-forms for a chord.

AUTOMATIC CHROMATIC CONTINUITY. Produced by subjecting an initial chord, usually in the diatonic system, to a process whereby one or more voices move by semitones in but one direction at a time. See Vol. I, p. 544.

AUXILIARY TONE OR UNIT. A type of **Directional Unit** consisting of a chordal tone (or **Neutral Unit**) preceded by a tone that is one semitone, or two semitones, or a diatonic step removed. Distinguished from other types by the fact that the auxiliary need not belong to any pre-set scale or harmonic structure.

AXES. In general, lines of reference. 1. *Key-axis*: the particular pitch-level representing the first tone of the **Real Scale** in which the music is written. A shift in key-axis involves **Modulation** in the *modern* sense of the term. 2. *Primary axis* is the pitch-level, not necessarily the same as the key-axis, around which a melodic line moves; it is usually the pitch sounded for the greatest total duration in the course of a melody; a shift in primary axis involves modulation in the 16th century sense, or *modal* modulation. 3. *Secondary axis* in melody is an axis that has a specific direction and that describes the movement of the melodic line: specifically; *a* axis, up from the primary; *b*, down to the primary; *c*, up to the primary; and *d*, down from the primary. 4. *Balancing axes* are those leading toward the primary, that is, the *b* and *c* axes. 5. *Unbalancing axes* (*a* and *d*) lead away from the primary. 6. *Binary* axes are simultaneous pairs of secondary axes. 7. *Ternary axes* are sets of three simultaneous secondary axes. 7. *Axis of symmetry* is the "center," or line of reference around which a symmetrical structure is constructed. 8. *Axis of inversion* is the line of reference from which inverted intervals are reckoned in **Inversion.**

AXIAL COORDINATION. In melody, the process of composing a continuity of **Secondary Axes**; in counterpoint, the composition of properly interrelated groups of secondary axes for two or more **Correlated Melodies.**

AXIS RELATIONS. In general, the relations between two or more axes of two or more simultaneous **parts** or **strata** in music. Hence it may refer to the relation between melody and harmony, or, in counterpoint, to the relations between any pair of correlated melodies; specifically: (1) UU, unitonal-unimodal, that is, same key-signature, same mode (displacement) for both melodies; (2) UP, unitonal-polymodal, same key-signature, different displacements; (3) PU, polytonal-unimodal, different key-signatures, same displacements; (4) PP, polytonal-polymodal, different key-signatures, different modes.

B

b Denotes one of the secondary axes. See **Axes.**
ⓑ Denotes a position in **Quadrant Rotation.**
B Symbol used for *balancing* axis (see **Axes**).

BALANCING. The process of adding to a **Resultant** a duration such that, if the generators are *a* and *b*, the duration added is a (a—b); especially, in **Contraction Groups.**
BALANCING AXIS. See **Axes.**
BEAT TONES. See **Differential Tones.**
BINARY AXES. See **Axes.**
BINOMIAL. A group consisting of two elements.
BLOCK HARMONY. An arranger's term referring to the result of a process in which a melodic line is subjected to **Coupling** at the octave, after which the remaining functions, whatever they may be, of a chord—usually S(5) with the 13th added or S(7)—are inserted between the extremes of the octave.

C

c Denotes one of the secondary axes; see **Axes.**
ⓒ Denotes a position—backwards and upside down—in **Quadrant Rotation.**
C_0 Abbreviation for **Zero Cycle.**

C_3 Abbreviation for **Cycle of Thirds.**	**Positive Cycles**	C_{-3}	**Negative Cycles**
C_5 Abbreviation for **Cycle of Fifths.**		C_{-5}	
C_7 Abbreviation for **Cycle of Sevenths.**		C_{-7}	

CADENCE. A configuration in melody and/or harmony, used very frequently, which has the effect nf halting or retarding the movement and which, hence, is used to mark ends of divisions and subdivisions of form. *Melodic*: essential form of a melodic cadence is the tone of the **Primary Axis** immediately preceded by the next lower or next higher tone. *Harmonic*: essential form of a harmonic cadence is the key-axis root (**Tonic**) immediately preceded by the next lower or next higher root in the particular cycle in which the roots are moving; thus, each cycle (C_3, C_5, etc.), has its own two essential forms of cadence: the root above, or the root below, in the particular cycle or, many times, in some cycle foreign to the continuity. *Combined forms*, either for melodic cadences or harmonic cadences, are made of some combination of these elements, but with the axial element always last. In cases of so-called *half-cadence* or *deceptive cadence*, an axis other than the normal one is used.
CANTUS FIRMUS. A term from old contrapuntal theory, now used to designate what is *given* in a contrapuntal problem, usually reduced to abstract form by noting it in whole notes.
CF Symbol for **Cantus Firmus.**
CHORDAL FUNCTION ③, ⑤. In a structure, or chord, each tone may be denoted in relation to the root by a number—for example, the 3rd by a 3, and so forth. The term *function* is used to denote a 3 or a 5 or some other such interval consistently, and is used especially when **Transformations** of structure make it important to be able to identify the same interval in a series in which the position of the function may be constantly changing.
CHROMATIC ALTERATION MODULATION. See **Modulation.**
CHROMATIC GROUP. The fundamental group of three chords in **Chromatic Harmony** in which the first is diatonic, the second is chromatic, and the third is diatonic but not necessarily diatonic with respect to the key of the first. Shapes of these three are S(5) or S(7) in **Special Harmony**, but the second is preferably of S(7) structure.
CHROMATIC HARMONY. Not to be confused with harmony of some other type that has been subjected to **Chromatization.** The essence of chromatic harmony is the group of three tones chromatically related, expressible by $x - x\sharp - y$ (or: $x - x\flat - y$). Around each tone a **Chord** is built, a requirement being that the middle chord be of the S(7) shape. Thus the motion of chords in the continuity is determined by these groups of three. The same technique may be applied to two chromatic lines simultaneously, to three, or, in exceptional cases, to four. The groups of three may be consecutive or may overlap or may, in special cases, be simultaneous. The first and third chords are ordinarily diatonic, although they need not be diatonic with respect to the same **Pitch-Scale.** From this arises harmonic **Modulation** as a special case of chromatic harmony.

CHROMATIZATION. A process in which all whole steps (two semitones) in a part, whether the part is melodic or harmonic, are broken into two steps of one semitone each by insertion of the required chromatic.
CIRCULAR PERMUTATION. A type of **Permutation** produced by *displacing* the original group one step at a time until the original returns, as when *abc* is permuted circularly to produce *bca*, *cab*, and again *abc*.
CLIMAX. This, in terms of any given continuity, is the point at which the quantities are at maximum magnitude (negative magnitude producing a *negative* climax), no matter what the continuity is. Generally, it is the segment or segments in a composition where one or more or all **continuities** reach a maximum magnitude. *Attack climax*: maximum number of **Attacks.** *Dynamic climax*: maximum volume of tone. *Harmonic climax*: maximum number of strata and maximum number of parts with maximum permissible **Tension.** *Melodic climax:* maximum in time and in distance of pitch from the primary axis (see **Axes**). The psychological effect of the climax is heightened if the maximum magnitude is reached in a series of increasing "waves," each "wave" being higher than the last but falling back only to be succeeded by a greater magnitude until the maximum is reached (see **Resistance Forms**). The reverse of **Climax** so far as this "wave" movement is concerned is **Anti-Climax.**
CLOCK TIME. Time as measured on the clock, usually in seconds.
CLOCKWISE ↻ Circular motion in the same direction as that of the hands of a clock; used to differentiate in **Transformations** those in which, for example, 1-3-5 changes to 3-5-1, which is *clockwise* (as in $\overset{1}{5\ \ 3}$) from those in which 1-3-5 changes to 5-1-3, which is counter-clockwise (as in $\overset{1}{5\ \ 3}$).

CLOCKWISE POSITIONS. These are positions of structures which correspond to so-called open positions of chords. The functions of a positive structure are reckoned downwards.

CLOSED TONE. **Timbre** characterized by presence of relatively high number of **Harmonics.**
COEFFICIENT. A number by which some element in a series (which element may or may not be a number) is multiplied. *Coefficients of recurrence*: a series of coefficients used to control the number of times some element in music—a theme, or duration, or interval, or timbre, etc.—recurs.
COMBINATION TONES. See **Differential Tones.**
COMBINED HARMONIC CONTINUITY. A form of harmonic continuity consisting of segments of the various basic types—diatonic, chromatic, symmetric, diatonic-symmetric—the segments being frequently linked in some pattern by other segments. A **Hybrid** form.
COMMON CHORD. A term used in modulatory technique to denote a common-*name* chord, i.e., a chord the *names* of the pitches of which are the same as the names of the pitches of another chord without regard to accidentals.
COMMON PRODUCT. In rhythm, the number obtained by multiplying two or more numbers together, especially when **Resultants** are being derived.
COMMON UNIT METHOD OF MODULATION. See **Modulation.**
COMPLEMENTARY FACTOR. In calculating **Resultants,** the number of times a particular **Generator** recurs; found by dividing the particular generator into the **Common Product** of all the generators.
COMPOUND SIGMA. Two or more **Sigmae** interrelated by some form of **Interval Symmetry.**
CONFIGURATION. A general term meaning about the same as pattern, but including a time dimension; a selection of a specific number of specific elements arranged in a specific design.
CONSTANT B TRANSFORMATION. A special case of **Transformation.** The function denoted by *b* (usually the 3rd) remains constant while the other functions permute.
CONSTANT STRUCTURES. In harmony, use of the same chordal structure (or S) throughout a continuity. Characteristic of some types of **Symmetric Harmony.**

CONSTANT TRANSFORMATIONS. A type of **Transformation** of three or more functions, according to which one or more functions remain constant (limited by the total number of functions minus 2) while the others permute.

CONTINUITY. In music, a sequence of elements organized in time, usually of the same kind. For example, *harmonic* continuity is the sequence of harmonies considered as a whole; *orchestral* continuity is the sequence of **Orchestral Groups** considered as a whole, *dynamic* continuity is the sequence of degrees of loudness or softness considered as a whole. The principal continuities of which a composition is composed are denoted as follows:

$A^{\rightarrow}$	Attack continuity.	$\Delta^{\rightarrow}$	Density group continuity.
$T^{\rightarrow}$	Durational or rhythmic continuity.	$\Omega^{\rightarrow}$	Orchestral group continuity.
$\Gamma^{\rightarrow}$	Instrumental attack-form continuity.	$V^{\rightarrow}$	Dynamic group continuity.

$\Sigma^{\rightarrow}$ Harmonic continuity, complete, or *sigma* continuity.

CONTINUOUS IMITATION. In counterpoint, what is meant by canonic imitation; a single melody coexisting in two or more different strata of the continuity in different phases and at a constant velocity. See p. 778.

CONTINUUM. A **Continuity,** but with special emphasis on the concept of the continuity as a whole.

CONTRACTION GROUP. A rhythmic group consisting of two parts, the first of which is the resultant of two uniform periodicities with fractioning ($r_{a \div b}$) and the second of which is the resultant of simple synchronization ($r_{a \div b}$). More generally, any complex rhythmic group in which movement is from a longer to a shorter duration group, which groups contain only durations derived from the same style series. An *expansion group* consists of the same two elements, but with the longer one coming after the shorter one.

$$E = r_{a \div b} + r_{a \div b}.$$

CONTRAPUNTALIZED HARMONY. Contrapuntal continuity produced by (a) writing a harmonic continuity; (b) controlling the entrance and dropping out of individual parts by various density patterns; (c) subjecting individual parts to various kinds of melodic figuration.

CONTRAPUNTAL OSTINATO. Persistent recurrence of a segment of counterpoint accompanied in each repetition by some changing set of other musical elements, usually harmonization or additional contrapuntal lines.

CONTRARY CORRELATION. See **Correlation.**

CONTRARY MOTION. Simultaneous movement of two or more melodic lines in contrary directions.

CORRELATED MELODIES. Schillinger's term for counterpoint, but used in a somewhat broader sense including many types of counterpoint not known to classical writers on the subject. Two or more melodic lines correlated as to (1) rhythmic continuities; (2) axial and other melodic characteristics; (3) tonal and modal relations; (4) harmonic relations.

CORRELATION. There are three main types of correlation of (a) pitch-time ratios in melody; (b) density-time displacements in composition of variations of density groups. The three types are: *parallel* when quantities increase simultaneously at the same rate; *oblique* when one series of quantities increases or decreases while the second series remains constant; *contrary* when one series increases in value while the other series decreases.

CORRELATION OF PRIMARY AXES. The planning of the interval or intervals separating two or more primary axes (see **Axes**), especially in counterpoint. See **Axis Relations.**

COS MOTION. Means *cosine motion*; see **Sine Motion** and **Ascribed Motion.**

COSINE MOTION. See **Ascribed Motion.**

COUNTERCLOCKWISE ↺ See **Clockwise.**

COUNTERCLOCKWISE POSITION. "Close" positions of chords; see **Clockwise position.**

COUNTERMELODY. A second melody written in counterpoint to a given melody; an arranger's term is counter*part*.

COUNTERPOINT. See **Correlated Melodies.**

COUNTERPOINT TO GROUND MELODY. See **Ostinato.**

COUPLING. Adding to any sequence of tones, usually a melodic line, a parallel sequence either at some diatonic or absolute interval. *Diatonic:* the particular diatonic scale in use controls the exact shape of the interval of coupling. *Absolute:* the interval of coupling is measured in semitones rather than diatonically and remains constant throughout. Playing a melody in octaves is diatonic coupling at the octave. *Inward coupling:* the coupling lies below the upper parts and above the lower. *Outward coupling:* the couplings are constructed downward from the lowest part and upward from the uppermost part.

CP. Symbol ordinarily used for *counterpoint* or *countermelody*; occasionally for **Common Product.**

CROSSWISE TRANSFORMATION ✥ Denoted by d ✥ b (a above, c below); and, as the figure illustrates, function *a* transforms into function *c* while *c* transforms into *a*, the other pair, *b* and *d*, meantime changing places in the same way. See **Transformations.**

CYCLES. These are the C_3, C_5, C_7 and their negative counterparts in the system of **Diatonic Harmony** and, by extension, in **Symmetric Harmony** as well.

D.

d Denotes one of the secondary axes; see **Axes.**

ⓓ Denotes the fourth position—forward and upside down—in **Quadrant Rotation.**

Δ, δ, $\Delta^{\rightarrow}$ See **Delta**; symbols used in composition of **Density.**

D, d Symbols sometimes used in density formulae.

d_0 Symbol for "zero displacement" of a **Pitch-Scale.** See **Displacement.** The zero displacement is no displacement at all; d_1, d_2, etc., indicate successive displacements.

d_1 1. Ordinarily indicates a scale **Displacement.** 2. Occasionally used in **Correlated Melodies** to indicate a dissonance as part of a pattern.

DELAYED RESOLUTION. A reduction of tension of a pitch assemblage accomplished, in contrast to direct resolution, with some other assemblage intervening. For example, the direct resolution in counterpoint of a 7th may be to a 6th; when a 3rd intervenes between the 7th and 6th, the resolution is delayed.

DELTA. The Greek letter Δ, referring to **Density** (textural).

DENSITY. Aside from density in the general sense of **Saturation,** or of instrumental density as a part of **Orchestration,** the term refers very specifically to the patterns made by the **Strata** actually sounding in music from moment to moment, in relation to the maximum number of **Strata** in the **Sigma Continuity.** The simplest patterns, or **Density Groups,** are developed into compound density groups (denoted by the Greek letter, *delta*) and these in turn are composed into **Density Continuity** (denoted by $\Delta^{\rightarrow}$) which, when further compounded, becomes "the delta of a delta" (analogous to the **Sigma of Sigma**) denoted by $\Delta^{\rightarrow}(\Delta^{\rightarrow})$. A density group or compound may be subjected to *phasic rotation* by techniques which are fully explained in the text. These rotations are symbolized by the Greek letter, *phi*, or ϕ. When compounded, the rotations are symbolized by the Greek letter, *theta*, or Θ. The technique permits utmost control over the texture of orchestral sound. Essentially, it is the **Displacement** technique applied to two dimensions rather than one.

DENSITY OF INTERVAL. A quality similar to sonority measured roughly by the *average* number of tones sounding per octave of total range.

DIAD. A structure in harmony of but two parts.

DIATONIC. Used as an adjective, it denotes that the **Pitch-units** in question all correspond to those in some one **Diatonic Scale.**

DIATONIC HARMONY. In general, any harmony all the pitch units of which are members of, at any one time, the same **Diatonic Scale.** Specifically, one of the main types of which **Special Harmony** is composed. This is a type of harmony in which both the progressions of chords and the structures of the chords themselves are derived from the first **Expansion** of whatever scale is in use (E_1). But the term refers not alone to the seven-tone scales in general use, but also to scales (usually primitive scales) of fewer than seven tones. *Root movement* in diatonic harmony takes place in positive (reckon downward) or negative (reckon upward) cycles, the cycles being: C_3 ("cycle of the third"), downward by diatonic thirds; C_5, downward by diatonic fifths; C_7, downward by diatonic sevenths—which is the same as upward by diatonic seconds, of course. Negative forms of these cycles are measured upward instead of downward. Selection of these cycles, and the proportions and pattern in which they are used, influence profoundly the harmonic style of the resulting music. Terminal roots in the cycles constitute **Cadences.** *Structures* or chord shapes are selected from the E_1 of the scale so that the pitch-units conform to the given scale, whatever it may be. Voice-leading is effected by **Transformations** and **Doublings,** with occasional use of special pre-set **Groups of Chords.**

DIATONIC INTERVAL. An interval denoted conventionally—as a *second,* or *third,* etc,—the number of semitones in it being determined by the particular scale in effect at the time.

DIATONIC NOMENCLATURE. Naming of intervals as *unisons, seconds, thirds,* etc., in the conventional way. See **Symmetric Nomenclature.**

DIATONIC SCALE. A **Pitch-Scale** with the following characteristics: (1) it has but one **Tonic**; (2) its range is not more than one octave, as a scale; (3) no pitch-name (A, B, C, D, etc.) is used more than once in the scale; (4) the scale may have any one set of accidentals in its **Real Signature** at a time. The conventional form is that of the seven-tone major or minor scale; but the definition also includes (a) scales of fewer than seven tones conforming to the requirements given above; (b) modal scales that conform to the requirements.

DIATONIC-SYMMETRIC HARMONY. A **Hybrid** form, in which the **Roots** move in the **Cycles** of **Diatonic Harmony** but the chordal structures, as in **Symmetric Harmony,** follow a pattern independent of the diatonic system, being chosen usually for their particular sonorities. Schillinger calls this harmony Type II and bases it on chord structures employing variants of the diatonic triad 4 + 3—that is, 3 + 4 (minor), 4 + 4 (augmented) and 3 + 3 (diminished).

DIFFERENCE TONES. See **Differential Tones.**

DIFFERENTIAL TONES. Tones produced by a pair of tones sounding together. The **Frequency** of a differential tone is equal to the frequency of the higher tone of the pair minus the frequency of the lower tone.

DIRECTIONAL UNIT. A group of tones attached to and including a **Neutral Unit in General Harmony.** A neutral unit is a chordal tone of a structure. The directional unit always has the neutral unit as a member and must consist of at least one other tone. This other tone is either a semitone, or two semitones, or a diatonic step removed from the neutral unit—and any additional tones in the directional unit must either lead into the neutral unit or into some other tone which itself is a leading tone. Using these directional units sequently, the requirement is that the neutral unit or chordal tone be sounded *last.* Directional units in various forms constitute the general form of all melodic figuration and, indeed, of melody itself. What is important about them, so far as style is concerned, is the answers to these questions: (1) which neutral units are equipped with directionals? (2) what is the interval or intervallic pattern of the directional unit? (3) in what direction (upward or downward) are they constructed?

DISPLACEMENT. The process of forming new groups by rearranging (permuting) the elements of the original group one place at a time, as when *cdefg* becomes *defgc.* Each element is shifted one place to the left, and the leftmost element is shifted to the extreme right. In this case *c, d, e, f, g* would be indicated by d_0, and called **zero displacement,** while *d, e, f, g, c* would, be indicated by d_1, called *first displacement.* Displacement scales of the natural major scale on C yield the various so-called ecclesiastic modes.

DISSONANT INTERVALS. In classical theory, the diatonic unison, octave, fifth, sixth and third (sometimes the fourth, especially when occurring in inner voices or when supported by a third below it) are regarded as consonant; all other intervals are regarded as dissonant. For these older concepts, however, Schillinger substitutes the notion of *tension* of interval; substitutes the notion of *reduction of tension* for the classical concept of *resolution*; requires reduction only of such intervals as are acoustically of higher tension than the diatonic third; and points out that conventionally consonant intervals become dissonant in low register and that conventionally dissonant intervals become acoustically consonant in high register.

DISTRIBUTIVE CUBE. See **Distributive Powers.**

DISTRIBUTIVE INVOLUTION GROUP. Groups of numbers consisting of a polynomial raised to some power and arranged distributively. See **Distributive Powers.**

DISTRIBUTIVE POWERS. A series of values derived by raising any polynomial to any power but keeping it in distributive form. For example, the *non*-distributive square of 3 + 2 is 25, but the distributive square is 9 + 6 + 6 + 4. The binomial *a* + b, squared is normally $a^2 + 2ab + b^2$, but *distributively* it is aa + ab + ba + bb. Distributive powers are used frequently by Schillinger as a means of controlling durations and other characteristics of music.

DISTRIBUTIVE SQUARE. See **Distributive Powers.**

DOUBLE PARALLEL CHROMATICS. One variety of **Chromatic Harmony.**

DOUBLING. When a **Chordal Function** appears twice in the same **Chord,** it is said to be **Doubled.** Not to be confused with **Coupling.**

DURABILITY OF TONE. In **Orchestration,** the characteristic of the tone that depends essentially on its duration, judged according to a **Scale** the extremes of which are *legatissimo* and *staccatissimo.*

DURATION. Extent in time, measured chronologically in **Clock Time** and musically in the time-values of notes; measured on a **Graph** by extension along the **Abscissa.**

DURATIONAL CONTINUITY. A **Continuity** composed of **Durational Groups.**

DURATION GROUP. A group of one or more durations in time.

DYNAMIC CONTINUITY. A **Continuity** composed of **Dynamic Groups.**

DYNAMIC GROUP. A group of degrees of volume, such as *pp-mf-ppp.*

DYNAMIC MARKS. Marks such as *pp, mf,* etc. Schillinger stresses that these marks now give directions to the performers, whereas in Beethoven's time they indicated the dynamics as experienced by the listener.

DYNAMICS. In orchestration, the adjustment of other factors to the desire for loudness or softness of tone. Schillinger differentiates between volume of tone as measured logarithmically by **Amplitude.** and the *psychological* effect of loudness which can be created by multiplicity of **Strata** and the resulting large number of upper **Harmonics.**

E.

E_0 Symbol for a **Pitch-Scale** in "zero expansion," i. e., the scale cannot be contracted in the given system of tuning. E_1, E_2, etc., ordinarily indicate further **Expansions** of a scale. See **Tonal Expansion**; also Vol. I, p. 133. These same abbreviations are used occasionally for various **Expositions** in **Thematic Continuity.**

ECCLESIASTICAL MODES. A system of denoting various diatonic scales worked out in mediaeval times on the basis of a misunderstanding of the much earlier Greek modal system. For both the Greek and the Ecclesiastical system, Schillinger substitutes the general procedure of **Displacement,** so that no matter what scale is used (denoted d_0, or "zero displacement") the successive displacements or modes (denoted d_1, d_2, etc.) may be readily derived and controlled.

ELEMENT. In a **Series** or **Sequent Group,** the single items of which the series or group is made, whatever may be the nature of these items, whether numeral or musical or both.

EQUAL TEMPERAMENT. A tuning system (developed by Andreas Werckmeister in 1691) according to which the octave is divided into 12 pitches by relating the frequencies of each pitch to a logarithmic series consisting of the twelfth root of 2, ($\sqrt[12]{2}$) which 2 is successively raised to the zero power, first power, square, cube, fourth power and so on up to the 12th power. Remaining pitches are derived by octave duplication. See pp. 101, 102 and 146.

EXPANSION. E_0, E_1, E_2, etc., denote a segment of music, especially a **Pitch-Scale,** subjected to **Geometrical Projection** so that the pitch is increased by some constant factor, the results being interpreted either exactly or tonally (diatonically). See **Geometrical Projections** and **Tonal Expansion.**

EXPANSION GROUP. See **Contraction Group.**

EXPOSITION. Essentially, a **Thematic Group,** or "setting forth" of a **Thematic Unit**; used especially in contrapuntal forms. Schillinger tends to substitute **Thematic Group** for this term in the latter portion of his manuscript.

F.

FORMS OF RESISTANCE. See **Resistance Forms.**

FRACTIONING. The process of splitting a **Duration** into fragments, usually in proportion to some polynomial of the **Style Series.**

FRAGMENTATION. In composition of thematic continuity, the use of a selected fragment of the total theme in order to shorten the duration of a particular thematic group.

FREQUENCY. In acoustics, the rate of vibration of a vibrating medium; expressed in terms of vibrations per second.

FRULATO. Flutter-tonguing.

FUNCTION. See **Chordal Function.**

FUNDAMENTAL HARMONY SCALE. See **Harmony Scale, Fundamental.**

FUNDAMENTAL TONE. In acoustics, the pitch produced by vibration of the whole of the vibrating medium, in contrast to **Harmonics** produced by vibrations of segments of the medium.

G.

G6 In **Diatonic Harmony** a special **Group of Chords** used as a unit. See p. 415.

G$\frac{6}{4}$ A special **Group of Chords** used as a unit. See p. 427.

GENERAL HARMONY. Schillinger's term for his technology for the development of **Pitch-Assemblages** and for organization of these into sequent groups. It is a technology embracing all the *tonal* material of music—*tonal*, as distinct from temporal (rhythmic) or instrumental material. A pitch-assemblage is any set of **Pitch-Units** or tones taken together; it means what is meant by *chord*, except that (1) a pitch-assemblage frequently includes a great many more tones than are found in the chords of conventional harmony. (2) the arrangement of these tones need not correspond to the conventional structures built on diatonic thirds. The tones of a pitch-assemblage may sound sequently—one after the other—as well as simultaneously. The complete harmonic continuity is denoted by $\Sigma^{\rightarrow}$ (to be read, "sigma continuity"). It is made up of a series of individual pitch-assemblages, each denoted by the Greek letter, *sigma*, Σ. Each sigma is, in turn, composed of a number of substructures or *strata*, each denoted by S, and each S consists of one or more *units* (usually called **parts,** denoted by p). The significant factors are: (1) the movement of the root tones of each stratum; (2) the pattern of intervals by which the various tones of each stratum are grouped around the root; (3) the spacing of strata; (4) the **Transformations** to which strata are subjected; and, finally, (5) the presence or absence of **Directional Units** within each stratum. Special aspects of general harmony are discussed under **Special Harmony** and under other subheadings in this glossary.

GENERATOR. A pattern of durations (usually a monomial) used in combination with another pattern to produce a new durational pattern, known as a **Resultant.** More simply, a series of sounds, notes or attacks of given duration.

GEOMETRICAL MUTATIONS. See **Geometrical Projections.**

GEOMETRICAL PROJECTIONS. A fundamental technique for variation. Any theme may be subjected to **Quadrant Rotation** to produce four forms: the original; the original backward in time; the original upside down as to pitch and backward in time; and the original upside down as to pitch and forward in time. The pitch may be multiplied by some factor, such as 2, 3, 4, etc., resulting in **Expansion.** The reverse process results in **Contraction;** but often this cannot be realized in our tuning system. Durations may also be increased or contracted. When the process of pitch expansion is done precisely, that is, with a graph divided into semitones, the results are called *geometrical*; when it is done diatonically, that is, with a graph divided according to some diatonic scale, the results are called tonal. The special forms resulting from these processes are thus: **Geometrical Expansion, Geometrical Contraction, Tonal Expansion, Tonal Contraction, Quadrant Rotation.** Temporal contraction and expansion are terms for operations on the time dimension as noted above.

GRAPH. A means of representing music by denoting the pitch of each tone according to the distance measured vertically and the duration of each tone by the distance measured horizontally. Paper ruled in small squares is usually used. Graphing may be (and preferably should be) *absolute*, that is, pitches should be measured in semitones in the 12-tone system; but it may be done diatonically, that is, having each pitch-line represent a tone of a **Diatonic Scale.**

GRAPHING. Specifically, notation of music in graph form, with the ordinate (up-down coordinate) representing pitch and the abscissa (left-right coordinate) representing time.

GROUND BASS. See **Ostinato.**

GROUND MELODY. See **Ostinato.**

GROUP. Used in the usual sense, except that it should be kept in mind that a group may consist of but one element and even, under some circumstances, of zero elements.

GROUP OF CHORDS. In **Diatonic Harmony** and occasionally in **Symmetric Harmony,** a *group* is a pre-set sequence of chords handled outside the prevailing system of cycles. They represent progressions influenced strongly by contrapuntal considerations. See page 415 ff.

H.

H Symbol used ordinarily for a harmonic structure, or chord.

H$^{\rightarrow}$ Symbol used for **Harmonic Continuity.**

HARMONIC. Used to refer to tonal materials in their mathematical connotation, i.e., pertaining to simple ratios. Not to be confused with "harmony" in its musical connotation, i.e., simultaneous pitch-assemblages varied in sequence.

HARMONIC CONTINUITY. A series of sequent **Pitch-Assemblages** arranged after each other in time in a certain order. Denoted either as $H^{\rightarrow}$ or, more fundamentally, as $\Sigma^{\rightarrow}$.

HARMONIC CORRELATION. See **Correlation of Primary Axes.**

HARMONIC INTERVALS. Two tones sounding simultaneously, in contrast to **Melodic Intervals.**

HARMONIC PROGRESSION. The pattern in which **Pitch-Assemblages (Chords)** follow one another, controlled especially by the pattern of roots; see **General Harmony.**

HARMONICS. Subcomponents of a sound wave, often called **Partials,** resulting from physical factors which convert a simple sound wave (or **Sine** wave) into a wave of more complex form. In music the term is used ordinarily to refer to one or more members of the **Natural Harmonic Series** in relation to a particular **Fundamental Tone,** and, in orchestration, to tones produced by stringed and certain other instruments.

HARMONIZATION OF HARMONY. A process by which, to a given harmonic continuity, one or more additional *harmonic* continuities are developed.

HARMONY. In the composition of music, the science of **Pitch-Assemblages** treated both individually (one by one) and in sequent groups (one after another). The foundation of Schillinger's harmony is **General Harmony,** which is the technology of all possible systems of harmony. A special variety of **General Harmony** is the kind of harmony usually (but not exclusively) found in Western music. **Special Harmony** in turn consists of four main types: **Diatonic Harmony; Diatonic-Symmetric Harmony; Symmetric Harmony;** and **Chromatic Harmony.**

HARMONY SCALE, FUNDAMENTAL. The E_1 (first **Expansion**) of any scale in b position as to **Quadrant Inversion**; from this are derived the various cyclic forms of root progression. If E_0 is *c-d-e-f-g-a-b-c*, then E_1 is *c-e-g-b-d-f-a-c*, and E_1 (b) is *c-a-f-d-b-g-e-c*.

HARMONY, TYPES OF. Schillinger classifies harmony by types as follows: Type I. Diatonic; Type II. Diatonic-Symmetric; Type III. Symmetric; Type IV. Chromatic.

HETEROGENEITY. Characteristic of **Groups** when the timbres in a single group are different, in contrast to **Homogeneity.**

HEXAD. A chord of six tones.

HOMOGENEITY. A characteristic of **Groups** of timbres when the timbres are similar, in contrast to **Heterogeneity.**

HYBRID. A term used to denote mixtures of type, as in *hybrid rhythmic style* (a mixture of groups deriving from more than one style-series); *hybrid harmonic continuity* (a mixture of more than one type of harmonic continuity). *Hybrid 5-part harmony* ordinarily consists of normal 4-part harmony to which an extra stratum (of one part; Sp = 1) has been added.

HYBRID HARMONIC CONTINUITY. Continuity composed of more than one main type of harmonic continuity; as in the mixture of diatonic and chromatic, for example.

I.

I Abbreviation for *interval*.

I Abbreviation ordinarily used for **Instrumental Group**.

IDENTICAL MOTIF One of three methods of melodic **Modulation**; a melodic pattern in one key is followed by the same pattern in the new key.

IDENTITY OF INTERVAL METHOD. A means of deriving from a given **Pitch-Scale** one or more additional scales that possess the same intonational characteristics; the intervals of the original scale are permuted so that all appear in the derivative scale but in a different order.

IDENTITY OF PITCH-UNITS METHOD. A method of deriving additional and related **Pitch-Scales** from a given scale. Some or all of the pitch-units of the given scale are used, but in different sequence. **Displacements** ("modes") of scales, for example, have the same pitch-units as the scales from which they are derived, but in a different order.

INDIRECT MODULATION. Any type of sequence in which the ultimate **Key-Axis** is reached by way of one or more intermediate keys in some fashion *other than* that by which the intermediate keys represent a one-by-one accumulation of flats or sharps. The one-by-one movement toward a "sharp" key is along the pattern, C-G-D-A-E-B-F#-C#-G#, and there is a similar pattern, in fifths downward, rather than upward, for "flat" keys. Any modulatory movement that *departs from* this pattern is called *indirect*.

INDIRECT RESOLUTION. See **Delayed Resolution**.

INSTRUMENTAL FORM. Schillinger never uses this term to indicate what is commonly known as "musical form," but rather to suggest the modification of a tonal continuity by various sequences of attack for actual performance on an instrument. It is a factor of great importance in composition. See **Attack-Form**.

INTERFERENCE. A phenomenon observed in all fields of wave motion—sound, light, radio. It refers to the crossing or synchronization of two waves which results in a third wave that is the summation of the two. Schillinger uses the term to refer to the combination of two continuously repeating sounds of different durations. *The Theory of Rhythm* (Book I) is based on this phenomenon .... Interference is also used by Schillinger in another sense. When two or more groups of elements consisting of nonidentical numbers of terms are combined by pairing, the two do not "come out even," so that the process must be repeated a number of times until the two terminate together. This is the fundamental form of interference and may be applied on many levels in music.

INTERLUDE. Although Schillinger occasionally uses this term in the conventional way to indicate a "bridge" or "passage" connecting two **Expositions** of a **Thematic Unit**, he prefers to treat all segments, no matter how episodic, as **Thematic Groups**.

INTERVAL SYMMETRY. Used of **Strata** or **Sigmae**, this term means that two or more strata or sigmae are separated as to pitch level so that the pattern of intervals determining the degree of separation is *symmetrical*.

INTONATION. Schillinger regards the fundamental material of music as being essentially *temporal* (that is, consisting of time elements), or *tonal* (that is, dealing with frequencies or pitches). Intonation or "intonational" are used throughout the text to refer to the pitches and pitch material in contrast to the temporal or durational material.

INTONATIONAL MODIFICATION. A generalized description of the several techniques of producing variations based on changes in the **Pitch-Units** of a **Thematic Unit**, specifically permutation of pitch-units; modal transposition or other scale modification by change of accidentals in **Real Signatures**; **Tonal Expansions**, **Quadrant Rotation** and **Geometrical Projections** in general; development of **Directional Units**; change in range of **Tension** in relation between harmony and melody, or **Reharmonization**.

INVARIANT OF INVERSION. In inversions (see **Geometrical Projections**), especially of chords, the element (tone) which does not change—i. e., the axis around which inversion takes place.

INVERSION. See **Quadrant Rotation, Geometrical Projections** and **Tonal Inversion**.

K.

KEY-AXIS. See **Axes**.

L.

LEADING TONE. A tone which inevitably moves to an adjacent tone, especially to a **Tonic**, a primary axis (see **Axes**), or a **Neutral Unit**. See p. 1169.

LINEAR COMPOSITION. Assembly of **Pitch-Units** and **Durations** into a **Melody** by the axial method, usually by **Graphing**.

LOGARITHM. A mathematical term referring to the **Power** to which a certain constant base must be raised to produce a given number.

LOGARITHMIC RELATION. Interrelation between two series corresponding to the interrelation between the series of cardinal numbers and their logarithms.

M.

M Symbol used ordinarily for a melodic form, or **Sectional Scale**. Also used occasionally to indicate a *major tetrachord*.

M_1 M, meaning *melody*, is used with subscript numerals when more than one melody is in question, as in **Correlated Melodies**.

m_1, m_2 Abbreviations for two minor tetrachord forms.

MAJOR GENERATOR. In the making of **Resultant** rhythms, the larger of two generators.

MANIFOLD. A set of elements which is itself the result of selection and from which further selection can be made; the manifold determines the limitations on musical material of some kind.

MEAN, ARITHMETICAL. See **Arithmetical Mean**.

MELODIC FIGURATION. A process by which a **Harmonic Continuity** is converted into continuity having some characteristics of counterpoint but less highly organized; used by Schillinger in contrast to **Melodization**. The technique consists of subjecting one or more parts of the continuity to alteration by means of **Directional Units** developed for each **Neutral Unit**. The elements of melodic figuration may be classified according to 1) direction (ascending, descending), 2) chordal function (1-13), 3) adherence to scale, and 4) number of elements employed simultaneously.

MELODIC INTERVAL. Two tones considered as sounding one after the other.

MELODIC PATTERN. Specifically in Schillinger's system, the pattern of secondary axes (see **Axes**) in melody without special regard to the pitch and time dimensions; denoted by **MP**.

MELODIZATION. Construction of a melodic continuity in correlation with a given harmonic continuity.

MELODY. A special case of a **Pitch-Scale** possessing a higher degree of organization, especially a primary axis and a number of secondary axes (see **Axes**) arranged with a view to **Climax** and with a view to certain general forms of **Trajectorial Motion**. Melodies may differ as to the degree of organization introduced into them; they may take complex forms related to the patterns used for **Thematic Continuity**.

MINOR GENERATOR. In the synchronization of two generators (usually two uniform periodicities), the generator of lower numerical value.

MODAL TRANSPOSITION. Alteration of the mode (or scale displacement), effected practically by changing the **Real Signature**.

MODERNIZED PRIMITIVE. An **Original Primitive** scale subjected to development by a technique that converts the span of the original scale into a span for a symmetrical harmonic scale.

MODIFIED RECURRENCE GROUP. In composition of thematic sequences, a sequence in which some one polynomial group recurs but with the elements of the polynomial subjected to permutations in each recurrence.

MODULATION. A general process for shift of primary and/or key-axes (see **Axes**). Melodic modulation affects the melodic line only and may involve only a change or **Displacement** of mode. Harmonic modulation involves a shift of key axis and of **Real Signature**. The general process is called configurational modulation, aiming at neutralization of the previous key and establishment of the new; it takes practical form in three general methods: (1) **Common Unit Method,** emphasis of tones common to the two keys; (2) **Chromatic method,** singling out of the tones not in common and chromatic alteration of these; (3) **Identical Motif Method,** sounding of a conspicuous motif in one key and then in the second key, uniting the two by the common motif. In harmony the method takes practical form in (1) chromatic modulations, discussed as a variety of **Chromatic Harmony;** and (2) symmetrical modulations. Choice of key-axis is determined by pattern; see **Indirect Modulations.**

MONOMIAL. A group consisting of but one element.

MONOMIAL PERIODICITY. A series composed of a repetition of the same (**Monomial**) number, applied usually to **Durations.**

MONOTHEMATIC. A composition with but a single **Thematic Unit.**

N.

NATURAL HARMONIC SERIES. This is the set of overtones or partials produced by a single tone, the original tone (or fundamental) being included in the series as the first term.

NEGATIVE CYCLES. The standard cycles of diatonic harmony, but measured in an upward direction rather than downward. See **Cycles.**

NEGATIVE FORMS OF STRUCTURES. A chord reckoned downward instead of upward. In harmony of negative cycles, negative forms are used; they derive from positive-cycle harmony in the *b* (backward) quadrant inversion. See **Negative Cycles.**

NEUTRAL MELODIC FIGURATION. Melodic figuration achieved without regard to any pre-set melodic forms, but rather developed by selection of devices used in any combination. A term used in contrast to **Thematic Melodic Figuration.**

NEUTRAL UNIT. A *chordal* tone in a **Structure** in **General Harmony.**

NOMOGRAPHY. Any scientific system of recording natural phenomena; in particular, graphic notation of music.

O.

ω Symbol for **Omega** (small). Ω Symbol for **Omega** (capital).

OBLIQUE CORRELATION. See **Correlation.**

OCTAVE DUPLICATION. Derivation of one pitch from another so that the derived pitch is distant by one or more octaves from the initial pitch.

OMEGA. The Greek letter used in its capital and small forms to designate orchestral groups and orchestral thematic units.

OPEN POSITION. A structure is said to be in *closed position* when the numbers of its functions when read downward proceed in a counter clockwise direction; any other distribution of these functions, especially in a clockwise direction, is called an *open position.* *Extra-open* position means that there is room for two such intermediate functions.

OPEN TONE. A timbral description, denoting a tone characterized by a very small quantity of partials, ideally with no partials at all.

ORCHESTRAL CONTINUITY. A **Continuity** formed of one or more sequent **Orchestral Groups.**

ORCHESTRAL GROUP. A group of timbres in orchestration, selected (a) with regard to **Homogeneity** or **Heterogeneity;** and (b) with regard to one or more of these three factors: 1, a type of a **Timbre;** 2, **Dynamics;** 3, **Durability of tone.**

ORCHESTRATION. In music, the science of individual characteristics of sound-producing instruments and ways of combining them; specifically, in this system, the science of composing **Orchestral Continuity** and correlating it with the other **Continuities** of which music is made. Subject to the limits of what is practically possible for the instruments used, **Orchestral Groups** are formed in various combinations or for various purposes. These are assembled into the continuity.

ORDER. A practical term referring to the way one thing comes after another, but difficult to define rigidly. *Higher order* refers to a process of any kind which is performed on the results of another process of the same kind; for example, *squaring a square,* or *grouping a group,* is a *higher order* operation.

ORDINATE. In **Graph,** the measurement vertically, up and down, denoting pitch in music.

ORGAN POINT. See **Pedal Point.**

ORIGINAL PRIMITIVE. A term denoting a **Pitch-Scale** or scales used in some type of primitive music, the scale being in its original form (usually of fewer than seven pitches) but being subjected, for purposes of contemporary music, to adjustment to equal temperament tuning.

OSTINATO. Persistent repetition of a group of any kind, while other musical components change; usually, a *melodic ostinato* (repeated melody with changing harmonic continuities), a *contrapuntal ostinato* (repeated contrapuntal continuity i.e., a repeated melody combined with changing countermelodies), a *harmonic ostinato* (repeated configuration of successive chords, variously melodized) or a *rhythmic ostinato* (repeated durational group).

P

ϕ See **Phi.**

p Symbol ordinarily used for part, either in orchestration or in harmony.

P Abbreviation occasionally for **Permutation.**

P. A. Abbreviation for primary axis; see **Axes.**

PARALLEL CHROMATICS. See **Chromatic Harmony.**

PARALLEL CORRELATION. See **Contrary Correlation.**

PART. In harmony, a specific layer in the harmony, such as the "second from the bottom", "third from the bottom," etc. A part (p) is an element of a **Stratum** in **General Harmony,** and also an element in instrumental continuity.

PARTIALS. See **Harmonics.**

PART-MELODIZATION. Use of one or more parts of one or more strata in general harmony, as a source of melodic shapes.

PASSING TONES. *Chromatic* passing-tones are the result of inserting a half-step movement into a **Melodic Interval** originally of a whole step; *diatonic* passing tones are tones inserted into an interval of a third or more, and converting the interval into seconds. All passing tones, whether diatonic or chromatic, are special forms of **Directional Units.**

PEDAL POINT. Used by Schillinger for the most part in the conventional sense, but with special observations as to *location* in the thematic continuity, use in producing **Climax,** and means of determining what structures are permissible for use in a pedal point.

PENTAD. A chord of five tones.

PENTANOMIAL. A group consisting of five elements.

PERIODICITY. The continuous repetition of notes, sounds, or attacks. *Uniform* periodicity means that the groups of attacks are of identical duration. Such groups may include one or more terms. When they include only one term—a series of quarter notes, eighth notes, etc., then we have monomial periodicity. Uniform periodicity may, however, involve groups of more than one term.

PERMUTATION. The process of rearranging the members of a group as to sequence. *General* permutations (or logical permutations) are exhaustive, that is, they involve every possible arrangement that can be made. Circular permutations constitute a special set within the general set involving clockwise or counterclockwise patterns of alteration; see **Circular Permutations.** Permutation is a fundamental process (applied to **Pitch-Units,** intervals, chordal structures, durations, or any element) for development of groups.

PHASIC ROTATION . In the composition of densities, a process for variation by displacement of any given group, the displacement taking place along the time-axis, or along the density axis, or both.

PHI ϕ The Greek letter used to refer to phasic rotation; see **Density.**

PITCH. The "highness" or "lowness" of a tone as measured by its **Frequency.** "Concert *a*" today is 440.6 per second.

PITCH-AGGREGATIONS. See **Structure.**

PITCH-NAME. The name of a pitch in alphabetical terminology (A, B, C, etc).

PITCH-SCALES. A sequence of pitch-units in order of increasing or decreasing frequency of pitch. Schillinger's classification of scales is in four groups. *Group One*: Scales with one tonic and not more than one octave in range. *Group Two*: Scales with one tonic and more than one octave in range; they are obtained by **Expansions** of scales in the first group. *Group Three*: Scales of more than one tonic and of not more than one octave in range. Scales are symmetrical, containing equal number of semitones between tonics. *Group Four*: Scales of more than one tonic, and more than one octave in range; these scales are symmetrical. In the most generalized form, scales of groups one and two are regarded as special cases of symmetrical scales in which the points of symmetry are one or more octaves apart. Groups three and four are further classified according to the number of tonics.

PITCH-TIME RATIO. In melody, the ratio between the maximum pitch to which a secondary axis (see **Axes**) rises or falls, and the time it takes the axis to reach this point. See also **Contrary Correlation.**

PITCH-UNIT. A **Pitch** or tone; any one of the tones that go to make up a **Manifold** of pitches, usually determined first by a **Tuning System** and then by **Pitch-Scale.**

PLOTTED MELODY. A melody constructed by the graphing (or plotting) method: one or more primary axes are located; groups of secondary axes are developed to the primary axis or axes; a rhythm is constructed; the rhythm is superimposed on the secondary axes, the result being interpreted in terms of a selected scale or of a given harmonic continuity.

POLYMODAL. In describing interrelations of two melodic lines, this term indicates that the two are not in the same mode or displacement. See **Unimodal.**

POLYNOMIAL. A group consisting of more than one element.

POLYTHEMATIC. A composition with more than one **Thematic Unit.**

POLYTONAL. In describing interrelations of two or more melodic lines, this term indicates that the **Real Key** of each line is different. Note that Schillinger's concept of *Polytonal* is somewhat different from the conventional use of the term to describe music in which different keys are used simultaneously.

POWER. The result of multiplying a number by itself a designated number of times. A *zero* power of any number is the number, 1. The *first* power of a number is the number itself. A *negative* power is the power of the number divided into the integer, 1. In Schillinger's system, powers are almost always used as **Distributive Powers.**

POWER SERIES. A **Series** in which the terms are successive **Powers** of some constant number.

PP, PU. See **Axis Relations.**

PRESELECTION. Same as **Selection,** but with emphasis on the fact that the decisions are made some time in advance of actual composition. See **Pre-Set.**

PRE-SET. This adjective, of considerable importance in Schillinger's system, means that the characteristics of some factor in a musical continuity are determined in advance of actual composition, the "settings" being chosen according to specific desired effects.

PRIMARY AXIS. See **Axes.**

PRIME NUMBER SERIES. A series composed of cardinal numbers which are divisible without remainder only by the integer 1 and themselves.

PRIMITIVE. See **Original Primitive, Stylized Primitive, Modernized Primitive.**

PROGRESSION. See **Harmonic Progression.**

PROGRESSIVE SYMMETRY. A form of **Thematic Sequence** in which the successive groups first "grow" by the addition of more and more themes, then "decline" by the subtraction of more and more themes, the whole being arranged symmetrically.

PYRAMIDS. An arrangers' term denoting an orchestral *arpeggio*, each tone of which, once sounded, is sustained—the whole being produced by successive entrances of instruments on various chordal tones.

Q

Q Symbol ordinarily used for *quality* in construction of **Quality Scales.**

QUADRANT ROTATION. Once music has been reduced to graph form, the original graph (denoted as ⓐ) will produce three additional forms. These are: ⓑ the original backward in time; ⓒ the original backward in time and upside down as to pitch; ⓓ the original forward in time but upside down in pitch. In this process the intervals are calculated exactly in semitones and they are reckoned from some specific tone selected as the **Axis Of Inversion.** When the intervals are calculated in the diatonic manner, the result is **Tonal Inversion.**

QUADRUPLE PARALLEL CHROMATICS. Special form used in **Chromatic Harmony.**

QUANTITATIVE SCALE. A scale developed from a chromatic (or occasionally symmetric) harmonic continuity, the scale consisting of a selected set of pitch units occurring in the continuity. It is enough that tones selected be frequent enough to afford a good choice in melodization; this means that in some cases a tone appearing frequently may be omitted in order to simplify the scale finally chosen. This is also a technique for diatonic melodization of chromatic harmonic continuity.

R.

r Symbol for **Resultant.**

R Abbreviation for single-reed tone in **Orchestration.**

RR Abbreviation for double-reed tone in **Orchestration.**

RANGE OF TENSION. In melodization of harmony or in harmonization of melody, the maximum variation permitted in **Tension.** Minimum tension is present so far as this relation is concerned when the melodic tone is a tone also present in the harmony. The range of tension may be **Pre-Set** as a means of controlling the harmonic style of the music.

REAL SCALE. See **Real Signature.**

REAL SIGNATURE. In conventional music notation, all signatures used are those associated with major diatonic scales, and these constitute the key-signatures as they appear on the staves. When scales other than major or natural minor are used, however, the written notes acquire a uniform set of accidentals which, if arranged in signature form, would constitute the *real signature.* A melody written in harmonic minor starting on *c*, for example, has a conventional signature of three flats (as for E♭ major) but has a *real* signature consisting of E♭, and A♭ *only.* There is no reason, except convention, for not making the real signature function as the actual signature on the staff—and, indeed, a very few composers sometimes do this. Real signatures may have both flats and sharps. See page 123.

RECTIFICATION. 1). Chromatic alteration of a chordal tone in chromatic harmony made necessary by the chromatic alteration of some other tone. The necessity arises from the need to avoid major seconds or augmented thirds (perfect fourths). The tone that has been rectified is not required to resolve, by a further semitone, in contrast to the requirement for the tone originally modified. 2). In rhythmic treatment of harmonic continuity, rectification refers to that point in time where all voices finally arrive together at the points required by the new chord, after various movements of voices and mixtures of adjacent chords resulting from different rhythms in the several parts have occurred.

RESISTANCE FORMS. Melodic or harmonic (stratum) motion that corresponds to the increases and decreases of movement characteristic of a specific force overcoming a specific resistance. Ordinarily, some type of **Rotary Movement.**

RESOLUTION OF DISSONANCES. In Schillinger's generalized contrapuntal technique, the practice of dividing intervals into hard-and-fast classes labelled "consonance" and "dissonance" is abandoned in favor of a graded classification according to tension. With this he introduced the principle of "resolution" of high-tension intervals by reduction of tension. Unless intentionally dissonant counterpoint is desired, intervals of a tension higher than the thirds need only to have their tensions reduced, not necessarily in the classical manner. But for production of counterpoint of the classical type, various additional procedures are to be followed, the set of procedures depending on the period-style of counterpoint desired. The main criteria are (1) judgments at various periods in musical history as to what intervals *require* resolution; (2) judgments as to the *period of time* in which the resolution must be accomplished; (3) judgments as to what movements of parts constitute an acceptable resolution.

RESOLUTION OF INTERVALS. See **Resolution of Dissonances.**

RESULTANT. In **Rhythm**, the pattern of **Durations** that results when two or more **Periodicities** (usually but not always **Monomial**) are **Synchronized**. The periodicities are called *generators*.

RESULTANT OF ACCELERATION. A special form of **Resultant** in which an **Acceleration Series** is synchronized with itself backwards.

RHYTHM. The organization in time of the durations involved in music. In Schillinger's system, rhythm refers not only to what is ordinarily called rhythm, that is, the division of time within a single measure or small group of measures (**Fractional Rhythm**), but also to the way in which the measures themselves are organized into groups (**Factorial Rhythm**). According to the *fractional* technique, a single duration of time of any length is subdivided binomially (into two parts), trinomially (into three parts), or polynomially (into *n* parts) according to one or more **Style-Series**. The results of this subdivision, or fractioning, are the rhythm. The results may be subjected to factorial technique, by which a group of durations is developed into larger groups. Such results may be distributed in a number of ways over a number of simultaneous instrumental parts. Every aspect of music is, in Schillinger's system, controlled fundamentally by his rhythmic techniques. . . . . Schillinger does not restrict the concept of rhythm to time and the durations of attacks. He deals also with 1). instrumental rhythm—the pattern according to which instruments enter and leave an ensemble; 2). intonational rhythm—the pattern of pitches in a phrase; and 3). harmonic rhythm—the pattern of harmonic groups in a sequence.

RHYTHM OF CHORD-PROGRESSION. The pattern that consists, one after the other, of the durations in which each successive chord or pitch-assemblage is being sounded, simultaneously or sequently. Practically, the rhythm of changes in the pitch of the root.

ROOT (ROOT TONE). The particular tone from which all other tones of a **Pitch-Assemblage** or **Pitch-Scale** are derived and/or reckoned. Used of a **Pitch-Scale**, it refers always to the **Real Key**.

ROTATION. See **Quadrant Rotation**.

ROTARY MOVEMENT. Movement of a melody or a stratum circulating above and below an **Axis** which, when graphed, produces a wave-like curve. May be based on simple circular or **Sine** forms, or on spirals of various sorts, mainly those representing some **Summation Series**.

RUBATO. An alteration in the durations of tones, ordinarily accomplished by the performer in deviation from the written notation. Regarded by Schillinger as best denoted in actual notation, and as best accomplished by introducing a standard unit of deviation, by which unit a balanced binomial may be unbalanced, or an unbalanced binomial may be balanced.

S.

Σ See **Sigma**.

S(5). A structure (chord) corresponding to the normal triad of conventional diatonic harmony. $S_1(5)$ is the *major* triad (a major third topped by a minor third, or, in **Symmetric Notation**, $4+3$); $S_2(5)$, *minor* triad; $S_3(5)$, *augmented* triad; $S_4(5)$, *diminished* triad.

S(7). This denotes a seventh-chord shape. In **Special Harmony** the specific varieties, correlated with their normal terminology and intervals (reading upward in semitones), are: $S(7)_1$ or $S_1(7)$, *major* seventh, 4-3-4; $S(7)_2$, *minor* seventh, 3-4-3; $S(7)_3$, *large* seventh, 4-3-3; $S(7)_4$, *small* seventh, 3-3-4; $S(7)_5$, *diminished* seventh, 3-3-3; $S(7)_6$, *augmented* I, 4-4-3; $S(7)_7$, *augmented* II, 3-4-4

Sp. To be read, "stratum equals (or consists of) one part."

S2p. A stratum consisting of *two* parts.

SATURATION. The degree of concentration of some element in a given continuity. Complete saturation refers to presence of the element in maximum possible quantity. Temporal saturation refers specifically to concentration of an element in time. See **Temporal Saturation**.

SATURATION OF WAVE. The degree to which **Harmonics** are present, taken along with their intensities, in characterizing **Timbre**.

SCALE. As used by Schillinger, scale does not necessarily refer to **Pitch-Scale**, but rather to any scalewise arrangement of elements according to increase or decrease in some characteristic, as, for example, a quality scale in orchestration—in which timbral elements are arranged according to increase of some timbral characteristic, such as closed tone.

SCALE FAMILIES. See **Pitch-Scales**.

SCALE OF TENSION. **Tension** as measured in harmony by the distance of the **Function** representing the melodic tone from the functions in the harmony on a scale 1-3-5-7-9-11-13 & . For a 1-3-5 chord, for example, lowest tension is 1-3-5, as functions in the melody; next higher are the adjacent functions, 7 and 13; with 9 next higher and 11 highest of all. Different ranges of tension result in different melodic styles.

SCORED INTERFERENCES. See **Interference**.

SECONDARY AXIS. See **Axes**.

SECTIONAL SCALE. A **Pitch-Scale** built in the symmetric fashion by selecting a number of **Tonics** that symmetrically split one or more octaves, then attaching to each such tonic an identical intervallic pattern of semitones and/or whole tones or larger intervals so that no pattern overlaps the next higher tonic.

SELECTION. Used in the usual sense of the word, but denoting the act in composition of music by which the composer, confronted by all possible choices arranged in a systematic way, decides which particular resources he will use. In a selective continuity of any kind, for example, certain elements are chosen deliberately by reason of their effects and are combined in proportions that correspond to the relative emphasis the composer wishes to give them.

SELECTIVE. See **Selection**.

SELECTIVE CYCLIC CONTINUITY. Harmonic continuity in which certain cycles are chosen and used in selected quantities in order to produce continuity of desired characteristics; used in contrast to non-selective or casual continuity, in which the "selection" is made literally from chord to chord, as in older music (16th century) which is, as to cycle, in general *non-selective*.

SELECTIVE SYSTEM. Music is composed by successive steps of selection, that is to say, from the total manifold of all possible frequencies, certain frequencies are *selected* to comprise the manifold known as the tuning system; then, from the tuning system itself, certain pitch-units are *selected* to form a scale; and from the scale, certain other selections are made. Schillinger refers particularly to two types of selective systems—primary and secondary. The primary system is a given system of tuning while the secondary system is a scale or melody within the primary or tuning system.

SEMANTIC. Used by Schillinger to refer simply to meaning, and not necessarily to the evolution of meaning.

SEQUENCE. A group or set of elements arranged and considered with special regard to the *order* in which they come after each other, and usually without regard to the **Durations** attached to each.

SEQUENT GROUP. A group of elements that occur one after the other in time.

SERIES. A group of quantities, one after the other; or a group of any elements consecutively; usually each element is related in some constant way to the other elements.

SIGMA. The Greek letter (Σ) used essentially to denote a large structure, as distinct from smaller structures (usually of not more than four tones or **Neutral Units**) in **General Harmony**. The *sigma* is the same as some **Tonal Expansion** (or, sometimes, **Geometrical Expansion**) of a **Pitch-Scale**. Ordinarily, the E_1 (either tonal or geometric, depending on the type of harmony) of a diatonic scale is used.

SIGMA CONTINUITY. Denoted by ($\Sigma^{\rightarrow}$). A sequent group of *sigmae*, used to denote the full tonal score and frequently to denote the patterns in which instrumental **Attack-Forms** are grouped.

SIGMAE. Plural form of **Sigma**.

SIGMA FAMILIES. Denoted by Σ (13), these are sigmae which consist of a root, 3rd, 5th, 7th, 9th, 11th, and 13th, the thirds involved being of all possible shapes. The result of various patternings of intervals is a series of different sigmae, each of which may become the source of a complete harmonic style through application of the techniques of **General Harmony**.

SIGMA OF SIGMA. See **Compound Sigma**, Σ (Σ).

SIMPLE HARMONIC MOTION. Melodic motion corresponding to the simple harmonic series (as used mathematically), or, practically, scalewise motion.

SIN MOTION. *Sin* is the conventional abbreviation for the mathematical ratio, *sine*. In the Schillinger System, *sin motion* is the same as **Ascribed Motion.** Used in contrast to *cos motion*, meaning cosine motion, which is the same as **Inscribed Motion.**

SINE. A mathematical ratio used in analysis of soundwaves and other types of cyclic motion. See **Sin Motion.**

SPECIAL HARMONY. The harmony associated with most of Western music, based on the E_1 of those scales which use all seven **Pitch-Names** with but one set of accidentals at a time. Schillinger uses this term, in contrast to **General Harmony,** to denote a narrow range of harmonic techniques corresponding to "classical" harmonic practice, but with considerable amplification of the range of device. His **General Harmony** includes **Special Harmony** as one type, and it, in turn, embraces **Diatonic Harmony, Diatonic-Symmetric Harmony Symmetric Harmony** and **Chromatic Harmony.**

SPEED. The number of **Attacks** in relation to total time; specifically, the number of basic time units, t, contained in the total duration.

SPLIT UNITS. In rhythm, the result of dividing a single duration by some divisor; extended to a technique by which the selection of units to be split is controlled by permutation or by coefficients of recurrence.

STATISTICAL SCALE. See **Quantitative Scale.**

STOPPED TONE. In **Orchestration, Quality Scales** or **Timbral Scales,** one of three general timbre intermediate between **Open** and **Closed Tone.**

STRATA. Plural form of **Stratum.**

STRATA HARMONY. A term meaning harmonic continuity in which a large number of parts are grouped into **Strata** and handled accordingly. See **General Harmony.**

STRATUM. One of the elements in **General Harmony** (or strata harmony, as it is frequently called). A stratum consists of one or more **Neutral Units** (rarely more than four, bowever), each neutral unit being a tooe. From one or more of these neutral units, **Directional Units** may be developed. The pattern of neutral units within a single stratum is denoted in relation to the root of the stratum itself.

STRUCTURE. In general, any pattern of elements, organized either in pitch or io time, or both. Specifically, when denoted by S, a Pitch-Assemblage or chord consisting of **Neutral Units** and sometimes **Directional Units,** with emphasis on the exact shape (pattern of intervals, binding together the **Neutral Units.** One or more such structures (which are, of course, the equivalent in **General Harmony of Strata**) constitute a **Sigma.**

STYLE. In the Schillinger system, the style of a composition is the result of the individual styles of the component continuities, the maio factors being *intonational* style, controlled by **Pitch-Scale** and its expansions into **Sigmae;** and *temporal* style, or **Rhythm,** controlled by **Style Series.** But many other aspects are also factors in the final style, especially those connected with **General Harmony.**

STYLE SERIES. This is a series which functions, in the Schillinger system, as the source of all families of temporal or rhythmic style, and consists of the following:

$$\frac{1}{0}\cdots\cdots\frac{1}{4},\ \frac{1}{3},\ \frac{1}{2},\ \frac{1}{1},\ \frac{2}{1},\ \frac{3}{1},\ \frac{4}{1}\cdots\cdots\frac{n}{1}$$

It may be compressed into simply:

$$\frac{1}{1},\ \frac{2}{2},\ \frac{3}{3},\ \frac{4}{4},\ \frac{5}{5},\ \frac{6}{6},\ \frac{7}{7},\ \frac{8}{8}\cdots\cdots\frac{n}{n}$$

The denominators control **Fractional Rhythm;** tbe numerators, **Factorial Rhythm.** The manner in which generation of a family of **Durational Groups** takes place is the following: a **Monomial,** sucb as 4 (for $\frac{1}{4}$ series), is split asymmetrically (or 3-1) by the smallest unit of deviation the number affords; it is then synchronized with itself run backward (i. e., 3-1 is synchronized with 1-3) to produce a trinomial (1-2-1). All permutations of the trinomial are combined to produce a new polynomial (in thia case, 1-1-1-1). The terms of the new polynomial (up to the point where uniformity is reached) are again permuted and "interfered"—and so on. The resulting durational groups constitute the raw material of the style. The numerators function as multiplicands in making larger continuities. Schillinger tends to use tbe term *time-series* of this process when it controls details of larger form, reserving the term *style-series* for fractional rhythms.

STYLIZED PRIMITIVE. An **Original Primitive** scale developed according to techniques that are essentially diatonic.

SUBTRACTION TONES. See **Differential Tones.**

SUMMATION SERIES. Any series of numbers in which the third and all subsequent terms are the total of the two immediately preceding terms; or, by extension, the total of any pre-set number of immediately preceding terms. *First summation series*: 1, 2, 3, 5, 8, 13, 21 etc. *Second summation series*: 1, 3, 4, 7, 11, 18 etc. Also known as Fibonacci Series.

SYMMETRICAL NOMENCLATURE. Naming of intervals by the number of semitones, rather than as seconds, thirds, etc. That is to say, what by symmetric nomenclature is a 5 would be a 4 (perfect 4) in diatooic nomenclature.

SYMMETRIC HARMONIZATION. Harmonizatioo of one or more melodies, the resulting harmonic continuity being of the symmetric type.

SYMMETRIC HARMONY. A system of harmony in which the roots of the chords move by patterns outside the diatonic system and computed in semitones; more specifically, a variety of tbe above in which the roots move in typical patterns, the patterns being: movement by semitones, denoted as $\sqrt[12]{2}$; by whole steps, denoted as $\sqrt[6]{2}$; by minor thirds (3 semitones), denoted as $\sqrt[4]{2}$; by major thirds (4 semitones, denoted as $\sqrt[3]{2}$; by augmented fourths (6 semitones), denoted as $\sqrt[2]{2}$. Movement of the root by an octave or unison is also technically a symmetric movement under the most generalized form. With the root moving as described, the specific tonal structures or chords are pre-set without relation either to any diatonic scale or to the tonal material of the pattern of roots, the chord forms being chosen usually for their acoustical sonority. **Transformations** ("voice leading") take place by permutation.

SYMMETRIC ROOTS. Patterns of root movement in **Symmetric Harmony.**

SYMMETRIC SCALES. Pitch-Scales, frequently of more than one octave in range, formed by a series of **Tonics** arranged symmetrically to which tonics are added one or more additional tones in a standard, pre-set interval relation. Schillinger describes two types of symmetric scales: Group III: range of less than one octave and containing equal number of semitones (2, 3, 4, 6) between tones; Group IV: range of more than one octave and containing equal number of semitones (8, 9, 10, 11) between tones.

SYMMETRY. A characteristic appertaining to any pattern, requiring that the whole pattern be susceptible to reversal without the pattern being thereby changed.

SYNCHRONIZATION. The process of making two series (usually **Durational Groups**) occupy the same period of time; performed by reducing each to a common denominator. **Interference** results unless the series are identical.

T.

T. Symbol ordinarily used for time. Occasionally a symbol for **Tonic.**

θ. See **Theta.**

τ. See **Tau.**

$T^{\rightarrow}$. Symbol used for **Durational Continuity,** or sequent group of durations.

TAU. The Greek letter (τ), used to denote a unit of deviation in the notation of durations, especially in calculating **Rubato, Fermata,** minor changes in tempo, etc.

TEMPORAL ORGANIZATION. The organization of all details of a composition or section thereof in time; in particular, the organization of factorial and fractional continuities; see **Rhythm.**

TEMPORAL SATURATION. See **Saturatinn** in geoeral. In the Schillinger System, increasing temporal saturation is achieved by having more and more **Thematic Groups** in a given continuity. This sometimes involves contrapuntal arrangements of the groups of the type known as stretto in older counterpoint, in which the thematic group has not yet come to an end before another thematic group (usually the same as the first) begins.

TENSION. The degree of dissonance (1) in a **Harmonic Interval,** or (2) between melody and harmony. The latter varies as to **Range of Tensinn,** which may be set narrowly or broadly, and as to degree of tension, itself, which may be kept high—around the 7, 9, 11 functions—or low—around the 1, 3, 5. The harmonic aspects of tension derive from the simpler intervallic aspects.

TERNARY AXES. See **Axes.**

TETRAD. A structure in harmony of four parts.

THEMATIC CONTINUITY. A sequence of **Thematic Grnups,** organized in some pattern: direct recurrence, modified recurrence, symmetrical recurrence, etc. It controls the fundamental *musical form* of the composition as well as the emphasis given to various types of **Thematic Units.**

THEMATIC GROUP. A **Thematic Unit** existing in one of its potential forms in a *specific* period of time.

THEMATIC MELODIC FIGURATION. A process for melodic figuration of barmony whereby pre-set melodic forms are introduced into the successive chords of the given harmonic continuity.

THEMATIC SEQUENCE. A **Thematic Continuity,** but with special emphasis on the serial order in which **Thematic Groups** follow one another, and without regard to the durations attached to each.

THEMATIC UNIT. A configuration of elements in music cbosen for its susceptibility to temporal and intonational modification. It may or may not be composed exclusively of tonal elements; it may also consist of **Density** or **Orchestral** groups; or of any other element in music. It may be rhythmic, melodic, harmonic, or contrapuntal. It may consist of some combination of elements, in which case it will have one element as a major component, around which the other elements (minor components) are organized. It is the basic ingredient of **Thematic Groups.**

THEME. In a composition, a **Thematic Grnup** in which the **Thematic Unit** is exhibited at its maximum duration io time; a subject.

THETA. The Greek letter referring to a compound rotation group in **Density,** and in general, to a **Density Continuity** with emphasis on phasic rotation.

TIMBRAL. The adjective refers to *timbre.*

TIMBRE. The quality or "color" of tone resulting from the interaction of all frequencies and intensities constituting a sound wave.

TONAL EXPANSION. **Expansinn** carried out in terms of a specific diatonic scale. Contrast witb the result of carrying out the same **Expansion** process geometrically, i.e., measuring in semitones rather than in diatonic intervals. Various degrees of expansion are denoted as E_1 ("first expansion"), E_2 ("second expansion"), etc,. especially when referring to expansions of diatonic scales. The first expansion is obtained through circular permutation *over* one pitch-unit of the original scale; the second expansion, over two pitch-units; etc.

TONAL INVERSION. A process for variation proceeding in much the same way as that used in **Quadrant Rotation,** except that the iotervals are calculated diatonically rather than absolutely, so that the result—in contrast to the result of some of the quadrant rotations—is adjusted to the key of the original.

TONE SYMBOLS. ○ See **Open Tnne.** ● See **Closed Tone.** ⊕ See **Stopped Tone.**

TONES OF THE DIFFERENCE. See **Differential Tones.**

TONIC. The first tone of a **Pitch-Scale** and, occasionally by extension, tbe first tooe or root of a *sigma. Two-tonic system*; a system of **Pitch-Scales** or **Harmonic Progressions** based on two tonics, usually related as C to F# or $\sqrt[2]{2}$. *Three-tonic system*: relationship usually as C, E, Ab, or $\sqrt[3]{2}$. *Four-tonic system*: relationship as C, Eb, Gb, A, or $\sqrt[4]{2}$. A five-tonic system does not exist in equal temperament tuning. *Six-tonic system*: relationship usually as C, D, E, F#, G#, A#, or $\sqrt[6]{2}$. *Twelve-tonic system*: relationsbip usually as all tones of tbe chromatic scale io succession, or $\sqrt[12]{2}$.

TRAJECTORIAL MOTION. Melodic motion analyzed from the viewpoint of the trajectories it outlioes when graphed; more specifically, these trajectories analyzed in relation to **Primary Axes** and **Secondary Axes.**

TRANSFORMATIONS. The general form of what is conventionally called "voice leading," but used in a much broader sense of the transformation of any Pitch-Assemblage (abcde, for example) into another (a'b'c'd'e', for example). *Parallel* transformations lead each function in the initial assemblage to the corresponding function in the next assemblage; in all other transfnrmations, the ioitial set of fuoctions transforms to a set of functions that represents some **Permutation** of the second set. In so-called **Constant Function Transformations,** in which one (or more) functions lead by parallels, the remaining functions leading by permutation. In general, transformations are classified as **Clockwise** or **Counterclockwise,** with **Crosswise** appearing as a special form of both.

TRIAD. A structure in harmony of but three parts; conventionally, but not necessarily, the familiar triad of ordinary diatonic harmooy.

TRINOMIAL. A group consisting of three elements.

TRIPLE PARALLEL CHROMATICS. One variety of **Chromatic Harmony.**

TRUE PRIMITIVE. See **Original Primitive.**

TUNING RANGE. The range of tones as they actually sound that an instrument can produce for orchestral use.

TUNING SYSTEM. In music only certain pitches from among all the possible pitches are utilized. The particular set of pitches selected for use is the *tuning system,* or primary selective system. Various systems have been nr are in use, but the system known as **Equal Temperament** is the basis of the ootation of most Occidental music and is the basis of the Schillinger system. See pp. 101, 102 and 144.

U.

U. Symbol used for *unbalancing* axis (see **Axes**).

UNBALANCING AXIS. See **Axes.**

UNIMODAL. Describes axial relations of two melodic lines when each is in the same mode of its particular key; the keys need not necessarily be the same, but the modes (displacements) must be identical.

UNITONAL. Describes interrelation of two or more melodic lines (especially in counterpoint) as to the **Real Key** of each, and means that each is in the same key, although not necessarily in the same mode. See **Pnlytnnal.** See. **Unimodal.**

UU, UP. See **Axis Relations.**

V.

V→ Symbol for **Dynamic Continuity.**

V. A symbol used in orchestration for *volume,* or dynamics.

v. A dynamic unit, See **V.**

VARIABLE DIFFERENCE SERIES. A **Series** in which each term is composed of a base that increases to some pattern, to which is added a secood number that also increases according to some pattern; essentially, tbe sum of two series.

VARIABLE DOUBLINGS. In harmony, a technique by whicb varying chordal functions (the 1, 3, or 5 usually) are selected for dnubling.

VARIABLE STRUCTURES. In harmony, the use of more than one structure (or "shape") nf chord io a cootinuity.

VOICE LEADING. The trajectnry or path followed by a **Part**; see especially **Transformations.**

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VITA

Joseph Schillinger was born in Kharkov, Russia, September 1, 1895, and died in New York on March 23, 1943. At the age of 5 he manifested interest in design, dramatics and verse; at 10 he was experimenting in play-writing and music. He was educated at the Classical College and entered the St. Petersburg Conservatory in 1914. In 1917 he was graduated from the class in composition, after which he studied conducting under N. N. Cherepnin.

In 1918 he was appointed senior instructor in composition at the Kharkov Academy of Music; in 1920 he was made professor, and the following year, dean of the faculty of composition. In the same period he served as head of the music department of the Board of Education of the Ukraine. From 1922 to 1926 he acted as consultant to the Leningrad Board of Education. Beginning in 1925, and for three years thereafter, he served as professor and member of the State Institute of the History of Arts at Leningrad. In 1927 he was commissioned to make phonograms of the folk music of the Georgian tribes in the Caucasus, and he succeeded in recording folk songs previously unknown to the world of music. In this period his pedagogical responsibilities multiplied and he also served as senior instructor of the State Central Technicum of Music. From 1926 to 1928 he was vice-president of the Leningrad branch of the International Society for Contemporary Music. During this period he organized and directed the first Russian jazz orchestra.

In November 1928 Schillinger came to the United States on invitation of the American Society for Cultural Relations with Russia. Shortly after his arrival he began collaborating with Leon Theremin on research in musical acoustics, and the application of electronics to tonal production. For six years, from 1930 to 1936, he taught at various American universities and schools of art and music. From 1930 to 1932 he was a lecturer at the David Berend School of Music. In 1932 and 1933 he lectured at the New School for Social Research. In 1934 he gave lectures at the Florence Cane School of Art, American Institute for the Study of Advanced Education, and American Institute of the City of New York. In 1934 he became a member of the faculty of Teachers College, Columbia University, serving in three different departments: music, fine arts and mathematics. The Mathematics Museum of Teachers College placed on permanent exhibition, in 1934, certain geometrical designs which he evolved as part of his Theory of Design. In 1936 he lectured at New York University. In July of the same year he became an American citizen.

Schillinger's major musical compositions include works for orchestra, voice, string instruments and piano. *March of the Orient*, Op. 11, was composed in 1924, and performed by the Leningrad State Philharmonic, as well as the Persymphans; during the seasons of 1926-27 and 1927-28 it was played by the Cleveland Symphony, Nikolai Sokoloff conducting. *Symphonic Rhapsody*, Op. 19, was composed on commission to celebrate the tenth anniversary of the Soviet Union. After performances in Moscow and Leningrad, it was given its premiere in the Western hemisphere by the Philadelphia Orchestra under Leopold Stokowski. In

1929 Schillinger wrote *First Airphonic Suite*, Op. 21, for RCA Theremin and orchestra. The first performances were given by the Cleveland Orchestra under Sokoloff. The following year, on commission by RCA, Schillinger wrote the *North Russian Symphony*, Op. 22, for radio performances.

Among his outstanding piano works are the *Five Movements for Piano*, Op. 12; *Excentriade*, Op. 14; *Sonata Rhapsody*, Op. 17; and Funeral March. His *Sonata for Violin and Piano*, Op. 9, received its first performance in Kharkov in 1922 with Nathan Milstein.

Schillinger's two major theoretical works are the *Mathematical Basis of the Arts*, and the *Schillinger System of Musical Composition*. The former work represents the first scientific theory of the arts, and presents the application of his foundation ideas to the spatial as well as tonal arts. *Kaleidophone*, a manual of pitch scales in relation to chord structures, was published in 1940. Articles on various subjects may be found in *Modern Music*, *Experimental Cinema*, *Tomorrow*, *Metronome*, *1938 Proceedings of the Music Teachers National Association* and *1938 Annual Meeting Papers of the American Musicological Society*. Schillinger left in manuscript, essays and articles, including *Musofun* (a book of musical games) and *Graph Method of Dance Notation*.

The publication of the *Schillinger System of Musical Composition* has been long awaited because of Schillinger's influence on American music for radio and motion pictures—an influence exerted through the prominent composers, conductors, arrangers and music directors who studied privately with him.